

4. Phenomenology of nucleon spin contents

Thomas' proposal toward the resolution of the nucleon spin puzzle

Several years ago, Thomas carried out an analysis of the proton spin contents in the context of the **refined cloudy bag (CB) model**, and concluded that the **modern spin discrepancy** can well be resolved in terms of the **standard features of the nonperturbative structure of the nucleon**, i.e.

- (1) **relativistic motion** of valence quarks
- (2) **pion cloud** required by chiral symmetry
- (3) **exchange current** contribution associated with **one-gluon-exchange (OGE)** hyperfine interactions

supplemented with the effect of **QCD scale evolution**.

(1) relativistic effect

$$\Delta\Sigma^Q \rightarrow 0.65 \times \Delta\Sigma^Q, \quad 2L^Q \rightarrow 0.35$$

- **lower (p-wave) components** of relativistic wave functions -

(2) pion cloud effects

physical nucleon = “bare nucleon” + pion cloud



3 valence quarks of nucleon core

bare nucleon probability $Z \sim 1 - P_{N\pi} - P_{\Delta\pi} \sim 0.7$

$$P_{N\pi} \sim 0.20 - 0.25$$

$$P_{\Delta\pi} \sim 0.05 - 0.10$$

reduction of quark spin fraction $\Delta\Sigma^Q$

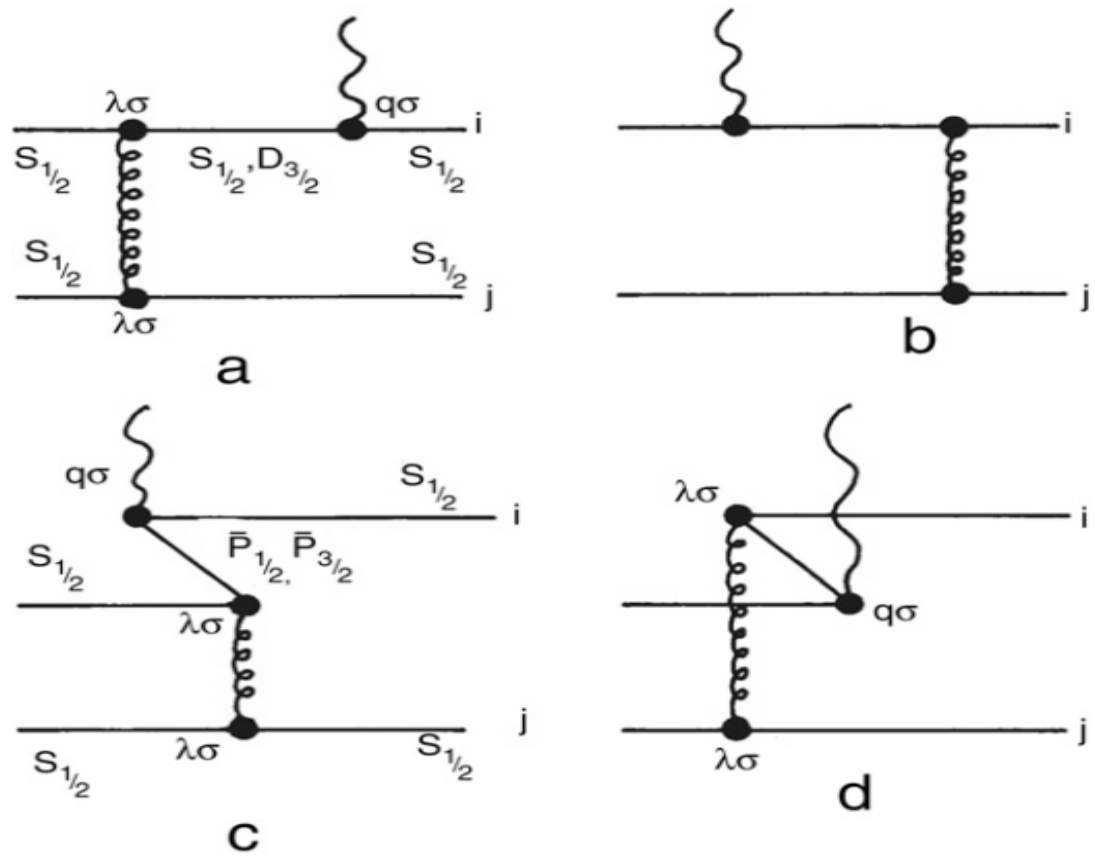
partial cancellation

$$\Delta\Sigma^Q \rightarrow \left(Z - \frac{1}{3} P_{N\pi} + \frac{5}{3} P_{\Delta\pi} \right) \Delta\Sigma^Q$$



main factor of reduction !

(3) one-gluon exchange correction (Myhrer and Thomas)



$$\Delta\Sigma^Q \rightarrow \Delta\Sigma^Q - 3G$$

$$G \propto \alpha_S \times (\text{certain bag model matrix elements})$$

predictions of the refined CB model (CBM)

- A. W. Thomas, Phys. Rev. Lett. 101 (2009) 102003.

	$2 L^u$	$2 L^d$	$\Delta\Sigma$
Non-relativistic	0	0	1.00
Relativistic	0.46	-0.11	0.65
+ OGE	0.52	-0.02	0.50
+ Pion Cloud	0.50	0.12	0.38

main features

$$\Delta\Sigma^Q \sim 0.4$$

$$L^u + L^d \sim 0.6, \quad L^u - L^d > 0$$

[Caution]

- CBM prediction corresponds to low energy model scale. $\leftrightarrow \sqrt{Q^2} \sim 400 \text{ MeV}$
- The quark OAM is a strongly scale-dependent quantity !

Leading-order (LO) evolution equation for quark OAM

- X. Ji, J. Tang, and P. Hoodbhoy, Phys. Rev. Lett. 76 (1996) 740.

- flavor singlet channel

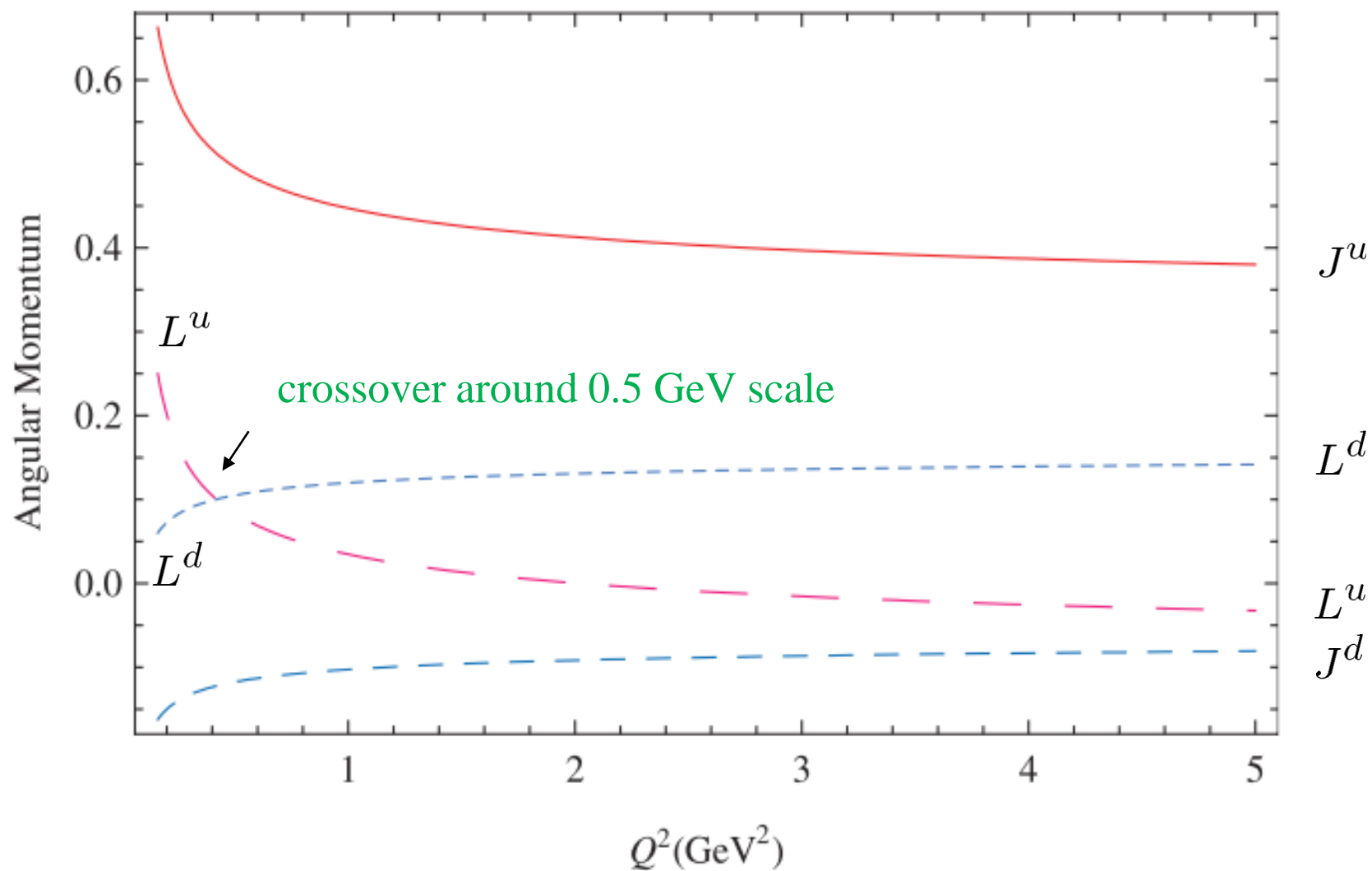
$$\begin{aligned} L^Q(t) &= -\frac{1}{2} \Delta\Sigma^Q + \frac{3n_f}{16 + 3n_f} \\ &+ \left(\frac{t}{t_0}\right)^{-2(16+3n_f)/9\beta_0} \left(L^Q(t_0) + \frac{1}{2} \Delta\Sigma^Q - \frac{1}{2} \frac{3n_f}{16 + 3n_f} \right) \\ L^G(t) &= -\Delta G(t) + \frac{16}{16 + 3n_f} \\ &+ \left(\frac{t}{t_0}\right)^{-2(16+3n_f)/9\beta_0} \left(L^G(t_0) + \Delta G(t_0) - \frac{1}{2} \frac{16}{16 + 3n_f} \right) \end{aligned}$$

