Perturbative QCD and Hard Processes

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Summary of lecture two

- QCD factorization has been extremely successful in predicting and interpreting high energy scattering data with the momentum transfer > 2 GeV

- PQCD factorization approach is mature, NLO calculations are available for most observables, NNLO are becoming available for the search of new physics

- Direct photon data are still puzzling and chalenging

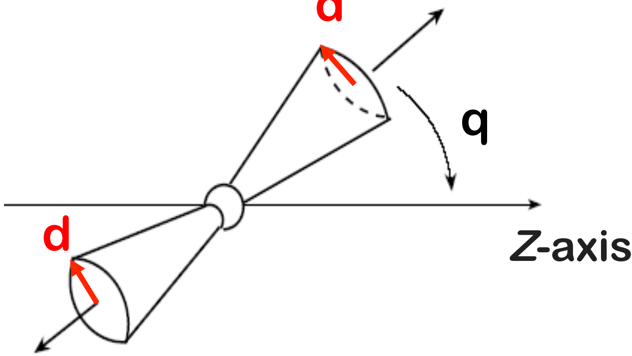
- NLO PDFs are very stable now, and NNLO PDFs are becoming available

- New ideas: Lattice QCD calculation of partonic structure of hadrons

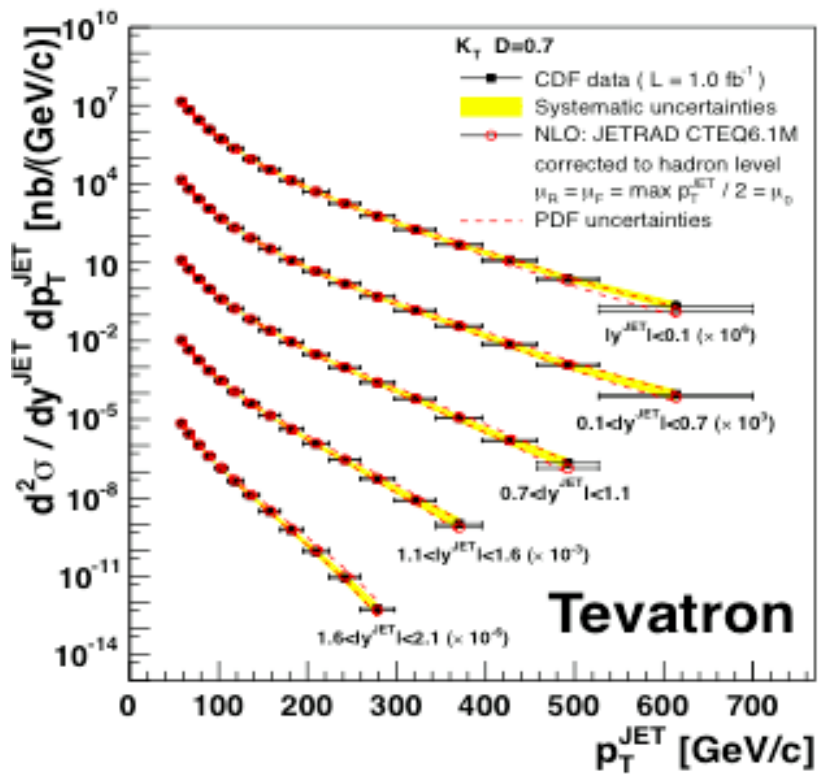
QCD hard processes with multiple scales,
hadron structure beyond PDFs, quantum correlation between hadron spin and its confined parton motions, … ?

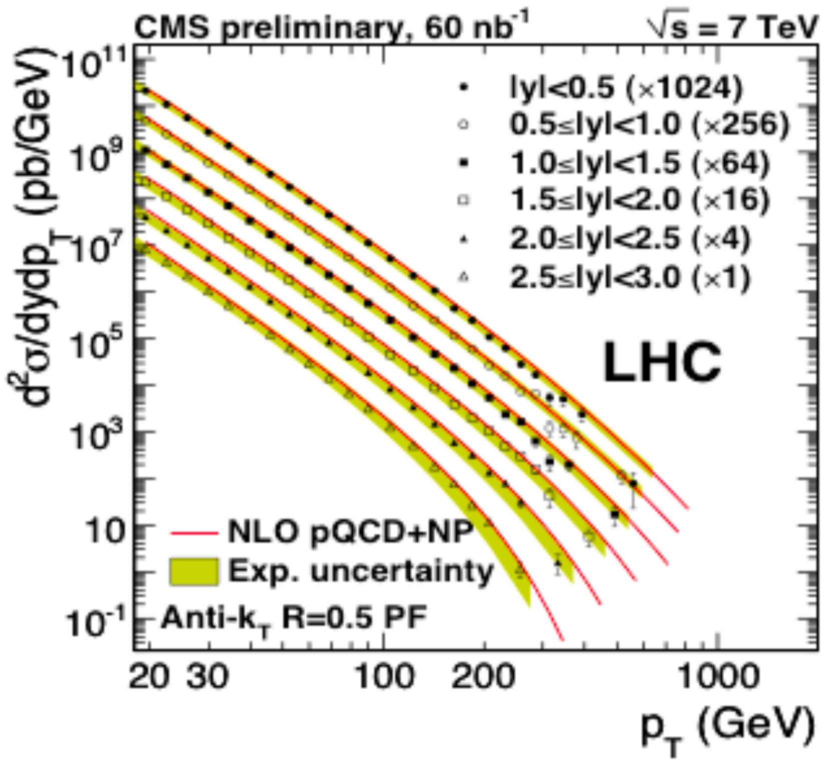
Jets

- ❑ **Definition:**
  - ✧ Inclusive cross section with limited phase space
  - ✧ "footprints" or "trace" of quarks and gluons



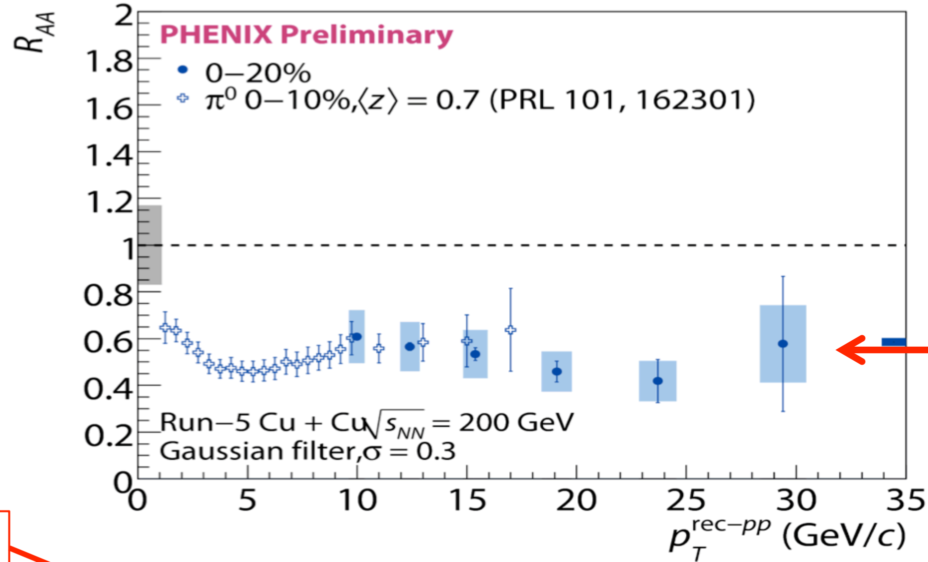
Sterman & Weinberg, PRL 1977

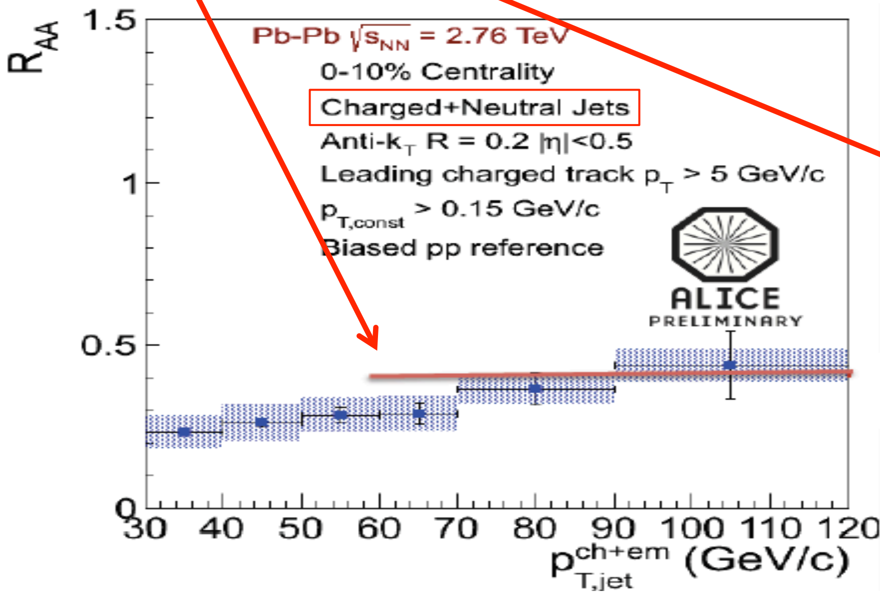


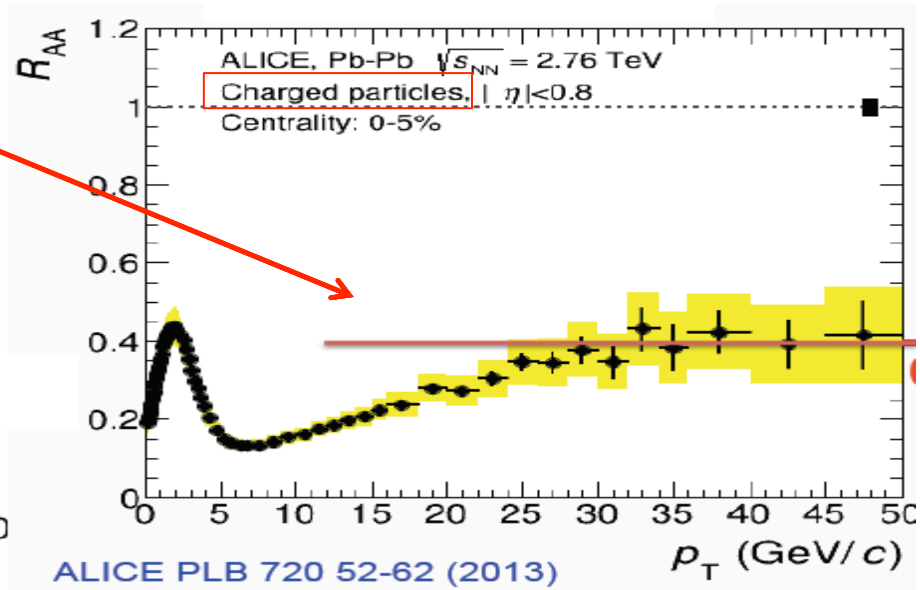


Suppression of jets – Jet quenching

- Jets vs. leading hadron:

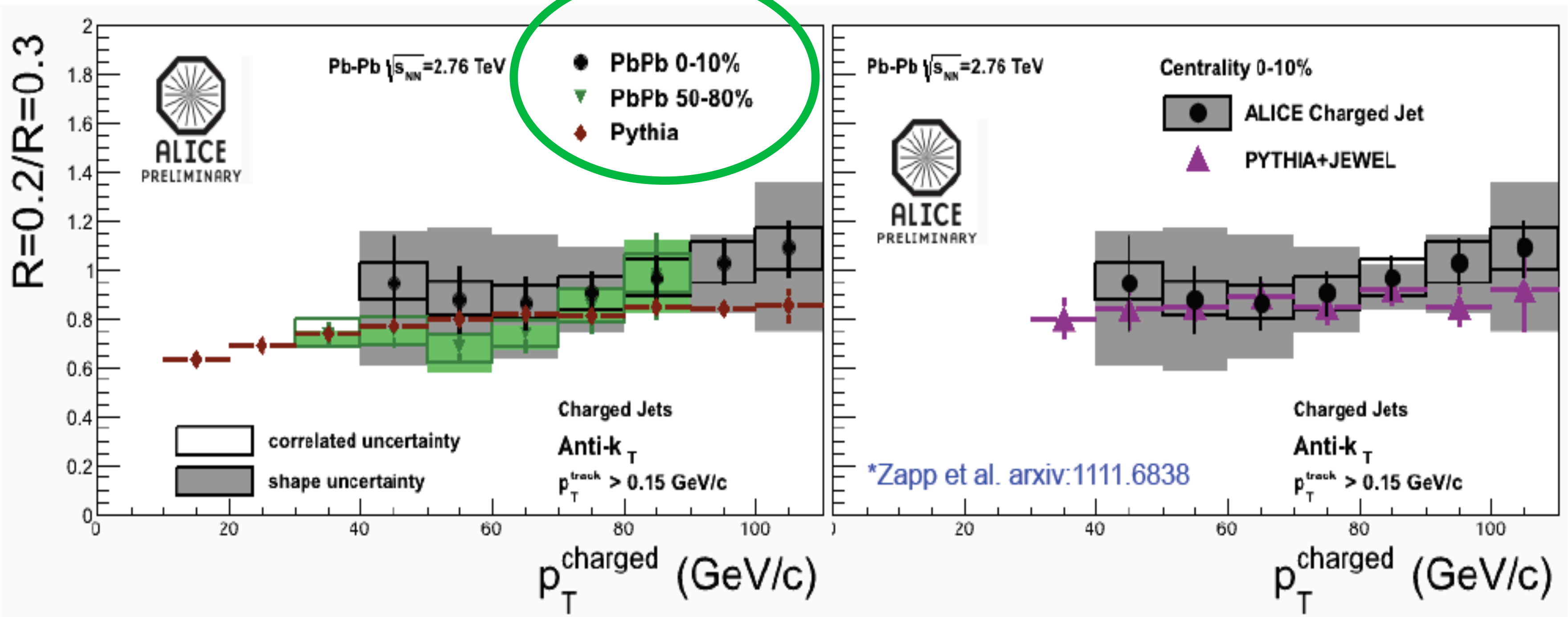






Role of Jet's cone size

❑ Cone size dependence of Jet quenching:



Ratio is consistent with vacuum jets for peripheral and central collisions?

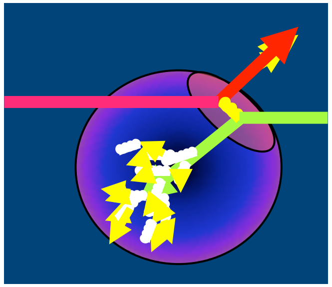
Multiple scattering ➔ radiation ➔ energy loss ➔ cone size ➔ …

Where does the lost energy go?

❑ **Medium induced radiation:**

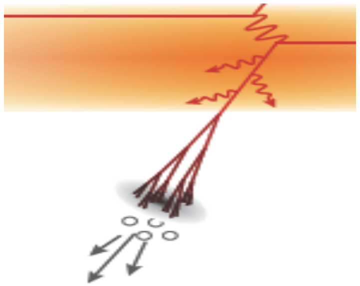
✧ Small angle in/near cone

No suppression if the cone is bigger enough!



✧ Thermalize with the medium:

Radiation is gone!



✧ Broaden the jet

Jet cone dependence!

❑ **Where does the lost energy go?**

We do not know, since we did not keep track of every particles

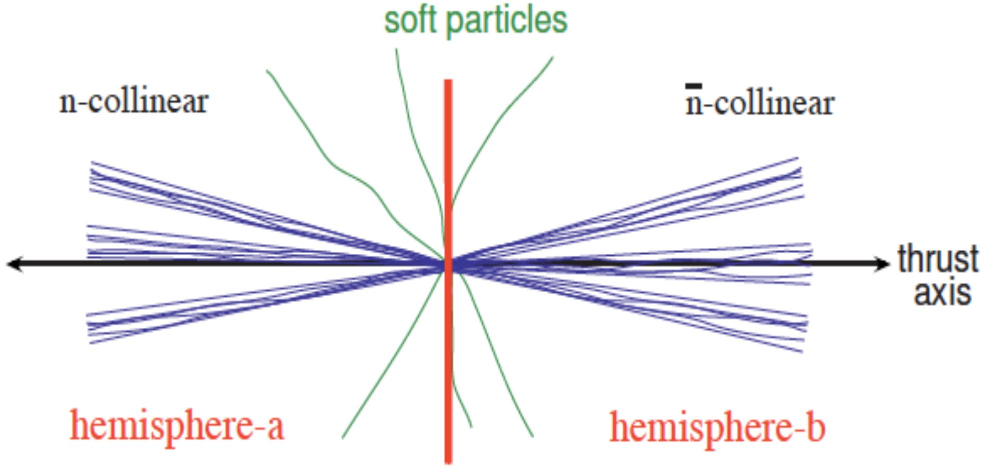
❑ **What if we do keep track of every particles?**

We should know the full event shape!

Event shapes

- ❑ **Event shapes are theoretically cleaner (more inclusive!):**
- ❑ **Thrust, as an example:**

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$



- ✧ **Two jet configurations obtained in the limit:**

$$T \to 1$$

- ✧ **Resummation of logarithms of (1-T), corresponds to a resummation of the jet veto logs**
- ✧ **Structure of resummation is simpler, *no jet algorithm dependence* (jet algorithm dependence begins at NNLO with two emissions)**

N-Jettiness

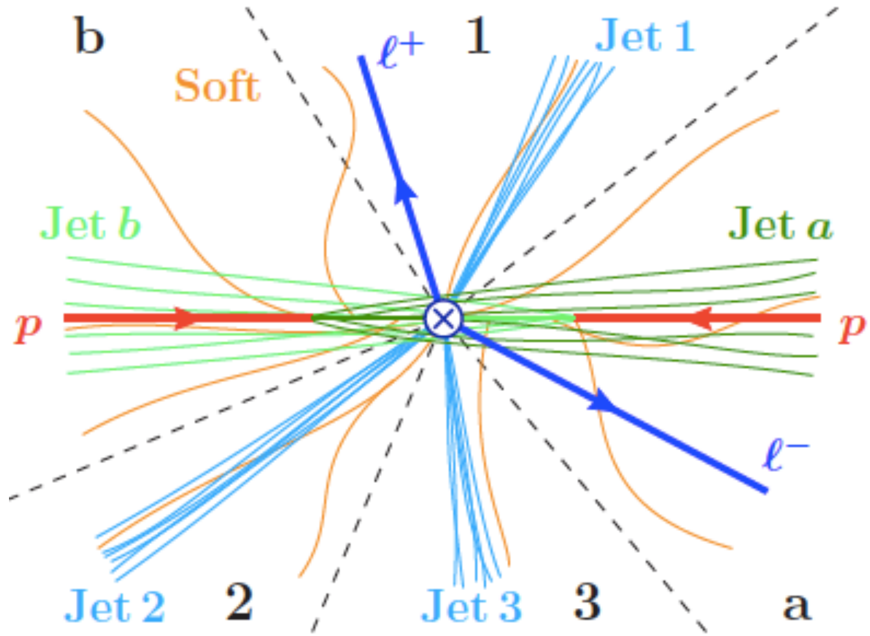
- Event structure:

  $pp \to$ leptons plus jets

- N-Jettiness:

  (Stewart, Tackmann, Waalewijin, 2010)

$$\tau_N^i = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$



The sum include all final-state hadrons *excluding* more than N jets

Allows for an event-shape based analysis of multi-jets events (a generalization of Thrust), and is complementary to jets

- N-infinitely narrow jets – isolated single hadron(s) (jet veto):

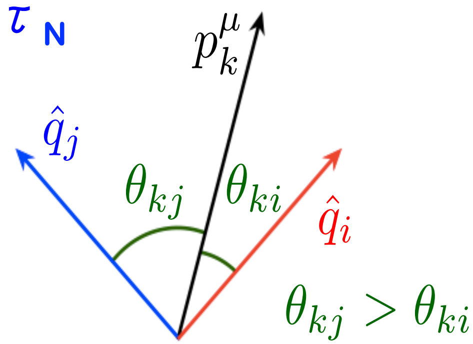
  As a limit of N-Jettiness: $\tau_N \to 0$

N-Jettiness – implementation

- **Steps for implementation:**
  - Use a standard jet algorithms to find N-jets
  - Initial reference vectors = momenta of the N-jets + hadron beam directions

    (reference vectors are the only information used from the jet algorithm)
  - Calculate value for the N-jettiness global event shape: τ_N

    (new reference directions from the minimization)
  - Select events with N narrow well-separated jets and impose veto on additional jets



- **New "jet" momenta = sum of momenta in jet regions**

$$P_i^\mu = \sum_k p_k^\mu \prod_{j \neq i} \theta(\hat{q}_j \cdot p_k - \hat{q}_i \cdot p_k)$$

- **N-jettiness momentum = sum of jettiness from each region:**

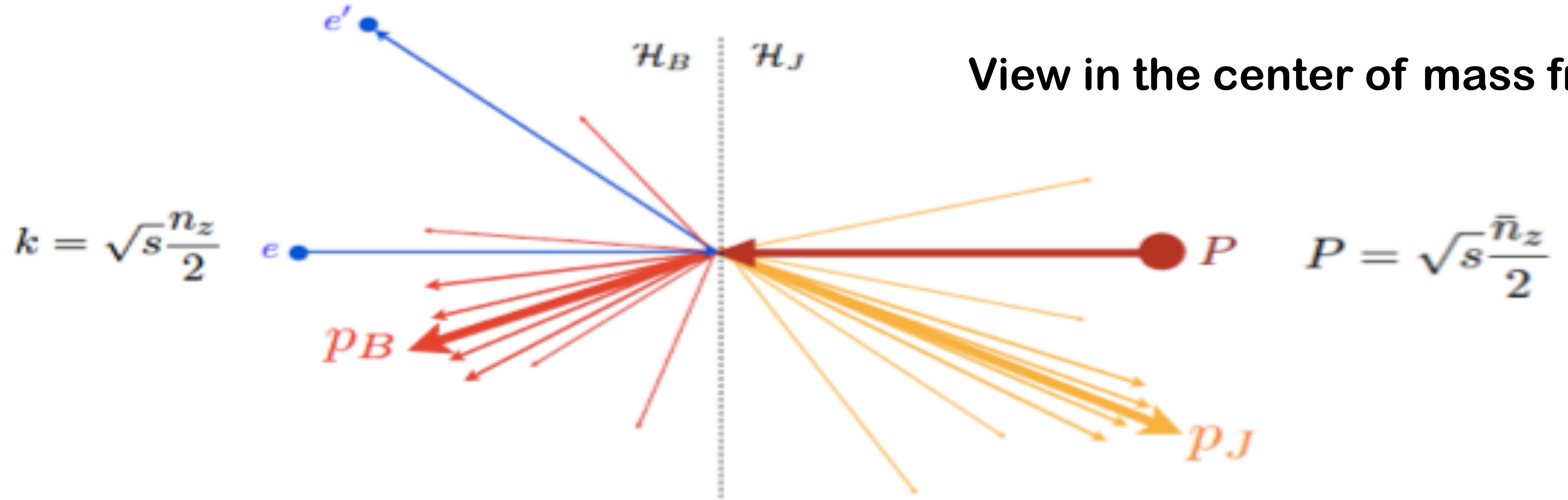
$$\mathcal{T}_N = \sum_i \mathcal{T}_N^i \equiv \sum_i 2\hat{q}_i \cdot P_i$$

- **Dependence on Jet algorithms is power suppressed**

1-Jettiness cross section in DIS

Kang, Mantry, Qiu, 2012

View in the center of mass frame



$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Very much "like" the calculation for the "Thrust"

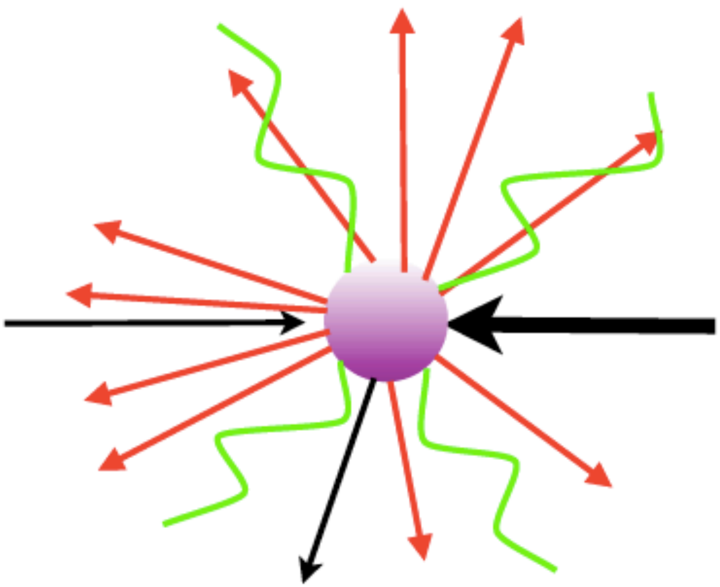
(Minimization vs maximization!)

