Perturbative QCD and Hard Processes

Jianwei Qiu
Brookhaven National Laboratory
Stony Brook University

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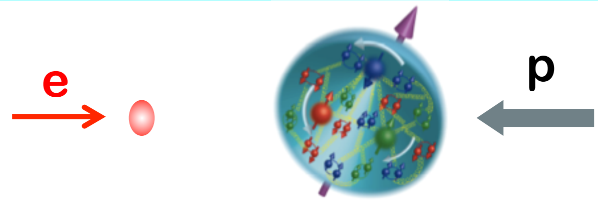
Summary of lecture one

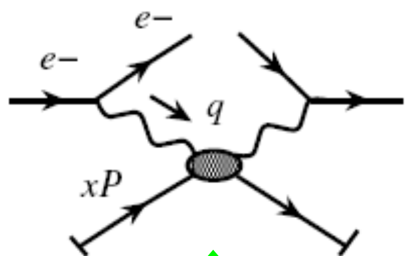
- QCD is a SU(3) color non-Abelian gauge theory of quark and gluon fields
- QCD perturbation theory works at high energy because of the Asymptotic Freedom
- Perturbative QCD calculations make sense only for infrared safe (IRS) quantities – e^+e^- total cross section
- Jets in high energy collisions provide us the "trace" of energetic quarks and gluons
- Factorization is necessary for pQCD to treat observables (cross sections) with "identified hadrons"
- Predictive power of QCD factorization relies on the universality of PDFs (or TMDs, GPDs, …), the calculations of perturbative coefficient functions – hard parts

From one hadron to two hadrons

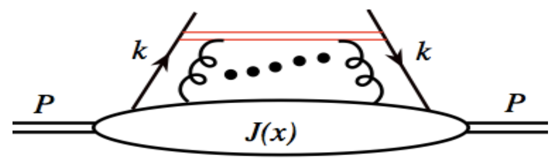
- **One hadron:**



$\sigma_{\rm tot}^{\rm DIS} \sim$



$\otimes$

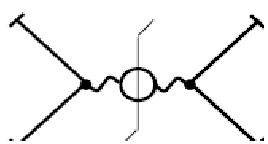


$+ \, O\left(\frac{1}{QR}\right)$

Hard-part Probe	Parton-distribution Structure	Power corrections Approximation

- **Two hadrons:**

$\sigma_{\rm tot}^{\rm DY} \sim$



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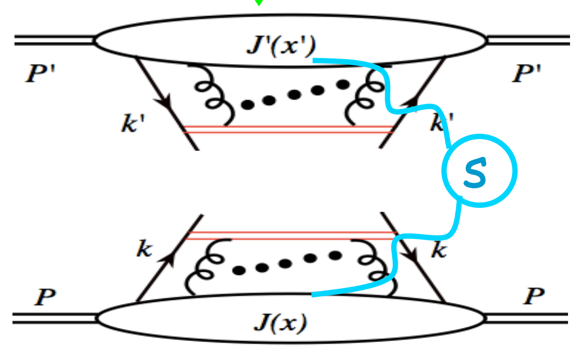
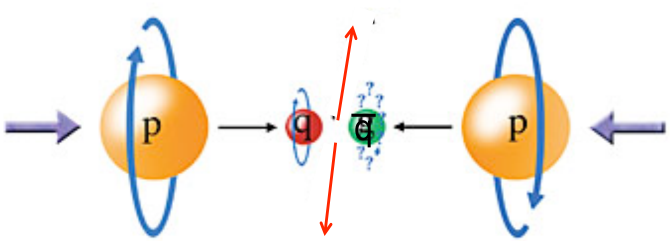


$+ \, O\left(\frac{1}{QR}\right)$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

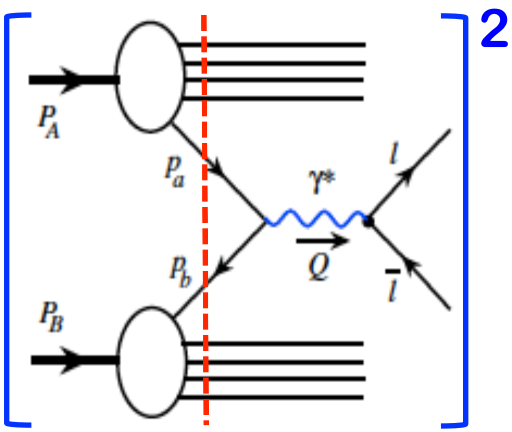
❑ Drell-Yan mechanism:

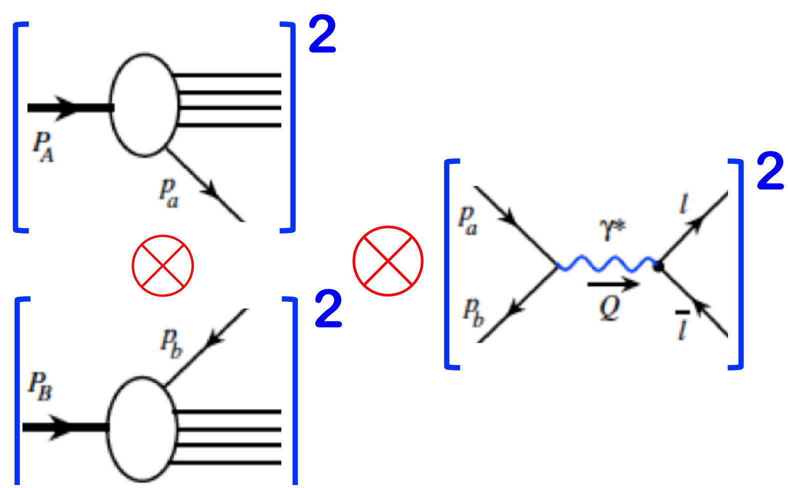
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X$$ **with** $q^2 \equiv Q^2 \gg \Lambda^2_{\rm QCD} \sim 1/{\rm fm}^2$

Lepton pair – from decay of a virtual photon, or in general, a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:





$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha^2_{em}}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

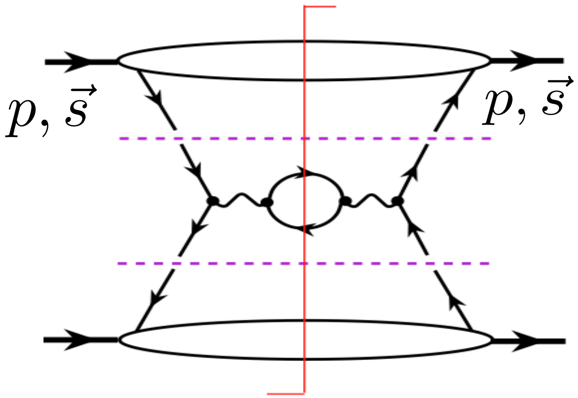
Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

$$x_A = \frac{Q}{\sqrt{S}}e^{y} \qquad x_B = \frac{Q}{\sqrt{S}}e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD

- **Spin decomposition – cut diagram notation:**



$\Leftarrow$ all γ structure: $\gamma^\alpha,\ \gamma^\alpha\gamma^5,\ \sigma^{\alpha\beta}(\text{or } \gamma^5\sigma^{\alpha\beta}),\ I,\ \gamma^5$

$\Leftarrow$ all γ structure: $\gamma^\alpha,\ \gamma^\alpha\gamma^5,\ \sigma^{\alpha\beta}(\text{or } \gamma^5\sigma^{\alpha\beta}),\ I,\ \gamma^5$

- **Factorized cross section:**

$$\sigma(Q,\vec{s}) \pm \sigma(Q,-\vec{s}) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle \pm \langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$$

- **Parity-Time reversal invariance:**

$$\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle = \langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle$$

- **Good operators:**

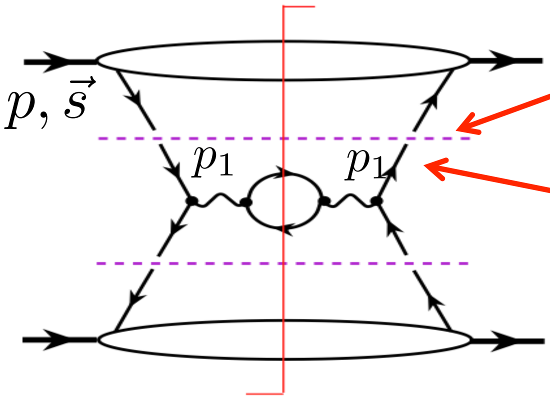
$$\langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle = \pm\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

"+" for spin-averaged cross section → PDFs:

$$\langle p,\vec{s}|\overline{\psi}(0)\gamma^+\psi(y^-)|p,\vec{s}\rangle, \qquad \langle p,\vec{s}|F^{+i}(0)F^{+j}|p,\vec{s}\rangle(-g_{ij})$$

Drell-Yan process in QCD

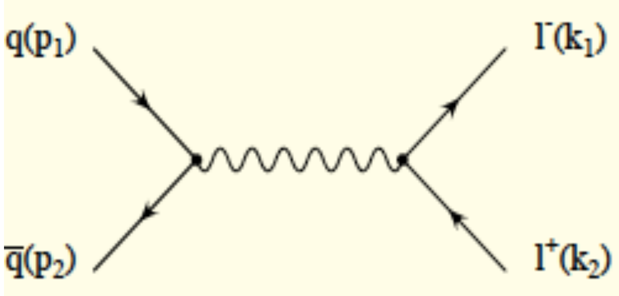
❑ Spin-averaged cross section – Lowest order:



$$\frac{1}{2p^+}\gamma^+\,\delta(x-p_1^+/p^+)dx$$

$$\frac{1}{2}\gamma\cdot p=\frac{1}{2}\sum_s u_s(p)\overline{u}_s(p)$$

$$\hat{s}=(p_1+p_2)^2=Q^2$$



❑ Lowest order partonic cross section:

$$\overline{\Sigma}\,|M|^2=\frac{e_q^2e^4}{\hat{s}^2}\left[\frac{1}{2}\right]\left[\frac{1}{2}\right]3\left\{\frac{1}{3}\right\}\left\{\frac{1}{3}\right\}\mathrm{Tr}[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu]\mathrm{Tr}[\not{k}_1\gamma_\nu\not{k}_2\gamma_\mu]=\left\{\frac{1}{3}\right\}e_q^2e^4(1+\cos^2\theta)$$

$$PS^{(2)}=\frac{d^2k_1}{(2\pi)^3 2E_1}\frac{d^2k_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-k_1-k_2)=\frac{1}{16\pi}d\cos(\theta)$$

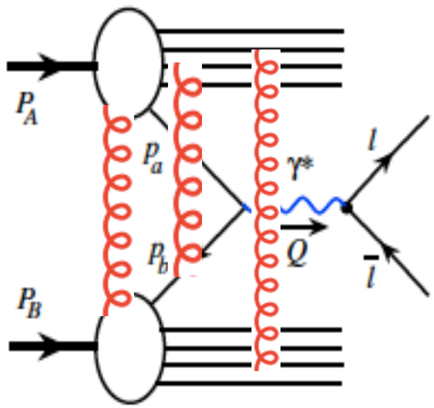
$$\sigma(q\bar{q}\to l^+l^-)=\left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2\equiv\sigma_0$$

❑ Drell-Yan cross section:

$$\frac{d\sigma}{dQ^2dy}=\Sigma_q\int dx_A\,dx_B\,\phi_{q/A}(x_A)\phi_{\bar{q}/B}(x_B)\left[\left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2\right]\delta(Q^2-\hat{s})\,\delta(y-\frac{1}{2}\ln(\frac{x_A}{x_B}))$$

Drell-Yan process in QCD

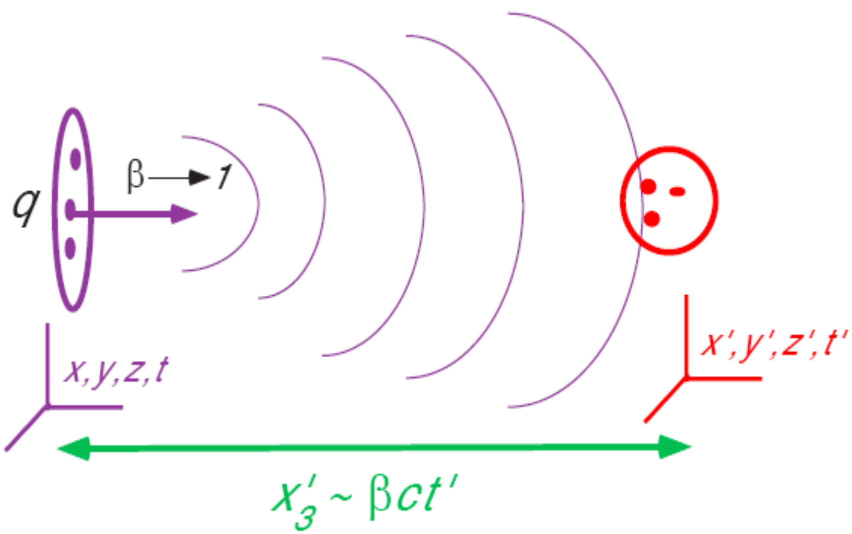
❑ Beyond the lowest order:



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

❑ Factorization – power suppression of soft gluon interaction:



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$ $\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$ $\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

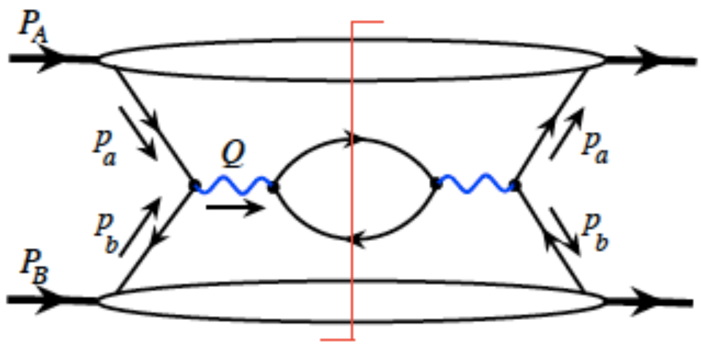
Drell-Yan process in QCD

❑ Factorization – approximation:

Collins, Soper, Sterman, 1988

- ✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

- ✧ Maintain the universality of PDFs:

  Long-range soft gluon interaction has to be power suppressed

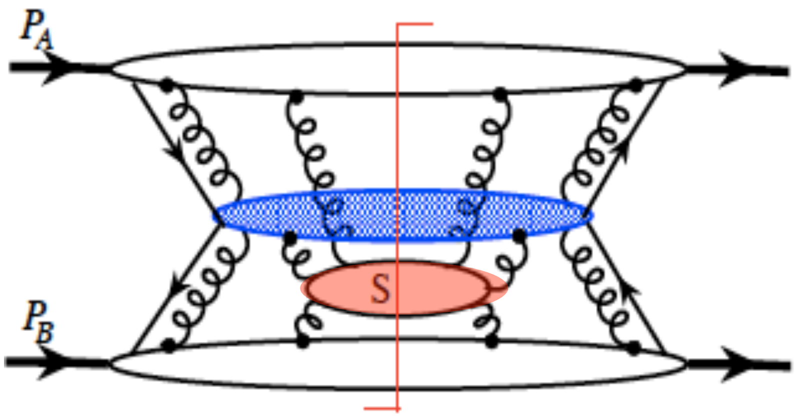
- ✧ Infrared safe of partonic parts:

  Cancelation of IR behavior
  Absorb all CO divergences into PDFs

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

Drell-Yan process in QCD

❑ **Leading singular integration regions (pinch surface):**



Hard: all lines off-shell by Q

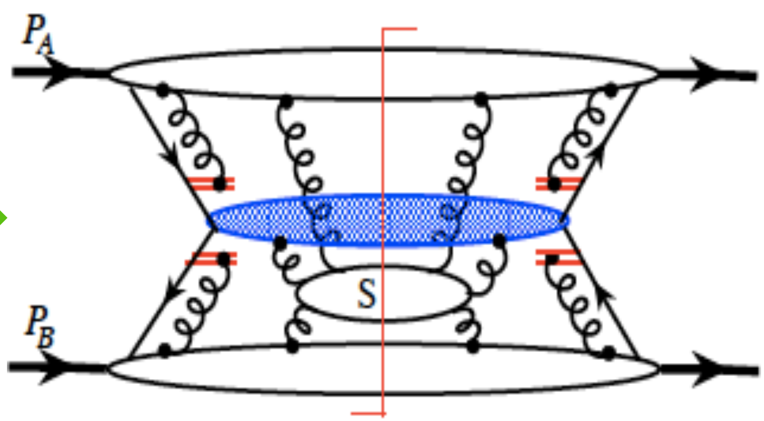
Collinear:
✧ lines collinear to A and B
✧ One "physical parton" per hadron

Soft: all components are soft

❑ **Collinear gluons:**

✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$

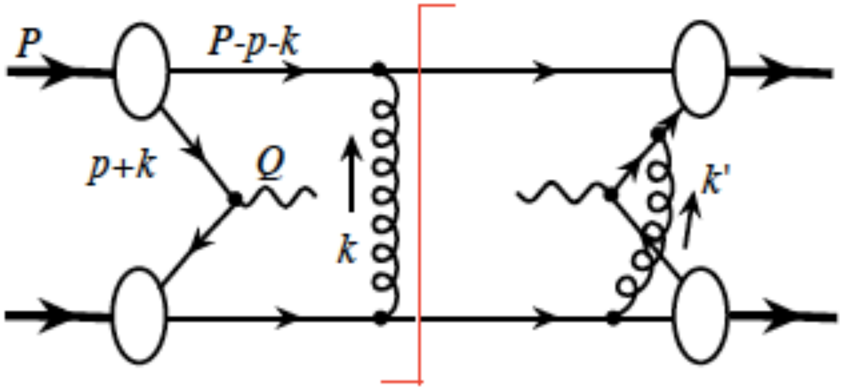
✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD

❑ Trouble with soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

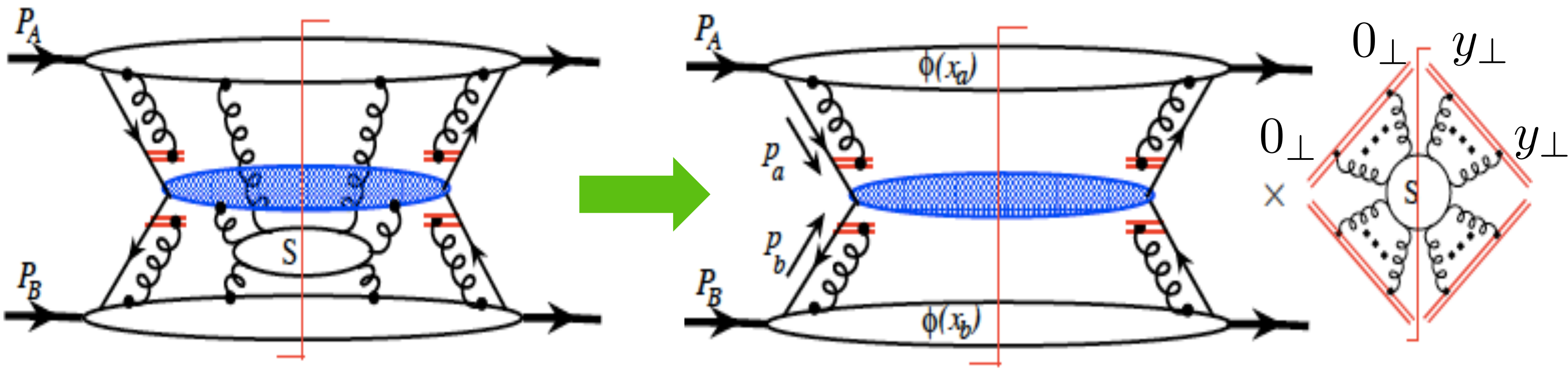
- ✧ **Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B**

- ✧ **The soft gluon approximations (with the eikonal lines) need $k^{\pm}$ not too small. But, $k^{\pm}$ could be trapped in "too small" region due to the pinch from spectator interaction:** $k^{\pm} \sim M^2/Q \ll k_{\perp} \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^{\pm}$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$
$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^y \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

The factorization arguments are independent of the spin states of the colliding hadrons

→ same formula with polarized PDFs for γ^*,W/Z, H^0...