- flavor non-singlet channel

$$L^{(NS)}(t) + \frac{1}{2} \Delta\Sigma^{(NS)} = \left(\frac{t}{t_0}\right)^{-32/9\beta_0} \left(L^{(NS)}(t_0) + \frac{1}{2} \Delta\Sigma^{(NS)} \right)$$

$$(NS) = u - d, \quad \text{or} \quad u + d - 2s$$

refined CBM predictions of Thomas



- A. W. Thomas, Phys. Rev. Lett. 101 (2008) 102003.

A remarkable feature is a **crossover** of L^u and L^d around 500 MeV scale !

This crossover is absolutely **necessary** for Thomas' scenario to hold, **because**

(1) Refined CBM prediction at low energy scale :

$$L^u - L^d > 0 \quad \text{at} \quad \sqrt{Q^2} \simeq 400 \text{ MeV}$$

(2) Asymptotic boundary condition dictated by the QCD evolution equation :

$$\lim_{Q^2 \rightarrow \infty} (L^u - L^d) = -g_A^{(3)}/2 < 0$$

See next page

Thomas then claims that, owing to this **crossover**, the predictions of the refined CBM **after taking account of QCD evolution** is qualitatively consistent with the recent lattice QCD data given at $Q^2 = 4 \text{ GeV}^2$;

- $L^u(\text{LHPC}) = -0.196 \pm 0.044$, $L^d(\text{LHPC}) = +0.200 \pm 0.044$
- $L^{u+d}(\text{LHPC}) \sim 0.06$ (**small**) $\Leftrightarrow L^{u+d}(\text{CB model}) \sim 0.11$

We shall show below that **this statement is not necessarily justified** !

[Note] on the asymptotic boundary condition for $L^u - L^d$

$$\lim_{Q^2 \rightarrow \infty} (L^u - L^d) = -g_A^{(3)} / 2$$

- M. W. and Y. Nakakoji, Phys. Rev. D77 (2008) 074011.

Leading-order evolution eq. for flavor nonsinglet channel

$$L^{u-d}(t) + \frac{1}{2} \Delta \Sigma^{u-d} = \left(\frac{t}{t_0} \right)^{-32/9\beta_0} \left(L^{u-d}(t_0) + \frac{1}{2} \Delta \Sigma^{u-d} \right)$$

with $\beta_0 = 11 - 3n_f / 2$

Since right-hand-side becomes 0 as $t \rightarrow \infty$, we find that

$$\lim_{t \rightarrow \infty} L^{u-d}(t) = -\frac{1}{2} \Delta \Sigma^{u-d} = -\frac{1}{2} g_A^{(3)} \sim -0.6$$

neutron beta-decay coupling constant !

Our (nearly) model-independent analysis of proton spin

- M. W. , Eur. Phys. J. A44 (2010) 297 ; ibid. A46 (2010) 327.

based on

- M. W. and T. Kubota, Phys. Rev. D60 (1999) 034020.
- M. W., Phys. Rev. D67 (2003) 034005.
- M. W. and Y. Nakakoji, Phys. Rev. D74 (2006) 054006.
- M. W. and Y. Nakakoji, Phys. Rev. D77 (2008) 074011.

We try to carry out the analysis of the nucleon spin contents **as model-independently as possible** !

[**Starting point**] most general and robust nucleon spin sum rule in QCD

$$J^Q + J^G = \frac{1}{2} \quad (Q \equiv u + d + s + \dots)$$

The point is that this **decomposition** can be made purely **experimentally** through the **GPD analyses** (X. Ji, 1997).

Ji's sum rule

$$J^Q = \frac{1}{2} \left[\langle x \rangle^Q + B_{20}^Q(0) \right], \quad (Q = u + d + \dots)$$

with

$$\langle x \rangle^Q = \int_0^1 x H^Q(x, \xi = 0, t = 0) dx \quad : \quad \text{quark momentum fraction}$$

$$B_{20}^Q(0) = \int_0^1 x E^Q(x, \xi = 0, t = 0) dx \quad : \quad \text{anomalous gravitomagnetic moment}$$

♣ Once J^Q is known, J^G is automatically known from $J^G = 1/2 - J^Q$

For flavor decomposition, we also need non-singlet combination

$$J^{(NS)} = \frac{1}{2} \left[\langle x \rangle^{(NS)} + B_{20}^{(NS)}(0) \right]$$

with

$$J^{(NS)} = J^{u-d}, \quad \text{or} \quad J^{u+d-2s}$$

1st key observation

♣ Ji showed that $\langle x \rangle^q$ and J^q obey exactly the **same evolution equation** !

At the leading order (LO)

$$2 J^Q(Q^2) = \frac{3 n_f}{16 + 3 n_f} + \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{2(16+3 n_f)/9 \beta_0} \left(2 J^Q(Q_0^2) - \frac{3 n_f}{16 + 3 n_f} \right)$$

with $\beta_0 = 11 - \frac{2}{3} n_f$ and **similarly** for $\langle x \rangle^Q$

$$2 J^{(NS)}(Q^2) = \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{32/9 \beta_0} 2 J^{(NS)}(Q_0^2)$$

and **similarly** for $\langle x \rangle^{(NS)}$

- ♣ The **next key observation** is that the quark and gluon **momentum fractions** are basically **known quantities** at least **above** $Q^2 \simeq (1 \sim 2) \text{ GeV}^2$, where the framework of **pQCD** can be trusted !
- ♣ **Neglecting** small contribution of **strange quarks**, which is not essential for our qualitative discussion below, we are then left with **two unknowns** :

$$B_{20}^{u+d}(0) \quad \text{or} \quad B_{20}^{u-d}(0)$$

- ♣ Fortunately, the available predictions of lattice QCD for **these quantities** corresponds to the **renormalization scale** $Q^2 = 4 \text{ GeV}^2$, which is high enough for the framework of **pQCD** to work.
- ♣ An **interesting idea** is then to use the **QCD evolution equation** to estimate the nucleon spin contents **at lower energy scales** of nonperturbative QCD.

inverse or downward evolution !

