2.3. "Canonical" or "Mechanical" decomposition ?

Historically, it was a common belief that the canonical OAM appearing in the Jaffe-Manohar decomposition would not correspond to observables, because they are not gauge-invariant quantities.

This nebulous impression did not change even after a gauge-invariant version of the Jaffe-Manohar decomposition a la Bashinsky and Jaffe appeared in 1999.

However, the impression has changed drastically after Lorcé and Pasquini showed that the canonical quark OAM can be related to a certain moment of a quark distribution function in a phase space, called the Wigner distribution.

$$\rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp; \mathcal{W}) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\,\Delta_\perp \cdot \boldsymbol{b}_\perp} \frac{1}{2} \int \frac{dz^-\, d^2 z_\perp}{(2\pi)^3} e^{i(x\bar{P}^+ z^- - \boldsymbol{k}_\perp \cdot \boldsymbol{z}_\perp)} \times \langle P'^+, \frac{\Delta_\perp}{2}, S \,|\, \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ \mathcal{W}\left[-\frac{z}{2}, \frac{z}{2}\right] \psi\left(\frac{z}{2}\right) |\, P^+, -\frac{\Delta_\perp}{2}, S \rangle \,|_{z^+=0}$$

$x = k^+ / \bar{P}^+$, $\quad \boldsymbol{k}_\perp$: transverse momentum

$\mathcal{W}$: gauge-link, $\quad \boldsymbol{b}_\perp$: impact parameter

According to them, a natural definition of quark OAM density in the phase-space

$$L^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W}) \;=\; (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)^3 \; \rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W})$$

After integrating over x, $\boldsymbol{k}_\perp$, and $\boldsymbol{b}_\perp$, they found a remarkable relation

$$\langle L^3 \rangle^{\mathcal{W}} \;=\; \int dx\, d^2k_\perp\, d^2b_\perp\, L^3(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W}) \;=\; - \int dx\, d^2k_\perp \frac{\boldsymbol{k}_\perp^2}{M^2} F^q_{1,4}(x, 0, \boldsymbol{k}^2_\perp, 0, 0, \mathcal{W})$$

where

$$\begin{aligned} \rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W}) \;&=\; F^q_{1,1}(x, \boldsymbol{k}^2_\perp, \boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp, \boldsymbol{b}^2_\perp ; \mathcal{W}) \\ &\quad - \frac{1}{M^2} (\boldsymbol{k}_\perp \times \nabla_{\boldsymbol{b}_\perp})_z\, F^q_{1,4}(x, \boldsymbol{k}^2_\perp, \boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp, \boldsymbol{b}^2_\perp ; \mathcal{W}) \end{aligned}$$

A delicacy here is that the Wigner distribution ρ^q generally depends on the chosen path of the gauge-link $\mathcal{W}$ connecting the points $z/2$ and $-z/2$.

As shown by a careful study by Hatta, with the choice of a staple-like gauge-link in the light-front direction, corresponding to the kinematics of the semi-inclusive reactions or the Drell-Yan processes, the above quark OAM turns out to coincide with the (GI) canonical quark OAM not the mechanical OAM :

$$\langle L^3 \rangle^{\mathcal{W} = LC} \;=\; L_{can}$$

This observation holds out a hope that the canonical quark OAM in the nucleon would also be a measurable quantity, at least in principle.

However, in a recent paper

- A. Courtoy et al., Phys. Lett. B731 (2014) 141.

Courtoy et al. throws a serious doubt on the practical observability of the Wigner function F_{14}^{q} appearing in the above intriguing sum rule.

According to them, even though F_{14}^{q} may be nonzero in particular models and also in real QCD, its observability would contradict several observations :

- it drops out in both the formulation of GPDs and TMDs ;
- it is nonzero only for imaginary values of the quark-proton helicity amplitudes.

Their observations suggest that F_{14}^{q} would not appear in the cross section formulas of any DIS processes at least in the leading order approximation.

It appears to us that this takes a discussion on the observability of the canonical OAM back to its starting point ?

An interesting question :

$$\langle L^3 \rangle^{\mathcal{W}=LC} = L_{can} \quad \Rightarrow \quad \text{Why ?}$$