Cross section with a single hard scale

- **Partonic hard parts:**

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO

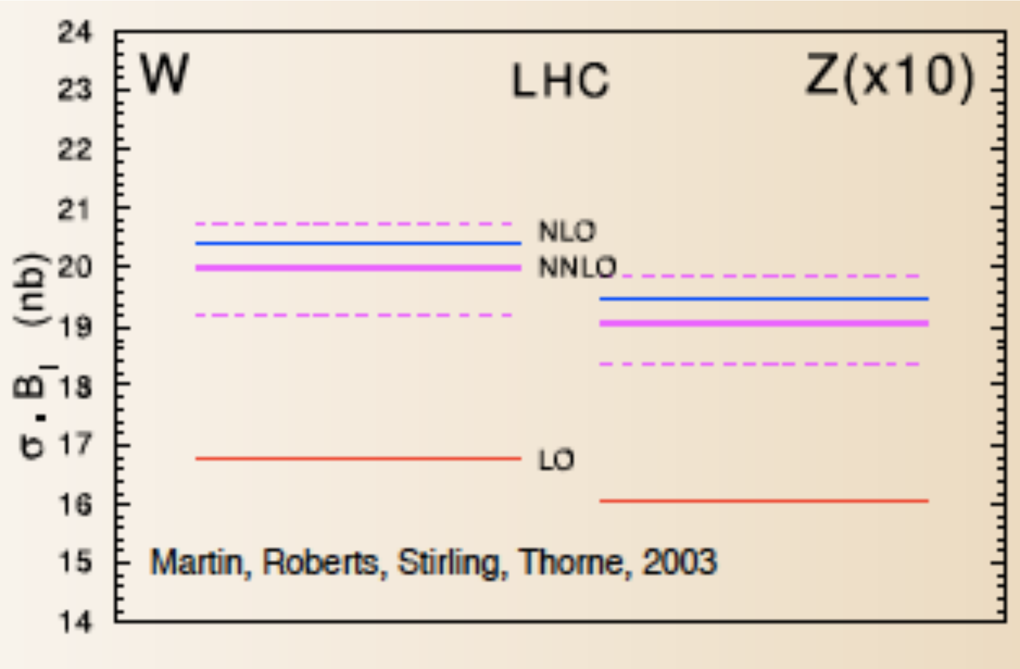
- **NNLO total x-section** $\sigma(AB \to W, Z)$:

(Hamberg, van Neerven, Matsuura; Harlander, Kilgore 1991)

✧ **Scale dependence:**

a few percent

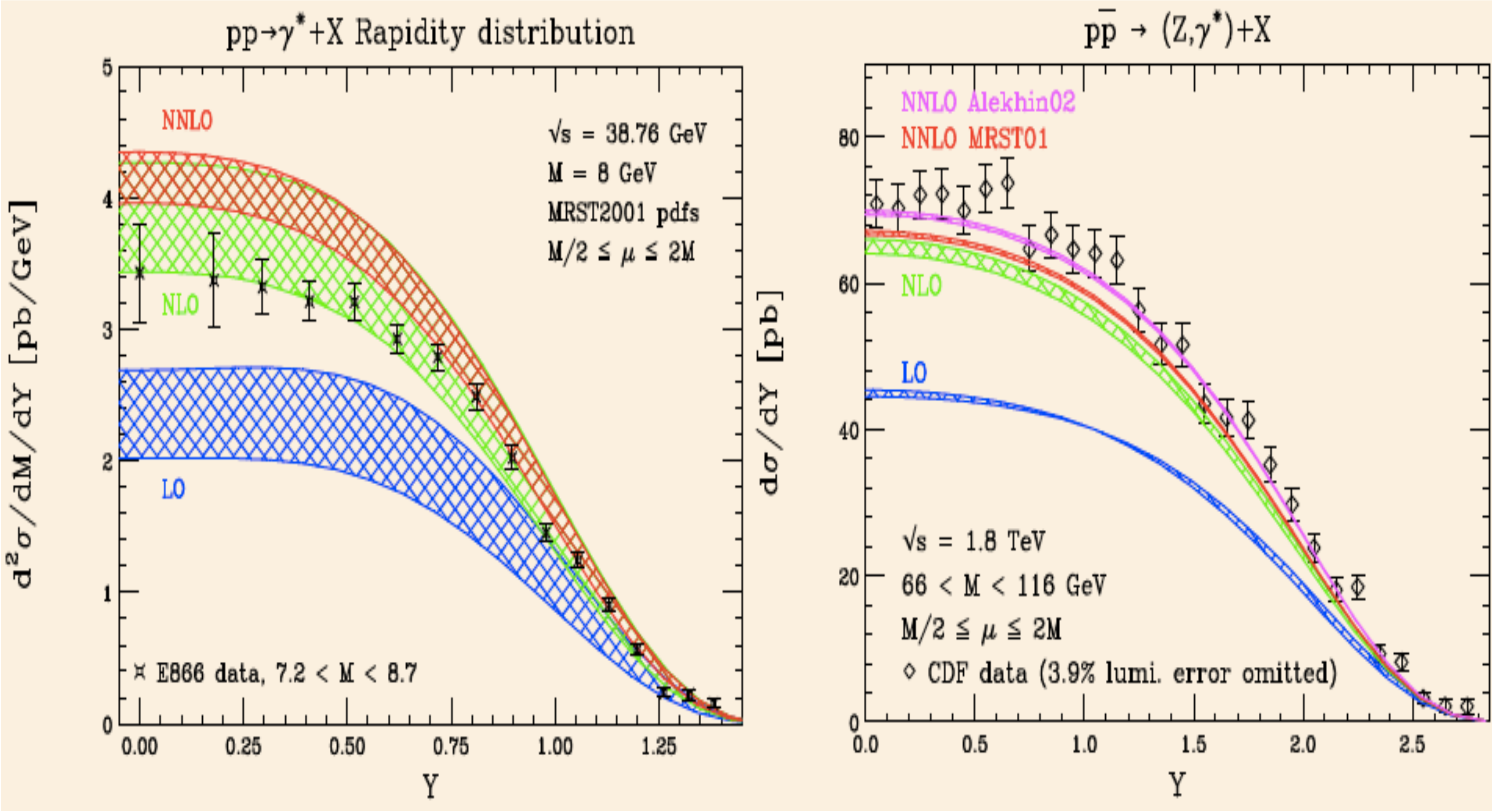
✧ **NNLO K-factor is about 0.98 for LHC data, 1.04 for Tevatron data**



Cross section with a single hard scale

- NNLO differential x-section:

Anastasiou, Dixon, Melnikov, Petriello, 2003-05



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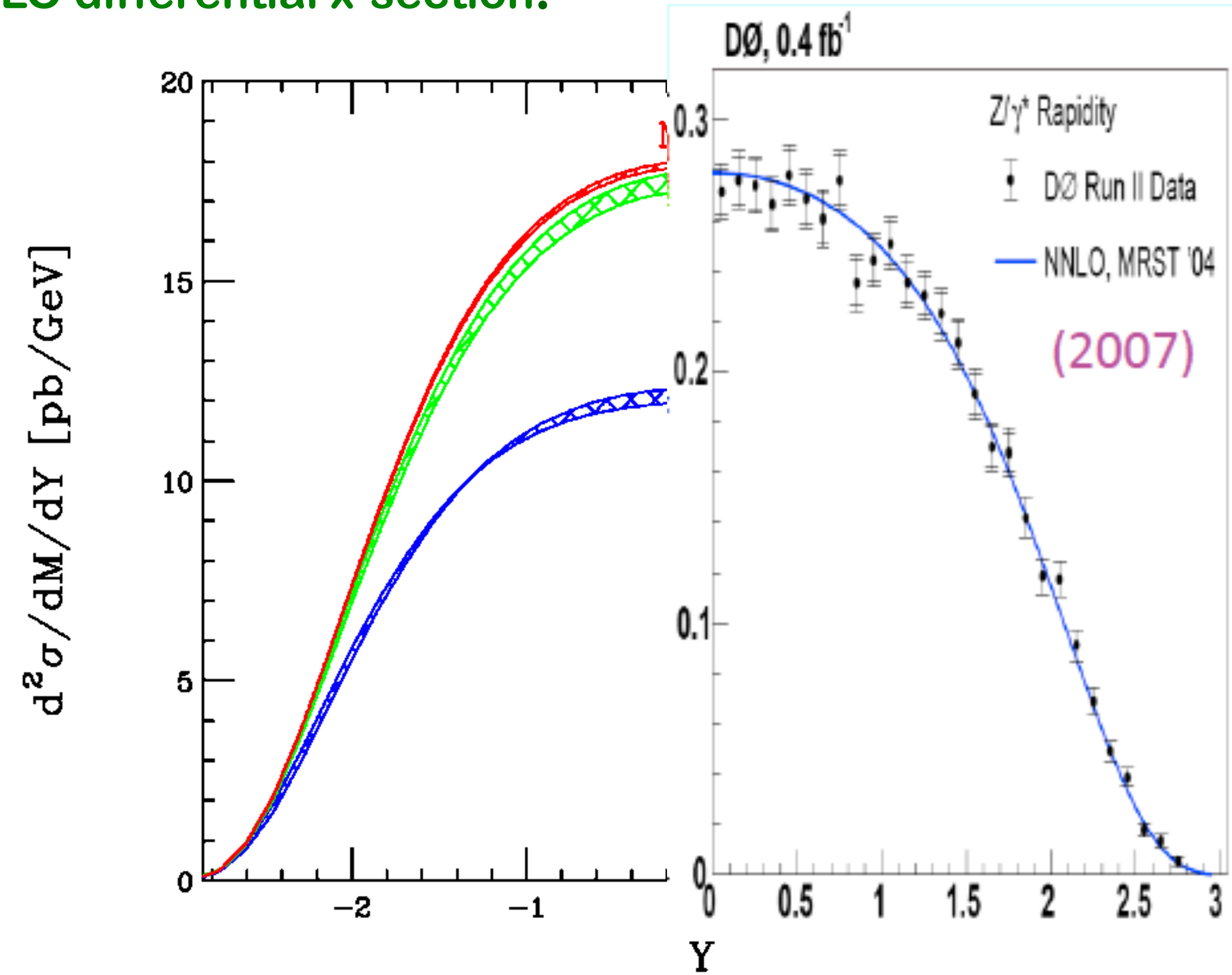
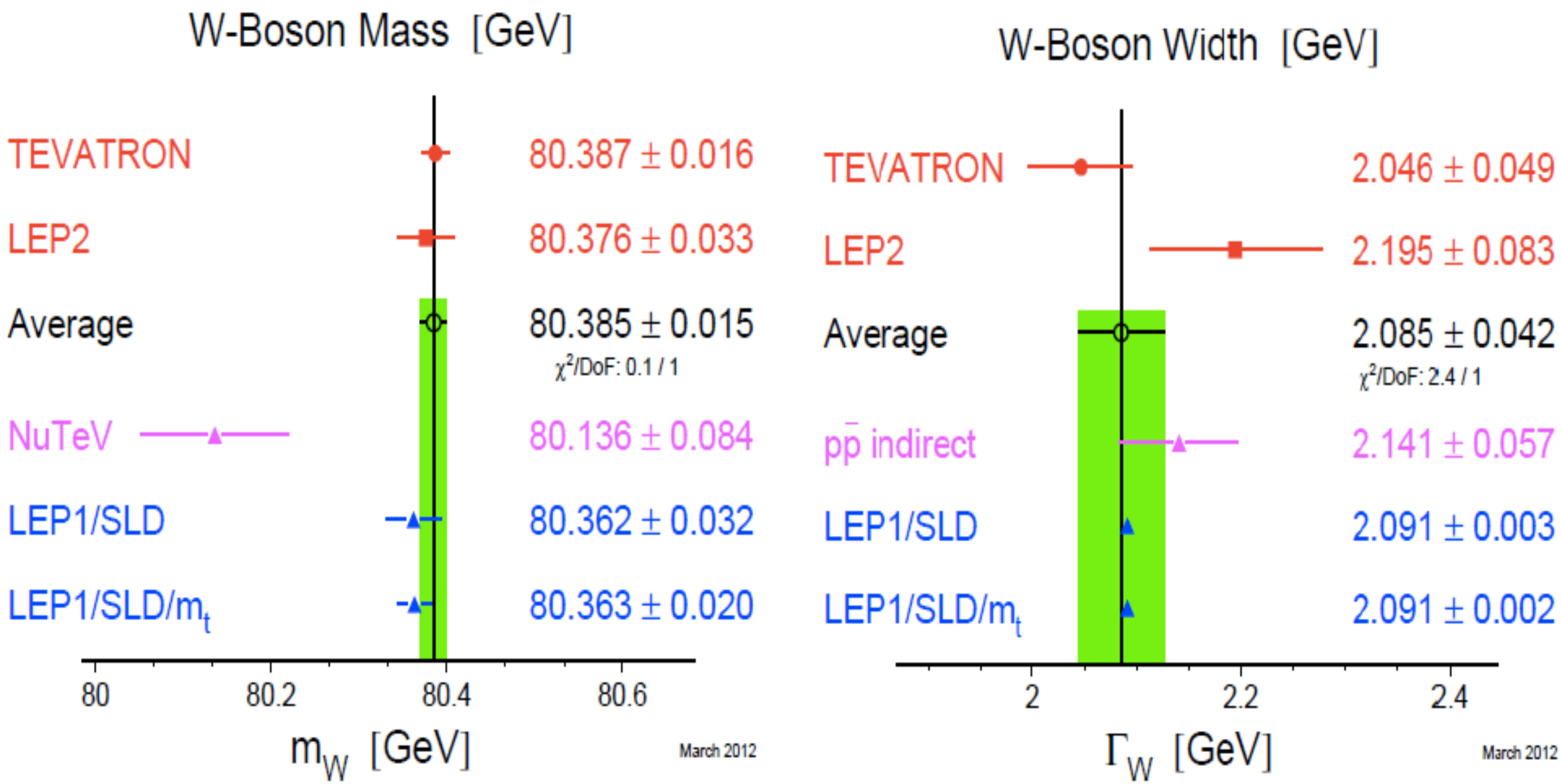


Cross section with a single hard scale

Fernando Febres Cordero, CTEQ SS2012

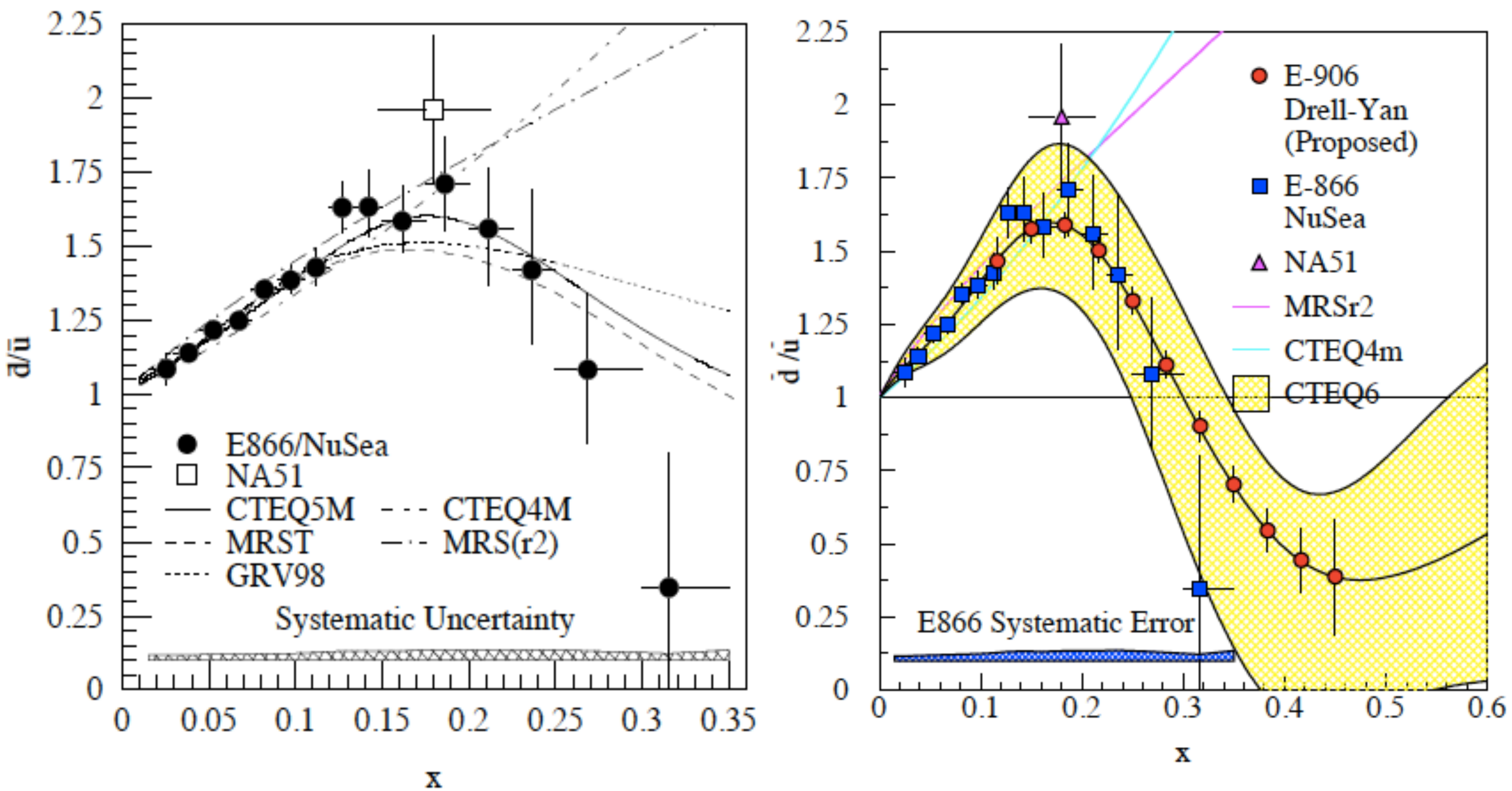
- **W mass & width:**



Cross section with a single hard scale

❑ **Flavor asymmetry of the sea:**

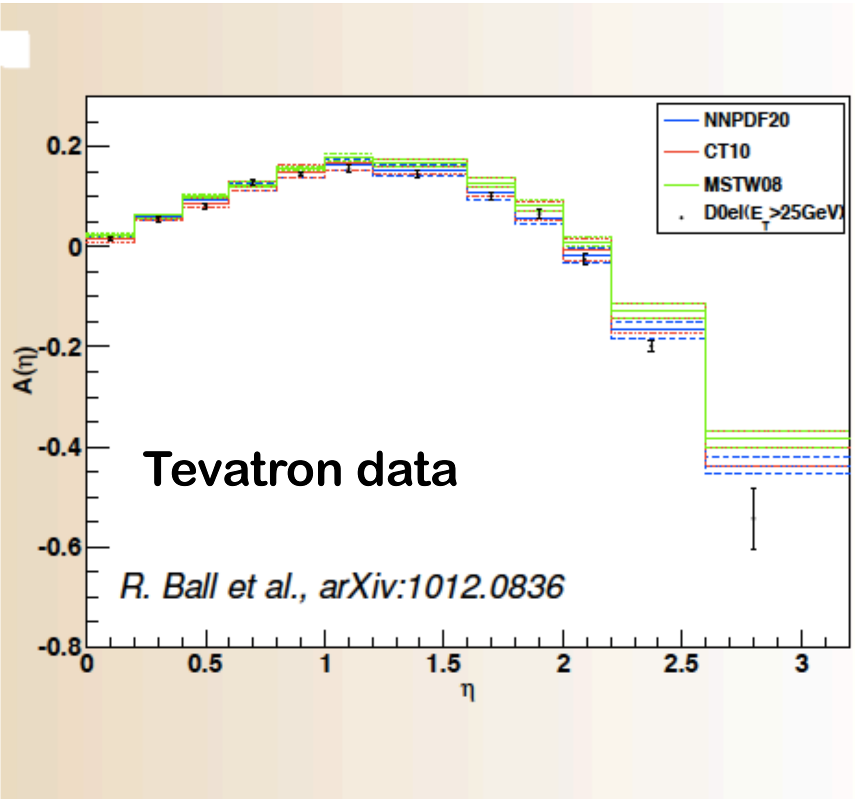
$$\sigma_{DY}(p+d)/2\sigma_{DY}(p+p) \simeq \left[1 + \bar{d}(x)/\bar{u}(x)\right]/2$$