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{du\, dP_{J_T}\, d\tau_1}$$

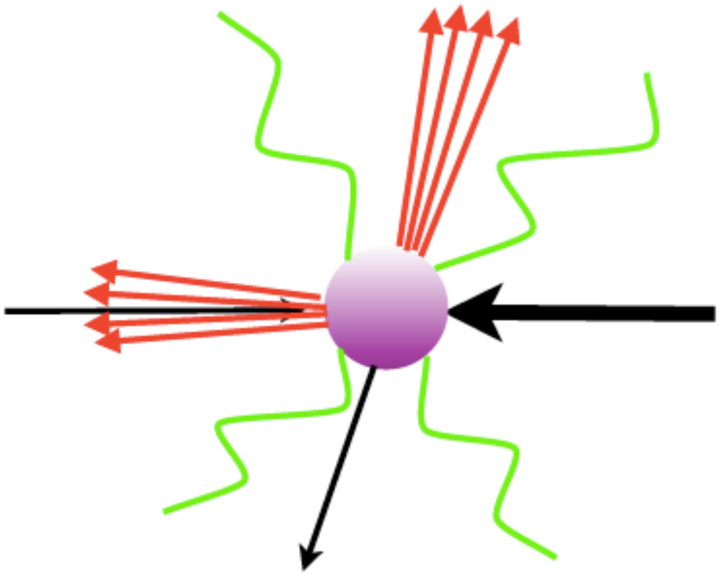
1-jettiness: global event shape

Event shape with 1-Jettiness

- Configurations of large and small 1-jettiness:



$$\tau_1 \sim P_{J_T}$$



$$\tau_1 \ll P_{J_T}$$

- 1-jettiness distributions can be a probe of nuclear structure and dynamics.

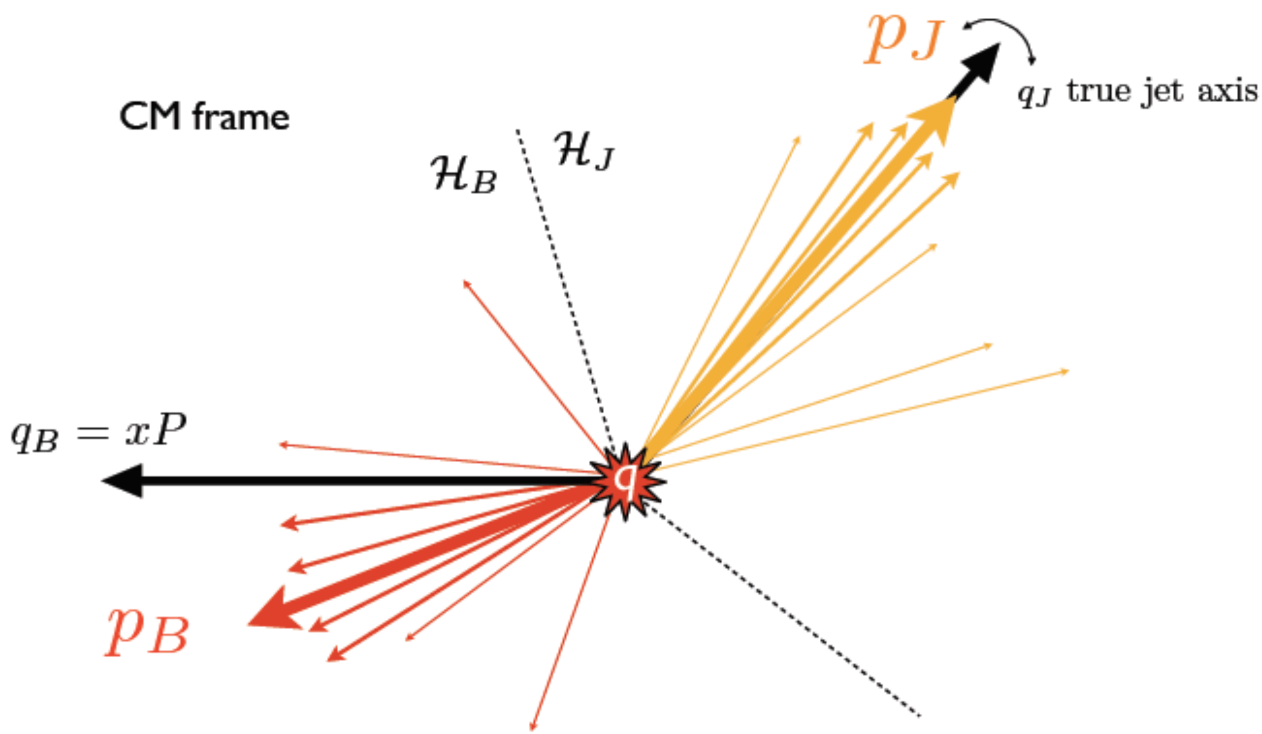
Most importantly, the radiation pattern following the additional scattering

Three ways to define the 1-jettiness

τ_1^a

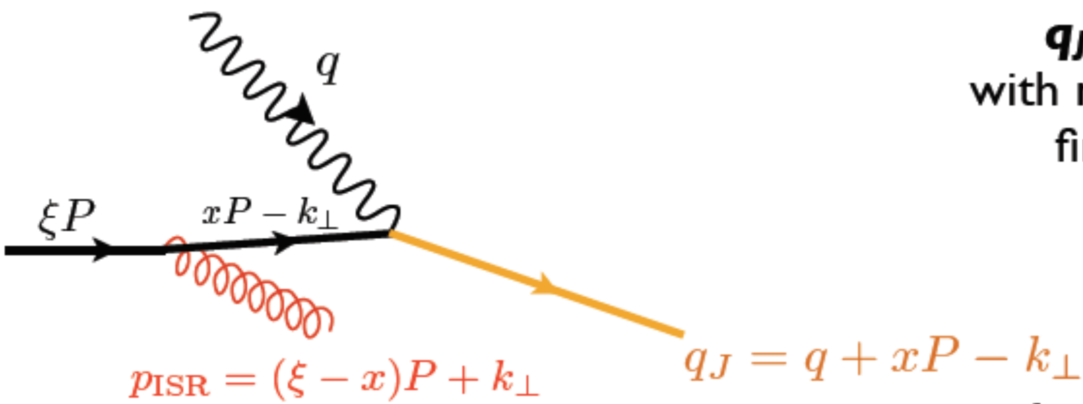
$$q_B = xP$$
$$q_J = \text{true jet axis}$$

Kang, Mantry, Qiu (2012)



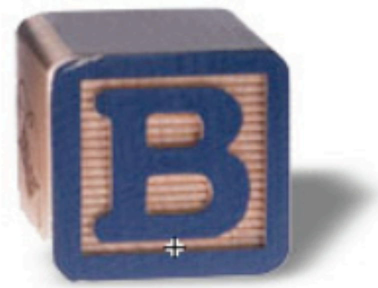
q_J is **A**ligned with the jet momentum, with no relative label transverse momentum: find by jet algorithm or minimization

→ depends on momenta of final-state hadrons

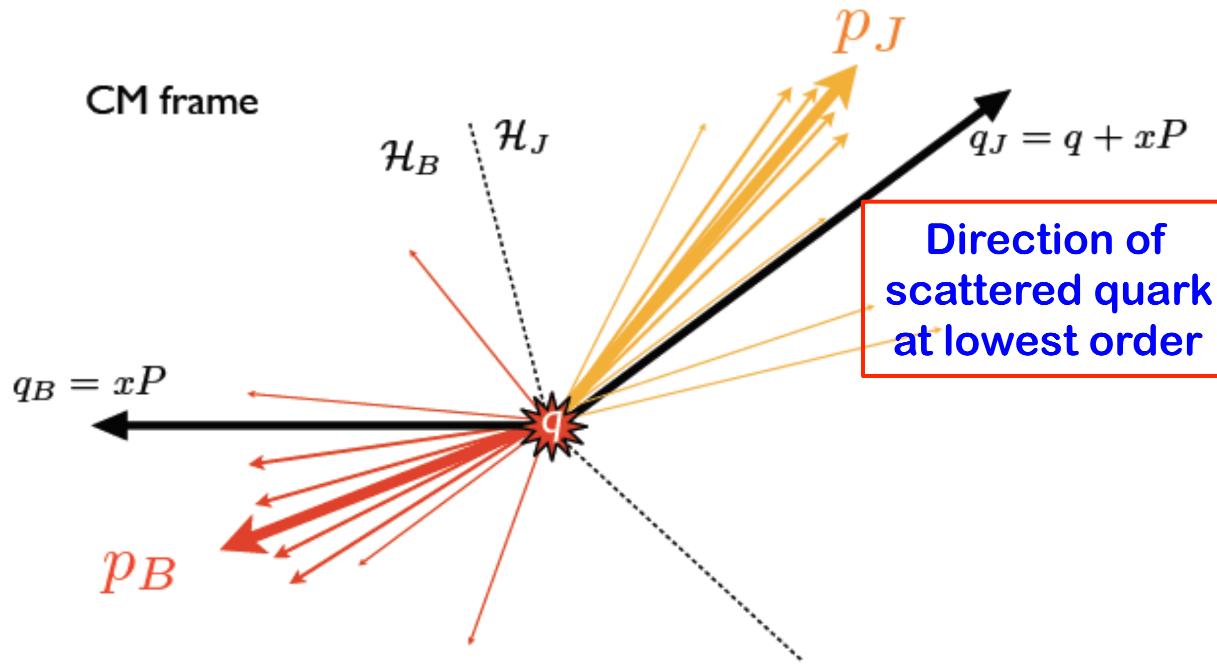


$k_\perp \sim Q\lambda$

Three ways to define the 1-jettiness



τ_1^b



$$q_B = xP$$

$$q_J = q + xP$$

same as DIS thrust
by Antonelli, Dasgupta, Salam
(1999)

$\boldsymbol{q_J}$ no longer exactly aligned with jet, but simpler in that $q+xP$ is given only by lepton and initial-state proton momenta

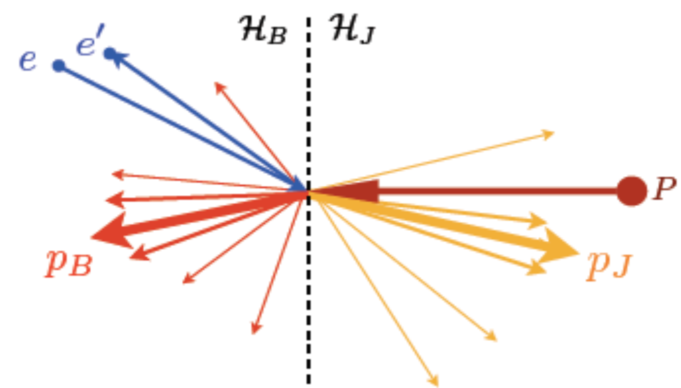
Boost

Breit frame:

$$q = (Q, 0, 0, Q)$$

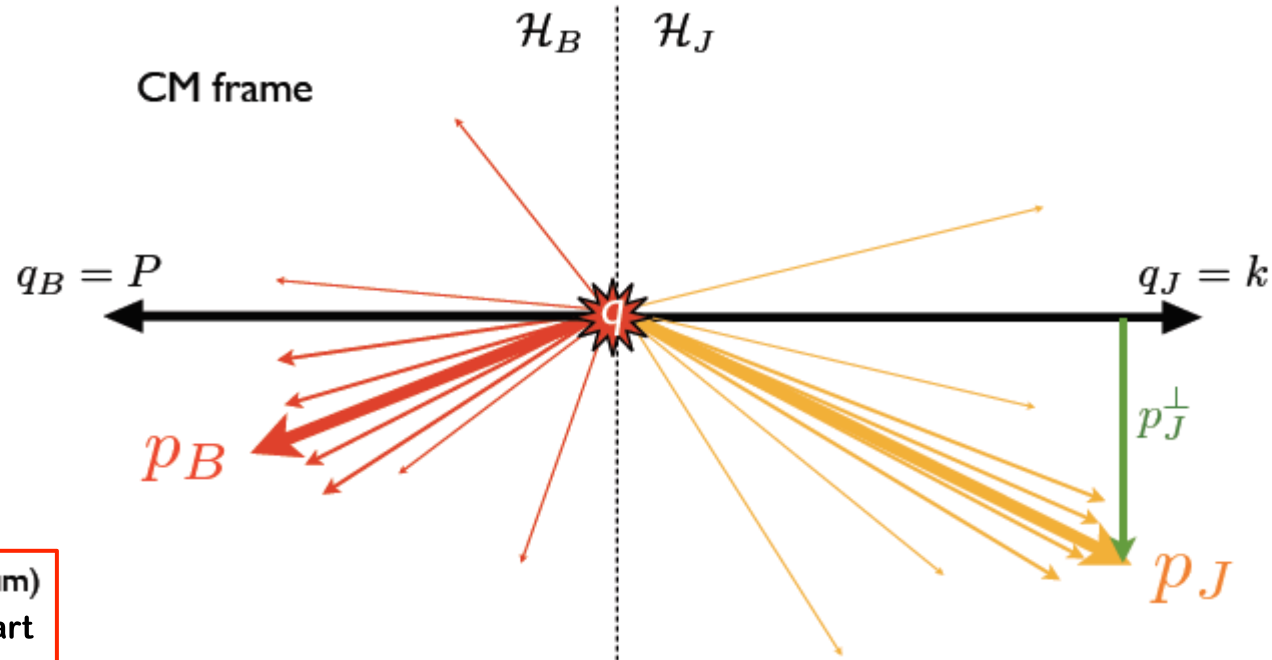
$$q_B = Q\bar{n}_z \quad q_J = Qn_z$$

1-jettiness regions are hemispheres in Breit frame



Three ways to define the 1-jettiness

τ_1^c



$$q_B = P$$
$$q_J = k$$

(electron momentum)
Kang, Lee, Stewart 2013

measures thrust in back-to-back hemispheres in **C**enter-of-momentum frame

momentum transfer ***q*** itself has a nonzero transverse component:

$$q = y\sqrt{s}\frac{n_z}{2} - xy\sqrt{s}\frac{\bar{n}_z}{2} + \sqrt{1-y}\,Q\hat{n}_\perp$$

seemingly simplest definition: *in practice* hardest to calculate!

Restriction: $p_J^\perp$ has to be small for 1-jettiness τ_1^c to be small $\Rightarrow 1 - y \sim \lambda^2$

Tree-level 1-jettiness distribution

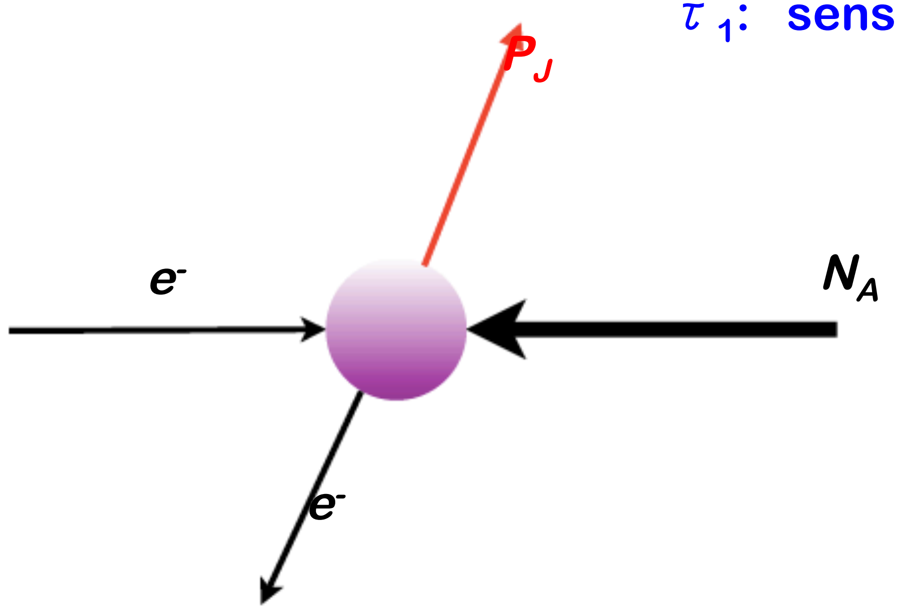
Kang, Mantry, Qiu, PRD (2012)

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$

Two scales observables!

