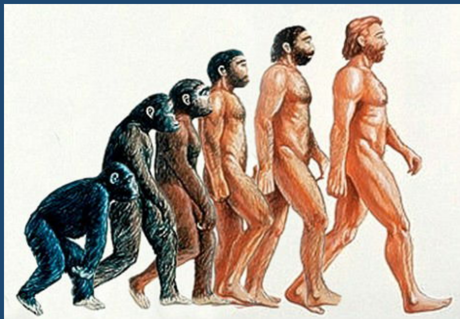

RIKEN 2koma plus seminar
Wako
July 1-2, 2015

Structure evolutions in exotic nuclei and nuclear forces

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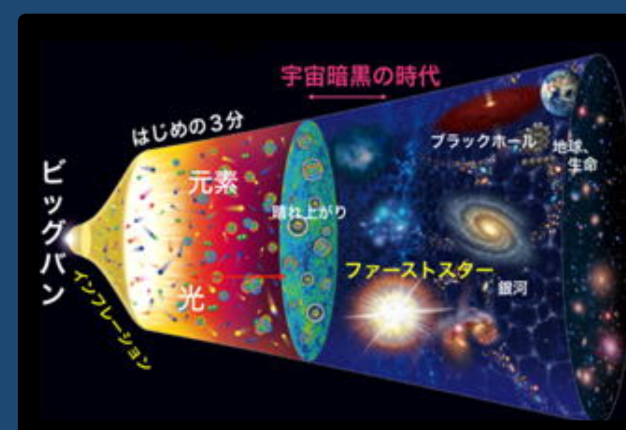
NSCL

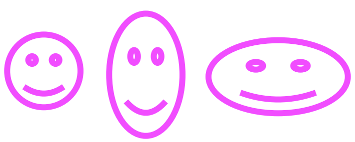
KU LEUVEN



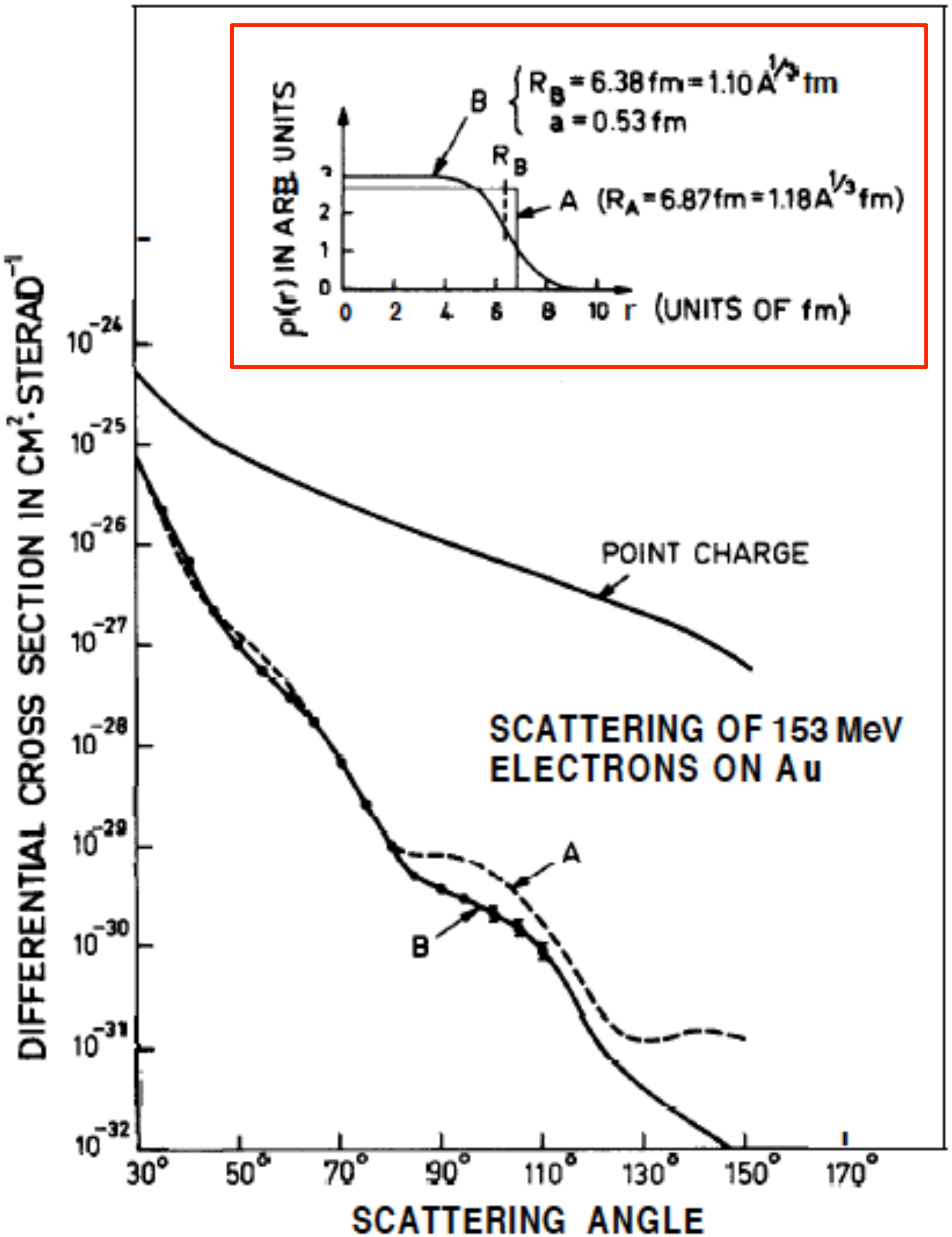
Outline

1. Introduction
2. Shell model and monopole interaction
3. Shell evolution and tensor force
4. Multiple quantum liquid in exotic nuclei
5. Shell evolution and three-nucleon force
6. Summary

Difference between stable and exotic nuclei

	stable nuclei	exotic nuclei
life time	infinite or long	short
number	~300	7000 ~ 10000
properties		?
density	constant inside (density saturation)	
shell	same magic numbers (2,8,20,28, ... *(1949)*)	
shape	shape transition 	

Density : Simple model works for all (stable) nuclei.



double constancy
- inside a nucleus
- among nuclei

Experimental data and theoretical analysis taken from Hahn, Ravenhall and Hofstadter (1956), etc.

Fig 2-1, Nuclear Structure, A. Bohr and B.R. Mottelson (1969)

Shell structure & magic numbers : another constancy

Eigenvalues of HO potential

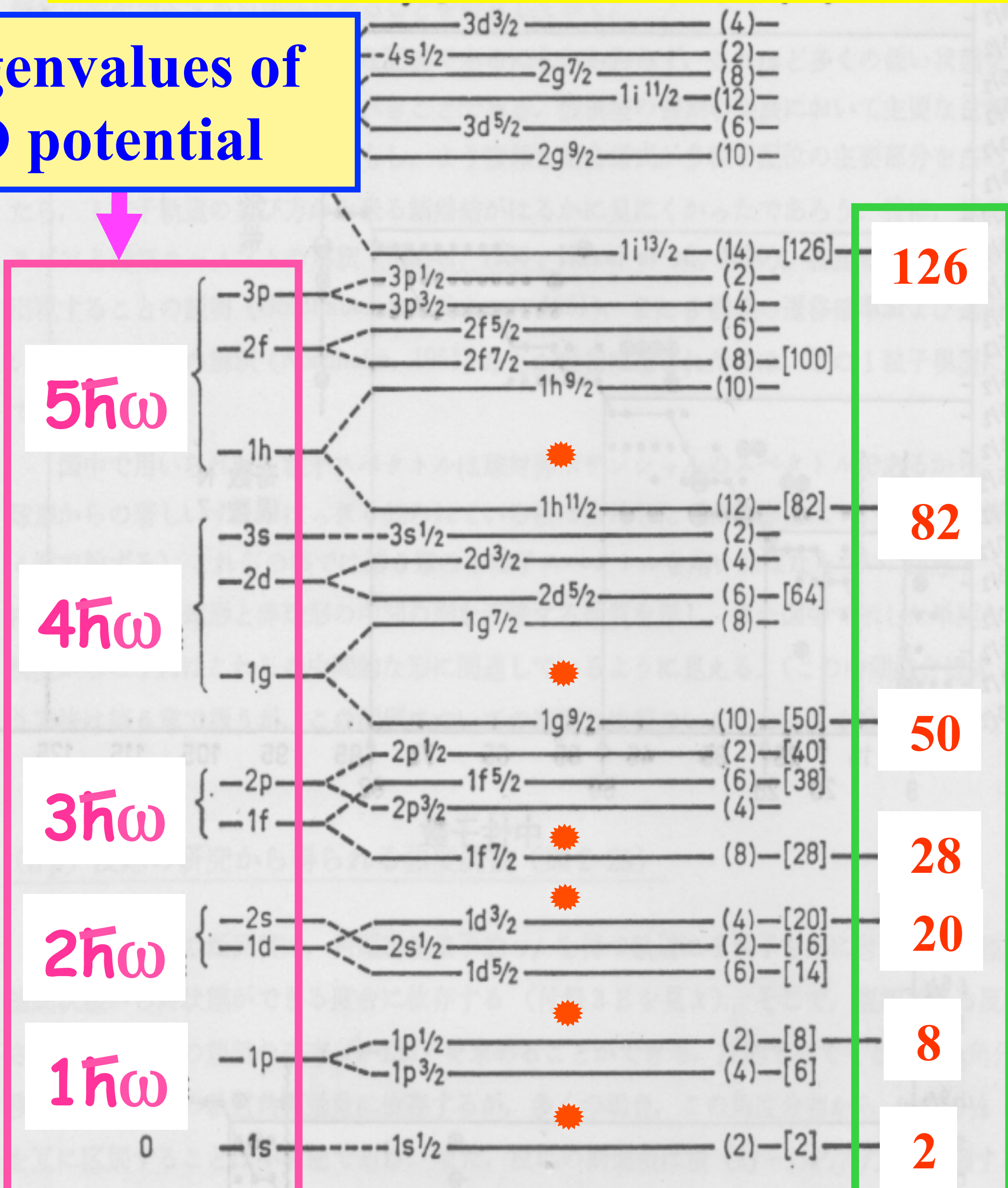
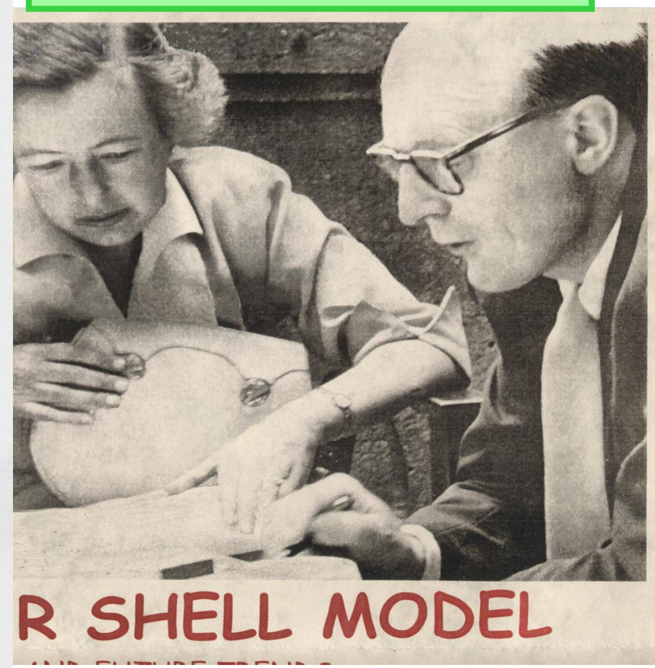


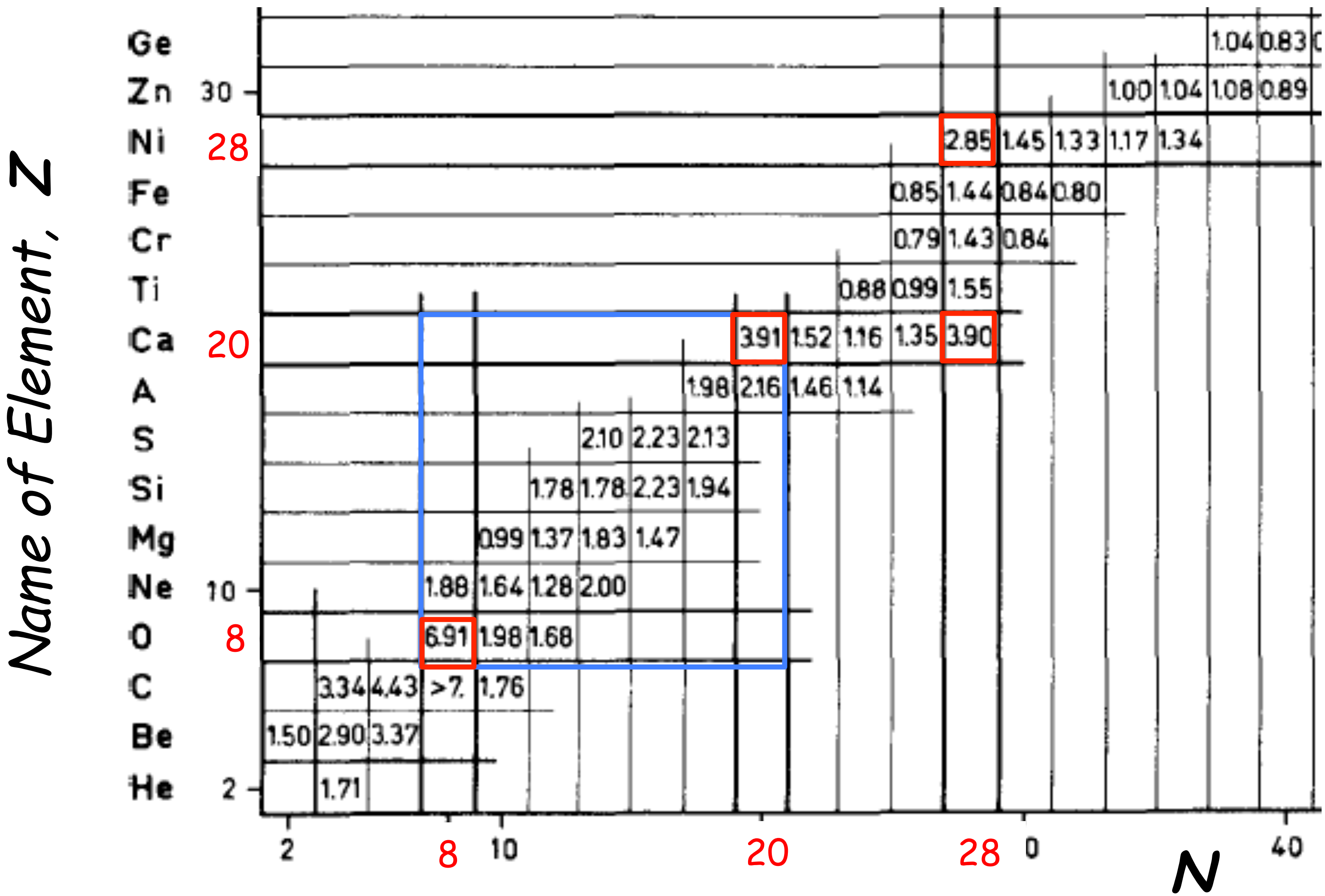
図2-23　1粒子軌道の順序．図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった．

Magic numbers by Mayer and Jensen (1949)



Excitation energy of the first 2^+ state

high at magic numbers (shown in red)



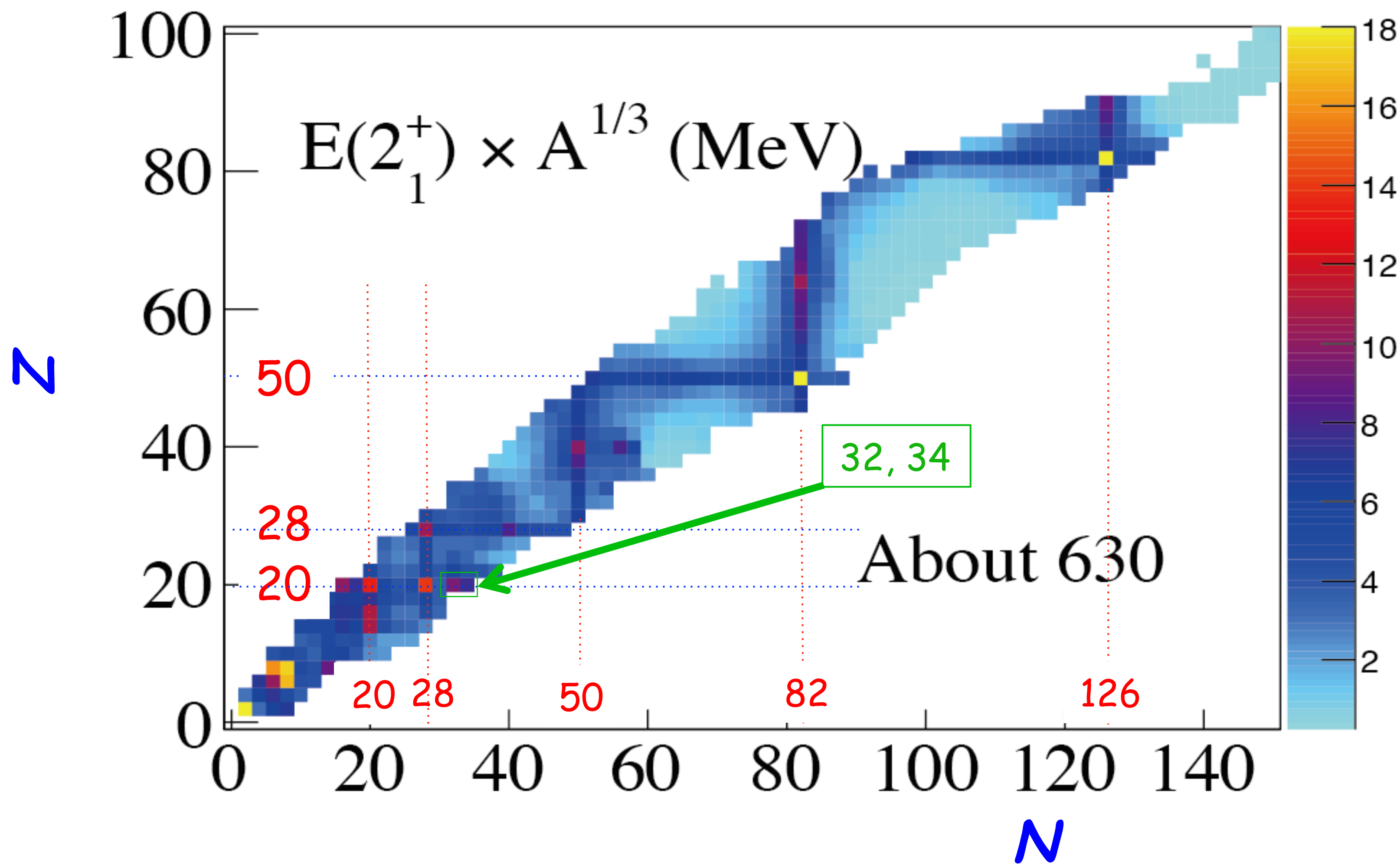
From Nuclear Structure, A. Bohr and B.R. Mottelson (1969)

Courtesy from Pieter Doornenbal

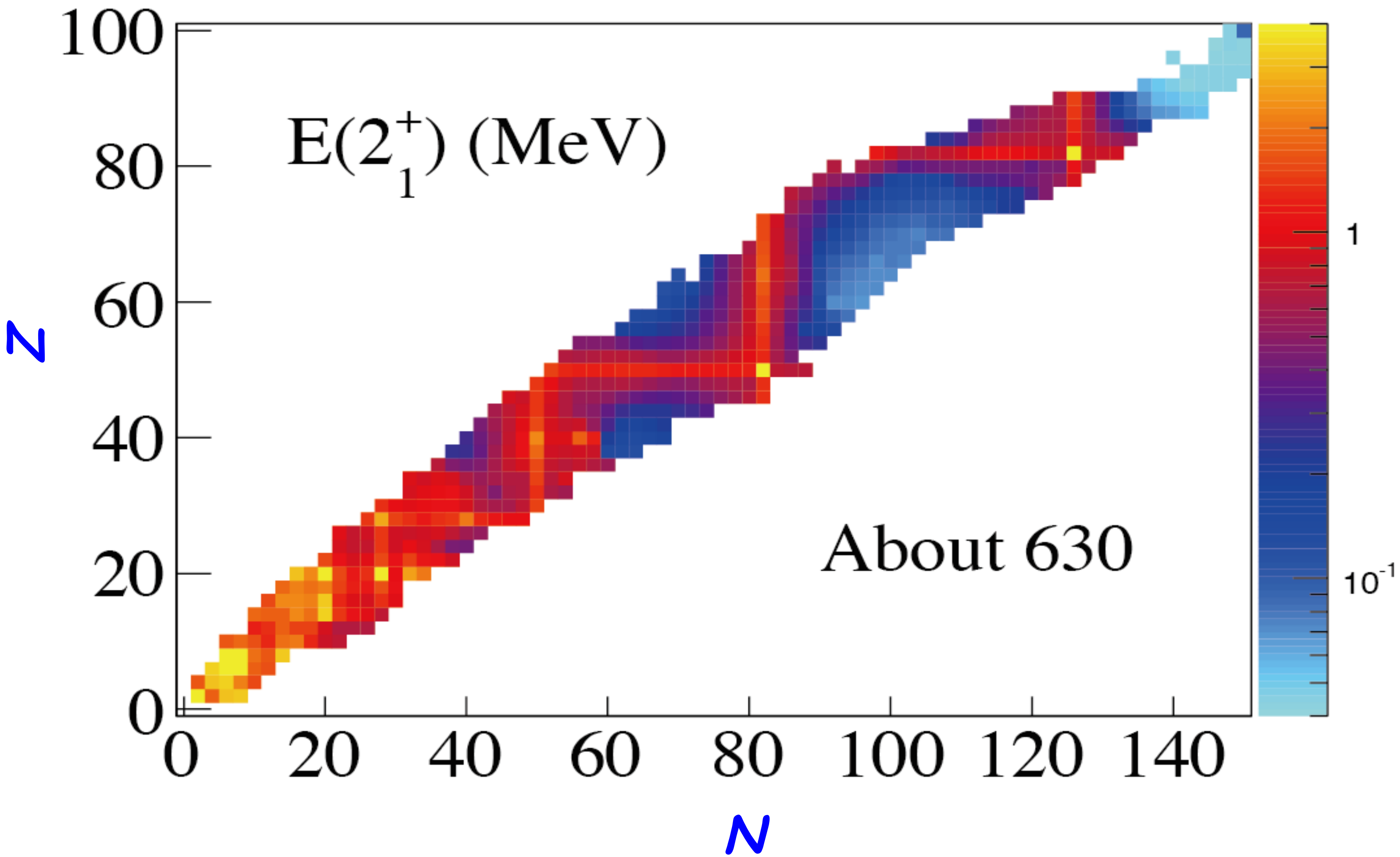
2+ levels x A 1/3

Z, N even numbers only

Red numbers : Conventional magic numbers



2+ levels (unscaled)

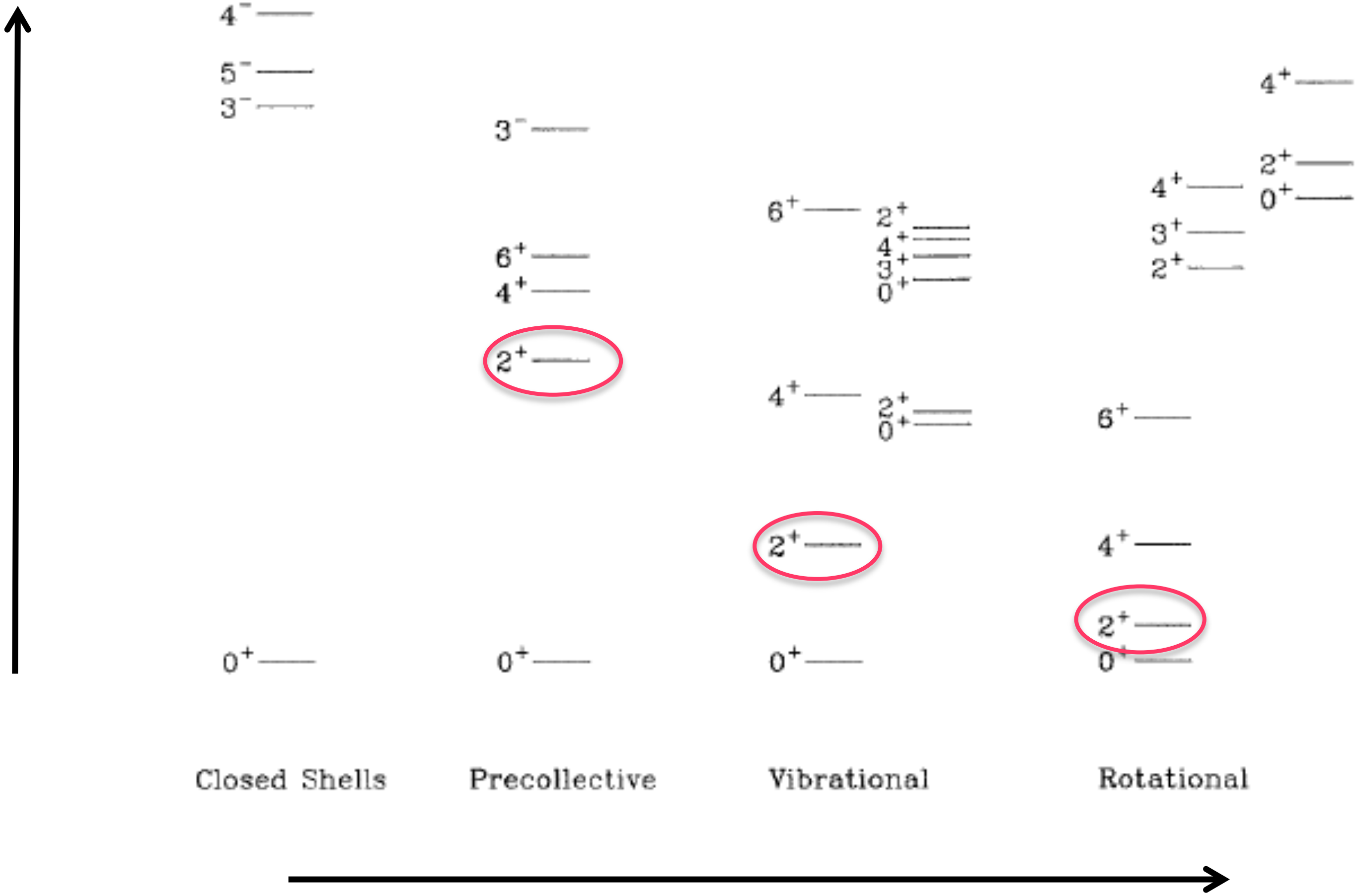


Schematic picture of shape evolution (sphere to ellipsoid)

- monotonic pattern throughout the nuclear chart -

one "shape" per one nucleus in many stable nuclei

excitation energy



Distance from the nearest closed shell in *N* or *Z*

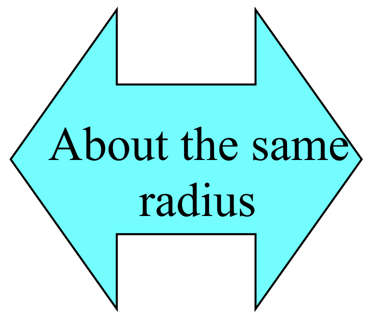
From Nuclear Structure from a Simple Perspective, R.F. Casten (2001)

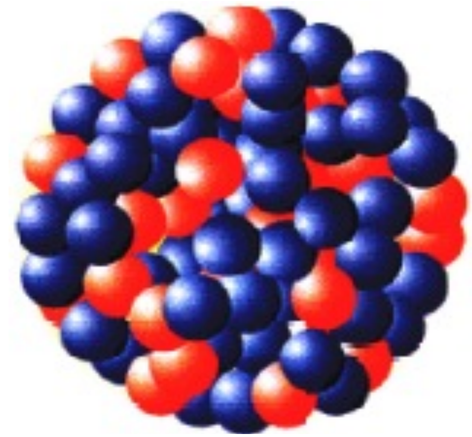
Anomalies or exceptions
have been observed
in exotic nuclei,
however.