average transverse momentum and longitudinal OAM of quarks

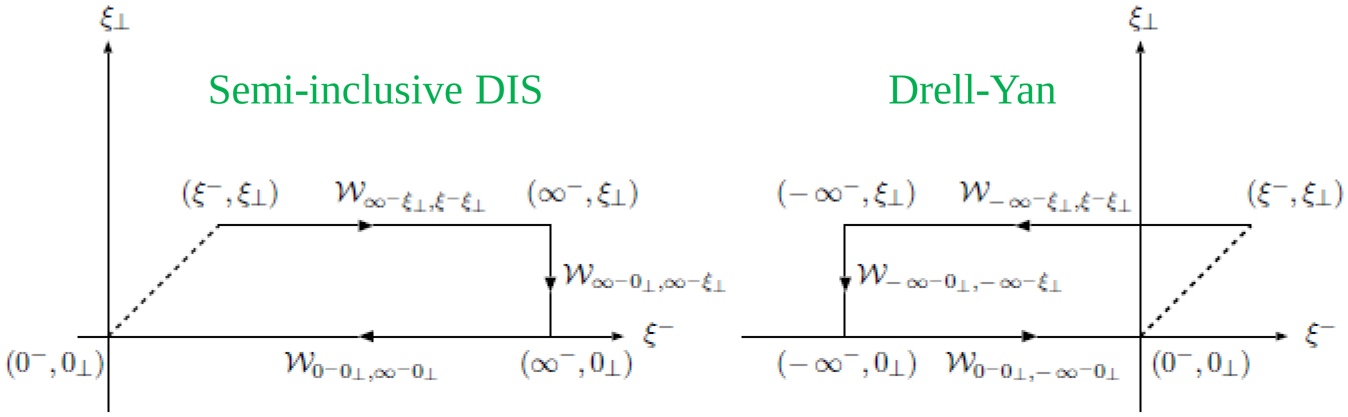
$$\langle \boldsymbol{k}^i_\perp \rangle^{\mathcal{W}} = \int dx \int d^2\boldsymbol{b}_\perp \int d^2\boldsymbol{k}_\perp \; \boldsymbol{k}^i_\perp \; \rho(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp ; \mathcal{W})$$

$$\langle L^3 \rangle^{\mathcal{W}} = \int dx \int d^2 b_\perp \int d^2 k_\perp \; (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)^3 \; \rho(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp ; \mathcal{W})$$

with

$\rho(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W})$ = generally gauge-link-path dep. Wigner distribution

2 paths with physical interest



(1) future-pointing staple-like LC path $\mathcal{W}^{+LC}$ (2) past-pointing staple-like LC path $\mathcal{W}^{-LC}$

Burkardt showed the relation

FSI or ISI ↗

$$\langle k_\perp^i \rangle^{\pm LC} = \langle k_\perp^i \rangle_{mech} + \langle k_\perp^i \rangle_{int}^{\pm LC}.$$

where

$$\langle k_\perp^i \rangle_{mech} = \frac{1}{2\,p^+} \langle p, s \,|\, \bar{\psi}(0) \,\frac{1}{i}\, D_\perp^i(0)\, \psi(0) \,|\, p, s \rangle.$$

while

$$\langle k_\perp^i \rangle_{int}^{\pm LC} = -\frac{1}{2\,p^+} \int_0^{\pm\infty} d\eta^- \times \langle p, s \,|\, \bar{\psi}(0)\, \mathcal{W}[0^- \mathbf{0}_\perp, \eta^- \mathbf{0}_\perp]\, g\, F^{+i}(\eta^-, \mathbf{0}_\perp)\, \mathcal{W}[\eta^- \mathbf{0}_\perp, 0^- \mathbf{0}_\perp]\, \psi(0) \,|\, p, s \rangle.$$

In the LC gauge, $\mathcal{W} \to 1,$ and

$$-\sqrt{2}\, g\, F^{+y} = g\,(E^y - B^x) = g\,[\,\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})\,]^y$$

Then, $\langle k_\perp^i \rangle_{int}^{+LC}$ can be interpreted as the change of transverse momentum of the struck quark by color Lorentz force when it leaves the target after being struck by the virtual photon in the semi-inclusive DIS processes.

Similarly, for the average longitudinal OAM

FSI or ISI

$$\langle L^3 \rangle^{\pm LC} \;=\; \langle L^3 \rangle_{mech} \;+\; \langle L^3 \rangle^{\pm LC}_{int},$$

where

$$\langle L^3 \rangle_{mech} \;=\; \mathcal{N} \int d^2 r_\perp$$
$$\times\; \langle p, s \,|\, \bar{\psi}(0^-, \boldsymbol{r}_\perp)\, \gamma^+ \, \epsilon^{ij}_\perp \, r^i_\perp \, \frac{1}{i} \, D^j_\perp(\boldsymbol{r}_\perp)(0^-, \boldsymbol{r}_\perp)\, \psi(0^-, \boldsymbol{r}_\perp) \,|\, p, s \rangle$$

while

$$\langle L^3 \rangle^{\pm LC}_{int} \;=\; -\mathcal{N} \int d^2 r_\perp \int_0^{\pm\infty} d\eta^- \; \epsilon^{ij}_\perp \, r^i_\perp \; \langle p, s \,|\, \bar{\psi}(0^-, \boldsymbol{r}_\perp)\, \gamma^+$$
$$\times\; \mathcal{L}[0^-\, \boldsymbol{r}_\perp, \eta^-\, \boldsymbol{r}_\perp]\, g\, F^{+j}[\eta^-, \boldsymbol{r}_\perp]\, \mathcal{L}[\eta^-\, \boldsymbol{r}_\perp, 0^-\, \boldsymbol{r}_\perp]\, \psi(0^-, \boldsymbol{r}_\perp) \,|\, p, s \rangle.$$

change from the previous case

Lorentz force ⇨ torque by Lorentz force

$$T^z(r^-, \boldsymbol{r}_\perp) \;\equiv\; -g \left(x\, F^{+y}(r^-, \boldsymbol{r}_\perp) \;-\; y\, F^{+x}(r^-, \boldsymbol{r}_\perp) \right)$$

Hatta showed that, due to the parity and time-reversal (PT) symmetry,

$$\langle L^3 \rangle^{-LC} = \langle L^3 \rangle^{+LC} = \langle L^3 \rangle_{can}$$

That is, the average longitudinal OAM defined through the Wigner distribution coincide with the GI canonical momentum (not the mechanical one) and it is independent of the two processes.