- ♣ A natural question is how far down to the low energy scale we can trust the framework of **perturbative renormalization group equations**.
- ♣ Leaving this fundamental question aside, one may continue the **downward evolution** up to the scale μ^2 , where the gluon momentum fraction becomes 0.

$$\langle x \rangle^Q = 1, \quad \langle x \rangle^g = 0 \quad : \text{unitarity-violating limit}$$

- ♣ By starting with the MRST2004 values, $\langle x \rangle^Q = 0.579$, $\langle x \rangle^g = 0.421$ at $Q^2 = 4 \text{ GeV}^2$, we find :

$$\mu^2 \simeq 0.070 \text{ GeV}^2 \quad \text{with LO evolution eq.}$$

$$\mu^2 \simeq 0.195 \text{ GeV}^2 \quad \text{with NLO evolution eq.}$$

- ♣ As advocated by Mulders and Pollock, these scales can be regarded as **matching scale** with **low energy effective quark models** without gluon degrees of freedom.
- ♣ Here, we take slightly more **conservative standpoint** that matching scale would be **somewhere** between μ^2 and 1 GeV^2 .
- ♣ At any rate, one can at the least say that **downward evolution below** this **unitarity-violating limit** μ^2 is absolutely **meaningless**, since $\langle x \rangle^g < 0$ there !

Now, we concentrate on getting reliable information on **two unknowns** :

$$B_{20}^{u+d}(0) \quad \text{and} \quad B_{20}^{u-d}(0)$$

(A) **Isvector part** $B_{20}^{u-d}(0)$

- lattice QCD results given at $Q^2 = 4 \text{ GeV}^2$

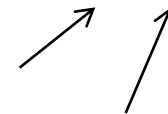
$$B_{20}^{u-d}(0) = 0.274 \pm 0.037$$

$$B_{20}^{u-d}(0) = 0.269 \pm 0.020$$

LHPC 2008

QCDSF-UKQCD 2007

close to each other !



- CQSM 2008 (M.W. and Y. Nakakoji)

$$B_{20}^{u-d}(0) \simeq 0.289$$

evolved from $Q^2 = 0.30 \text{ GeV}^2$ to 4.0 GeV^2 with NLO evolution eq.

To avoid starting energy dependence of CQSM estimate, here we simply use the central value of **LHPC 2008** :

$$B_{20}^{u-d}(0) = 0.274 \quad \text{at} \quad Q^2 = 4 \text{ GeV}^2$$

(B) Isoscalar part $B_{20}^{u+d}(0)$

Lattice QCD predictions are **sensitive** to the used method of χ PT and **dispersed** !

$$B_{20}^{u+d}(0) = -0.094 \pm 0.050 \quad : \quad \text{LHPC2008 with covariant baryon } \chi\text{PT}$$

$$B_{20}^{u+d}(0) = +0.050 \pm 0.049 \quad : \quad \text{LHPC2008 with heavy baryon } \chi\text{PT}$$

$$B_{20}^{u+d}(0) = -0.120 \pm 0.023 \quad : \quad \text{QCDSF-UKQCD2007}$$

From the analysis of forward limit of unpolarized GPD $E^{u+d}(x, \xi, t)$ within the CQSM, the 2nd moment of which gives $B_{20}^{u+d}(0)$, a reasonable theoretical bound for $B_{20}^{u+d}(0)$ is obtained (M.W. and Y. Nakakoji, 2008)

$$0 \geq B_{20}^{u+d}(0) \geq -0.12 \quad (= \kappa^{p+n})$$

This works to **exclude** some range of lattice QCD predictions !

In the following analysis, we therefore regard $B_{20}^{u+d}(0)$ as an **uncertain constant within the above theoretical bound**.

Once $J^q (q = u, d, s)$ is known, the (mechanical) quark OAM L^q is easily obtained by subtracting the known longitudinal quark polarization :

$$L^q(Q^2) = J^q(Q^2) - \frac{1}{2}\Delta\Sigma^q$$

$\Delta\Sigma^q (q = u, d, s) \quad : \quad \text{scale indep. at L0}$
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For $\Delta\Sigma^{u+d+s}$, we simply use here the central value of HERMES analysis :

$$\Delta\Sigma^Q = 0.33$$

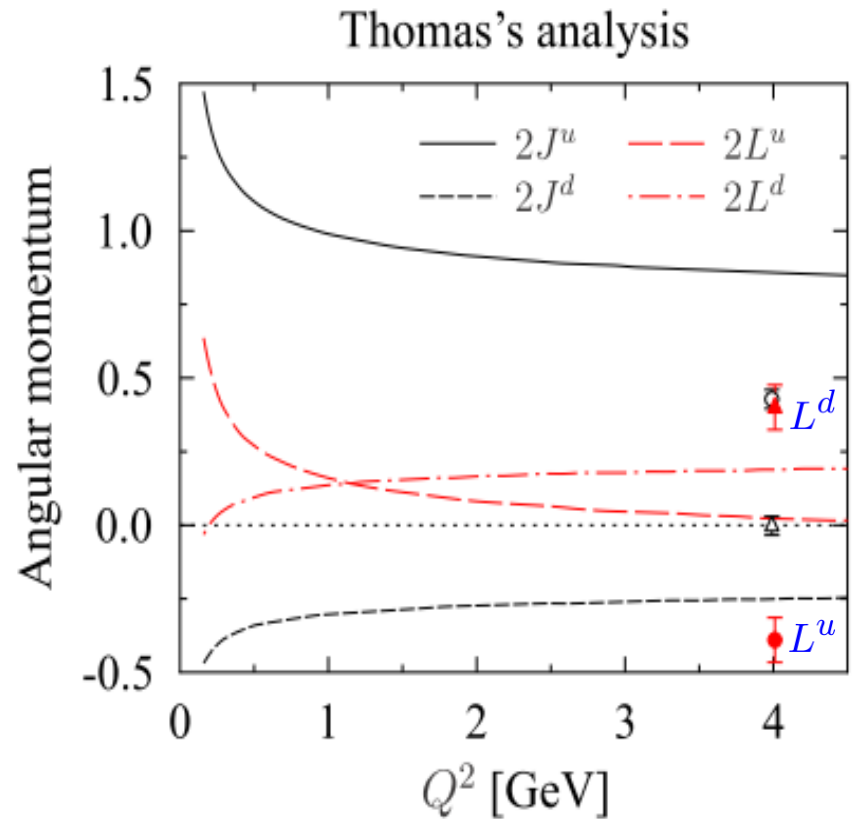
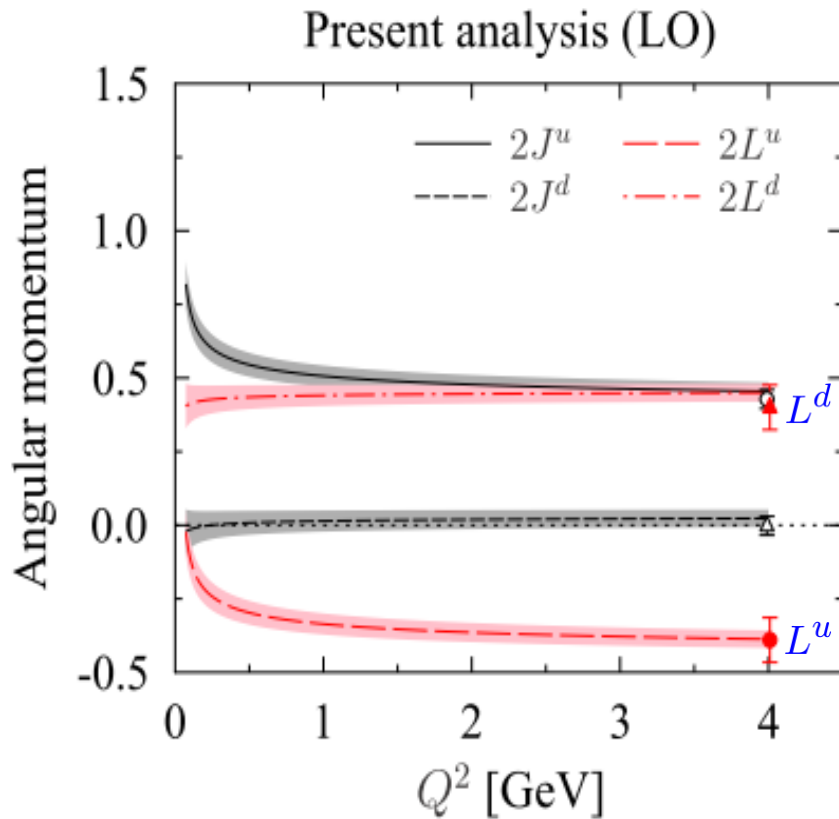
Flavor non-singlet combinations : fairly precisely known

$$\begin{aligned} \Delta\Sigma^{u-d} &\equiv g_A^{(3)} = 1.269 \pm 0.003 \\ \Delta\Sigma^{u+d-2s} &\equiv g_A^{(8)} = 0.586 \pm 0.031 \end{aligned}$$

summary of complete initial conditions at $Q^2 = 4 \text{ GeV}^2$

$\langle x \rangle^Q = 0.579, \quad \langle x \rangle^{u-d} = 0.158, \quad \langle x \rangle^s = 0.041$ $B_{20}^{u-d}(0) = 0.274, \quad -0.12 \leq B_{20}^Q(0) = B_{20}^{u+d-2s}(0) \leq 0$ $\Delta\Sigma^Q = 0.33, \quad \Delta\Sigma^{u-d} = 1.27, \quad \Delta\Sigma^{u+d-2s} = 0.586$

J^u, J^d , and L^u, L^d as functions of Q^2



Data at $Q^2 = 4 \text{ GeV}^2$ are from [LHPC2008](#).