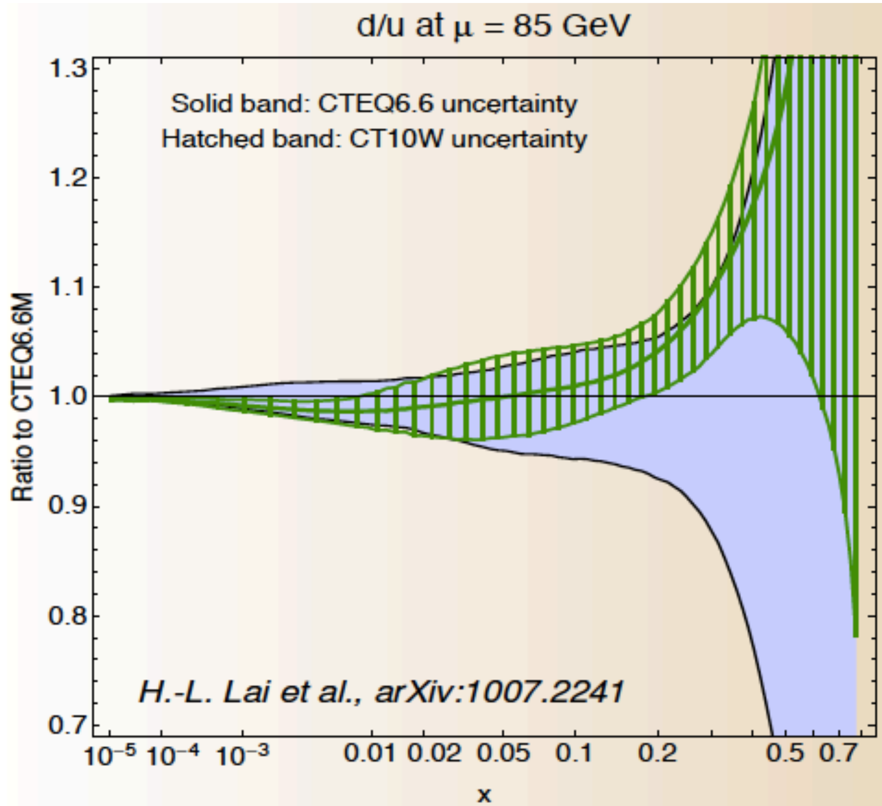


Cross section with a single hard scale

- **Charged lepton asymmetry:** $y \rightarrow y_{\rm max}$

$$A_{ch}(y_e) = \frac{d\sigma^{W^+}/dy_e - d\sigma^{W^-}/dy_e}{d\sigma^{W^+}/dy_e + d\sigma^{W^-}/dy_e} \longrightarrow \frac{d(x_B,M_W)/u(x_B,M_W) - d(x_A,M_W)/u(x_A,M_W)}{d(x_B,M_W)/u(x_B,M_W) + d(x_A,M_W)/u(x_A,M_W)}$$





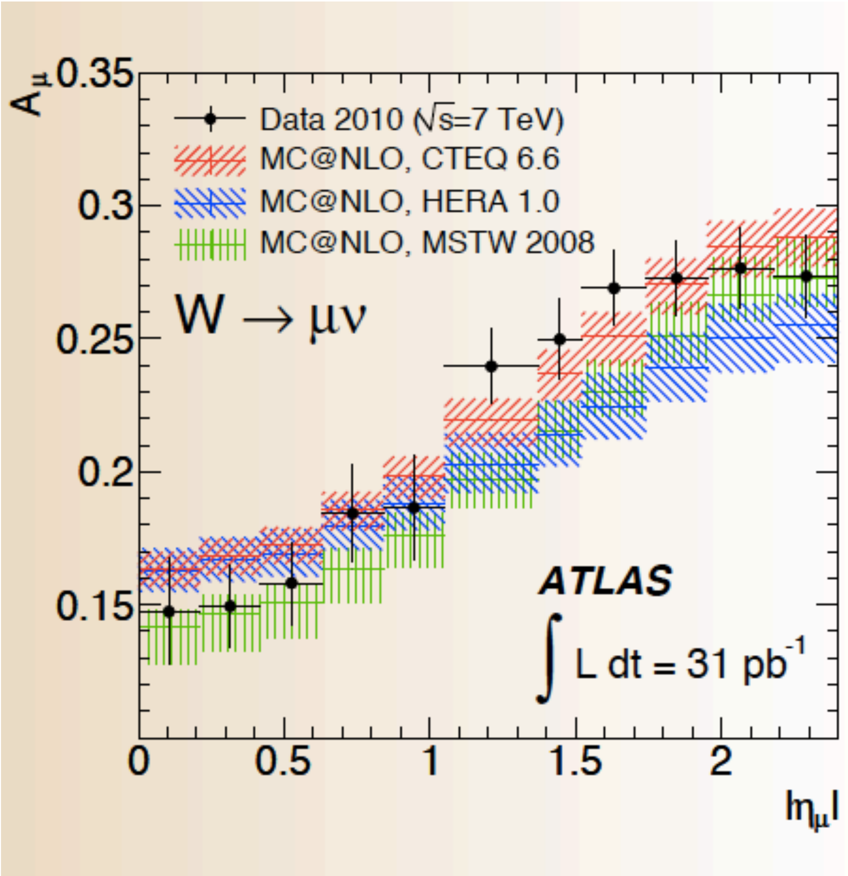
The A_{ch} data distinguish between the PDF models, reduce the PDF uncertainty

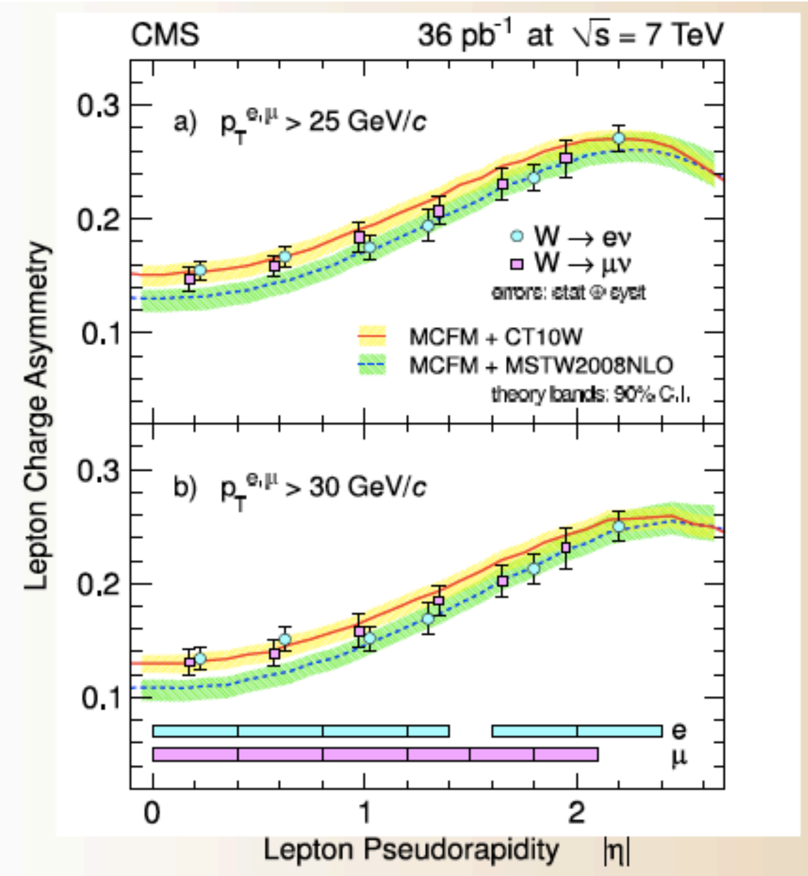
D0 – W charge asymmetry

Cross section with a single hard scale

❑ Charged lepton asymmetry: $y \to y_{\max}$

$$A_{ch}(y_e) = \frac{d\sigma^{W^+}/dy_e - d\sigma^{W^-}/dy_e}{d\sigma^{W^+}/dy_e + d\sigma^{W^-}/dy_e} \longrightarrow \frac{d(x_B,M_W)/u(x_B,M_W) - d(x_A,M_W)/u(x_A,M_W)}{d(x_B,M_W)/u(x_B,M_W) + d(x_A,M_W)/u(x_A,M_W)}$$



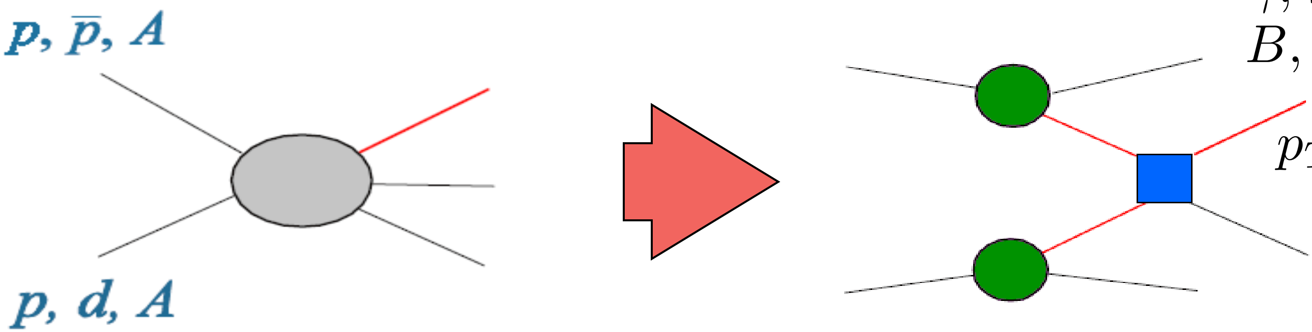


Sensitive both to d/u at x > 0.1 and u/d at x ~ 0.01

Factorization for more than two hadrons

Nayak, Qiu, Sterman, 2006

- Factorization for high p_T single hadron:



$\gamma, W/Z, \ell(s), \mathrm{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

$p_T \gg m \gtrsim \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x, \mu_F^2) \otimes \phi_{B\to b}(x', \mu_F^2)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z, \mu_F^2)$$

✧ Fragmentation function: $D_{c\to C}(z, \mu_F^2)$

✧ Choice of the scales: $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Why photons?

- Photon is a EM probe:

  It can be produced at any stage of the collision

  It does not interact strongly once produced

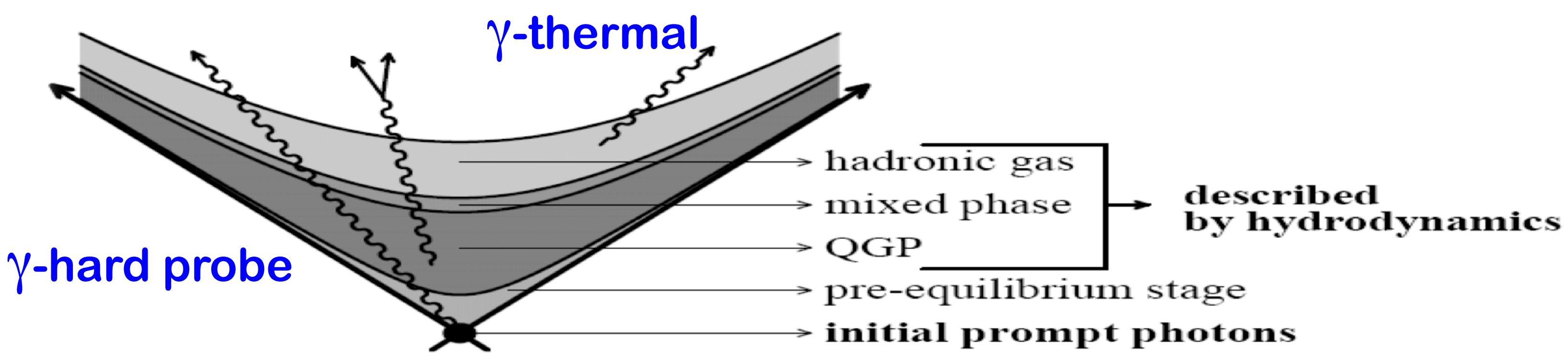
- Good probe of short-distance strong interaction:

  Isolated or "direct" photon is produced at a distance $1/p_T \ll$ fm

  "snap shot" of what happened at the distance scale $1/p_T$

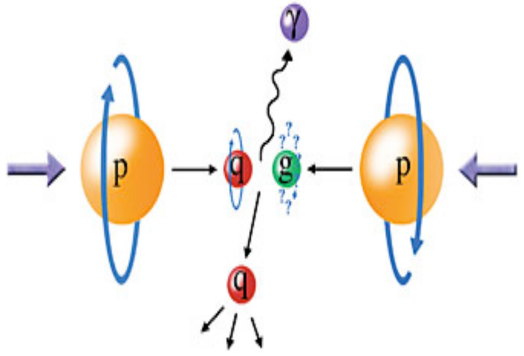
  Key signal, as well as background of Higgs production: $H^0 \to \gamma + \gamma$

- Photon can tell the full history of heavy ion collision:



Theory behind the high p_T photon

- Production mechanism – leading power factorization:



$$\frac{d\sigma_{AB}}{dy dp_T^2} = \int dx f_{a/A}(x,\mu) \int dx' f_{b/B}(x',\mu) \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu))}{dy dp_T^2}$$

$$+ \text{frag contribution} + \mathcal{O}\left(\frac{1}{p_T^n}\right)$$

Hard part: $\hat{\sigma}_{ab}(\alpha_s(\mu)) = \hat{\sigma}^0_{ab}\alpha_s^m(\mu) + \hat{\sigma}^1_{ab}(\log(\mu))\alpha_s^{m+1}(\mu) + \ldots$

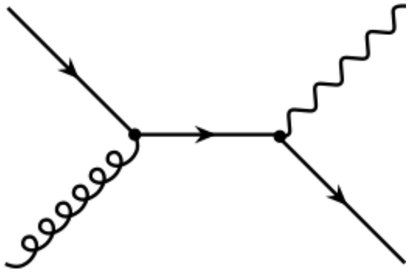
- Predictive power:
  - Short-distance part is Infrared-Safe, and calculable
  - Long-distance part at the leading power is Universal – PDFs, FFs
- Factorization and renormalization scale dependence:
  - NLO is necessary
- Power correction could be important at low p_T

Direct photon is sensitive to gluon

❑ Sensitive to gluon at the leading order – hadronic collision:

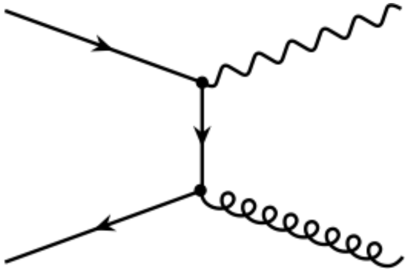
✧ Lowest order direct $\mathcal{O}(\alpha_{em}\alpha_s)$:

Compton: $q(\bar{q}) + g \to \gamma + q(\bar{q})$



+ ...

Annihilation: $q + \bar{q} \to \gamma + g$



+ ...