One might expect that a similar relation holds also for the average transverse mom :

$$\langle k_\perp^i \rangle^{\pm LC} \stackrel{?}{=} \langle k_\perp^i \rangle_{can}$$

with the definition of the GI canonical transverse momentum as

$$\langle k_\perp^i \rangle_{can} = \frac{1}{2\,p^+} \langle p, s \,|\, \bar{\psi}(0)\,\gamma^+ \frac{1}{i} D^i_{\perp,pure}(0)\,\psi(0)\,|\, p, s \rangle$$

In fact, Lorce claims in a recent paper that the momentum variable in the Wigner distribution refers to the canonical momentum not the mechanical momentum.

In the following, we show that this statement is not always true and we will give universally correct physical interpretation of the average transverse momentum as well as the average longitudinal OAM defined through the Wigner distribution.

To this end, we first recall the fact that, according to Hatta, there can be plural choices for defining the physical component of the gluon in the decomposition

$$A^{\mu} = A^{\mu}_{phys} + A^{\mu}_{pure}$$

Choice (I) : corresponds to retarded and advanced B.C. for gauge field

$$A^i_{phys}(0) \equiv - \int_{-\infty}^{+\infty} d\eta^- \, (\pm \theta(\pm \eta^-)) \times \mathcal{L}[0^- \mathbf{0}_\perp, \eta^- \mathbf{0}_\perp] \, g \, F^{+i}(\eta^-, \mathbf{0}_\perp) \, \mathcal{L}[\eta^- \mathbf{0}_\perp, 0^- \mathbf{0}_\perp],$$

Choice (II) : corresponds to asymmetric B.C. for the gauge field

$$A^i_{phys}(0) \equiv - \frac{1}{2} \int_{-\infty}^{+\infty} d\eta^- \, \epsilon(\eta^-) \times \mathcal{L}[0^- \mathbf{0}_\perp, \eta^- \mathbf{0}_\perp] \, g \, F^{+i}(\eta^-, \mathbf{0}_\perp) \, \mathcal{L}[\eta^- \mathbf{0}_\perp, 0^- \mathbf{0}_\perp],$$

Remarkably, in the case of average longitudinal OAM, any of the above choices for A^i_{phys} gives the same answer for $\langle L^3 \rangle^{\pm LC}$, which coincides with the canonical OAM of quarks. (Hatta, 2012)

This is related to the PT-even nature of the quantity $\langle L^3 \rangle$.

However, it is not necessarily true for $\langle k^i_\perp \rangle^{\pm LC}$.

For choice (I), one can show

$$\begin{aligned}\langle k_\perp^i \rangle^{\pm LC} &= \frac{1}{2\,p^+}\langle p,s\,|\,\bar{\psi}(0)\,\gamma^+\,\frac{1}{i}\,D_\perp^i(0)\,\psi(0)\,|\,p,s\rangle \\ &+ \frac{1}{2\,p^+}\langle p,s\,|\,\bar{\psi}(0)\,\gamma^+\,g\,A^i_{phys}(0)\,\psi(0)\,|\,p,s\rangle \\ &= \frac{1}{2\,p^+}\langle p,s\,|\,\bar{\psi}(0)\,\gamma^+\,\frac{1}{i}\,D^i_{\perp,pure}(0)\,\psi(0)\,|\,p,s\rangle,\end{aligned}$$

so that, one formally have

$$\langle k_\perp^i \rangle^{\pm LC} = \langle k_\perp^i \rangle_{can},$$

However, we already know that the average transverse momentum corresponding to the future- and past-pointing stale-like LC path have different signs

$$\langle k_\perp^i \rangle^{-LC} = -\,\langle k_\perp^i \rangle^{+LC}$$

Then, the canonical transverse momentum defined as above is not a universal quantity, i.e. it is process-dependent quantity.