- Ph. Haegler et al. (LHPC Collaboration), Phys. Rev. D77 (2008) 094502.

One sees that the **difference** between the results of the two analyses is **quite large**, which also means that the **agreement** between Thomas' results and the lattice QCD predictions is **not so good** as he claimed .

♣ The most significant difference appears in the **quark OAM** !

♣ Thomas' analysis shows a **crossover** of L^u and L^d around 0.5 GeV scale.

It originates from the fact that

- refined CB model predicts $L^u - L^d > 0$ at low energy model scale.
- QCD evolution dictates that $\lim_{Q^2 \rightarrow \infty} (L^u - L^d) < 0$.

♣ In contrast, **no crossover** of L^u and L^d is observed in our analysis.

L^d remains to be larger than L^u even down to the **unitarity-violating limit**.

- ♣ One might suspect that the **uncertainties of the initial conditions** given at $Q^2 = 4 \text{ GeV}^2$ might alter this remarkable conclusion.

It is clear by now, however, that the problem exists for the **isovector channel**, for which the **uncertainties are fairly small**. In fact, in the r.h.s. of the relation

$$L^{u-d} = \frac{1}{2} \left[\langle x \rangle^{u-d} + B_{20}^{u-d}(0) \right] - \frac{1}{2} \Delta \Sigma^{u-d}$$

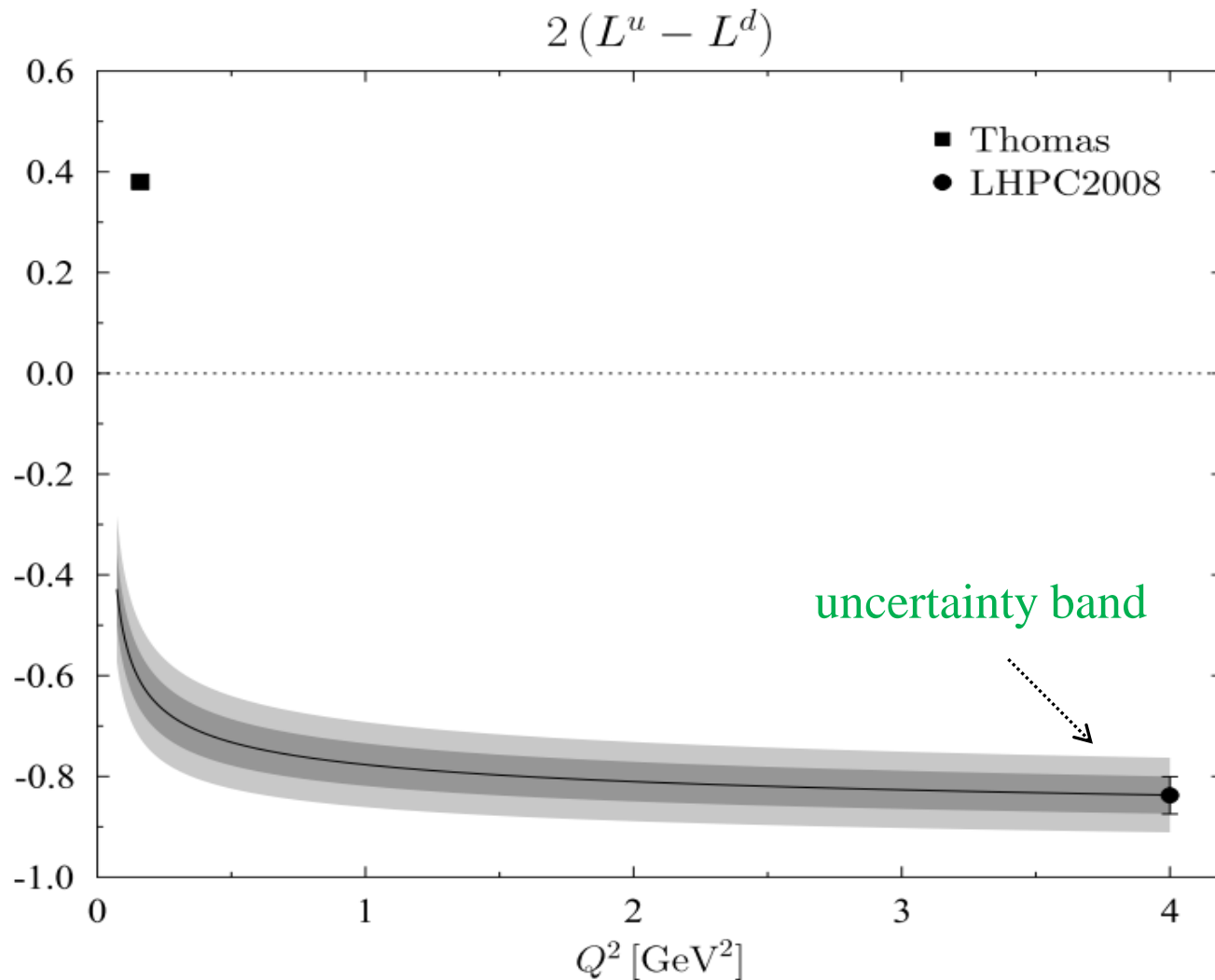
uncertainties

$$\begin{aligned} \Delta \Sigma^{u-d} &= 1.2695 \pm 0.0029 && (\sim 0.29\%) \\ \langle x \rangle^{u-d} &= 0.158 && (< 1\%) \end{aligned}$$

Main uncertainty comes from **the isovector anomalous gravito-magnetic moment !**

$$\begin{aligned} B_{20}^{u-d}(0) &= 0.274 \pm 0.037 \quad (13.5\%) && : \text{ LHPC2008} \\ &= 0.269 \pm 0.020 \quad (7.4\%) && : \text{ QCDSF-UKQCD2009} \\ &= 0.289 && : \text{ CQSM} \end{aligned}$$