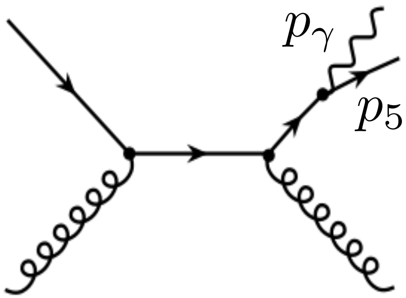
✧ Compton dominates in pp collision:

$$f_{g/p}(x,\mu^2) \gg f_{\bar{q}/p}(x,\mu^2)$$ **for all x**

Direct photon production could be a good probe of gluon distribution

Complication from high orders

❑ Final-state collinear singularity:



$$\overline{\sum}|M(qg\to\gamma qg)|^2 \approx \frac{\alpha_{em}}{2\pi}\mathcal{P}^{(0)}_{q\to\gamma}(z)\,\frac{1}{s_{\gamma q}}\overline{\sum}|M(qg\to qg)|^2$$

$$\mathcal{P}^{(0)}_{q\to\gamma}(z) = \frac{1+(1-z)^2}{z}$$

$$s_{\gamma q} = (p_\gamma + p_5)^2 \to 0 \quad \textbf{when} \quad p_\gamma \parallel p_5$$

An internal quark line goes on-shell signaling long-distance physics

❑ Fragmentation contribution:

$$\frac{d\sigma^{\rm Frag}_{AB\to\gamma}}{dy\,dp_T^2} = \sum_{abc}\int\frac{dz}{z^2}D_{c\to\gamma}(z,\mu)\int dx\,f_{a/A}(x,\mu)\int dx'\,f_{b/B}(x',\mu)\,\frac{d\hat{\sigma}^{\rm Frag}_{ab\to c}}{dy\,dp_T^2}$$

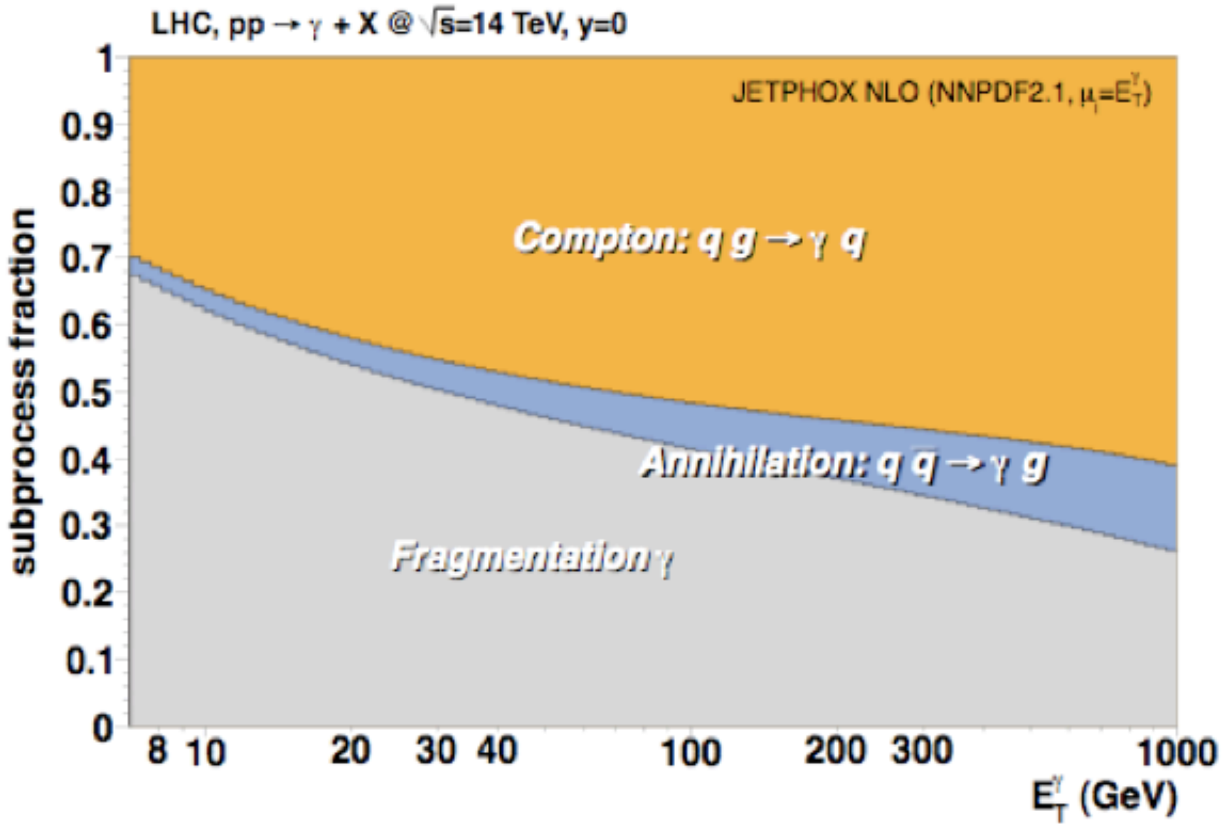
❑ Photon fragmentation functions – inhomogeneous evolution:

$$\frac{\partial D_{c\to\gamma}(z,\mu)}{\partial\log(\mu)} = \frac{\alpha_{em}}{2\pi}\mathcal{P}_{c\to\gamma}(z) + \sum_{a=q\bar{q}g}\frac{\alpha_s}{2\pi}P_{ac}(z)\otimes D_{a\to\gamma}(z,\mu)$$

Size of fragmentation

Campbell, CTEQ SS2013

❑ Inclusive direct photon:



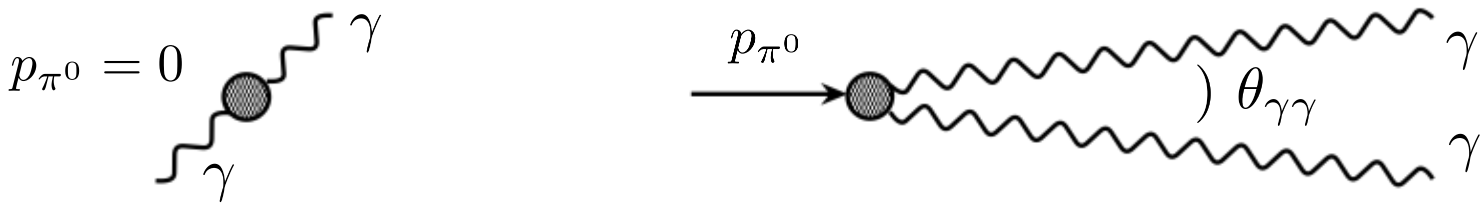
✧ Production at NLO – available, e.g., in MCFM and JETPHOX (shown here)

✧ Fragmentation contribution is huge for inclusive production:

$\sigma^{Frag} / \sigma^{Total}$ > 50% at pT=20 GeV @ LHC (role of FF!)

Complication from the measurement

- Separation the signal photon from $\pi^0 \to \gamma\gamma$:



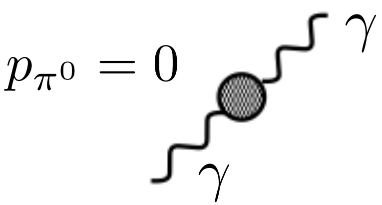
  - When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases
  - Two photons could be misidentified as one photon at high p_T

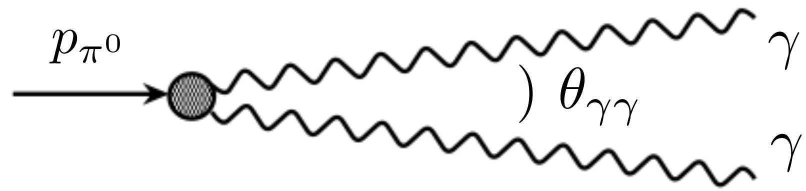
- Isolation cut – algorithms (like jet):
  - Cone algorithm – reduction of fragmentation contribution

    Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

Complication from the measurement

- **Separation the signal photon from $\pi^0 \to \gamma\gamma$:**





  - ✧ When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases
  - ✧ Two photons could be misidentified as one photon at high p_T

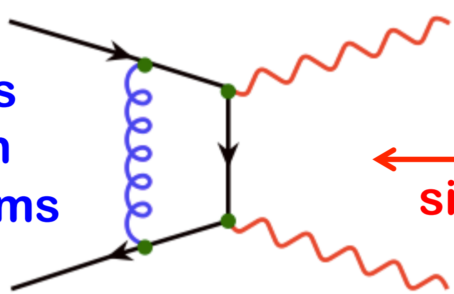
- **Isolation cut – algorithms:**

Needed for IR safety

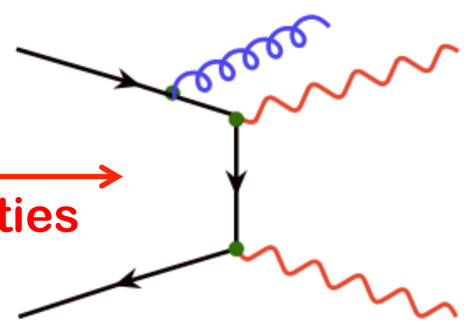
  - ✧ Cone algorithm – reduction of fragmentation contribution

    Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

Isolation has No effect on virtual diagrams



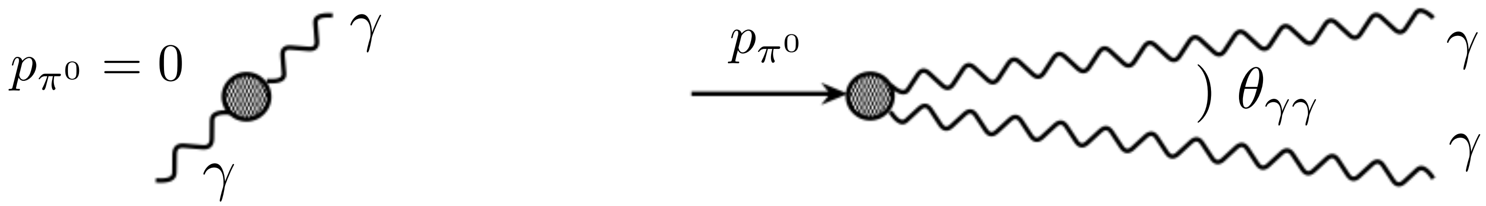
Soft singularities



Isolation limits phase space of real diagrams

Complication from the measurement

- Separation the signal photon from $\pi^0 \to \gamma\gamma$:



  - ✧ When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases
  - ✧ Two photons could be misidentified as one photon at high p_T

- Isolation cut – algorithms:
  - ✧ Cone algorithm – reduction of fragmentation contribution

    Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

  - ✧ Modified cone algorithm – NO fragmentation contribution

$$\sum_{R_{j\gamma} \in R_0} E_T(\mathrm{had}) < \epsilon_h p_T^\gamma \left(\frac{1 - \cos R_{j\gamma}}{1 - \cos R_0} \right)$$

  - ✧ Parton is softer as it closer to photon
  - ✧ No contribution at CO singularity

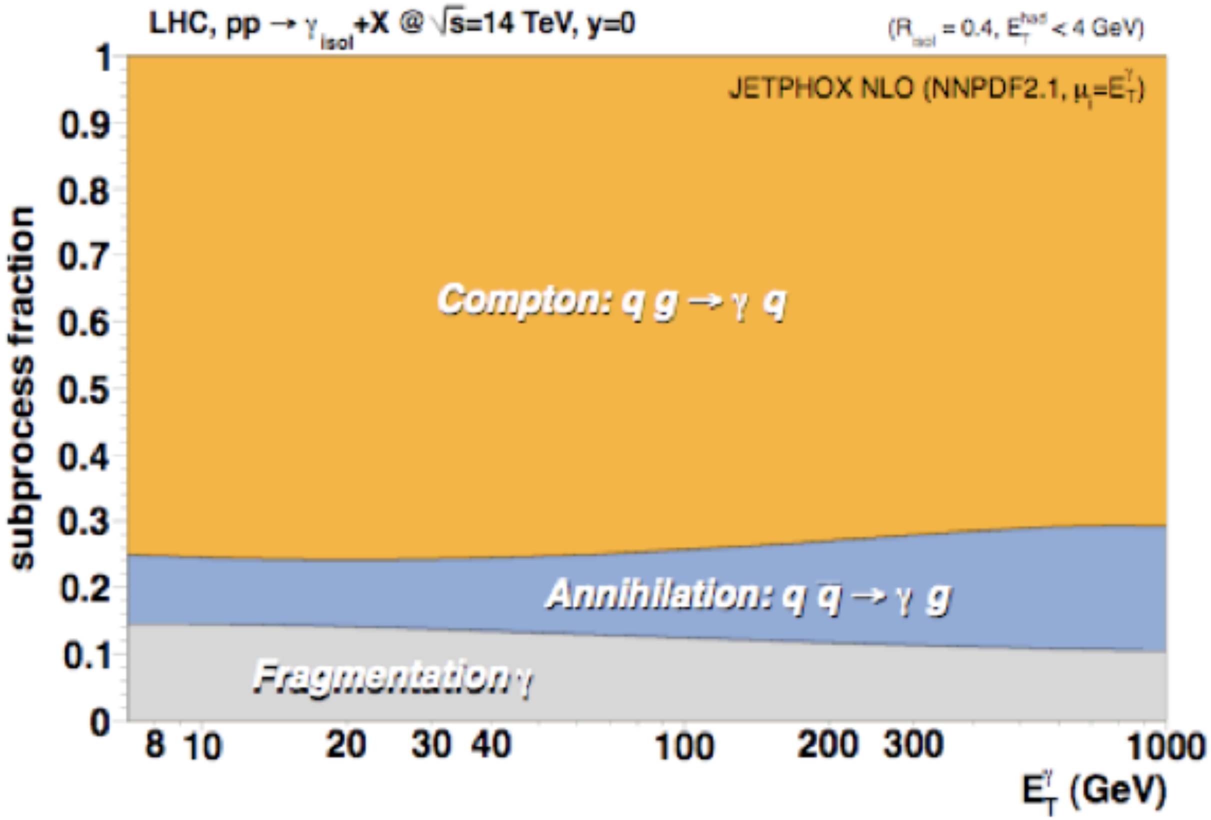
    Hard to implement experimentally (detector resolution)

S. Frixione, 1998

Size of fragmentation

Campbell, CTEQ SS2013

❑ **Isolated direct photon:**



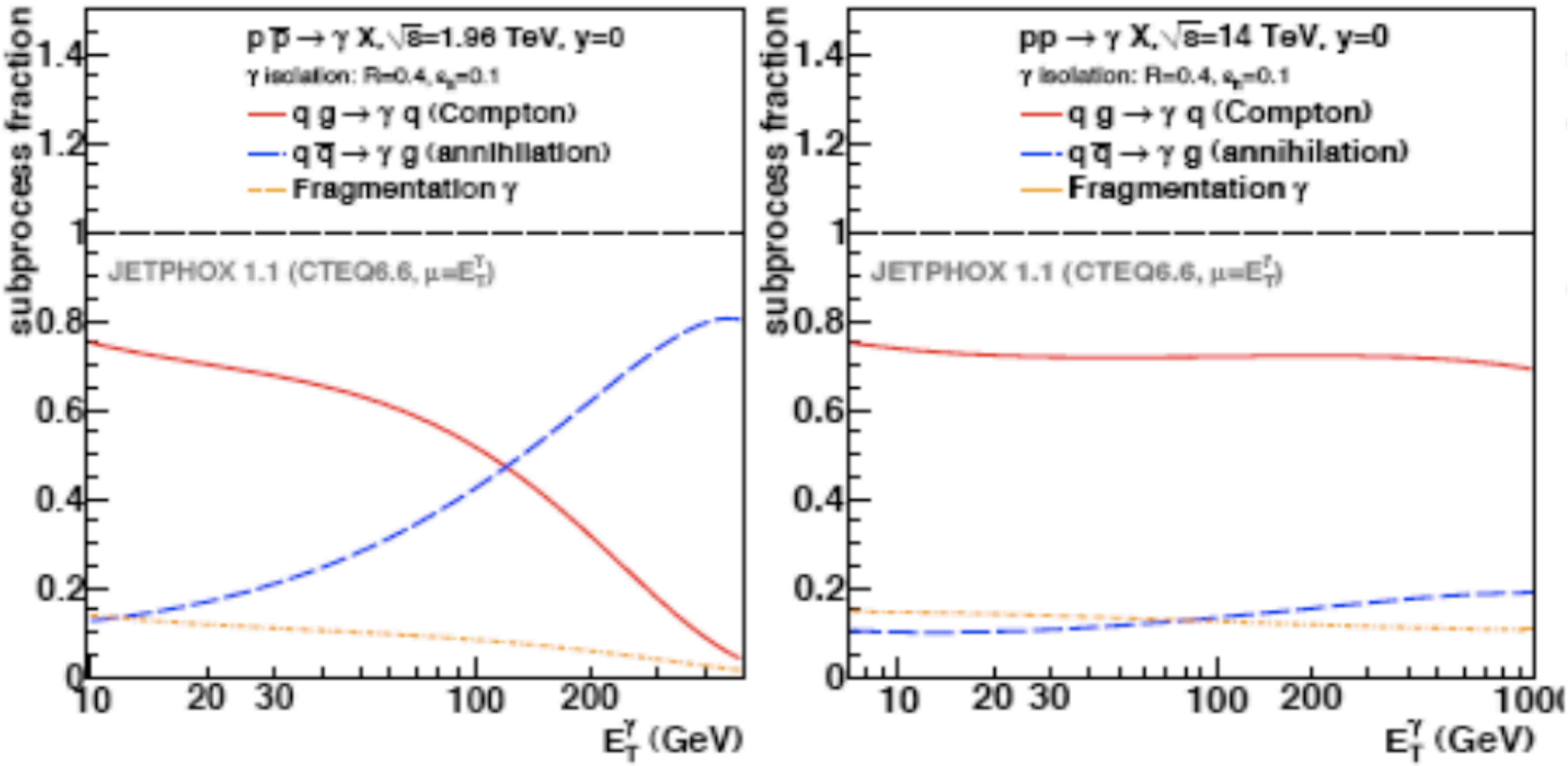
✧ Isolation removes the most of fragmentation contribution! (down to 10%)

✧ About 75% of production rate is from gluon initiated subprocesses

Potentially, a useful probe of gluon PDF

Role of gluon in pp collision

❑ pp vs $p\bar{p}$:



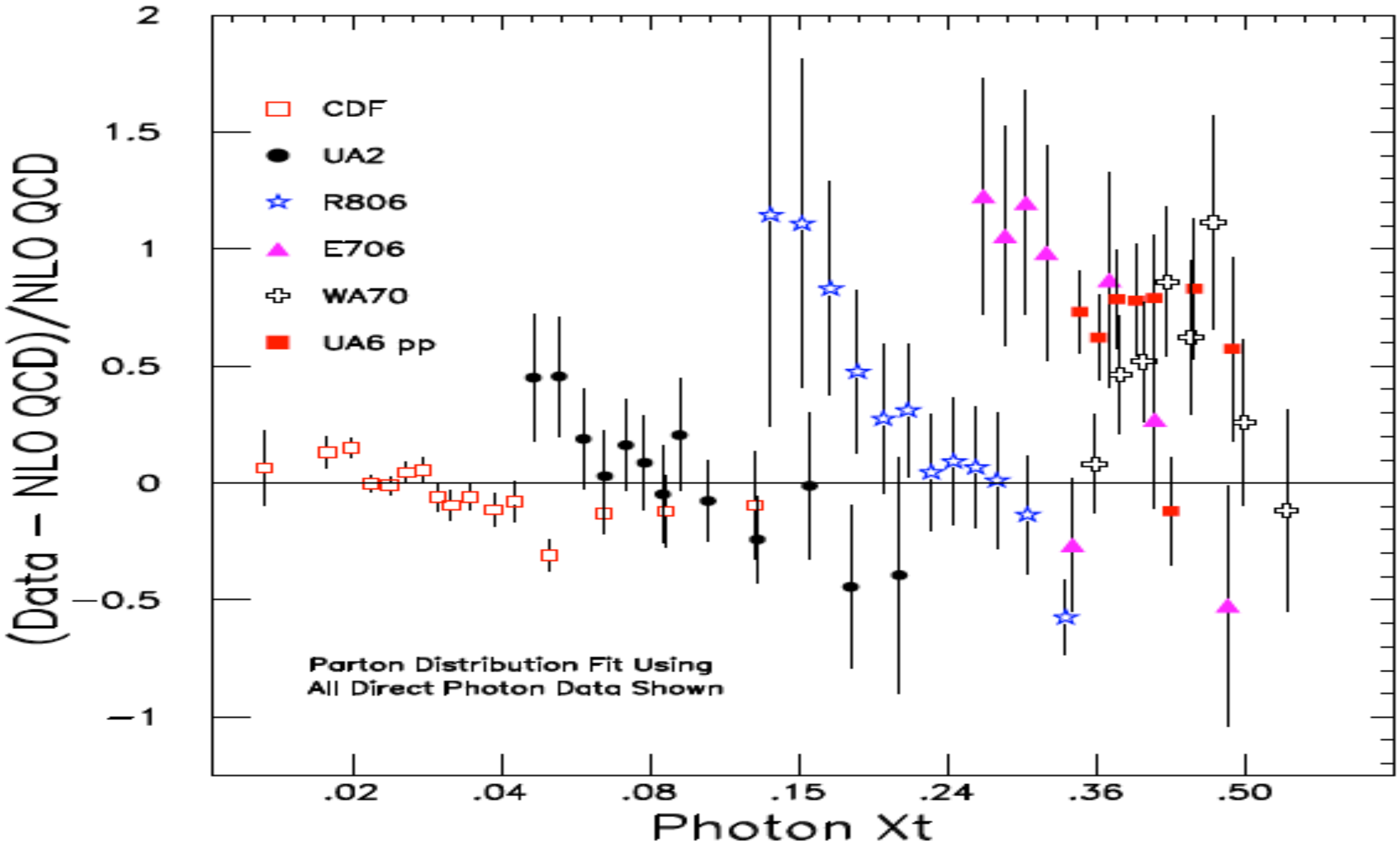
✧ Dominant role of the gluon in pp collision!

