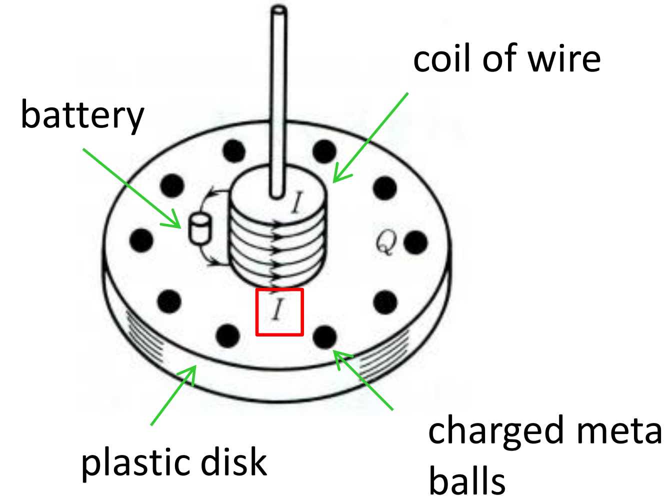
Backup Slides

[Backup slide A] A short review of the **Feynman paradox**

1. Initially, the disk is at rest.

2. Shut off the electric current at some moment.



Question

Does the disk begin to rotate, or does it continue to be at rest ?

Answer (A)

- ♣ Since an electric current is flowing through the coil, there is a magnetic flux along the axis.
- ♣ When the current is stopped, due to the electromagnetic induction, an electric field along the circumference of a circle is induced.
- ♣ Since the charged metal ball receives forces by this electric field, the disk begins to rotate !

Answer (B)

- ♣ Since the disk is initially at rest, its angular momentum is zero.

- ♣ Because of the conservation of angular momentum, the disk continue to be at rest !

⇓

2 totally conflicting answers !

Feynman's paradox

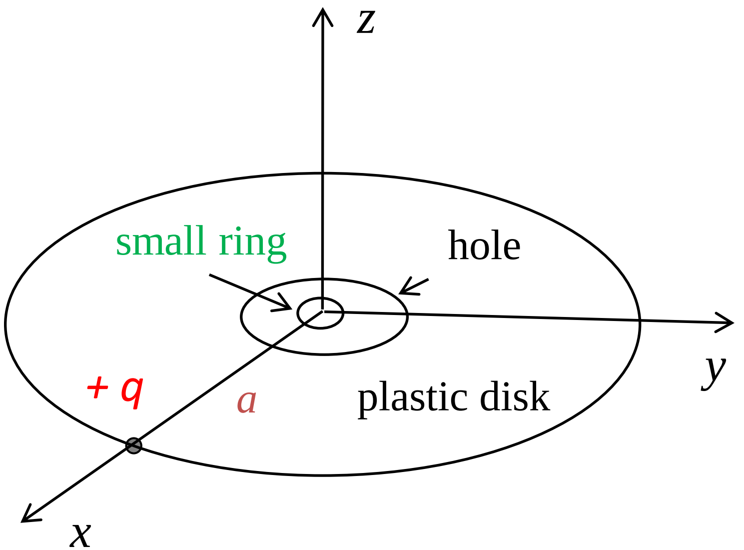
The paradox is resolved, if one takes account of the angular momentum carried by the electromagnetic field or potential generated by an electric current !

$$\boldsymbol{L}_{e.m.} = \int \boldsymbol{r} \times \rho \, \boldsymbol{A} \, d^3 r$$

The answer (A) is correct !

[Backup Slides B] A simplified model of the Feynman paradox

- J.M. Aguirregabiria and A. Hernandez, Eur. J. Phys. 2 (1981) 168.



- A current I is flowing in a small (nearly point-like) ring so that it has a magnetic moment

$$\boldsymbol{m} = m\,\boldsymbol{e}_z$$

- A charge $+q$ is located at

$$\boldsymbol{r} = (a, 0, 0)$$

- This disk is initially at rest.

