Perturbative QCD and Hard Processes

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Summary of lecture three

- Event shape – jettiness is a new powerful observable for studying the pattern of QCD (medium induced) radiation

- Heavy quarkonium production is still a very fascinating subject challenging our understanding of QCD bound states

- The need to have a heavy quark pair, heavy quarkonium production is an ideal place to study QCD power corrections, coherent multiple scatterings, …

- TMD factorization of two-scale observables (one large, one small) provides a new and unique probe to "see" the confined motion: the large scale to pin down the parton d.o.f. while the small scale to probe the nonperturbative structure as well as the motion

- Proton spin provides another controllable "knob" to help isolate various physical effects

Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation:

- **Cross section:**

Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma_{AB}^{(2)}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma_{AB}^{(3)}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma_{AB}^{(4)}(Q,\vec{s}) + \cdots$$

- **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ **– Positive definite**

- **Asymmetries or difference of cross sections:**

  - **both beams polarized** A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

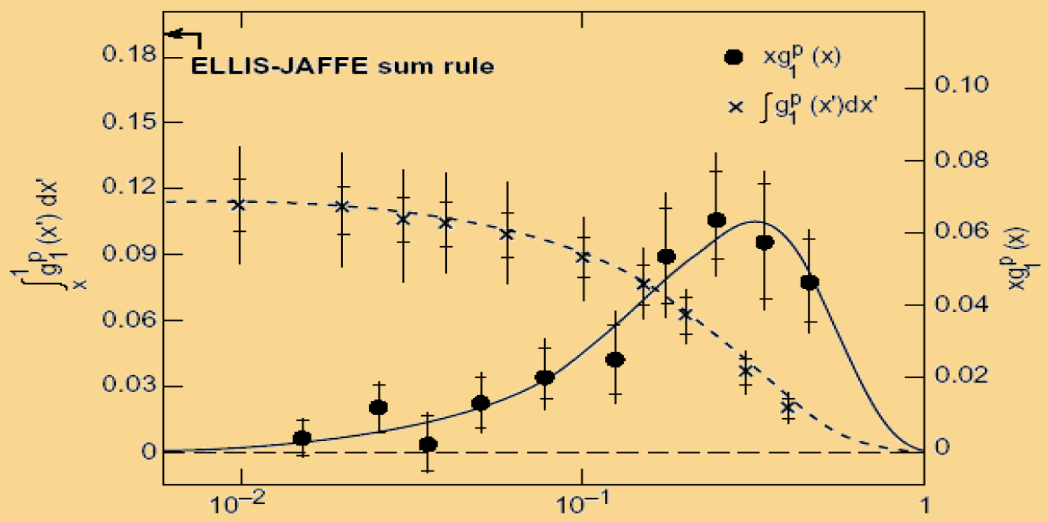
  - **one beam polarized** A_L, A_N **– Not necessary positive!**

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Proton "spin crisis" – excited the field

❑ EMC (European Muon Collaboration '87) – "the Plot":



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

- ✧ **Combined with earlier SLAC data:** $\int_0^1 g_1^p(x)dx = 0.126 \pm 0.018$
- ✧ **Combined with:** $g_A^3 = \Delta u - \Delta d$ **and** $g_A^8 = \Delta u + \Delta d - 2\Delta s$ **from low energy neutron & hyperon β decay**

➡ $$\Delta\Sigma = \sum_q \left[\Delta q + \Delta\bar{q}\right] = 0.12 \pm 0.17$$

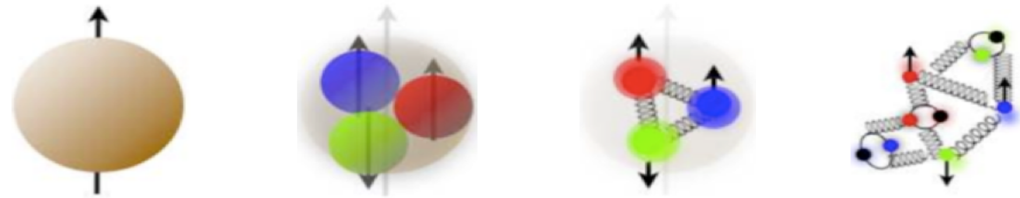
❑ "Spin crisis" or puzzle:

- ✧ **Strange sea polarization is sizable & negative**
- ✧ **Very little of the proton spin is carried by quarks**

➡ **New era of spin physics**

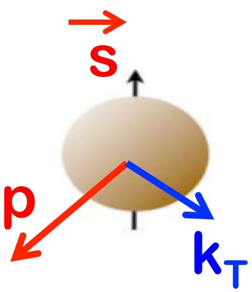
Questions driving the spin physics

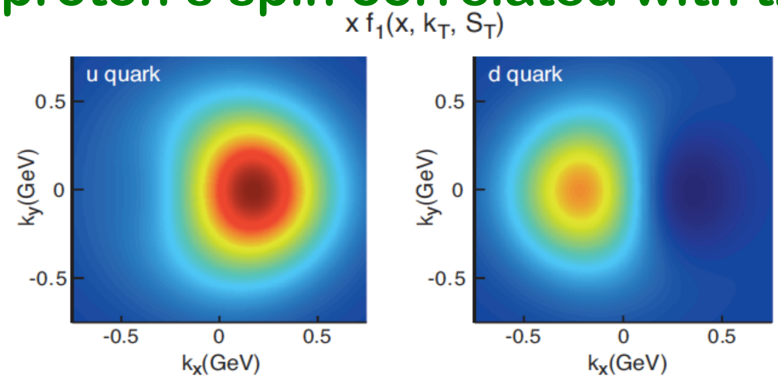
❑ How do quarks/gluons + their dynamics make up the proton spin?

$S(\mu) = \frac{1}{2}$  $\mu \Rightarrow \infty$

Helicity distributions + orbital contribution

❑ How is proton's spin correlated with the motion of quarks/gluons?

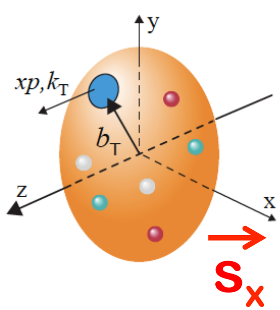


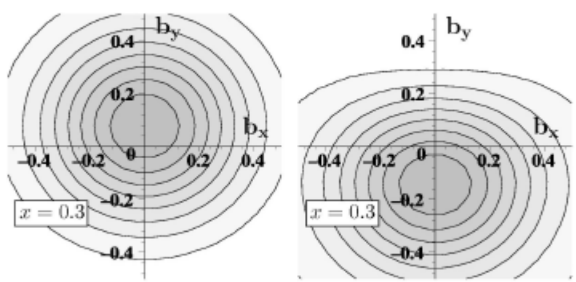


Deformation of parton's *confined motion* when hadron is polarized?

TMDs!

❑ How does proton's spin influence the spatial distribution of partons?





Deformation of parton's *spatial distribution* when hadron is polarized?

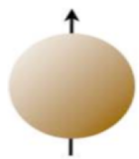
GPDs!

Current understanding for Proton Spin

- **The sum rule:** $$S(\mu) = \sum_f \langle P,S|\hat{J}_f^z(\mu)|P,S\rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$
  - **Infinite possibilities of decompositions – connection to observables?**
  - **Intrinsic properties + dynamical motion and interactions**

- **An incomplete story:**

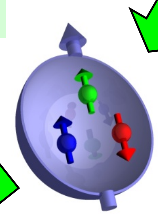
See Prof. Wakamatsu's talk

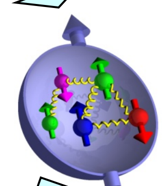


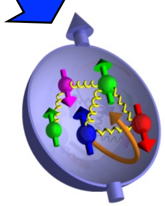
Proton Spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …







Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$
$$\sim 30\%$$

Sea quarks?

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$
$$\sim 20\%(\text{with RHIC data})$$

Orbital Angular Momentum of quarks and gluons
Little known

Net effect of partons' transverse motion?

Two roles of the proton spin program

❑ Proton is a composite particle:

Spin is a consequence of internal dynamics of the bound state

For example, the nucleon-nucleon interaction and shell structure determines the observed nuclear spin states

➡ Decomposition of proton spin in terms of quark and gluon d.o.f.

helps understand the dynamics of a fundamental QCD bound state

– Nucleon is a building block all hadronic matter

(> 95% mass of all visible matter)

See Prof. Wakamatsu's talk

❑ Use the spin as a tool – asymmetries:

Cross section is a probability – classically measured

Spin asymmetry – the difference of two cross sections involving two different spin states

Asymmetry could be a pure quantum effect!

Spin decomposition

- The "big" question:

  If there are infinite possibilities, why bother and what do we learn?

- The "origin" of the difficulty/confusion:

  QCD is a gauge theory: a pure quark field in one gauge is a superposition of quarks and gluons in another gauge

- The fact:

  None of the items in all spin decompositions are direct physical observables, unlike cross sections, asymmetries, …

- Ambiguity in interpretation – two old examples:

  - Factorization scheme:

$$F_2(x,Q^2) = \sum_{q,\bar{q}} C_q^{\mathrm{DIS}}(x,Q^2/\mu^2) \otimes q^{\mathrm{DIS}}(x,\mu^2)$$

No glue contribution to F_2?

  - Anomaly contribution to longitudinal polarization:

$$g_1(x,Q^2) = \sum_{q,\bar{q}} \widetilde{C}_q^{\mathrm{ANO}} \otimes \Delta q^{\mathrm{ANO}} + \widetilde{C}_g^{\mathrm{ANO}} \otimes \Delta G^{\mathrm{ANO}}$$

$$\Rightarrow \quad \Delta\Sigma \longrightarrow \Delta\Sigma^{\mathrm{ANO}} - \frac{n_f \alpha_s}{2\pi} \Delta G^{\mathrm{ANO}}$$

Larger quark helicity?

Spin decomposition

- Key for a good decomposition – sum rule:
  - ✧ Every term can be related to a physical observable with controllable approximation – "independently measurable"

    DIS scheme is ok for F2, but, less effective for other observables

    Additional symmetry constraints, leading to "better" decomposition?

  - ✧ Natural physical interpretation for each term – "hadron structure"
  - ✧ Hopefully, calculable in lattice QCD – "numbers w/o distributions"

The most important task is,

Finding the connection to physical observables!

See also Prof. Wakamatsu's talk
for interesting physics insides

Basics for spin observables

❑ Factorized cross section:

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$$

❑ Parity and Time-reversal invariance:

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

❑ IF: $\langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

or $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

Operators lead to the "+" sign ➡ spin-averaged cross sections

Operators lead to the "-" sign ➡ spin asymmetries

❑ Example:

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \;\Rightarrow\; q(x)$$

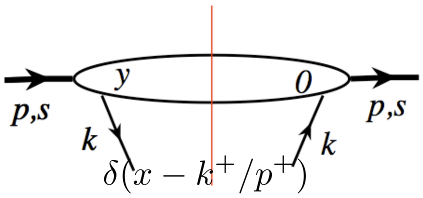
$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \;\Rightarrow\; \Delta q(x)$$

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \;\Rightarrow\; \delta q(x) \to h(x)$$

$$\mathcal{O}(\psi,A^\mu) = \frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow \Delta g(x)$$

Leading power quark distributions

❑ Spin projection for leading power quark distributions:



$$\Rightarrow\ \phi(x,p,s)_{ij} = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle p,s|\overline{\psi}_i(0)\,\psi_j(y)|p,s\rangle$$

$$\Rightarrow\ \phi(x,p,s) = \frac{1}{2}\gamma\cdot p\{q(x) - 2\lambda\Delta q(x)\gamma_5 + h(x)\gamma_5\gamma\cdot s_T\}$$

$$\text{wih nucleon helicity}\quad \lambda = \pm 1/2$$

Spin-averaged quark distribution: $q(x)$

Quark helicity distribution: $\Delta q(x)$

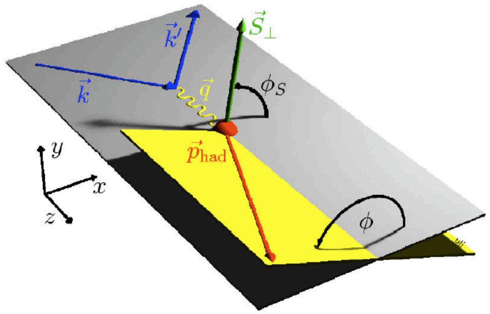
❑ Transversity distribution $h(x)$ – chiral odd:

Spin projection operator is even in gamma matrices)

Need two for a contribution to the cross section or asymmetry:

Drell-Yan: $A_{TT} \propto h(x_1) \otimes h(x_2)$

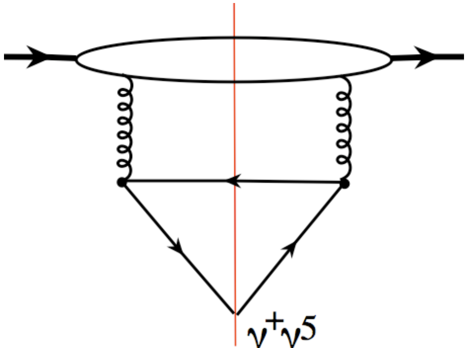
SIDIS: $A_T^{\sin(\phi+\phi_s)} \propto h(x) \otimes D_{\text{Collins}}(z)$



Soffer's bound: $q(x) + \Delta q(x) \geq 2|h(x)|$

Earlier "solution" to the "crisis"

- **Large ΔG to cancel the "true" Δq:**



$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_\| | \overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0) | P, s_\| \rangle$$

$$\Delta\Sigma \to \Delta\Sigma - \frac{n_f \alpha_s(Q^2)}{2\pi}\Delta G(Q^2)$$

- **What value of ΔG is needed?**

$$\Delta G(Q^2) \sim 2$$ **at Q ~ 1 GeV**

- **Question: How to measure ΔG independently?**

  - ✧ **Precision inclusive DIS**
  - ✧ **Jets in SIDIS**
  - ✧ **Hadronic collisions – RHIC spin, …**

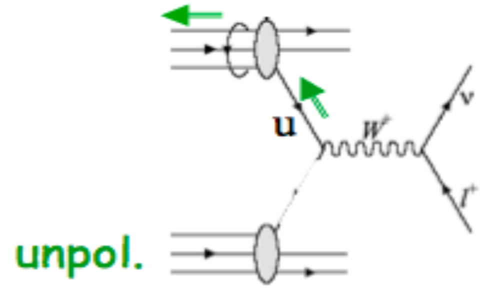
Lead to the first goal of RHIC spin program