✧ Even more dominance in the forward region!

Compare with data from different expt's

❑ **CTEQ global analysis:**

CTEQ Huston et al.



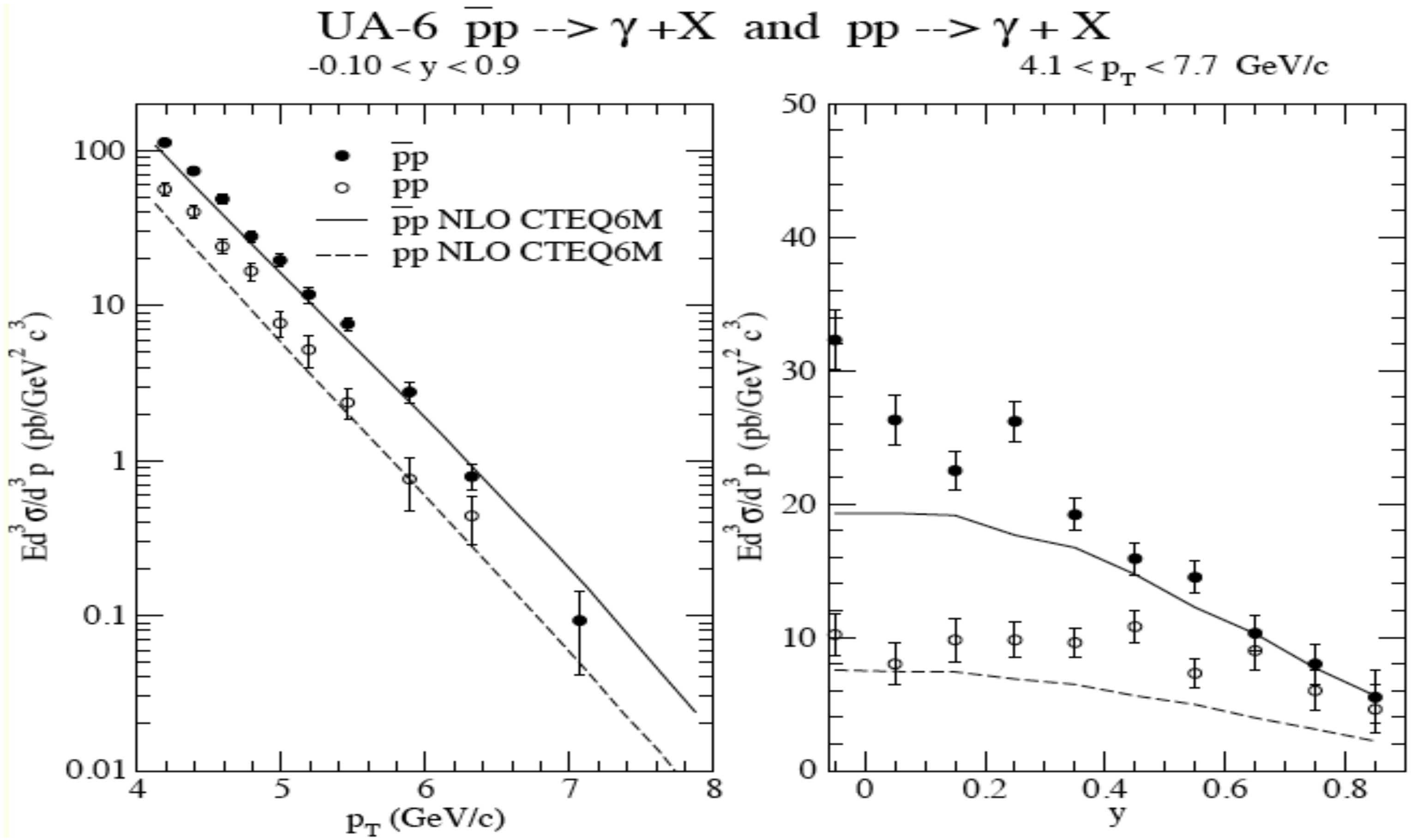
$$x_T = \frac{2p_T}{\sqrt{s}}$$

✧ **Neither PDFs nor photon FFs can significantly improve the shape**

✧ **Direct photon data were excluded from most global fits**

Experiments with both pp and $p\bar{p}$

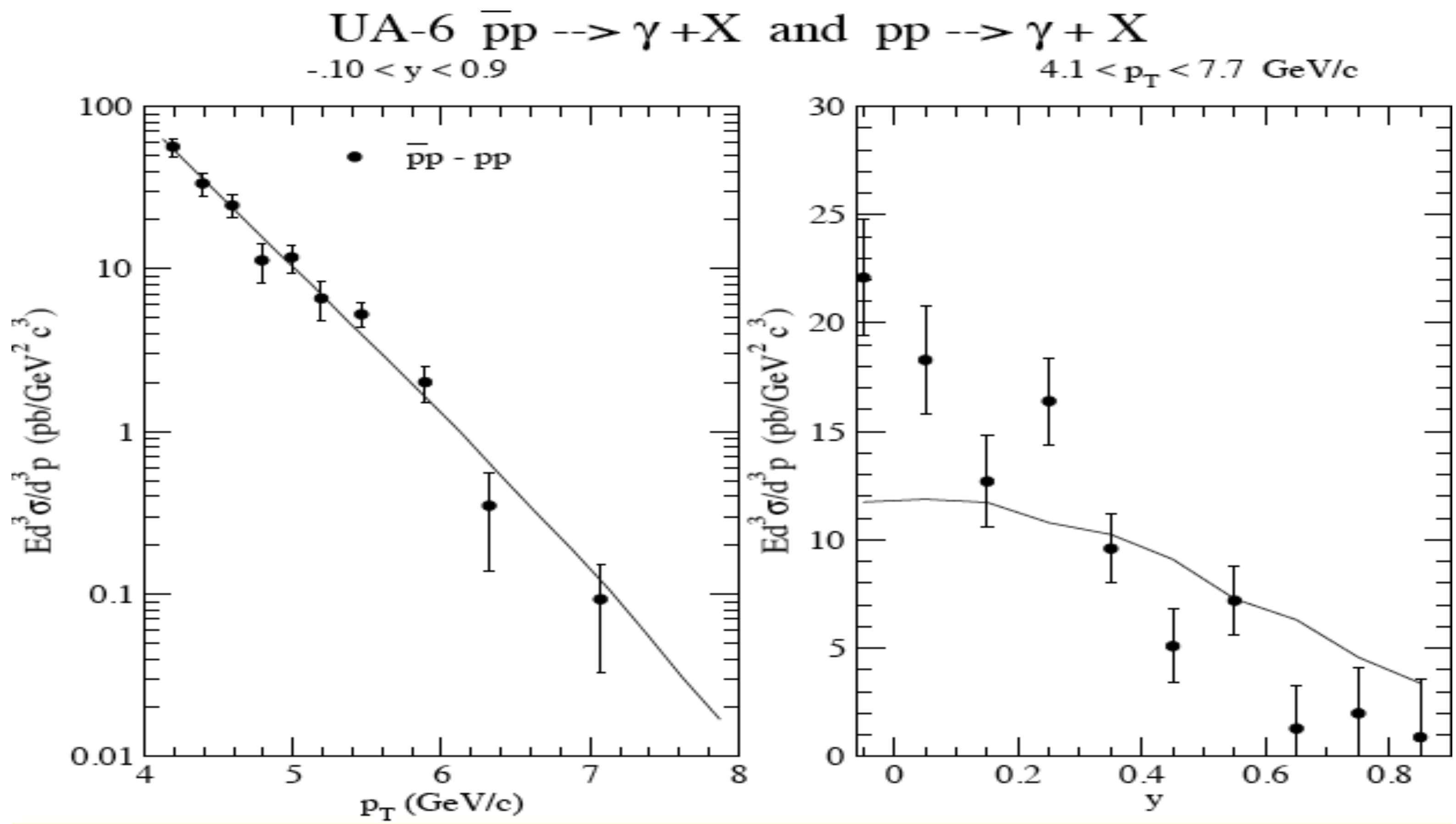
- **UA6:** both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV



- ✧ **Theory curves are below the data**
- ✧ **Rapidity curves are flatter**

Role of gluon distribution?

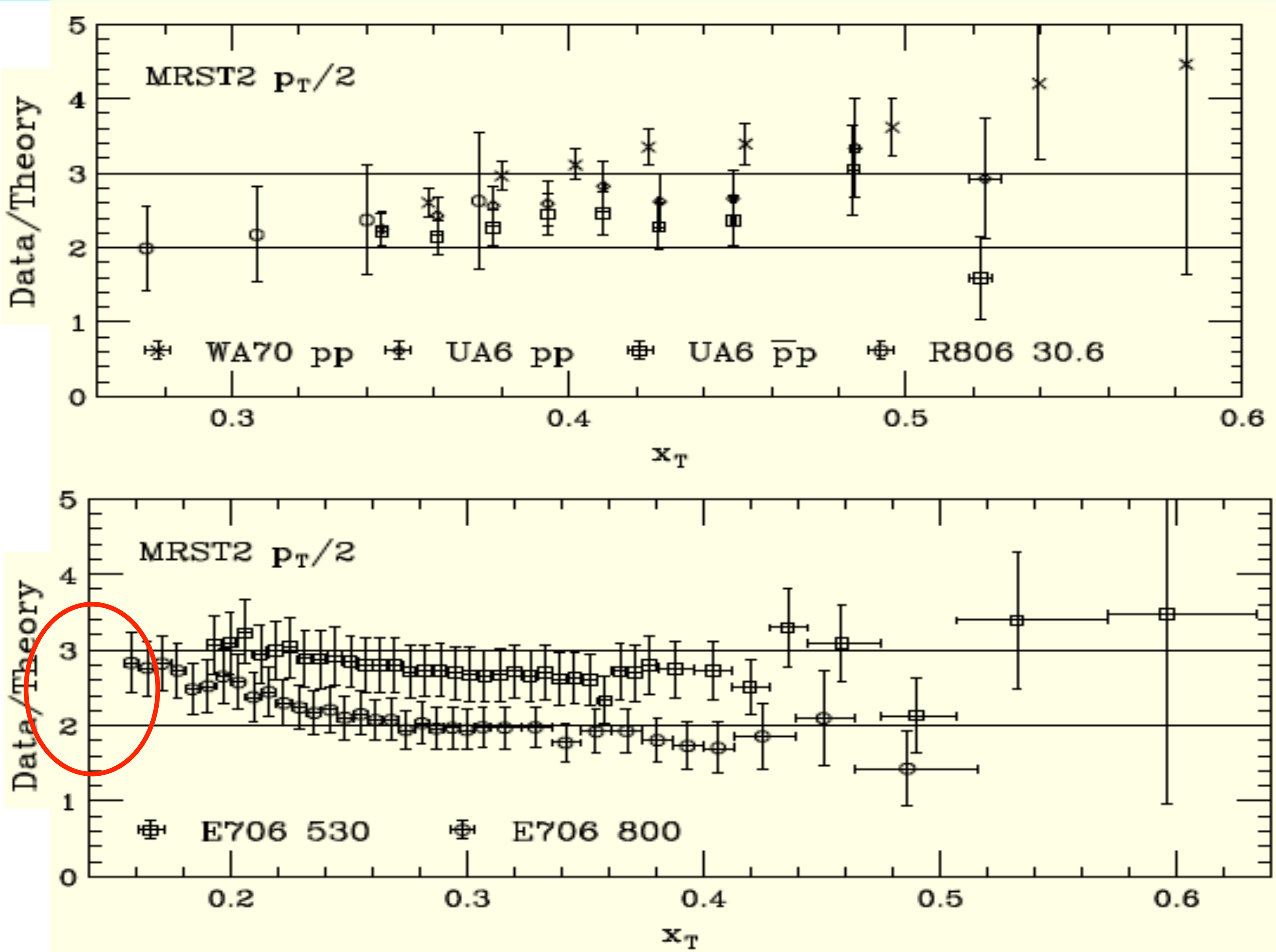
❑ **UA6: $\bar{p}p$ - pp** both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV



✧ **NO gluon contribution to the difference!**

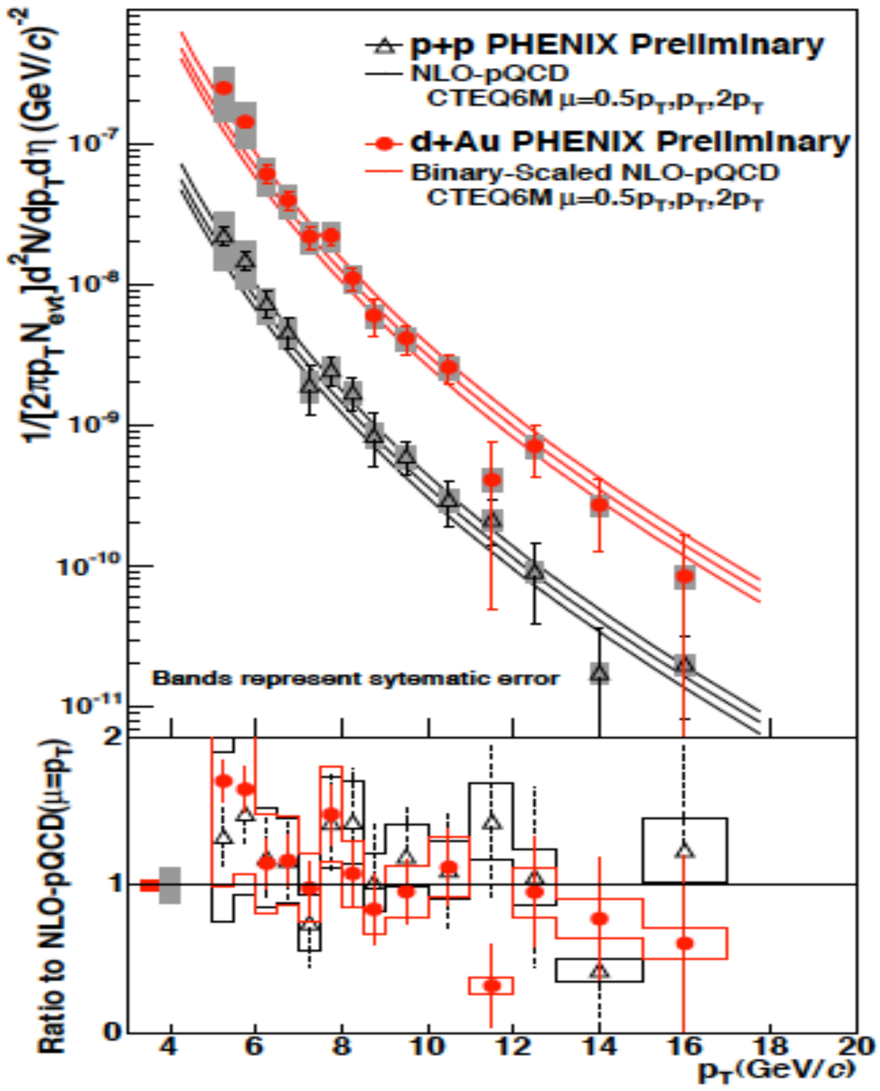
✧ **Theory matches the data better – role of gluon?**