Neutron halo

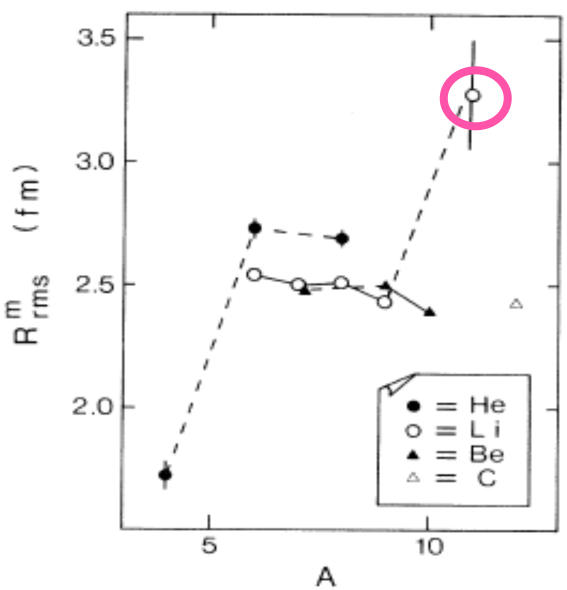
Strong tunneling of loosely bound excess neutrons

^{11}Li





^{208}Pb



. 24 PHYSICAL REVIEW LETTERS 9 DECEMBER 1985

f Interaction Cross Sections and Nuclear Radii in the Light *p*-Shell Region

I. Tanihata,[a] H. Hamagaki, O. Hashimoto, Y. Shida, and N. Yoshikawa
Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188, Japan

K. Sugimoto,[b] O. Yamakawa, and T. Kobayashi
ıce Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

and

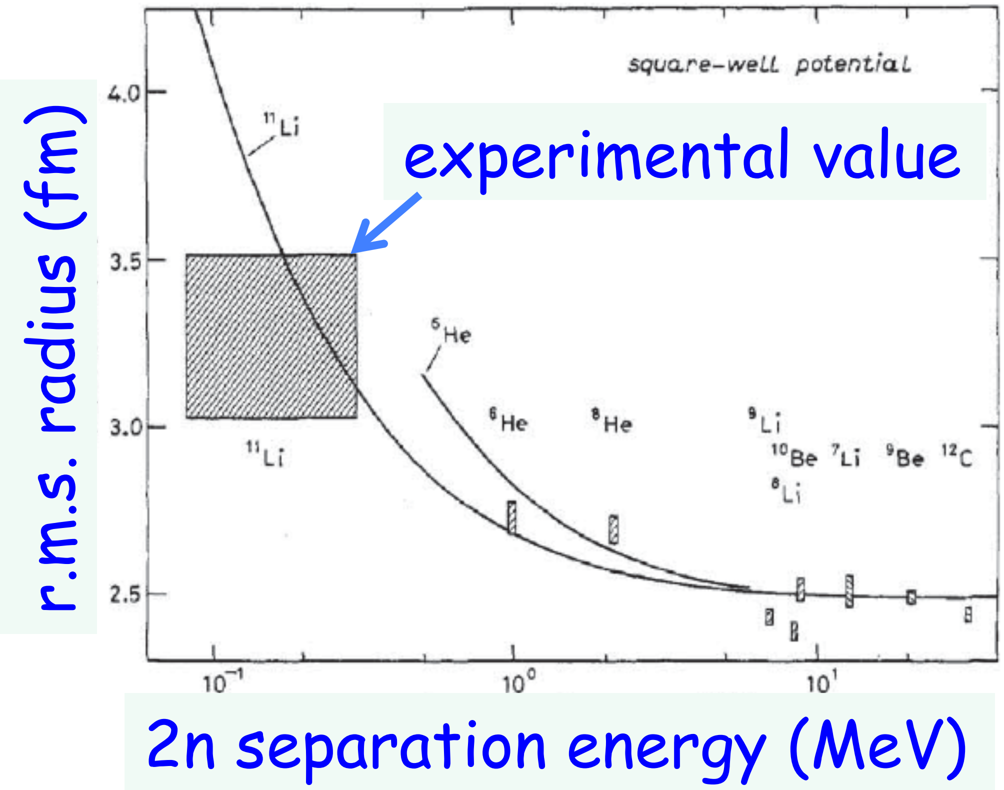
N. Takahashi

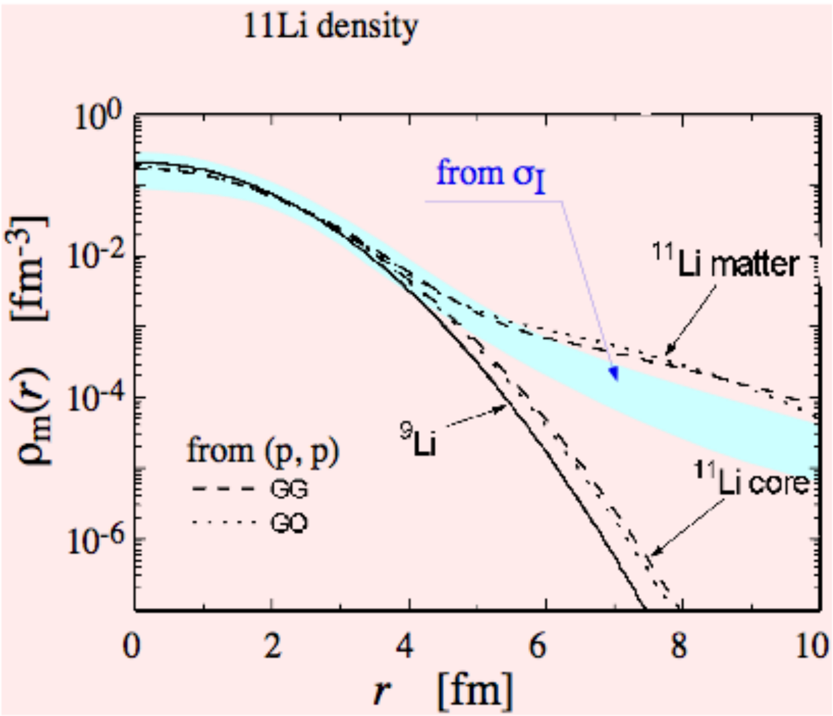
EUROPHYSICS LETTERS

Europhys. Lett., 4 (4), pp. 409-414 (1987)

The Neutron Halo of Extremely Neutron-Rich Nuclei.

P. G. HANSEN (*) (§) and B. JONSON (**)





Interaction cross section data

I. Tanihata et al. Phys. Lett. B 289 (1992) 261.

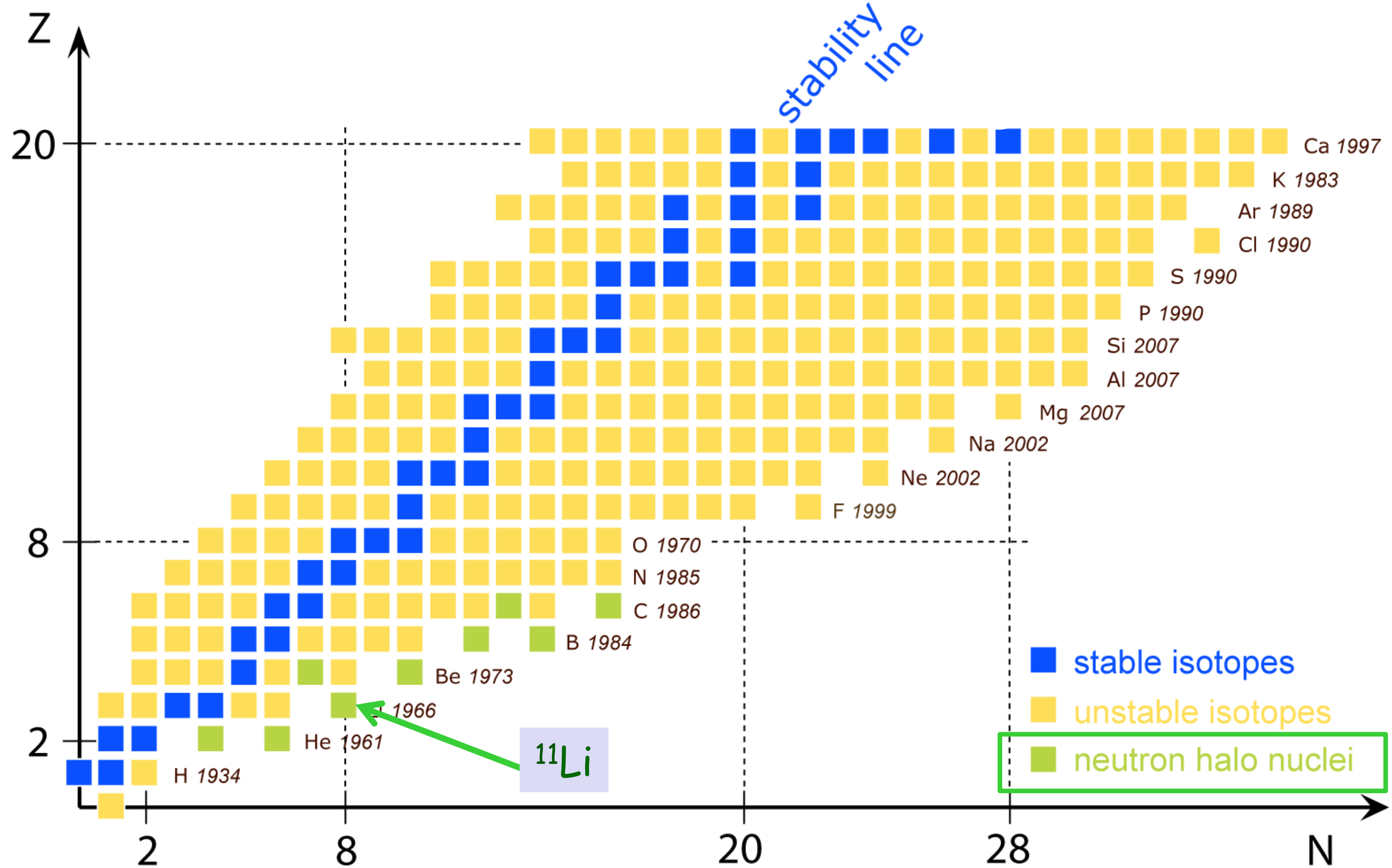
I. Tanihata et al. Phys. Lett. B 287 (1992) 307.

Proton scattering data:

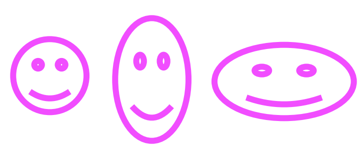
G. D. Alkazov et al., Phys. Rev. Lett. 78(1997) 12

P. Egelhof et al., Euro. Phys. J. A, 15 (2002) 27.

Neutron Halo nuclei on the nuclear chart



Difference between stable and exotic nuclei

	stable nuclei	exotic nuclei
life time	infinite or long	short
number	~300	7000 ~ 10000
properties		
density	constant inside (density saturation)	low-density surface (halo, skin)
shell	same magic numbers (2,8,20,28, … *(1949)*)	?
shape	shape transition 	

Anomaly in levels (deformation)

β-DECAY SCHEMES OF VERY NEUTRON-RICH SODIUM ISOTOPES AND THEIR DESCENDANTS

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M. DE SAINT-SIMON, C. THIBAULT and F. TOUCHARD

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BP 1, F-91406 Orsay, France

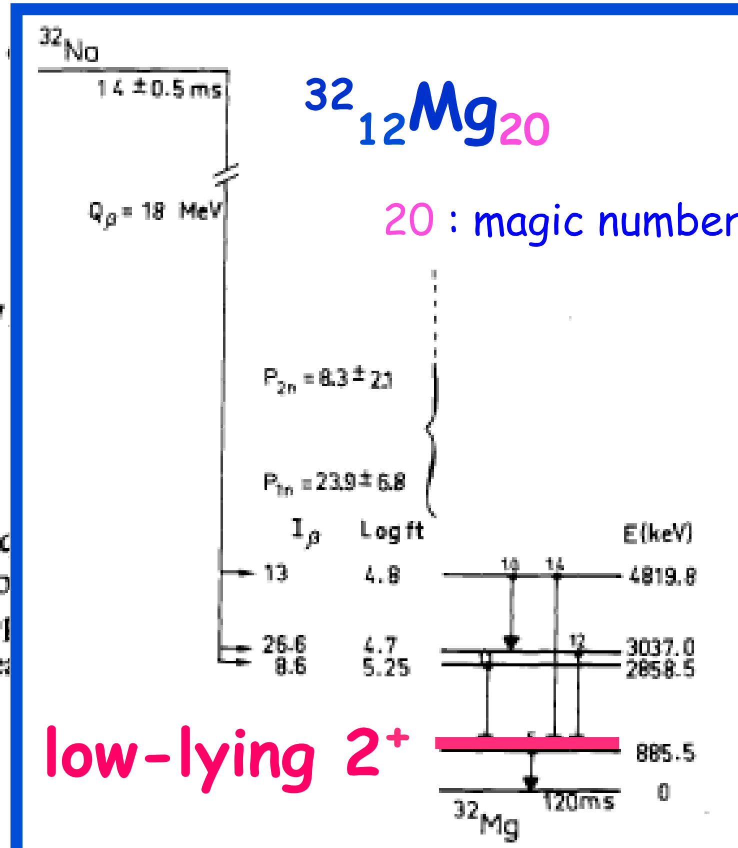
and

M. EPHERRE

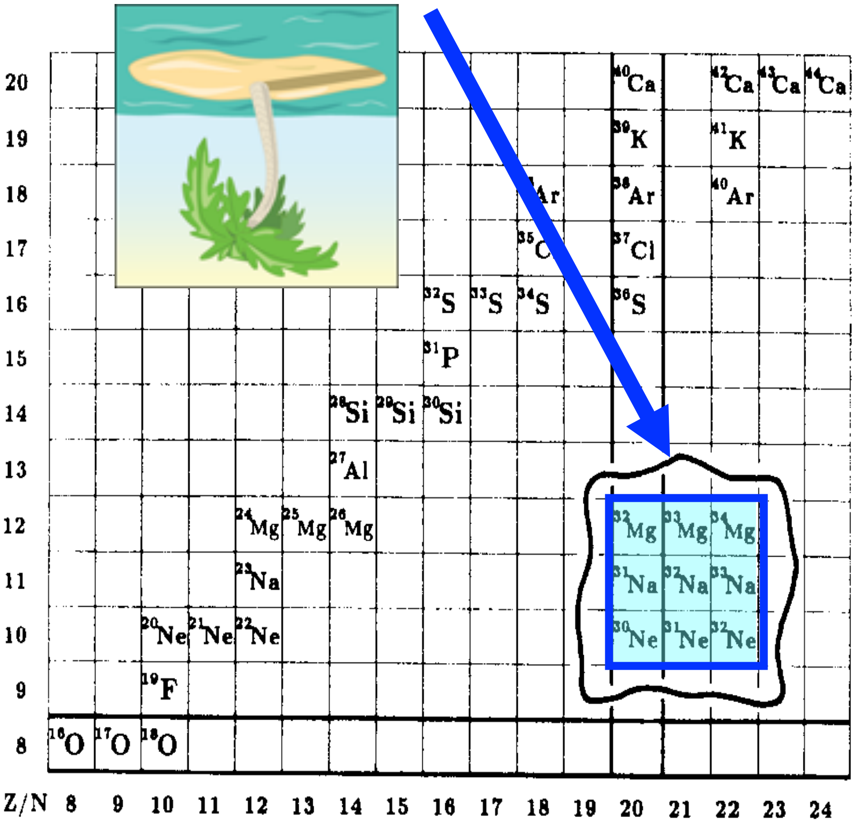
Laboratoire René Bernas and CERN, Division EP, CH-121

Received 6 February 1984

Abstract: The γ-activities from the β-decay of Na isotopes up
fragmentation and analysed through mass-spectrometry technic
from their Mg descendants. The I_γ intensities, the β-delayed
and the I_β intensities are measured. Decay schemes are pro
location of the first 2^+ level of ^{32}Mg, the occurrence of a nucle



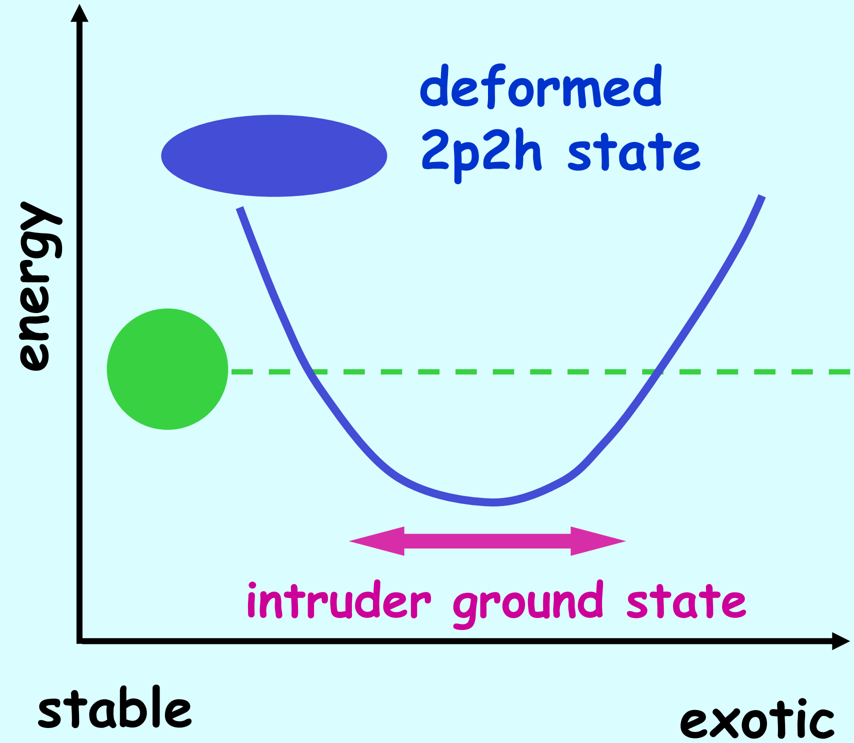
Island of Inversion

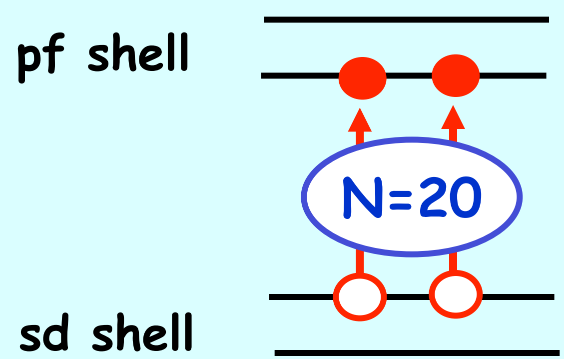


9 nuclei:
Ne, Na, Mg with N=20-22

Phys. Rev. C 41, 1147 (1990), Warburton, Becker and Brown

Basic picture was





gap ~ constant

An older anomaly as a combination of halo + deformation + single-particle energy + configuration mixing

PHYSICAL REVIEW VOLUME 113, NUMBER 2 JANUARY 15, 1959

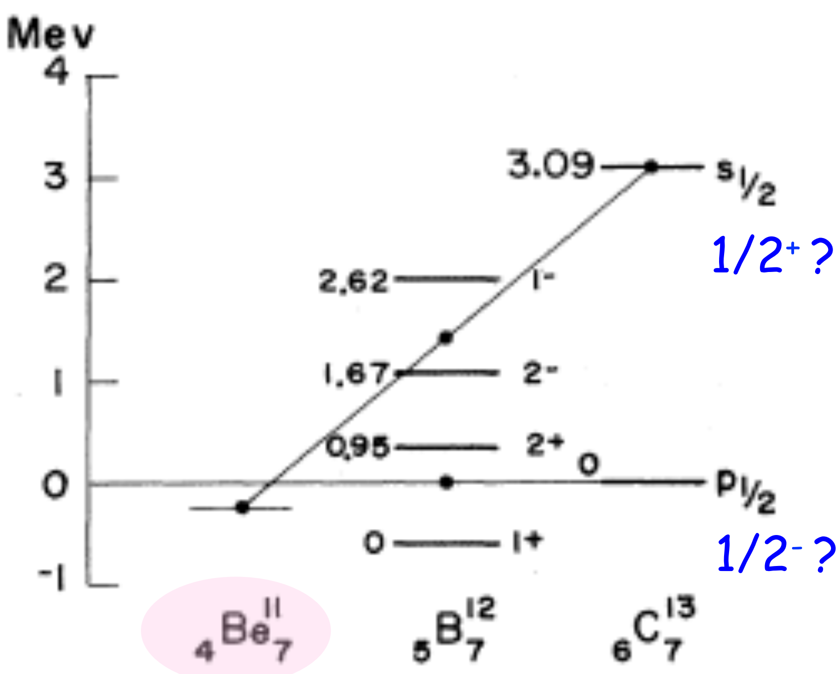
Beta Decay of Be^{11}

D. H. WILKINSON* AND D. E. ALBURGER

VOLUME 4, NUMBER 9 PHYSICAL REVIEW LETTERS MAY 1, 1960

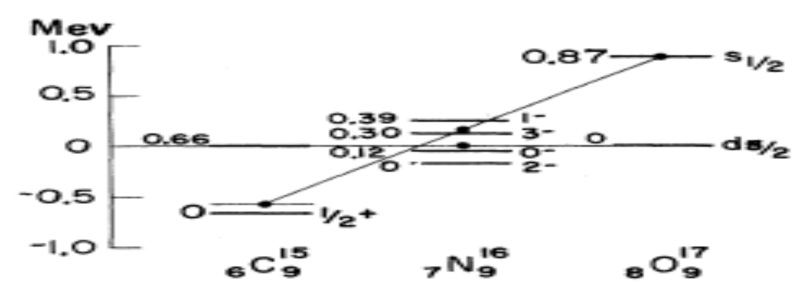
ORDER OF LEVELS IN THE SHELL MODEL AND SPIN OF Be^{11}*

I. Talmi and I. Unna



Questions from this empirical analysis,

- What is actually changed ?
- What is the mechanism ?
- Does it suggest new physics ?



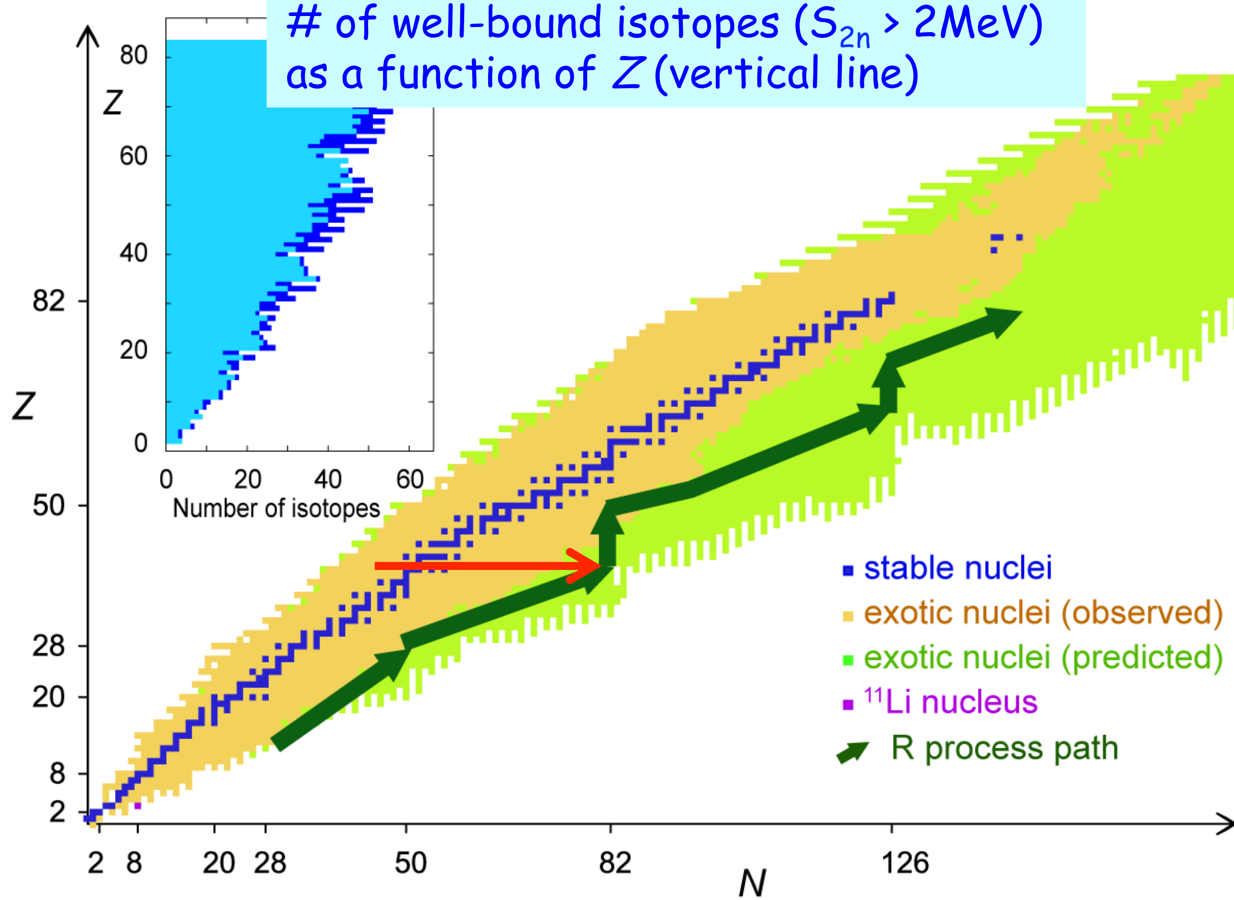
Around the year 2000, …

Neutron halo is a tunneling effect
→ physics with extremely low density and momentum

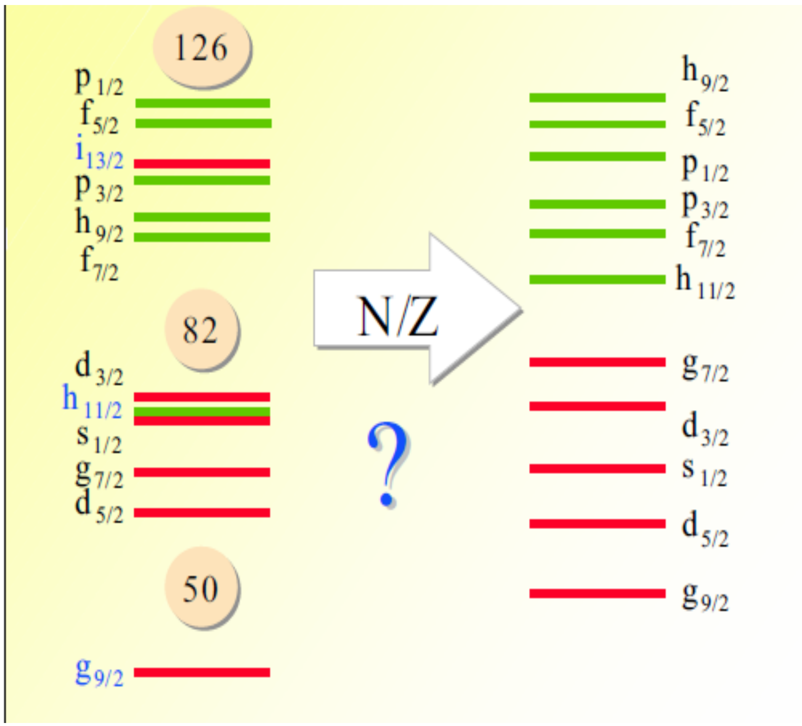
Can there be something new with (almost) normal density (and momentum) but unbalanced proton-neutron ratios ?

Fermi levels of protons and neutrons change independently, over many exotic nuclei.

Nuclear forces can play leading roles !



… not an effect due to loose binding



RIA Physics White Paper

shell structure. The bunching of the energy levels that is endemic to shell structure depends on the form and the shape of the average mean field potential in which the hadrons are moving. With a diffuse surface region, the spin-orbit force may be weakened. Some

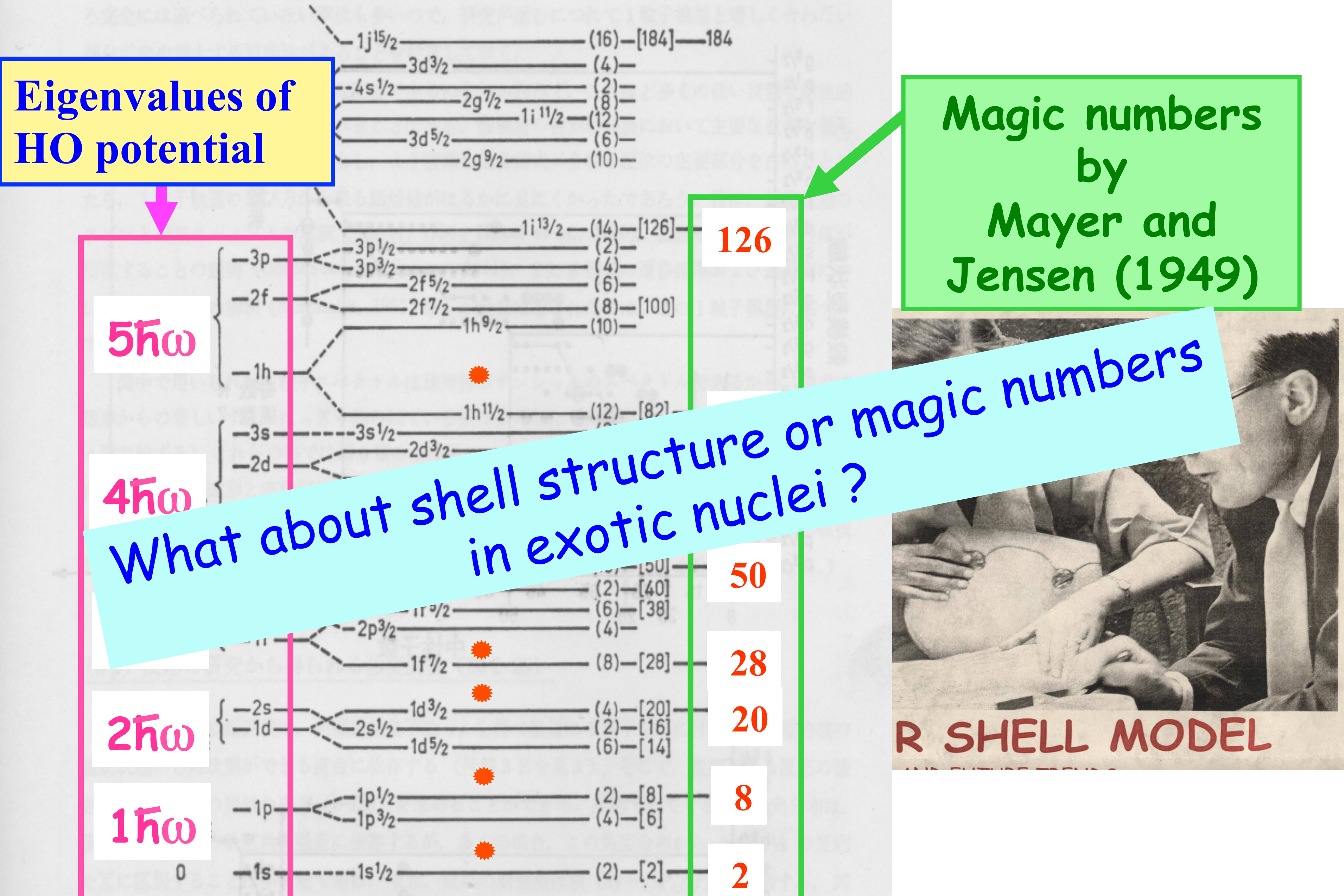


図2-23 1粒子軌道の順序．図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった．

From undergraduate nuclear-physics course,

(a) density saturation
+ (b) short-range *NN* interaction
+ (c) spin-orbit splitting

(a) & (b) → Woods-Saxon type potential
→ Harmonic Oscillator potential

→ Mayer-Jensen's magic number
with rather constant gaps
(except for gradual A dependence)

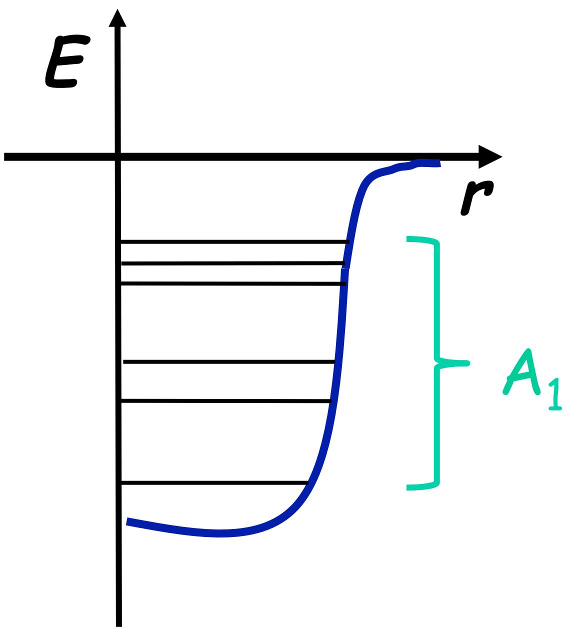
Nuclear forces, which are not included in the above argument, may change this "robust" feature.