Sea quark polarization – RHIC W program

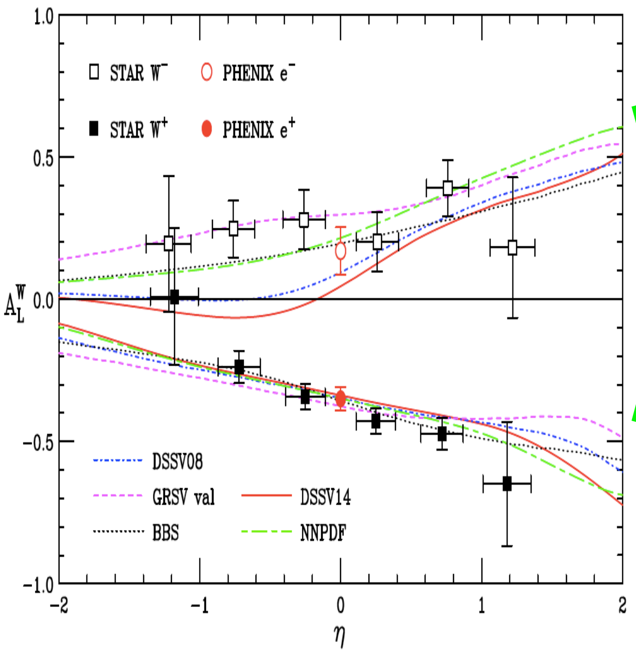
- Single longitudinal spin asymmetries:

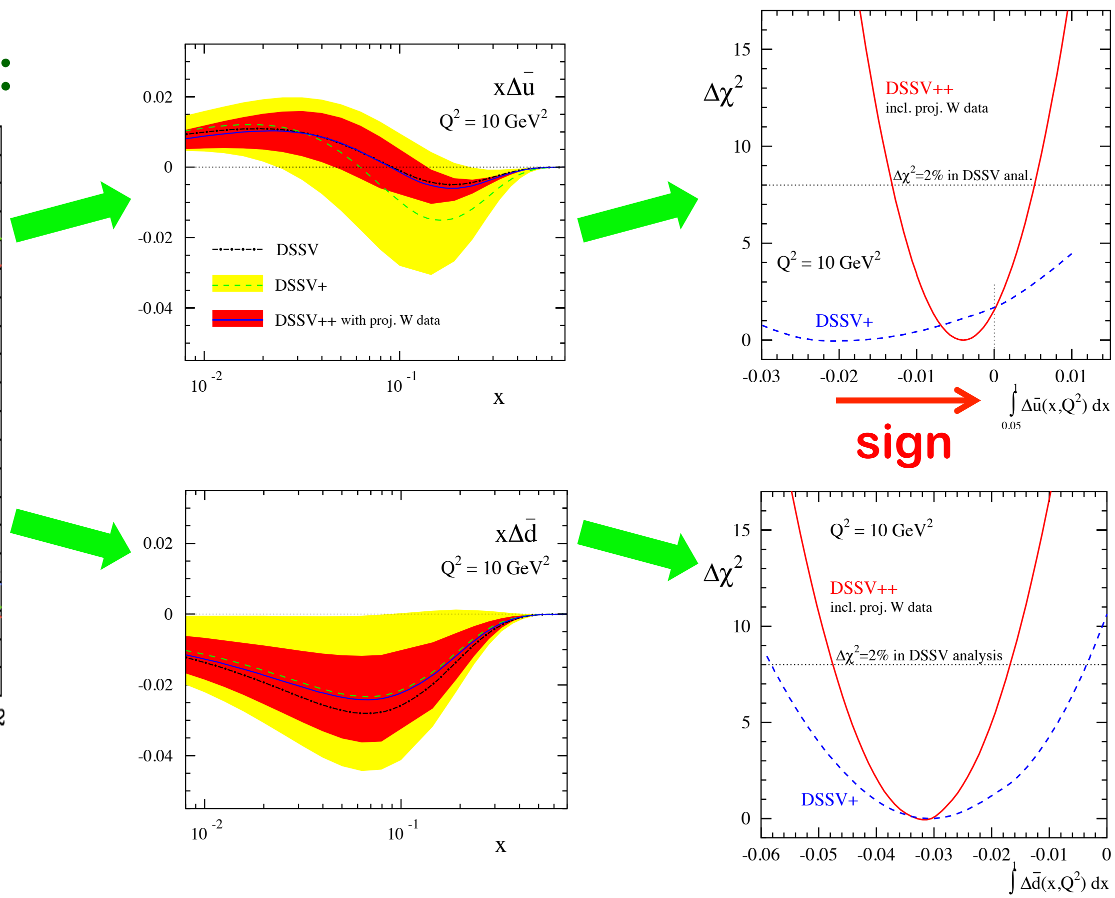
$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s)$$

Parity violating weak interaction



- From 2013 RHIC data:

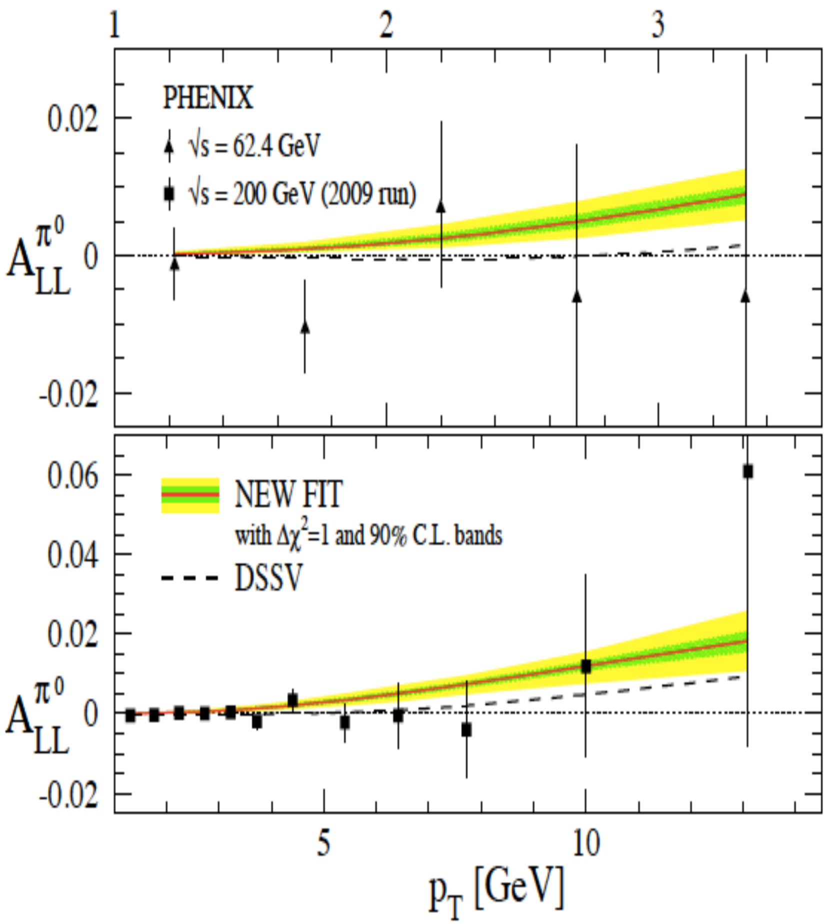


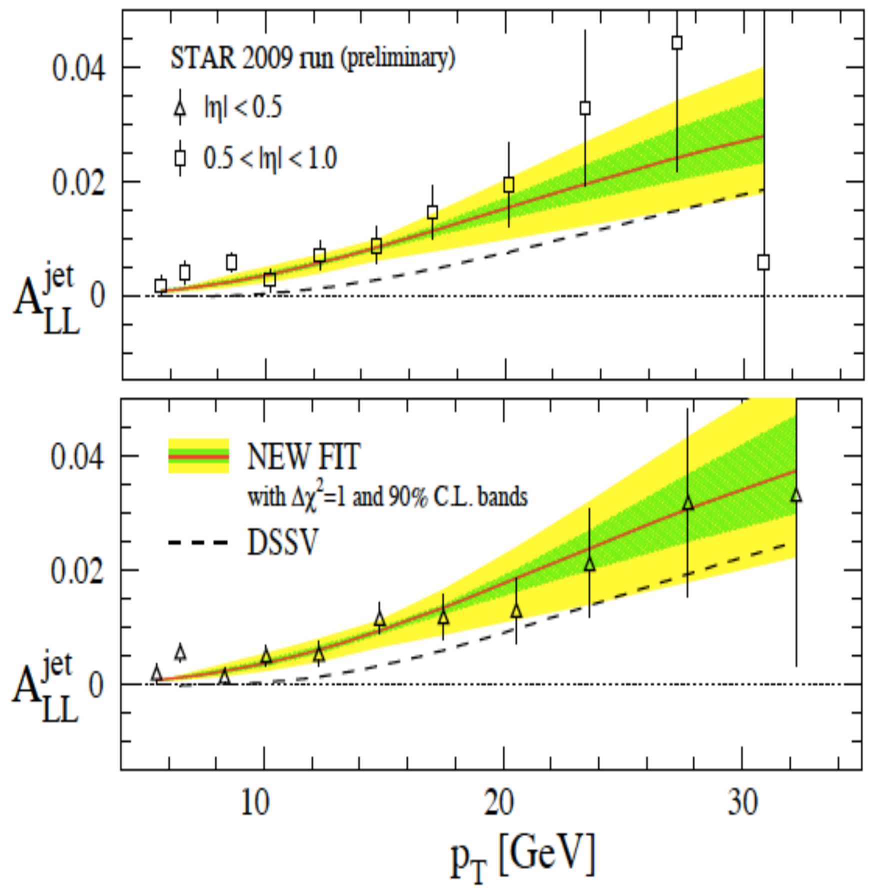


Gluon helicity contribution – RHIC data

❑ RHIC 2009 data:

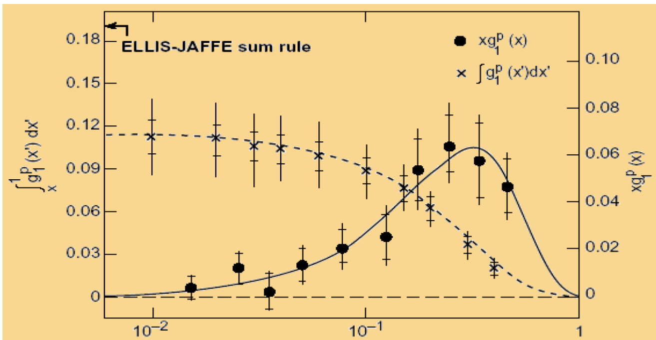
Jet/pion production at RHIC – gluon helicity:

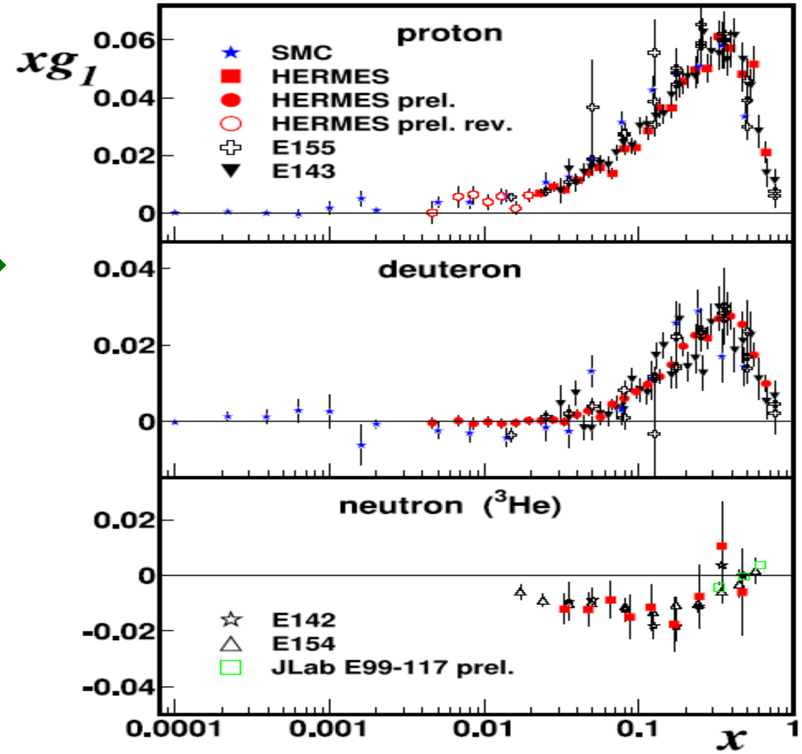




Inclusive DIS data

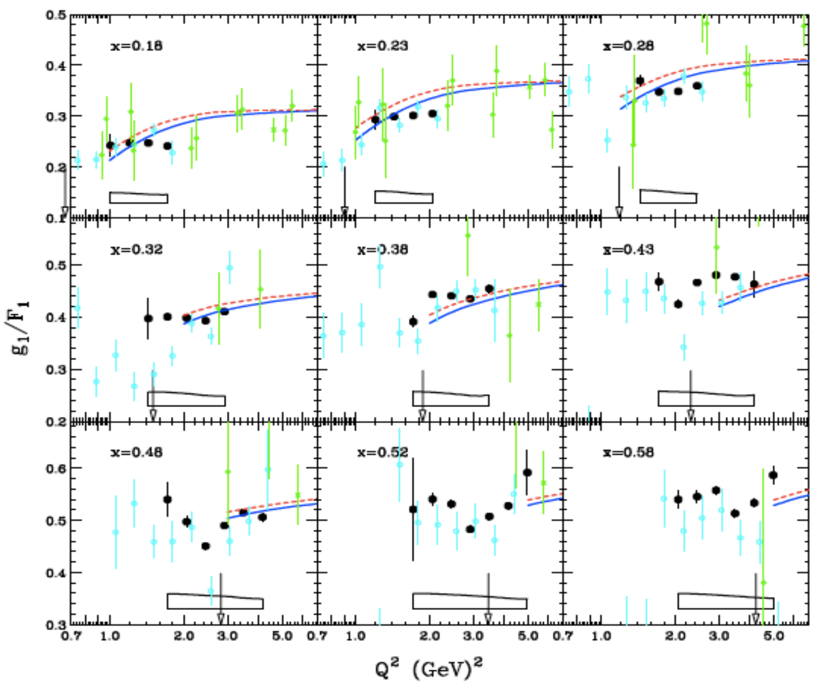
- ❑ **The "Plot" is greatly improved:**



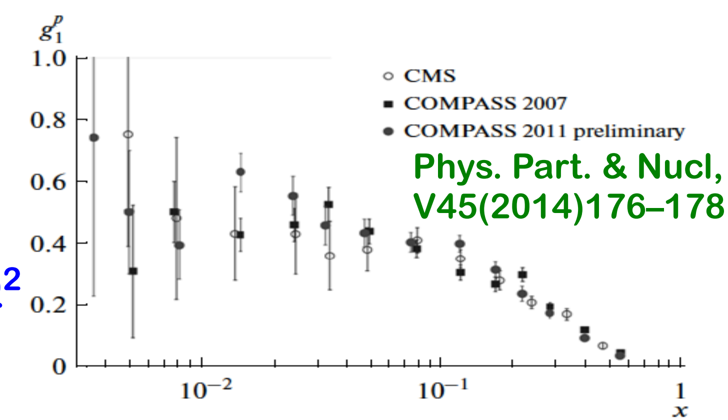


JLab/CLAS

arXiv:1404.6231



Lower Q^2 HT's



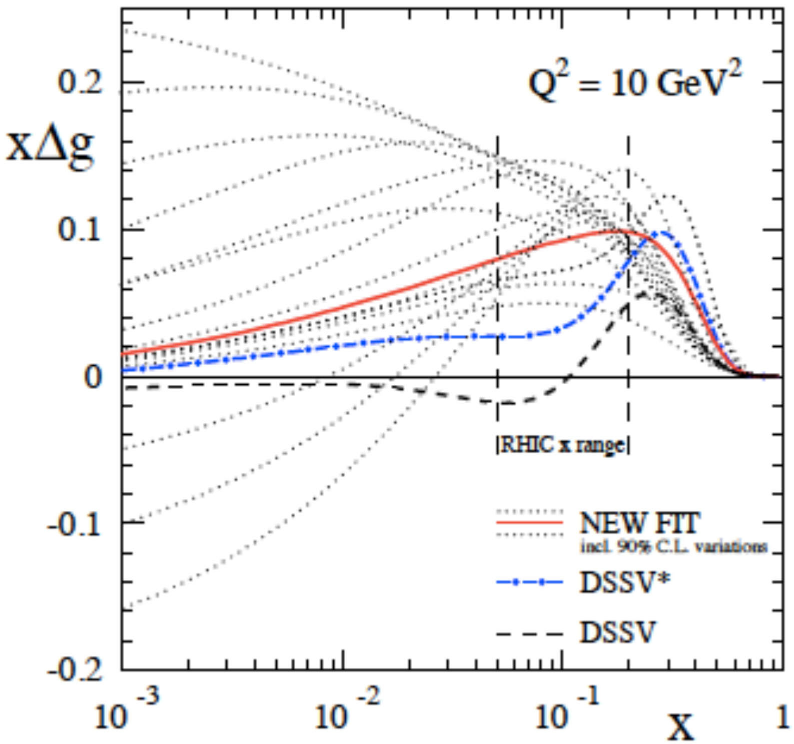
Phys. Part. & Nucl, V45(2014)176–178

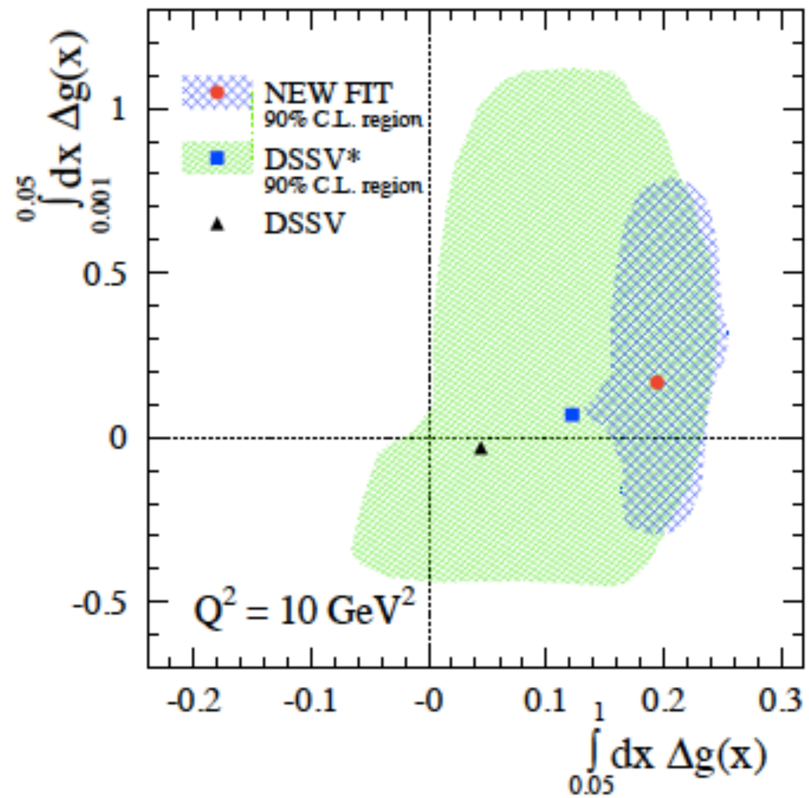
Global QCD analysis of helicity PDFs

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014) 012001

results featured in Sci. Am., Phys. World, ...

❑ Impact on gluon helicity:



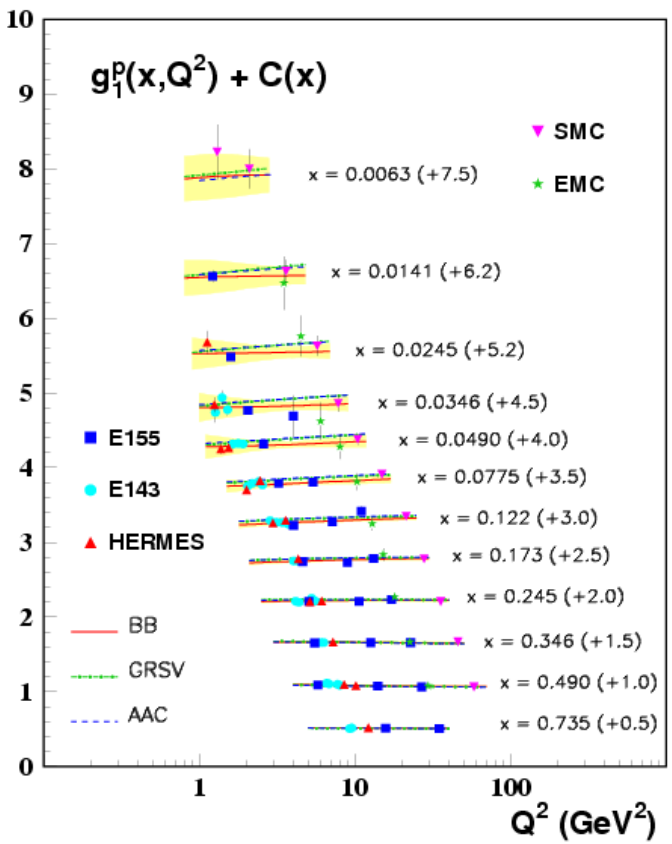


- ✧ Red line is the new fit
- ✧ Dotted lines = other fits with 90% C.L.

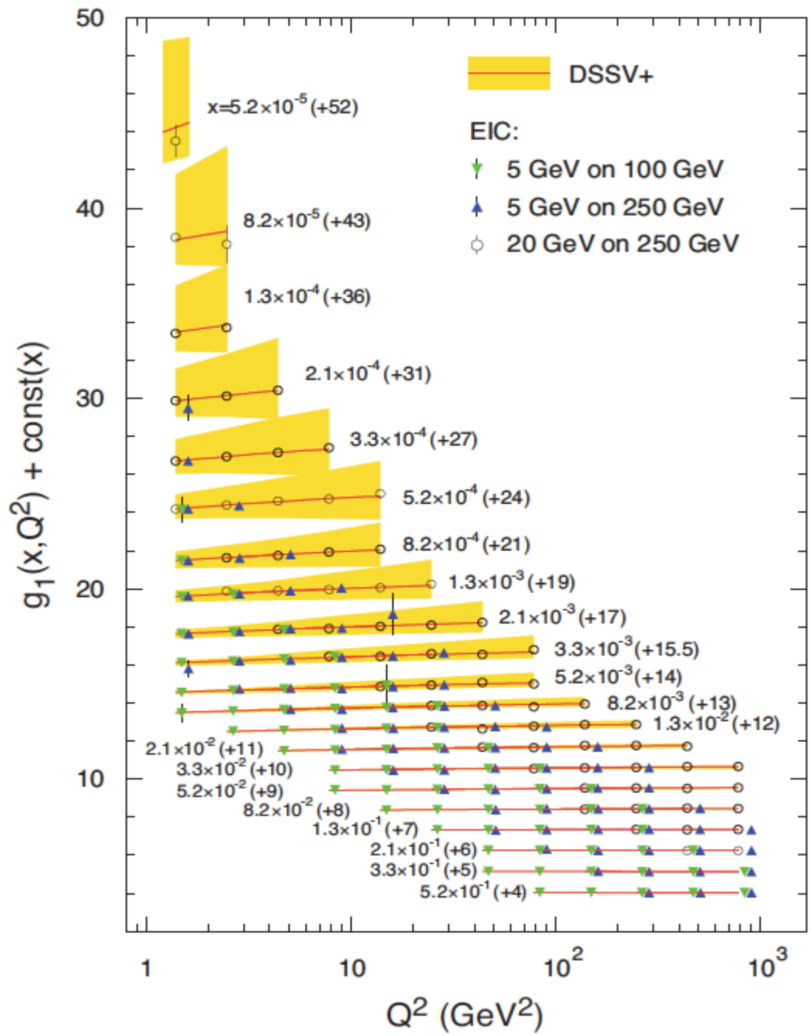
- ✧ 90% C.L. areas
- ✧ Leads ΔG to a positive #

The Future: Challenges & opportunities

- The power & precision of EIC:



at EIC



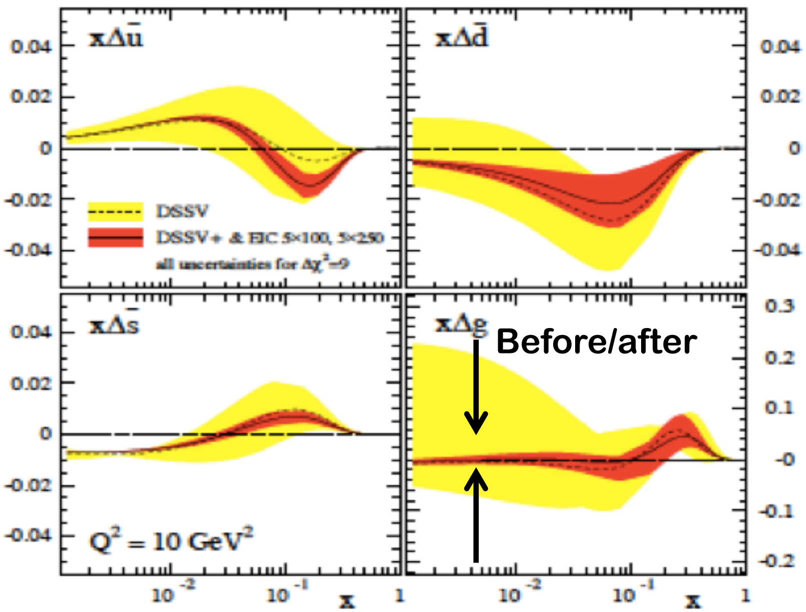
- Reach out the glue:

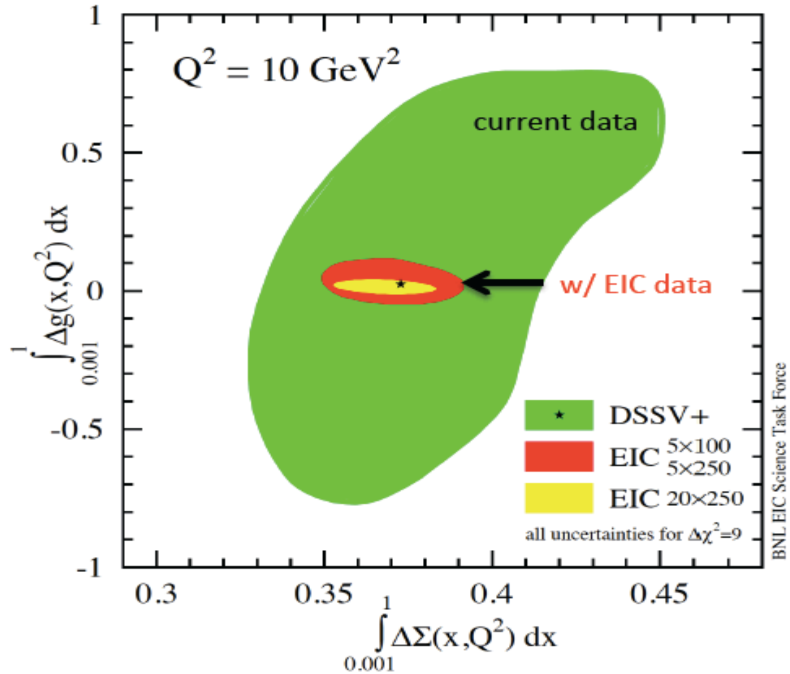
$$\frac{dg_1(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g(x,Q^2) + \cdots$$

The Future: Challenges & opportunities

- ❑ **One-year of running at EIC:**

Wider Q^2 and x range including low x at EIC!





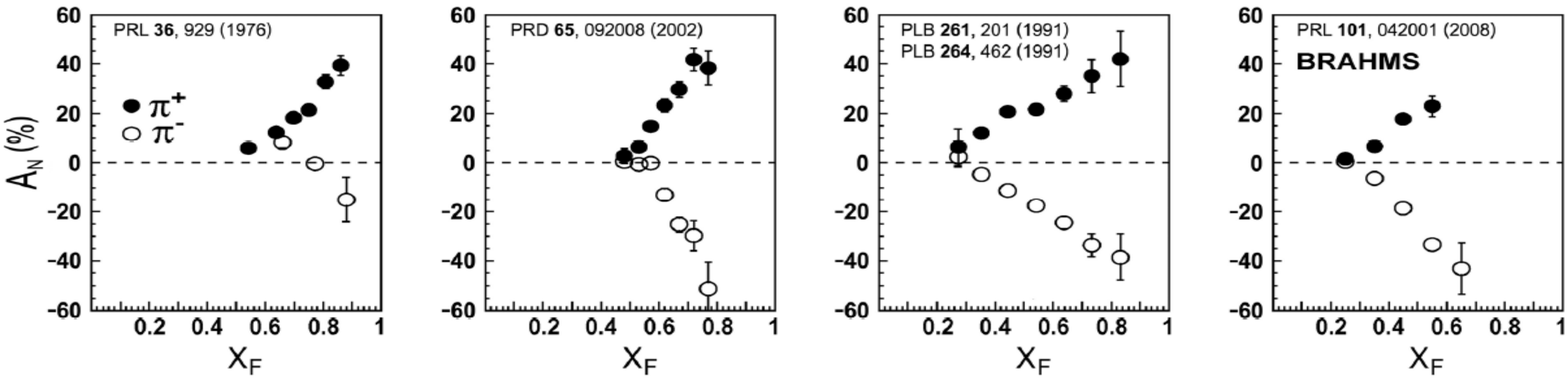
No other machine in the world can achieve this!