More natural would be the choice (II). In this case, by using the identity,

$$\pm\,\theta(\pm\,\eta^-) \;=\; \frac{1}{2}\,[\,\epsilon(\eta^-)\;\pm\;1]$$

we can show

$$\langle k_\perp^i \rangle^{\pm LC} \;=\; \frac{1}{2\,p^+}\,\langle p,s\,|\,\bar{\psi}(0)\,\gamma^+\,\frac{1}{i}\,D^i_{\perp,pure}(0)\,\psi(0)\,|\,p,s\rangle \quad \longrightarrow \quad \langle k_\perp^i \rangle_{can}$$

$$\mp\,\frac{1}{4\,p^+}\int_{-\infty}^{+\infty} d\eta^-\,\langle p,s\,|\,\bar{\psi}(0)\,\gamma^+\,\mathcal{W}[0^-,\eta^-]\,g\,F^{+i}(\eta^-)\,\mathcal{W}[\eta^-,0^-]\,\psi(0)\,|\,p,s\rangle,$$

which means that

$$\langle k_\perp^i \rangle^{\pm LC} \;\neq\; \langle k_\perp^i \rangle_{can}$$

In any case, the present consideration confirms non-universal nature of the statement by Lorce that the momentum variable in the Wigner distribution refers to the canonical momentum not the mechanical momentum.

In our opinion, the above-mentioned arbitrariness in the definition of the canonical transverse momentum is an indication of its mathematical or theoretical nature in contrast to more physical nature of mechanical transverse momentum.

What is universally correct physical interpretation of Wigner-distribution-based definitions of the average transverse momentum and longitudinal OAM, then ?

Since the physical statement should be independent of the ambiguity in the definition of the physical component of the gluon field, or the definitions of the canonical transverse momentum, it is convenient to go back to the expression of Burkardt.

$$\langle k_\perp^i \rangle^{\pm LC} = \langle k_\perp^i \rangle_{mech} + \langle k_\perp^i \rangle_{int}^{\pm LC}$$

where

$$\langle k_\perp^i \rangle_{int}^{\pm LC} = -\frac{1}{2p^+} \int_0^{\pm\infty} d\eta^- \times \langle p, s \,|\, \bar{\psi}(0)\, \mathcal{W}[0^-\, \mathbf{0}_\perp, \eta^-\, \mathbf{0}_\perp]\, g\, F^{+i}(\eta^-, \mathbf{0}_\perp)\, \mathcal{W}[\eta^-\, \mathbf{0}_\perp, 0^-\, \mathbf{0}_\perp]\, \psi(0) \,|\, p, s \rangle.$$

Here, we can say from PT symmetry that

$$\langle k_\perp^i \rangle_{mech} = 0$$

so, after all

$$\langle k_\perp^i \rangle^{\pm LC} = \langle k_\perp^i \rangle_{int}^{\pm LC}$$

As is well known, this FSI or ISI interaction term can be related to the gluon pole term of the twist-3 quark-gluon correlation function known as Efremov-Teryaev-Qiu-Sterman (ETQS) function as

$$\langle k_\perp^i \rangle^{\pm LC} = \langle k_\perp^i \rangle^{\pm LC}_{int} = \frac{1}{2} \epsilon_\perp^{ij} s_\perp^j (\mp \pi) \int dx \, \Psi_F(x, x) \quad \cdots \quad (A)$$

with the definition

$$\begin{aligned} &\int \frac{d\xi^-}{2\pi} \int \frac{d\eta^-}{2\pi} e^{i p^+ \xi^- x} e^{i p^+ \eta^- (x' - x)} \\ &\times \langle ps \,|\, \bar{\psi}(0) \gamma^+ \mathcal{W}[0^-, \eta^-] \, g \, F^{+i}(\eta^-) \, \mathcal{W}[\eta^-, \xi^-] \, \psi(\xi) \,|\, ps \rangle \\ &= \frac{1}{p^+} \epsilon_\perp^{ij} s_\perp^j \, \Psi_F(x', x) + \cdots \end{aligned}$$

On the other hand, the average transverse momentum defined by the Wigner distribution can be expressed also with the TMD based on the relation

$$\begin{aligned} \langle k_\perp^i \rangle^{\pm LC} &= \int dx \int d^2 b_\perp \int d^2 k_\perp \, k_\perp^i \, \tilde{\rho}^{\gamma^+}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) \\ &= \int dx \int dk_\perp \, k_\perp^i \, \tilde{\rho}^{\gamma^+}(x, \boldsymbol{k}_\perp, \Delta_\perp) \Big|_{\Delta_\perp = 0} \end{aligned}$$

Using the standard parametrization of GTMD (Meissner-Metz-Schlegel, 2009)

$$\tilde{\rho}^{\gamma^+}(x, \boldsymbol{\Delta}_\perp, \boldsymbol{k}_\perp) = \frac{1}{2p^+}\bar{u}(p,s)\left[\gamma^+ F_{11} + \frac{i\,\sigma^{i+}\Delta_\perp^i}{2M_N}(2F_{13} - F_{11}) + \frac{i\,\sigma^{i+}k_\perp^i}{2M_N}F_{12} + \frac{i\,\sigma^{ij}k_\perp^i\Delta_\perp^j}{M_N^2}F_{14}\right]u(p,s)$$

one can show that

$$\langle k_\perp^i \rangle^{\pm LC} = -\frac{1}{2}\epsilon_\perp^{ij}s_\perp^j \int dx \int d^2k_\perp \frac{\boldsymbol{k}_\perp^2}{M_N} f_{1T}^\perp(x, \boldsymbol{k}_\perp^2) \quad \cdots \quad (B)$$