  The vector potential $\boldsymbol{A}$ at a point $\boldsymbol{r}$ created by the small ring is

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\boldsymbol{r}}{r^3}$$

- Now, the magnetic moment is slowly decreased.

  The induced electric fields $\boldsymbol{E} = -\partial\boldsymbol{A}/\partial t$ has a tangential component.

$$E_\phi = -\frac{\mu_0}{4\pi}\frac{\dot{m}}{a^2} \quad \text{at} \quad \boldsymbol{r} = (a, 0, 0)$$

Torque

$$N_z = a \times q\,E_\phi = -\frac{\mu_0}{4\pi}\frac{q\,\dot{m}}{a}$$

When m becomes 0, the angular momentum of the disk is

$$L_z = \int N_z \, dt = -\frac{\mu_0 \, q}{4 \, \pi \, a} \int_m^0 \dot{m} \, dt = \frac{\mu_0 \, q \, m}{4 \, \pi \, a}$$

However, since the angular momentum of the disk is initially zero and if it must be conserved, the disk must be at rest.

basically the **Feynman paradox**

We must consider the angular momentum carried by the e.m. field (or potential)

$$\boldsymbol{L}_{e.m.} = \frac{1}{c^2} \int \boldsymbol{r} \times \boldsymbol{S} \, dV = \frac{1}{\mu_0 \, c^2} \int \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B}) \, dV$$

Using the identity

$$\boldsymbol{C} \times (\nabla \times \boldsymbol{D}) + \boldsymbol{D} \times (\nabla \times \boldsymbol{C}) = (\nabla \cdot \boldsymbol{C}) \boldsymbol{D} + (\nabla \cdot \boldsymbol{D}) \boldsymbol{C} + \nabla \cdot \boldsymbol{T}$$

$$\boldsymbol{r} \times \nabla \cdot \boldsymbol{T} = \nabla \cdot \boldsymbol{R}$$

with

$$T_{ij} = (\boldsymbol{C} \cdot \boldsymbol{D}) \, \delta_{ij} - (C_i \, D_j + C_j \, D_i)$$

$$R_{ij} = \varepsilon_j^{kl} \, x_k \, T_{il}$$

we can write as

$$\boldsymbol{L}_{e.m.} = \varepsilon_0 \int \boldsymbol{r} \times [\boldsymbol{E} \times (\nabla \times \boldsymbol{A})] \, dV$$
$$= \int (\boldsymbol{r} \times \rho \boldsymbol{A}) \, dV + \varepsilon_0 \int [(\nabla \cdot \boldsymbol{A}) \boldsymbol{r} \times \boldsymbol{E}] \, dV + \varepsilon \int \nabla \cdot \boldsymbol{Q} \, dV$$

with

$$Q_{ij} = \varepsilon_j^{kl} x_k [(\boldsymbol{E} \cdot \boldsymbol{A}) \delta_{li} - (E_l A_i + E_i A_l)]$$

The 2nd term vanishes, since $\boldsymbol{A}$ satisfies $\nabla \cdot \boldsymbol{A} = 0$.

Using the Gauss law, the 3rd term also vanishes, since $\boldsymbol{Q} \rightarrow 1/r^3$.

Then, noting that $\rho = q \, \delta^{(3)}(\boldsymbol{r} - \boldsymbol{a})$, we get

$$\boldsymbol{L}_{e.m.} = \int (\boldsymbol{r} \times \rho \boldsymbol{A}) \, dV = q \, \boldsymbol{r} \times \boldsymbol{A}(\boldsymbol{a})$$

That is

$$\boldsymbol{L}_{e.m.} = \frac{\mu_0 q}{4 \pi a^3} \boldsymbol{a} \times (\boldsymbol{m} \times \boldsymbol{a}) = \frac{\mu_0 q m}{4 \pi a} \boldsymbol{e}_z$$

This exactly coincides with the previously-derived angular momentum of the plastic disk in the final state !

-- OAM carried by e.m. field or potential --