- ❑ **Ultimate solution to the proton spin puzzle:**
  - ✧ ***Precision measurement of $\Delta g(x)$ – extend to smaller x regime***
  - ✧ ***Orbital angular momentum contribution – measurement of GPDs!***

Single transverse-spin asymmetry

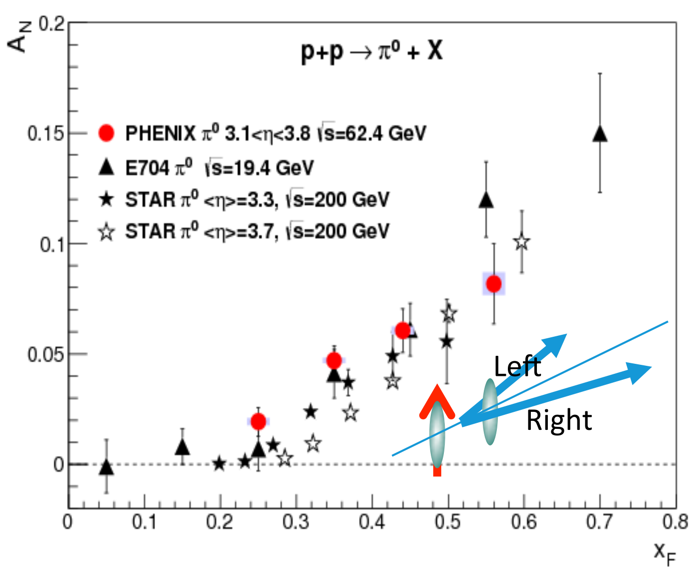
- ❑ A_N - consistently observed for over 35 years (~ 0 in parton model)!

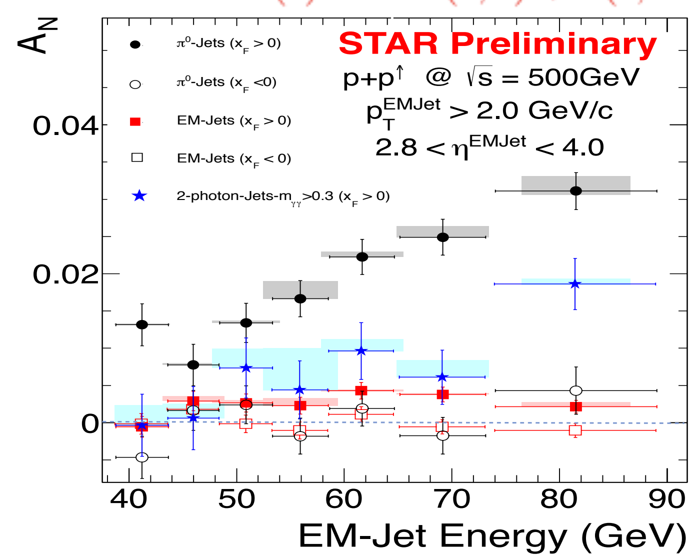
ANL – 4.9 GeV **BNL – 6.6 GeV** **FNAL – 20 GeV** **BNL – 62.4 GeV**



- ❑ Survived the highest RHIC energy:

$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$



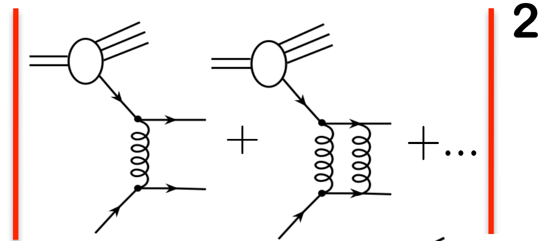


Do we understand it?

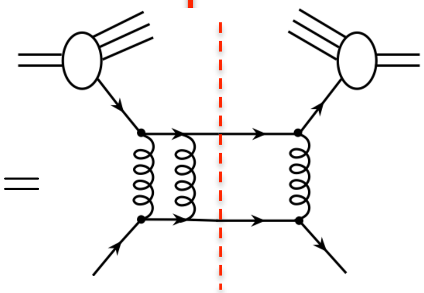
Kane, Pumplin, Repko, PRL, 1978

❑ Early attempt:

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) =$



$\propto \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

❑ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

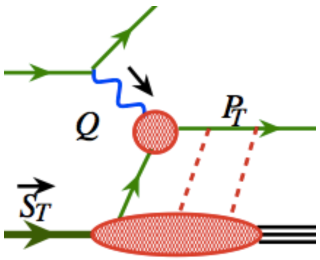
❑ Vanish without parton's transverse motion:

A direct probe for parton's transverse motion,

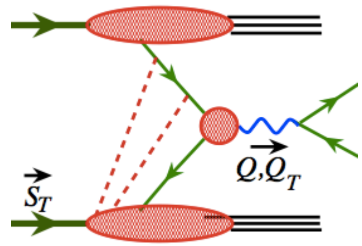
Spin-orbital correlation, QCD quantum interference

Current understanding of SSAs

- Two scales observables – $Q_1 >> Q_2 \sim \Lambda_{QCD}$:



SIDIS: $Q>>P_T$

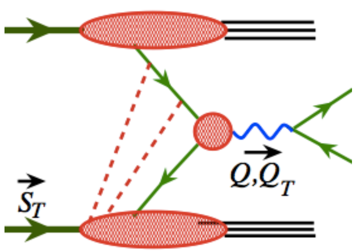


DY: $Q>>Q_T$

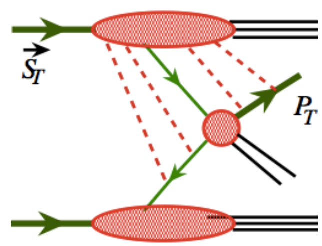
TMD factorization
TMD distributions

Direct information on parton k_T

- One scale observables – $Q >> \Lambda_{QCD}$:



DY: $Q \sim Q_T$

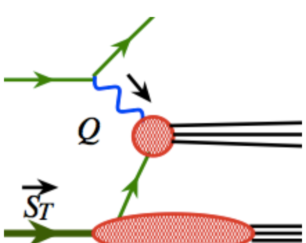


Jet, Particle: P_T

Collinear factorization
Twist-3 distributions

Information on moments of parton k_T

- Symmetry plays important role:



Inclusive DIS
Single scale
Q

Parity
Time-reversal

$A_N = 0$

Factorized Drell-Yan cross sections

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}}\, e^{y} \qquad x_B = \frac{Q}{\sqrt{s}}\, e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu)$$
$$+\mathcal{O}(1/Q, 1/q_\perp)$$

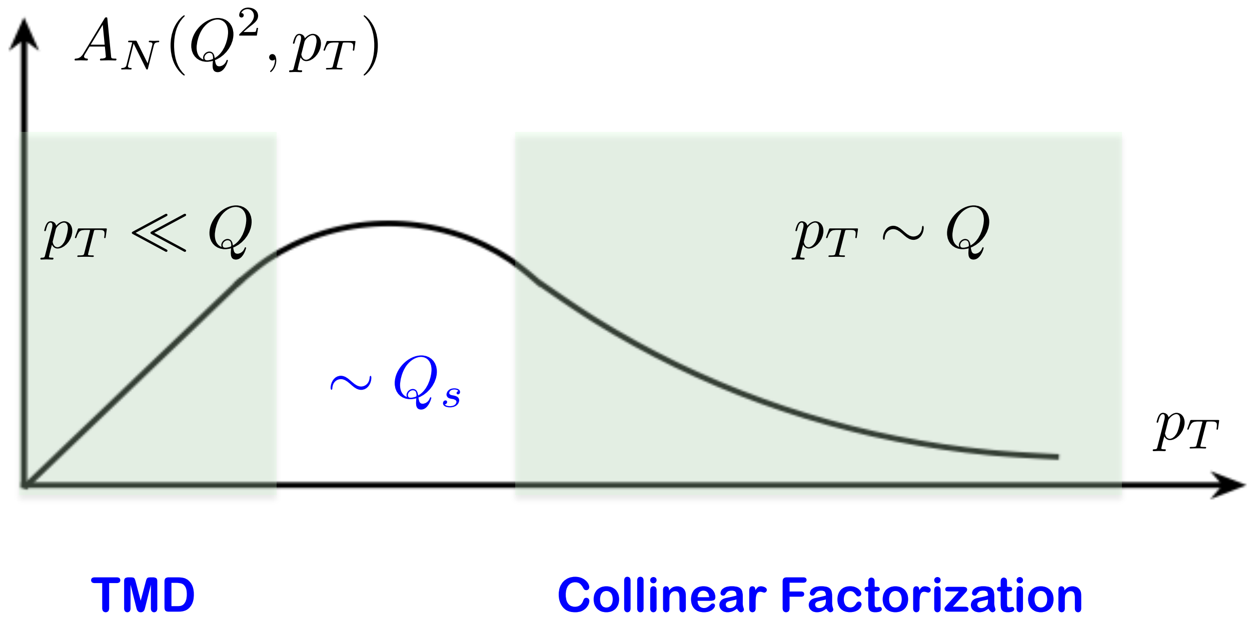
- **Spin dependence:**

The factorization arguments are independent of the spin states of the colliding hadrons

➡ **same formula with different distributions for γ*,W/Z, H^0…**

Drell-Yan from low p_T to high p_T

❑ Covers both double-scale and single-scale cases:



❑ TMD factorization to collinear factorization:

Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan

Two factorizations are consistent in the overlap region: $\Lambda_{\rm QCD} \ll p_T \ll Q$

A_N finite – requires correlation of multiple collinear partons

No probability interpretation! New opportunities!

Most notable TMD distributions

- Sivers function – transverse polarized hadron:

Sivers function

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

- Boer-Mulder function – transverse polarized quark:

$$f_{q,s_q/p}(x,\boldsymbol{k}_\perp) = \frac{1}{2} f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q^\uparrow/p}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$= \frac{1}{2} f_{q/p}(x,k_\perp) - \frac{1}{2}\frac{k_\perp}{M} h_1^{\perp q}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

Boer-Mulder function

Affect angular distribution of Drell-Yan lepton pair

Most notable TMD distributions

- Collins function – FF of a transversely polarized parton:

$$D_{h/q,s_q}(z,\boldsymbol{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_h}H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

Collins function

- Fragmentation function to a polarized hadron:

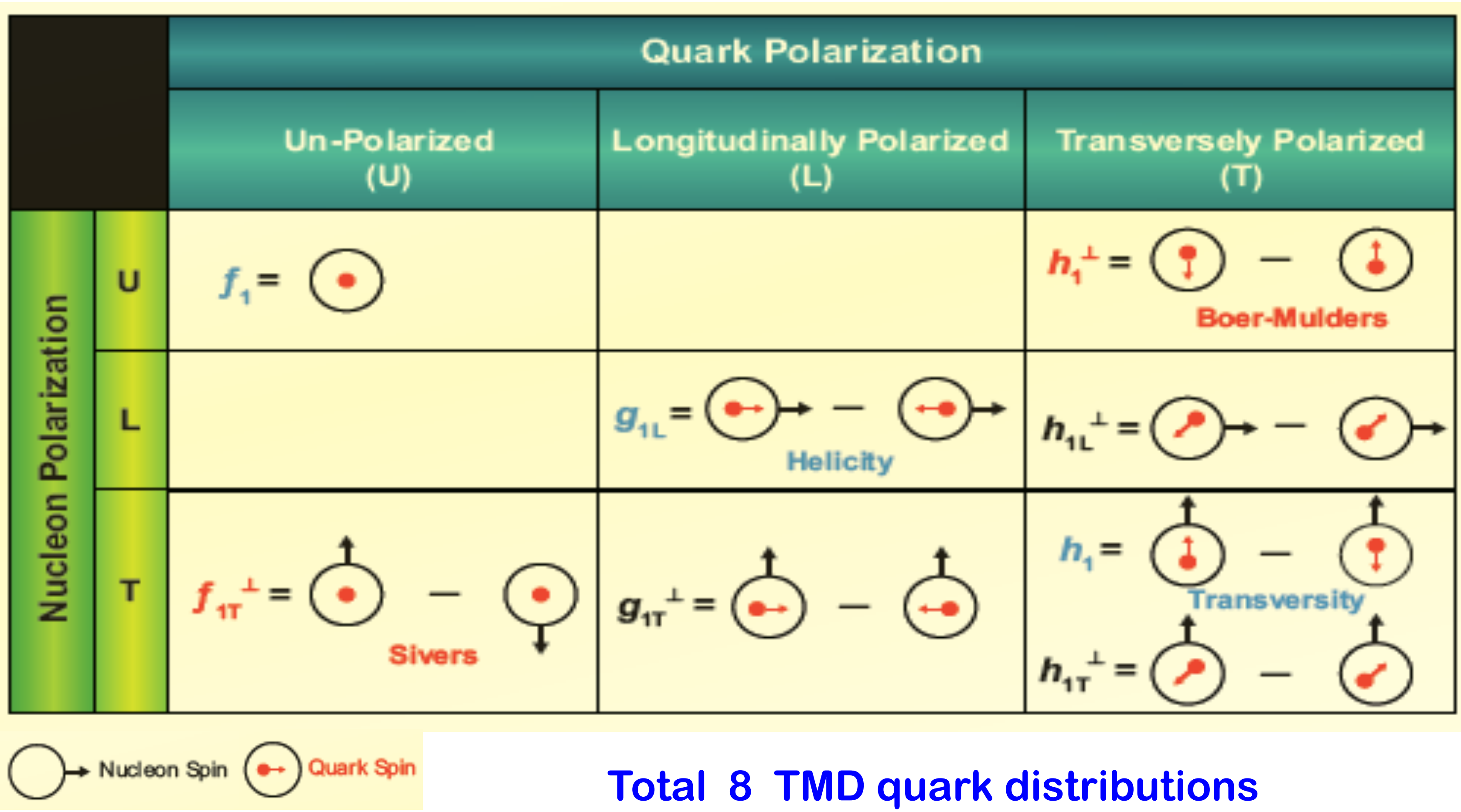
$$D_{\Lambda,S_\Lambda/q}(z,\boldsymbol{p}_\perp) = \frac{1}{2}D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{\Lambda^\uparrow/q}(z,p_\perp)\,\boldsymbol{S}_\Lambda\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$= \frac{1}{2}D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_\Lambda}D_{1T}^{\perp q}(z,p_\perp)\,\boldsymbol{S}_\Lambda\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