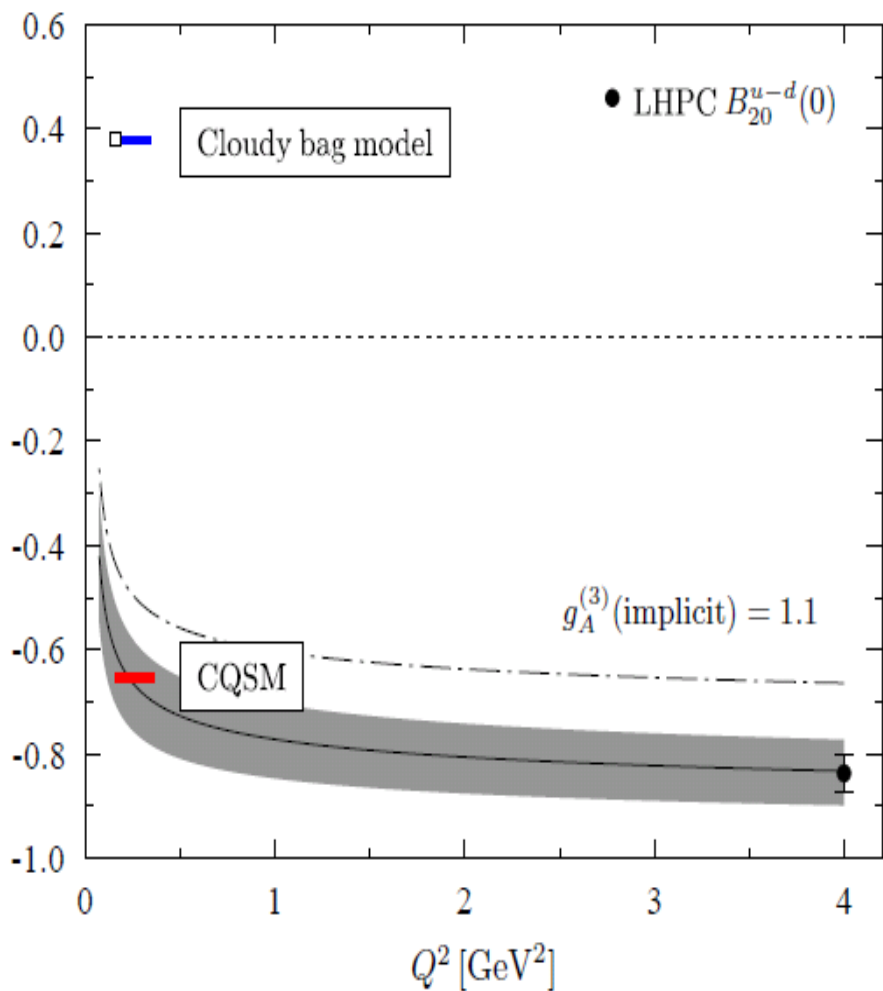
uncertainty estimate



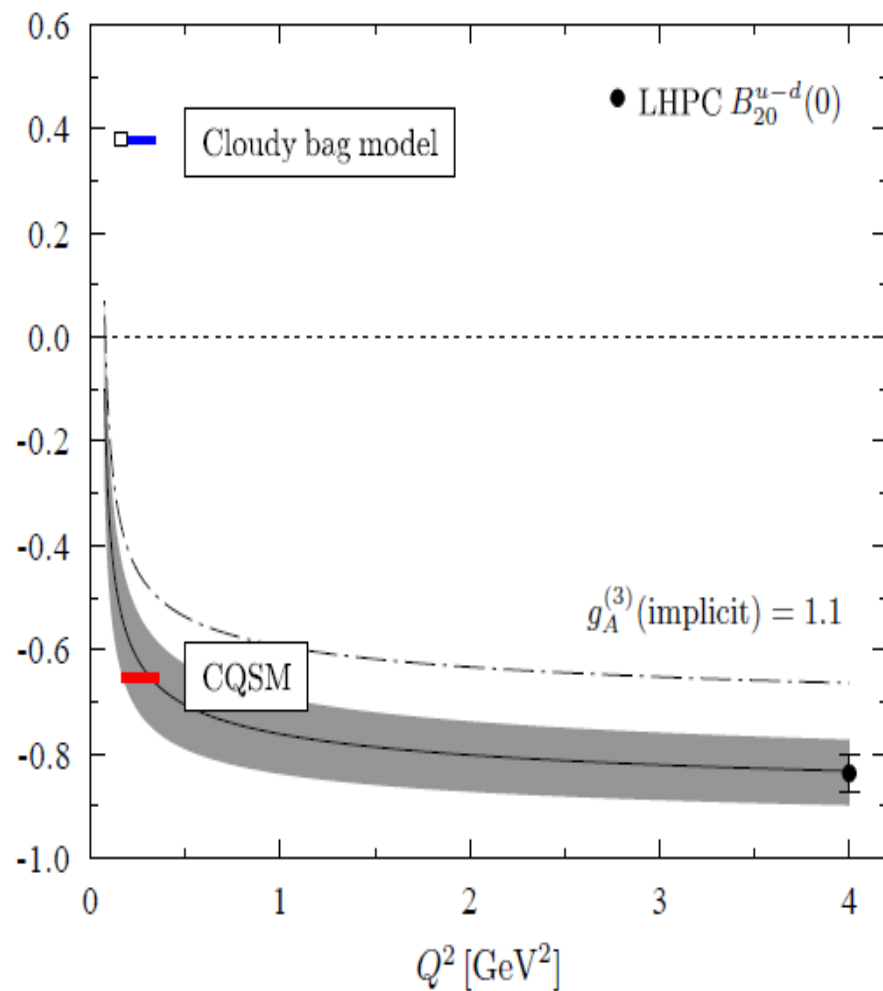
$L^u - L^d$ remains **negative** even down to the lower energy scale close to the unitarity-violating bound !

sensitivity to the magnitude of QCD coupling constant

(a) $2(L^u - L^d)$ at LO with $\alpha_S(Q^2 = 4 \text{ GeV}^2) = 0.284$



(b) $2(L^u - L^d)$ at LO with $\alpha_S(Q^2 = 4 \text{ GeV}^2) = 0.320$



Also interesting would be a **direct comparison** with the **empirical information** on J^u and J^d extracted from the recent **GPD analyses**.

See figure in the next page.

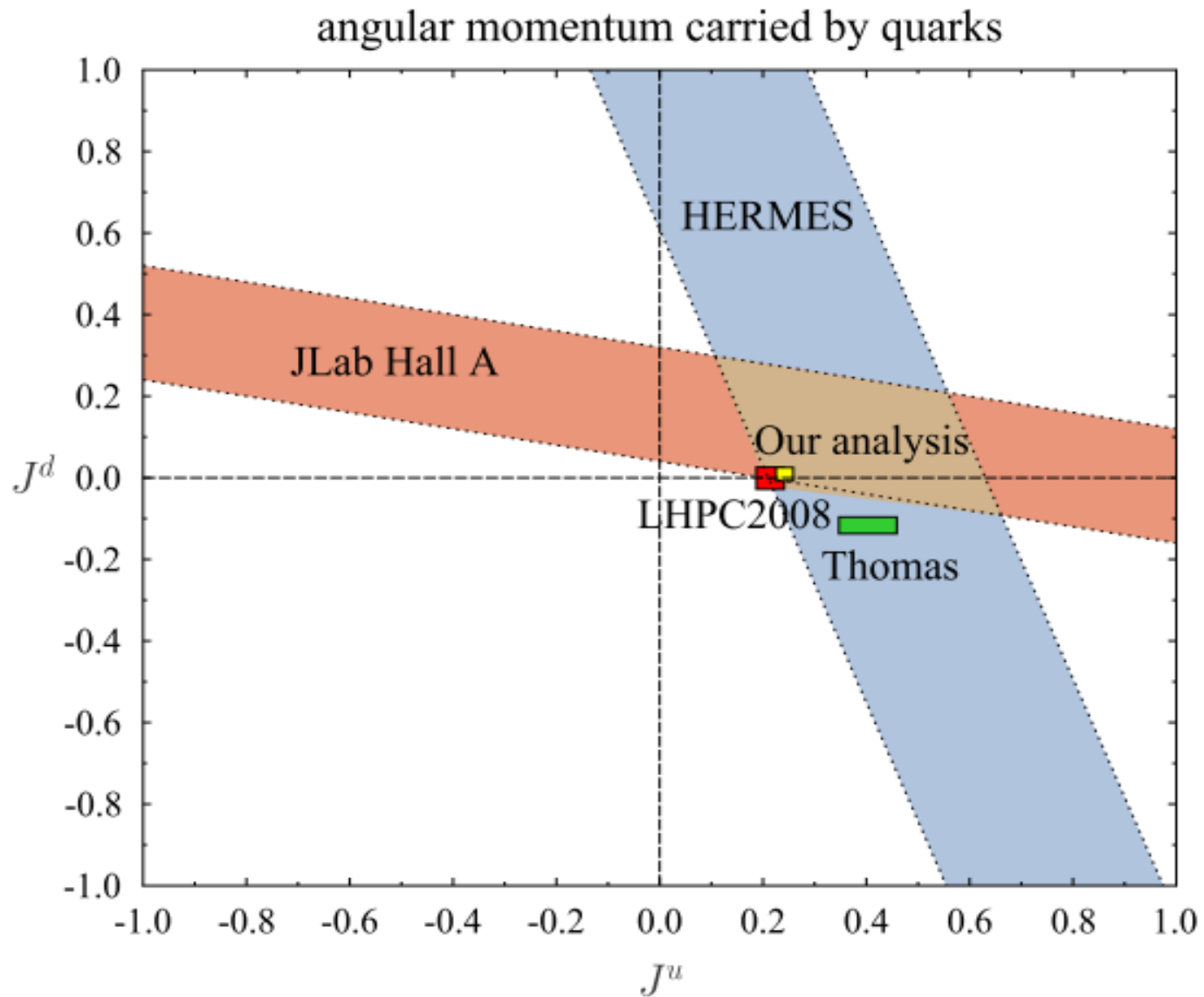
One sees that, by construction, the result of our semi-phenomenological analysis is fairly close to that of the lattice QCD simulations.

On the other hand, the result of Thomas' analysis significantly deviates from the other two, and outside the **error-band of JLab data**.

Typical features of Thomas' predictions

- J^u is **sizably larger** than the other two.
- J^d is a little smaller than the other two.