Same excess seen in π^0 production

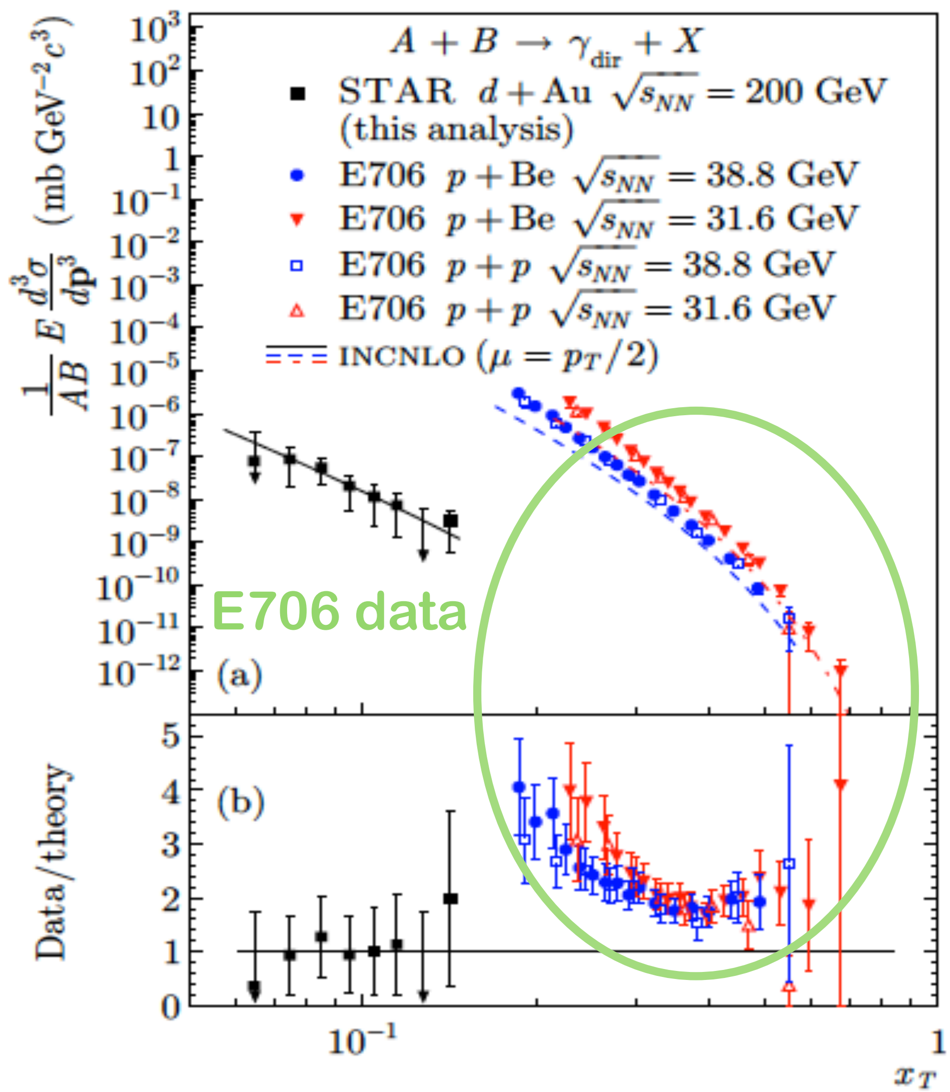


Theory works well at RHIC energy

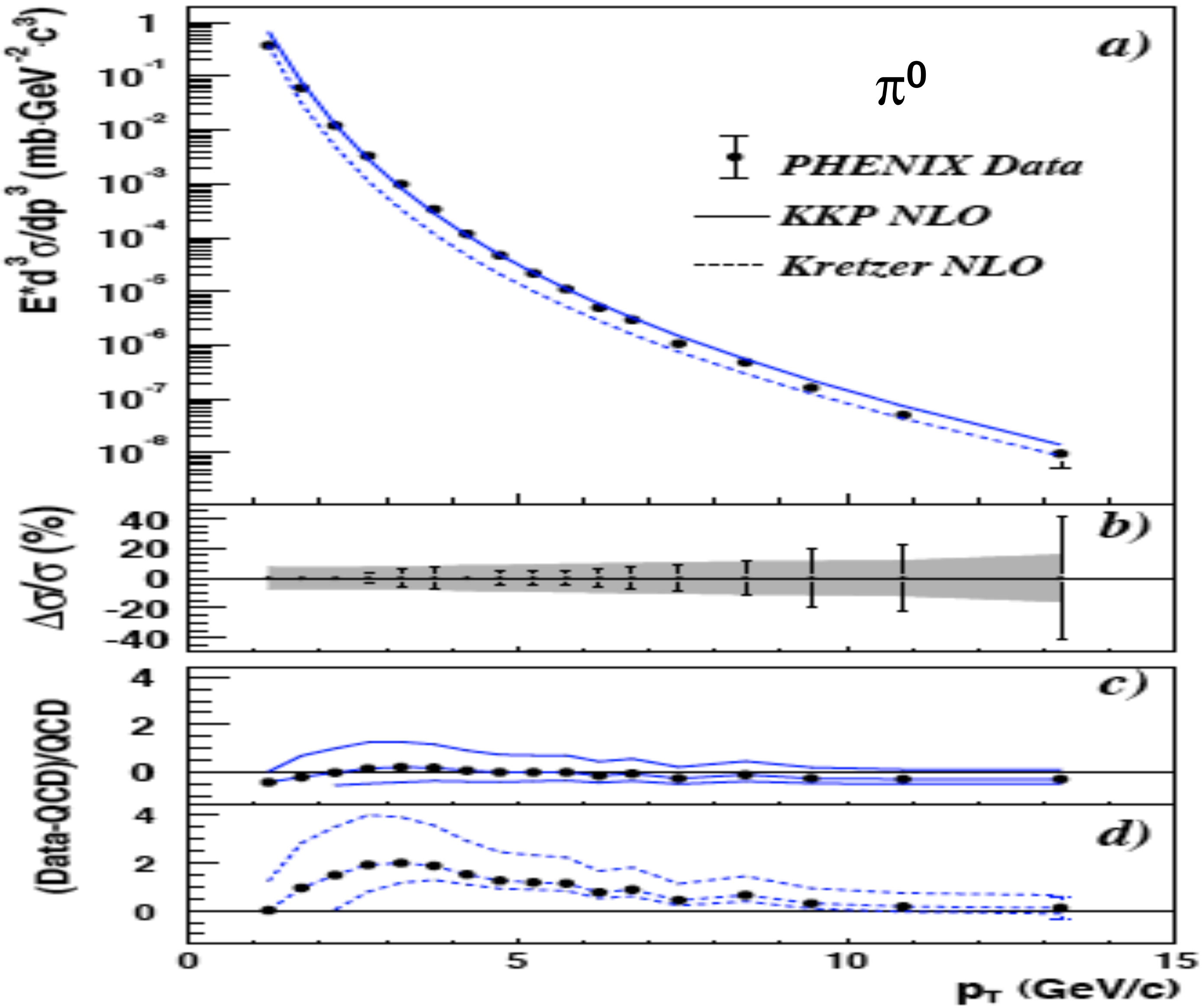
PHENIX



STAR



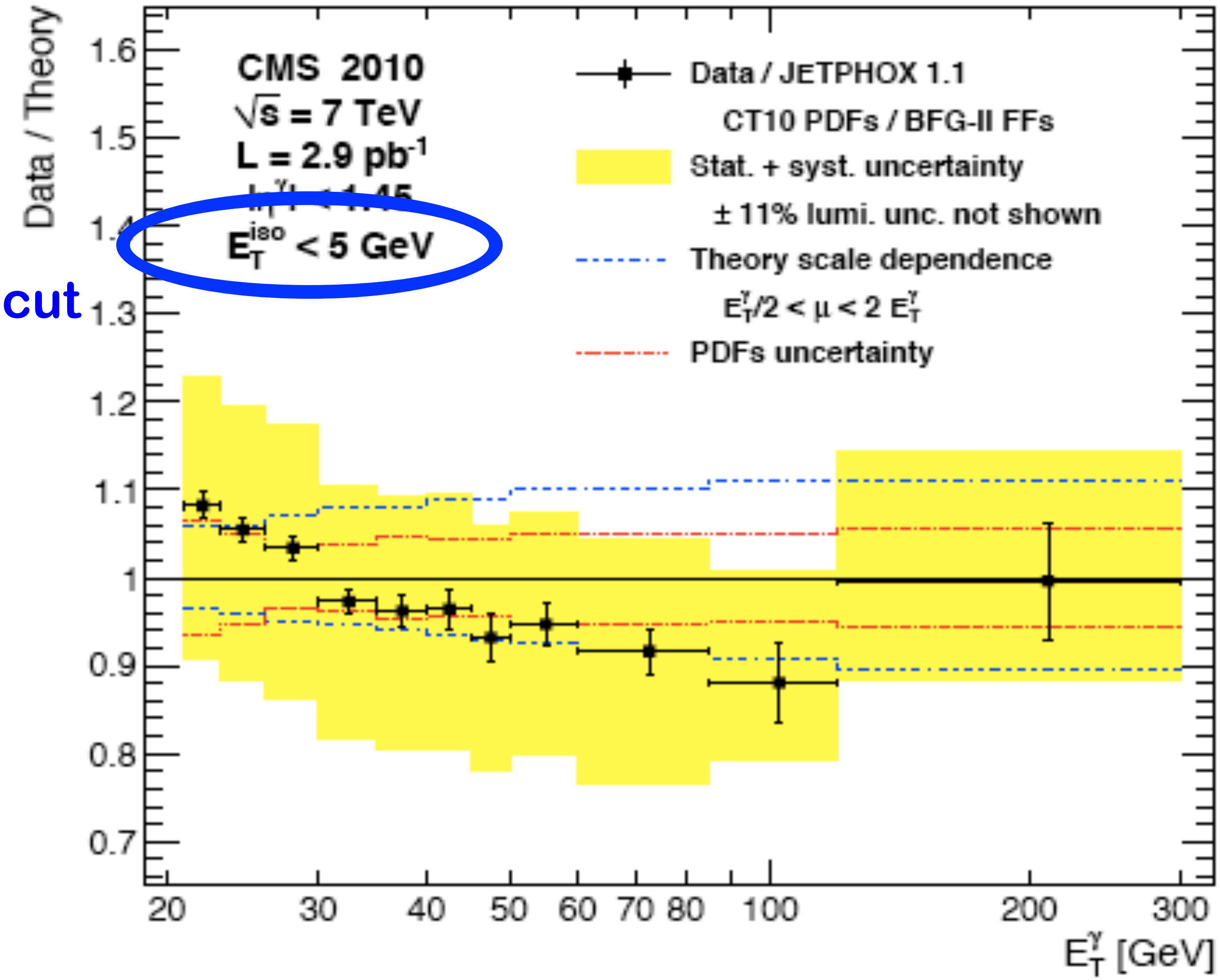
But, works at RHIC energy



How about at the LHC?

- CMS:

Isolation cut

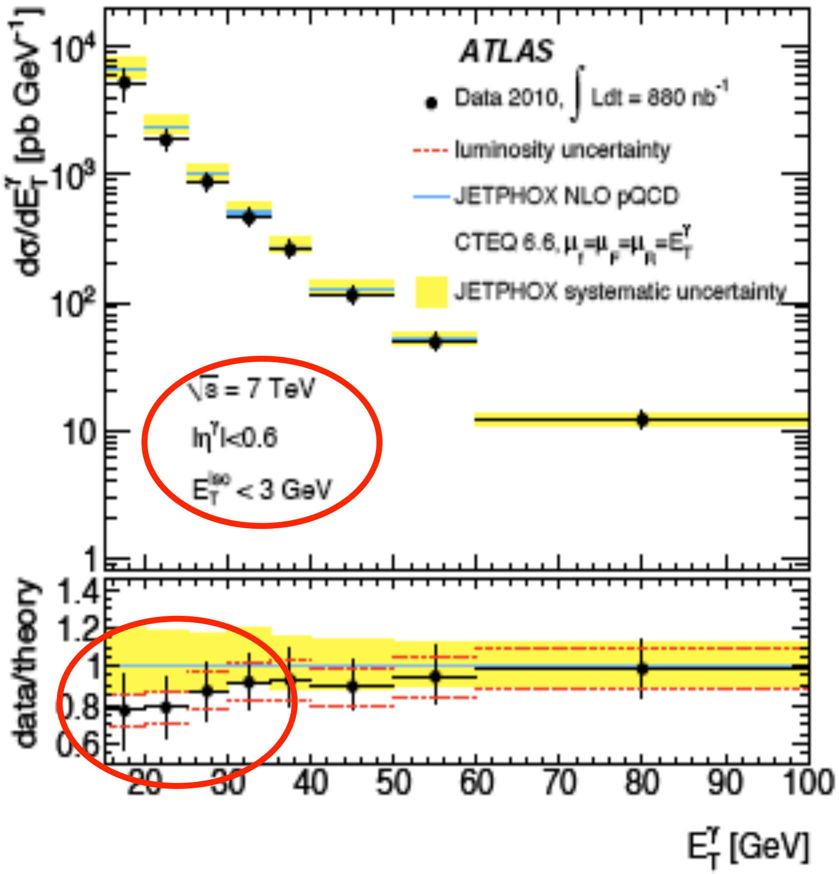


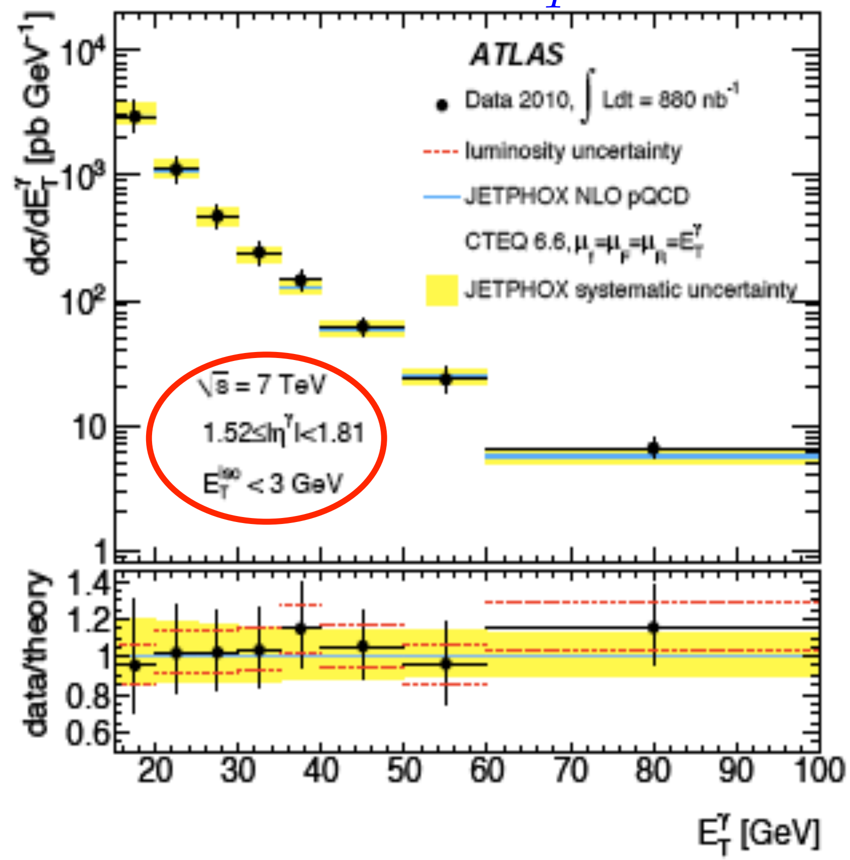
✧ Shape in x_T – within the PDF uncertainty?

Rapidity dependence at the LHC

❑ **ATLAS:**

Note: CMS has $E_T^{\text{iso}} < 5 \text{ GeV}$





✧ **Data seems to be lower than theory at central** η^γ **and small** E_T^γ

Overall consistency is better at collider energies!

Where do we stand?

- Agreement between theory and data improves with increasing energy and is excellent at $\sqrt{s}$ = 200 GeV

- Situation with fixed target direct photon data is confusing:
  - ✧ Disagreement between experiments
  - ✧ A reassessment of systematic errors on the existing fixed target photon experiments might help resolve the discrepancies

- We need an improved method of calculating single particle inclusive cross sections in the fixed target energy
  – Threshold resummation helps

- All experiments see an excess of data over theory at fixed target energies, but, less than theory at low pT at the LHC

More data from the LHC should help (the gluon dominance)!

Global QCD analyses – test of pQCD

- **Factorization for observables with identified hadrons:**
  - **One-hadron (DIS):**

$$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

  - **Two-hadrons (DY, Jets, W/Z, …) :**

$$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

  - **DGLAP Evolution:**

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

- **Input for QCD Global analysis/fitting:**
  - **World data with "Q" > 2 GeV**
  - **PDFs at an input scale:** $\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$

Input scale ~ GeV

Fitting paramters

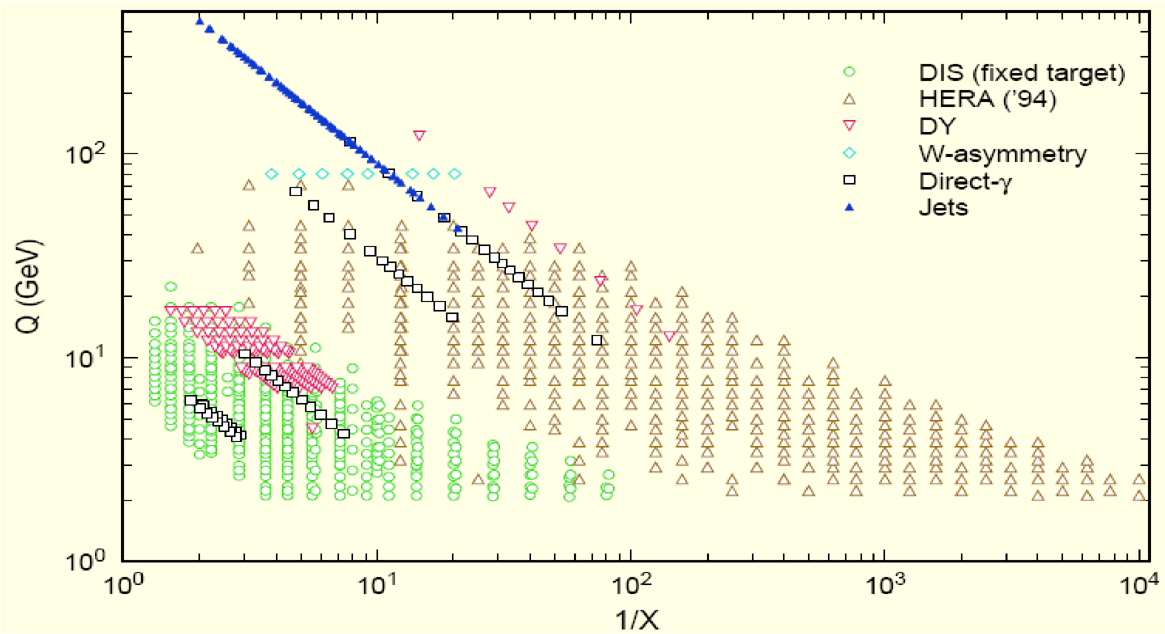
Global QCD analysis for PDFs

Input DPFs at Q_0
$\varphi_{f/h}\left(x,\{a_j\}\right)$

Vary $\{a_j\}$

Minimize Chi^2

DGLAP



$\varphi_{f/h}(x)$ **at $Q>Q_0$**

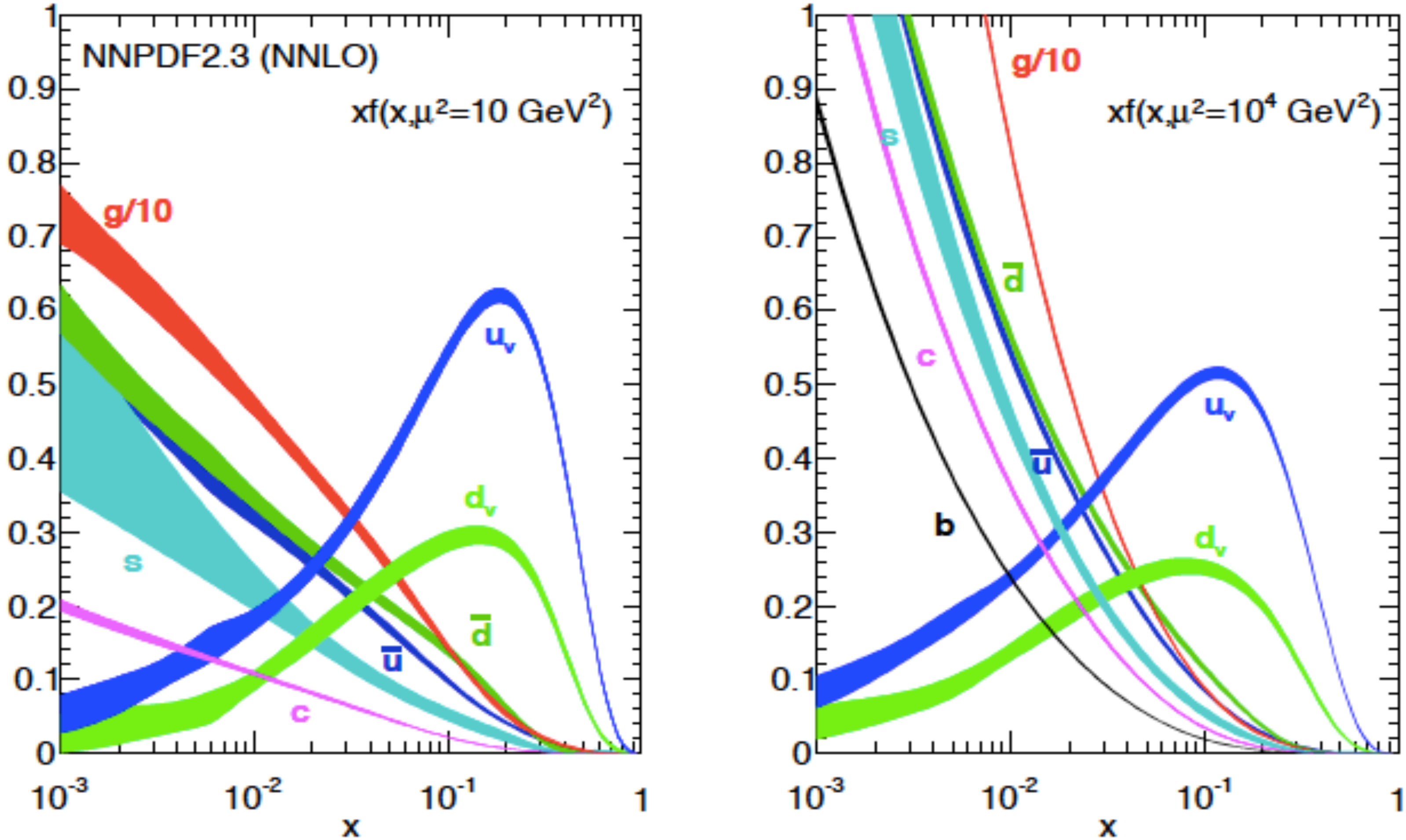
QCD calculation

Comparison with Data at various x and Q

Procedure: Iterate to find the best set of $\{a_j\}$ for the input DPFs

PDFs of a spin-averaged proton

- ❑ Modern sets of PDFs @NNLO with uncertainties:



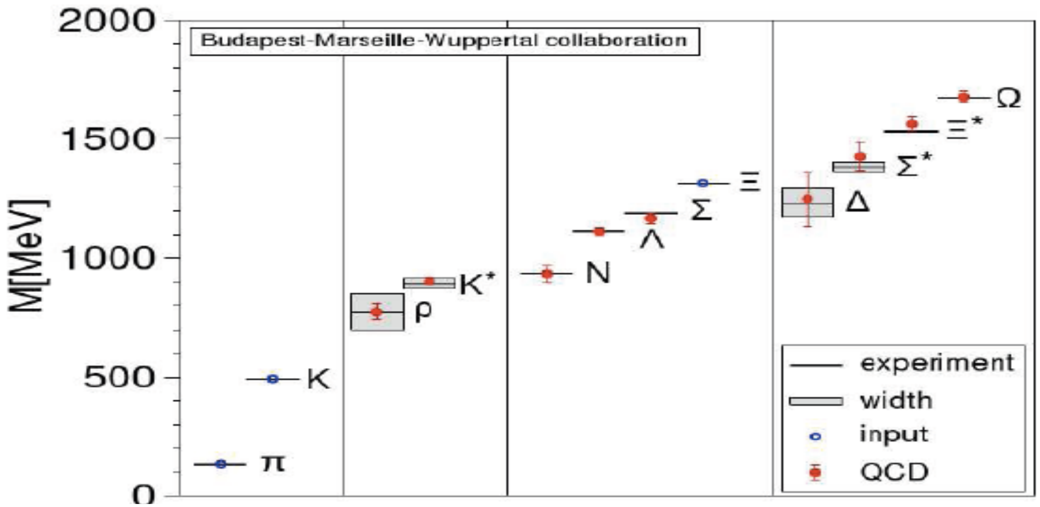
K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)

Consistently fit almost all data with Q > 2GeV

Successes of QCD

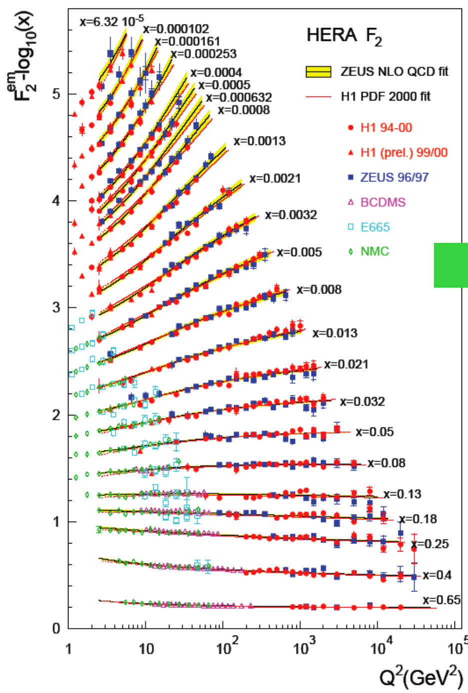
- @low energy:

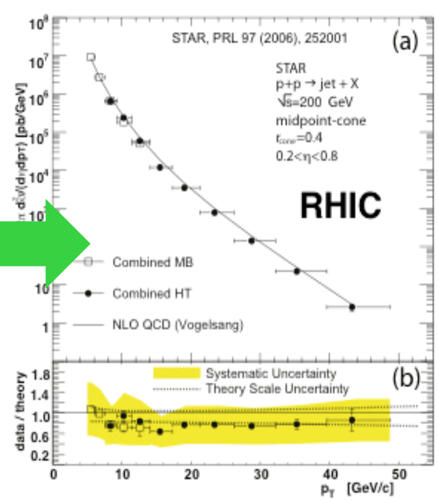
  Hadron mass spectrum from lattice QCD

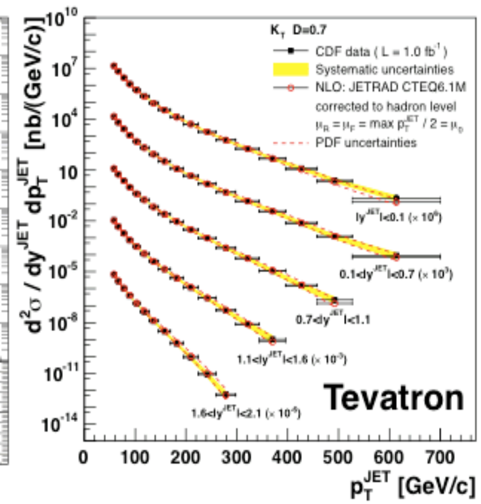


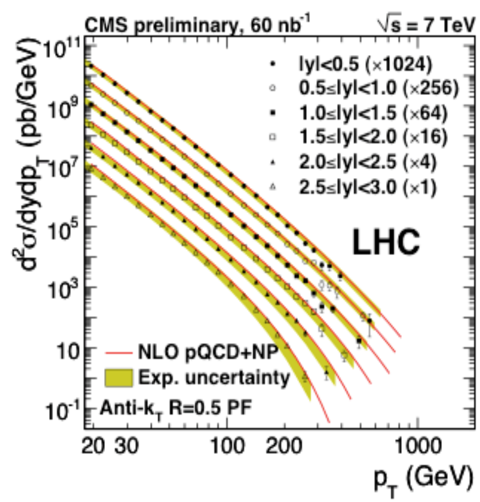
- @high energy:

  Asymptotic freedom + perturbative QCD



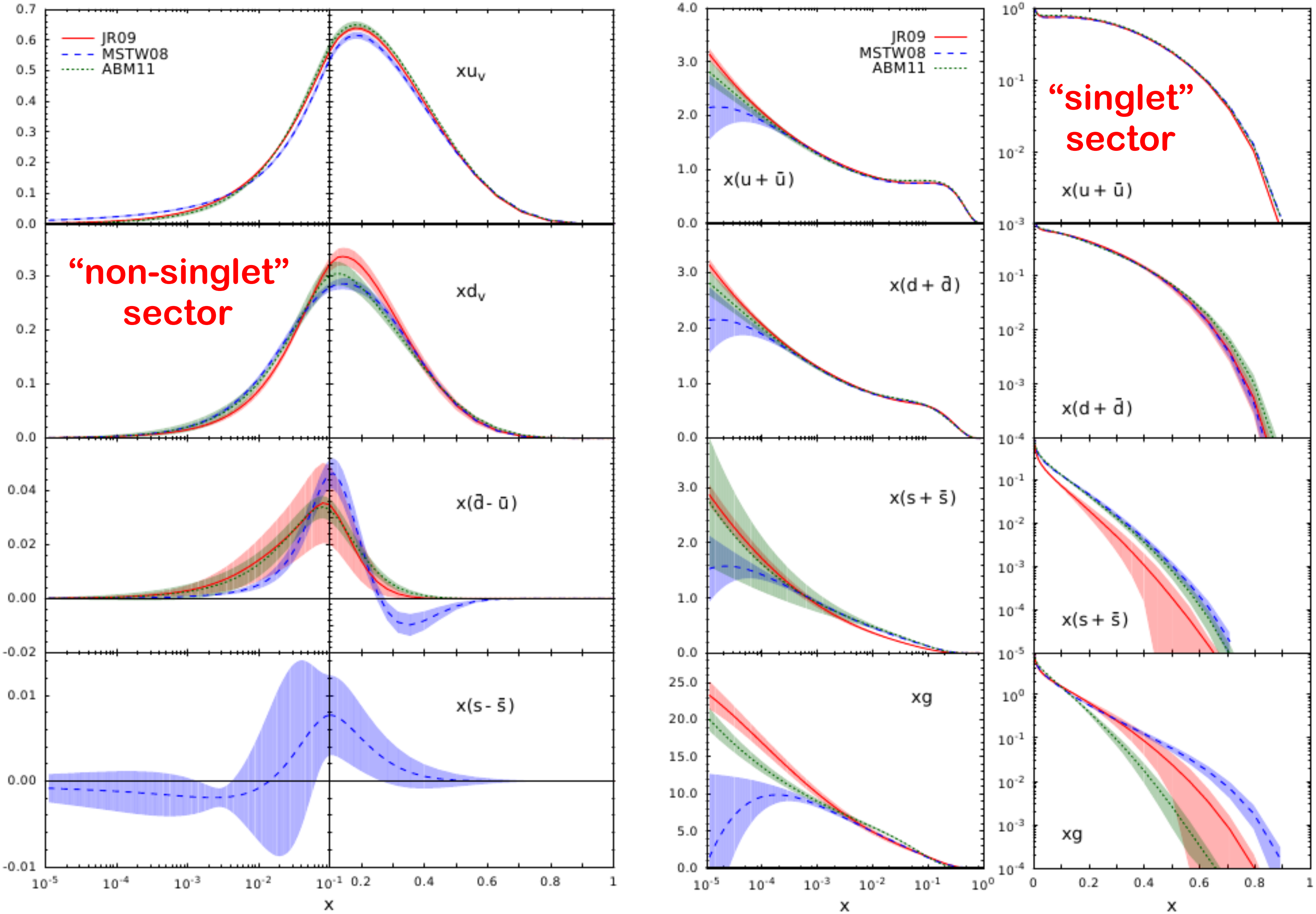






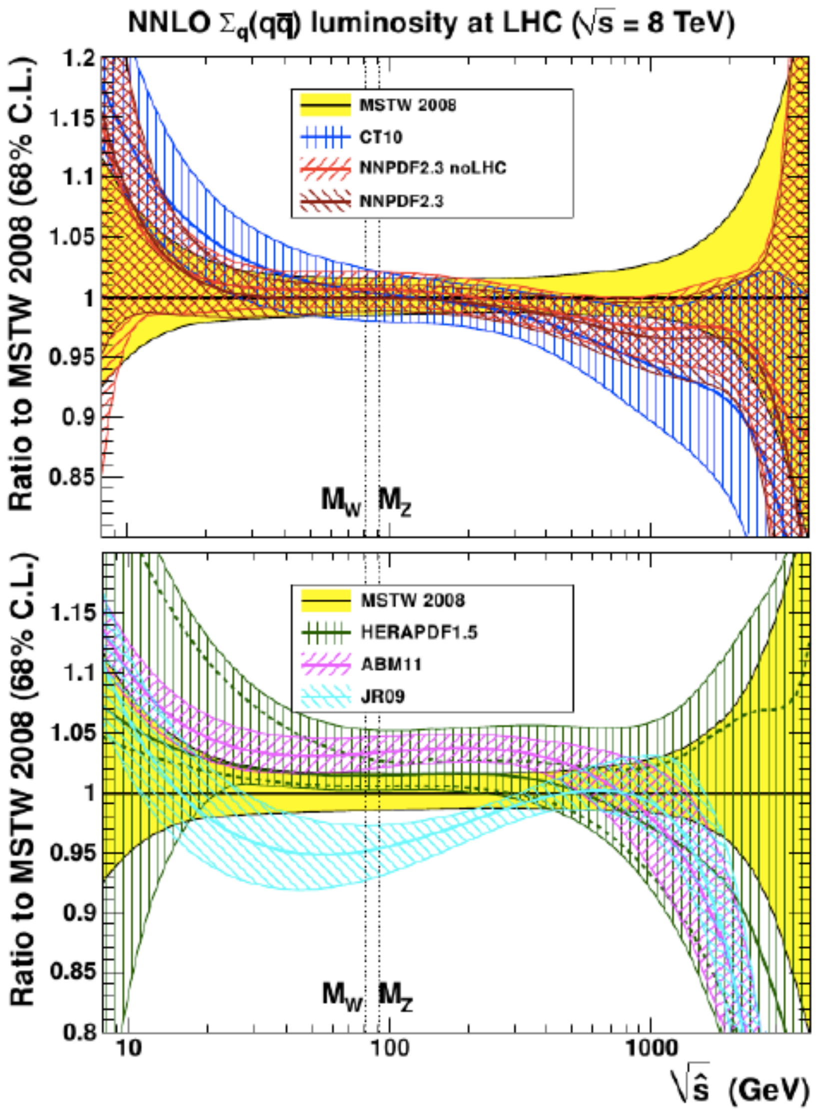
Measure e-p at 0.3 TeV (HERA)
Predict p-p and p-p̄ at 0.2, 1.96, and 7 TeV

Uncertainties of PDFs

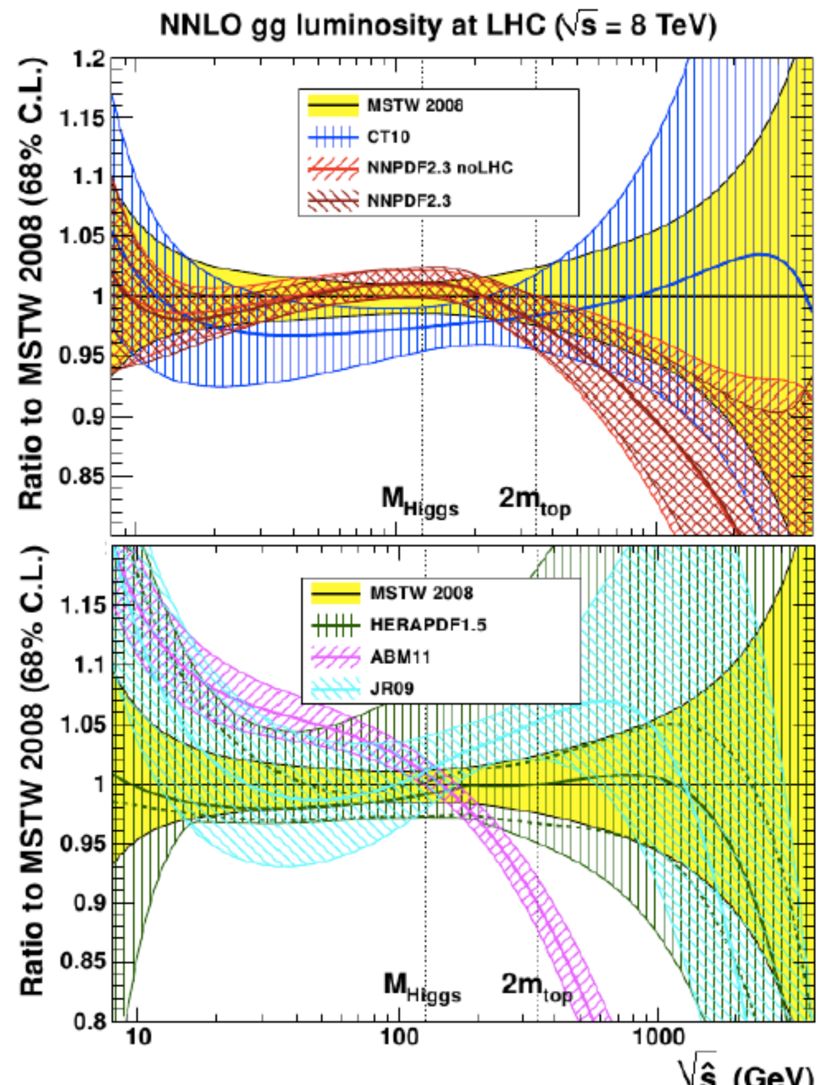


Partonic luminosities

q - qbar



g - g



Improvement from resummation

- Beyond the Born term (lowest order), partonic hard-parts are NOT unique, due to the PDFs' scheme dependence

- Same parton-level PDFs should be used for calculations of partonic parts of all observables

- All partonic hard parts have: $P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2}\right)$

  Suggests to choose the scale: $\mu_F^2 \sim Q^2$

- Hard parts have potentially large logarithms:

$$\ell n(x), \quad \frac{1}{(1-x)_+}, \quad \left(\frac{\ell n(1-x)}{1-x}\right)_+$$

⟹ Resummation of the large logarithms

PDFs at large x

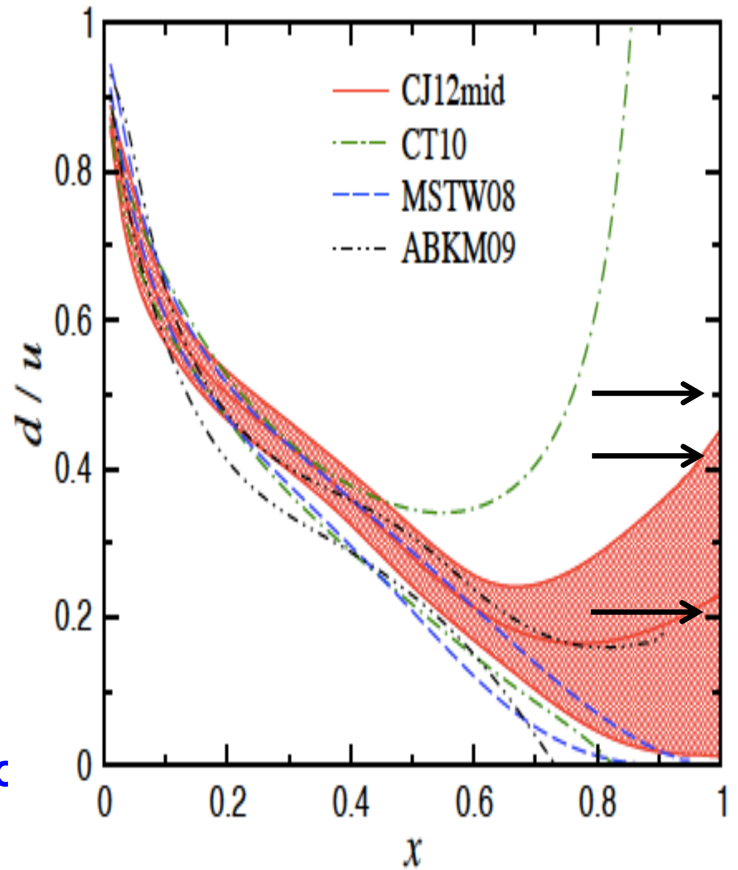
❑ **Testing ground for hadron structure at x → 1:**

✧ $d/u \to 1/2$ — **SU(6) Spin-flavor symmetry**

✧ $d/u \to 0$ — **Scalar diquark dominance**

✧ $d/u \to 1/5$ — **pQCD power counting**

✧ $d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$ — **Local quark-hadrc duality**



PDFs at large x

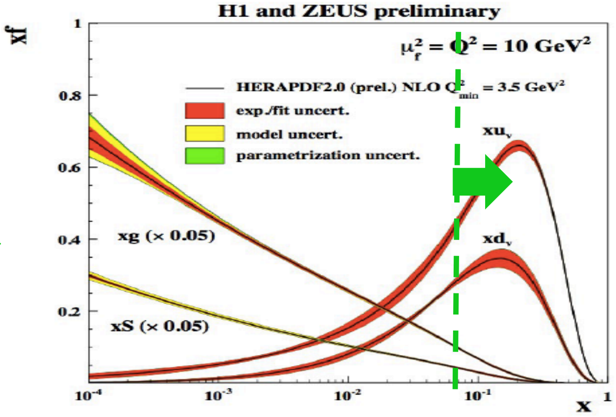
❑ **Testing ground for hadron structure at x → 1:**

✧ $d/u \to 1/2$	**SU(6) Spin-flavor symmetry**	✧ $\Delta u/u \to 2/3$ $\Delta d/d \to -1/3$
✧ $d/u \to 0$	**Scalar diquark dominance**	✧ $\Delta u/u \to 1$ $\Delta d/d \to -1/3$
✧ $d/u \to 1/5$	**pQCD power counting**	✧ $\Delta u/u \to 1$ $\Delta d/d \to 1$
✧ $d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$	**Local quark-hadron duality**	✧ $\Delta u/u \to 1$ $\Delta d/d \to 1$

Can lattice QCD help?