Unpolarized parton fragments into a polarized hadron - Λ

Other TMD distributions

- ❑ **Quark TMDs – rich quantum corelations:**



Total 8 TMD quark distributions

- ❑ **Gluon TMD distributions, …**

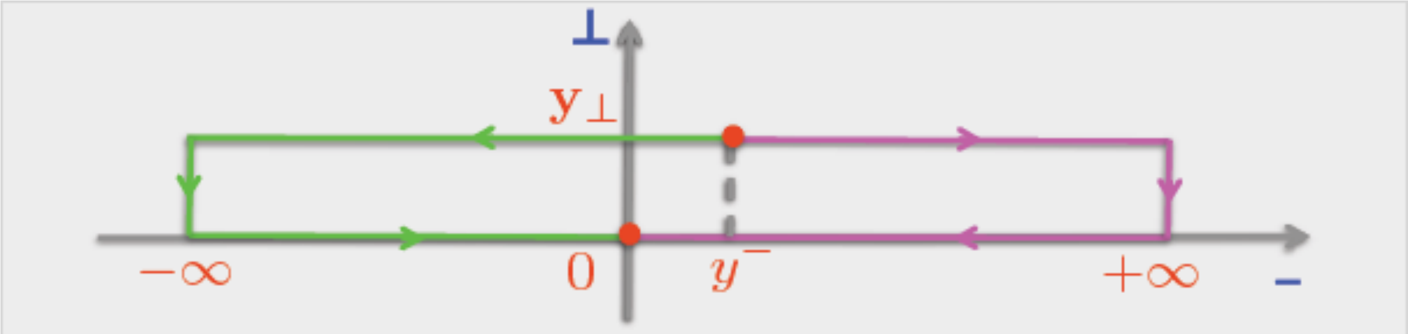
Production of quarkonium, two-photon, …

Process dependence of TMDs

❑ **The form of gauge link is a result of factorization:**

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+ y^- - i\,\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p, \vec{S} | \overline{\psi}(0^-, \mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2} \psi(y^-, \mathbf{y}_\perp) | p, \vec{S} \rangle$$

- SIDIS: $\Phi_n^\dagger(\{+\infty, 0\}, \mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(+\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\})\Phi_n(\{+\infty, y^-\}, \mathbf{y}_\perp)$
- DY: $\Phi_n^\dagger(\{-\infty, 0\}, \mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(-\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\})\Phi_n(\{-\infty, y^-\}, \mathbf{y}_\perp)$



$$f_{q/h^\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, \vec{S})$$

Collinear factorized PDFs are process independent

Critical test of TMD factorization

- **Parity – Time reversal invariance:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- **Definition of Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

- **Modified universality:**

$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x,k_\perp)$$

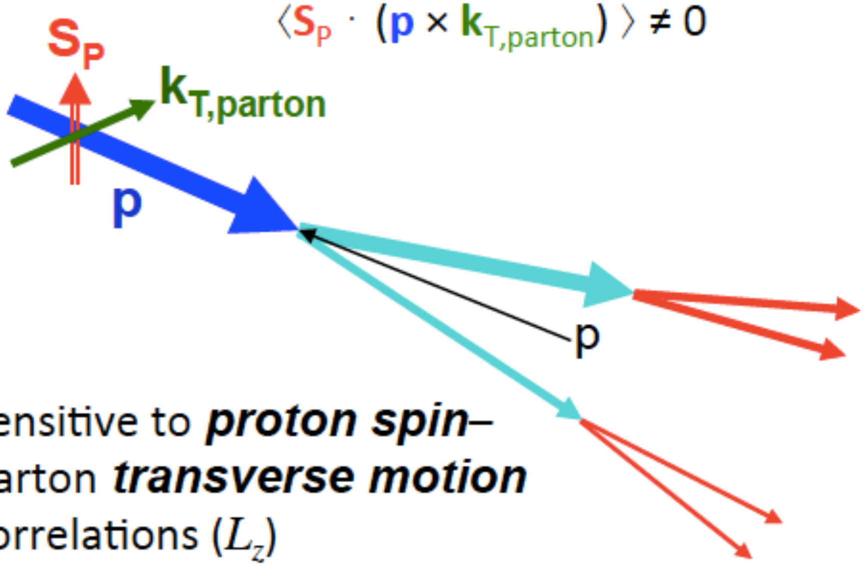
Same applies to TMD gluon distribution

Spin-averaged TMD is process independent

Transverse motion and TMDs

Sivers mechanism: asymmetry in the forward jet or γ *production*

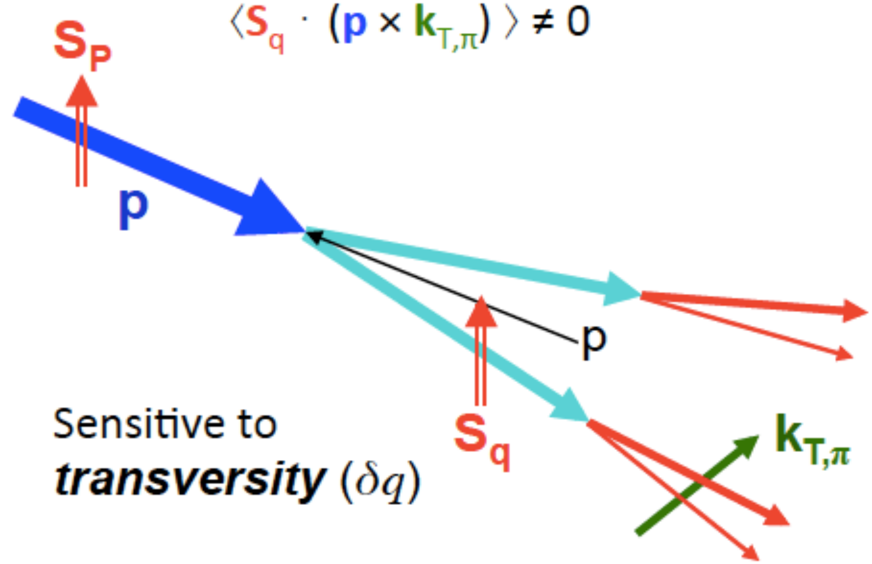
$\langle S_p \cdot (p \times k_{T,\mathrm{parton}}) \rangle \neq 0$



Sensitive to ***proton spin–parton transverse motion*** correlations (L_z)

Collins mechanism: asymmetry in the forward jet *fragmentation*

$\langle S_q \cdot (p \times k_{T,\pi}) \rangle \neq 0$



Sensitive to ***transversity*** (δq)

Inclusive hadrons:

Observed transverse single-spin asymmetries could arise from the **Sivers effect** or **Collins effect**, or from a **linear combination of the two**

Sivers or Collins ~ sin(ϕ_S)

ϕ_S—*angle between spin and event plane*

Evolution equations for TMDs

Boer, 2001, 2009, JI, Ma, Yuan, 2004
Idilbi, et al, 2004, Kang, Xiao, Yuan, 2011
Aybat, Collins, Qiu, Rogers, 2011
Aybat, Prokudin, Rogers, 2012
Idilbi, et al, 2012, Sun, Yuan 2013, …

❑ Collins-Soper equation:

– b-space quark TMD with γ*

$$\frac{\partial \tilde{F}_{f/P^{\uparrow}}(x,\mathrm{b_T},S;\mu;\zeta_F)}{\partial \ln\sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)\tilde{F}_{f/P^{\uparrow}}(x,\mathrm{b_T},S;\mu;\zeta_F) \qquad \tilde{K}(b_T;\mu) = \frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{\tilde{S}(b_T;y_s,-\infty)}{\tilde{S}(b_T;+\infty,y_s)}\right)$$

❑ RG equations:

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \qquad \frac{d\tilde{F}_{f/P^{\uparrow}}(x,\mathrm{b_T},S;\mu;\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}_{f/P^{\uparrow}}(x,\mathrm{b_T},S;\mu;\zeta_F).$$

❑ Evolution equations for Sivers function:

$$F_{f/P^{\uparrow}}(x,k_T,S;\mu,\zeta_F) = F_{f/P}(x,k_T;\mu,\zeta_F) - F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)\frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

CS: $$\frac{\partial \ln \tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)}{\partial\ln\sqrt{\zeta_F}} = \tilde{K}(b_T;\mu) \qquad \tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp f}(x,b_T;\mu,\zeta_F)}{\partial b_T}$$

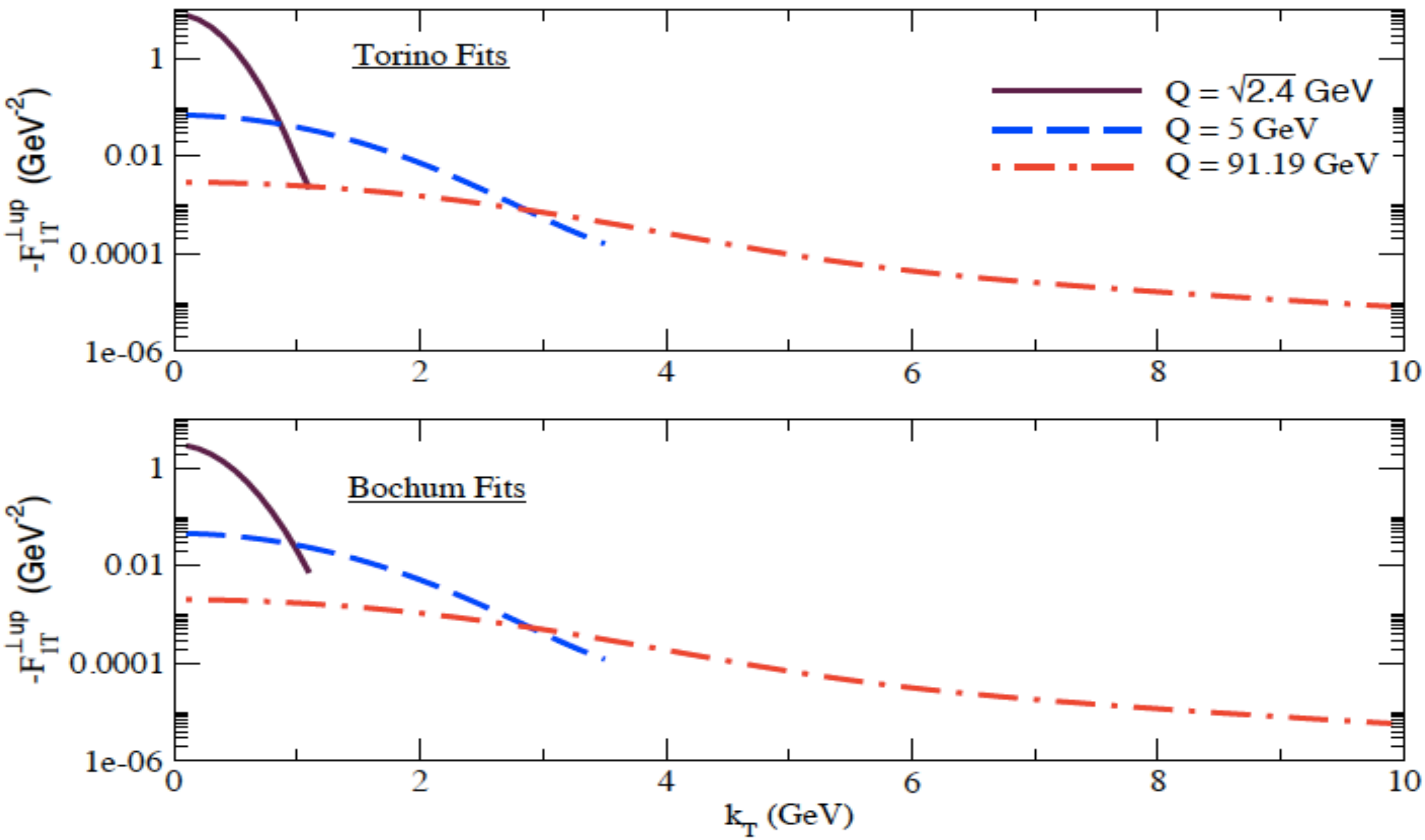
RGs: $$\frac{d\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \Longrightarrow \quad \frac{\partial\gamma_F(g(\mu);\zeta_F/\mu^2)}{\partial\ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

Scale dependence of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

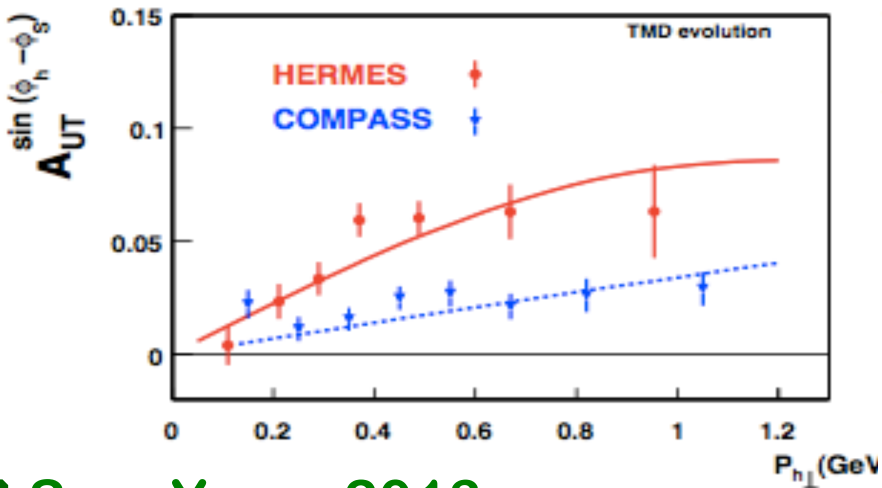
- Up quark Sivers function:

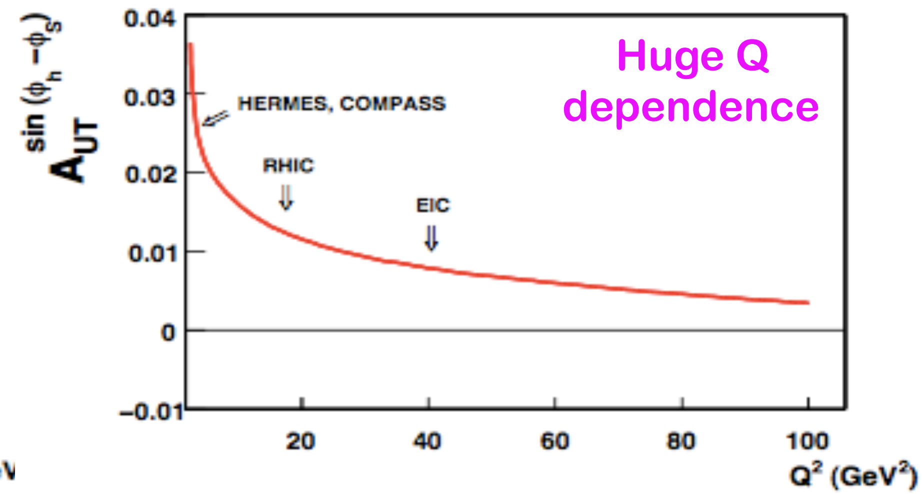


Very significant growth in the width of transverse momentum

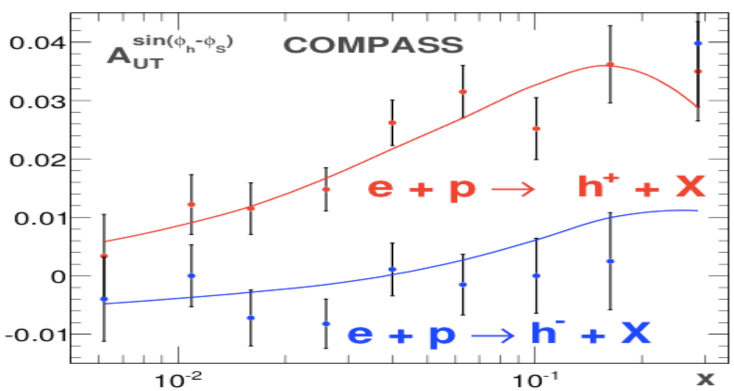
Nonperturbative input to Sivers function