Here, $f_{1T}^\perp$ is the famous Sivers function related to the imaginary part of GTMD F_{12} as

$$f_{1T}^\perp(x, \boldsymbol{k}_\perp^2) = \mathrm{Im}\, F_{12}(x, \xi = 0, \boldsymbol{k}_\perp^2, \boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp = 0, \boldsymbol{\Delta}_\perp^2 = 0)$$

Comparison of (A) and (B) gives the famous relation between the Sivers function and the ETQS function as (Boer, Mulders, Pijlman, 2003)

$$\int d^2k_\perp \frac{\boldsymbol{k}_\perp^2}{M_N} f_{1T}^\perp(x, \boldsymbol{k}_\perp^2) = \mp\pi\,\Psi_F(x,x)$$

In any case, the physical picture obtained from the above consideration is clear.

For clarity, let us take the semi-inclusive DIS case as a concrete example.

$$\langle k_\perp^i \rangle^{+LC} = \langle k_\perp^i \rangle_{mech} + \langle k_\perp^i \rangle_{int}^{+LC}$$

Initially, the average transverse momentum of quarks inside the nucleon,, which is nothing but the manifestly GI mechanical transverse momentum, is zero.

$$\langle k_\perp^i \rangle_{mech} = 0$$

Through FSI, the ejected quark acquires nonzero transverse momentum

$$\langle k_\perp^i \rangle^{+LC} = \langle k_\perp^i \rangle_{int}^{+LC} \neq 0$$

From this fact, one can conclude that the average transverse momentum of quarks defined by the Wigner distribution represents the asymptotic momentum of a quark after it leaves the target.

Exactly the same interpretation holds also for the average longitudinal OAM.

Again, it is convenient to go back to the expression of Burkardt.

$$\langle L^3 \rangle^{+LC} \;=\; \langle L^3 \rangle_{mech} \;+\; \langle L^3 \rangle^{+LC}_{int},$$

Initially, the average longitudinal OAM of quarks inside the nucleon is nothing but the manifestly GI mechanical OAM, which is generally nonzero.

$$\langle L^3 \rangle_{mech} \;\neq\; 0$$

Through FSI, the ejected quark receives additional OAM change.

The average longitudinal OAM defined by the Wigner distribution represents the sum of these two pieces of OAM.

On the other hand, we already know the fact that

$$\langle L^3 \rangle^{+LC}_{int} \;=\; \langle L^3 \rangle_{pot}$$

so that

$$\langle L^3 \rangle^{+LC} \;=\; \langle L^3 \rangle_{mech} \;+\; \langle L^3 \rangle_{pot} \;=\; \langle L^3 \rangle_{\text{"}can\text{"}}$$

Now we understand the reason why this last relation holds.

$$\langle L^3 \rangle^{+LC} \; = \; \langle L^3 \rangle_{``can"}$$

For, according to our general rule, the average longitudinal OAM $\langle L^3 \rangle^{+LC}$, defined by the Wigner distribution, must represent the asymptotic OAM of the quark after leaving the spectator in SIDIS reaction.

It is only natural that this quark OAM well separated from the original nucleon center reduces to the "canonical" OAM, which is basically the free quark OAM.

It is also clear that this quark OAM is not the one which is carried by the quarks inside the nucleon.

In other words, the "generalized canonical OAM" of Chen decomposition is not an intrinsic property of the nucleon, but the fact is that

$$\langle L^3 \rangle_{``can"} \; = \; \texttt{intrinsic OAM} \; + \; \texttt{FSI}$$

Let us repeat again what we have found.

$$\langle L^3 \rangle^{+LC} \; = \; \langle L^3 \rangle_{mech} \; + \; \langle L^3 \rangle_{pot} \; = \; \langle L^3 \rangle_{\text{"}can\text{"}}$$

Initially, in the nucleon, the average OAM of quarks is obviously the manifestly gauge-invariant mechanical OAM $\langle L^3 \rangle_{mech}$.

Through FSI, the ejected quark acquires potential angular momentum $\langle L^3 \rangle_{pot}$, which was originally stored in the gluon part.

As a consequence, the final OAM of the ejected quark becomes the "canonical" OAM, which is basically free quark OAM.

Now we hope everybody convinces that what represents the intrinsic OAM of quarks in the nucleon is

mechanical OAM **not** generalized "canonical" OAM of Chen et al.

The latter is not an intrinsic property of the nucleon structure.

Because our conclusion is fairly different from the naïve picture believed by many members of the canonical momentum party, there may be some persons who need more explanation.

After all, what makes the problem complicated is FSI or ISI, which comes into the game through the transverse gauge-link.