P_T: localized probe

τ_1: sensitive to event shape

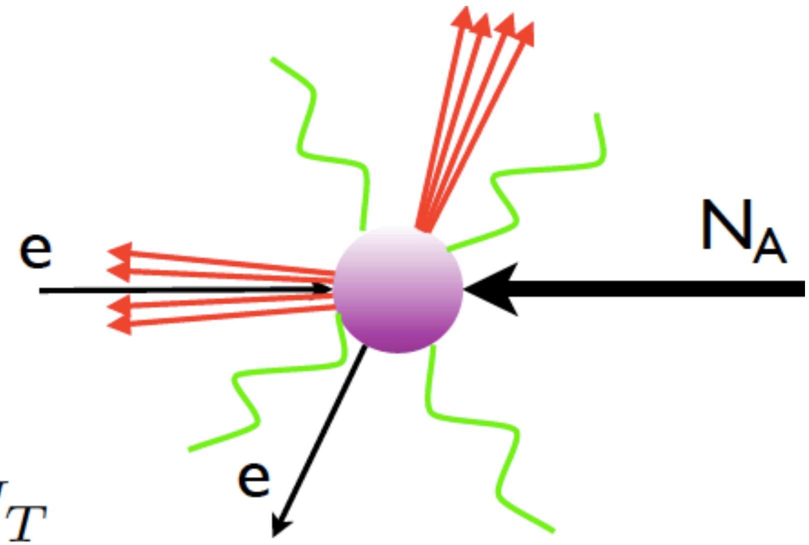


- Tree-level distribution in 1-jettiness:

$$\frac{d^3\sigma^{(0)}}{dy dP_{JT} d\tau_1} = \sigma_0 \delta(\tau_1) \sum_q e_q^2 \frac{1}{A} f_{q/A}(x_A, \mu)$$

Hierarchy of energy scales

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$



- Hierarchy of scales:

$$Q_s, \Lambda_{QCD} \ll \tau_1 \ll P_{J_T}$$

Nuclear scales

1-jettiness

Jet transverse momentum

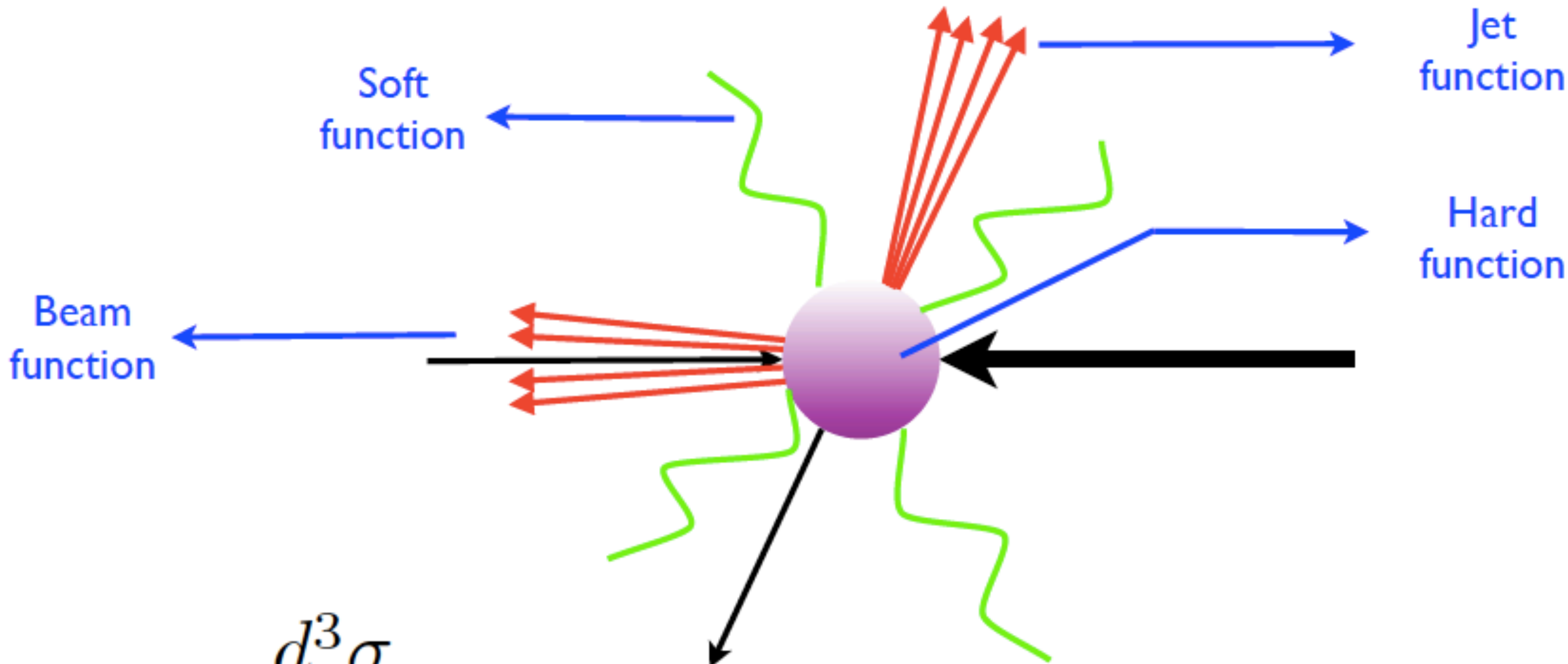
Hard	$\mu_H \sim P_{J_T}$
Beam, Jet	$\mu_B \sim \mu_J \sim \sqrt{\tau_1 P_{J_T}}$
Soft	$\mu_S \sim \tau_1$
Nuclear	$Q_s^2(A) \sim A^\alpha \Lambda_{QCD}^2$

- Jet-veto Sudakov logarithms:

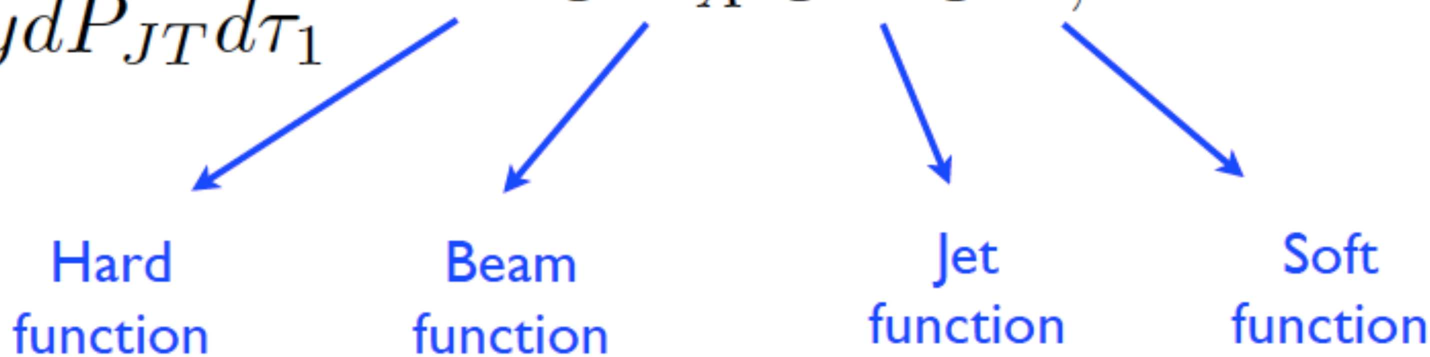
$$\sim \alpha_s^n \ln^{2n}(\tau_1 / P_{J_T})$$

Factorization – SCET

- Schematic form of factorization:



$$\frac{d^3\sigma}{dy\,dP_{JT}\,d\tau_1} \sim H \otimes B_A \otimes J \otimes \mathcal{S},$$



Factorized cross section

- Detailed form of factorization:

$$\frac{d^3\sigma}{dy dP_{JT} d\tau_1} = \frac{\sigma_0}{A}\sum_{q,i} e_q^2 \int_0^1 dx \int ds_J \int dt_a$$

Hard function → $$\times H(xAQ_e P_{J_T} e^{-y}, \mu; \mu_H)\delta\left[x - \frac{e^y P_{J_T}}{A(Q_e - e^{-y}P_{J_T})}\right]$$

Jet function → $$\times J^q(s_J, \mu; \mu_J) B^q(x, t_a, \mu; \mu_B)$$ ← Beam function

$$\times \mathcal{S}\left(\tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S\right),$$ ← Soft Function

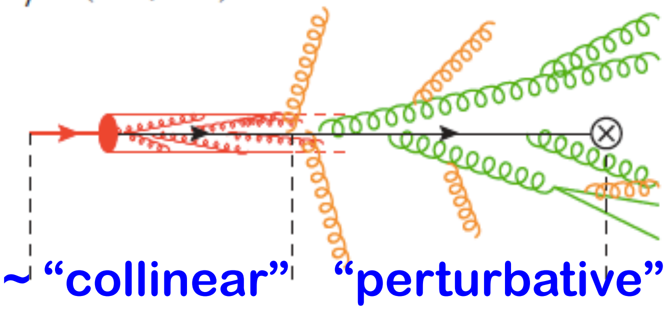
- Beam function matching onto the PDF:

(Fleming,Leibovich,Mehen; Jouttenus, Stewart, Tackmann, Waalewijin)

$$B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} \mathcal{I}^{qi}\left(\frac{x}{z}, t_a, \mu; \mu_B\right) f_{i/A}(z, \mu_B)$$

- Tree-level matching:

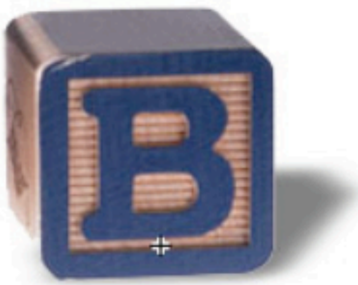
$$B^q(x, t_a, \mu_B) = \delta(t_a) f_{q/A}(x, \mu_B)$$



Differences between the three definitions

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J\, dt_B\, dk_S\, \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

Z. Kang, Mantry, Qiu, 2012



$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp\, dt_J\, dt_B\, dk_S\, \delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

D. Kang, Lee, Stewart, 2013

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^c} = H(Q^2,\mu)\int d^2p_\perp\, dt_J\, dt_B\, dk_S\, \delta\left(\tau_1^c - \frac{t_J}{Q^2} - \frac{t_B}{xQ^2} - \frac{k_S}{\sqrt{x}Q}\right) \times J_q(t_J - (\mathbf{q}_\perp + \mathbf{p}_\perp)^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

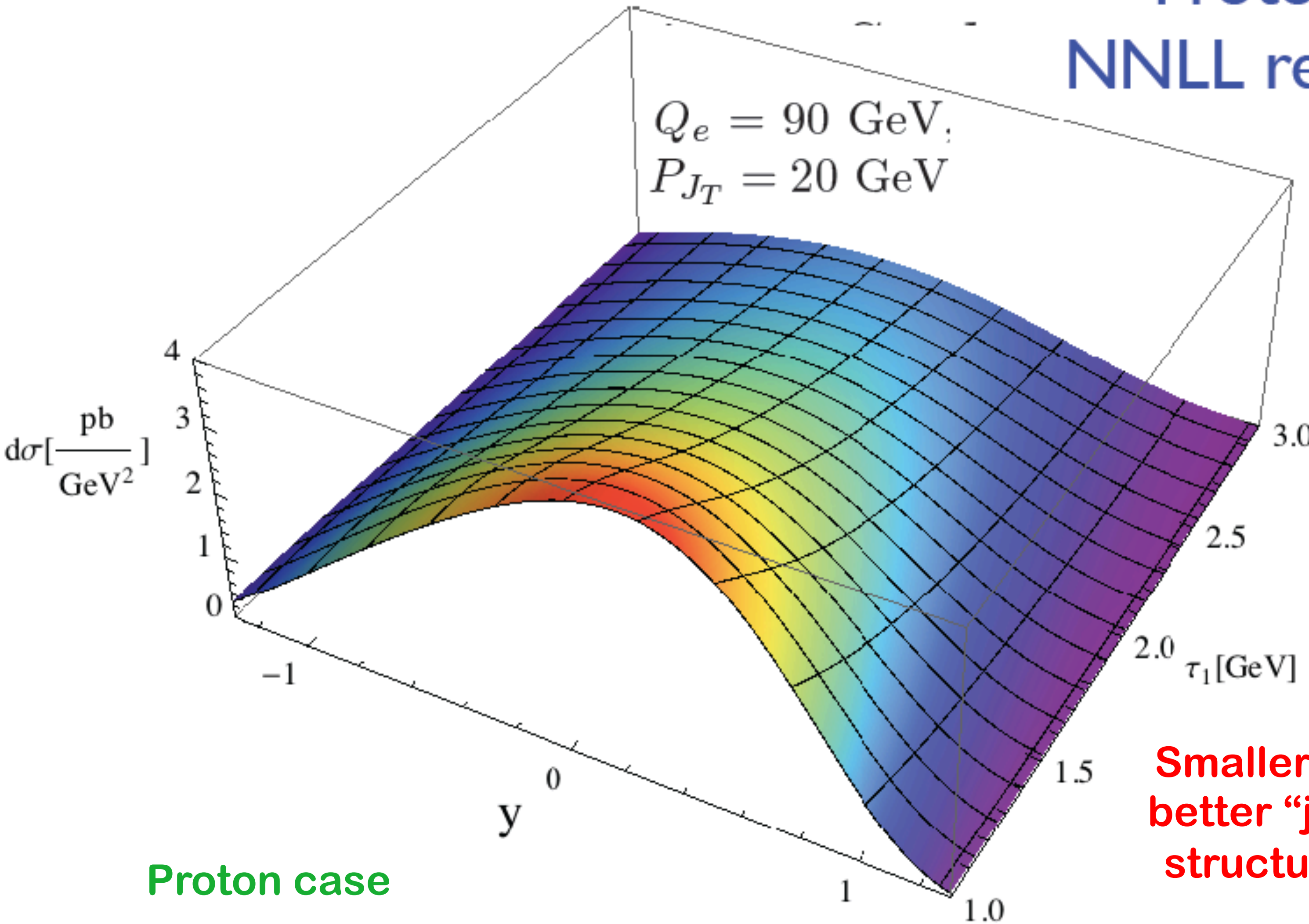
D. Kang, Lee, Stewart, 2013

1-jettiness and rapidity distribution

- One can study distributions in the space of :

$$\{A, Q_e, P_{J_T}, y, \tau_1\}$$

Proton target:
NNLL resummation



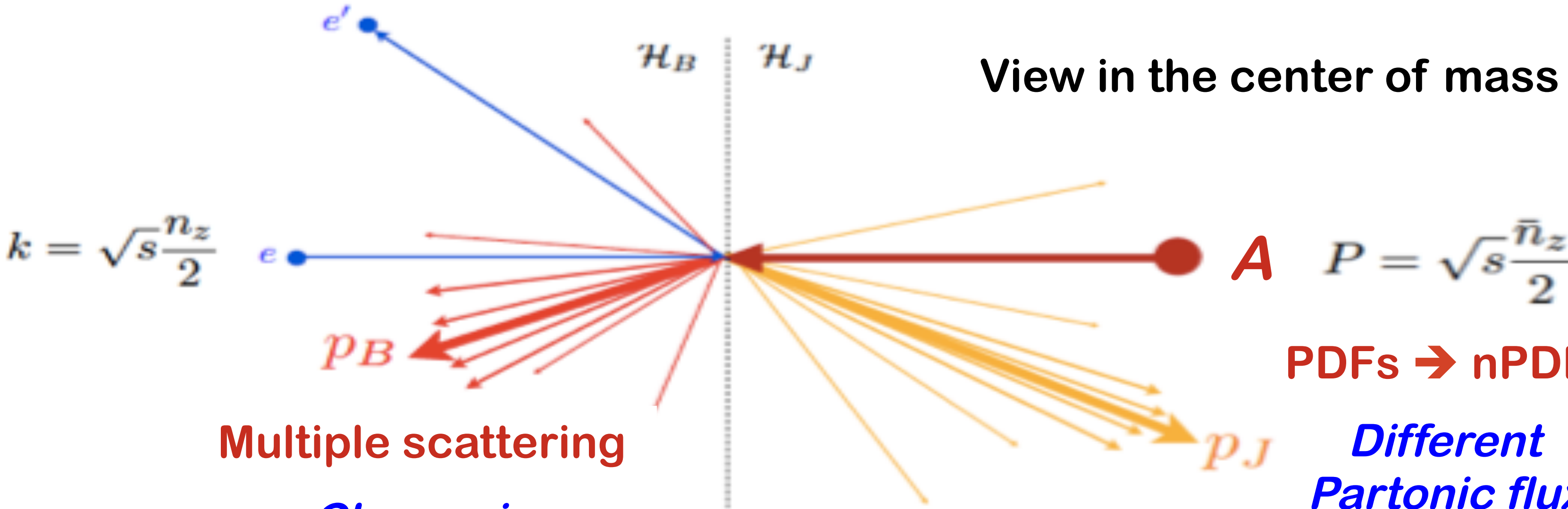
Larger τ
less “jet” structure

Smaller τ
better “jet” structure

Proton case

1-Jettiness cross section in e+A DIS

Kang, Mantry, Qiu, 2012, 2013



View in the center of mass frame

PDFs → nPDFs

Different Partonic flux

Multiple scattering

Change in radiation pattern

❑ Same definition:

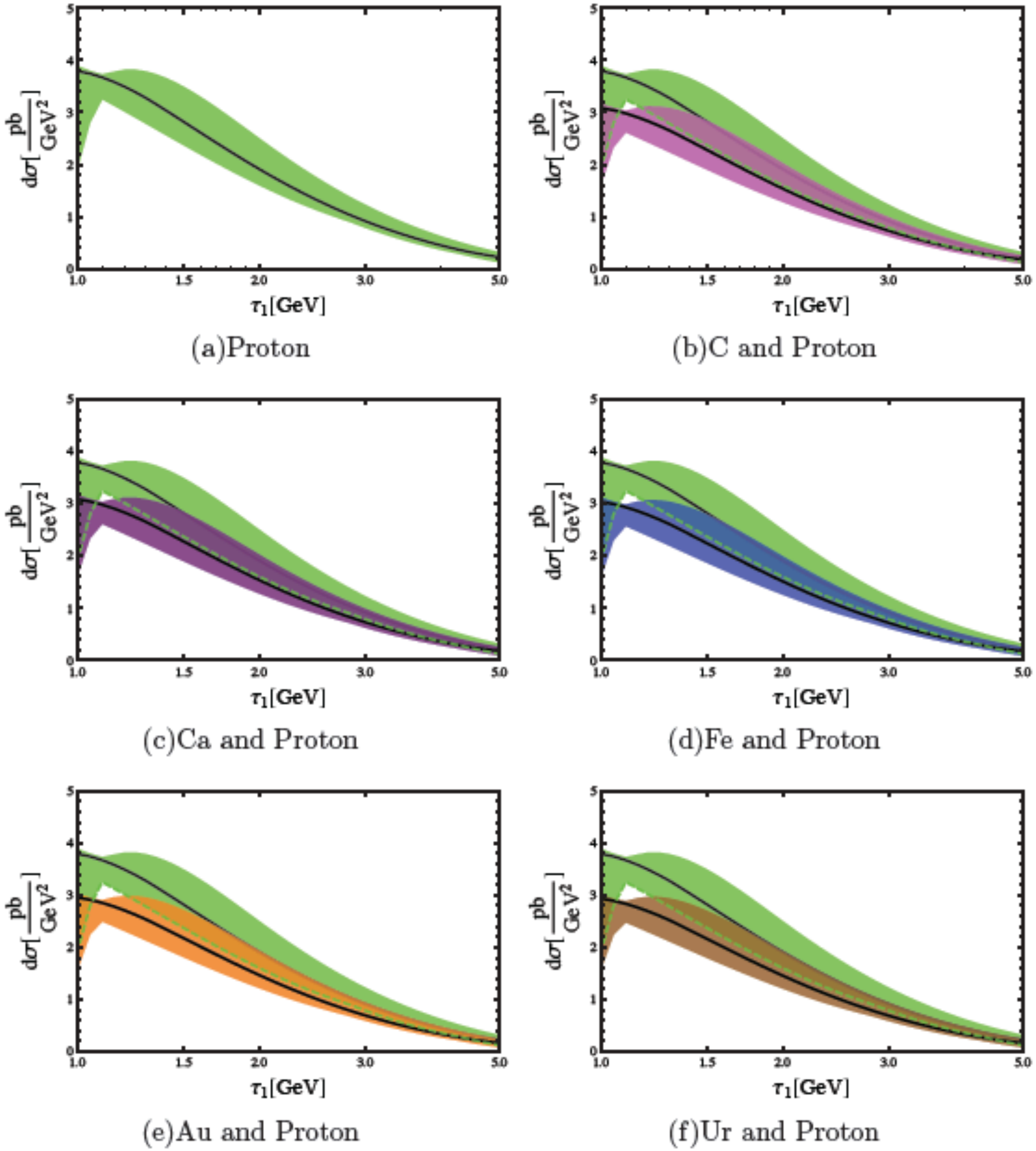
$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Additional variable: *A*

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{du\, dP_{J_T}\, d\tau_1}$$

1-jettiness: global event shape

1-jettiness distribution in e+A for various nuclei



(a)Proton

(b)C and Proton

(c)Ca and Proton

(d)Fe and Proton

(e)Au and Proton

(f)Ur and Proton

NNLL resummation

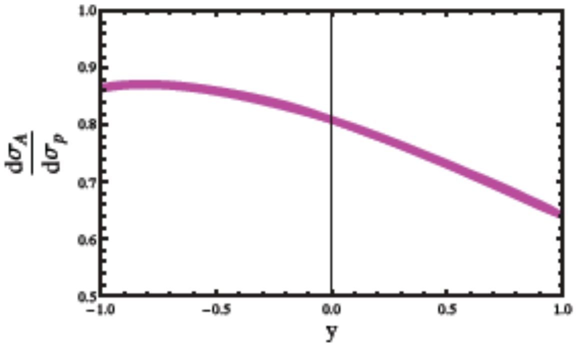
$Q_e = 90$ GeV,

$P_{J_T} = 20$ GeV

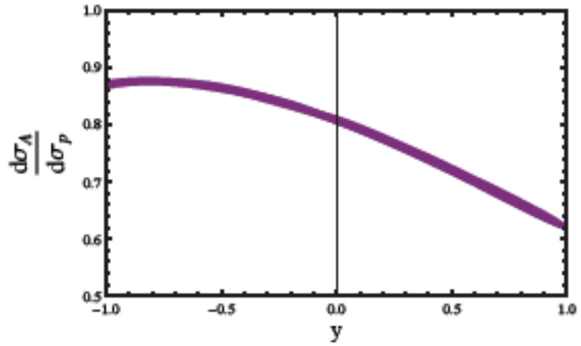
$y = 0$

Effect of nPDFs and smearing

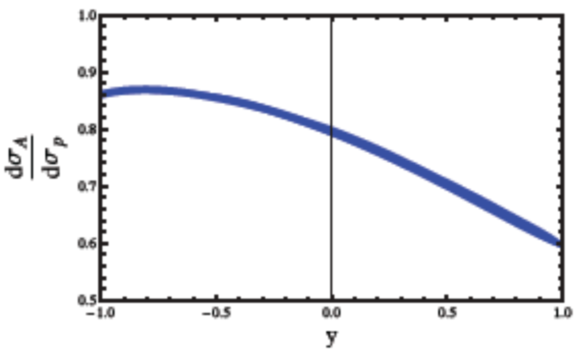
Jet rapidity: Nuclei over Proton



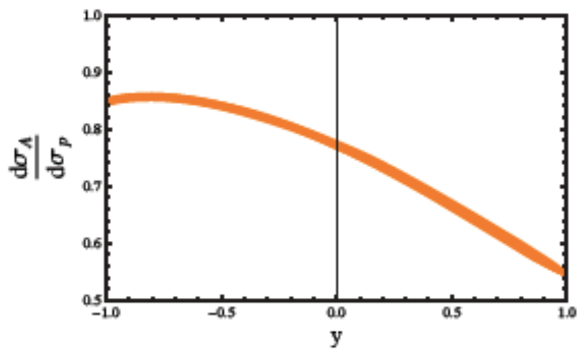
(a) C



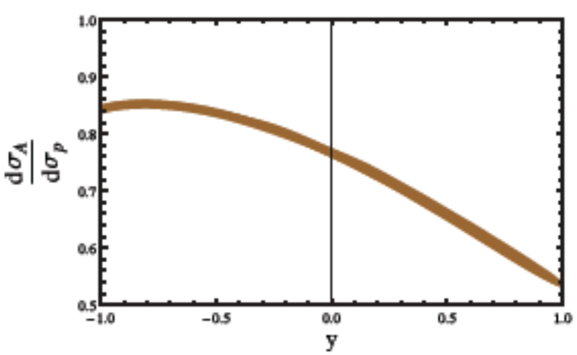
(b) Ca



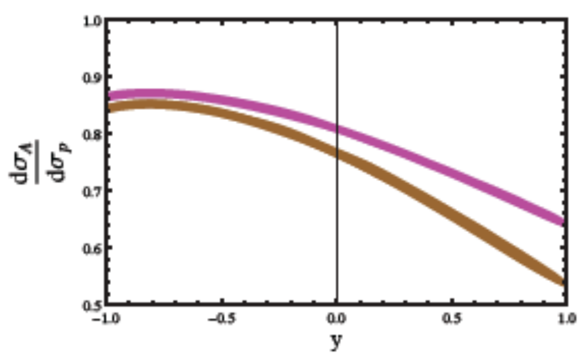
(c) Fe



(d) Au



(e) Ur



(f) C and Ur

$$R_A(\tau_1, P_{J_T}, y) = \frac{d\sigma_A(\tau_1, P_{J_T}, y)}{d\sigma_p(\tau_1, P_{J_T}, y)}$$