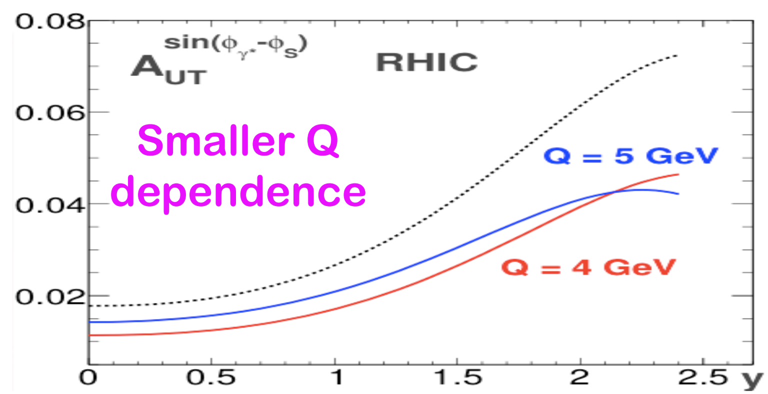
- Aybat, Prokudin, Rogers, 2012:





- Sun, Yuan, 2013:



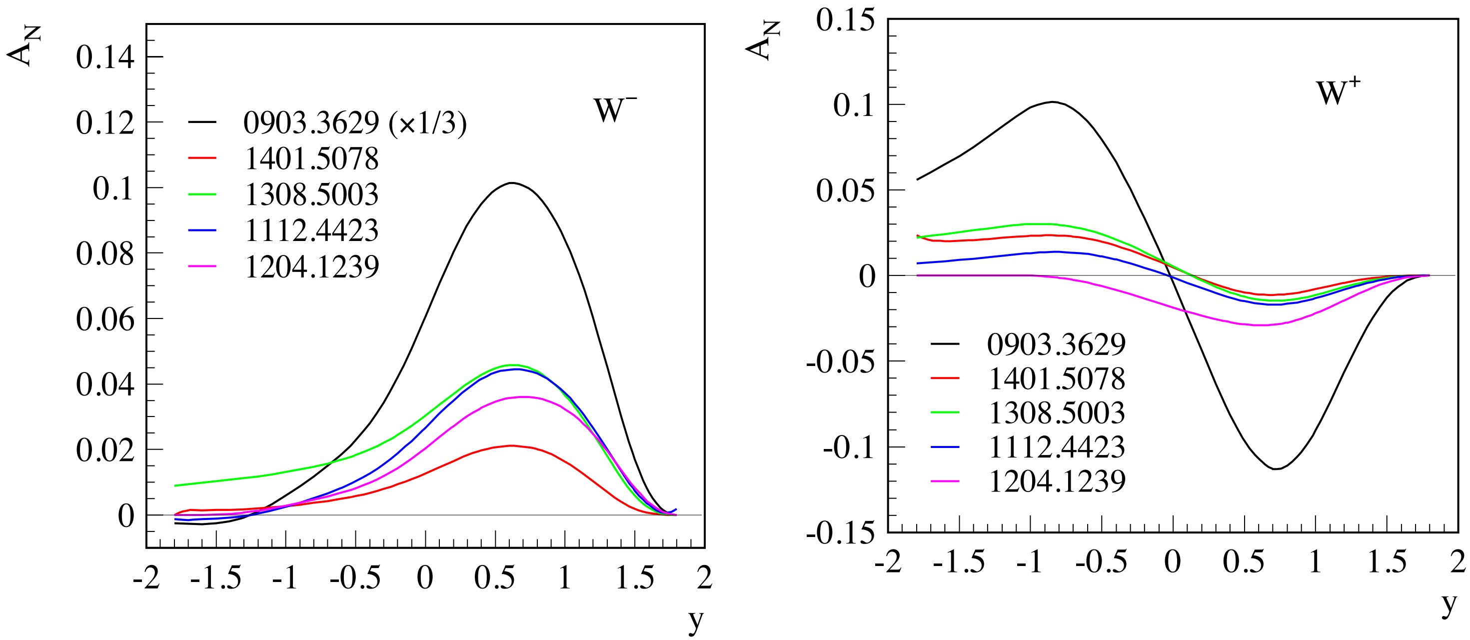


No disagreement on evolution equations!

Issues: Extrapolation to non-perturbative large b-region
Choice of the Q-dependent "form factor"

"Predictions" for A_N of W-production at RHIC?

- **Sivers Effect:**
  - Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons
  - QCD Prediction: Sign change of Sivers function from SIDIS and DY
- **Current "prediction" and uncertainty of QCD evolution:**



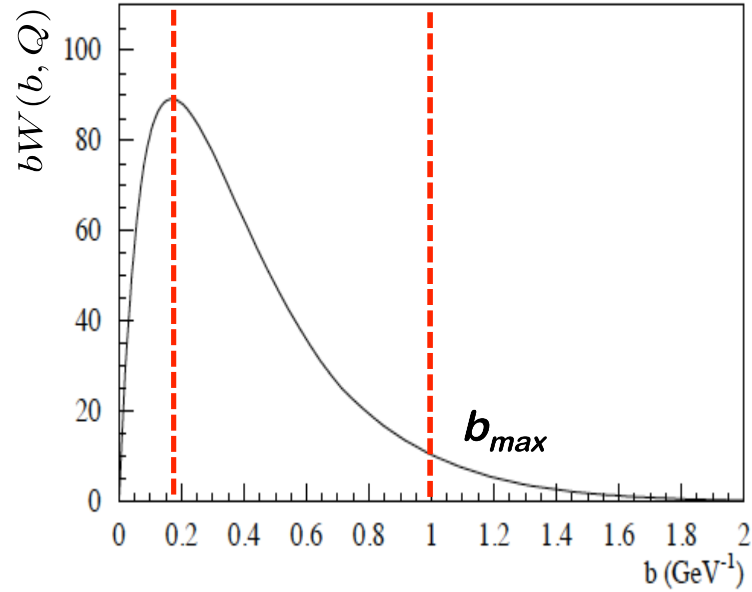
TMD collaboration proposal: Lattice, theory & Phenomenology

RHIC is the excellent and unique facility to test this (W/Z – DY)!

Evolution and extrapolation - I

❑ **Q-evolution is achieved in b-space:**

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T}\tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2,Q^2)$$



❑ **CSS formalism:**

$$\widetilde{W}(b,Q) \approx W^{\rm pert}(b_*,Q)\, e^{-\mathcal{F}^{\rm NP}(b,Q)}$$

✧ $$b_* = \frac{b}{\sqrt{1+b^2/b_{\rm max}^2}} \to b_{\rm max}, \quad \text{as} \quad b\to\infty$$

✧ $$W^{\rm pert}(b,Q) \propto e^{-S_p(b,Q)} [\mathcal{C}\otimes f]_q(x_A,b,\mu)\, [\mathcal{C}\otimes f]_{\bar{q}}(x_B,b,\mu)$$

✧ $$\mathcal{F}^{\rm NP} \approx b^2(a_1 + a_2\ln(Q/Q_0) + a_3\ln(x_A x_B) + \ldots) + \ldots$$

Issues:

Qiu, Zhang, PRL, PRD, 2001

✧ **Predictive power – universality of "form factor":** $\mathcal{F}^{\rm NP}(b,Q)$

$\ln(Q/Q_0)$ **- dependence is only valid when** $\ln(Q/Q_0) \gg (Q_0/Q)$

Not satisfied for HERMES, even COMPASS, data for $Q_0 \sim 2\,\text{GeV}$ ***!!!***

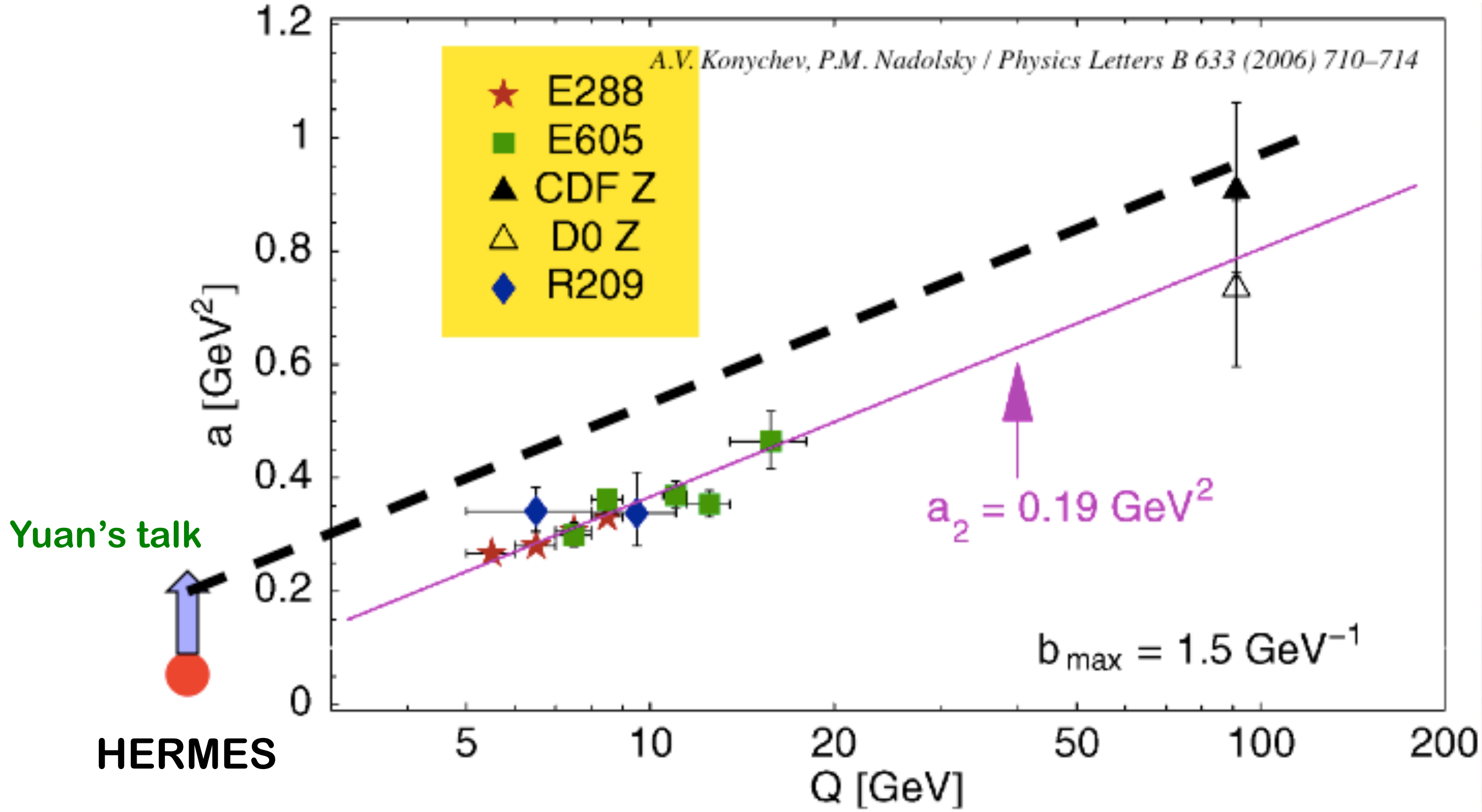
✧ **"Unwanted change" to small-b perturbative contribution**

b_* ***and*** $\mathcal{F}^{\rm NP}(b,Q)$ ***change*** $W^{\rm pert}(b,Q)$ ***in small-b region!!!***

Importance of the evolution - II

- Q-dependence of the "form factor" :

Konychev, Nadolsky, 2006



$$\mathcal{F}^{\mathrm{NP}}(b,Q) = a(Q^2)\, b^2$$

At Q ~ 1 GeV, ln(Q/Q_0) term may not be the dominant one!

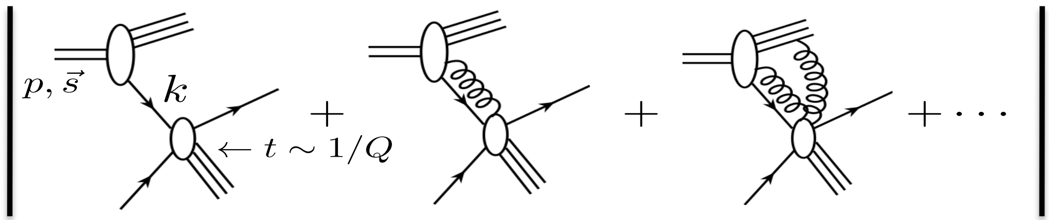
$$\mathcal{F}^{\mathrm{NP}} \approx b^2(a_1 + a_2 \ln(Q/Q_0) + a_3 \ln(x_A x_B) + \ldots) + \ldots$$

Power correction? ($(Q_0/Q)^n$-term? ***Better fits for HERMES data?***

How collinear factorization generates SSA?

❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto$



$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – Expansion

$$\sigma(Q,s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

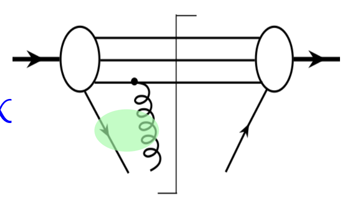
Too large to compete!

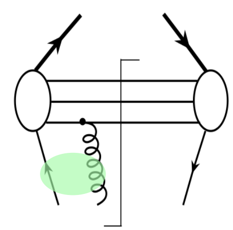
Three-parton correlation

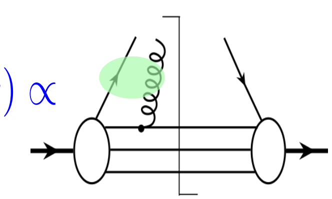
❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$ 

$D^{(3)}(z,z) \propto$ 

$T^{(3\sigma)}(x,x) \propto$ 

Qiu, Sterman, 1991, …

Kang, Yuan, Zhou, 2010

Kanazawa, Koike, 2000

Integrated information on parton's transverse motion!

Quantum interference between a single and a composite state

Inclusive single hadron production

❑ **One large scale:** $A(p_A, S_\perp) + B(p_B) \to h(p) + X$ **with $p_T \gg \Lambda_{QCD}$**

Three identified hadrons: $A(p_A, S_\perp),\ B(p_B),\ h(p)$

❑ **QCD collinear factorization:**

Qiu, Sterman, 1991, 1998, …

$$A_N \propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp)$$

$$= T^{(3)}_{a/A}(x,x,S_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}^T_{ab\to c} \otimes D_{h/c}(z)$$

$$+ \delta q_{a/A}(x,S_\perp) \otimes T^{(3\sigma)}_{b/B}(x',x') \otimes \hat{\sigma}^\phi_{ab\to c} \otimes D_{h/c}(z)$$

$$+ \delta q_{a/A}(x,S_\perp) \otimes \phi_{b/B}(x',x') \otimes \hat{\sigma}^D_{ab\to c} \otimes D^{(3)}_{h/c}(z,z)$$

Leading power contribution to cross section cancels!
Only one twist-3 distribution at each term!