This can be convinced if one consider the average longitudinal momentum defined by the Wigner distribution :

$$\langle x \rangle^{\mathcal{W}} \equiv \int dx \int d^2 b_\perp \int d^2 k_\perp \; x \; \rho(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp \, ; \, \mathcal{W})$$

In this case, the integration over $\boldsymbol{b}_\perp$ and $\boldsymbol{k}_\perp$ is trivial, and the gauge-link path dependence essentially disappears, thereby leading to the familiar result :

$$\langle x \rangle = \frac{1}{2\,p^+} \langle ps \,|\, \bar{\psi}(0) \, \gamma^+ \frac{1}{i} D^+ \, \psi(0) \,|\, ps \rangle$$

This is nothing but the manifestly gauge-invariant mechanical momentum.

$$\langle x \rangle_{mech}$$

In our general rule, the average longitudinal momentum of quarks defined by the Wigner distribution should represent the asymptotic momentum.

There is no discrepancy, however, since

$$\begin{aligned} \langle x \rangle &= \frac{1}{2\,p^+}\,\langle ps\,|\,\bar{\psi}(0)\,\gamma^+\,\frac{1}{i}\,D^+\,\psi(0)\,|\,ps\rangle \;=\; \langle x \rangle_{mech} \\ &= \frac{1}{2\,p^+}\,\langle ps\,|\,\bar{\psi}(0)\,\gamma^+\,\frac{1}{i}\,D^+_{pure}\,\psi(0)\,|\,ps\rangle \\ &- \frac{1}{2\,p^+}\,\langle ps\,|\,\bar{\psi}(0)\,\gamma^+\,A^+_{phys}\,\psi(0)\,|\,ps\rangle \\ &= \langle x \rangle_{\text{"}can\text{"}} \;-\; \langle x \rangle_{pot} \end{aligned}$$

and since we know that the FSI or the potential momentum term vanishes.

$$\langle x \rangle_{pot} \;=\; 0$$

♣ This is manifest in the LC gauge $A^+ = A^+_{phys} = 0$, and it is true in general gauge.

Namely, due to the vanishment of the FSI for the collinear momentum case,

$$\langle x \rangle_{mech} \;=\; \langle x \rangle_{\text{"}can\text{"}}$$

intrinsic momentum **asymptotic momentum**

We have reached clear understanding of the physical meaning of the two OAMs ;

mechanical OAM and "canonical" OAM

The remaining task is to judge the relative merits of these two OAM, or the two types of nucleon spin decomposition, from the observational standpoint.

We have already pointed out that the canonical quark OAM can be related to the Wigner distribution (or GTMD) F_{14} :

$$L^q_{can} \;=\; - \int dx\, d^2 k_\perp \frac{\boldsymbol{k}_\perp^2}{M^2} F^q_{1,4}(x, 0, \boldsymbol{k}_\perp^2, 0, 0) \;\Leftrightarrow\; \texttt{Wigner distribution}$$

The question is the observability of the Wigner distribution F_{14} .

In the DIS physics, the factorization theorem is an important criterion of observability (or quasi-observability) of PDFs, GPDs, and TMDs.

A shortage of the Wigner function F_{14} is that it totally drops out in both of the GPD and TMD factorizations.

- A. Courtoy et al., Phys. Lett. B731 (2014) 141.

The importance of the factorization theorem can easily be convinced if one remembers the nuclear theory.

It is widely-known that the nuclear wave functions, or the momentum distribution of the nucleon in a nucleus, are not direct observables.

This is because there is no factorization theorem in the theory of nuclear reactions, so that the nuclear structure information cannot be extracted model-independently.

Namely, the reaction mechanism or the effect of final-state interactions (FSI) is necessarily mixed up in this extraction process.

By the same reason, the decomposition of the total spin of a nucleus into the spin and orbital parts can be done only within a particular theoretical model.

This circumstance can be easily understood if one inspects the decomposition problem of the total angular momentum of the deuteron, as the simplest nucleus.

Non-observability of the OAM of a constituent in a composite particle

(- in the absence of factorization theorem -)

deuteron example

deuteron w.f. and S- and D-state probabilities

$$\psi_d(\boldsymbol{r}) \; = \; \left[u(r) \; + \; \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}} \, w(r) \right] \frac{\chi}{\sqrt{4\,\pi}}$$

$$P_S \; = \; \int_0^\infty u^2(r) \, r^2 \, dr, \quad P_D \; = \; \int_0^\infty w^2(r) \, r^2 \, dr$$

decomposition of the total deuteron spin

$$\langle J_3 \rangle \; = \; \langle (\boldsymbol{r} \times \boldsymbol{p})_3 \rangle \; + \; \langle S_3 \rangle$$

$$= \; \langle L_3 \rangle \; + \; \langle S_3 \rangle \; = \; \frac{3}{2} P_D \; + \; \left(P_S - \frac{1}{2} P_D \right) \; = \; 1$$

The OAM contribution to the net deuteron spin is proportional to P_D !

However, we know that the D-state probability is not a direct observable !

♣ The point is that bound state w.f.'s are not direct observables.

- R.D. Amado, Phys. Rev. C19 (1979) 1473.

♣ 2-body unitary transformation arising in the theory of meson-exchange currents can change the D-state probability, while keeping the deuteron observables intact.