NNLL resummation

$$Q_e = 90 \text{ GeV},$$

$$P_{J_T} = 20 \text{ GeV}$$

$$\tau_1 = 1.5 \text{ GeV}$$

$$x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}}$$

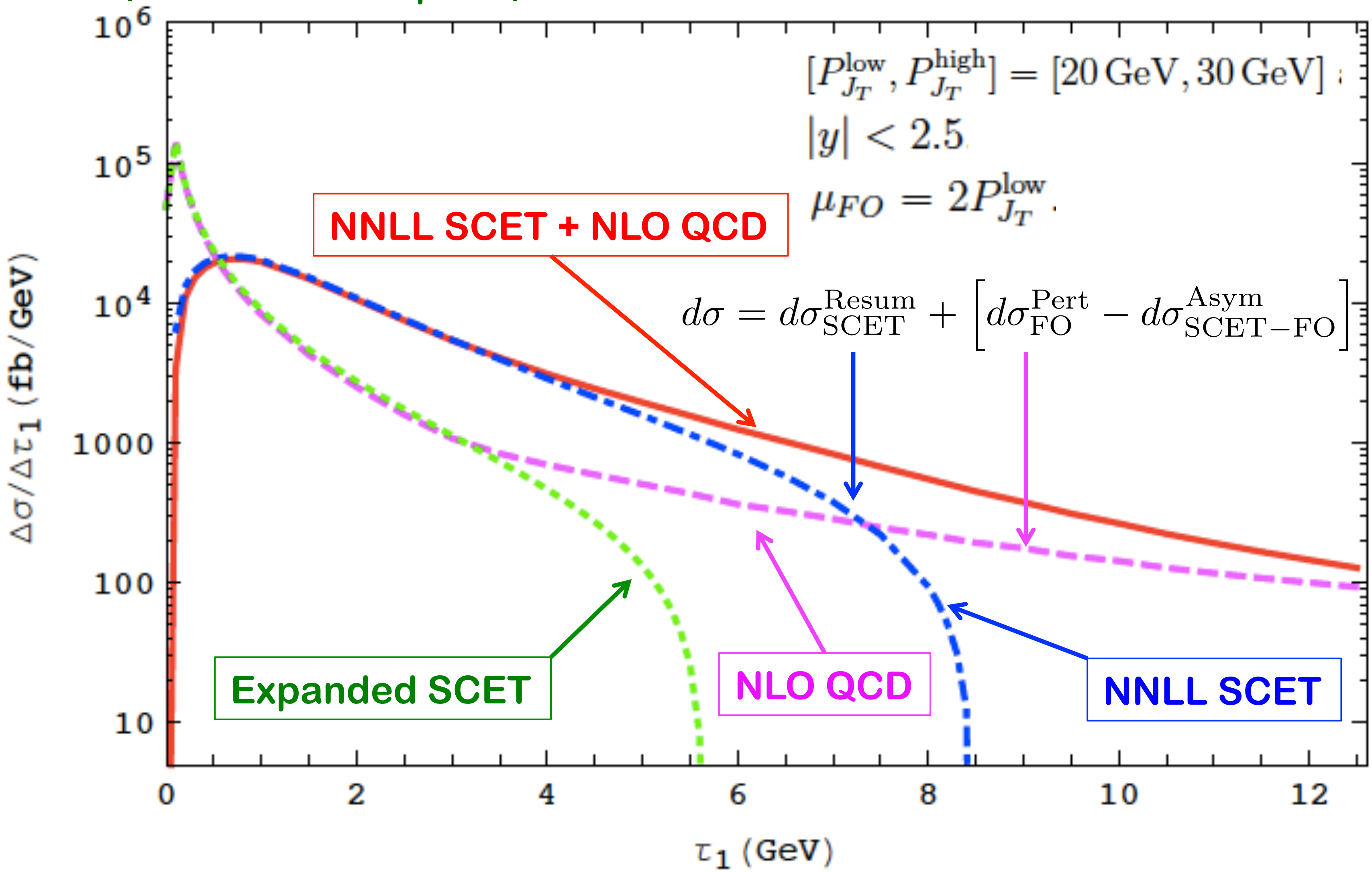
$$x_* \in [0.2, 0.7]$$

Effect of nPDFs and smearing

Matching from low τ to high τ

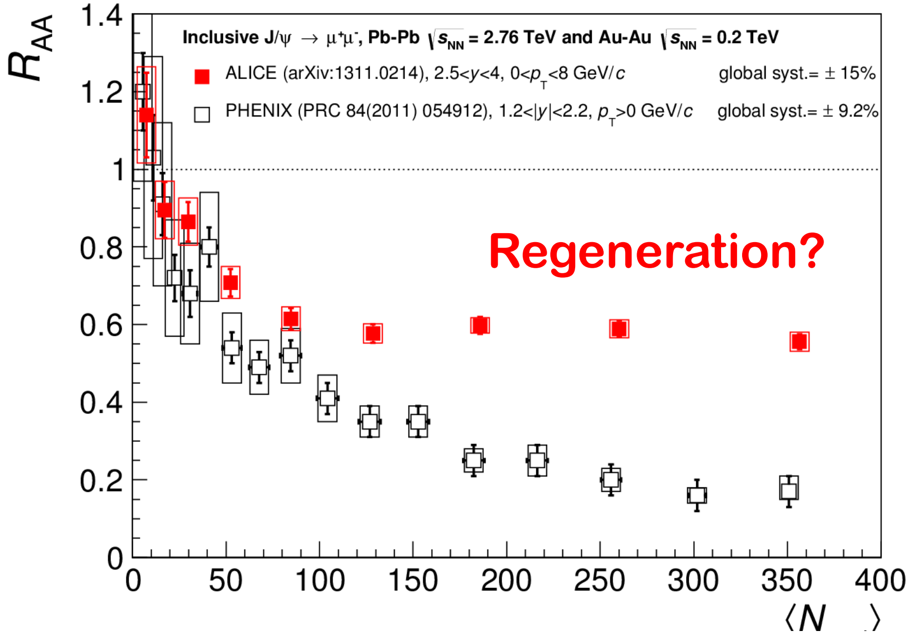
- Full spectrum in τ_1 on proton:

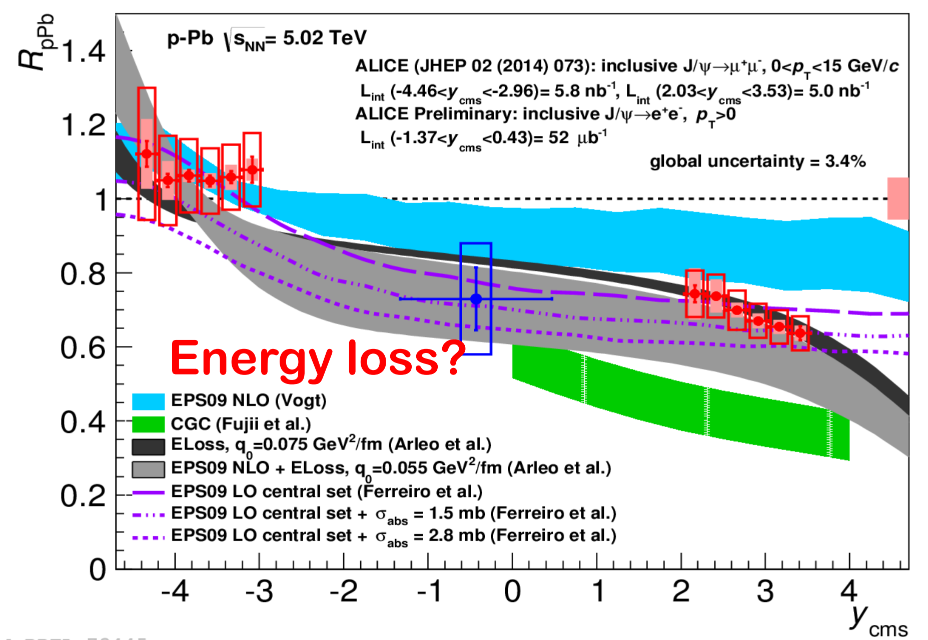
Kang, Liu, Mantry, 1312.0301

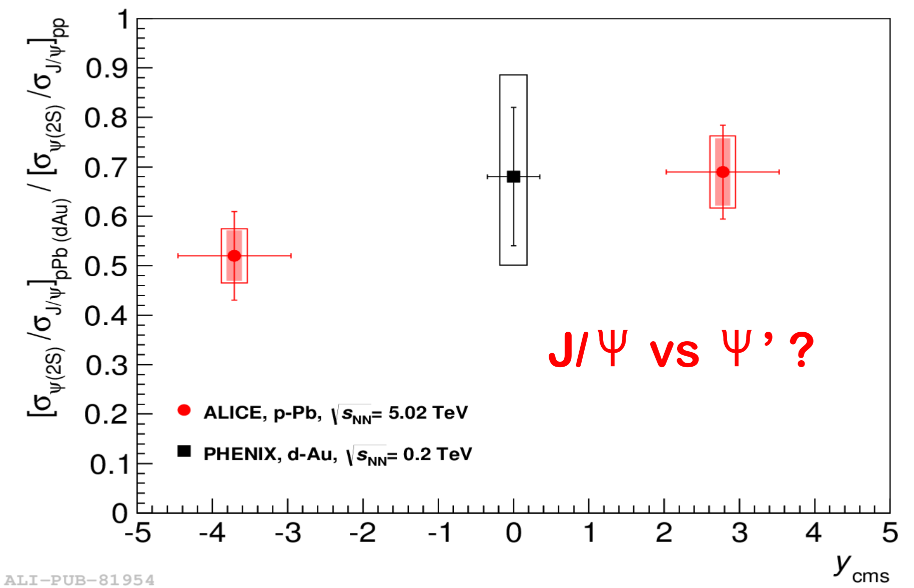


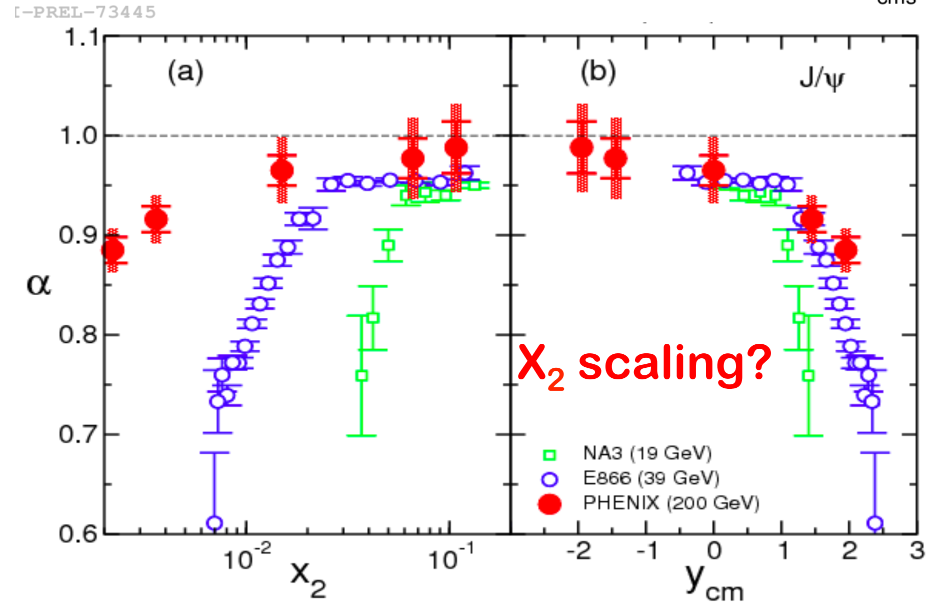
Multiple scattering = Broadening in τ_1 distribution

Heavy quarkonium puzzles – “suppression”

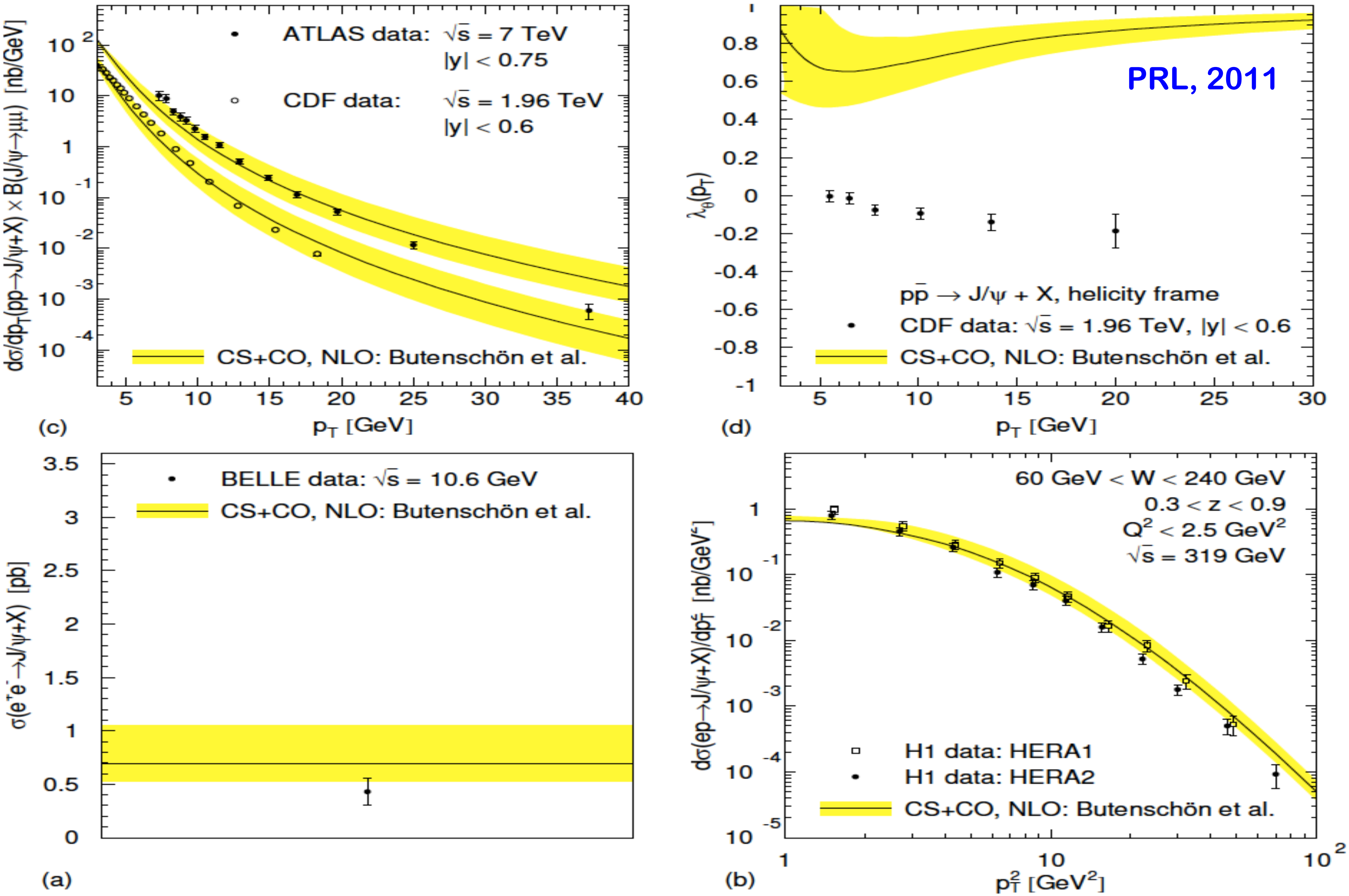




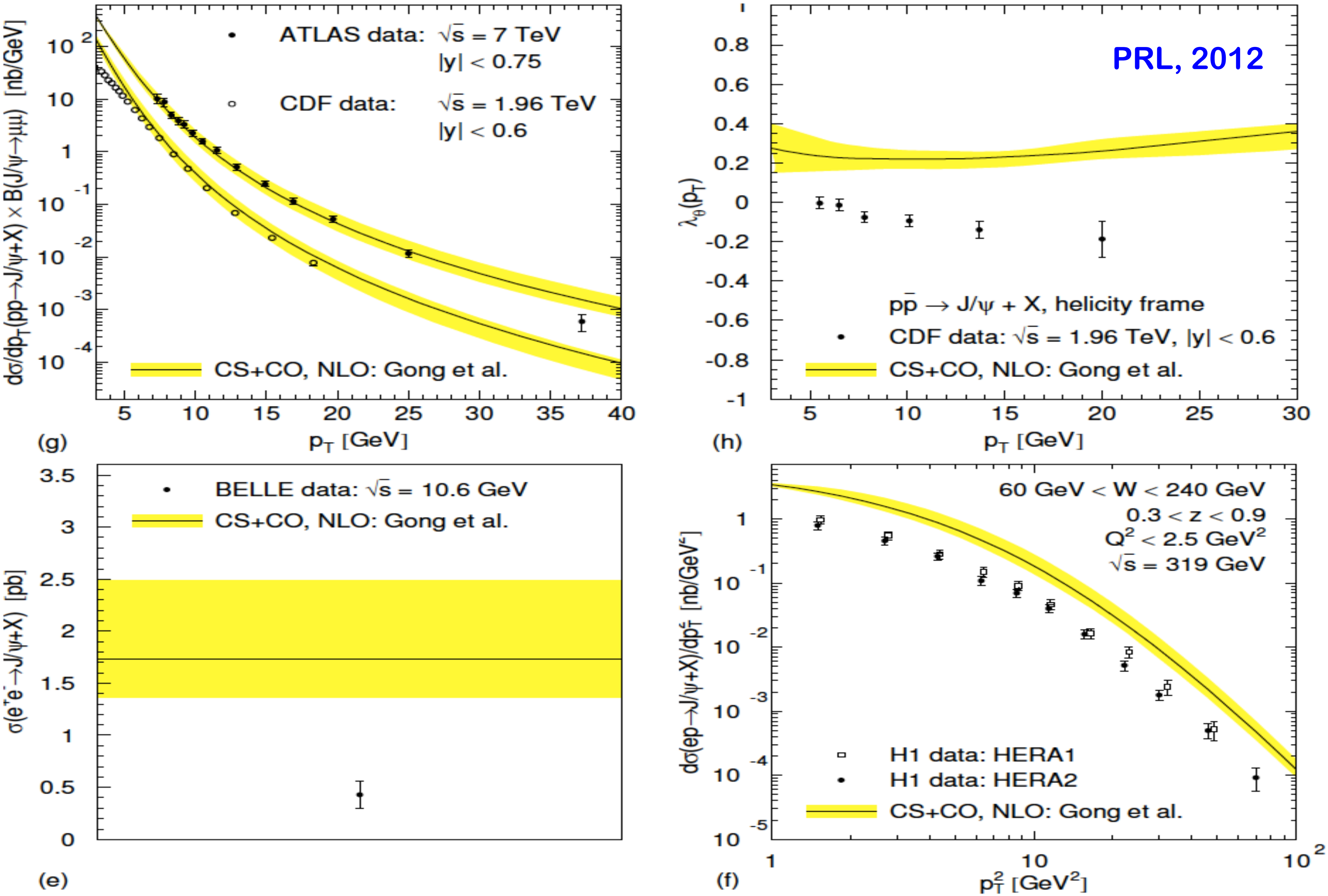




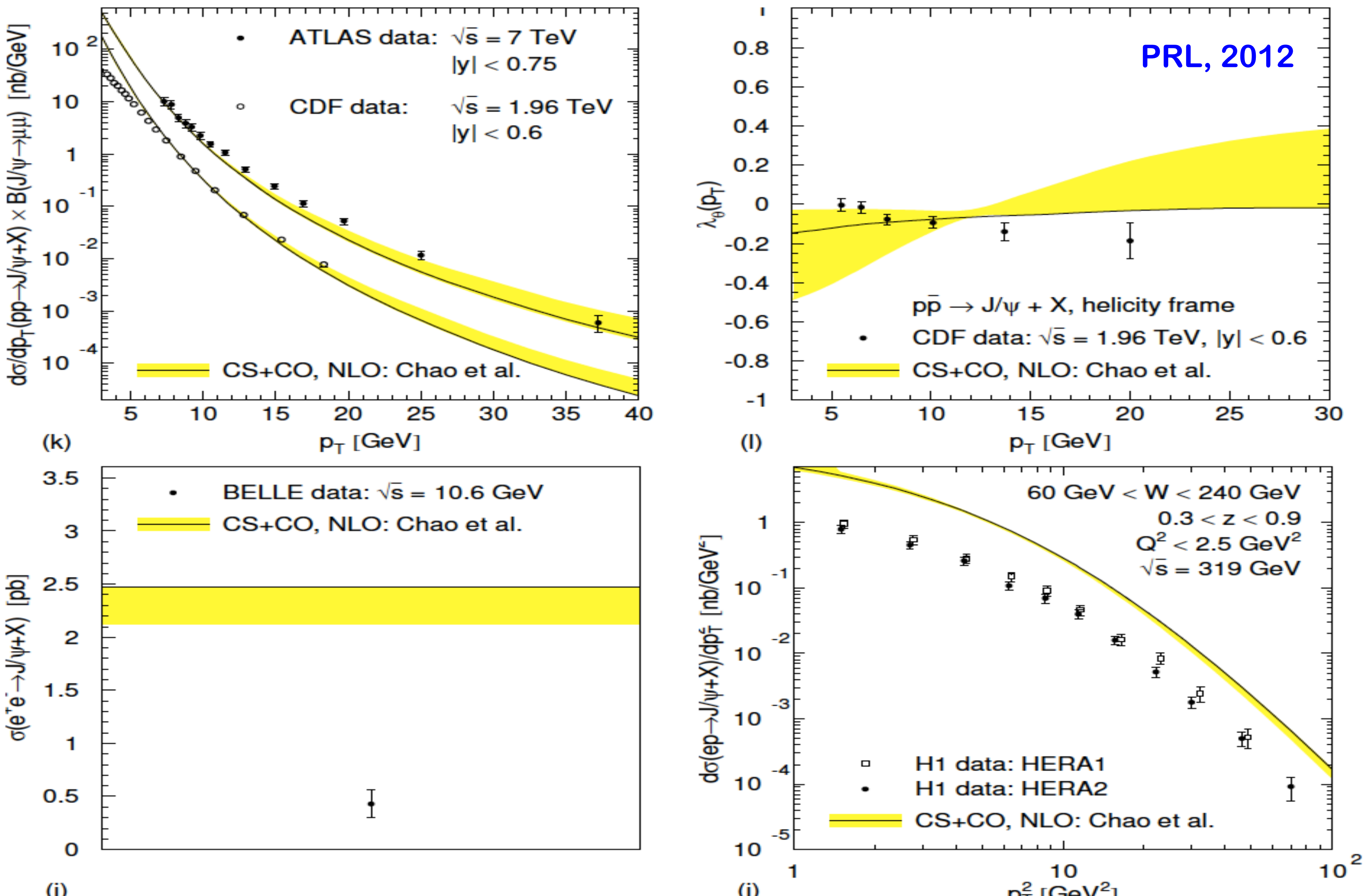
Production (NRQCD) – Butenschoen et al.