Comparison with GPD extraction of J^u , J^d



Anyhow, our semi-phenomenological analysis, which is consistent with empirical information as well as the lattice QCD data at high energy scale **indicates** that

$L^u - L^d$ remains **largely negative** !

even at low energy scale of nonperturbative QCD !

If this is really confirmed, it is a serious challenge to any **low energy models of nucleon**, because they must now explain

- **small** $\Delta\Sigma^Q$
- **largely negative** L^{u-d}

at the same time !

We might call the 2nd observation

another nucleon spin puzzle

because it is totally **incompatible with** the picture of the **standard quark model**, including the MIT bag model and the refined CB model of Thomas and Myhrer.

Is there **any low energy model** which can explain this **extraordinary feature** ?

Very curiously , the **CQSM** can !

We have been claiming long that the CQSM explain small quark spin fraction without any fine-tuning.

$$\Delta\Sigma^Q \simeq 0.35 \quad \text{at the model scale}$$

because of the **very nature of the model** (i.e. the nucleon as a **rotating hedgehog**)

Very interestingly, its prediction for $L^{u-d} \equiv L^u - L^d$ given in the paper

- M. W. and H. Tsujimoto, Phys. Rev. D71 (2005) 074001.

$$L^{u-d} \simeq -0.33 \quad \text{at the model scale}$$

perfectly matches the **scenario** emerged from the present **semi-empirical analysis** !

But why ?

The problem may have **deep connection** with the **definition of quark OAM** !

We have already argued that there are **2 kinds of quark OAMs**.

(1) **“mechanical”** quark OAM = Ji’s quark OAM

(2) **“canonical”** quark OAM = Jaffe-Manohar’s quark OAM

Remember the fact that the quark OAM defined through **GPDs** is the 1st one, i.e. the **“mechanical”** quark OAM .

We also recall that the **quark OAM** in the **Ji decomposition** contains **covariant-derivative**, so that it contains some **interaction effects with the gluon field**.

Since the **CQSM** is an **effective quark theory** that contains **no gauge field**, one might naively expect that there is no ambiguity in the definition of the quark OAM.

However, it turns out that this is not necessarily the case. The reason is probably that it is a **highly nontrivial interaction theory** of **quark fields**.

To explain it, we recall the past analyses of **GPD sum rules** within the **CQSM**.

CQSM analyses of GPD sum rules :

- **Isoscalar channel** : J. Ossmann et al., Phys. Rev. D71,034001 (2005).
- **Isovector channel** : M. W. and H. Tsujimoto, Phys. Rev. D71,074001 (2005).

Isoscalar case : 2nd moment of $E_M^{u+d}(x, 0, 0) \equiv H^{u+d}(x, 0, 0) + E^{u+d}(x, 0, 0)$

$$\frac{1}{2} \int_{-1}^1 x E_M^{u+d}(x, 0, 0) dx = L_f^{u+d} + \frac{1}{2} \Delta \Sigma^{u+d}$$

where

$$L_f^{u+d} \equiv \langle p \uparrow | \hat{L}_f^{u+d} | p \uparrow \rangle, \quad \Delta \Sigma^{u+d} \equiv \langle p \uparrow | \Delta \hat{\Sigma}^{u+d} | p \uparrow \rangle$$

with

$$\hat{L}_f^{u+d} = \int \psi^\dagger(x) [\mathbf{x} \times (-i \nabla)]_3 \psi(x) d^3x,$$

: free-field quark OAM operator

= canonical OAM ?

$$\Delta \hat{\Sigma}^{u+d} = \int \psi^\dagger(x) \Sigma_3 \psi(x) d^3x$$

Isvector case : 2nd moment of $E_M^{u-d}(x, 0, 0) \equiv H^{u-d}(x, 0, 0) + E^{u-d}(x, 0, 0)$

$$\int_0^1 x E_M^{u-d}(x, 0, 0) dx = \frac{1}{2} \Delta \Sigma^{(I=1)} + \left(L_f^{(I=1)} + \delta L^{(I=1)} \right)$$

with

$$\Delta \Sigma^{(I=1)} = \langle p \uparrow | \int d^3x \psi^\dagger(\mathbf{x}) \tau_3 \Sigma_3 \psi(\mathbf{x}) | p \uparrow \rangle$$

$$L_f^{(I=1)} = \langle p \uparrow | \int d^3x \psi^\dagger(\mathbf{x}) \tau_3 (\mathbf{x} \times \mathbf{p})_3 \psi(\mathbf{x}) | p \uparrow \rangle$$

$$\delta L^{(I=1)} = -M \frac{N_c}{18} \sum_{n \in occ} \langle n | r \sin F(r) \gamma^0 [\Sigma \cdot \hat{\mathbf{r}} \boldsymbol{\tau} \cdot \hat{\mathbf{r}} - \Sigma \cdot \boldsymbol{\tau}] | n \rangle$$

strange spin-isospin correlation

	valence	sea	total
$L_f^{(I=1)}$	0.147	- 0.265	- 0.118
$\delta L^{(I=1)}$	- 0.289	0.077	- 0.212
$L_f^{(I=1)} + \delta L^{(I=1)}$	- 0.142	- 0.188	- 0.33

Short Summary on $L^u - L^d$ anomaly

- ♣ We have estimated the orbital angular momentum of up and down quarks in the proton as functions of the energy scale, by carrying out a downward evolution of available information at high energy, to find that $L^u - L^d$ remains to be largely negative even at low energy scale of nonperturbative QCD ! We emphasized that, if it is really confirmed in near-future experiments, it may be called

another nucleon spin puzzle ?

because it absolutely contradicts the prediction of standard quark models !



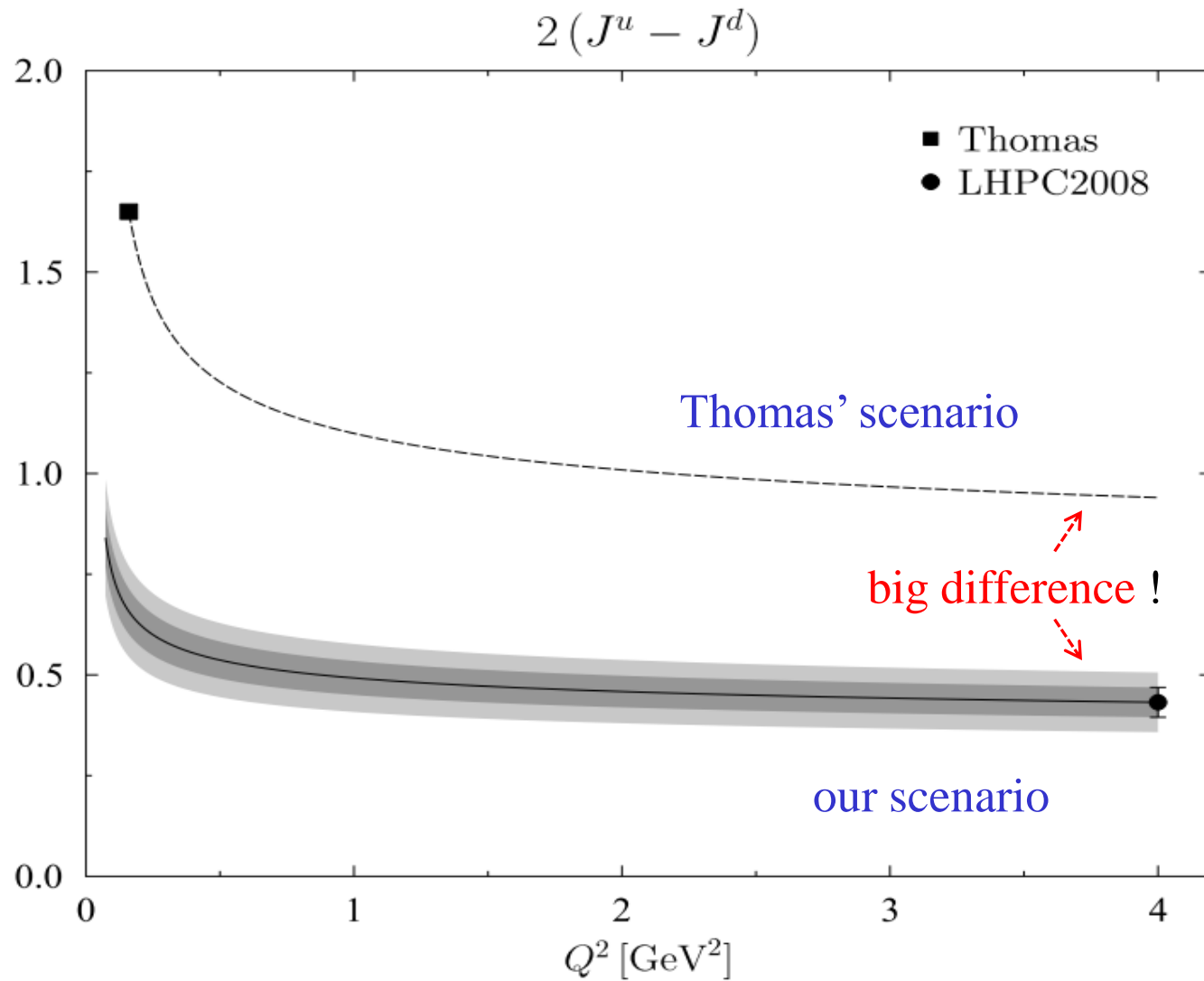
- ♣ Does the strong scale dependence of $L^u - L^d$ rescue this puzzle, as Thomas claims ?

or

- ♣ Is it an indication of a big difference between

“mechanical” quark OAM & “canonical” one ?