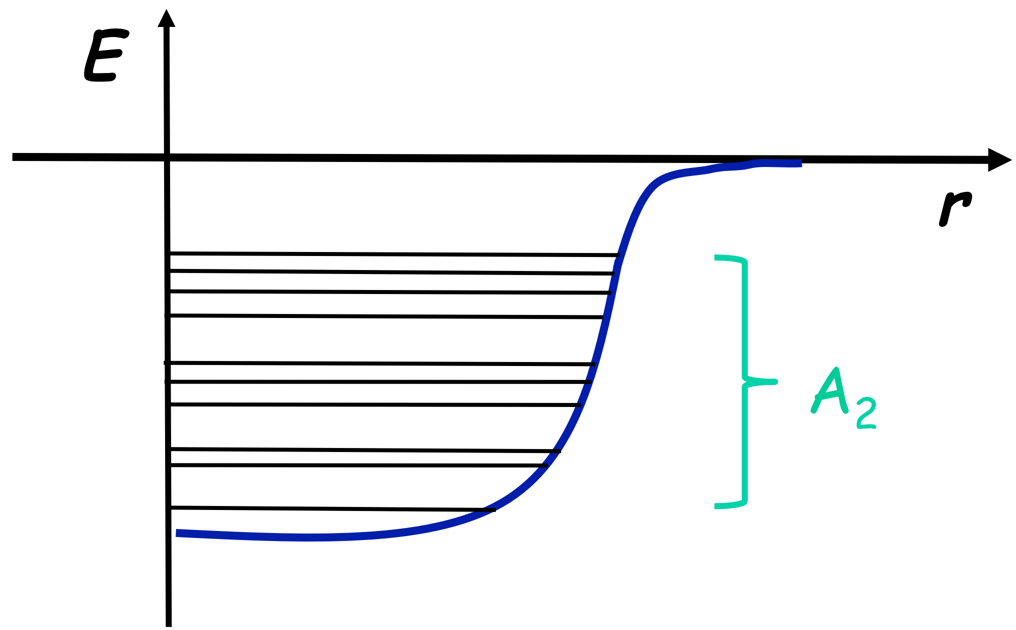
Single-particle states - starting point -

Mean potential becomes wider so as to cast *A* nucleons with the same separation energy.

light nuclei

heavy nuclei



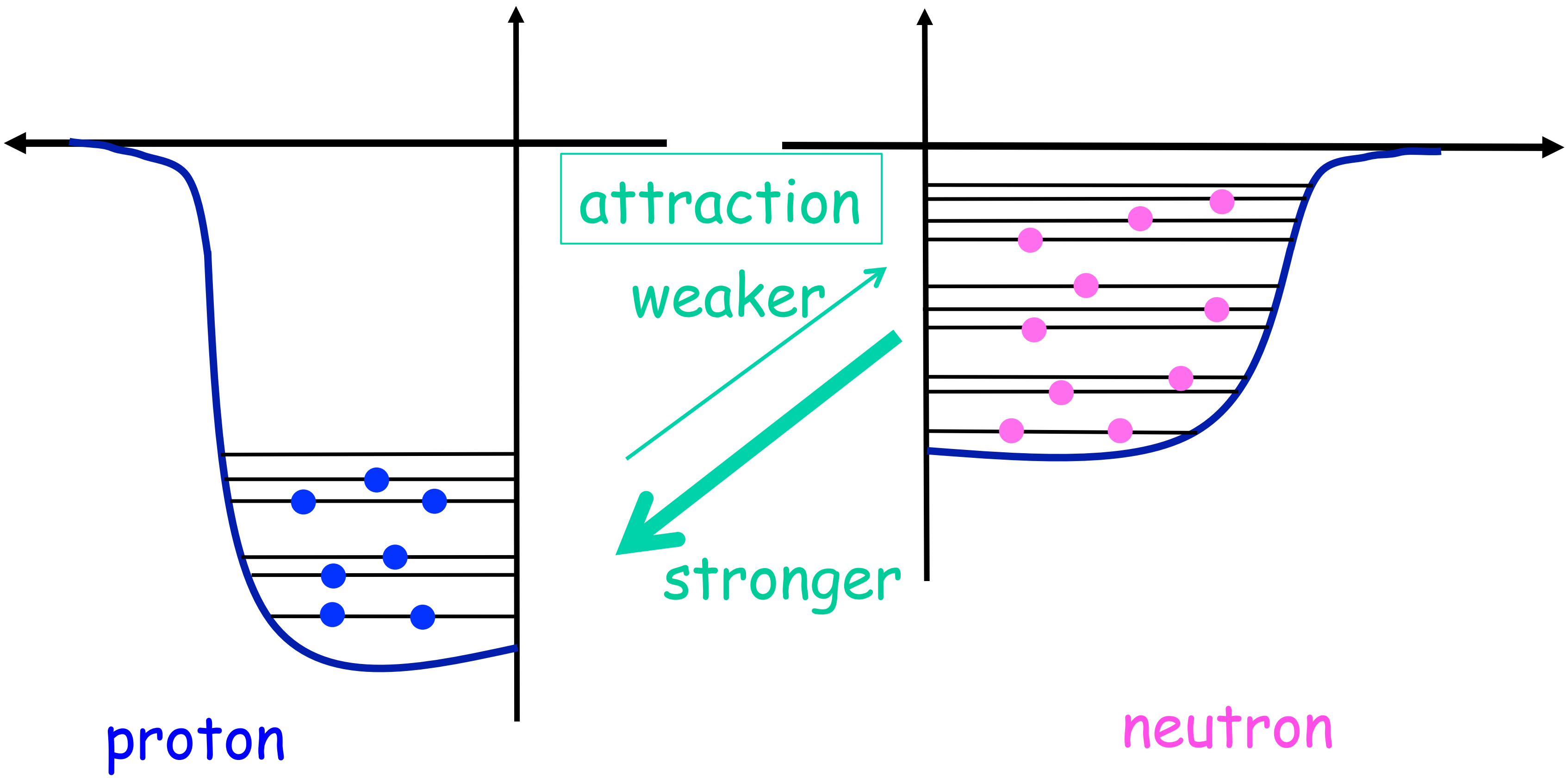


But, this is a story for stable nuclei.

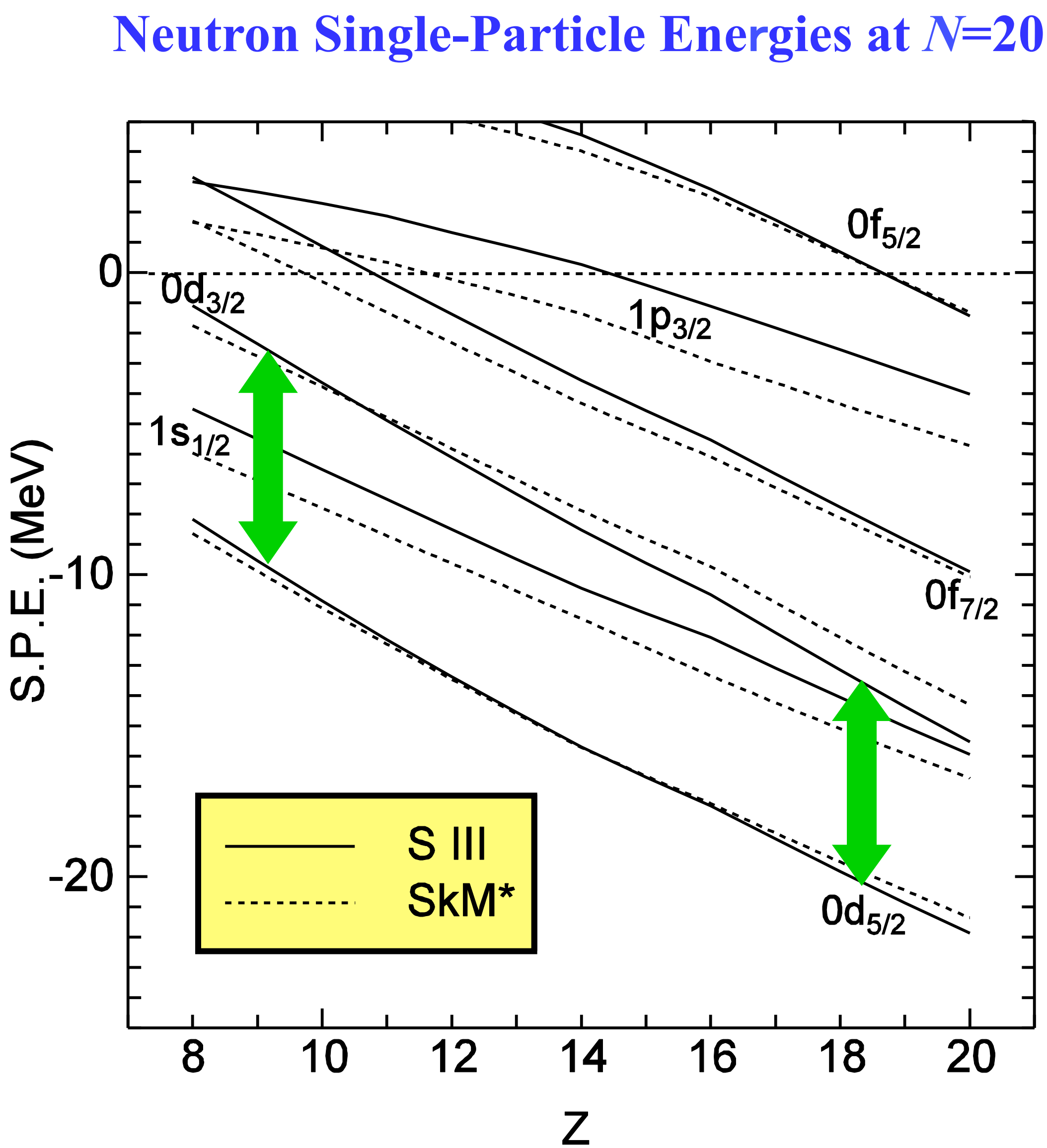
proton-neutron interaction
» proton-proton or neutron-neutron

If Z « N, protons are more bound.

Relative relations are preserved, because only the depth changes.



Realization in Hartree-Fock energies by Skyrme model



The shell structure remain rather unchanged

-- orbitals shifting together

-- change of potential depth

~ Woods-Saxon.

What about more characteristic effects
directly from nuclear forces
besides these “bulk” properties ?

Outline

1. Introduction
2. Shell model and monopole interaction
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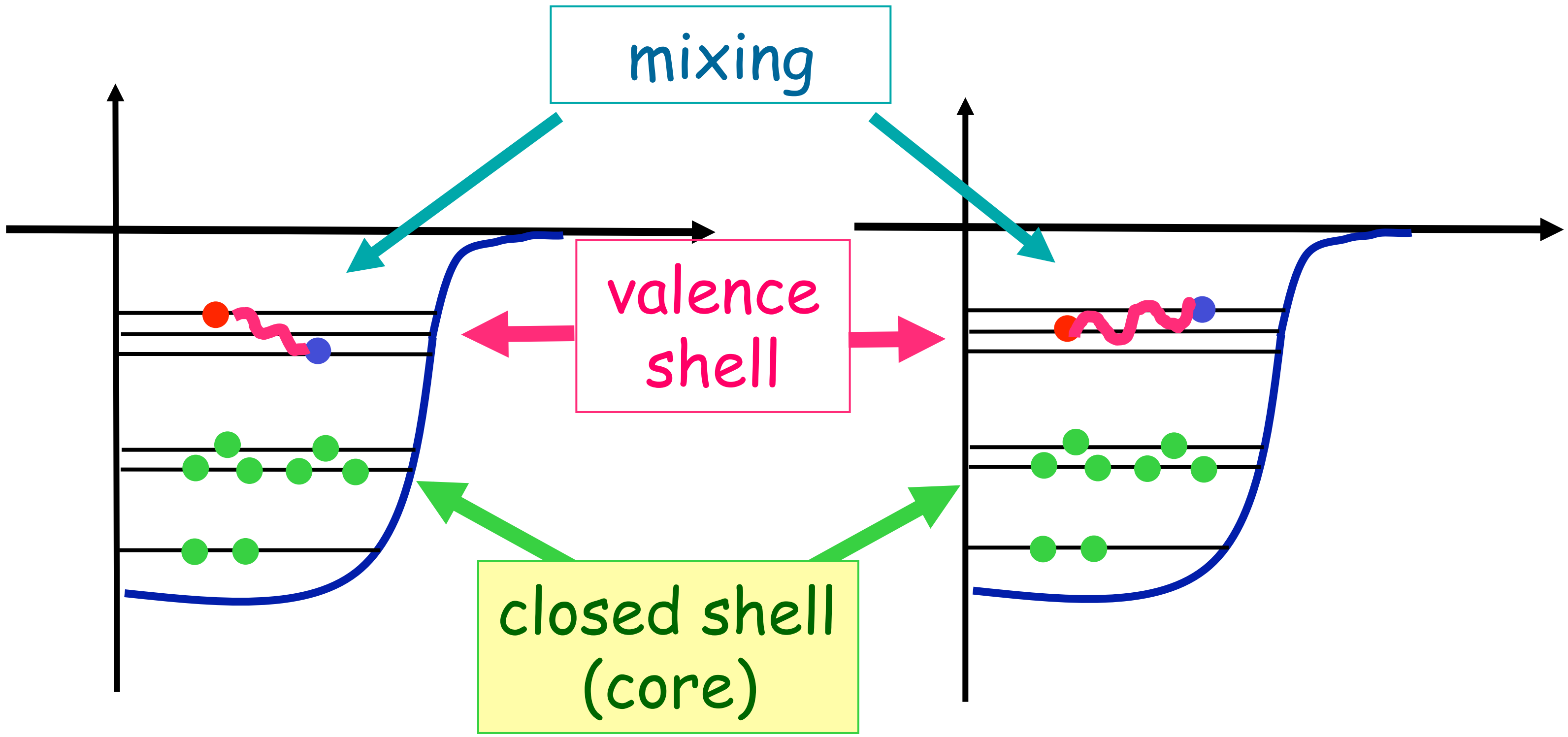
Nuclear shell model

A nucleon does not stay in an orbit for ever. The interaction between nucleons changes their occupations as a result of scattering.

Pattern of occupation of valence particles : configuration



Hamiltonian

$$H = \sum_i \epsilon_i n_i + \sum_{i,j,k,l} v_{ij,kl}\, a_i^\dagger a_j^\dagger a_l a_k$$

ε_i : single particle energy

$v_{ij,kl}$: two-body interaction matrix element

($i\ j\ k\ l$: single-particle states in valence shell)

How to get eigenvalues and eigenfunctions ?

Prepare Slater determinants $\phi_1, \phi_2, \phi_3, \ldots$ **which correspond to all configurations under consideration**

Step 1: Calculate matrix elements

$$< \phi_1 \,|\, H \,|\, \phi_1 >, \quad < \phi_1 \,|\, H \,|\, \phi_2 >, \quad < \phi_1 \,|\, H \,|\, \phi_3 >, \; \ldots.$$

where ϕ_1 , ϕ_2 , ϕ_3 are Slater determinants

In the second quantization,

closed shell

$$\phi_1 = a_\alpha^+ \; a_\beta^+ \; a_\gamma^+ \; \ldots.. | 0 >$$

$$\phi_2 = a_{\alpha'}^+ \; a_{\beta'}^+ \; a_{\gamma'}^+ \; \ldots.. | 0 >$$

$$\phi_3 = \ldots.$$

m-scheme representation of states

$$H = \Sigma_i \, \epsilon_i \, n_i + \Sigma_{i,j,k,l} \, v_{ij,kl} \, a_i^\dagger \, a_j^\dagger \, a_l \, a_k$$

Step 2 : Diagonalize the matrix of Hamiltonian, H

$$
H = \begin{pmatrix}
\langle \phi_1 |H| \phi_1 \rangle & \langle \phi_1 |H| \phi_2 \rangle & \langle \phi_1 |H| \phi_3 \rangle & \cdots \\
\langle \phi_2 |H| \phi_1 \rangle & \langle \phi_2 |H| \phi_2 \rangle & \langle \phi_2 |H| \phi_3 \rangle & \cdots \\
\langle \phi_3 |H| \phi_1 \rangle & \langle \phi_3 |H| \phi_2 \rangle & \langle \phi_3 |H| \phi_3 \rangle & \cdots \\
\langle \phi_4 |H| \phi_1 \rangle & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot
\end{pmatrix}
$$

Thus, we have solved the eigenvalue problem :

$$\mathbf{H}\,\Psi = \mathbf{E}\,\Psi$$

With Slater determinants $\phi_1, \phi_2, \phi_3, \ldots,$

the eigenfunction is expanded as

$$\Psi = c_1\,\phi_1 + c_2\,\phi_2 + c_3\,\phi_3 + \ldots..$$

c_i **probability amplitudes**

Model space : a set of orbits where the shell model calculation is done.

The model space is determined by
- character of the subject/object *larger preferred*
- computational ability *smaller preferred*

A typical choice:
model space = one major shell on top of the core.

Major shell : shell composed of orbits between two magic numbers

If magic numbers become uncertain, a very intriguing situation arises !

The closed shell (core) is treated as the vacuum.
Its effects are assumed to be included in the single-particle energies and the effective interaction.

Two-body interaction

A two-body state can be rewritten as

$$| j1, j2, J, M \rangle = \Sigma_{m1,\ m2} (j1, m1, j2, m2 \mid J, M) \, |j1, m1\rangle \, |j2, m2\rangle$$

Clebsch-Gordon coef.

Two-body matrix elements

$$\langle j1, j2, J, M \mid V \mid j3, j4, J', M' \rangle$$

J-scheme

$$= \Sigma_{m1,\ m2} (j1, m1, j2, m2 \mid J, M) \times \Sigma_{m3,\ m4} (j3, m3, j4, m4 \mid J', M') \times \langle j1, m1, j2, m2 \mid V \mid j3, m3, j4, m4 \rangle$$

m-scheme

Because the interaction V is a scalar with respect to the rotation, it cannot change J or M.

Only J=J' and M=M' matrix elements can be non-zero.

Hamiltonian

$$H = \Sigma_i \, \epsilon_i \, n_i + \Sigma_{i,j,k,l} \, v_{ij,kl} \, a_i^\dagger \, a_j^\dagger \, a_l \, a_k$$

ε_i : single particle energy

$v_{ij,kl}$: two-body interaction matrix element

($i\ j\ k\ l$: m-scheme state of valence shell)

↓

$v_{m1\ m2,\ m3\ m4}$

Two-body matrix elements

$$\langle j1, j2, J, \cancel{M} \mid V \mid j3, j4, J, \cancel{M} \rangle$$

are independent of M value, also because V is a scalar.

Two-body matrix elements are assigned by

j1, j2, j3, j4 and J.

Jargon : Two-Body Matrix Element = TBME

Because of complexity of nuclear force, one can not express all TBME's by a few *empirical* parameters.

As a result, TBME shows

- basic trend
- back ground which looks like random numbers

Basic trends include

pairing interaction* (nn, pp, i.e., T=1 channel)

monopole interaction (strong T=0, weak T=1 channels)

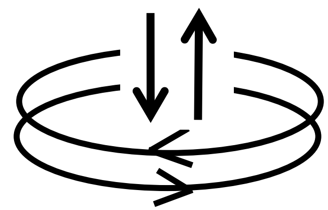
quadrupole interaction (pn, i.e., T=0 channel)

*Pairing interaction

$\langle j_1^2\ J=0 | V | j_2^2\ J=0 \rangle$ $\quad j_1^2 = j_1 \times j_1$

in T=1 channel

(identical particles)



time reversal states
-> strong attraction

No model for precise prediction so far

What is monopole interaction ?

Example : proton *jp* and neutron *jn* orbits

Proton-neutron interaction in m-scheme

$$V = \Sigma_{mp', mn', mp, mn} \langle mp' \, mn' \,|\, v \,|\, mp \, mn \rangle \; a^{+}_{mp'} \; a^{+}_{mn'} \; a_{mn} \; a_{mp}$$

$U_{jn}{}^{(K)}{}_{-M}$ ← rank K tensor ($a^{+}_{mn'} \, a_{mn}$)

$U_{jp}{}^{(K)}{}_{M}$ ← rank K tensor ($a^{+}_{mp'} \, a_{mp}$)

$$= \Sigma_K F^{(K)}(jp, jn) \; \Sigma_M (-1)^M \, U_{jp}{}^{(K)}{}_{M} \, U_{jn}{}^{(K)}{}_{-M}$$

$$= \Sigma_K F^{(K)}(jp, jn) \; \underbrace{(U_{jp}{}^{(K)} \cdot U_{jn}{}^{(K)})}_{\text{scalar product}}$$

K=0 monopole

K=1 dipole (angular mom.)

K=2 quadrupole

rank K unit tensor

$$U_{jp}{}^{(K)}{}_{M} = (2\,jp + 1)^{1/2}\,[\,a^{+}{}_{jp}\,\tilde{a}_{jp}\,]^{(K)}{}_{M}$$

$$= (2\,jp + 1)^{1/2}\,\Sigma_{mp',mp}\,(jp\ mp'\ jp\ mp\ |\ K\ M)\ a^{+}{}_{mp'}\,\tilde{a}_{mp}$$

direct product

In order to carry proper angular momentum property, $\tilde{a}_{mp} = (-1)^{jp+mp}\,a_{-mp}$ is used.

$mp' = -mp$

$$\langle \text{closed shell}\ |\ U_{jp}{}^{(K)}{}_{M}\ |\ \text{closed shell}\rangle$$

(*fully occupied orbit*)

$$= (2\,jp + 1)^{1/2}\,\Sigma_{mp'}\,(jp\ mp'\ jp\ {-mp'}\ |\ K\ 0)\,(-1)^{jp-mp'}$$

$$= (2\,jp + 1)\,\Sigma_{mp'}\,(jp\ mp'\ jp\ {-mp'}\ |\ K\ 0)\,(jp\ mp'\ jp\ {-mp'}\ |\ 0\ 0)$$

$$= (2\,jp + 1)\,\delta_{K,0}$$

Clebsch-Gordon coef. orthogonality condition

For closed shell, only K=0 (or monopole) remains.

rank K unit tensor

K=2 case

$$U_{jp}{}^{(K)}{}_{M} = (2\,jp + 1)^{1/2}\,[\,a^{+}{}_{jp}\,a\tilde{}_{jp}\,]^{(K)}{}_{M}$$

$$= (2\,jp + 1)^{1/2}\,\Sigma_{mp',mp}\,(jp\ mp'\ jp\ mp \mid K\ M)\,a^{+}{}_{mp'}\,a\tilde{}_{mp}$$

direct product

In order to carry proper angular momentum property, $a\tilde{}_{mp} = (-1)^{jp+mp}\,a_{-mp}$ is used.

mp' = -mp

<closed shell | $U_{jp}{}^{(K)}{}_{M}$ | closed shell> (*fully occupied orbit*)

$$= (2\,jp + 1)^{1/2}\,\Sigma_{mp'}\,(jp\ mp'\ jp\ {-mp'} \mid 2\ 0)\,(-1)^{jp-mp'}$$

$$= 0$$

$$(jp\ mp'\ jp\ {-mp'} \mid 2\ 0) = (-1)^{jp-mp'}\,(3\,(mp')^2 - jp\,(jp + 1))$$

Take the expectation value with respect to the proton and neutron closed shells (fully occupied orbits)

$$\langle p \,\&\, n \text{ c. s.} \mid \Sigma_K F^{(K)}(jp, jn) \, (U_{jp}^{(K)} \cdot U_{jn}^{(K)}) \mid p \,\&\, n \text{ c. s.} \rangle$$

$$= F^{(K)}(jp, jn)\,(2\,jp + 1)\,(2\,jn + 1)\,\delta_{K,0}$$

Because only K=0 (or monopole) remains for closed shells,

$$F^{(0)}(jp, jn)$$
$$= \langle p \,\&\, n \text{ c. s.} \mid \mathbf{V} \mid p \,\&\, n \text{ c. s.} \rangle / (2\,jp + 1)\,(2\,jn + 1)$$

p-n monopole interaction = $F^{(0)}(jp, jn)\; n_{jp}\; n_{jn}$

where n_{jp} & n_{jn} denote number operators for each orbit

Note $U_{jp}^{(0)} = - n_{jp}$, $U_{jn}^{(0)} = - n_{jn}$

Remarks

(i) Monopole interaction can be rewritten as

$F^{(0)}(jp, jn)$

$= \langle p \,\&\, n \text{ c. s.} \,|\, \mathbf{V} \,|\, p \,\&\, n \text{ c. s.} \rangle / (2\,jp + 1)(2\,jn + 1)$

$= \Sigma_{mp,\, mn} \langle mp,\ mn \,|\, V \,|\, mp,\ mn \rangle \,/\, \Sigma_{mp,\, mn}\ 1$

Summation over all possible orientation (as jp and jn are fixed)

total number of states

(ii) Monople effect is proportional to n_{jp} and n_{jn} ,
whereas other effects are vanished for closed shell.

↓

Multipole interaction

As N or Z is changed to a large extent in exotic nuclei, the shell structure is changed (evolved) by

- **Monopole component of the *NN* interaction**

$$v_{m;j,j'} = \sum_{k,k'} \langle jkj'k'|V|jkj'k'\rangle \Big/ \sum_{k,k'} 1,$$

→ **Averaged over possible orientations**

Linearity: Shift $\Delta\epsilon_j = v_{m;j,j'} n_{j'}$ $n_{j'}$: # of particles in j'

Poves and Zuker made a major contribution in initiating systematic use of the monopole interaction. (Poves and Zuker, Phys. Rep. 70, 235 (1981))

Basic feature of monopole interaction

- p-p, n-n or T=0, 1 monopole interactions are defined in a similar way (equations slightly more complicated):
  average over all possible orientations

- Equivalent definition

$$v_{m;j\,j'} = \Sigma_J\,(2J+1)\,\langle j_1, j_2, J\,|V|\,j_1, j_2, J\rangle \,/\, \Sigma_J\,(2J+1)$$

Average over all possible J

Example : next page

- Linear dependence

$$\Delta\epsilon_j = v_{m;j,j'}\,n_{j'}$$

This effect is accumulated as $n_{j'}$ increases.

Effects of multipole interactions are not linear

- Closed shell $n_{j'} = (2j' + 1)$

The effect becomes a change of single-particle energy.