Production (NRQCD) – Gong et al.



Production (NRQCD) – Chao et al.

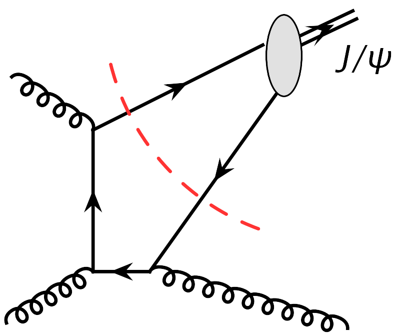


Why high orders in NRQCD are so large?

- **Consider J/ψ production in CSM:**

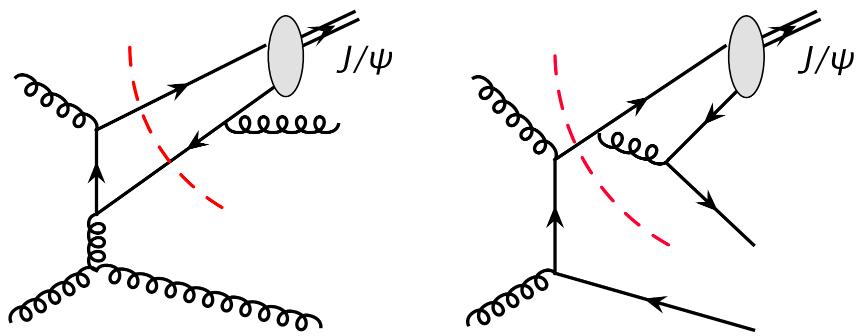
Kang, Qiu and Sterman, 2011

See also talk by H. Zhang



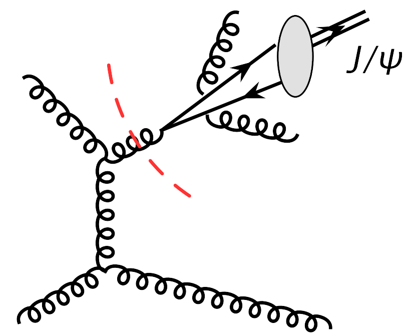
LO in α_s

NNLP $\propto \alpha_s^3 \frac{m_Q^4}{p_T^8}$



NLO in α_s

NLP in 1/P_T $\propto \alpha_s^4 \frac{m_Q^2}{p_T^6}$



NNLO in α_s

LP: $\propto \alpha_s^5 \frac{1}{p_T^4}$

- ✧ High-order correction receive power enhancement
- ✧ Expect no further power enhancement beyond NNLO
- ✧ $[\alpha_s \ln(p_T^2/m_Q^2)]^n$ ruins the perturbation series at sufficiently large p_T

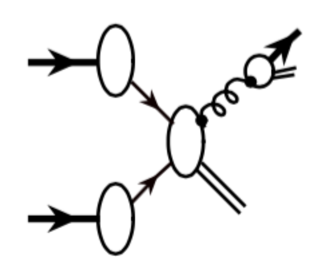
Leading order in α_s-expansion =\= leading power in 1/p_T-expansion!

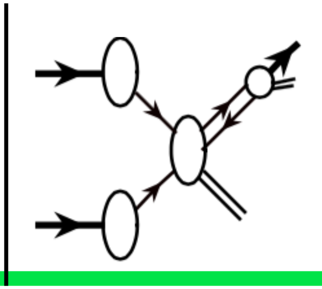
At high p_T, fragmentation contribution dominant

QCD factorization – Kang et al.

Kang, Ma, Qiu and Sterman, 2014

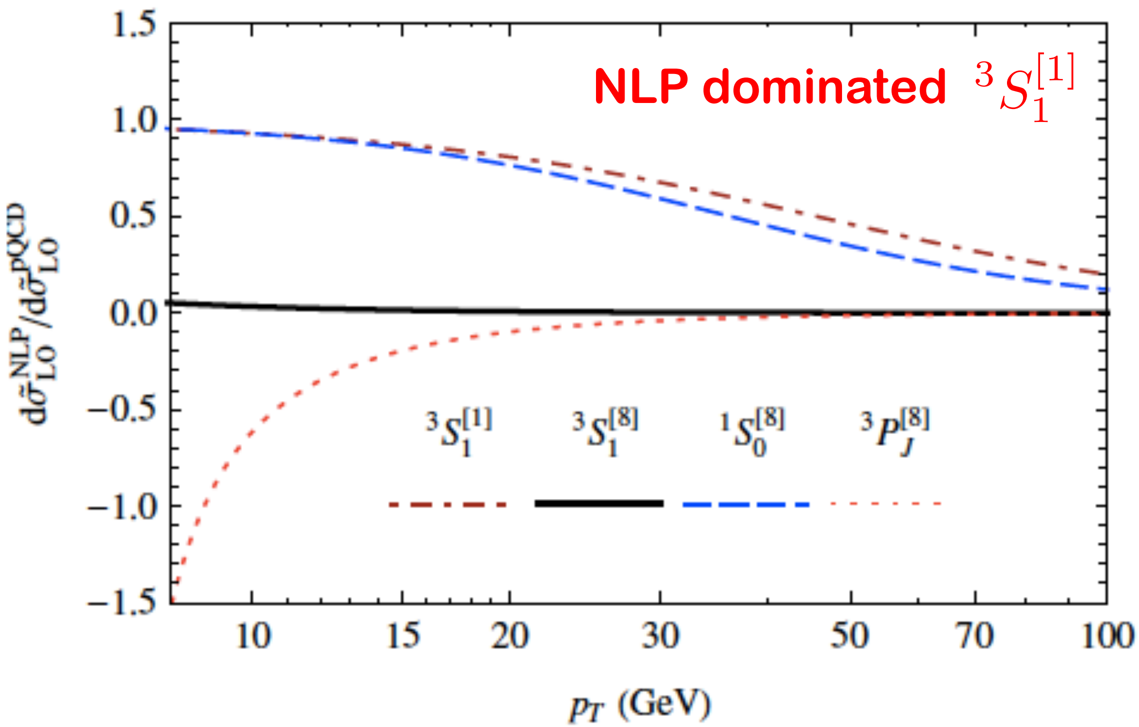
$$\frac{d\sigma_{AB\to H+X}}{dy\,dp_T^2} = \left| \quad \right|^2 + \left| \quad \right|^2 + \ldots$$





NLP

- Channel-by-channel, LP vs. NLP (both LO):



independent of NRQCD LDMEs

LP dominated

$^3S_1^{[8]}$ and $^3P_J^{[8]}$

NLP dominated

$^1S_0^{[8]}$

for wide pT

PT distribution is consistent with distribution of $^1S_0^{[8]}$

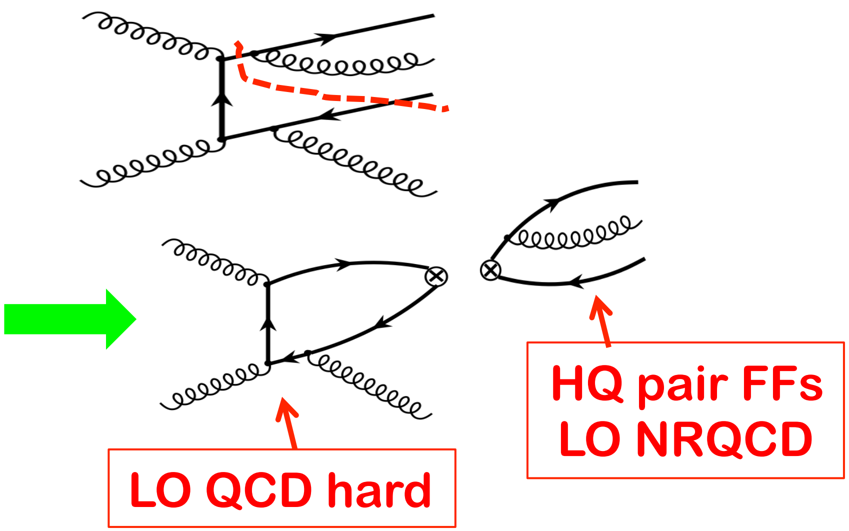
QCD Factorization = better controlled HO corrections!

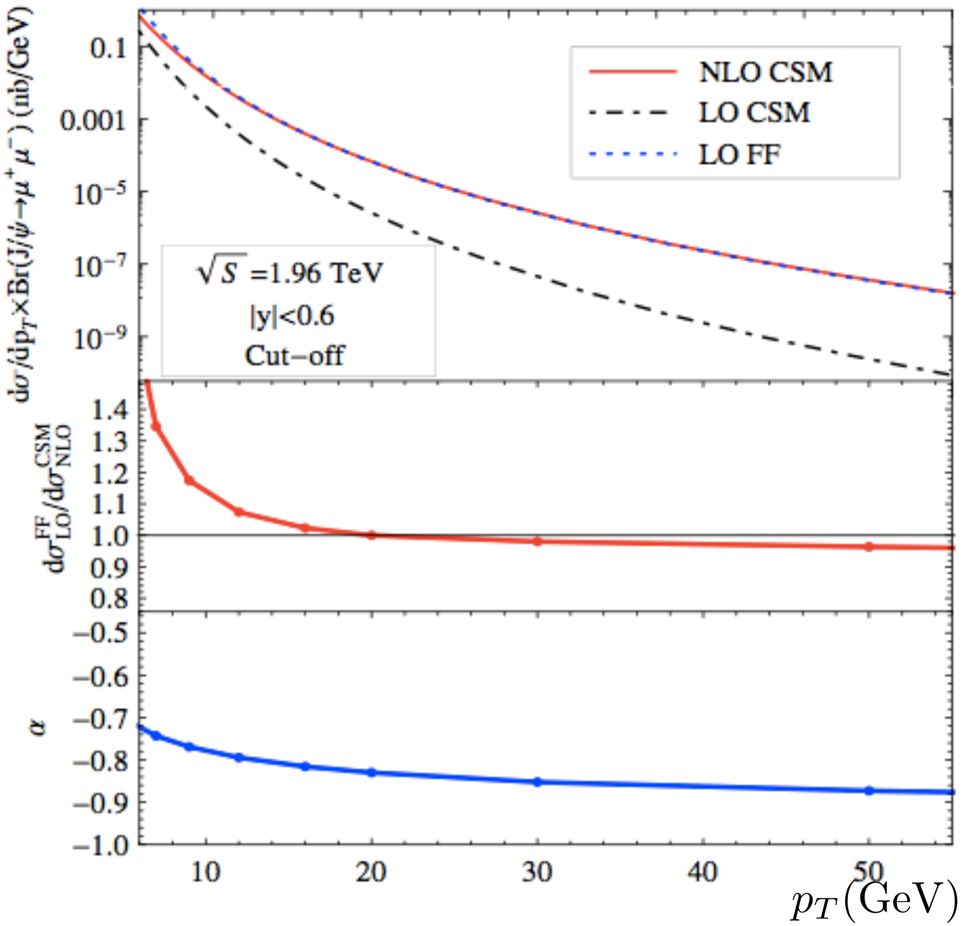
PRL, 2014

LO QCD factorization vs NLO NRQCD

Kang, Ma, Qiu and Sterman, 2014

❑ Color singlet as an example:





$$\sigma^{(\mathrm{NLO})}_{\mathrm{NRQCD}} \propto \Big[d\hat{\sigma}^{A(\mathrm{LO})}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(v8)]\to J/\psi}$$

$$+ d\hat{\sigma}^{S(\mathrm{LO})}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(a8)]\to J/\psi} \Big]$$

LO pQCD: reproduces NLO CSM rate for p_T > 10 GeV!

NLO pQCD can be done, while NNLO NRQCD is impossible!

QCD Factorization = better controlled HO corrections!

Matching from high p_T to low p_T

- **Matching if both factorizable:**

$$E_P \frac{d\sigma_{A+B\to H+X}}{d^3P}(P,m_Q) \equiv E_P \frac{d\sigma^{\rm QCD}_{A+B\to H+X}}{d^3P}(P,m_Q=0) + E_P \frac{d\sigma^{\rm NRQCD}_{A+B\to H+X}}{d^3P}(P,m_Q\neq 0) - E_P \frac{d\sigma^{\rm QCD-Asym}_{A+B\to H+X}}{d^3P}(P,m_Q=0)$$

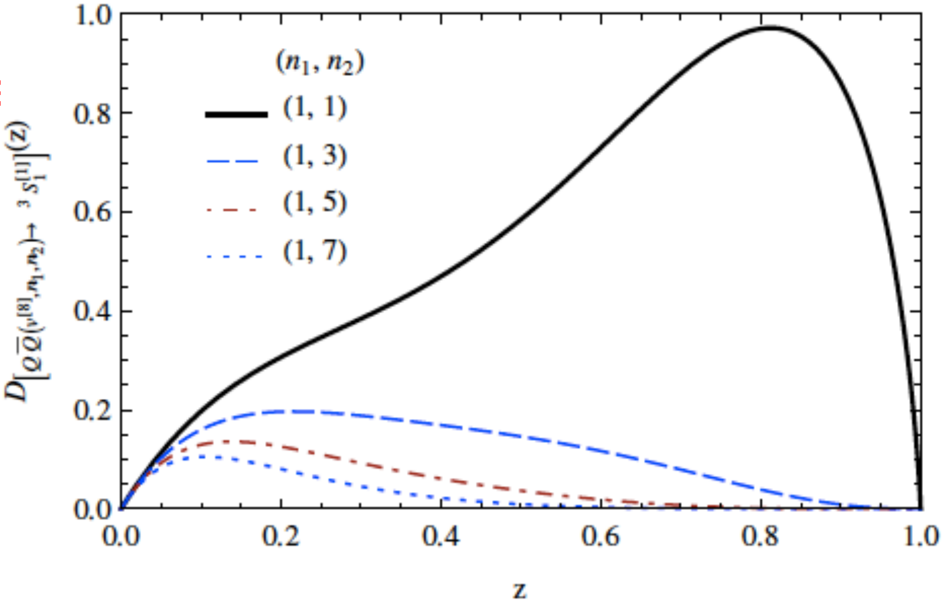
Mass effect + P_T region ($P_T \gtrsim m_Q$)

- **Fragmentation functions – nonperturbative!**

Responsible for "polarization", relative size of production channe

- **Model of FFs:**
  - ✧ NRQCD factorization of FFs
  - ✧ Express all FFs in terms of *a few* NRQCD LDMEs

$$\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z,\zeta_1,\zeta_2)$$

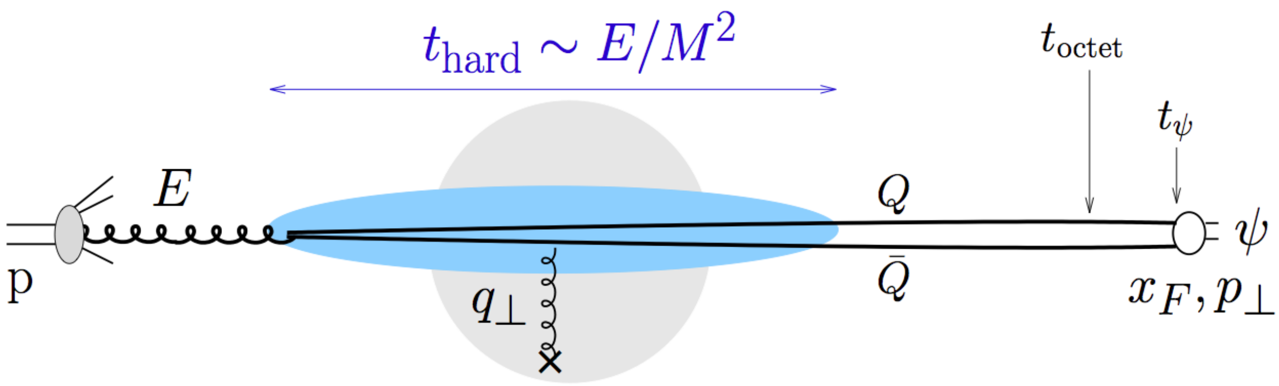


QCD factorization approach is ready to compare with Data

Multiple scattering – energy loss in p(d)+A

❑ Picture + assumptions:

Arleo, Peigne, 2012
Arleo, Kolevatov, Peigne, 2014



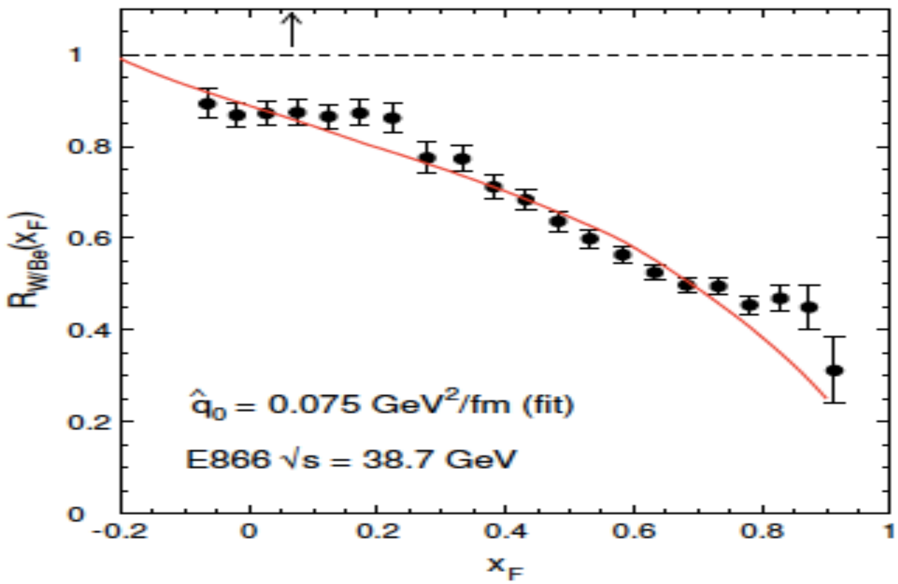
- Color neutralization happens on long time scales: $t_{\rm octet} \gg t_{\rm hard}$
- Medium rescatterings do not resolve the octet $c\bar{c}$ pair
- Hadronization happens outside of the nucleus: $t_\psi \gtrsim L$
- $c\bar{c}$ pair produced by gluon fusion

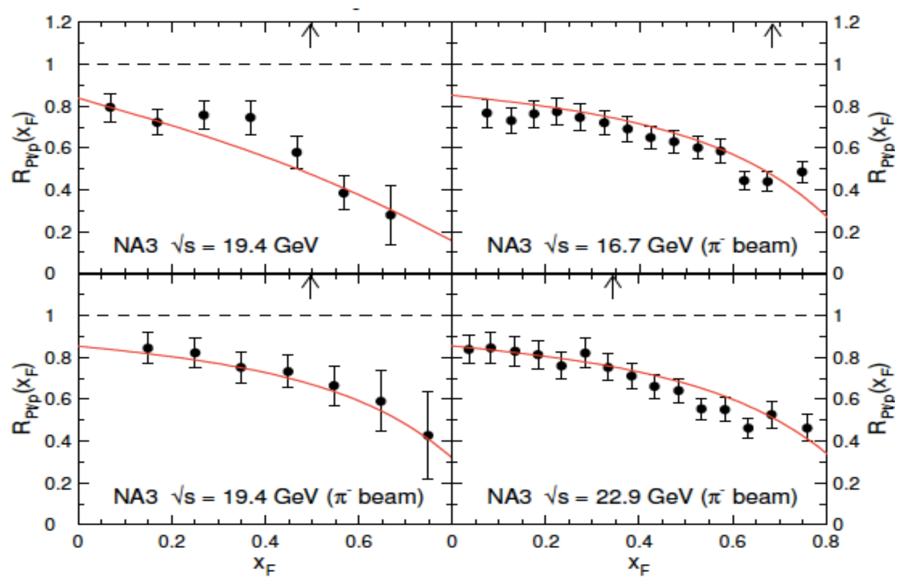
❑ Model energy loss:

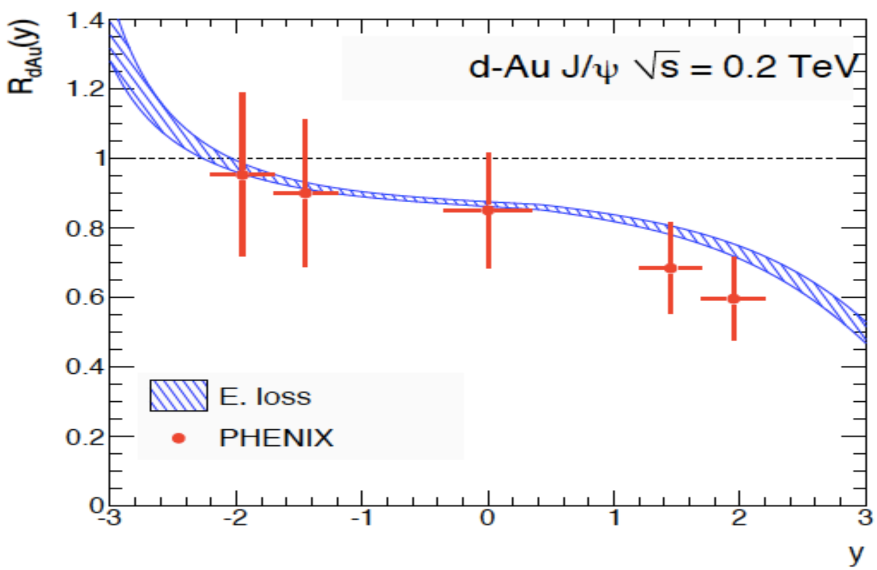
$$\frac{1}{A}\frac{d\sigma_{pA}}{dE}(E,\sqrt{s}) = \int_0^{\varepsilon_{\rm max}} d\varepsilon\, \mathcal{P}(\varepsilon, E)\, \frac{d\sigma_{pp}}{dE}(E+\varepsilon,\sqrt{s}) \qquad \hat{q}(x) \sim \hat{q}_0 \left(\frac{10^{-2}}{x}\right)^{0.3}$$

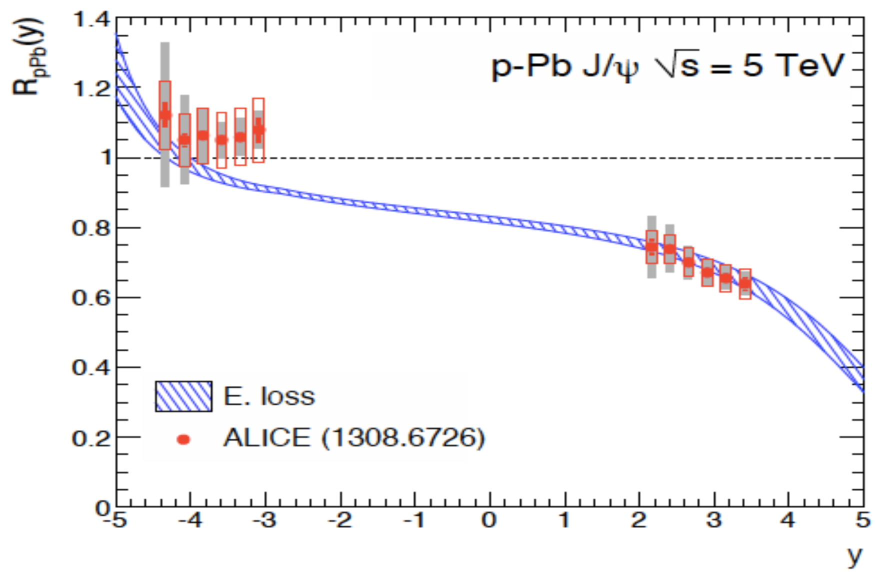
$\mathcal{P}(\varepsilon, E)$: **Quenching weight ~ scaling function of** $\sqrt{\hat{q}L}/M_\perp \times E$

A-dependence in rapidity y (x_F) in p(d)+A





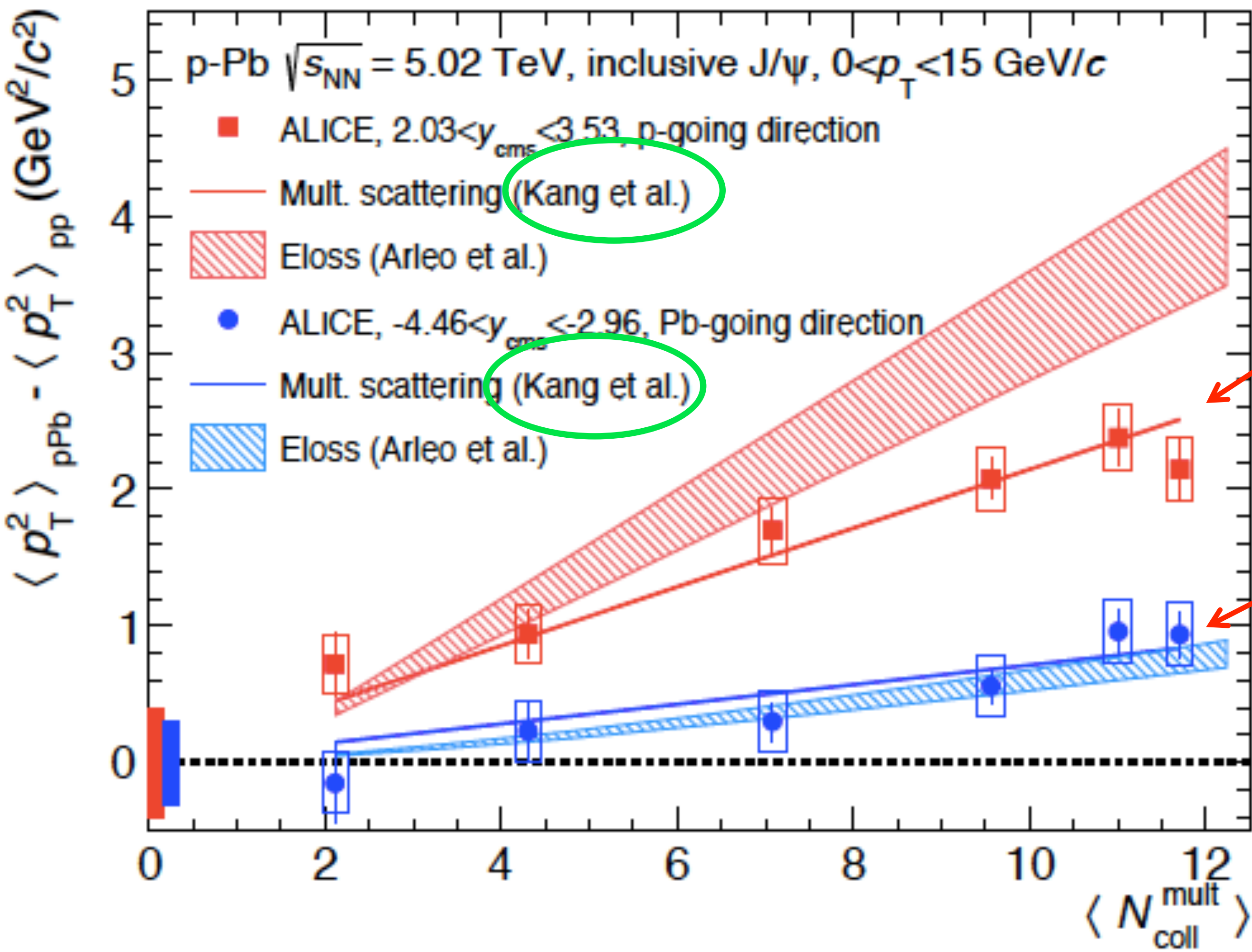




Broadening in p_T @ LHC

Kang, Qiu, 2013
+ in preparation

- Newly released ALICE data (1506.08808):



Forward

Predictions from QCD multiple scattering

Backward

QCD multiple scattering = consistent QCD power corrections

Cross section with two scales – resummation

$$Q_1^2 \gg Q_2^2 \gg \Lambda_{\rm QCD}^2, \quad Q_1^2 \gg Q_2^2 \gtrsim \Lambda_{\rm QCD}^2$$

- **Large perturbative logarithms:**

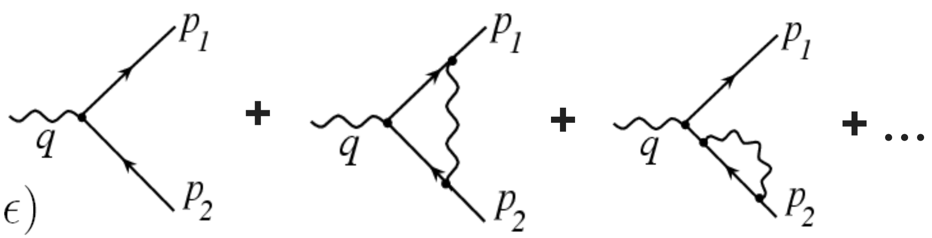
  $\alpha_s(\mu^2 = Q_1^2)$ **is small, But,** $\alpha_s(Q_1^2)\ln(Q_1^2/Q_2^2)$ **is not necessary small!**

- **Massless theory:**

  Two powers of large logs for each order in perturbation theory

  $\alpha_s(Q_1^2)\ln^2(Q_1^2/Q_2^2)$ **due to overlap of IR and CO regions**

- **Example – EM form factor:**



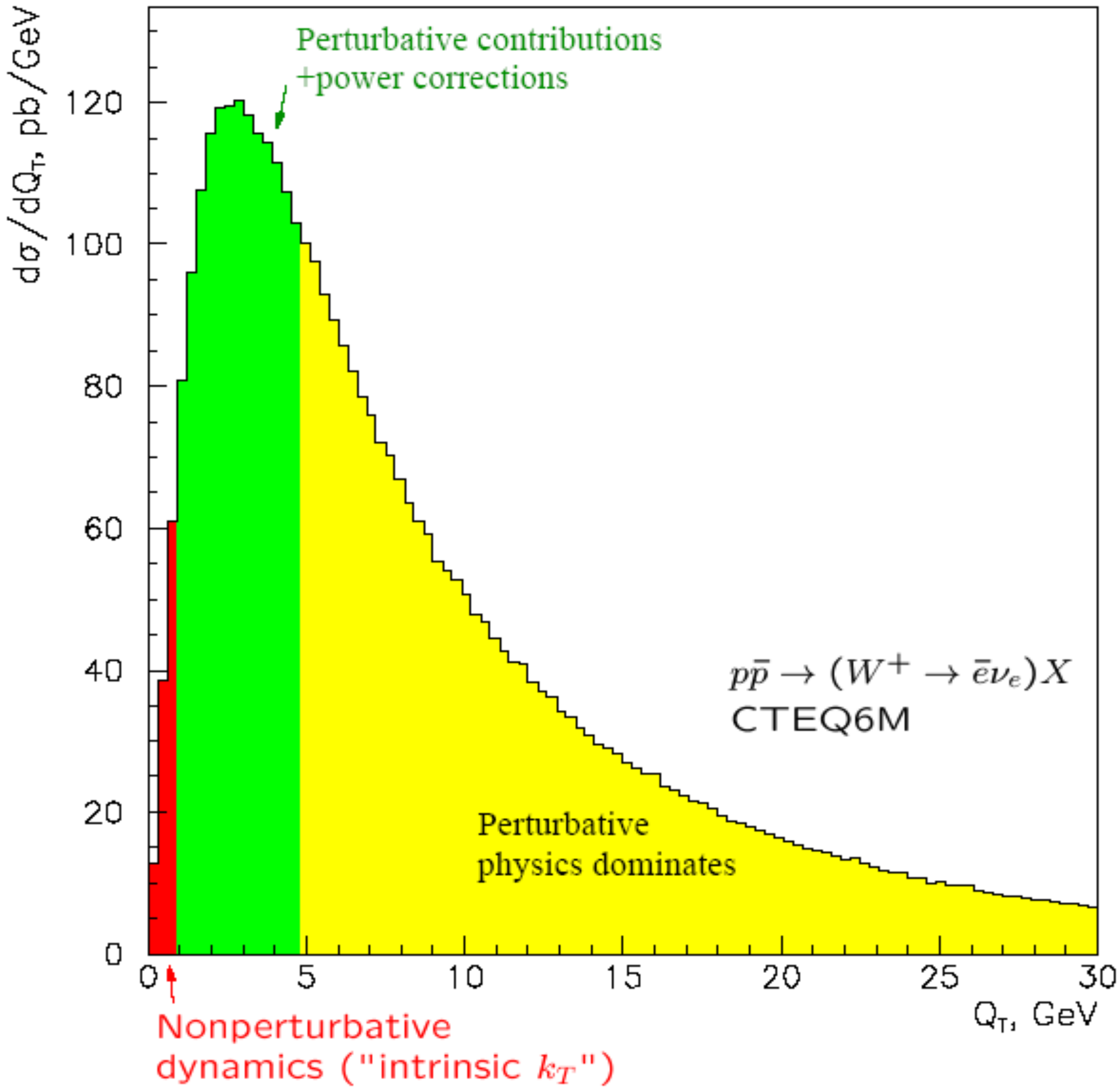
$$\Gamma_\mu(q^2,\epsilon) = -ie\mu^\epsilon\,\bar{u}\,(p_1)\gamma_\mu v(p_2)\,\rho(q^2,\epsilon)$$

$$\rho(q^2,\epsilon) = -\frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{-q^2-i\epsilon}\right)^\epsilon \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\left\{\frac{1}{(-\epsilon)^2}-\frac{3}{2(-\epsilon)}+4\right\}$$

$$= 1 - \frac{\alpha_s}{4\pi}C_F\,\ln^2(q^2/\mu^2) + \ldots$$ **Sudakov double logarithms**

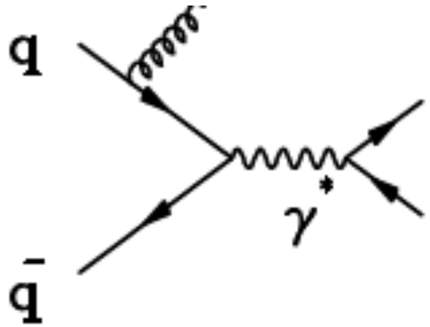
Common to all massless theories

Drell-Yan Q_T-distribution



Showing the different theoretical regions in momentum space

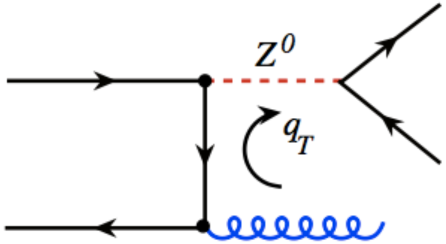
Drell-Yan type subprocess



Photon can be replaced by W, Z, Higgs, etc.

Leading double log contribution

- LO Differential Q_T-distribution as $Q_T \to 0$:



$$\frac{d\sigma}{dy\,dQ_T^2}\bigg|_{\rm LO} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

$$\Longrightarrow \int_0^{Q^2} \frac{d\sigma}{dy\,dQ_T^2}\bigg|_{\rm real+virutal} dQ_T^2 \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} + O(\alpha_s) \qquad \text{with } Q^2 \approx M_Z^2$$

- Integrated Q_T-distribution:

$$\int_0^{Q_T^2} \frac{d\sigma}{dy\,dp_T^2}\bigg|_{\rm real+virutal} dp_T^2 \equiv \left[\int_0^{Q^2} - \int_{Q_T^2}^{Q^2}\right] \frac{d\sigma}{dy\,dp_T^2}\bigg|_{\rm real+virutal} dp_T^2$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times \left[1 - \int_{Q_T^2}^{Q^2} 2C_F \frac{\alpha_s}{\pi} \frac{\ln\left(Q^2/p_T^2\right)}{p_T^2} dp_T^2\right] = \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times \left[1 - C_F\frac{\alpha_s}{\pi}\ln^2\left(Q^2/Q_T^2\right)\right]$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times \exp\left[-C_F\frac{\alpha_s}{\pi}\ln^2\left(Q^2/Q_T^2\right)\right]$$

Effect of gluon emission

Assume this exponentiates

Resummed Q_T distribution

- Differentiate the integrated Q_T-distribution:

$$\frac{d\sigma}{dy\,dQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \times \exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ln^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- Compare to the explicit LO calculation:

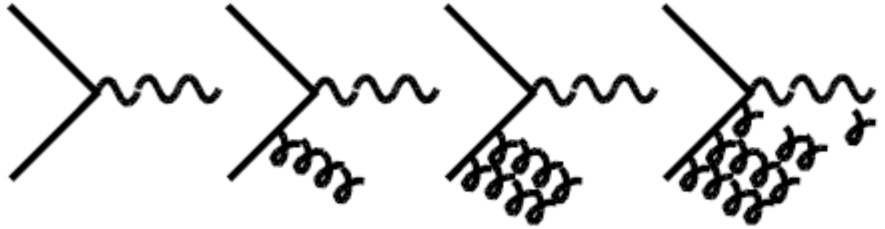
$$\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\text{LO}} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

Q_T-spectrum (as $Q_T \to 0$) is completely changed!

- We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \;\ldots$$

$L \propto \ln\left(Q^2/Q_T^2\right)$



Soft gluon emission treated as uncorrelated

Still a wrong Q_T-distribution

❑ **Experimental fact:** $\frac{d\sigma}{dy\,dQ_T^2} \Rightarrow$ finite [neither ∞ nor 0!] as $Q_T \to 0$

- Double Leading Logarithms Approximation (DLLA)

  radiated gluons are both soft and collinear with strong ordering in their transverse momenta

- Strong ordering in transverse momenta in DLLA
  - overly constrains the phase space ofthe emitted gluons
  - ignores the overall transverse momentum conservation

  $\Rightarrow$ DLLA over suppresses small Q_T region

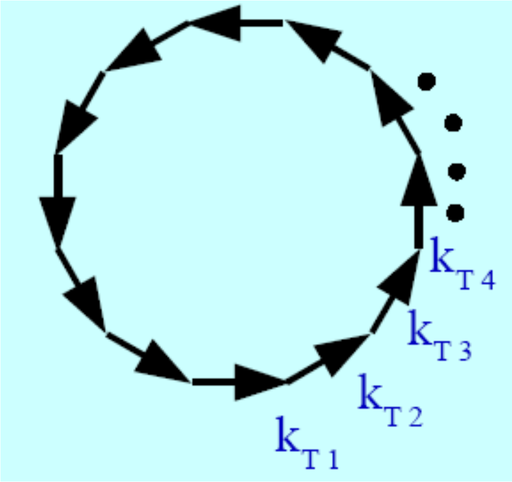
Resummation of uncorrelated soft gluon emission leads to a too strong suppression at $Q_T = 0$!

Still a wrong Q_T-distribution

- **Why?**

  Particle can receive many finite k_T kicks via soft gluon radiation yet still have $Q_T = 0$

  – Need a vector sum!



- **Subleading logarithms are equally important at $Q_T = 0$**
- **Solution:**

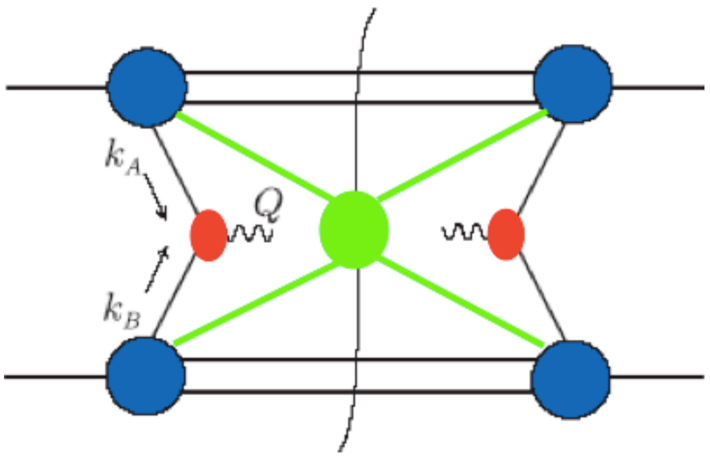
  To impose the 4-momentum conservation at each step of soft gluon resummation

  ⟹ TMD factorization

CSS b-space resummation formalism

Collins, Soper, Sterman, 1985

- TMD-factorized cross section:



$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2k_{A_T} d^2k_{B_T} d^2k_{s,T}}{(2\pi)^6}$$

$$\times P_{f/A}(\xi_a, k_{A_T}) P_{\bar{f}/B}(\xi_b, k_{B_T}) H_{f\bar{f}}(Q^2) S(k_{s,T})$$

$$\times \delta^2(\vec{Q}_T - \vec{k}_{A_T} - \vec{k}_{B_T} - \vec{k}_{s,T})$$

$$\delta^2(\vec{Q}_T - \prod_i \vec{k}_{i,T}) = \frac{1}{(2\pi)^2} \int d^2b \, e^{i\vec{b}\cdot\vec{Q}_T} \prod_i e^{-i\vec{b}\cdot\vec{k}_{i,T}}$$

- Factorized cross section in "impact parameter b-space":

$$\frac{d\sigma_{AB}(Q,b)}{dQ^2} = \sum_f \int d\xi_a d\xi_b \bar{P}_{f/A}(\xi_a, b, n) \bar{P}_{\bar{f}/B}(\xi_b, b, n) H_{f\bar{f}}(Q^2) U(b,n)$$

- Resummation: Two equations, two resummation of log's

$$\mu_{\text{ren}} \frac{d\sigma}{d\mu_{\text{ren}}} = 0 \qquad\qquad n^\nu \frac{d\sigma}{dn^\nu} = 0$$

CSS b-space resummation formalism

- Solve those two equations and transform back to Q_T:

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T}\tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2,Q^2)$$