- ♣ A key is a precise measurement of $2(J^u - J^d)$ at a few GeV scale.



Recent progress of Lattice QCD studies

Main conclusions of the LHPC and QCDSF-UKQCD Lattice QCD collaboration carried out several years ago can be summarized as follows :

LHPC and QCDSF-UKQCD Lattice QCD collaborations (2007-2010)

flavor sum	LHPC	QCDSF-UKQCD	CQSM ($Q^2 = 4 \text{ GeV}^2$)
$2J$	0.43	0.45	0.68
$\Delta\Sigma$	0.41	0.40	0.32
$2L$	0.01	0.05	0.36

$$2L^u \simeq -0.2, \quad 2L^d \simeq +0.2, \quad 2L^{u+d} \simeq 0 !!$$

Net OAM carried by **quarks** in the nucleon is **very small** !

However, these simulations include only **Connected Insertion (diagram) only** !.

Importance of **Disconnected Insertion** (DI)


- χ QCD Collaboration : M. Deka et al., Phys. Rev. D91 (2015) 014505.

flavor sum	CI (u+d)	DI (u+d+s)	sum		CQSM ($Q^2 = 4 \text{ GeV}^2$)
$2J$	0.63	0.09	0.72		0.68
$\Delta\Sigma$	0.62	- 0.36	0.25		0.32
$2L$	0.01	0.46	0.47	\longleftrightarrow	0.36

Lattice QCD predictions with **DI** correction became **closer to** those of **CQSM** !

- Lattice QCD predictions seems still **time-dependent** ? -

More complete Lattice QCD simulation must also pay attention to

- Finite volume effects
 - Extrapolation to physical pion mass
 - **non-quenched (full QCD) simulation**
 - ...
-  **next discontinuous change ?**

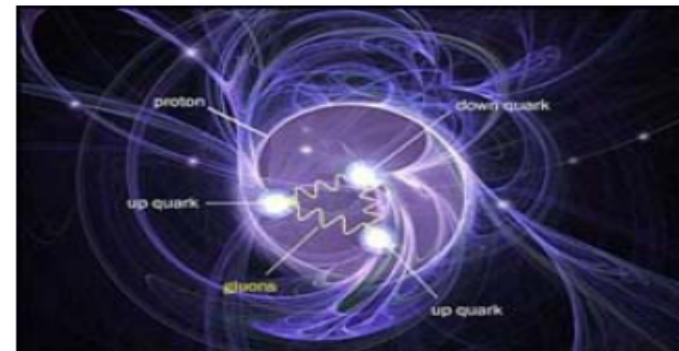
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Proton Spin Mystery Gains a New Clue

Physicists long assumed a proton's spin came from its three constituent quarks. New measurements suggest particles called gluons make a significant contribution

July 21, 2014 | By [Clara Moskowitz](#)

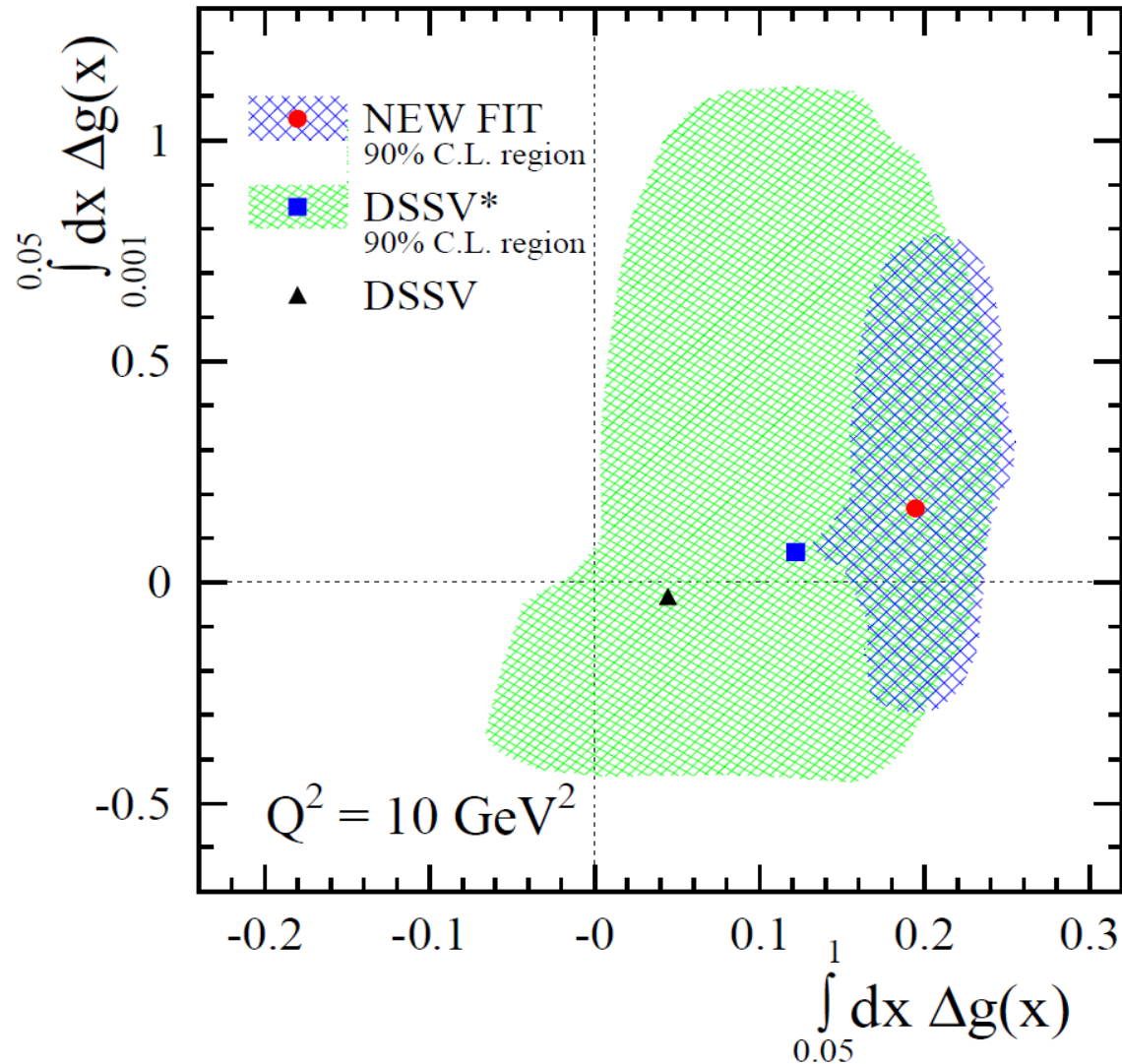
Protons have a constant spin that is an intrinsic particle property like mass or charge. Yet where this spin comes from is such a mystery it's dubbed the "proton spin crisis." Initially physicists thought a proton's spin was the sum of the spins of its three constituent quarks. But a 1987 experiment showed that quarks can account for only a small portion of a proton's spin, raising the question of where the rest arises. The quarks inside a proton are held together by **gluons** so scientists suggested perhaps they contribute spin. That idea now has support from a pair of studies analyzing the results of proton collisions inside the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory in Upton, N.Y.



Brookhaven National Laboratory

The recent global analysis of gluon polarization

- DSSV Collaboration, Phys. Rev. Lett. 113, 012001 (2014).