Lattice calculations of hadron structure

Lattice QCD



X-dep distributions

- **New ideas – from quasi-PDFs (lattice calculable) to PDFs:**
  - ✧ High P_z effective field theory approach:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Ji, et al., arXiv:1305.1539 1404.6680

  - ✧ QCD collinear factorization approach:

$$\tilde{q}(x,\mu^2,P_z) = \sum_f \int_0^1 \frac{dy}{y} \mathcal{C}_f\left(\frac{x}{y},\frac{\mu^2}{\bar{\mu}^2},P_z\right) f(y,\bar{\mu}^2) + \mathcal{O}\left(\frac{1}{\mu^2}\right)$$

Ma and Qiu, arXiv:1404.6860 1412.2688

Ishikawa, Qiu, Yoshida,

Parameter like $\sqrt{s}$

Factorization scale

High twist Power corrections

Unmatched potential: *PDFs of proton, neutron, pion, …, and TMDs and GPDs, …*

Summary of lecture two

- QCD factorization has been extremely successful in predicting and interpreting high energy scattering data with the momentum transfer > 2 GeV

- PQCD factorization approach is mature, NLO calculations are available for most observables, NNLO are becoming available for the search of new physics

- Direct photon data are still puzzling and chalenging

- NLO PDFs are very stable now, and NNLO PDFs are becoming available

- New ideas: Lattice QCD calculation of partonic structure of hadrons

Hadron structure beyond PDFs, quantum correlation between hadron spin and its confined parton motions, … ?

Backup slides

How to calculate the perturbative parts?

❑ **Use DIS structure function F_2 as an example:**

$$F_{2h}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right) + O\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

✧ **Apply the factorized formula to parton states:** $h \to q$

Feynman diagrams → $$F_{2q}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$ ← **Feynman diagrams**

✧ **Express both SFs and PDFs in terms of powers of** α_s**:**

0th order: $$F_{2q}^{(0)}(x_B,Q^2) = C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$

⇨ $$C_q^{(0)}(x) = F_{2q}^{(0)}(x) \qquad \varphi_{q/q}^{(0)}\left(x\right) = \delta_{qq}\delta\left(1-x\right)$$

1th order: $$F_{2q}^{(1)}(x_B,Q^2) = C_q^{(1)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right) + C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

⇨ $$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

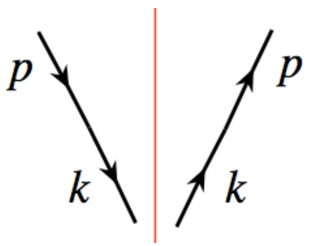
PDFs of a parton

- Change the state without changing the operator:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\, \psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow |\text{parton}(p)\rangle$ ➡ $\phi_{f/q}(x,\mu^2)$ – **given by Feynman diagrams**

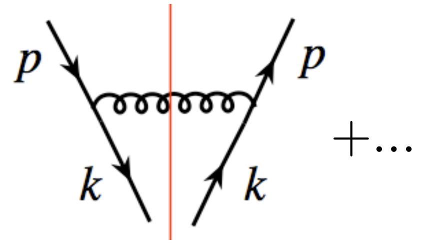
- Lowest order quark distribution:



  - ✧ From the operator definition:

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'}\int \frac{d^4k}{(2\pi)^4}\text{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)\\ &= \delta_{qq'}\;\delta(1-x)\end{aligned}$$

- Leading order in α_s quark distribution:



  - ✧ Expand to $(g_s)^2$ – logarithmic divergent:

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\,\delta(1-x)\right] + \text{UVCT}$$

UV and CO divergence

Partonic cross sections

❑ Projection operators for SFs:

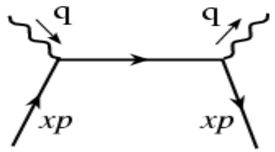
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x,Q^2\right)$$

$$F_1(x,Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2)$$

$$F_2(x,Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2)$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = xg^{\mu\nu}W_{\mu\nu,q}^{(0)} = xg^{\mu\nu}\left[\frac{1}{4\pi}\right.$$



$$\left.\right]$$

$$= \left(xg^{\mu\nu}\right)\frac{e_q^2}{4\pi}\mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p\gamma_\mu\gamma\cdot\left(p+q\right)\gamma_\nu\right]2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x\delta(1-x)$$

$$C_q^{(0)}(x) = e_q^2 x\delta(1-x)$$

NLO coefficient function – complete example

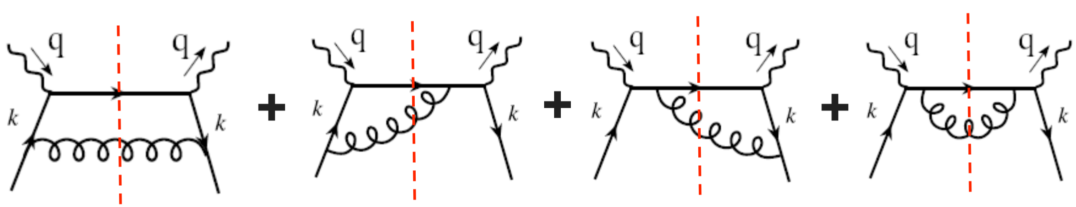
$$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

- **Projection operators in n-dimension:** $g_{\mu\nu}g^{\mu\nu} = n \equiv 4-2\varepsilon$

$$\left(1-\varepsilon\right)F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}$$

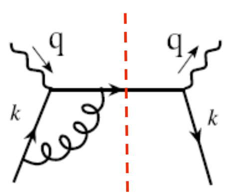
- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$

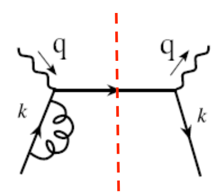


} **Real**

Virtual { +



+ c.c. (+



+ c.c. + UV CT)

- **Calculation:** $-g^{\mu\nu}W_{\mu\nu,q}^{(1)}$ and $p^\mu p^\nu W_{\mu\nu,q}^{(1)}$

Contribution from the trace of $W_{\mu\nu}$

- **Lowest order in n-dimension:**

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

- **NLO virtual contribution:**

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

- **NLO real contribution:**

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ln(1-z) f(1)$$

❑ One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu} W^{(1)}_{\mu\nu,q} = e_q^2 (1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$
$$+C_F\left[\left(1+x^2\right)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x)\right.$$
$$\left.\left.+3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

- One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W_{\mu\nu,q}^{(1)V} = 0 \qquad\qquad p^\mu p^\nu W_{\mu\nu,q}^{(1)R} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

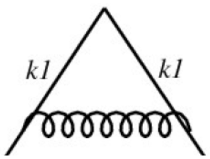
- One loop contribution to F_2 of a quark:

$$F_{2q}^{(1)}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}} P_{qq}(x)\left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as } \varepsilon \to 0$$

- One loop contribution to quark PDF of a quark:

$$\varphi_{q/q}^{(1)}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\mathrm{UV}} + \left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

❑ Common UV-CT terms:

✧ **MS scheme:** $\text{UV-CT}\big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$

✧ **$\overline{\text{MS}}$ scheme:** $\text{UV-CT}\big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1 + \varepsilon \ln(4\pi e^{-\gamma_E})\right)$

✧ **DIS scheme:** **choose a UV-CT, such that** $C_q^{(1)}(x, Q^2/\mu^2)\big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

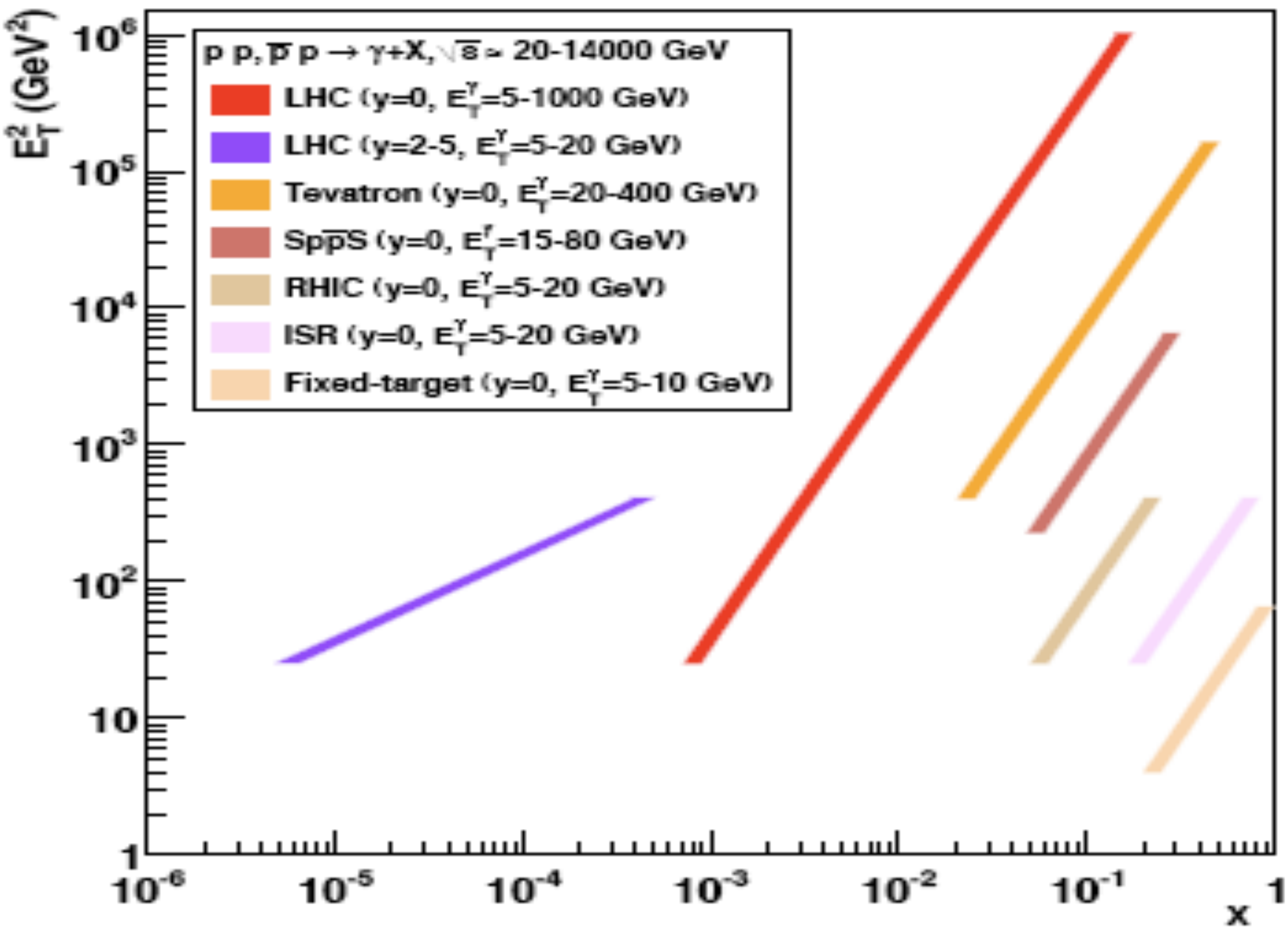
$$C_q^{(1)}(x, Q^2/\mu^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}\left(x, \mu^2\right)$$

$$C_q^{(1)}(x, Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{P_{qq}(x)\ln\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right)\right.$$

$$\left. + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Direct photon covers a wide range of x and Q^2

- Photon energy vs gluon momentum fraction x:



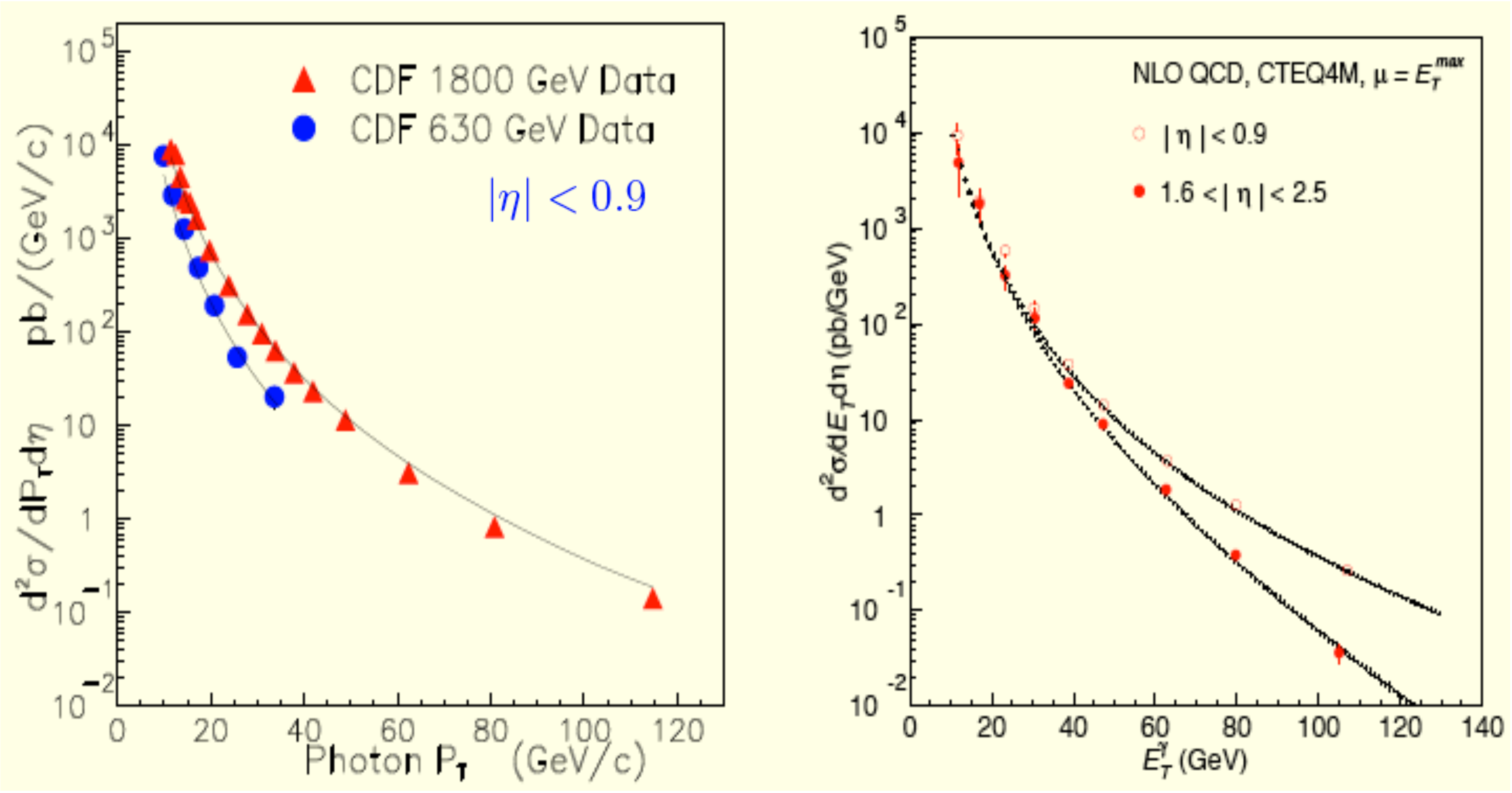
Ichou and D'Enterria, arXiv:1005.4529

Direct photon data

- ❑ **Fixed target energies** $\sqrt{s} = 20 - 40\ \text{GeV}$:
  - ✧ **With p_T = 3-10 GeV, data have high $x_T = \dfrac{2p_T}{\sqrt{s}}$**
  - ✧ **Challenge for NLO theory to fit data – wrong shape!**
- ❑ **Collider energies:**
  - ✧ **pp at ISR with** $\sqrt{s} = 44 - 62\ \text{GeV}$
  - ✧ **pp at CERN and Fermilab with** $\sqrt{s} = 540 - 1960\ \text{GeV}$
  - ✧ **$p\bar{p}$ at RHIC with** $\sqrt{s} = 200 - 500\ \text{GeV}$**, dA and AA as well**
  - ✧ **pp at LHC with** $\sqrt{s} = 7 - 14\ \text{TeV}$**, and PbPb as well**
- ❑ **Data sources:**
  - ✧ **Data review by W. Vogelsang and M.R. Whalley,**
    J. Phys. G23, Suppl. 7A, A1 (1997)
  - ✧ **Online database at http://durpdg.dur.ac.uk/HEPDATA**

Theory vs experimental data

❑ Tevatron data:



✧ Agreement looks good when plotted on a logarithmic scale

✧ QCD description of direct photon production works

QCD and hadrons