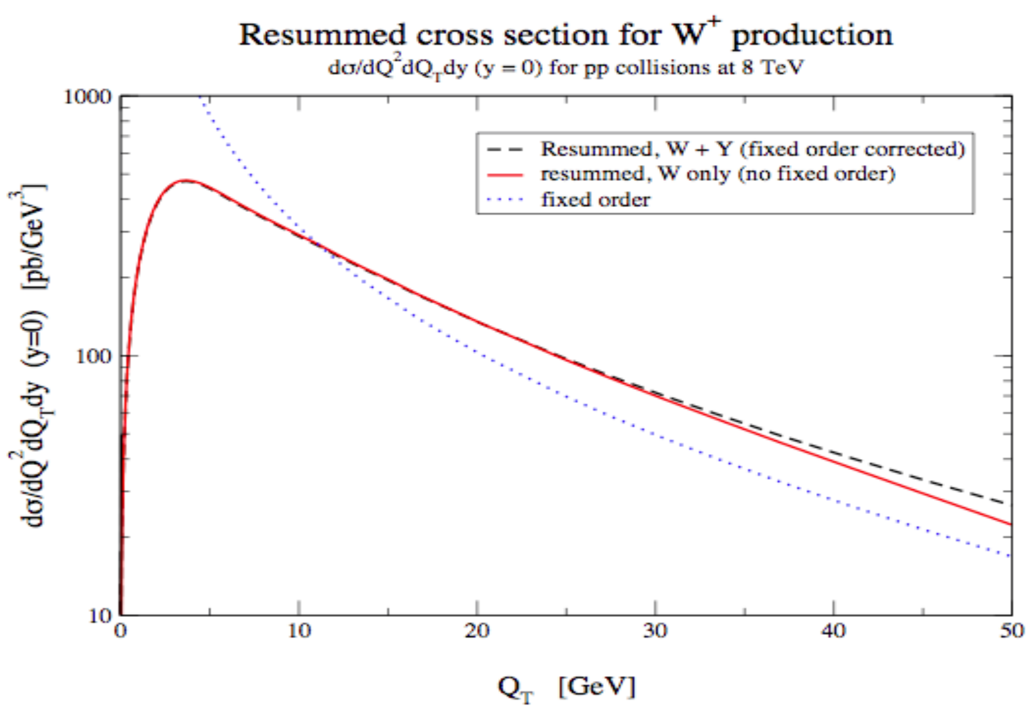
$$= \frac{1}{(2\pi)}\int_0^\infty db\, J_0(bQ_T)\, b\tilde{W}_{AB}(b,Q) + \left[\frac{d\sigma_{AB}^{(\mathrm{Pert})}}{dQ^2 dQ_T^2} - \frac{d\sigma_{AB}^{(\mathrm{Asym})}}{dQ^2 dQ_T^2}\right]$$

resummed

No large log's

- Role of each term:

implemented in **RESBOS** code



CSS b-space resummation formalism

- b-space distribution:

$$\tilde{W}_{AB}(b,Q) \equiv \sum_{i,j} \tilde{W}_{ij}(b,Q)\hat{\sigma}_{ij}(Q)$$

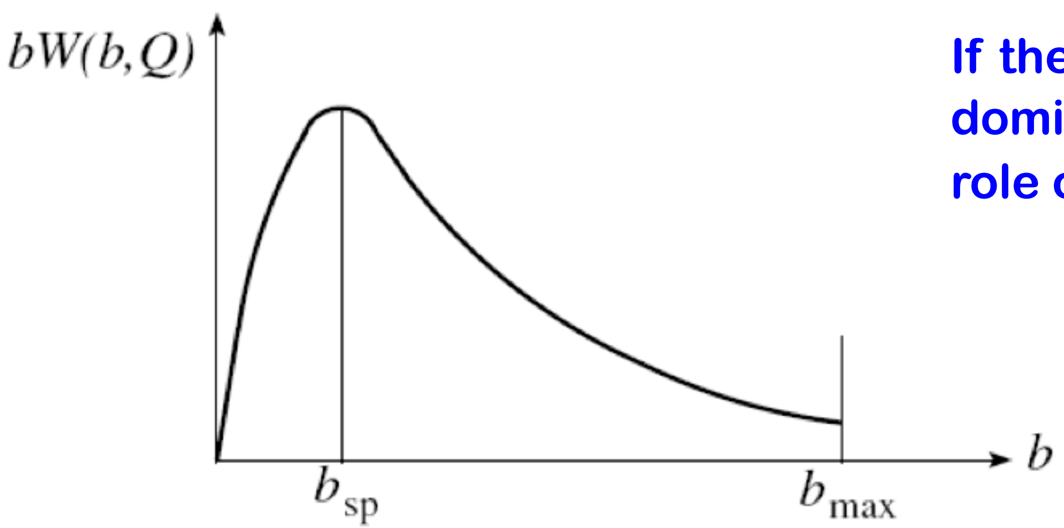
- Perturbative contribution ($b \ll 1/\Lambda_{\rm QCD}$):

$$W_{AB}^{\rm Pert}(b,Q) = \sum_{a,b,i,j} \sigma_{ij\to V}^{\rm LO} \left[\phi_{a/A} \otimes C_{a\to i}\right] \otimes \left[\phi_{b/B} \otimes C_{b\to j}\right] e^{-S(b,Q)}$$

Sudakov Form factor

Collinear PDFs

- Nonperturbative contribution from large b-region:



If the area under the curve is dominated by samll-b region, the role of large-b region is minimal

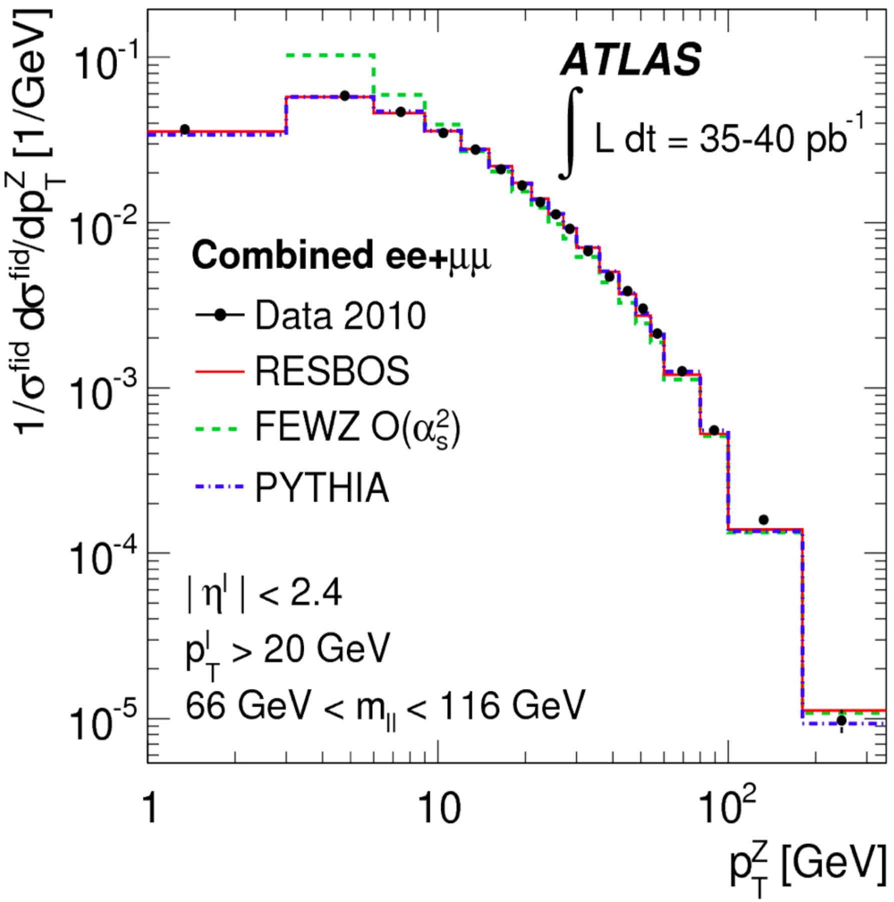
- Large Q, and/or
- Large $\sqrt{S}$

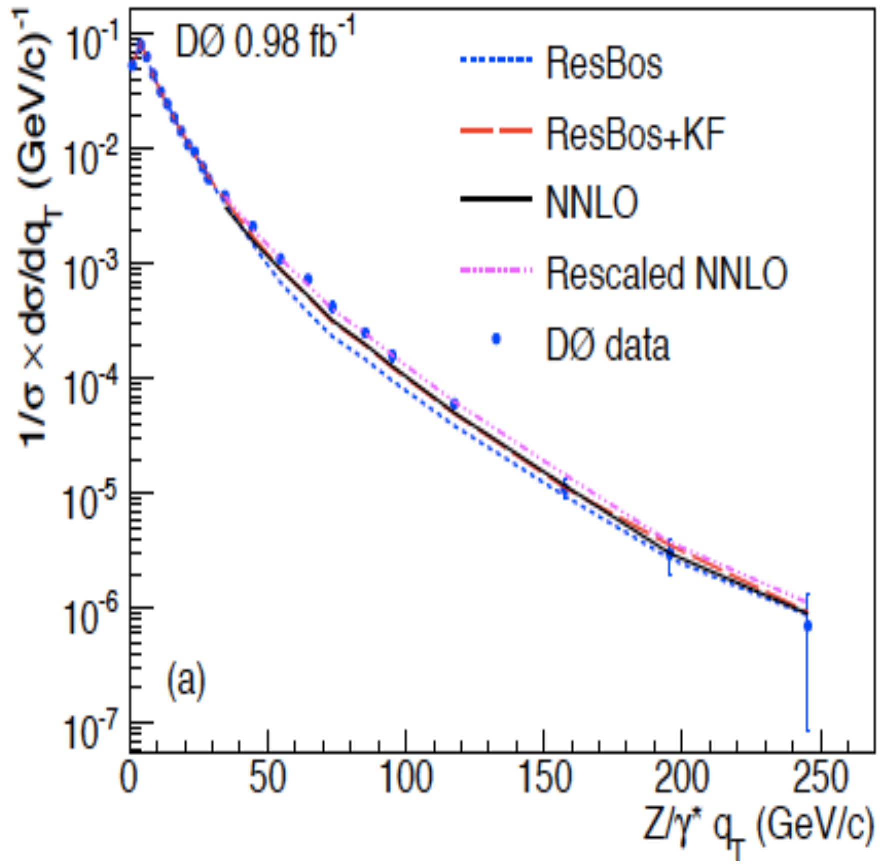
➡ *Absolute prediction!*

Qiu, Zhang, 2001

Phenomenology

- Compare with the LHC data:

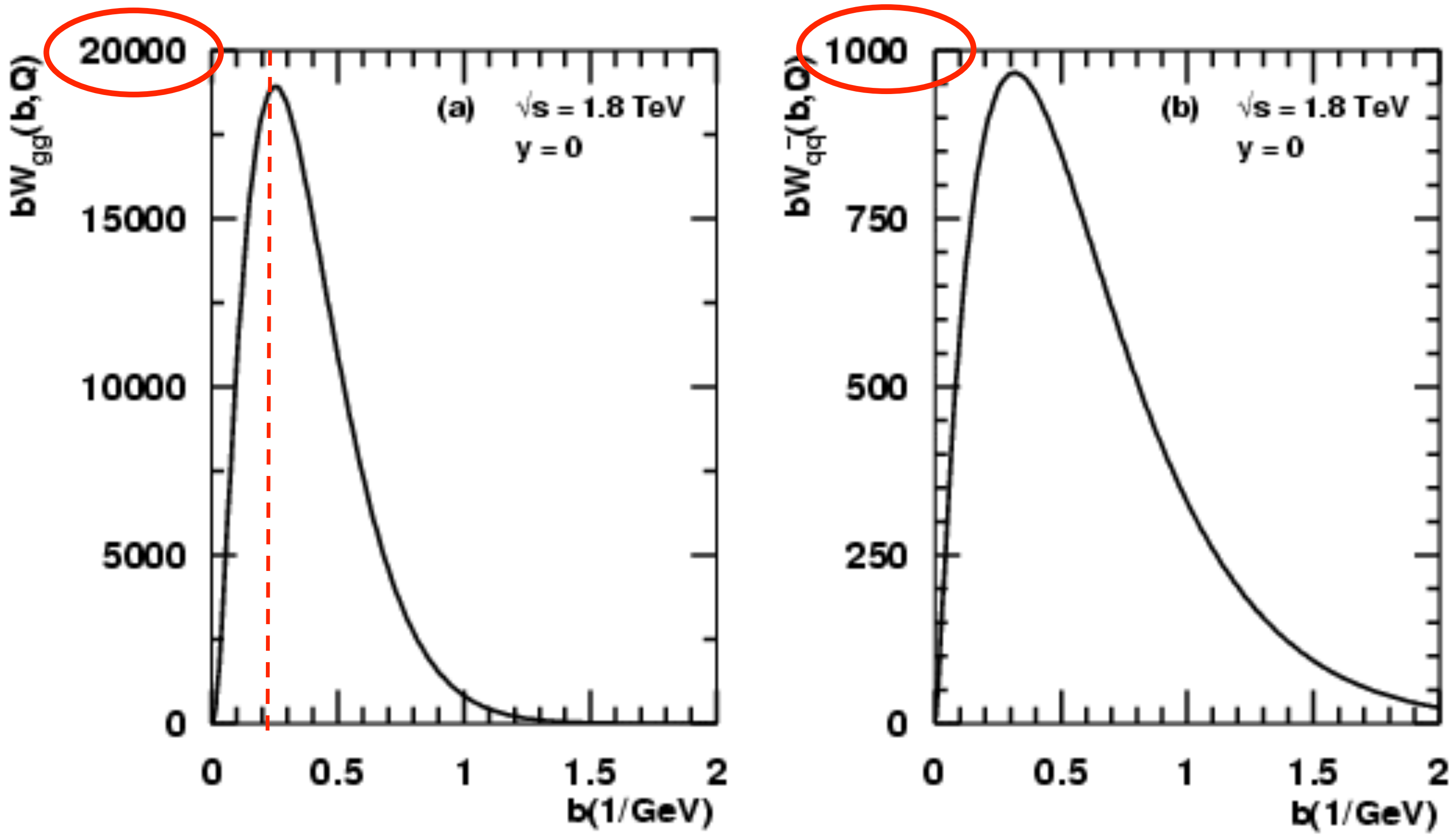




Phenomenology

Berger, Qiu, Wang, 2005

- Upsilon production (low Q, large phase space):



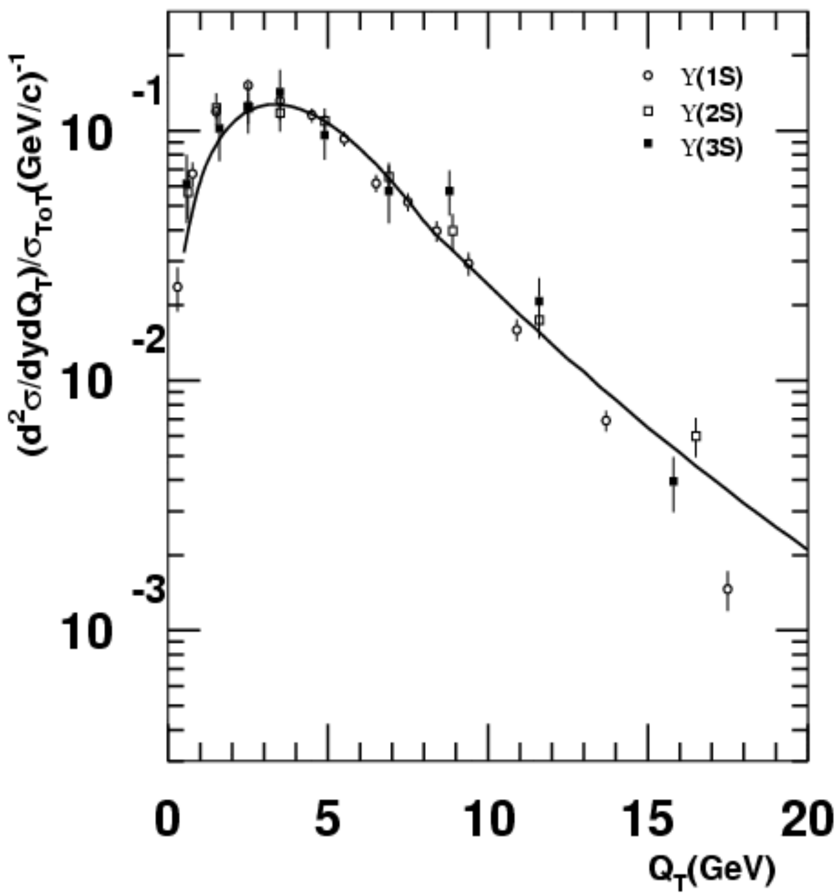
Gluon-gluon dominate the production

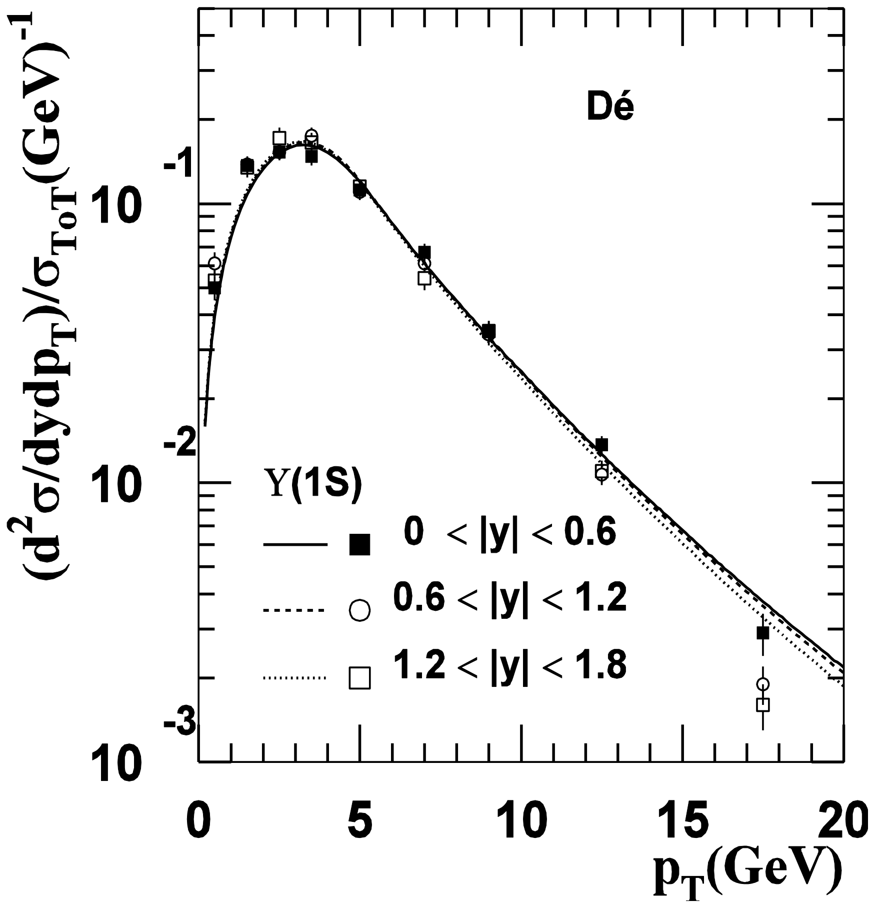
Dominated by perturbative contribution even M_Υ~10 GeV

Phenomenology

Berger, Qiu, Wang, 2005

- Prediction vs Tevatron data:





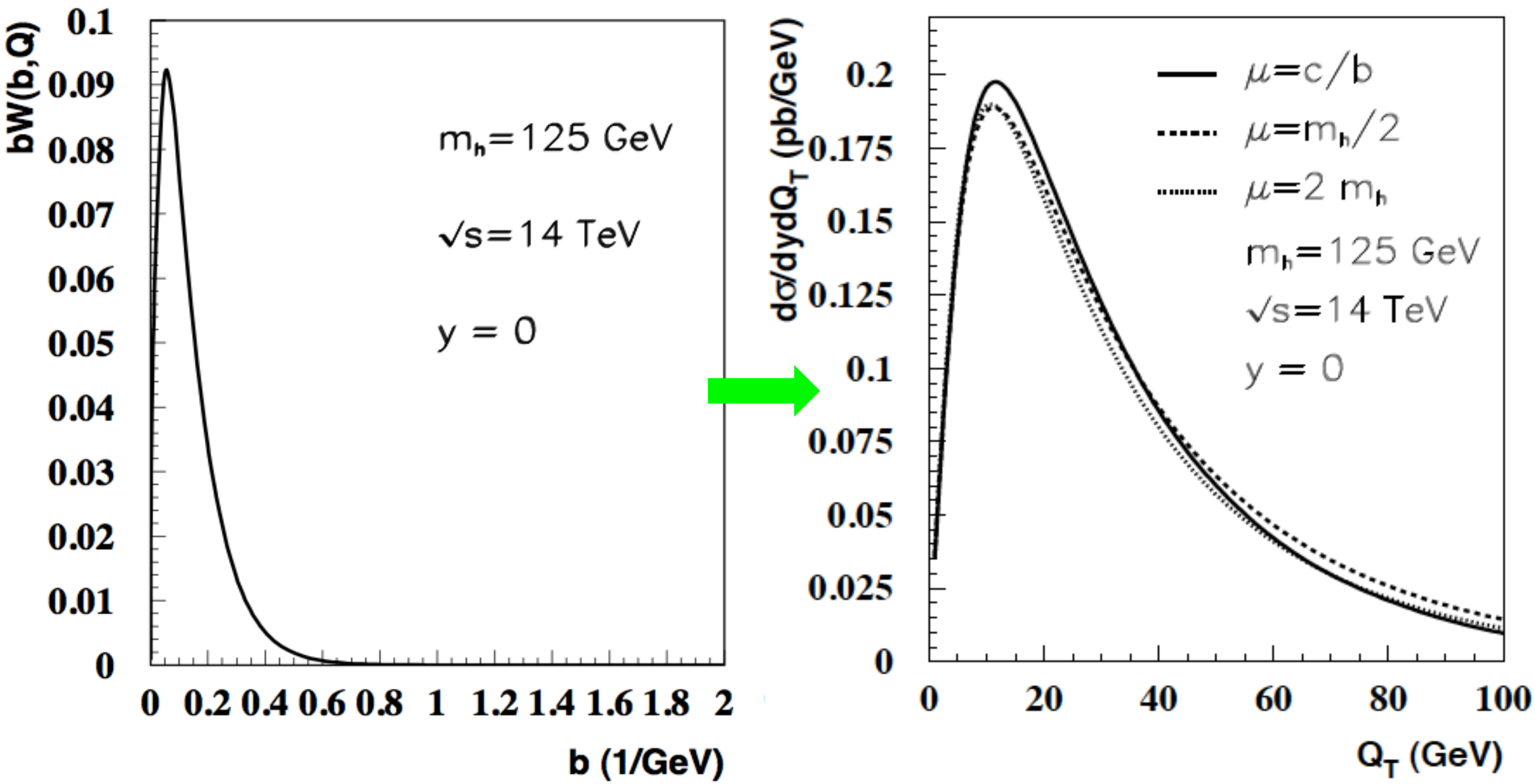
CDF Run-I data

DO Run-II data

Phenomenology

❑ Higgs at the LHC:

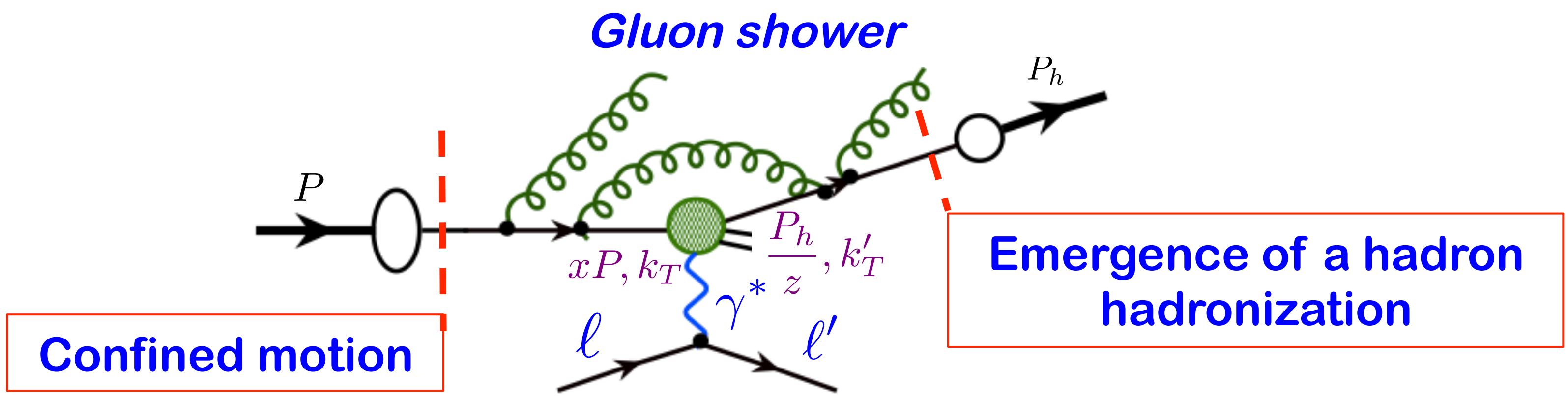
Berger, Qiu, 2003



Effectively NO non-perturbative uncertainty – Shower dominates!