DSSV results at $Q^2 = 10 \text{ GeV}^2$ can be summarized as

$$\int_{0.05}^1 \Delta g(x) dx = 0.194 \pm_{-0.060}^{+0.060}, \quad \int_{0.001}^{0.05} \Delta g(x) dx = 0.166 \pm_{-0.46}^{+0.62}$$

↓

$$\int_{0.001}^1 \Delta g(x) dx = 0.361 \pm_{-0.522}^{+0.683} \quad \text{likely to be positive ?}$$

[Caution !] scale dependent nature of gluon polarization

Once, we have confirmed that the analyses of the longitudinally polarized PDFs based on the CQSM appears to be consistent with the assumption :

$$\Delta g(x) \simeq 0 \quad \text{at} \quad Q_{ini}^2 = 0.30 \text{ GeV}^2$$

although

$$\langle x \rangle_G \simeq 0.2 \quad \text{at} \quad Q_{ini}^2 = 0.30 \text{ GeV}^2$$

[Evolution]

$Q^2 [\text{GeV}^2]$	0.30	1.0	4.0	10.0
$\Delta G(Q^2)$	0.0	0.21	0.40	0.51

From more rigorous viewpoint, the **validity of using evolution equation** might be justified only **at much higher energy scales**.

One may start with the MRST fit at $Q^2 = 4 \text{ GeV}^2$, which is safe enough scale :

$$\langle x \rangle_Q = 0.579, \quad \langle x \rangle_G = 0.421$$

Since lattice QCD indicates that $B_{20}^Q = -B_{20}^G$ is **small**, let us assume

$$\begin{aligned} 2 J_Q &= \langle x \rangle_Q + B_{20}^Q \simeq \langle x \rangle_Q \\ 2 J_G &= \langle x \rangle_G + B_{20}^G \simeq \langle x \rangle_G \end{aligned}$$

Other information

$$\Delta\Sigma \simeq 0.3 \quad : \quad \text{HERMES, COMPASS (nearly scale-indep.)}$$

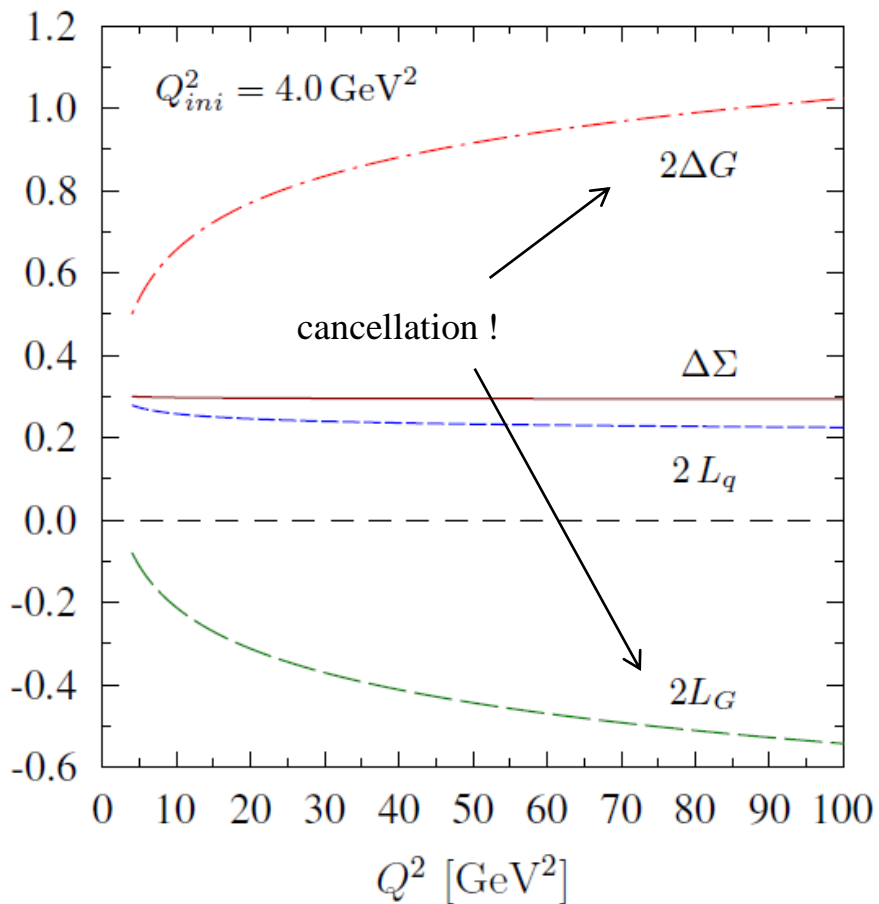
$$\Delta G(Q^2 = 4 \text{ GeV}^2) = 0.25 \quad : \quad \text{trial choice}$$

Solve **NLO evolution equations** for $(\Delta\Sigma, \Delta G)$ & (J_Q, J_G)

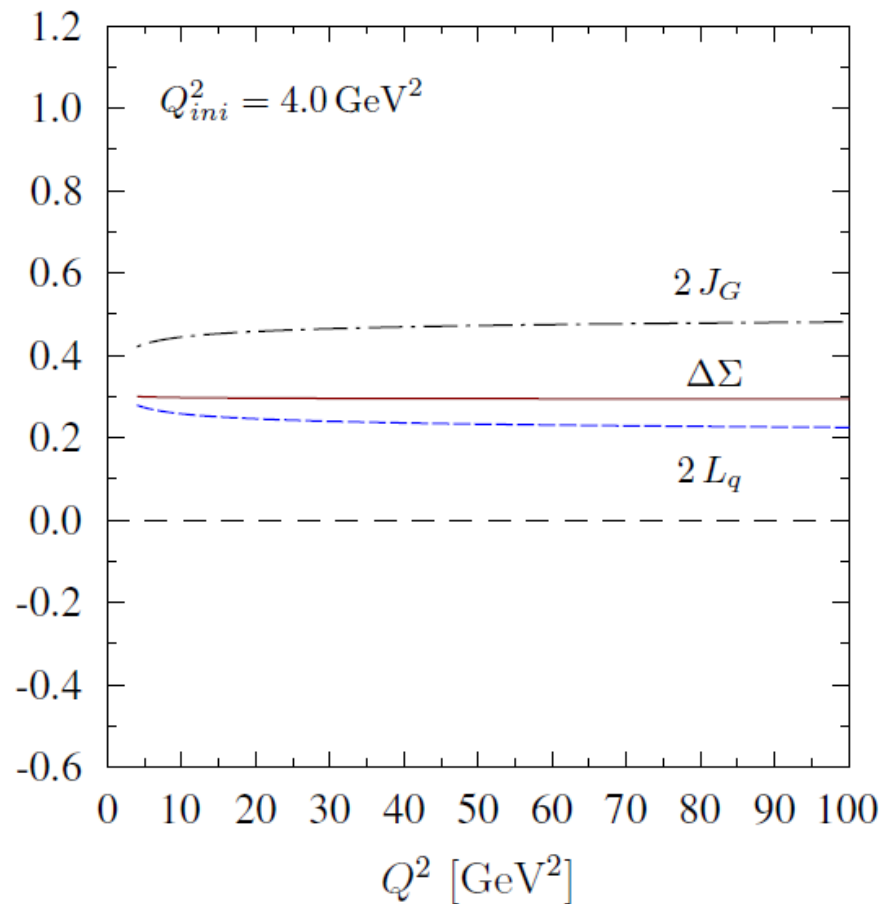


$$2L_Q(Q^2) = 2J_Q(Q^2) - \Delta\Sigma(Q^2) \quad \& \quad 2L_G(Q^2) = 2J_G(Q^2) - 2\Delta G(Q^2)$$

scale dependence of nucleon spin contents



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Decomposing J_G into ΔG and L_G is a highly **scale-dependent** operation ?

Short Summary on the gluon contribution to the nucleon spin

It is always very difficult to cleanly see gluon-related quantities, since gluon is **flavor-blind** and there is **no electroweak probes directly coupled to gluons**.

The extraction of the **gluon distributions**, including the unpolarized one and the longitudinally polarized one, is therefore **more or less indirect**.

Still, the recent DSSV global analysis has succeeded to give a useful constraint on the net longitudinal polarization of gluons. Although with large uncertainties, it indicates sizable **positive contribution of gluon spin** to the **net nucleon spin**.

However, one must be careful about the fact that the **decomposition of the total gluon angular momentum** into its **spin** and **orbital parts** is a strongly **scale-dependent operation**.

$$J^G \Rightarrow \Delta G + L^G \quad : \quad \text{scale-dependent decomposition !}$$