- J.L. Friar, Phys. Rev. C20 (1979) 325.

♣ The D-state probability, for instance, depends on the cutoff Λ of short range physics in an effective theory of 2-nucleon system.

- S.K. Bogner et al., Nucl. Phys. A784 (2007) 79.

⇩

See the figure in the next page !

Deuteron **D-state probability** in an effective theory

Bogner et al, 2007

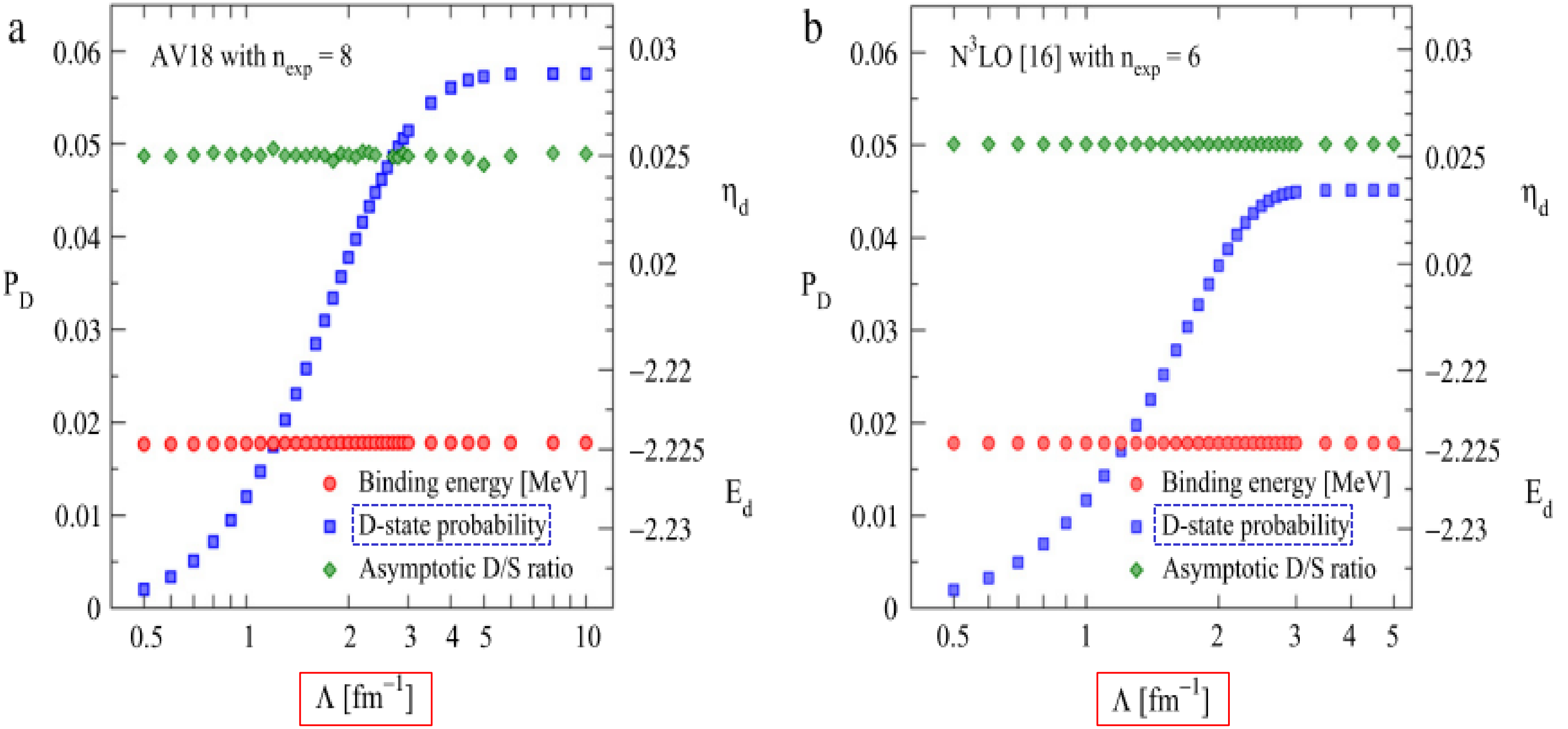


Fig. 57. D-state probability P_D (left axis), binding energy E_d (lower right axis), and asymptotic D/S-state ratio η_d (upper right axis) of the deuteron as a function of the cutoff [6], starting from (a) the Argonne v_{18} [18] and (b) the N^{3}LO NN potential of Ref. [20] using different smooth $V_{\text{low}\,k}$ regulators. Similar results are found with SRG evolution.

What about the observability of the mechanical OAM, then ?