❑ **Three-type contributions:**

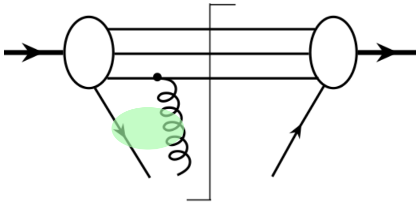
Spin-flip: **Twist-3 correlation functions, transversity distributions**

Phase: **Interference between the real part and imaginary part of the scattering amplitude**

Twist-3 correlation functions

❑ Twist-3 polarized correlation functions:

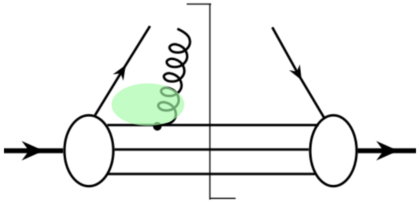
Efremov, Teryaev, 1982, …
Qiu, Sterman, 1991, …

$T^{(3)}(x, x, S_\perp) \propto$ 

Moment of Sivers function

❑ Twist-3 unpolarized correlation functions:

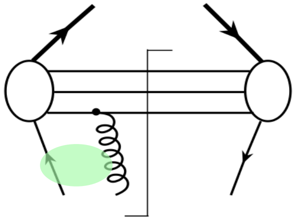
Kanazawa, Koike 2000, …

$T^{(3\sigma)}(x', x') \propto$ 

Moment of Boer-Mulders function

❑ Twist-3 fragmentation functions:

Kang, Yuan, Zhou, 2010

$D^{(3)}(z, z) \propto$ 

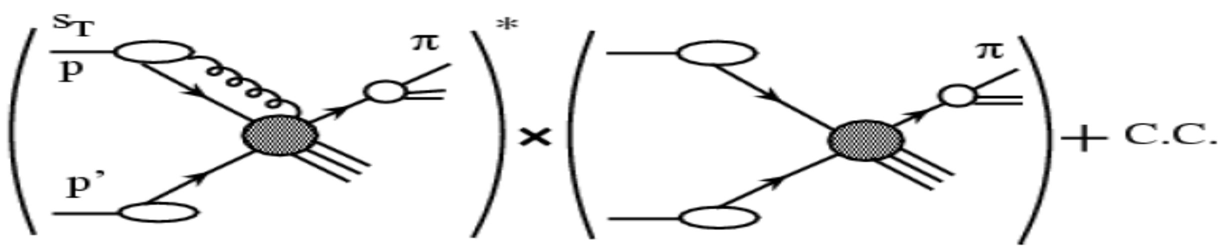
Moment of Collins function?

All these correlation functions have No probability interpretation!

Quantum interference between a single and a composite state

SSAs generated by twist-3 PDFs

- First non-vanish contribution – interference:



- Dominated by the derivative term – forward region:

$$E\frac{d\Delta\sigma}{d^3\ell} \propto \epsilon^{\ell_T s_T n \bar{n}}\, D_{c\to\pi}(z) \otimes \left[-x\frac{\partial}{\partial x}T_F(x,x)\right] \otimes \frac{1}{-\hat{u}}\left[G(x')\otimes\Delta\hat{\sigma}_{qg\to c} + \sum_{q'} q'(x')\otimes\Delta\hat{\sigma}_{qq'\to c}\right]$$

Qiu, Sterman, 1998, …

$$\boxed{A_N \propto \left(\frac{\ell_\perp}{-\hat{u}}\right)\frac{n}{1-x} \quad \text{if } T_F(x,x)\propto q(x)\propto(1-x)^n}$$

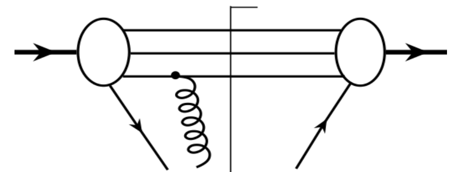
- Complete leading order contribution:

Kouvaris, Qiu, Vogelsang, Yuan, 2006

$$E_\ell\frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int_{z_{\min}}^1\frac{dz}{z^2}D_{c\to h}(z)\int_{x'_{\min}}^1\frac{dx'}{x'}\frac{1}{x'S+T/z}\phi_{b/B}(x')\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right)$$
$$\times\frac{1}{x}\left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right]H_{ab\to c}(\hat{s},\hat{t},\hat{u}).$$

Twist-3 distributions relevant to A_N

- Two-sets Twist-3 correlation functions:



No probability interpretation!

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+}{2} \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+\gamma^5}{2} \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle\, (i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

- Twist-2 distributions:
  - Unpolarized PDFs:

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

  - Polarized PDFs:

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

- Twist-3 fragmentation functions:

See Kang, Yuan, Zhou, 2010, Kang 2010

Test QCD evolution at twist-3 level

❑ Scaling violation – "DGLAP" evolution:

Kang, Qiu, 2009; Yuan, Zhou, 2009
Vogelsang, Yuan, 2009, Braun et al, 2009

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \underbrace{\begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}}_{(x,\, x+x_2,\, \mu,\, s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix}}_{(\xi,\, \xi+\xi_2;\, x,\, x+x_2,\, \alpha_s)} \underbrace{\otimes}_{\int d\xi \int d\xi_2} \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}$$

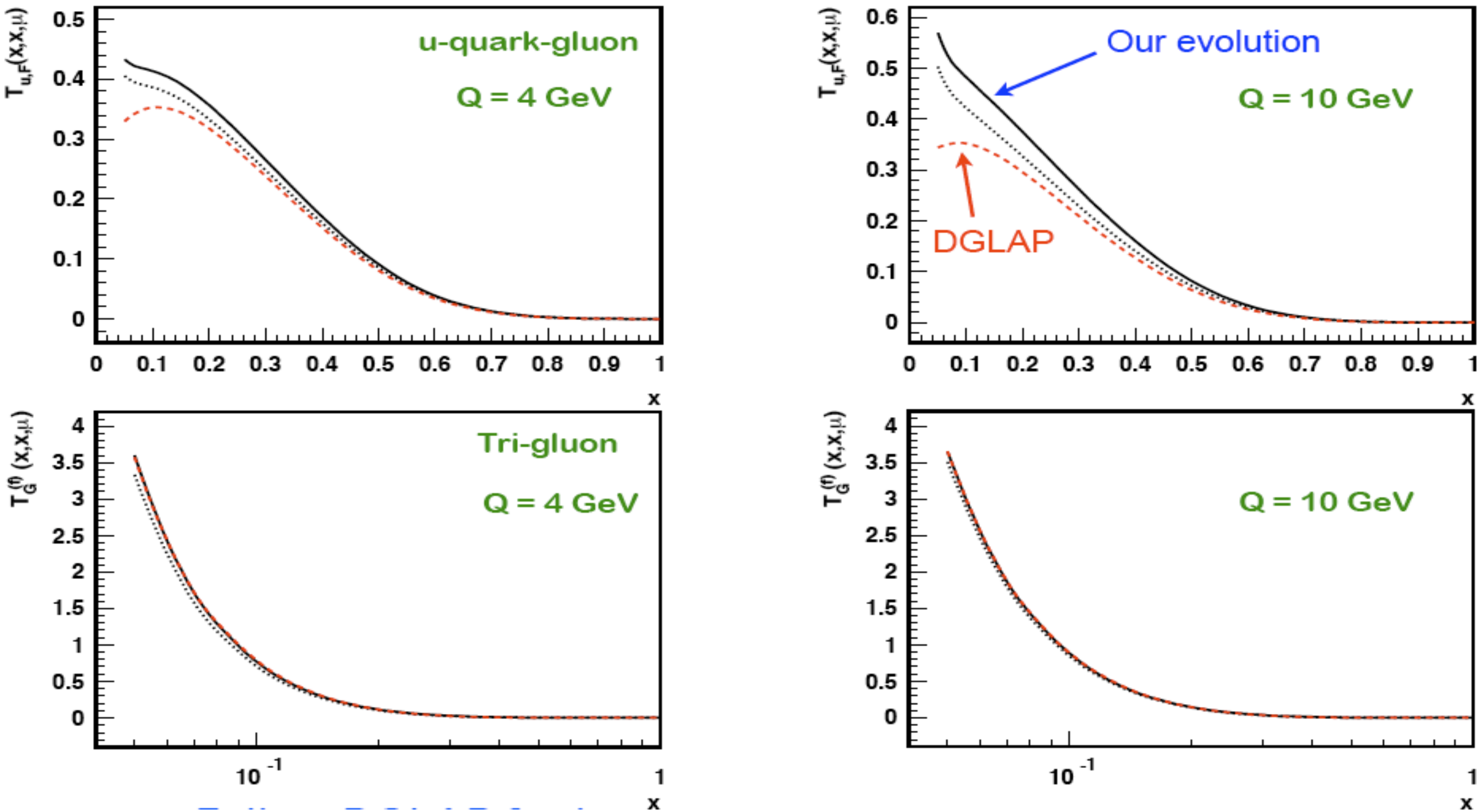
❑ Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\dfrac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\dfrac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\dfrac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

Scaling violation of twist-3 correlations?



✧ Follow DGLAP at large x

✧ Large deviation at low x (stronger correlation)