USD interaction

1 = d3/2

2= d5/2

3= s1/2

i	j	k	l	J	T	v
1	1	1	1	0	1	-2.1845
1	1	1	1	1	0	-1.4151
1	1	1	1	2	1	-0.0665
1	1	1	1	3	0	-2.8842
2	1	1	1	1	0	0.5647
2	1	1	1	2	1	-0.6149
2	1	1	1	3	0	2.0337
2	1	2	1	1	0	-6.5058
2	1	2	1	1	1	1.0334
2	1	2	1	2	0	-3.8253
2	1	2	1	2	1	-0.3248
2	1	2	1	3	0	-0.5377
2	1	2	1	3	1	0.5894
2	1	2	1	4	0	-4.5062
2	1	2	1	4	1	-1.4497
2	1	3	1	1	0	-1.7080
2	1	3	1	1	1	0.1874
2	1	3	1	2	0	0.2832
2	1	3	1	2	1	-0.5247
2	1	3	3	1	0	2.1042
2	2	1	1	0	1	-3.1856
2	2	1	1	1	0	0.7221
2	2	1	1	2	1	-1.6221
2	2	1	1	3	0	1.8949
2	2	2	1	1	0	2.5435
2	2	2	1	2	1	-0.2828
2	2	2	1	3	0	2.2216
2	2	2	1	4	1	-1.2363
2	2	2	2	0	1	-2.8197
2	2	2	2	1	0	-1.6321
2	2	2	2	2	1	-1.0020
2	2	2	2	3	0	-1.5012

T=0 monopole int. between $d_{3/2}$ and $d_{5/2}$

$-6.506 \times 3 = -19.518$

$-3.825 \times 5 = -19.125$

$-0.538 \times 7 = -3.766$

$-4.506 \times 9 = -40.554$

Sum -82.963

Sum of (2J+1) = 24

$V_{m:12} = -3.457$

More remarks on the monopole and multipole interactions

The monopole interaction is a component of a two-body interaction. It is not something added.

Monopole interaction changes (spherical) single-particle energies effectively according to the occupation of valence shell orbits.
When all valence orbits are fully occupied, a new closed shell is formed and the monopole interaction provides single-particle energies for this new closed shell.

In relation to the Nilsson model, monopole interaction shifts Nilsson levels at zero deformation, which are constant in the original Nilsson model.

Outline

1. Introduction
2. Shell model and monopole interaction
3. Shell evolution and tensor force
4. Multiple quantum liquid in exotic nuclei
5. Shell evolution and three-nucleon force
6. Summary

What parts of nuclear forces are relevant ?

$$v_{m;j\,j'} = \Sigma_J\,(2J+1)\,\langle j_1, j_2, J \,|\, V \,|\, j_1, j_2, J \rangle \,/\, \Sigma_J\,(2J+1)$$

This TBME becomes larger generally, if the overlap of radial wave functions of orbits j_1 and j_2 becomes larger.

The monopole interaction $v_{m;j\,j'}$ becomes stronger for central force with a short range.

The overlap of the radial wave functions are larger, if

- j_1 and j_2 are spin-orbit partner, *e.g.*, $d_{3/2}$ and $d_{5/2}$
- j_1 and j_2 are both high j orbits, *e.g.*, $f_{7/2}$ and $g_{9/2}$

What else ?

Proton-neutron interaction

A famous example : Federman-Pittel mechanism

Volume 69B, number 4 PHYSICS LETTERS 29 August 1977

TOWARDS A UNIFIED MICROSCOPIC DESCRIPTION OF NUCLEAR DEFORMATION

P. FEDERMAN
IFUNAM, Ap. Postal 20-364, México 20, D.F.

and

S. PITTEL
Bartol Research Foundation of The Franklin Institute, Swarthmore, Pennsylvania 19081, USA

Nuclear deformation, as it occurs in both light and heavy nuclei, is discussed in a unified microscopic shell-model framework. The short-range $^3S_1$ neutron-proton interaction plays an important role in this discussion.

the $T = 0$ attraction of nucleons in spin-orbit-partner orbitals may be even stronger than the attraction of two nucleons in the same orbitals. To illustrate this

The importance of the $^3S_1$ attraction between nucleons in the $1d_{5/2}$ and $1d_{3/2}$ orbitals can be seen experimentally in the spectrum of ^{18}F, which has only a single neutron and proton outside ^{16}O. The ^{18}F spec-

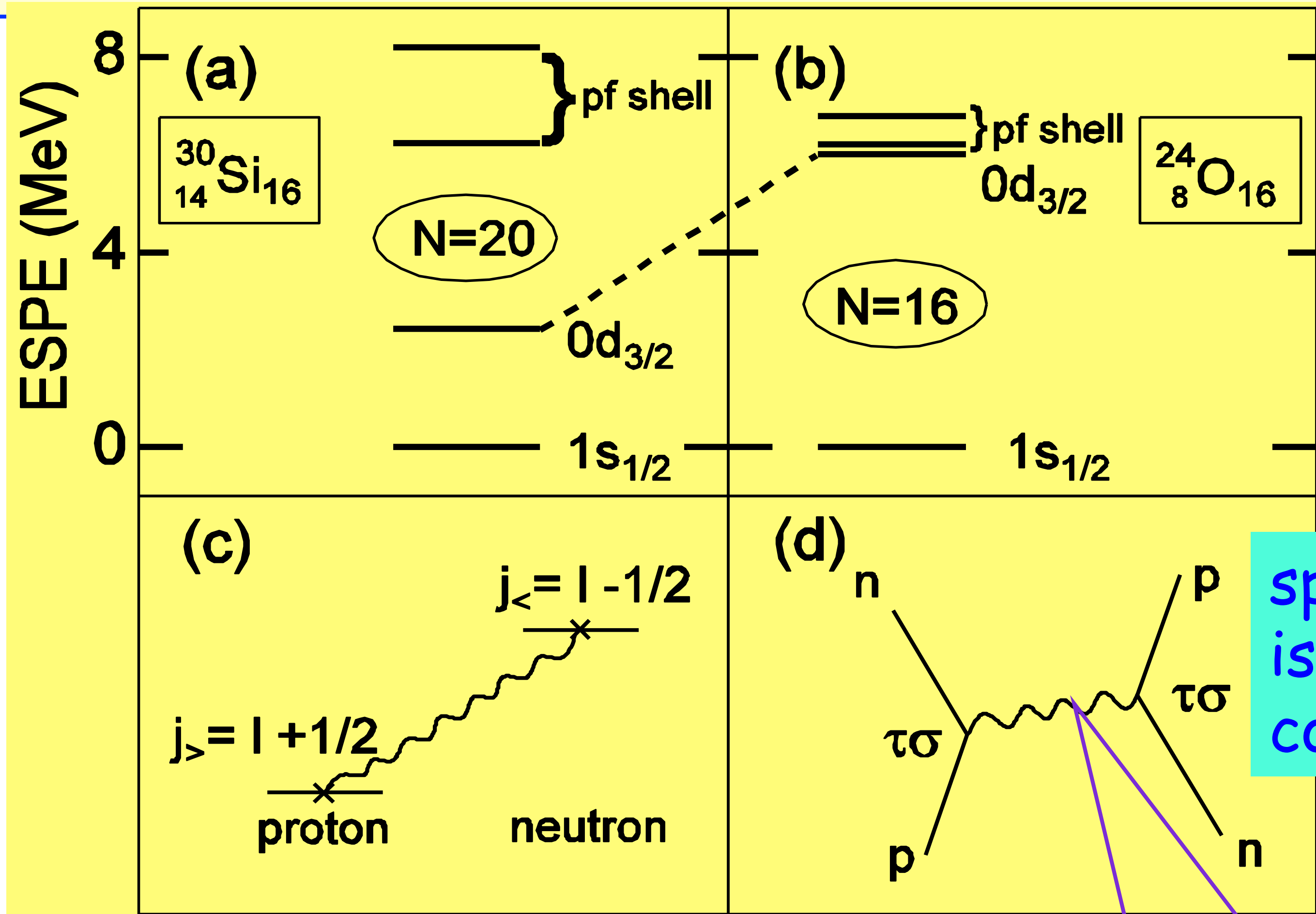
T=0 central force important

Overlap of radial wave function relevant

Spin-orbit-partner orbitals -> larger effect
underlying physics untouched

No monopole, no tensor … *close, but not quite*

Chronologically, the first attempt was based on the $\sigma\sigma\tau\tau$ central interaction with long range.



spin-flip-isospin-flip coupling

Otsuka et al. Phys. Rev. Lett. 87, 082502 (2001)

$\sigma\sigma\tau\tau$ central

Tensor

Evolution of shell structure due to the tensor force

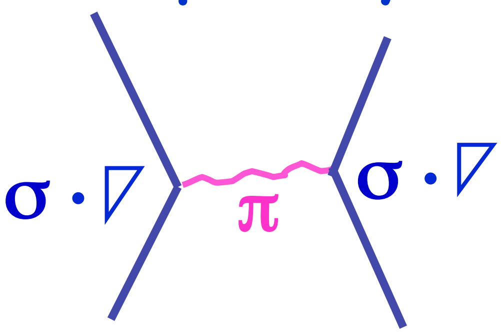
Tensor Interaction by pion exchange

$$V_T = (\tau_1 \tau_2)\,(\,[\sigma_1 \sigma_2]^{(2)}\, Y^{(2)}(\Omega)\,)\, Z(r)$$

contributes only to **S=1** states

relative motion

π meson : primary source



Proc. Phys. Math. Soc. Japan 17, 48 (1935)

ρ meson (~ π+π) : minor (~1/4) cancellation

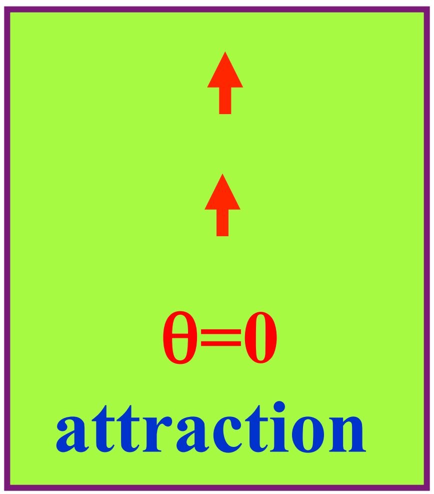
Ref: Osterfeld, Rev. Mod. Phys. 64, 491 (92)

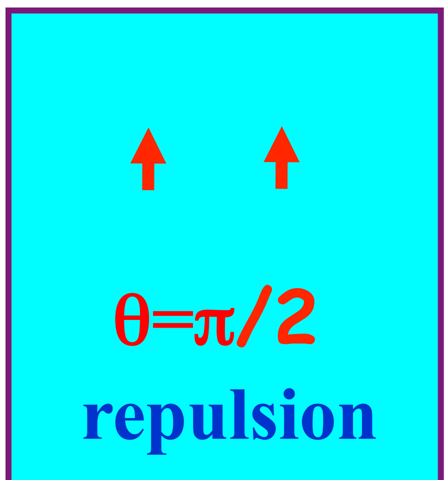
How does the tensor force work ?

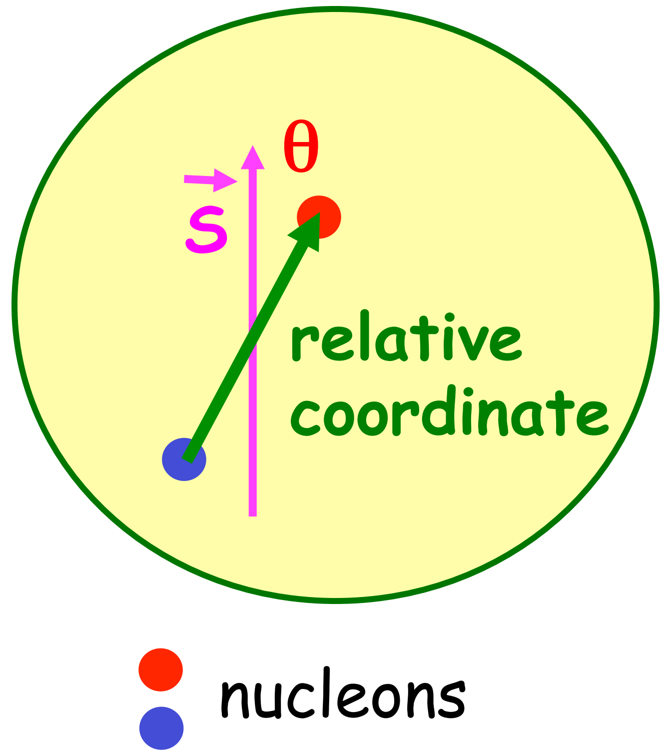
Spin of each nucleon ↑ is parallel, because the total spin must be S=1

The potential has the following dependence on the angle θ with respect to the total spin $\vec{S}$.

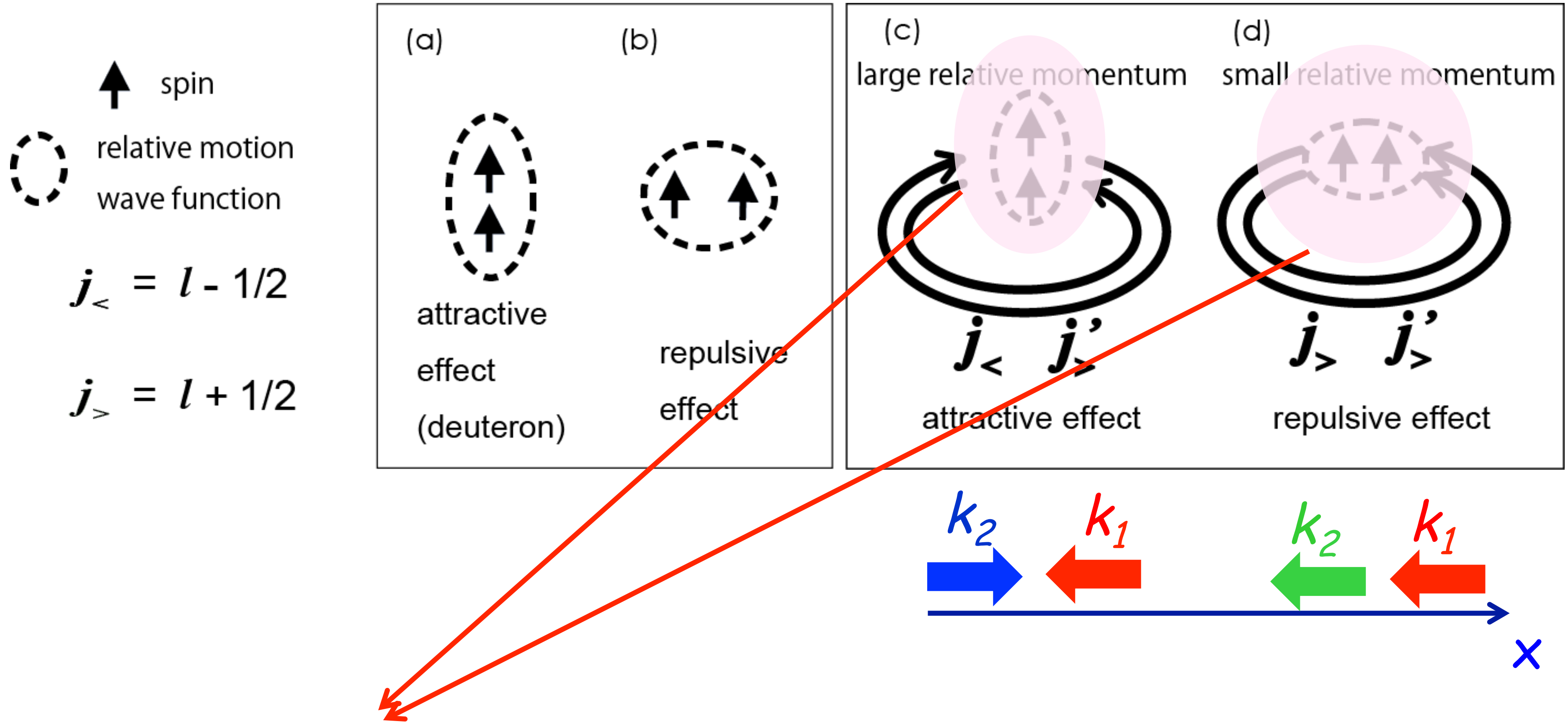
$$V \sim Y_{2,0} \sim 1 - 3\cos^2\theta$$







One-dimensional (x axis)collision model



$$\Psi \propto e^{ik_1x_1}e^{ik_2x_2} + e^{ik_2x_1}e^{ik_1x_2} = 2\,e^{iKX}\cos(kx)$$

$$K, k = k_1 \pm k_2, \quad X, x = (x_1 \pm x_2)/2$$

Note: wave functions in the y-z directions are "uniform".

One-dimensional (x axis) collision model – cont'd

Note: wave functions in the y-z directions are "uniform".

k relative momentum; $k = k_1 - k_2$

$$\Psi \propto e^{ik_1x_1}e^{ik_2x_2} + e^{ik_2x_1}e^{ik_1x_2} = 2\,e^{iKX}\cos(kx)$$

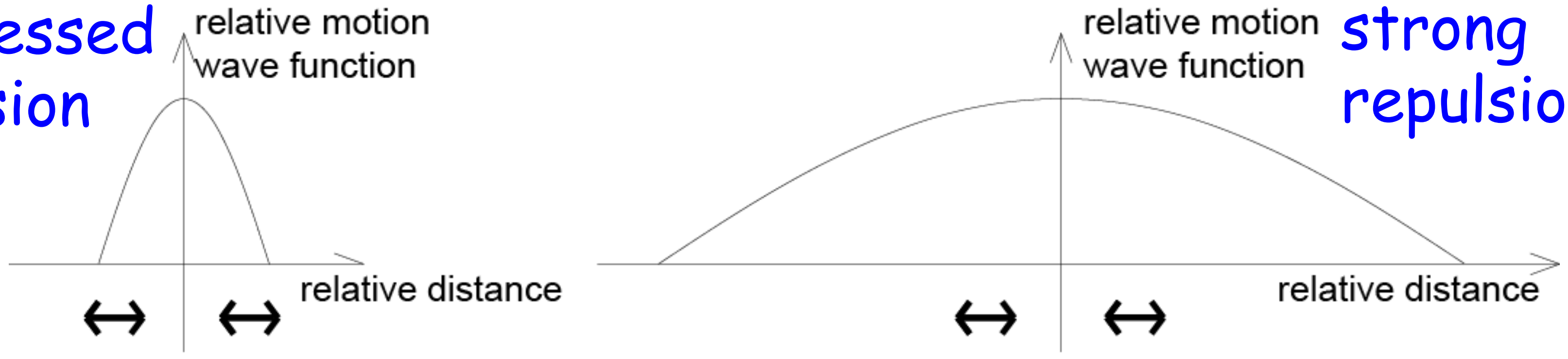
$$K, k = k_1 \pm k_2, \quad X, x = (x_1 \pm x_2)/2$$

k <~ 1.5 fm^{-1} (from Fermi momentum)
x <~ 1 fm (range of tensor force)
kx <~ 1.5 < $\pi/2$

(a) $j_>$ and $j'_<$ (b) $j_>$ and $j'_>$

k large — head-on collison; k small — parallel motion

suppressed repulsion; strong repulsion



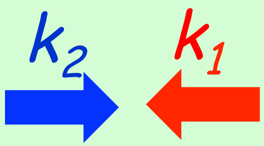
↔ region relevant to the repulsive effect by the tensor force

One-dimensional collision model - summary-

TO, Suzuki *et al*. PRL 95, 232502 (2005)
TO, Phys. Scr. T152, 014007 (2013)

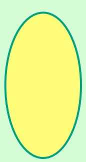
At collision point: $\Psi \propto e^{ik_1x_1}e^{ik_2x_2} + e^{ik_2x_1}e^{ik_1x_2} = 2\,e^{iKX}\cos(kx)$

$k = k_1 - k_2\,, \quad K = k_1 + k_2$

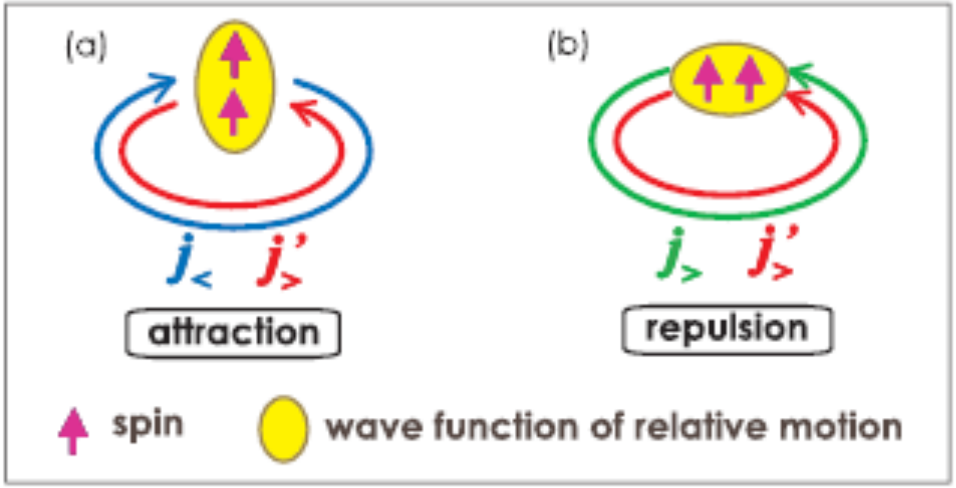


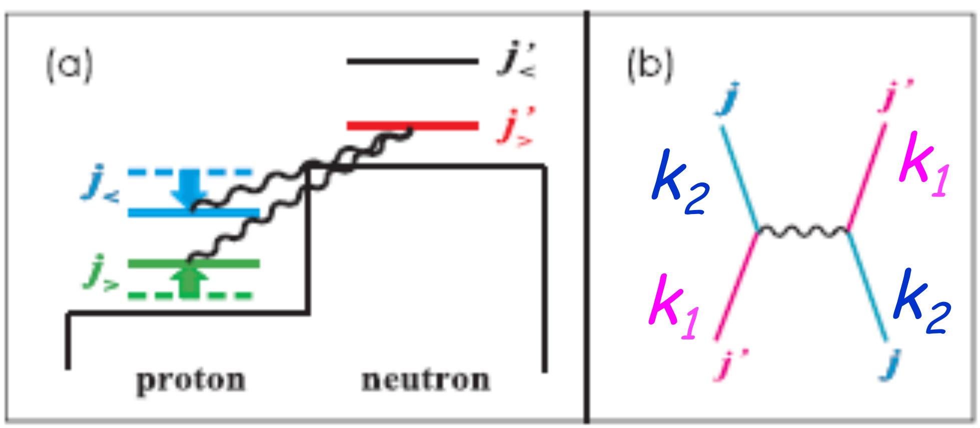
large relative momentum k

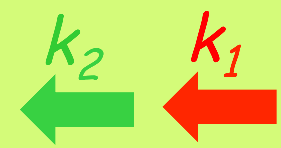
strong damping



wave function of relative coordinate

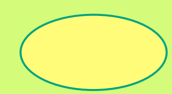






small relative momentum k

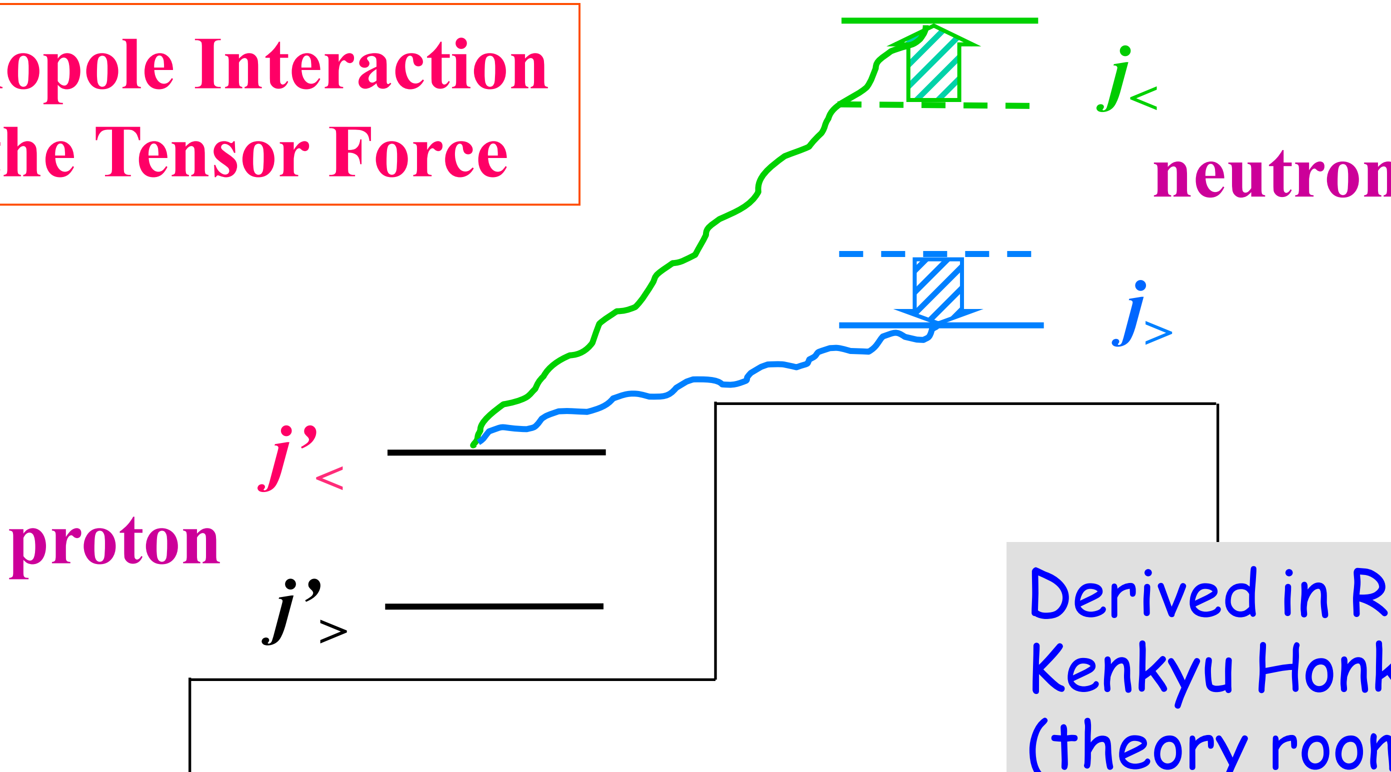
loose damping



wave function of relative coordinate

T. Otsuka et al., Phys. Rev. Lett. 95, 232502 (2005)

Monopole Interaction of the Tensor Force



Derived in RIKEN Kenkyu Honkan (theory room then)

Identity for tensor monopole interaction

$$(2j_> + 1)\; v_{m,T}^{(j' j_>)} + (2j_< + 1)\; v_{m,T}^{(j' j_<)} = 0$$

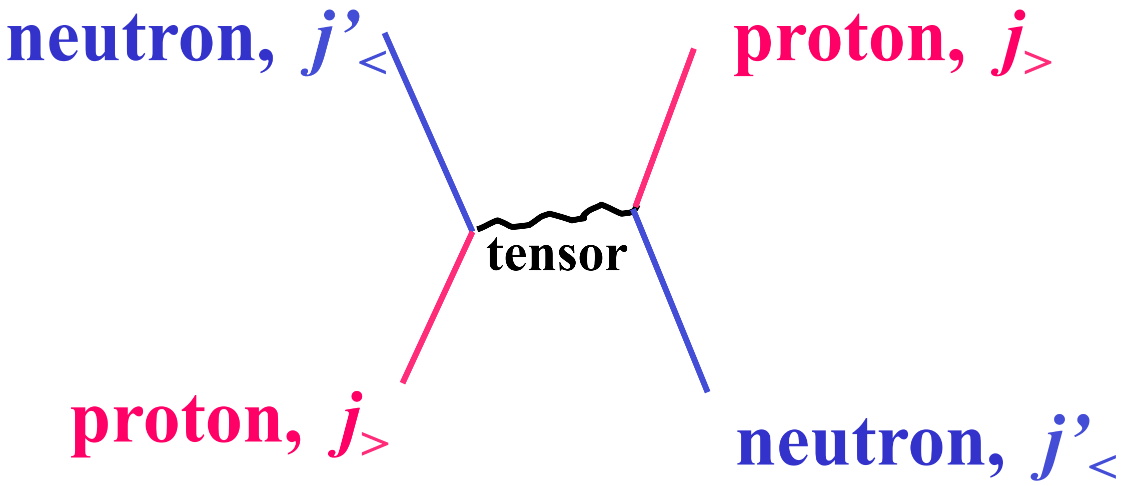
$v_{m,T}$: monopole strength for isospin T