Parton k_T at the hard collision

- Sources of parton k_T at the hard collision:



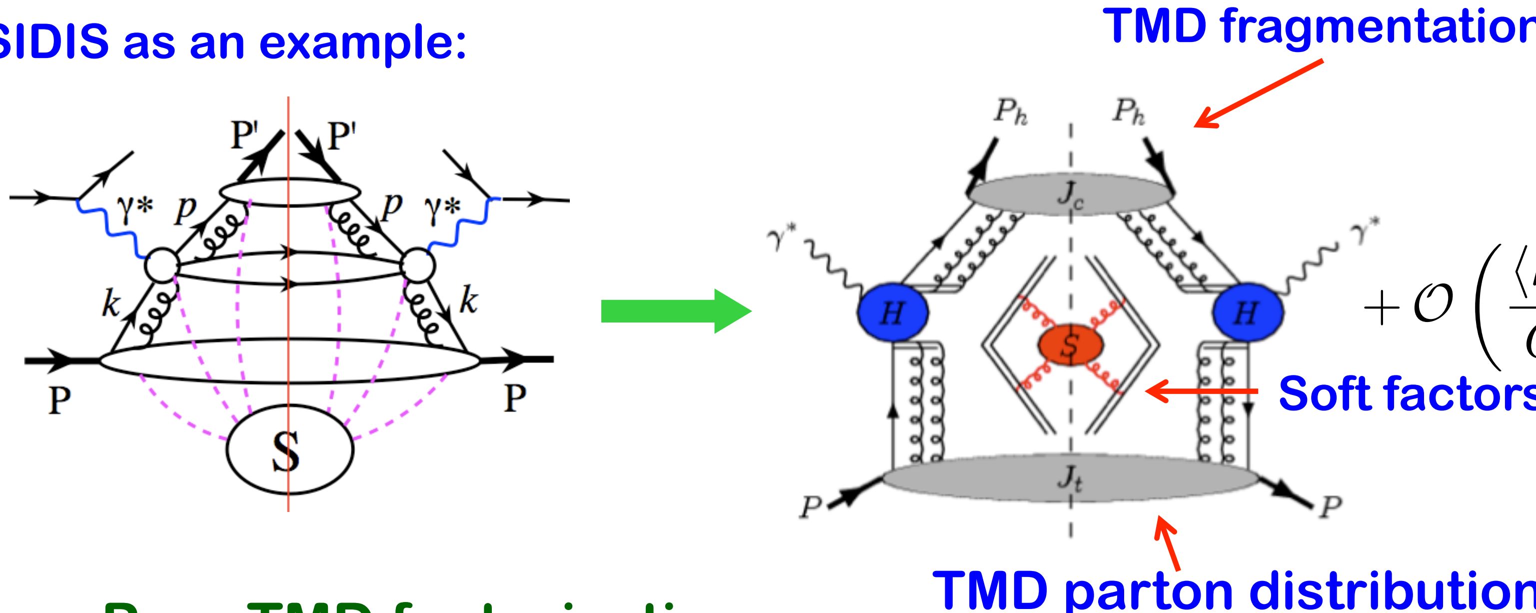
- Large k_T generated by the shower (caused by the collision):
  - ✧ Q^2-dependence – linear evolution equation of TMDs in b-space
  - ✧ The evolution kernels are perturbative at small b, but, not large b
  - ➡ The nonperturbative inputs at large b could impact TMDs at all Q^2

- Challenge: to extract the “true” parton’s confined motion:
  - ✧ Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs

Collinear vs TMD Factorization – SIDIS

- **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

- **Low P_{hT} – TMD factorization:**

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

- **High P_{hT} – Collinear factorization:**

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q,P_{h\perp},\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{P_{h\perp}},\frac{1}{Q}\right)$$

- **P_{hT} Integrated - Collinear factorization:**

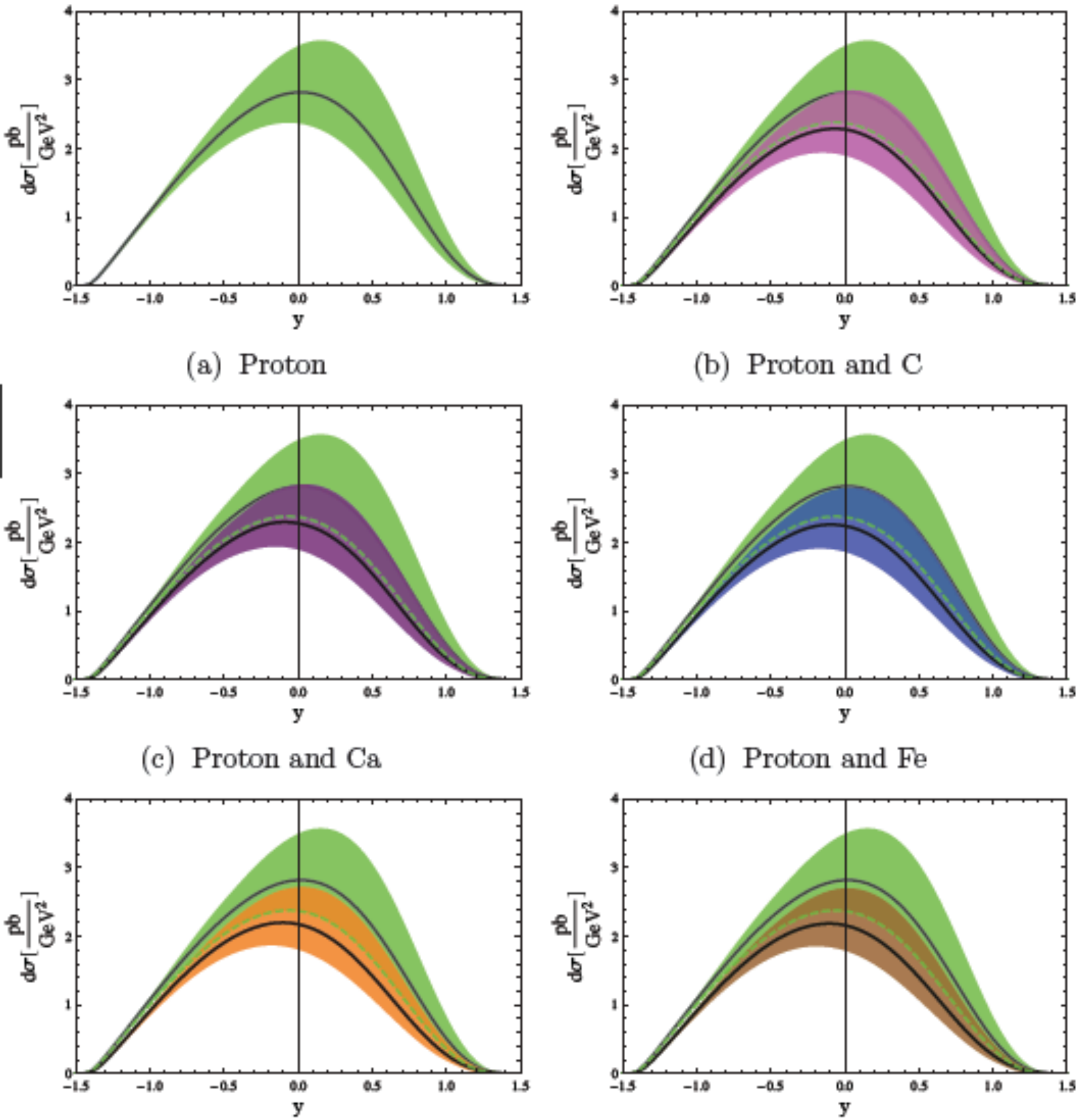
$$\sigma_{\rm SIDIS}(Q,x_B,z_h)=\tilde{H}(Q,\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{Q}\right)$$

Summary of lecture three

- Event shape – jettiness is a new powerful observable for studying the pattern of QCD (medium induced) radiation
- Heavy quarkonium production is still a very fascinating subject challenging our understanding of QCD bound states
- The need to have a heavy quark pair, heavy quarkonium production is an ideal place to study QCD power corrections, coherent multiple scatterings, …
- TMD factorization of two-scale observables (one large, one small) provides a new and unique probe to "see" the confined motion: the large scale to pin down the parton d.o.f. while the small scale to probe the nonperturbative structure as well as the motion
- Proton spin provides another controllable "knob" to help isolate various physical effects

Backup slides

Jet rapidity distributions in e+A for various nuclei



(a) Proton

(b) Proton and C

(c) Proton and Ca

(d) Proton and Fe

(e) Proton and Au

(f) Proton and Ur

NNLL resummation

$Q_e = 90$ GeV,

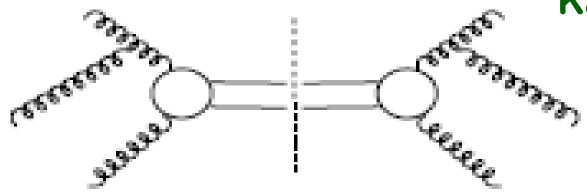
$P_{J_T} = 20$ GeV

$\tau_1 = 1.5$ GeV

Effect of nPDFs and smearing

Quarkonium P_T-broadening in p(d)+A

Kang, Qiu, PRD77(2008)



- ❑ **Initial-state only:**

$$\Delta\langle q_T^2\rangle^{(I)}_{J/\psi} = C_A\left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)$$

$$\Delta\langle q_T^2\rangle_{DY} \approx C_F\left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)$$

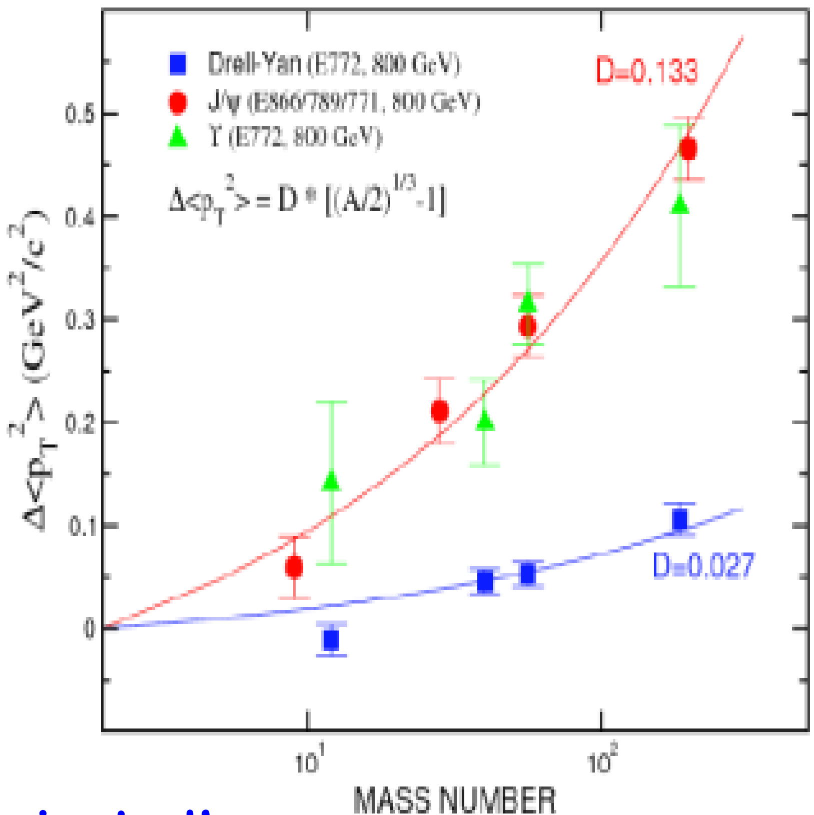
- ❑ **Experimental data from d+A:**

Clear $A^{1/3}$ dependence

But, wrong normalization!

$$\Delta\langle q_T^2\rangle^{(I)}_{J/\psi}\,/\,\Delta\langle q_T^2\rangle_{DY}\Big|_{thy} = C_A/C_F = 2.25$$

$$\Delta\langle q_T^2\rangle^{(I)}_{J/\psi}\,/\,\Delta\langle q_T^2\rangle_{DY}\Big|_{exp} = 0.133/0.027 \approx 4.9$$



Final-state effect – octet channel dominated!

Only depend on observed quarkonia

J.C.Peng, hep-ph/9912371

Johnson,et al, 2007

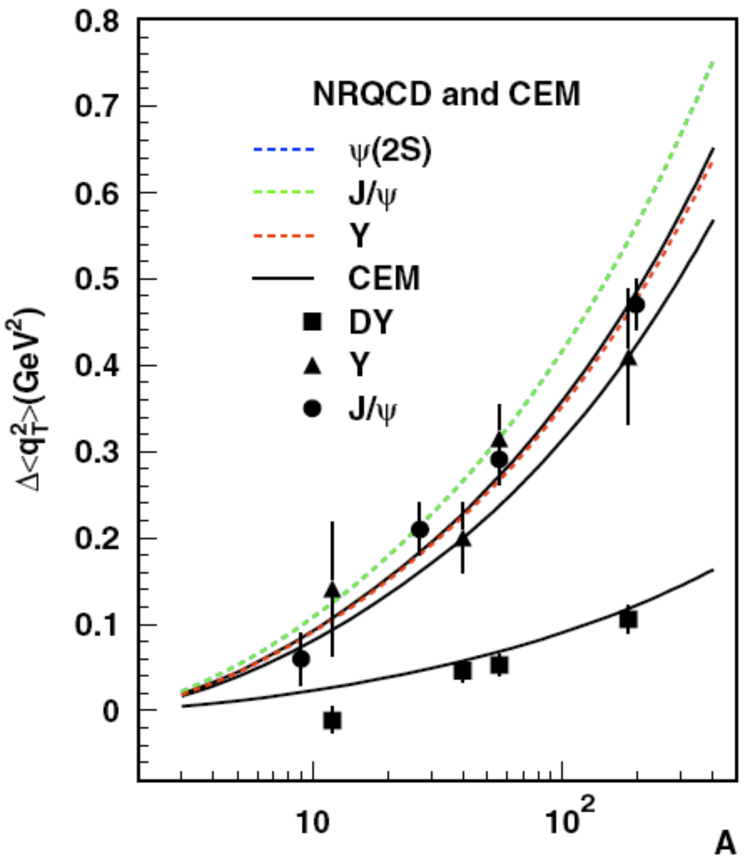
Broadening of heavy quarkonia in p(d)+A

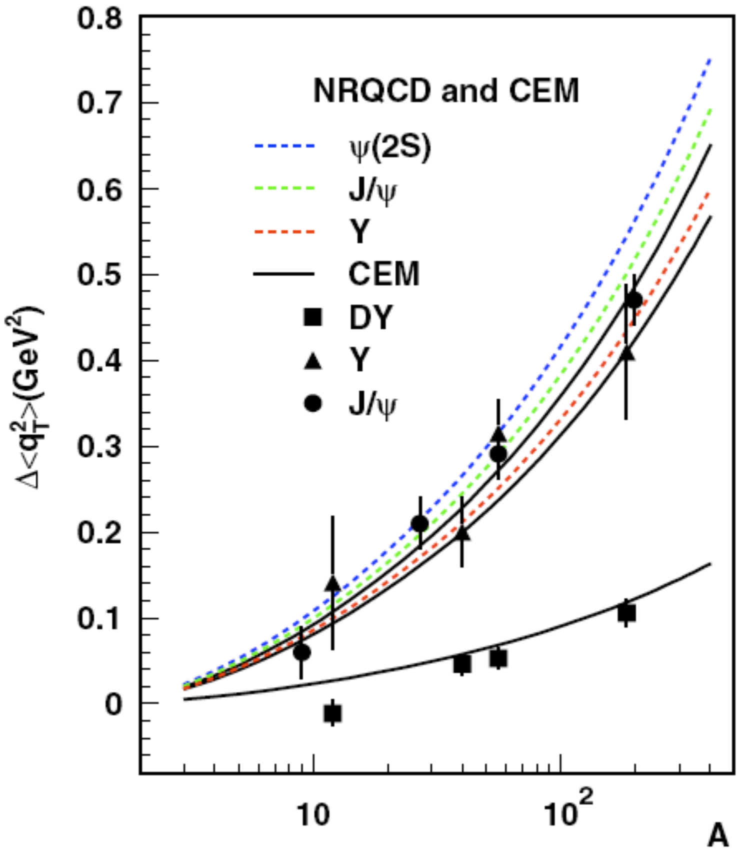
Kang, Qiu, PRD77(2008)

- ❑ Final-state effect is important:

$$\Delta\langle q_T^2\rangle_{J/\psi}^{(I+F)} \Big/ \Delta\langle q_T^2\rangle_{\mathrm{DY}}\Big|_{\mathrm{thy}} \approx 2C_A/C_F = 4.5$$

in both CEM and NRQCD



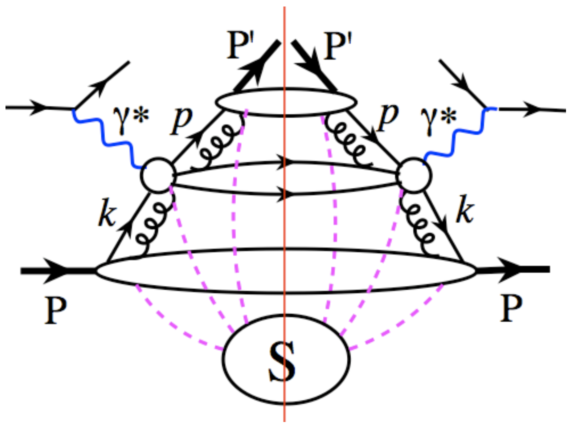


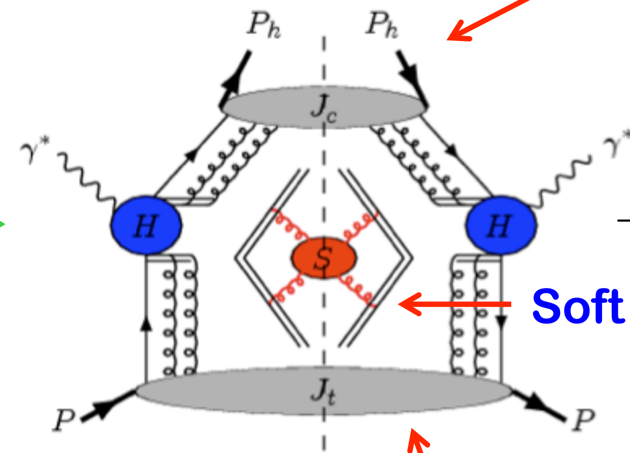
- ❑ Mass – independence, not very sensitive to the feeddown

Collinear vs TMD Factorization – SIDIS

- ❑ **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:





$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}, \frac{\langle p^2\rangle}{Q^2}\right)$$

TMD fragmentation

Soft factors

TMD parton distribution

- ❑ **Operator definition:**

$$\Phi^{[U]}(x, p_T; n, \mu) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,p\cdot\xi}\, \langle P,S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P,S\rangle_{\xi^+=0} + \mathrm{UVCT}(\mu)$$

$$\equiv \int^{\mathcal{O}(\mu^2)} dp^2\, \widetilde{\Phi}^{[U]}(p;n) = \int^{\infty} dp^2\, \widetilde{\Phi}^{[U]}(p;n) - \int_{\mathcal{O}(\mu^2)}^{\infty} dp^2\, \widetilde{\Phi}^{[U]}(p;n)$$

This operator definition is scheme dependent, & needed for calculating the short-distance hard coefficients, order-by-order, in perturbation theory

QCD and hadrons