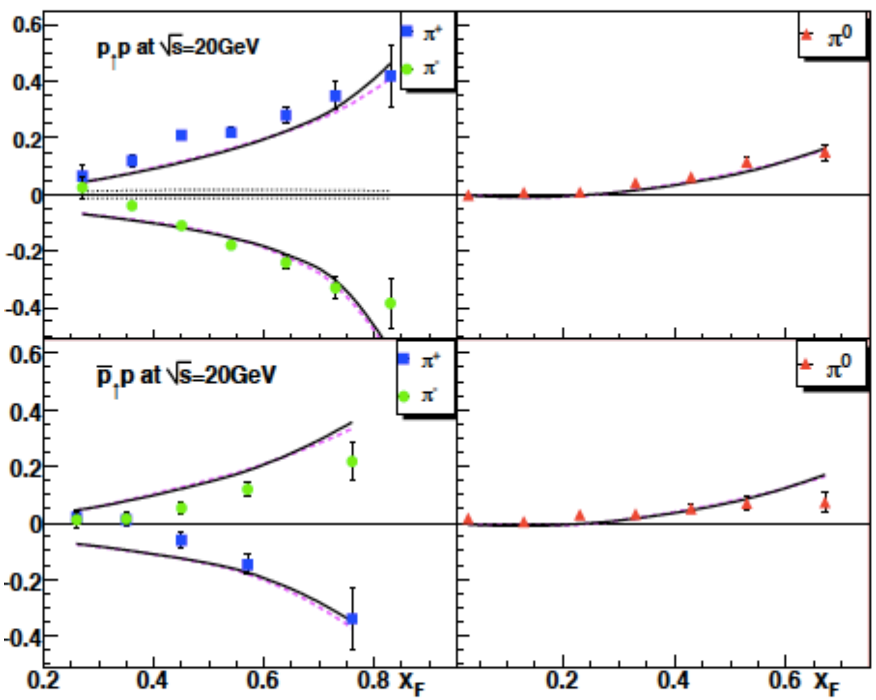
Kang, Qiu, PRD, 2009

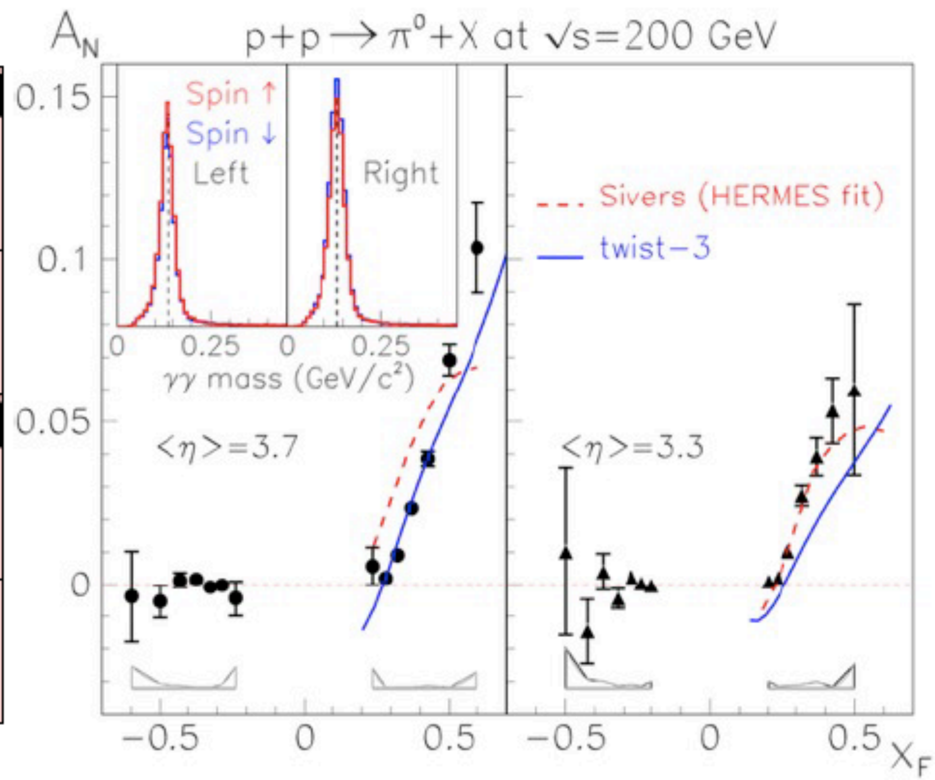
Fits with $T_F(x,x)$ only

Kouvaris-Qiu-Vogelsang-Yuan, 2006

- ❑ **Fitting both fixed target and collider data:**

$$T_{q,F}(x,x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \phi_q(x)$$





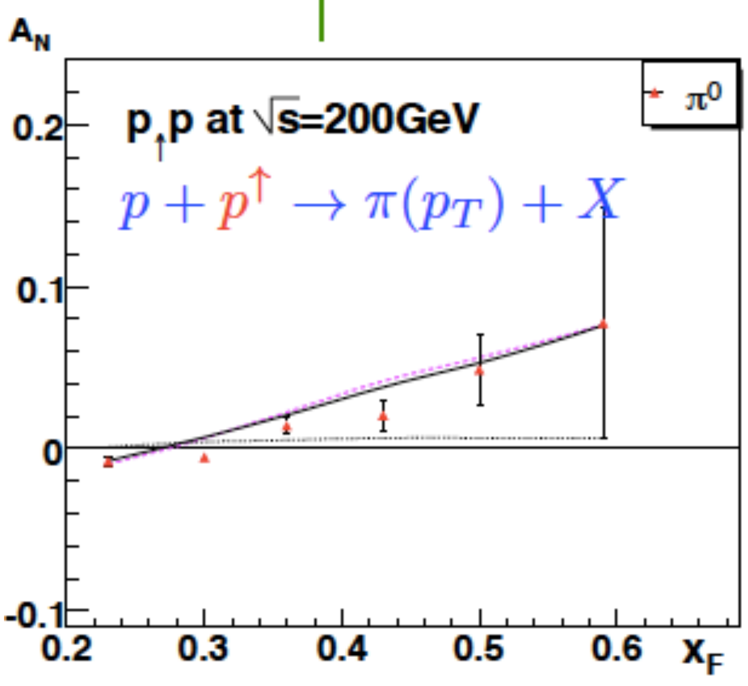
Connection between TMDs and Twist-3

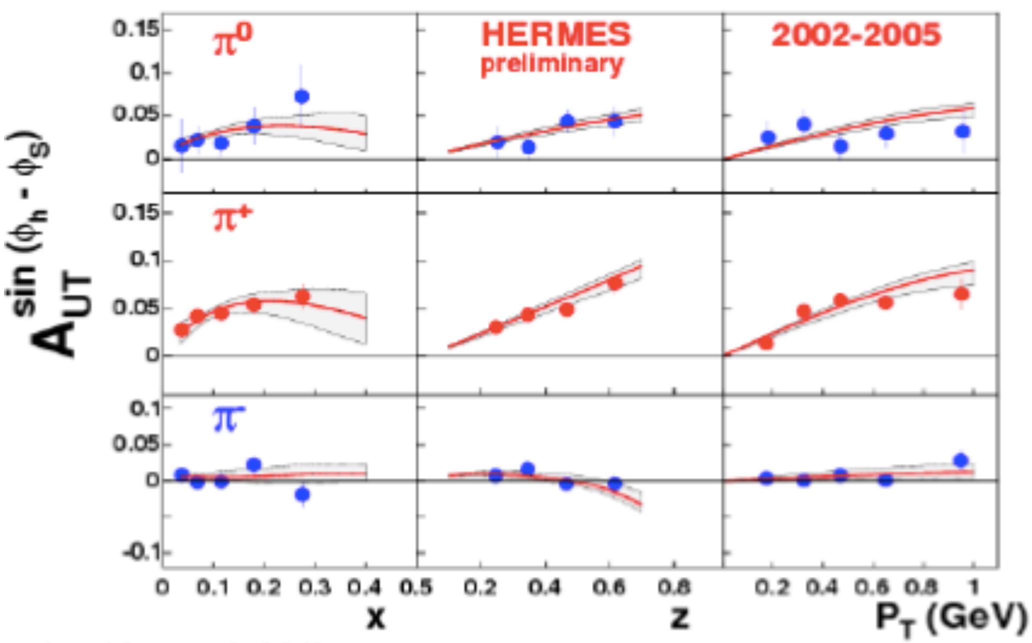
❑ Sivers function and twist-3 correlation:

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\rm SIDIS} \quad + \text{UVCT}$$

Boer, Mulders, Pijlman, 2003

Ji, Qiu, Vogelsang, Yuan, 2006



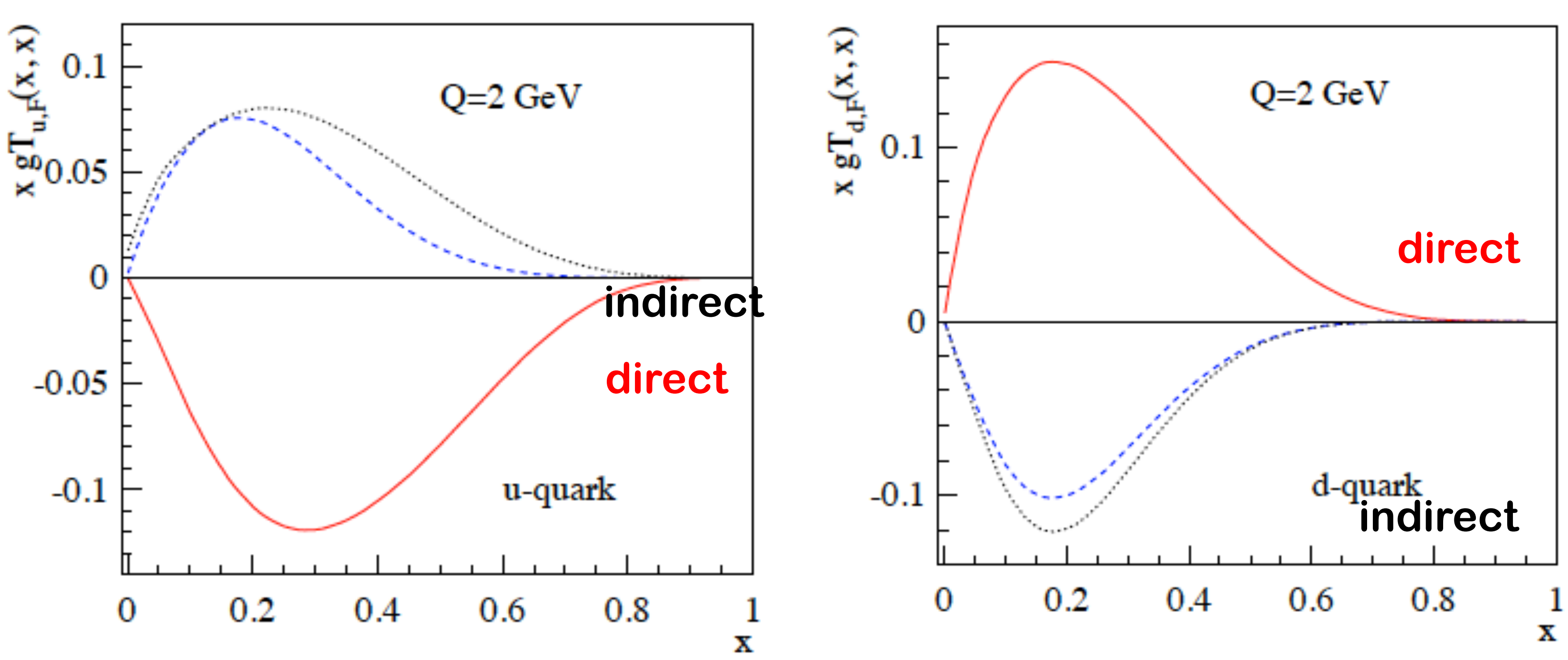


A “sign mismatch”

- “direct” and “indirect” twist-3 correlation functions:

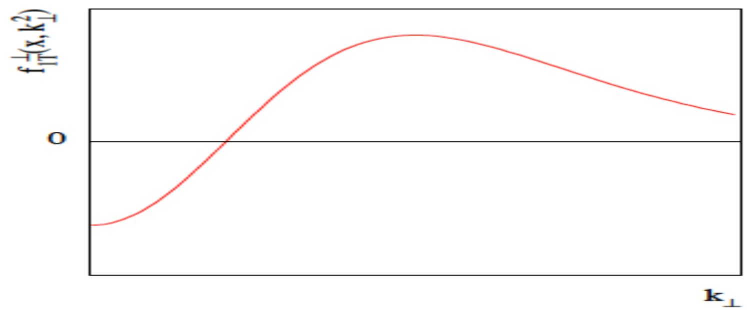
Kang, Qiu, Vogelsang, Yuan, 2011

Calculate $T_{q,F}(x,x)$ by using the measured Sivers functions



“Failed attempts”

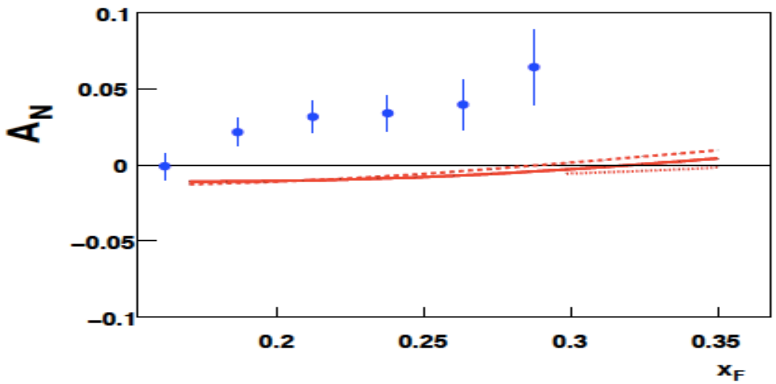
- A node in k_T-distribution:
  - Like the DSSV’s $\Delta G(x)$
  - HERMES vs COMPASS
  - Physics behind the sign change?

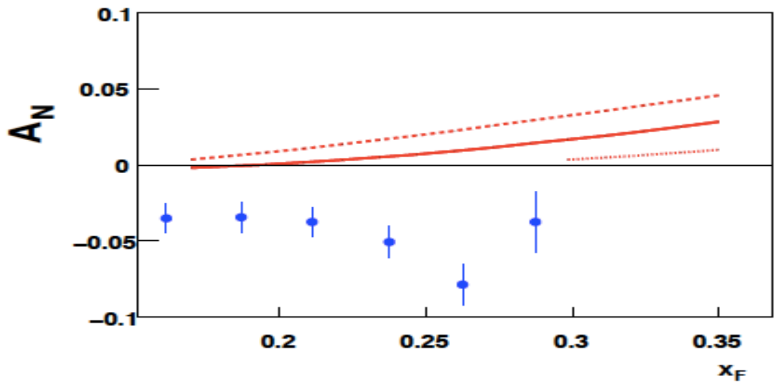


Unlikely, too small parameter space

Kang-Prokudin, PRD85, 2012

- A node in x-distribution:





Fail to describe BRAHMS data

Kang-Prokudin, PRD85, 2012

- Conclusion:

The sivers-type twist-3 contribution might not be the leading source of SSA of pion production – Twist-3 fragmentation contribution?

Add twist-3 fragmentation contribution

Metz, Pitonyak, PLB723 (2013)

- **Leading order results:**

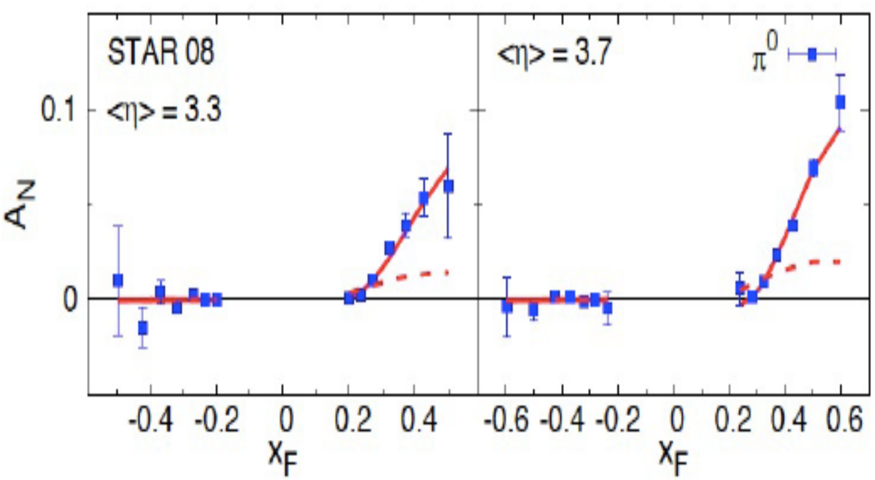
$$\frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\,\epsilon_{\perp\mu\nu}\,S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \frac{1}{-x\hat{u} - x'\hat{t}}$$

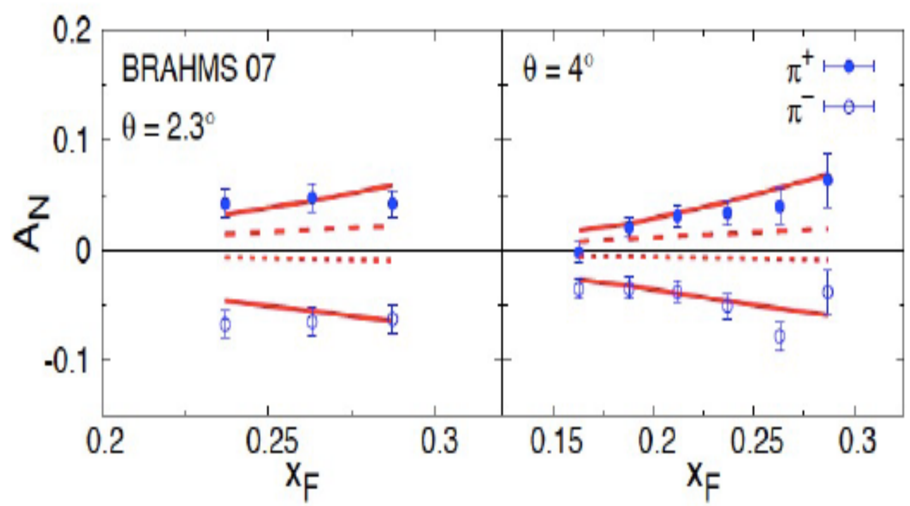
$$\times\,\frac{1}{x}\,h_1^a(x)\,f_1^b(x')\left\{\left(\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz}\right)S_{\hat{H}}^i + \frac{1}{z}\,H^{C/c}(z)\,S_H^i\right.$$

$$\left. + 2z^2 \int \frac{dz_1}{z_1^2}\,PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{C/c,\Im}(z,z_1)\,\frac{1}{\xi}\,S_{\hat{H}_{FU}}^i\right\}$$

Kanazawa, Koike, Metz, Pitonyak, PRC89, 2014

- **New fitting results:**

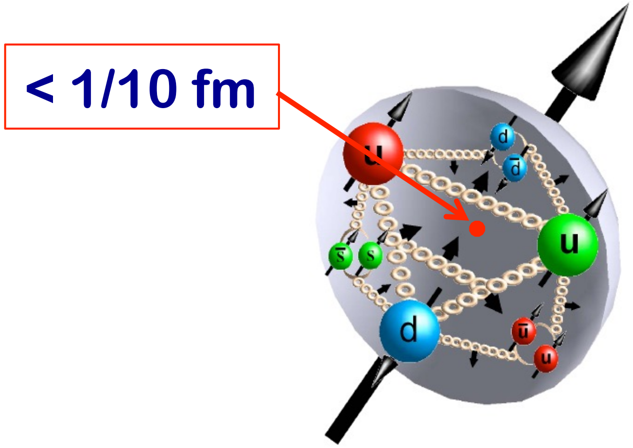




Without FF contribution

Summary

- Since the "spin crisis" in the 80th, we have learned a lot about proton spin – there is a need for orbital contribution
- Single transverse-spin asymmetry in real, and is a unique probe for hadron's internal dynamics – Sivers, Collins, twist-3, … effects
- Evolution of TMDs is still a very much open question! Better approach to non-perturbative inputs is needed
- QCD has been extremely successful in interpreting and predicting high energy experimental data!
- But, we still do not know much about hadron structure – a lot of work to do!



Thank you!

Backup slides

QCD and hadrons