Major features

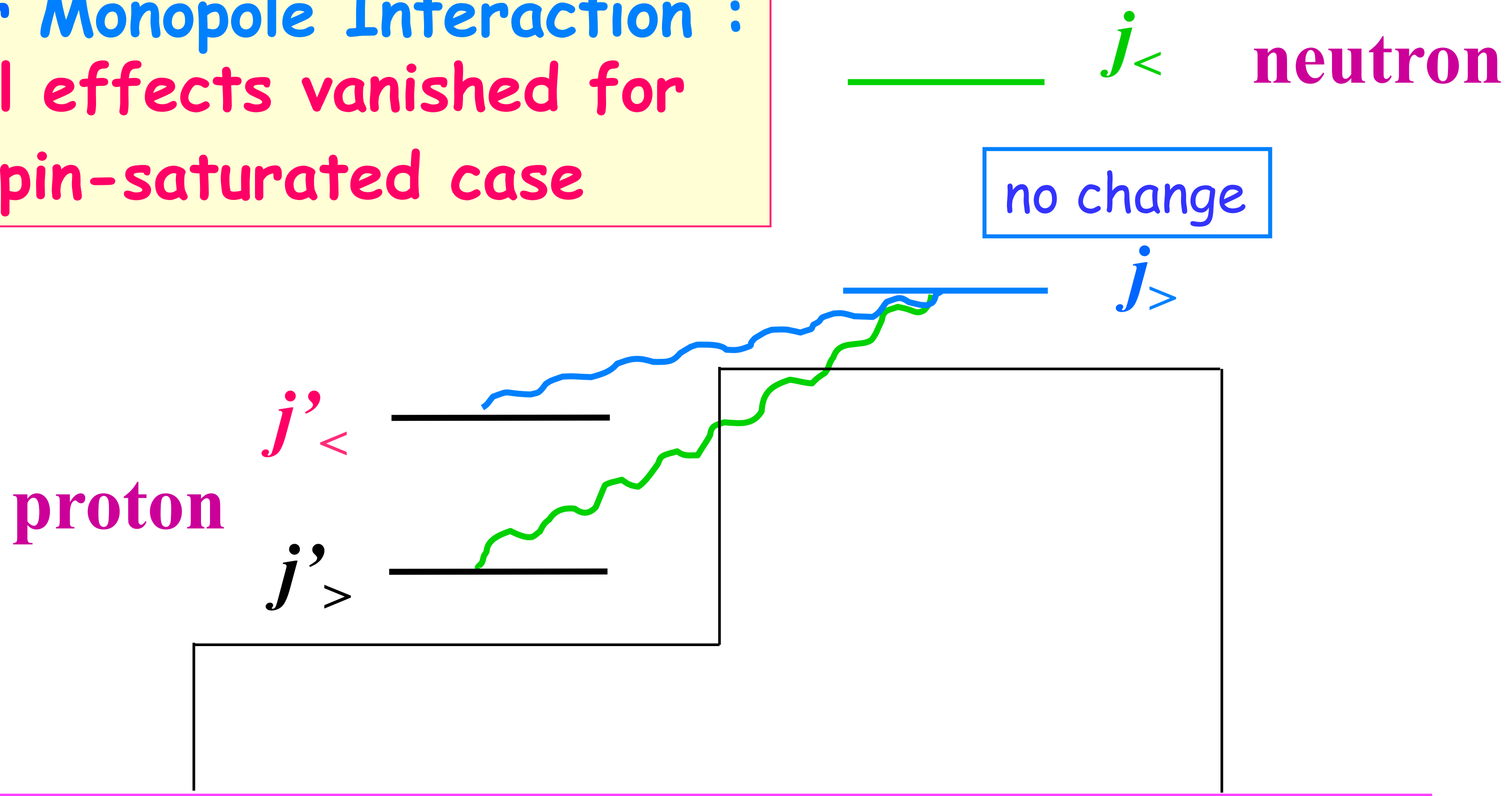
Opposite signs ⟹ spin-orbit splitting varied

T=0 : T=1 = 3 : 1 (same sign)

Only exchange terms (generally for spin-spin forces)



Tensor Monopole Interaction :
total effects vanished for spin-saturated case

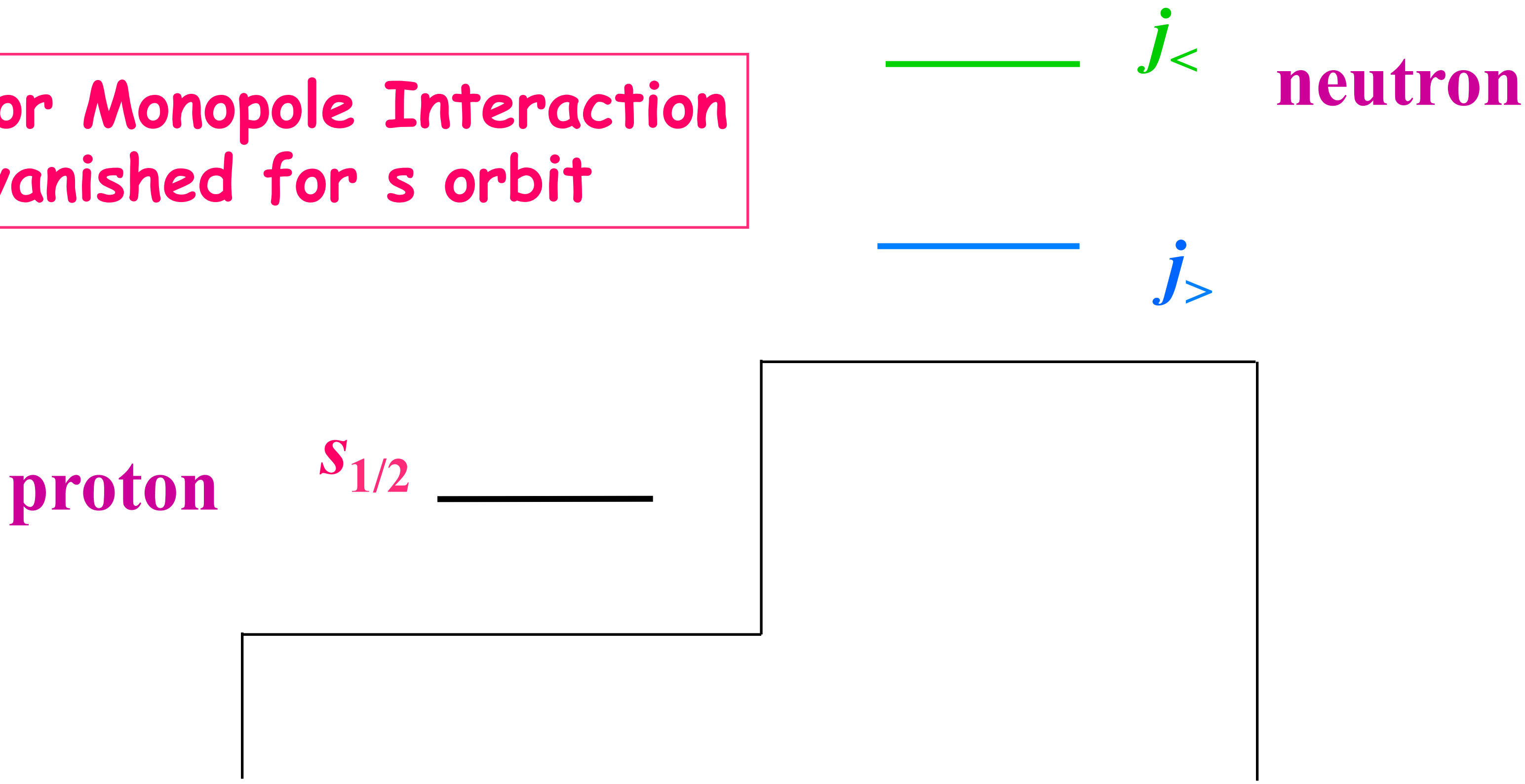


Same Identity with different interpretation

$$(2j_> +1)\ v_{m,T}^{(j'j_>)} + (2j_< +1)\ v_{m,T}^{(j'j_<)} = 0$$

$v_{m,T}$: monopole strength for isospin T

Tensor Monopole Interaction vanished for s orbit

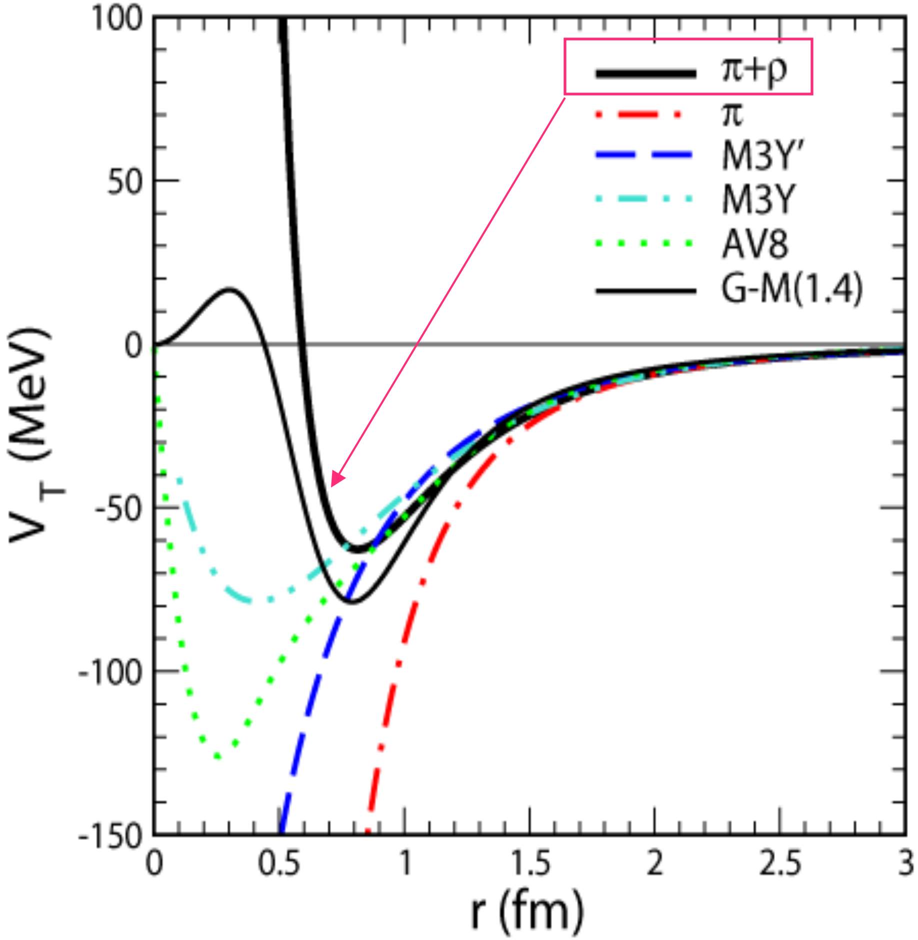


For s orbit, $j_>$ and $j_<$ are the same :

$$(2j_> +1)\ v_{m,T}^{(j'j_>)} + (2j_< +1)\ v_{m,T}^{(j'j_<)} = 0$$

$v_{m,T}$: monopole strength for isospin T

Tensor potential



tensor

⇩

no s-wave to s-wave coupling

⇩

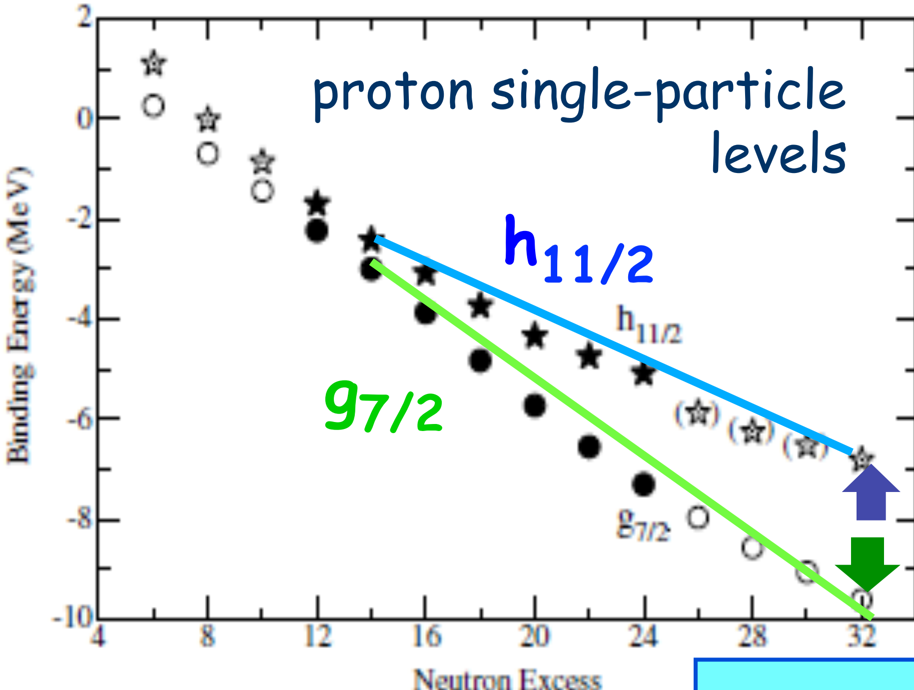
differences in short distance :
irrelevant

An example with $_{51}$Sb isotopes

VOLUME 92, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 23 APRIL 2004

Is the Nuclear Spin-Orbit Interaction Changing with Neutron Excess?

J. P. Schiffer,[1] S. J. Freeman,[1,2] J. A. Caggiano,[3] C. Deibel,[3] A. Heinz,[3] C.-L. Jiang,[1] R. Lewis,[3] A. Parikh,[3] P. D. Parker,[3] K. E. Rehm,[1] S. Sinha,[1] and J. S. Thomas[4]



Z =51 isotopes

change driven by neutrons in $1h_{11/2}$

$h_{11/2}$ - $h_{11/2}$ repulsive

$h_{11/2}$ - $g_{7/2}$ attractive

$\pi + \rho$ meson exchange tensor force

(splitting increased by ~ 2 MeV)

No mean field theory, (Skyrme, Gogny, RMF) explained this before.

TO *et al.*, PRL 95, 232502 (2005)

Tensor-force monopoles

in realistic interactions

Anatomy of effective shell-model interaction

GXPF1(A)

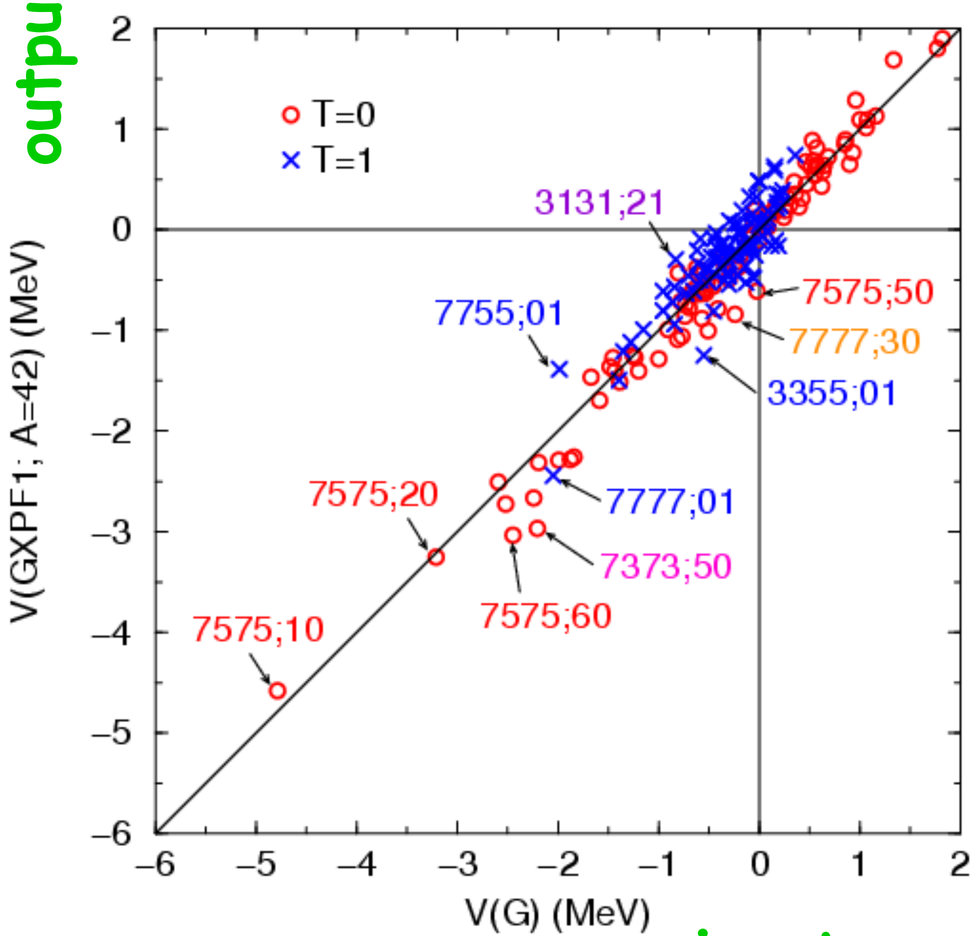
two-body matrix element

$< ab;\, JT \mid V \mid cd\, ;\, JT >$

$7 = f_{7/2},\ 3 = p_{3/2},\ 5 = f_{5/2},\ 1 = p_{1/2}$

G-matrix obtained from Bonn-C potential + 3^{rd} order Q_{box}

output



G-matrix input

As *N* or *Z* is changed to a large extent in exotic nuclei, the shell structure is changed (evolved) by

- **Monopole component of the *NN* interaction**

$$v_{m;j,j'} = \sum_{k,k'} \langle jkj'k'|V|jkj'k'\rangle \Big/ \sum_{k,k'} 1,$$

→ **Averaged over possible orientations**

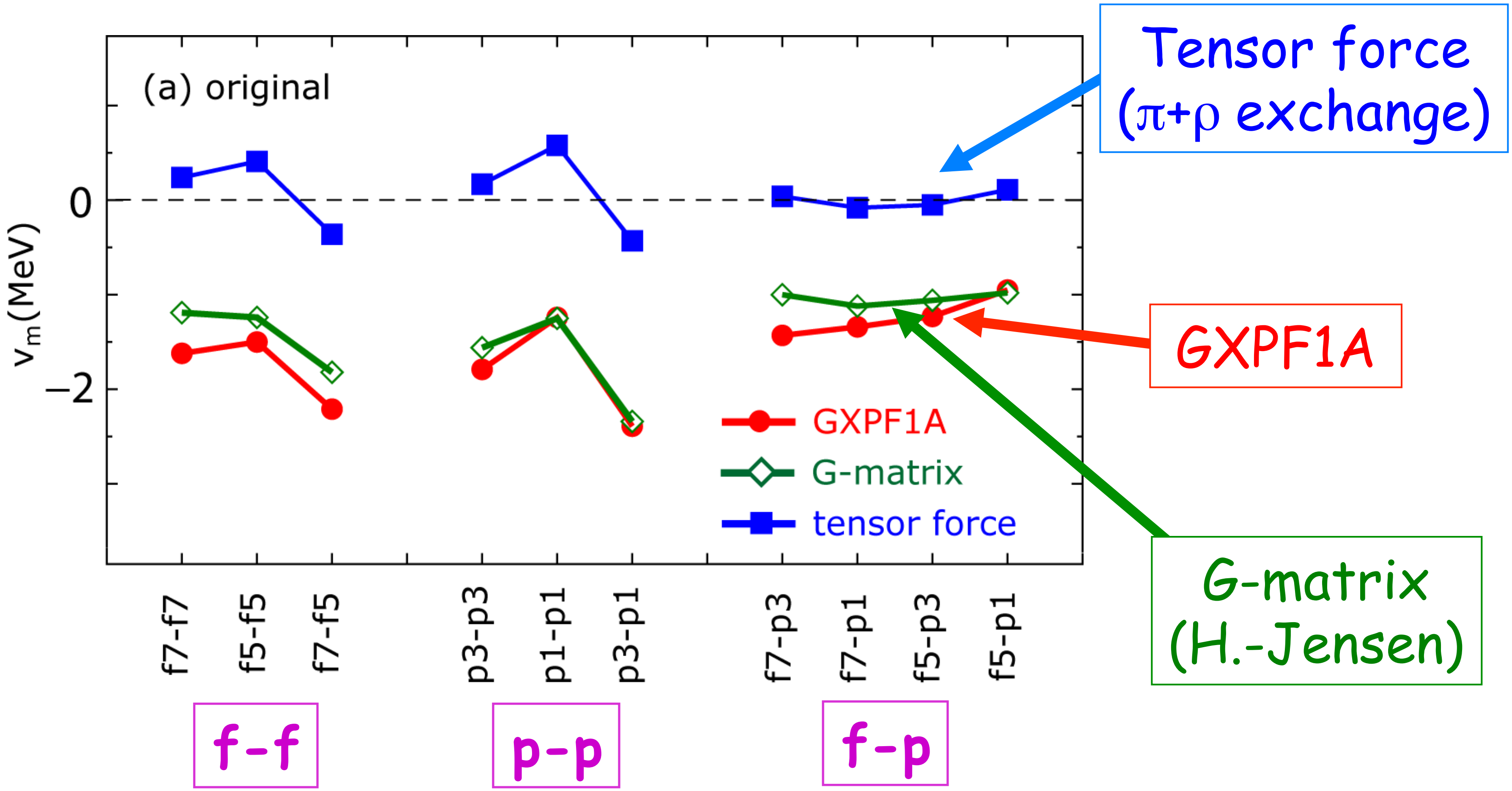
Selected for a Viewpoint in *Physics*

PRL 104, 012501 (2010) PHYSICAL REVIEW LETTERS week ending 8 JANUARY 2010

Novel Features of Nuclear Forces and Shell Evolution in Exotic Nuclei

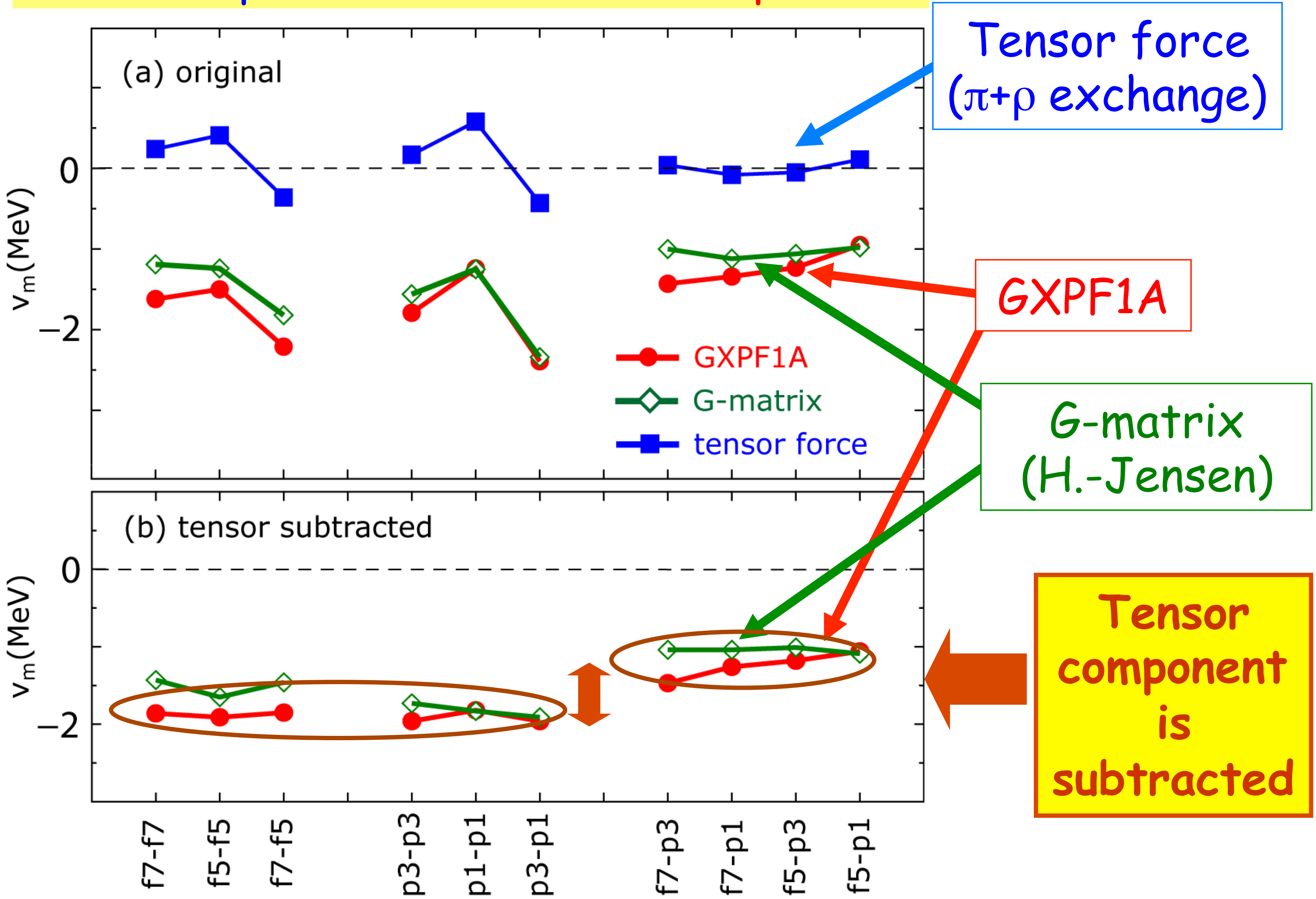
Takaharu Otsuka,[1,2] Toshio Suzuki,[3] Michio Honma,[4] Yutaka Utsuno,[5] Naofumi Tsunoda,[1] Koshiroh Tsukiyama,[1] and Morten Hjorth-Jensen[6]

T=0 monopole interactions in the pf shell

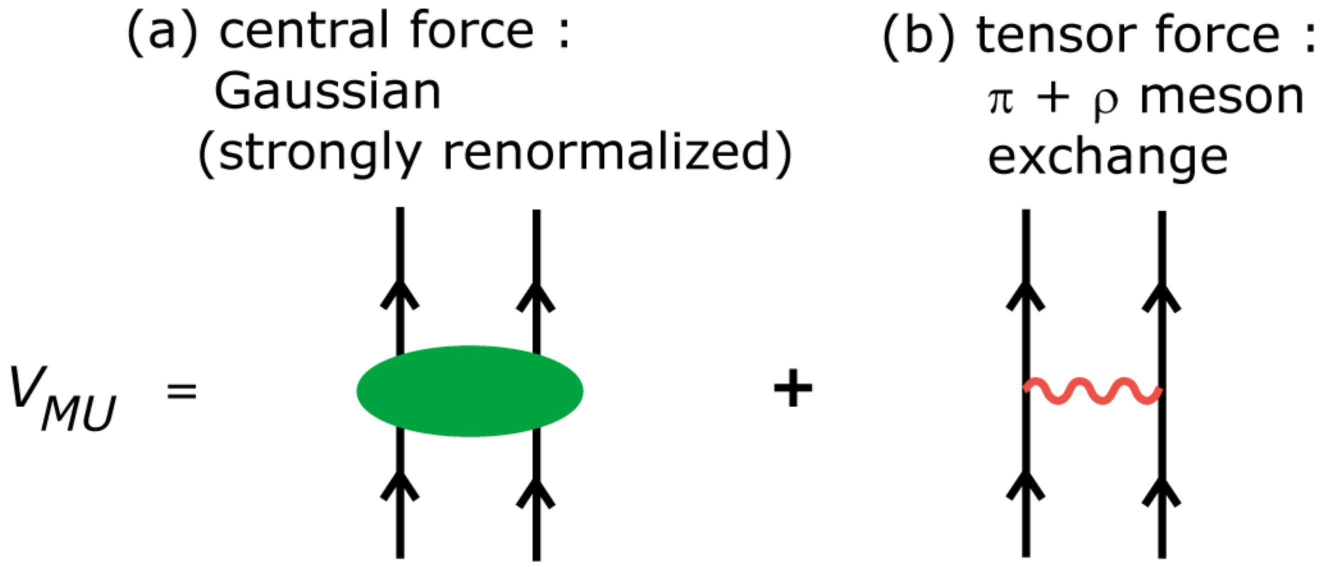


"Local pattern" ← tensor force

T=0 monopole interactions in the pf shell



Systematic description of monopole properties of exotic nuclei can be obtained by an extremely simple interaction as