We already know the relations :

$$L^q_{mech} = \frac{1}{2}\int_{-1}^{1} x\,[\,H^q(x,0,0) + E^q(x,0,0)\,]\,dx - \frac{1}{2}\int_{-1}^{1} \Delta q(x)\,dx$$

$$= J^q - \frac{1}{2}\Delta\Sigma^q \qquad : \quad \text{(Ji, 1979)}$$

$$L^G_{mech} = \frac{1}{2}\int_{-1}^{1} x\,[\,H^G(x,0,0) + E^G(x,0,0)\,]\,dx - \int_{-1}^{1} \Delta g(x)\,dx$$

$$= J^G - \Delta G \qquad : \quad \text{(Wakamatsu, 2010, 2011)}$$

These are naively expected relations, except for the following delicate point :

$$L^G_{mech} = L^G(JM) + L_{pot}$$

All the quantities appearing in the r.h.s. of the above relations are twist-2 GPDs and PDFs, so that the mechanical OAMs are in principle measurable quantities.

However, one might feel that this extraction is somewhat indirect, since both OAMs are given as differences of total angular momenta and spins.

At the twist-3 level, there is a direct relation, in which the mechanical OAM is given as a 2nd moment of the twist-3 GPD G_2 .

- Penttinen et al. (2000), Kiptily and Polyakov (2004), Hatta and Yoshida (2012)

$$L^q_{mech} \;=\; - \int x \, G^q_2(x, 0, 0) \, dx$$

It is very important to remember fact that this GPD G_2 sum rule, which gives the mechanical OAM, is derived from the following identity :

$$0 \;=\langle \bar{\psi}(0) \, \gamma^i \, \not{n} \, \mathcal{L}[0, \lambda] \, \not{D}(\lambda) \, \psi(\lambda) \rangle$$

with

$$\langle \cdots \rangle = \langle p', s' \, | \, \cdots \, | \, p, s \rangle$$

which hold owing to the QCD equation of motion :

$$\not{D}(\lambda) \, \psi(\lambda) \;=\; 0$$

To sum up

- GPD G_2 can in principle be extracted from GPD analyses.
- Wigner distribution $F_{1,4}$ drops out in both the TMD and GPD factorization !

The situation for canonical OAM is similar to the deuteron problem !

After all, what would be the crucial ingredient which discriminates the two cases ?

Now that both OAMs satisfy the gauge-invariance, the gauge-principle cannot say anything about the superiority and inferiority of the two.

In our opinion, a vital physical difference between these two OAMs is that the mechanical OAM (not the canonical OAM) appears in the equation motion with Lorentz force.

$$\frac{d}{dt} \boldsymbol{L}_{mech} = q \, \boldsymbol{r} \times [\, \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \,]$$

Remember that G_2 sum rule is obtained from the QCD equation of motion !

Summary statements on the nucleon spin decomposition problem

- We have clarified the fact that what plays a **key role** in the **gauge-invariant** decomposition problem of the nucleon spin is the **Lorentz-frame independence**, or **boost-invariance** along the direction of the nucleon momentum.

  After all, we can say that the Lorentz symmetry plays more crucial role than the gauge symmetry in the proper definition of the nucleon spin decomposition.

- We have also carried out a comparative analysis of two types of nucleon spin decomposition, which are characterized by two types of OAMs, i.e.

  “canonical” OAMs & “mechanical” OAMs

- We have advocated a viewpoint which favors the mechanical OAMs rather than the canonical OAMs, from the observational viewpoint.

  Again, it appears that the gauge-symmetry plays only a minor role in the difference between the (GI) canonical OAM and mechanical OAM.

  Physics lies in the fact that the latter not the former appears in the eq. of motion.

  More physical is mechanical OAM !

[Comment on the relation between Sivers function and quark OAM]

twist-3 quark-gluon correlation function

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} \, e^{i\lambda x} \, e^{i\mu(x'-x)}$$
$$\times \quad \langle p' s' \,|\, \bar{\psi}(0) \, \gamma^+ \, \mathcal{L}[0,\mu] \, g \, F^{+i} \, \mathcal{L}[\mu,\lambda] \, \psi(\lambda) \,|\, p \, s \rangle$$
$$= \; P^+ \, \epsilon_\perp^{ij} \, s_\perp^j \, \Psi_F(x,x') \; + \; \epsilon_\perp^{ij} \, \Delta_\perp^j \, s^+ \, \Phi_F(x,x') \; + \; \cdots$$

We have already shown the relation

ETQS func.

Sivers func.

$$\langle k_\perp^i \rangle^{\pm LC} \; = \; \frac{1}{2} \epsilon_\perp^{ij} \, s_\perp^j \, (\mp \pi) \int dx \, \Psi_F(x,x)$$
$$= \; -\frac{1}{2} \epsilon_\perp^{ij} \, s_\perp^j \int dx \int d^2 k_\perp \, \frac{\boldsymbol{k}_\perp^2}{M_N} \, f_{1T}^\perp(x, \boldsymbol{k}_\perp^2)$$

but

$$L_{pot}^q \; = \; \int dx \int dx' \, \mathcal{P} \, \frac{1}{x - x'} \, \Phi_F(x,x') \qquad \text{(Hatta, 2012)}$$

Note that

$\Psi_F(x,x') \;\Leftrightarrow\; \Phi_F(x,x')$: totally independent functions !