Parameters are fixed for all nuclei

V_{MU} : Monopole-based Universal Interaction

The central force is modeled by a Gaussian function

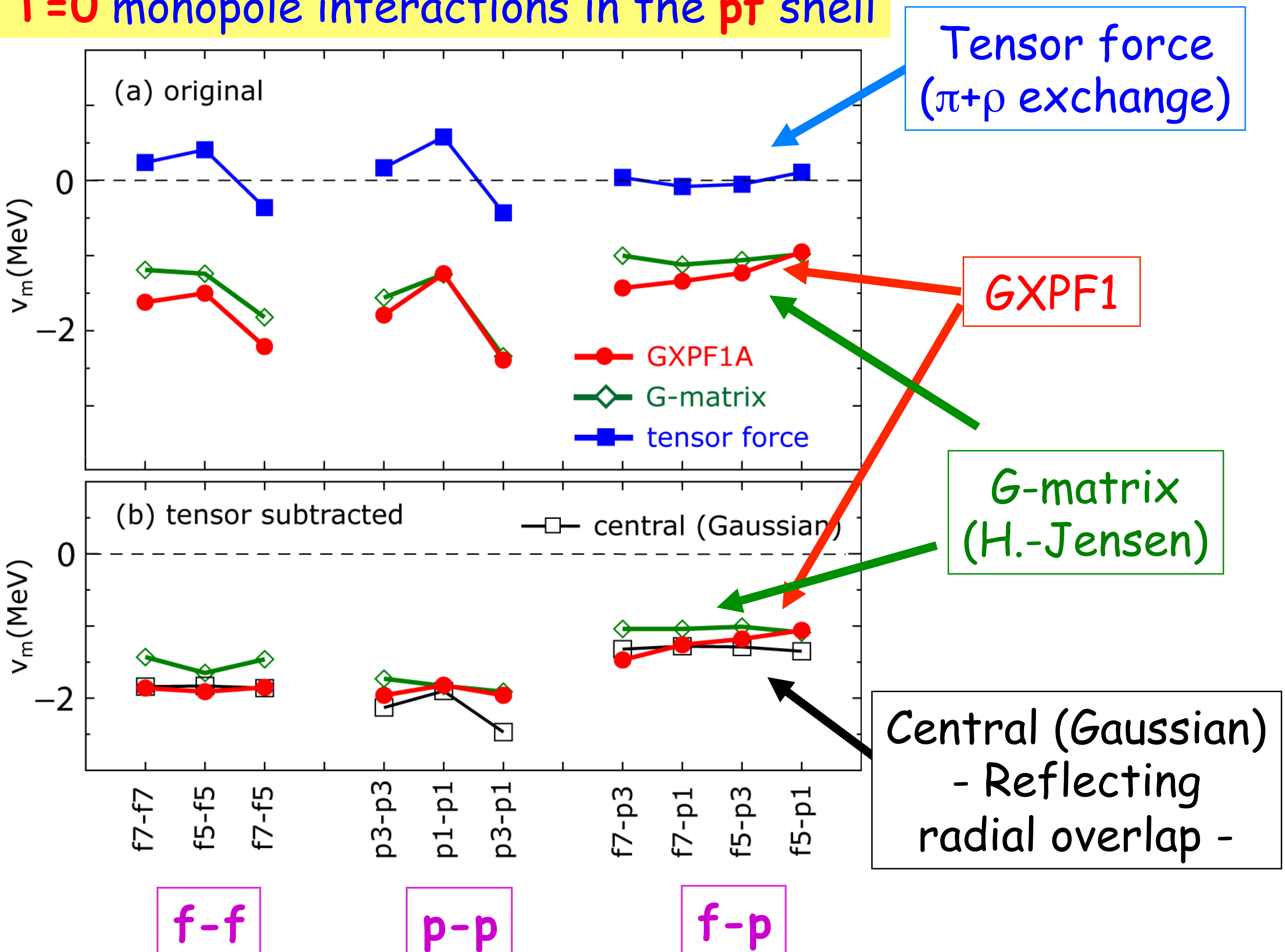
$$V = V_0 \exp(-(r/\mu)^2) \quad \text{(S,T dependences)}$$

with V_0 = -166 MeV, μ=1.0 fm,

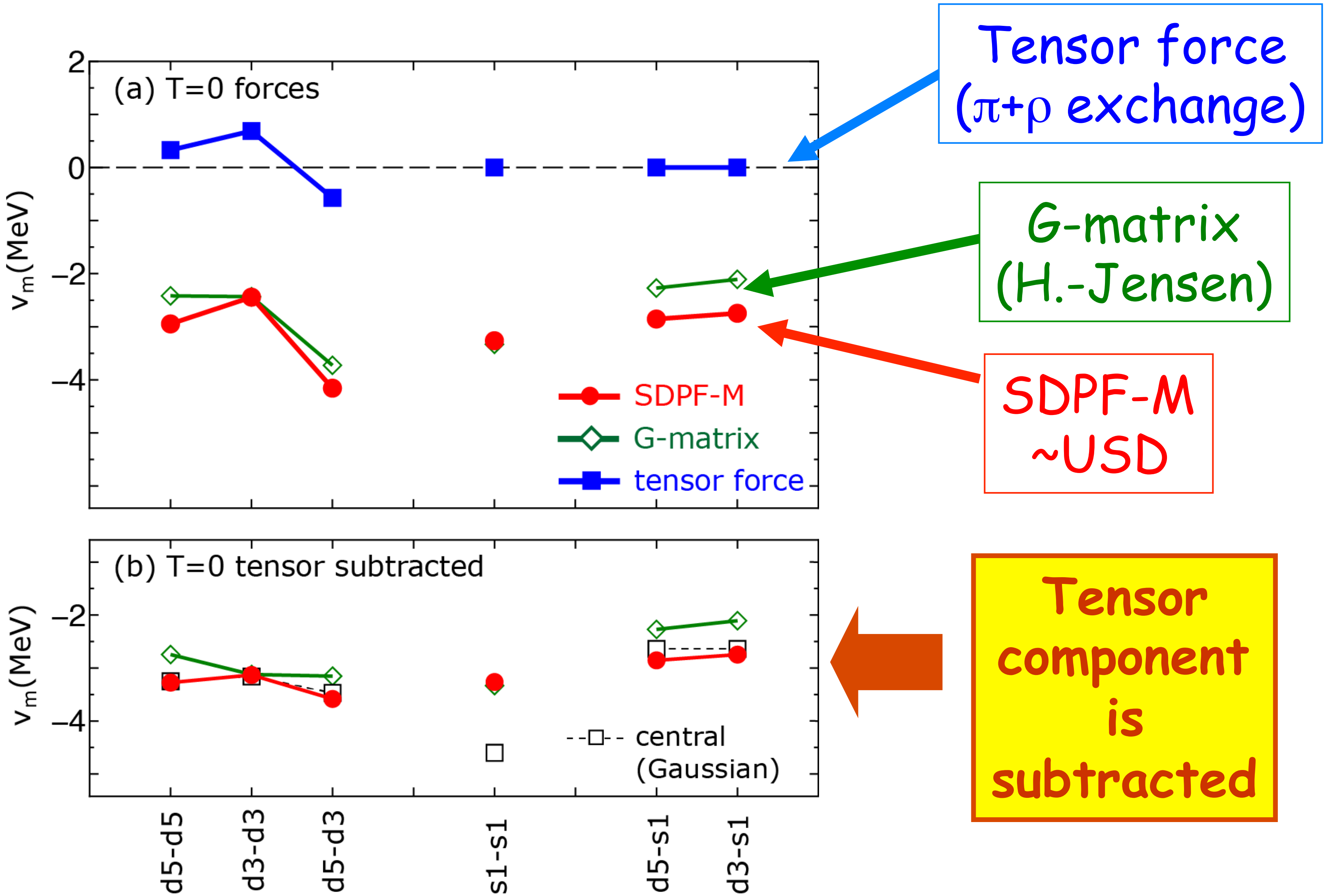
(S,T) factor	(0,0)	(1,0)	(0,1)	(1,1)
relative strength	1	1	0.6	-0.8

Can we explain the difference between f-f/p-p and f-p ?

T=0 monopole interactions in the pf shell



T=0 monopole interactions in the sd shell



The tensor part of the effective *NN* interaction for valence nucleons is similar to the bare tensor force.

bare tensor force : $\pi + \rho$ meson exchange

S. Weinberg,
PLB 251, 288 (1990)

Central force: strongly renormalized

tive potential gives a local coordinate-space two-nucleon potential:

$$V_{2\text{-nucleon}} = 2(C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta^3(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

$$-\left(\frac{2g_A}{F_\pi}\right)^2 (\boldsymbol{t}_1 \cdot \boldsymbol{t}_2)(\boldsymbol{\sigma}_1 \cdot \nabla_1)(\boldsymbol{\sigma}_2 \cdot \nabla_2) Y(|\boldsymbol{x}_1 - \boldsymbol{x}_2|)$$

$$-(1' \leftrightarrow 2'),$$

Tensor force is explicit

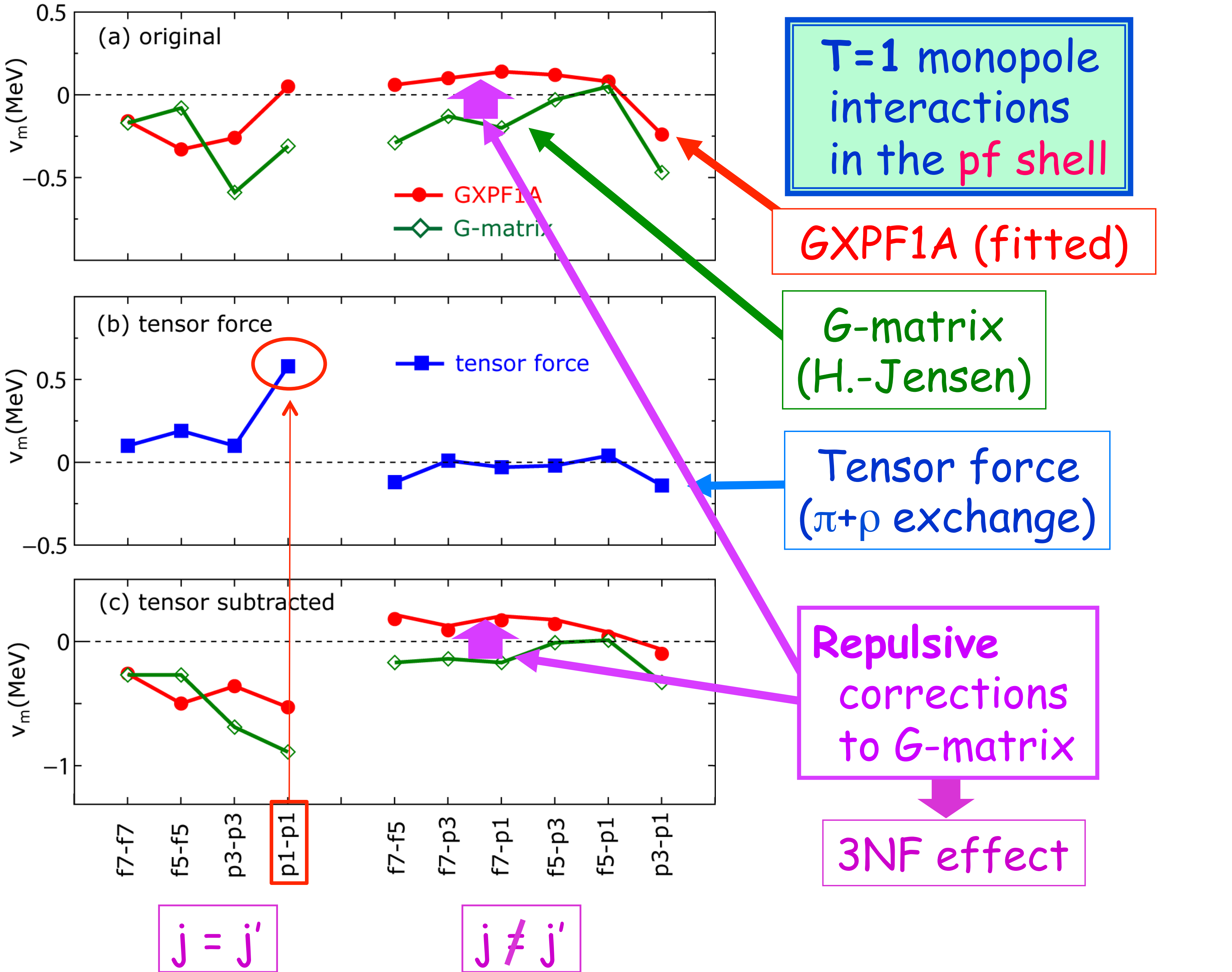
In nuclei

→ finite range

→ $\pi + \rho$ exchange

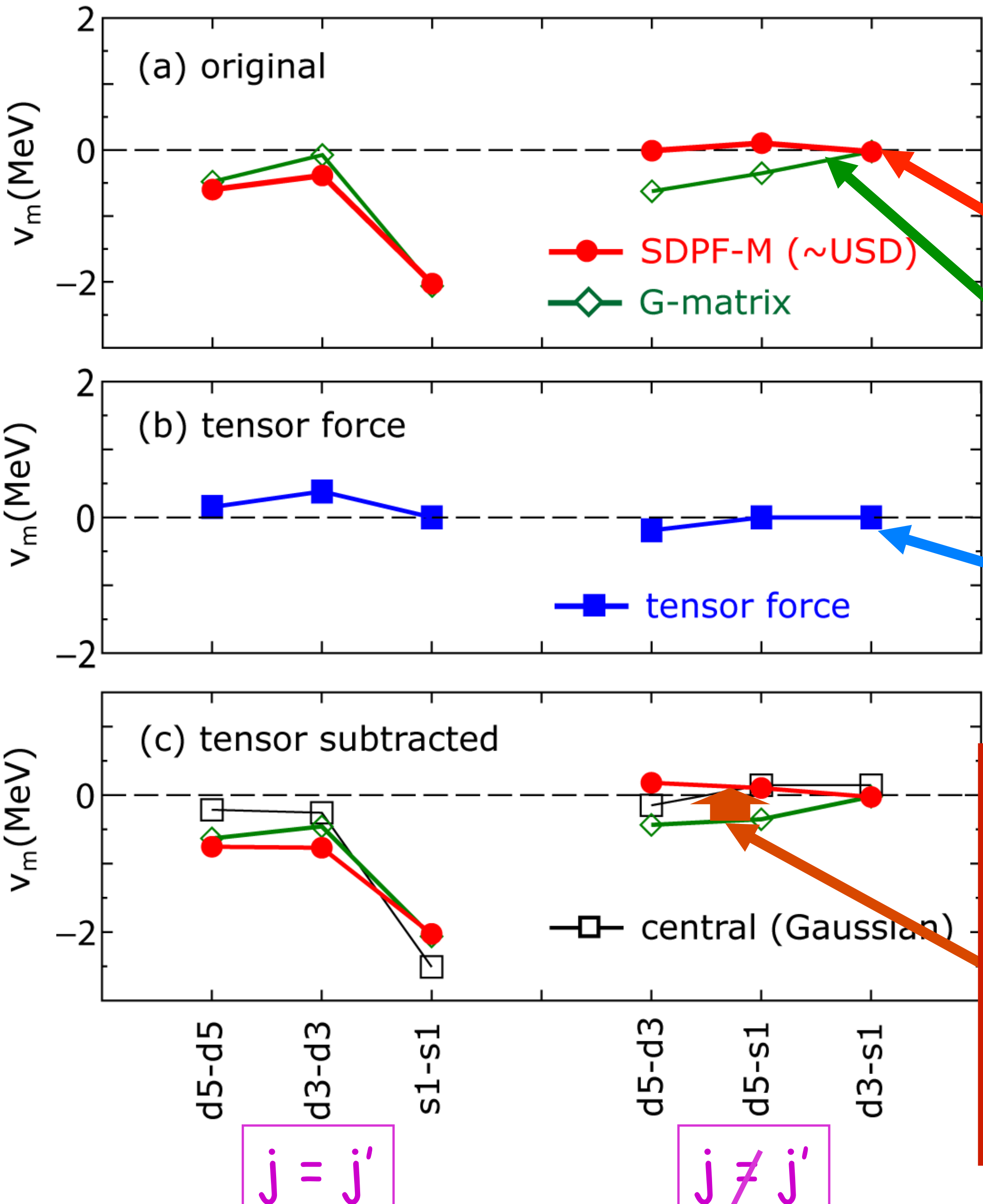
→ Chiral Perturbation of QCD

T=1 monopole interaction



T=1 monopole interactions in the pf shell
(a) original
GXPF1A
G-matrix
(b) tensor force
tensor force
(c) tensor subtracted
v_m(MeV)
0.5
0
−0.5
−1
f7-f7
f5-f5
p3-p3
p1-p1
f7-f5
f7-p3
f7-p1
f5-p3
f5-p1
p3-p1
GXPF1A (fitted)
G-matrix (H.-Jensen)
Tensor force (π+ρ exchange)
Repulsive corrections to G-matrix
3NF effect
j = j'
j ≠ j'

T=1 monopole interactions in the sd shell



SDPF-M (~USD)

G-matrix (H.-Jensen)

Tensor force ($\pi+\rho$ exchange)

Basic scale
~ 1/10 of T=0

Repulsive corrections to G-matrix (effect of 3-body force)
discussed later

Let's come back to Island of Inversion

with VMU interaction

Anomaly in levels (deformation)

β-DECAY SCHEMES OF VERY NEUTRON-RICH SODIUM ISOTOPES AND THEIR DESCENDANTS

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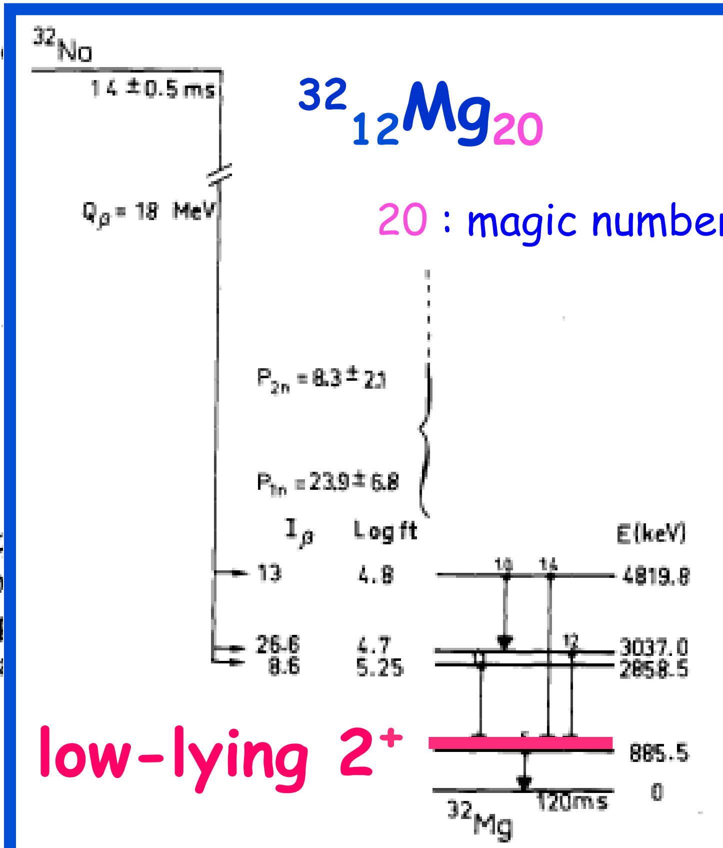
Laboratoire René Bernas and CERN, Division EP, CH-121

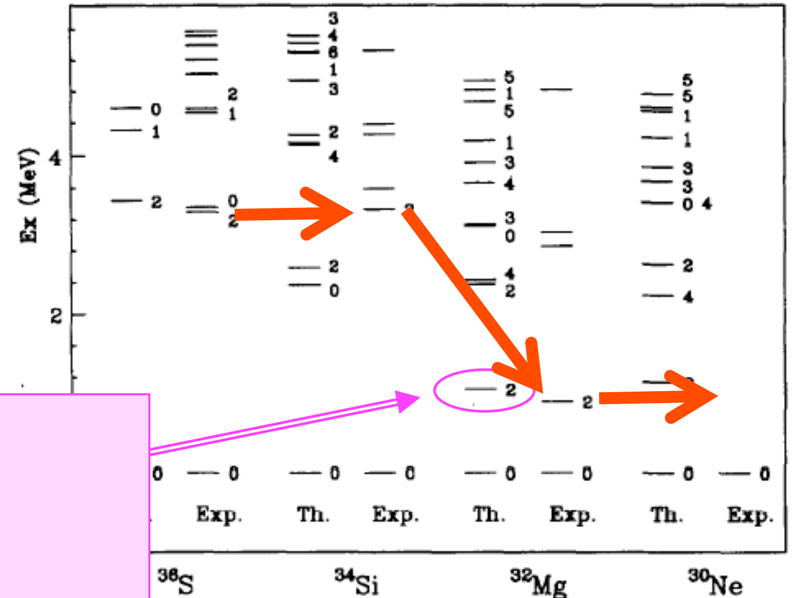
Received 6 February 1984

Abstract: The γ-activities from the β-decay of Na isotopes up
fragmentation and analysed through mass-spectrometry technic
from their Mg descendants. The I_γ intensities, the β-delayed o
and the I_β intensities are measured. Decay schemes are pro
location of the first 2^+ level of ^{32}Mg, the occurrence of a nucle

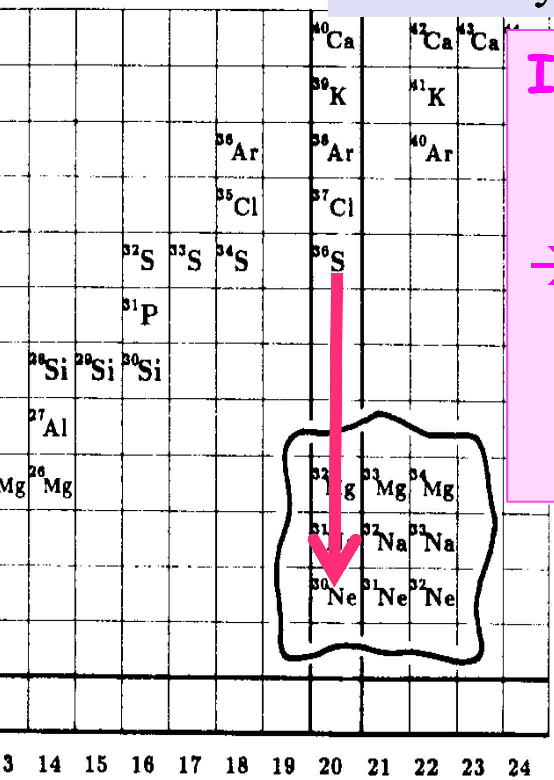


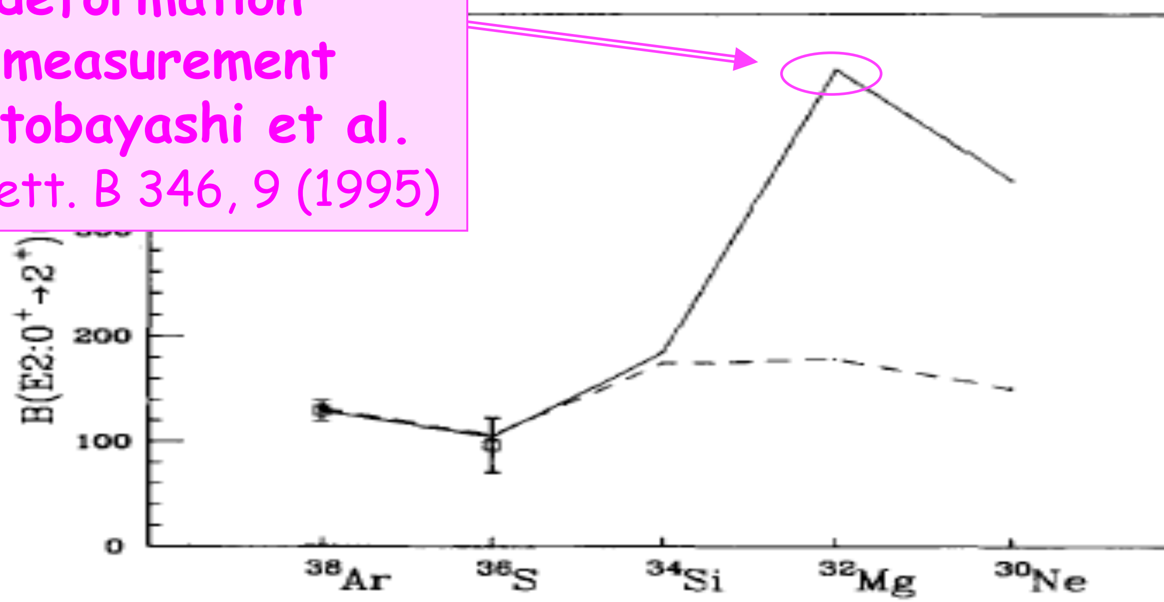
^{32}Mg case : famous example

2^+ experimental level :
D. Guillemaud-Mueller et al.
Nucl. Phys. A426, 37 (1984)



Intruder states
2p2h excitation
from sd shell
→ large deformation
B(E2) measurement
by Motobayashi et al.
Phys. Lett. B 346, 9 (1995)

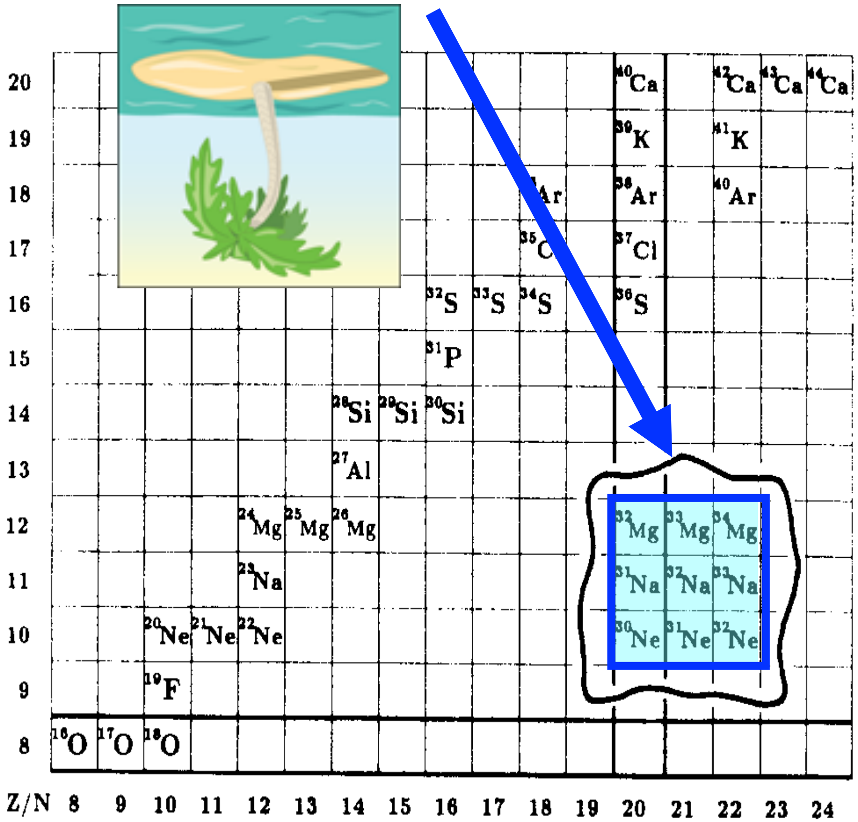




Fukunishi et al.,
Phys. Lett. 296B, 279 (1992)

← stable exotic →

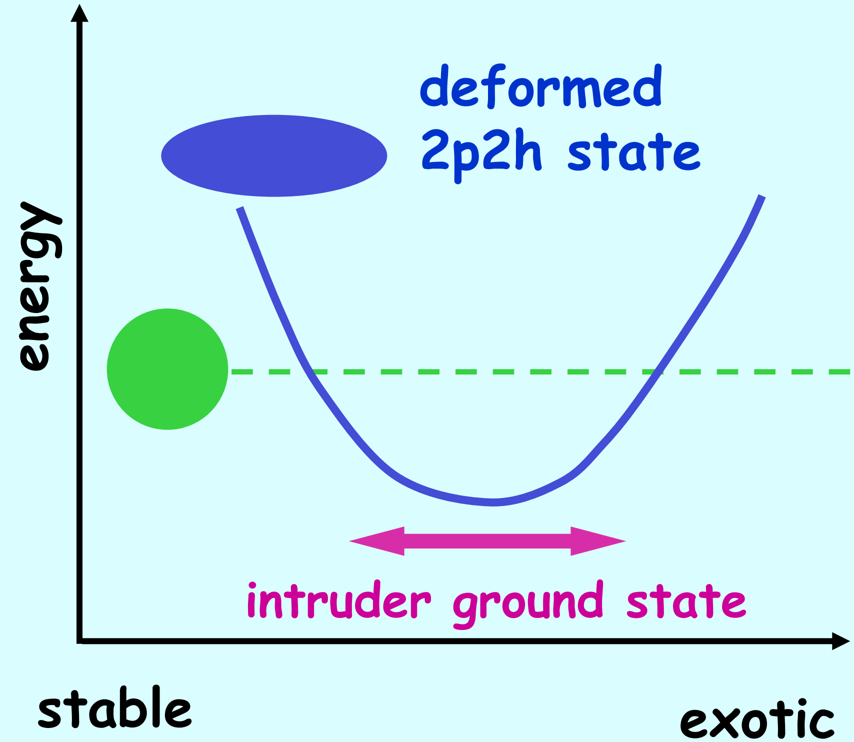
Island of Inversion

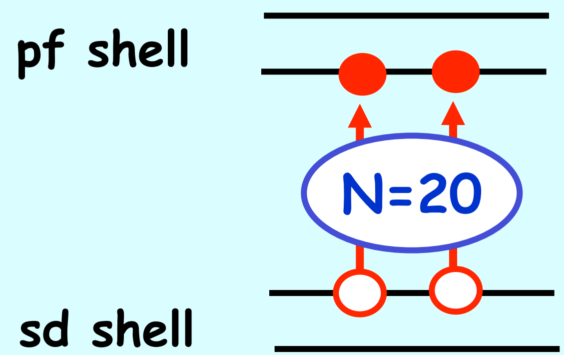


9 nuclei:
Ne, Na, Mg with N=20-22

Phys. Rev. C 41, 1147 (1990), Warburton, Becker and Brown

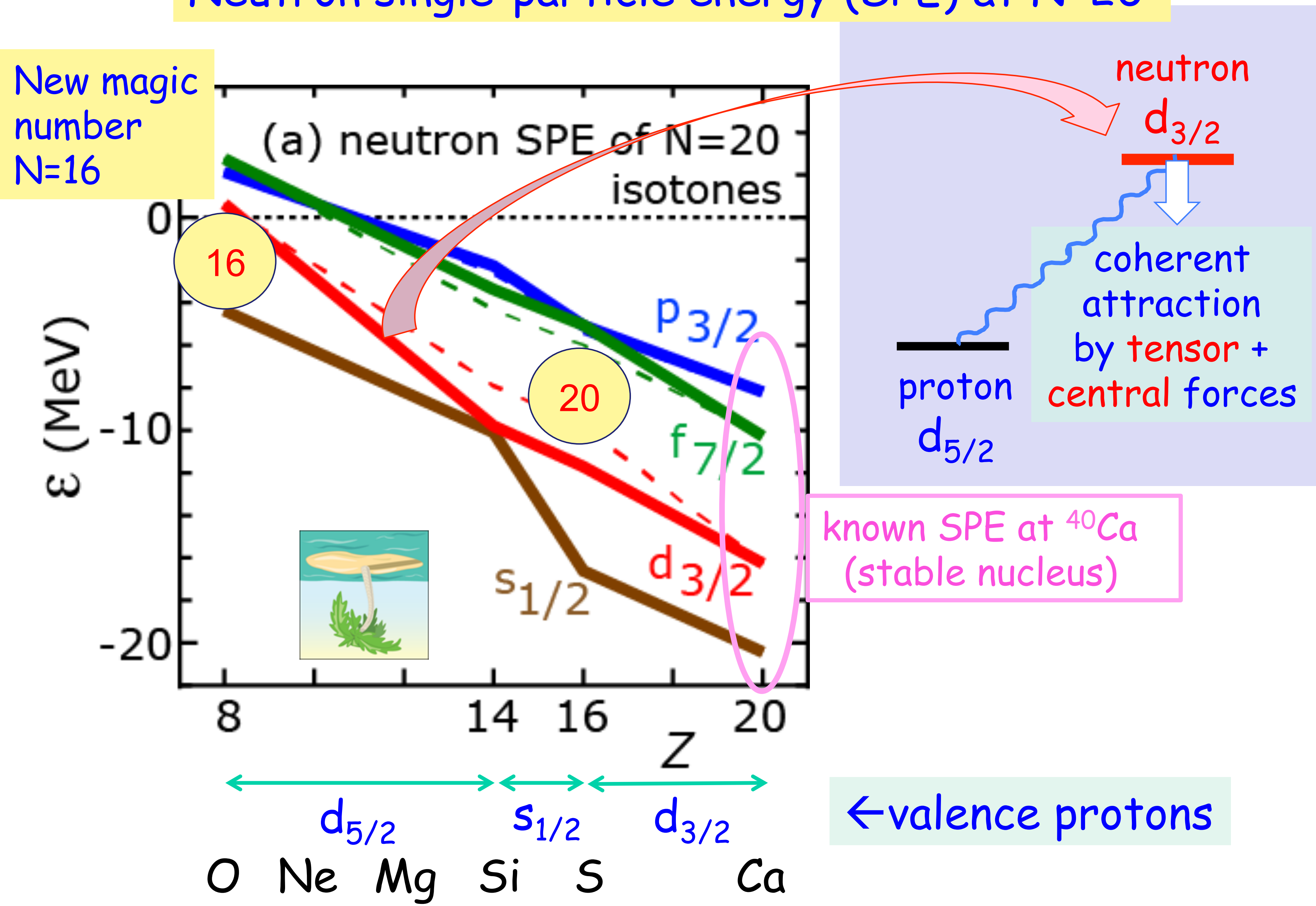
Conventional picture was





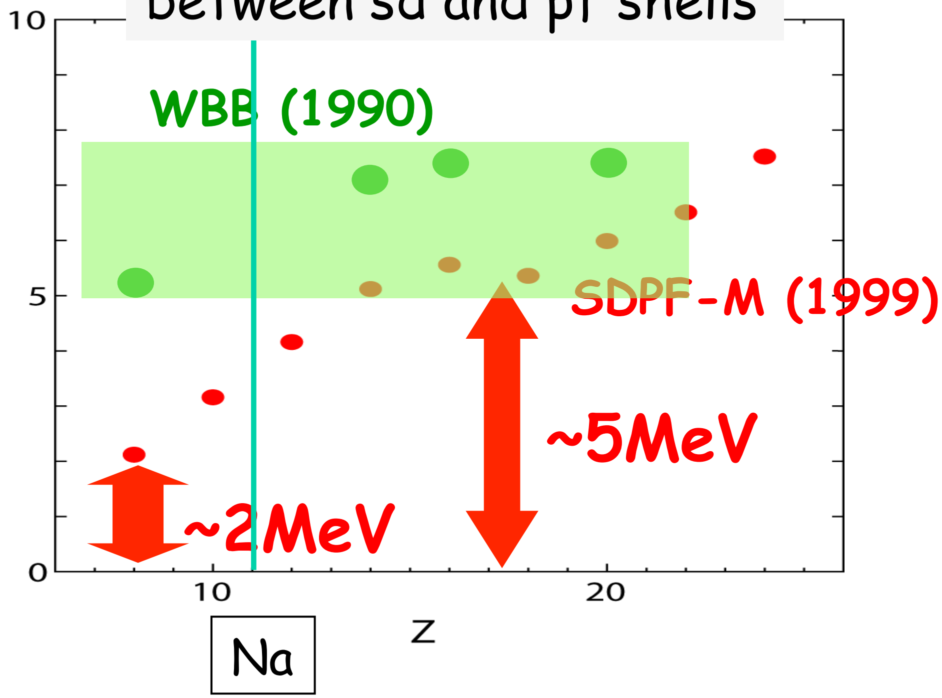
gap ~ constant

Neutron single-particle energy (SPE) at N=20



Shell gap evolves rather than staying constant

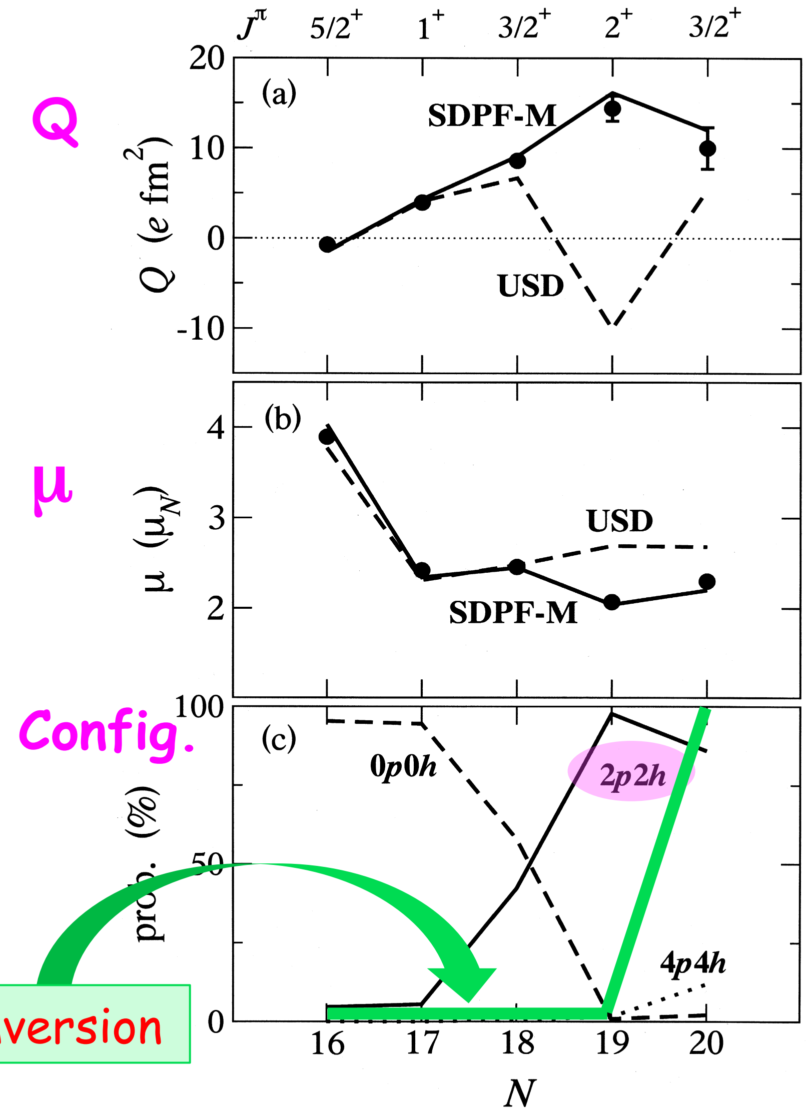
N=20 gap
between sd and pf shells



Phys. Rev. C 41, 1147 (1990),
Warburton, Becker and Brown

Na isotopes

Phys. Rev. C 70, 044307 (2004),
Y. Utsuno et al.

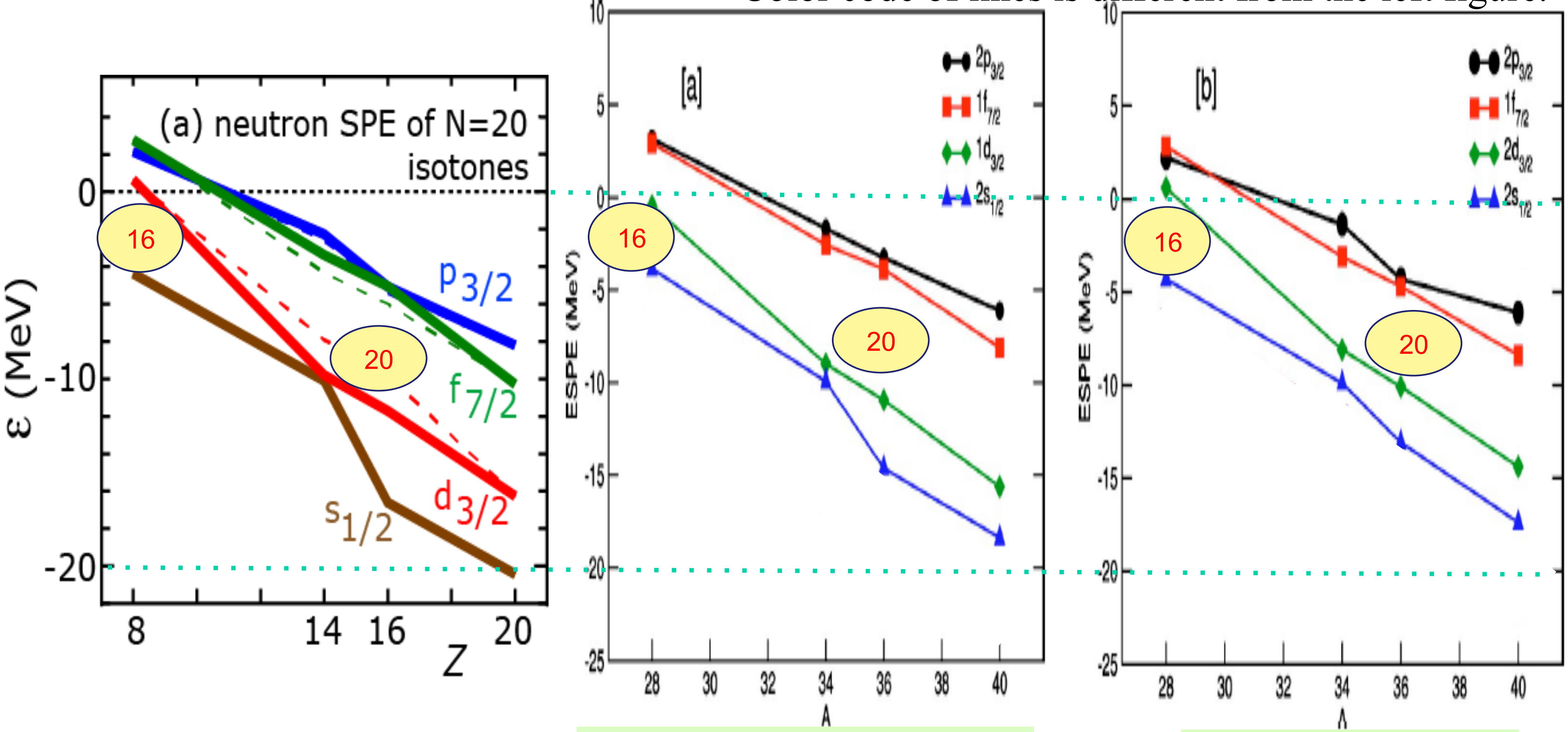


Older picture of Island of Inversion

Comparison to shell-model interactions

Shell-model interactions comprised of realistic forces show similar results, even if they have been constructed without knowing relevant mechanisms.

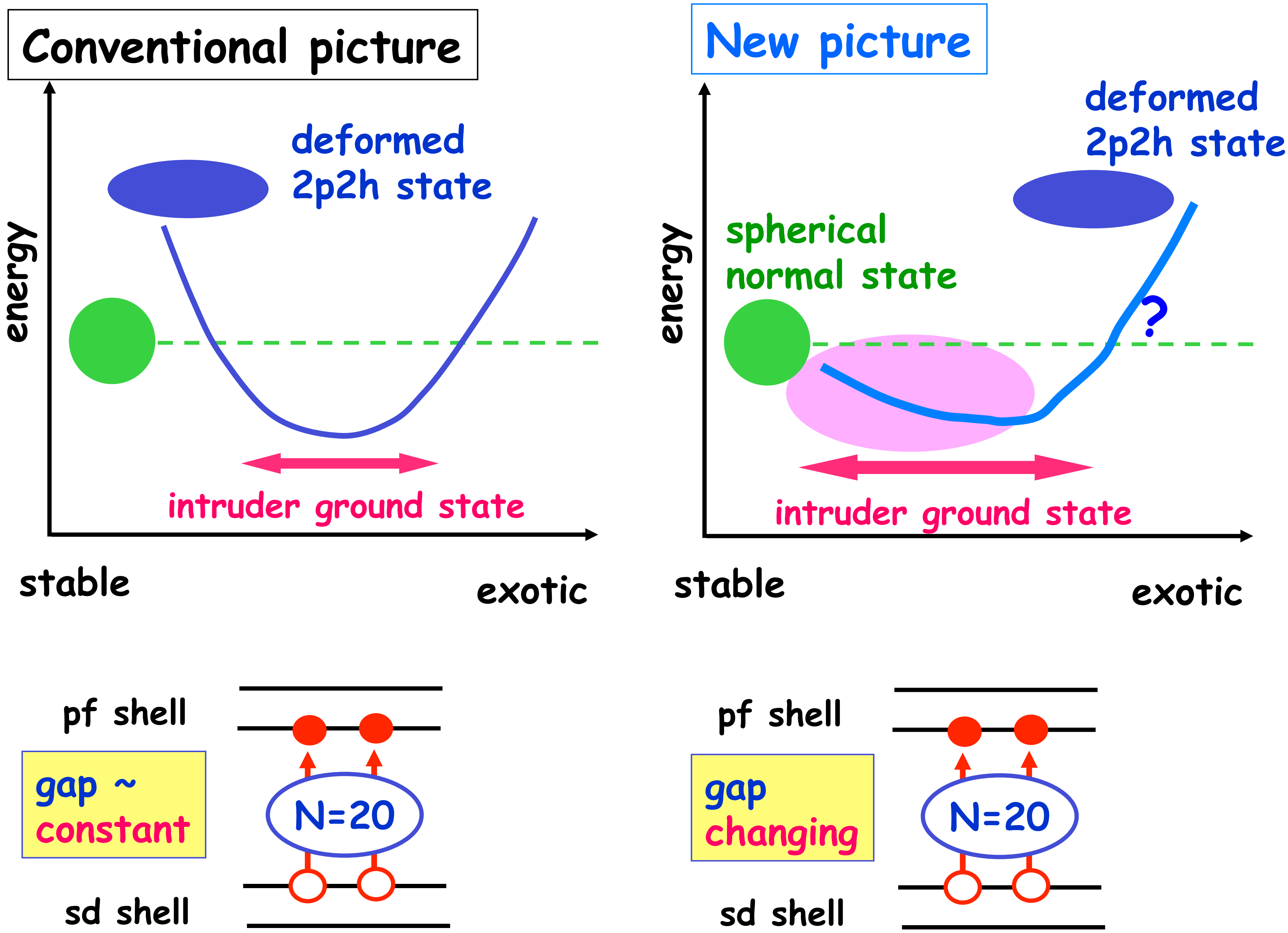
Color code of lines is different from the left figure.



Strasbourg SDPF-NR

Tokyo sdpf-M

Based on Fig 41, Caurier *et al.* RMP 77, 427 (2005)


Conventional picture
energy
deformed
2p2h state
intruder ground state
stable
exotic
pf shell
gap ~
constant
N=20
sd shell
New picture
energy
deformed
2p2h state
spherical
normal state
?
intruder ground state
stable
exotic
pf shell
gap
changing
N=20
sd shell

What is the boundary (shape) of the Island of Inversion ?

- Are there clear boundaries in all directions ?

- Is the Island really like the square ?

Which type of boundaries ?

Shallow
(diffuse & extended)

Steep (sharp)

Straight lines

Island of Inversion looks like ...

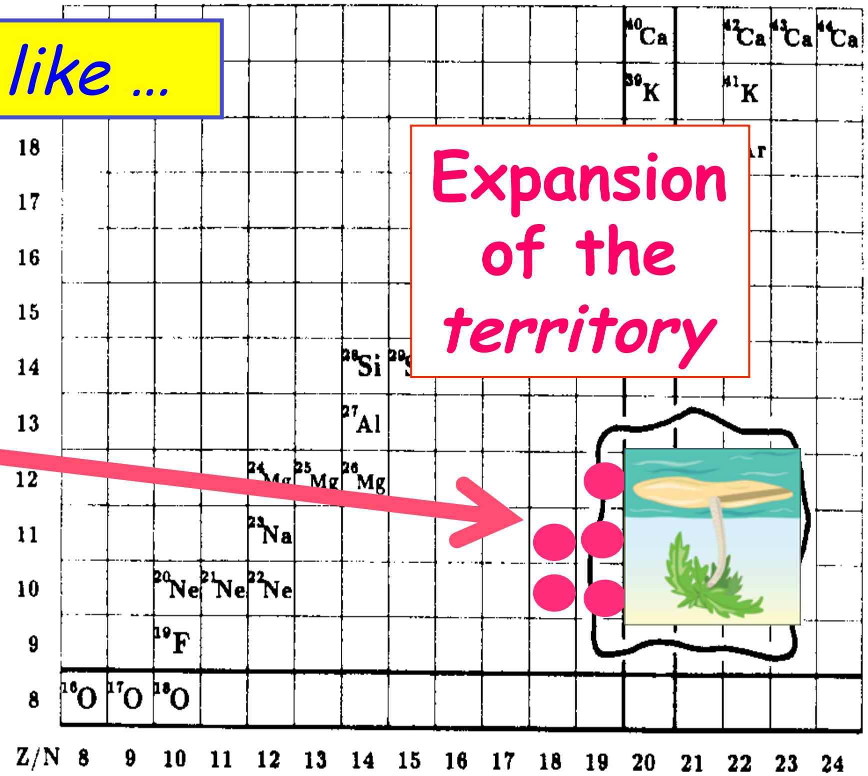
Experimental evidences :

Neyens *et al.* 2005 **Mg**

Tripathi *et al.* 2005 **Na**

Dombradi *et al.* 2006 **Ne**

Terry *et al.* 2007 **Ne**

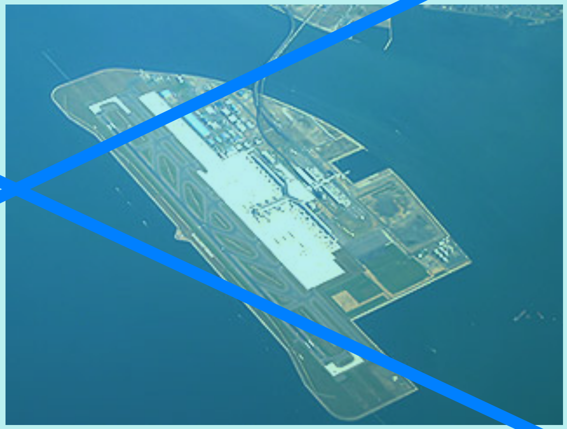


Shallow (diffuse & extended)

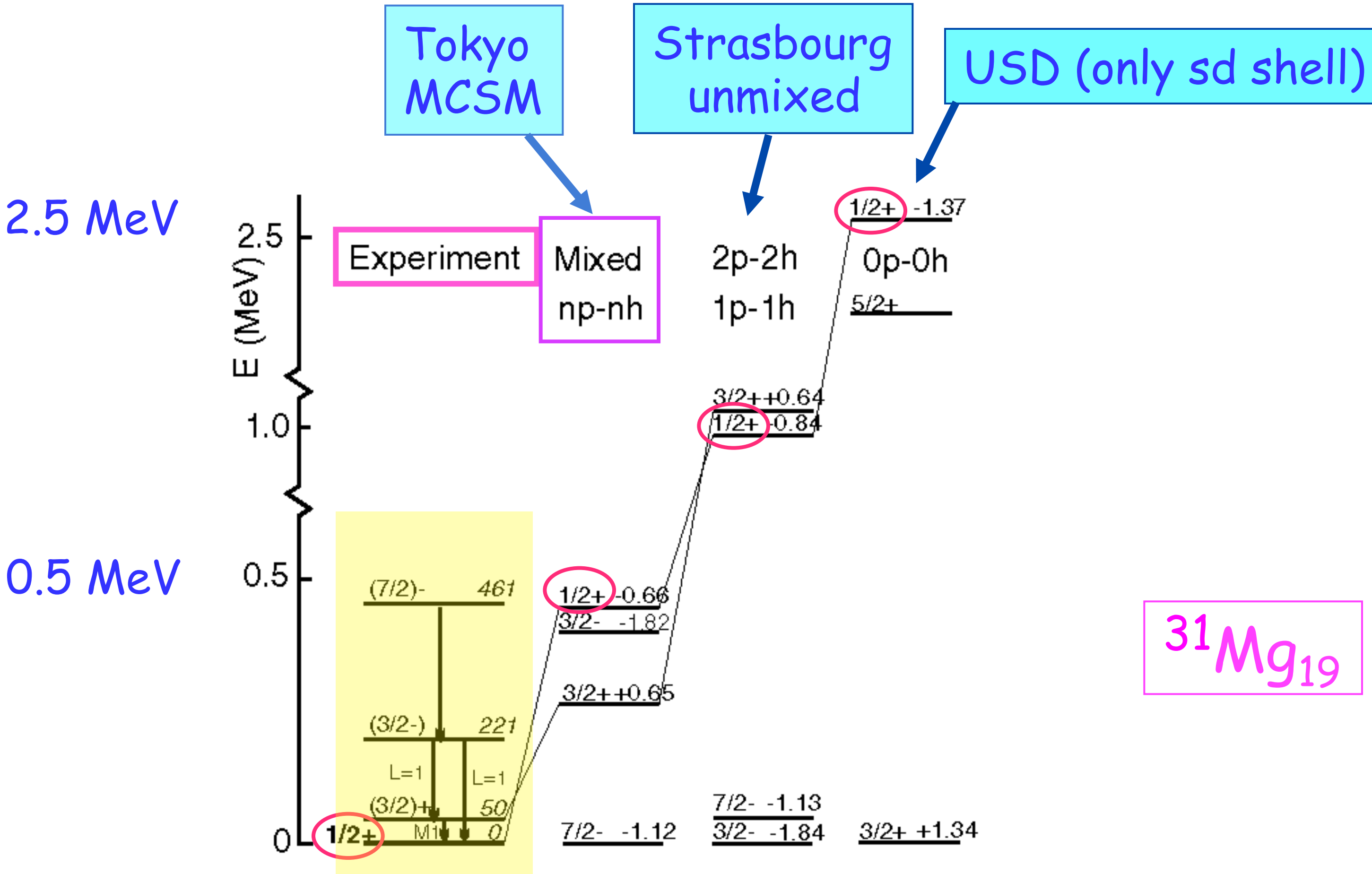
Island of Inversion ~ tropical paradise

Steep (sharp)

Straight lines



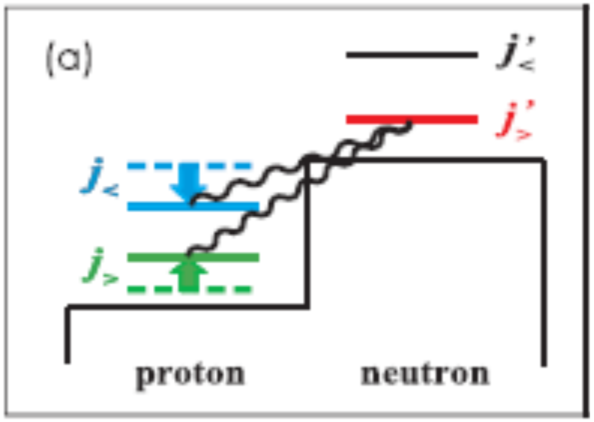
Phys. Rev. Lett. 94, 022501 (2005), G. Neyens, et al.



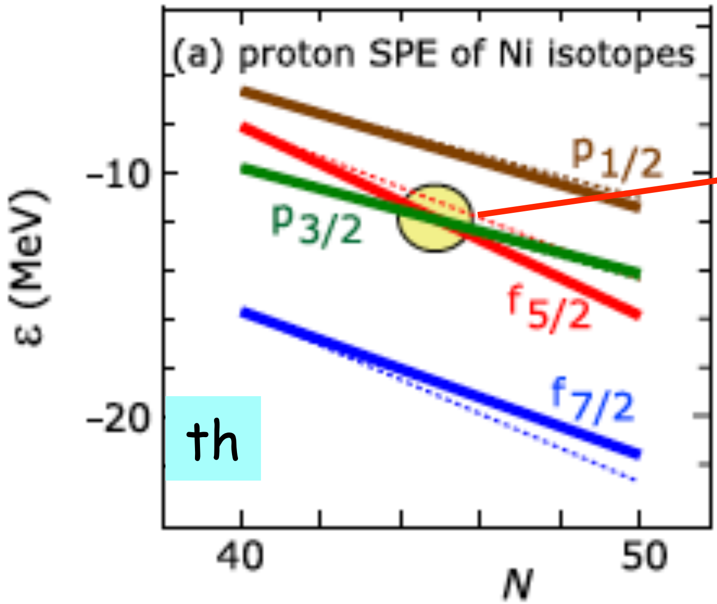
$^{31}Mg_{19}$

FIG. 3. Partial experimental level scheme of ^{31}Mg, with new spin/parity assignments, compared to various shell-model calculations (see text for details). The magnetic moments of theoretical levels are mentioned on the right (units μ_N).

An example of experimental test on the shell evolution

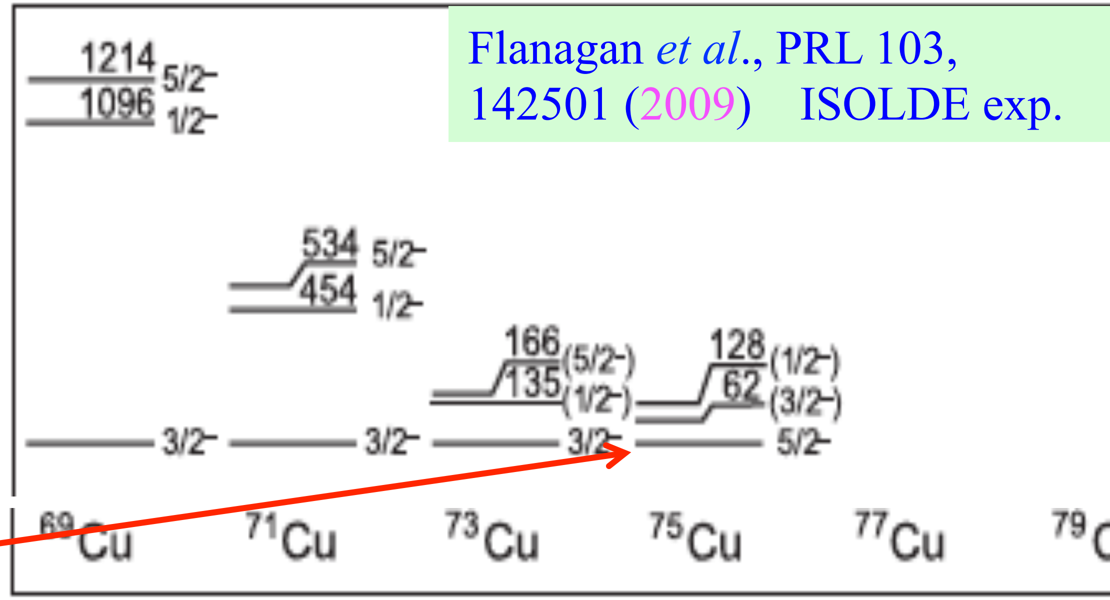


TO, Suzuki, *et al.*
PRL 95, 232502 (2005)



Proton $f_{5/2}$-$p_{3/2}$ inversion in Cu due to neutron occupancy of $g_{9/2}$

Flanagan *et al.*, PRL 103, 142501 (2009) ISOLDE exp.



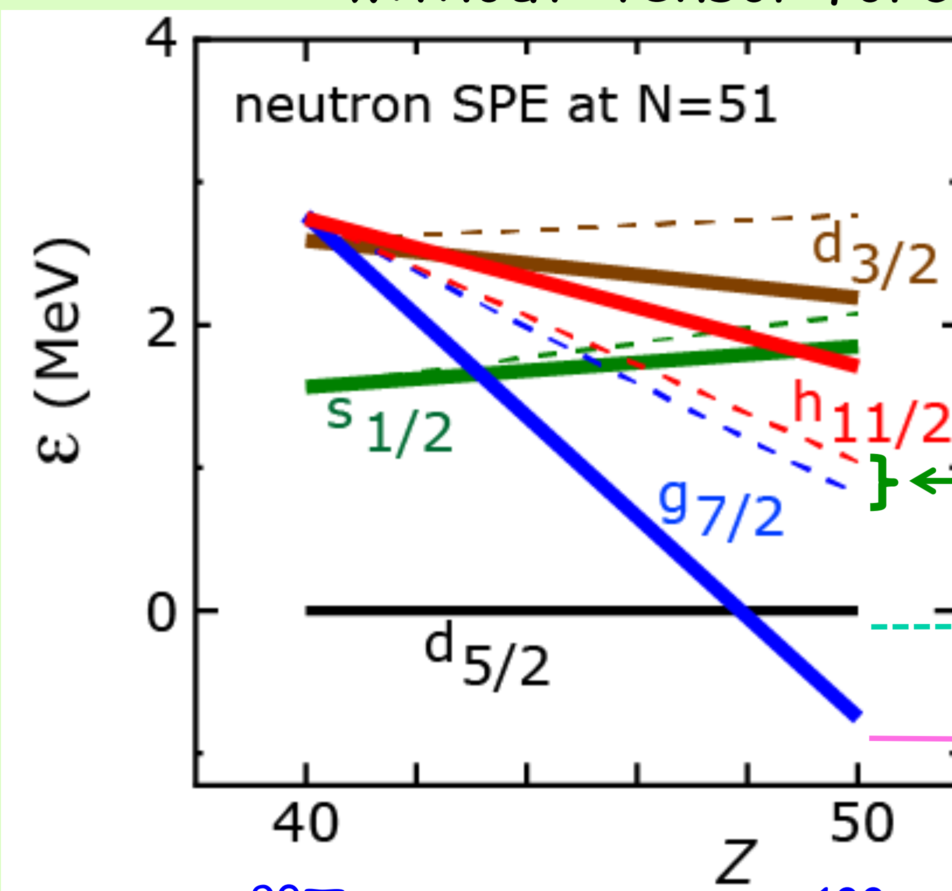
Franchoo *et al.*, PRC 64, 054308 (2001)
"level scheme … newly established for ^{71,73}Cu"
"… unexpected and sharp lowering of the $\pi f_{5/2}$ orbital"
"… ascribed to the monopole term of the residual int. .."

→ Finally a clean example of tensor-force driven shell evolution

Evolution of neutron shell from Z=40 to 50

—— with tensor force

- - - - - without tensor force



^{90}Zr (from exp.)

^{100}Sn (prediction)

^{101}Sn

open question of the ordering

Seweryniak et al., PRL 99, 022504 (2007)

Darby et al., PRL 105, 162502 (2010)

if no tensor force, $h_{11/2}$ & $g_{7/2}$ ~ degenerate

full tensor force

Central only: Fedderman-Pittel (1977)

New magic number 34 ?

Eigenvalues of HO potential

Magic numbers Mayer and Jensen (1949)

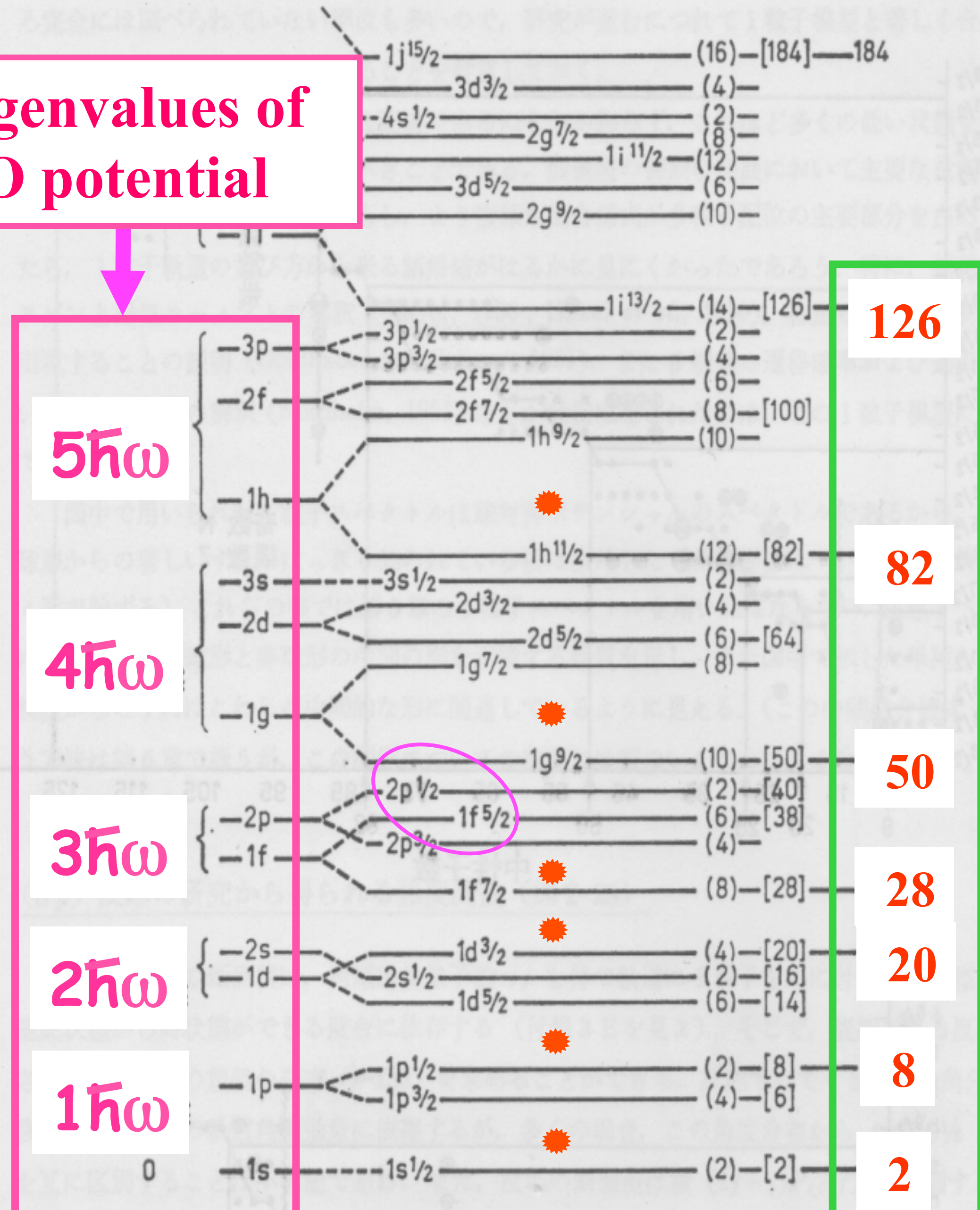
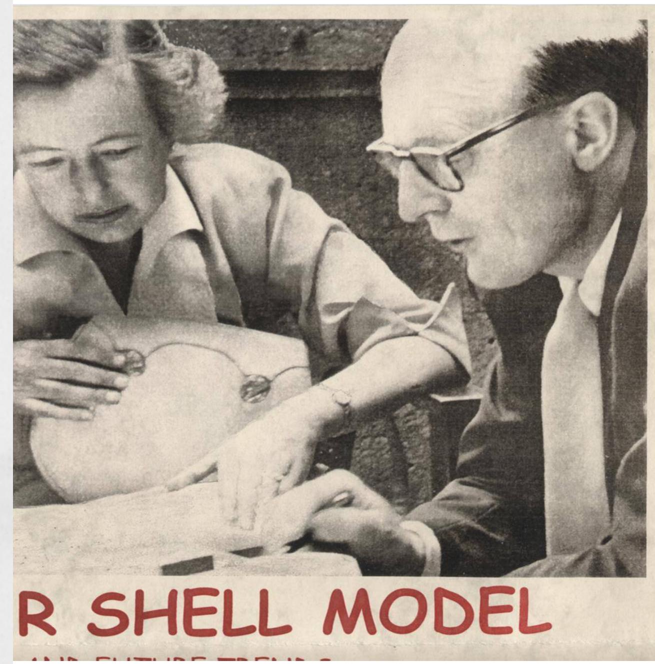


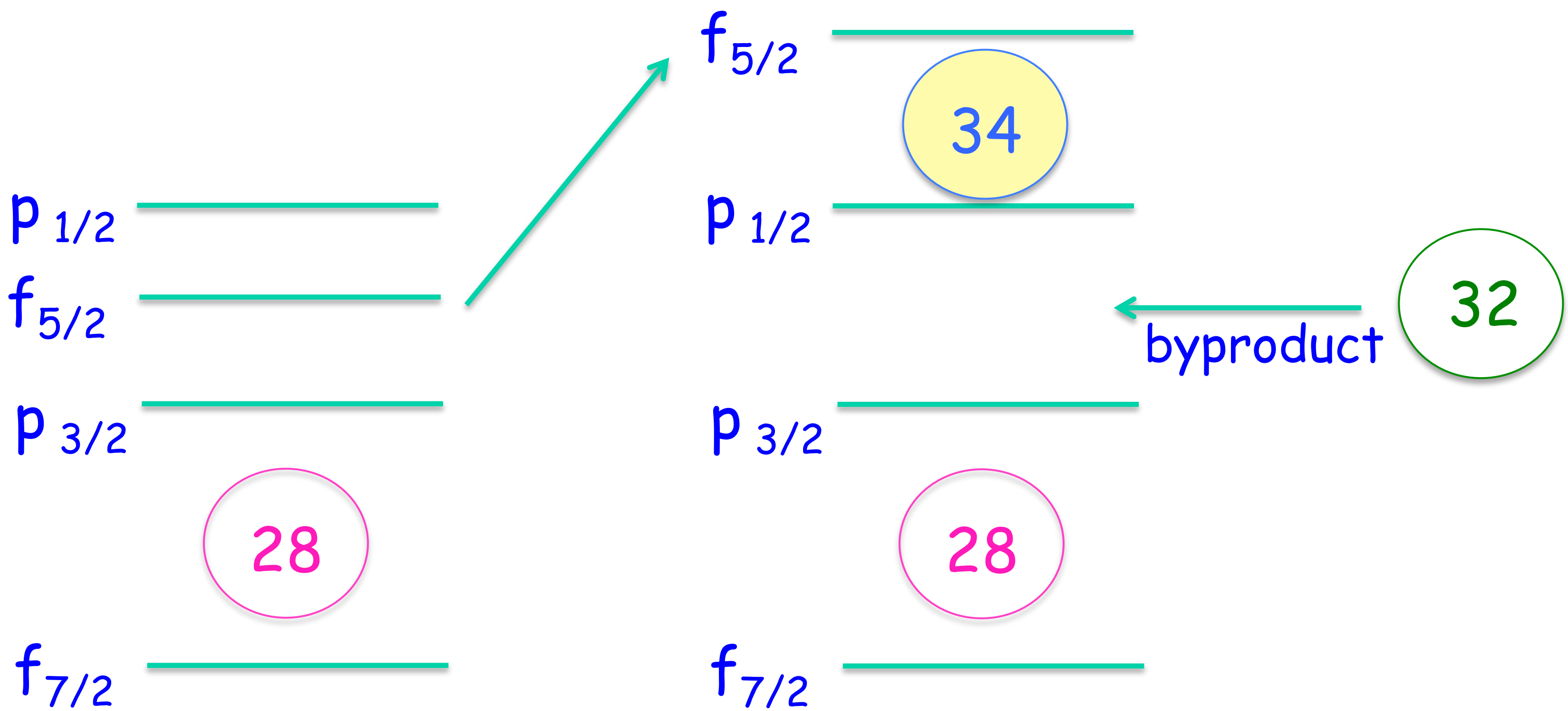
図2-23 1粒子軌道の順序. 図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった.



Basic picture

shell structure
for neutrons
in Ni isotopes
($f_{7/2}$ fully occupied)

N=34 magic number may appear
if proton $f_{7/2}$ becomes vacant (Ca)
($f_{5/2}$ becomes less bound)



N=34 magic number and the shell evolution due to proton-neutron interaction

neutron $f_{5/2}$ - $p_{1/2}$ spacing increases by ~0.5 MeV per one-proton removal from $f_{7/2}$, where tensor and central forces works coherently and almost equally.

note : $f_{5/2} = j_<$ $f_{7/2} = j_>$

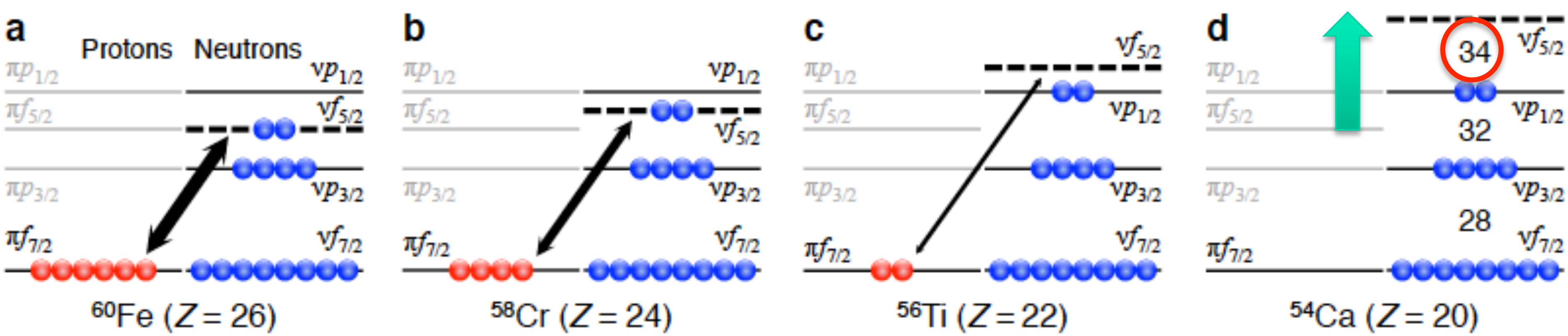


FIG. 1: **Schematic illustration highlighting the attractive interaction between the proton $\pi f_{7/2}$ and neutron $\nu f_{5/2}$ single particle orbitals for $N = 34$ isotones. a–c,** As protons are removed from the $\pi f_{7/2}$ orbital (from **a**, ^{60}Fe, through **b**, ^{58}Cr to **c**, ^{56}Ti), the strength of the π-ν interaction reduces, as represented by the decreasing width of the diagonal arrows, causing the $\nu f_{5/2}$ orbital to shift up in energy relative to the $\nu p_{3/2}$-$\nu p_{1/2}$ spin-orbit partners. Consequently, a significant shell closure presents itself at $N = 32$ in isotopes far from stability. **d,** The possibility of an additional shell closure at $N = 34$ for ^{54}Ca is presented. The $\nu f_{5/2}$ SPO is indicated as a bold-dashed line to help guide the eye.

Steppenbeck *et al.* Nature, 502, 207 (2013)

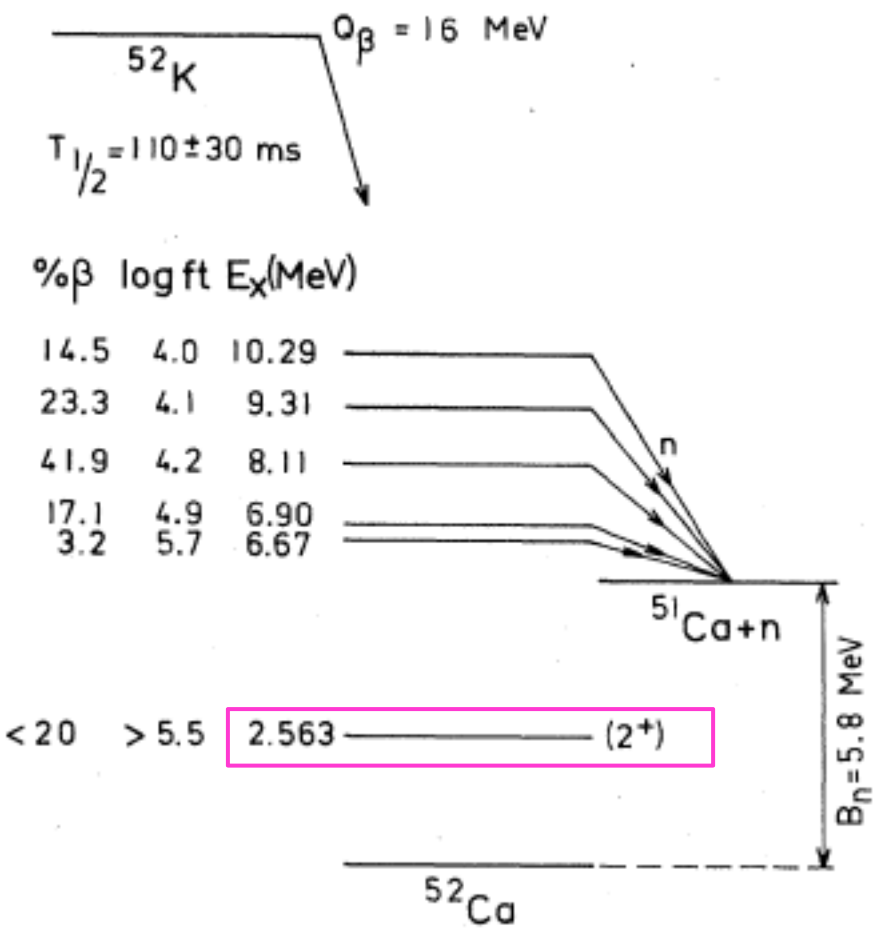


FIG. 5. Decay scheme of ^{52}K.

A large *N*=32 gap (high 2$^+$ level for ^{52}Ca) has been suggested since 1985, by Strasbourg experimental group.

ISOLDE experiment
Huck *et al*., "Beta decay of the new isotopes 52K, …" Phys. Rev. C 31, 2226 (1985).

Is there N=34 magic number ?

In comparison to N=32 magic number known experimentally for nearly 30 years.

Moving back to heavier nuclei, from the strong interaction in Fig. 1(c), we can predict other magic numbers, for instance, $N = 34$ associated with the $0f_{7/2}$-$0f_{5/2}$ interaction. In heavier nuclei, $0g_{7/2}$, $0h_{9/2}$, etc. are shifted upward in neutron-rich exotic nuclei, disturbing the magic numbers $N = 82, 126$, etc. It is of interest how the r process of nucleosynthesis is affected by it.

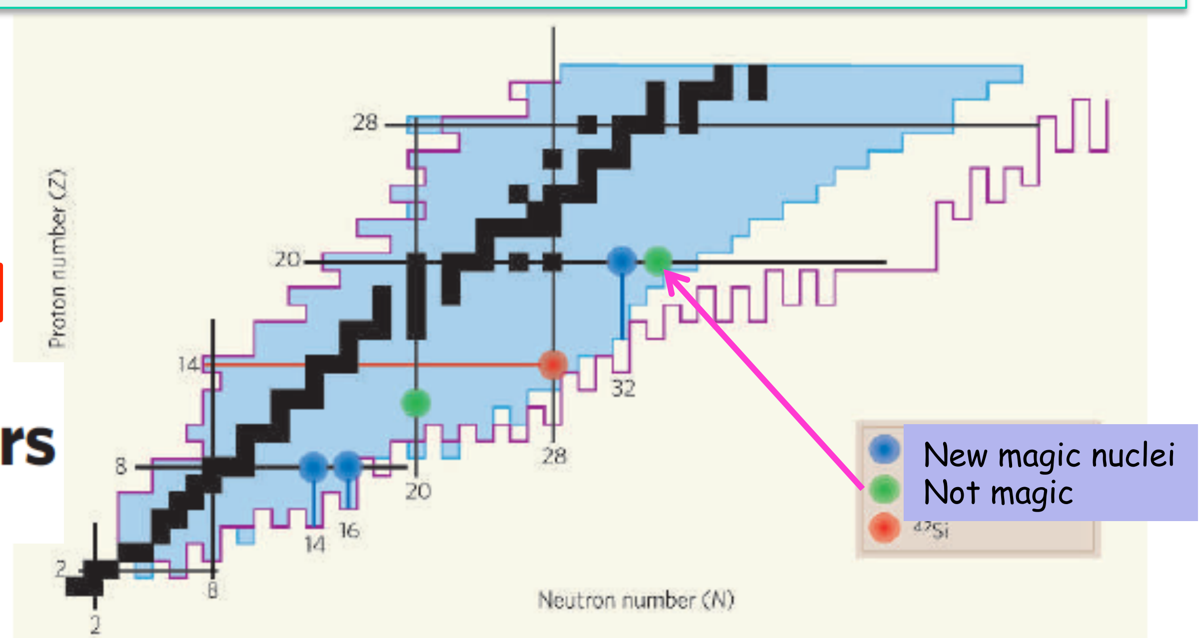
TO et al.
PRL 87 (2001)

NATURE|Vol 435|16 June 2005

NUCLEAR PHYSICS

Elusive magic numbers

Robert V. F. Janssens



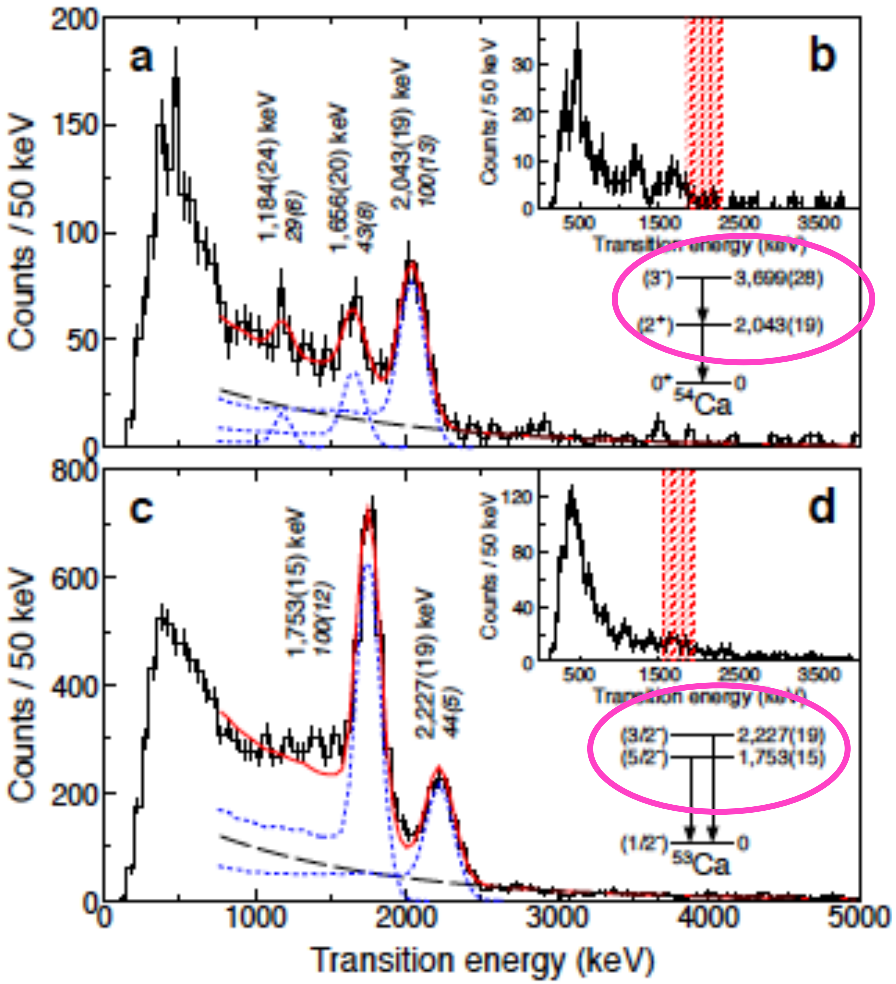
Experiment
by
RIBF

DALI-2
γ-ray detector

Neutron number 34 makes exotic calcium-54 isotopes doubly magic

PAGE 207

Experiment @ RIBF → Finally confirmed



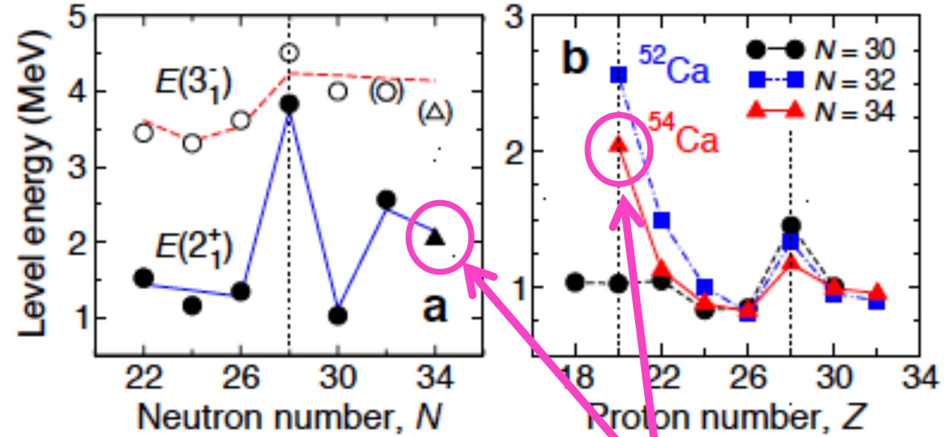


FIG. 4: **Systematics of excited-state e**[...]**even-even Ca isotopes and neighbouring n**[...]ergies of first 2^+ (closed symbols) and 3^- (open [...]els for even-even $^{42-54}$Ca isotopes [28]. The res[...]resent study are indicated by triangular marke[...]l and dashed lines are shell-model predictions of [...](3_1^-), respectively (see text for details). Tentat[...]ty assignments are enclosed by parentheses. **b,** $E(2_1^+)$ along the $N = 30$, 32 and 34 isotonic chains. The solid and dashed lines are intended to guide the eye. Vertical dotted lines represent the traditional magic numbers in both plots.

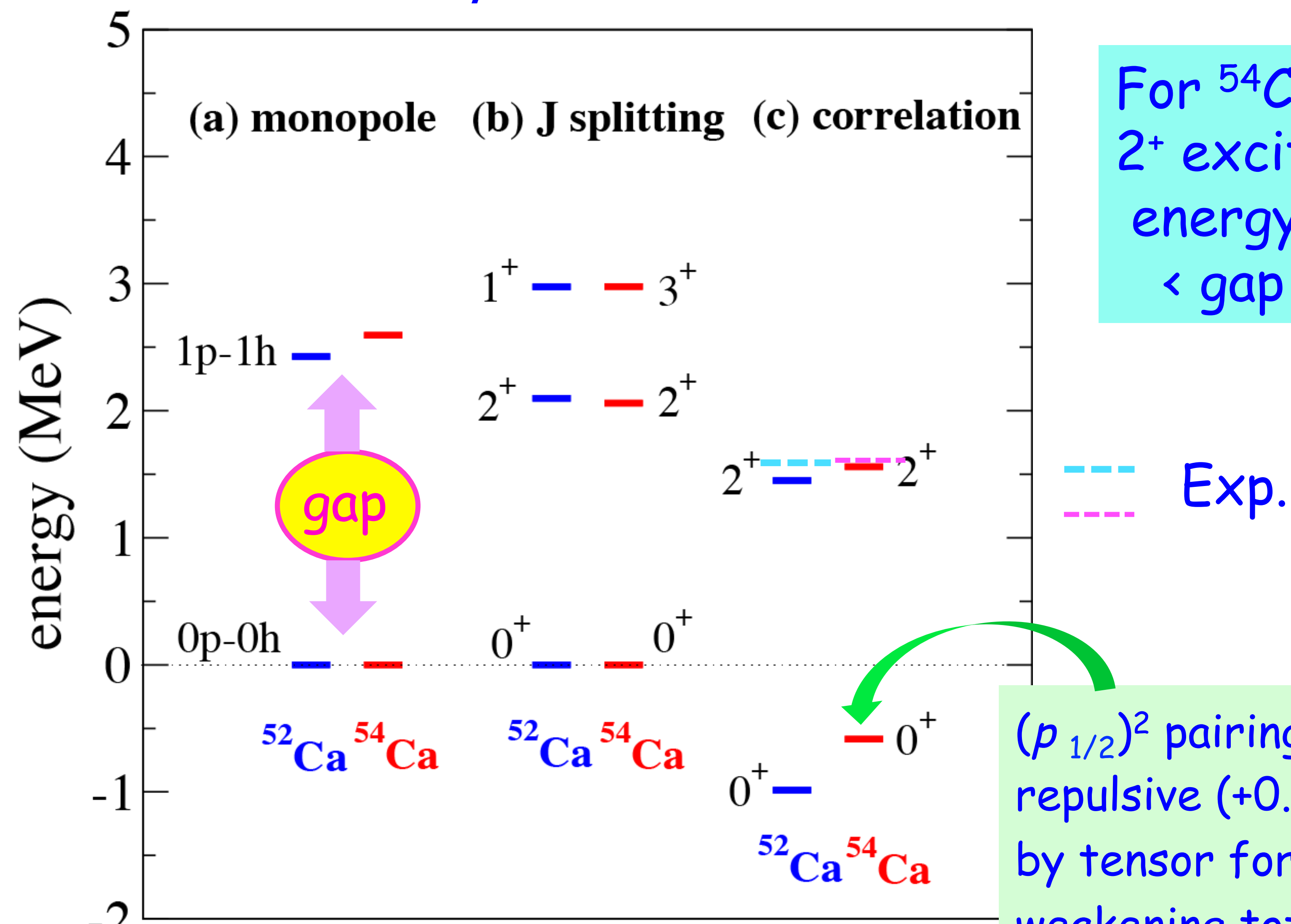
er-corrected γ-ray energy spectra. De-excitation γ rays measured in coinci-
^{54}Ca and **c,** ^{53}Ca reaction products. Peaks a

Steppenbeck *et al.* Nature, 502, 207 (2013)

ive intensities are indicated by italic fonts. The short-blue and long-black dashed

2+ energy level v.s. shell gap

Calculation by GXPF1Br interaction



For ^{54}Ca,
2^+ excitation energy
< gap energy

$(p_{1/2})^2$ pairing repulsive (+0.5 MeV) by tensor force, weakening total pairing

How can we identify "magic numbers" ?

First 2^+ level - see next page -

Nuclear force can change it keeping wave functions.

Info from wave functions

probability that the ground state is a closed shell

= "magic index" (*proposed now*)

Ni isotopes (theory predictions)

^{56}Ni 60% ^{68}Ni 53% ^{78}Ni 75%

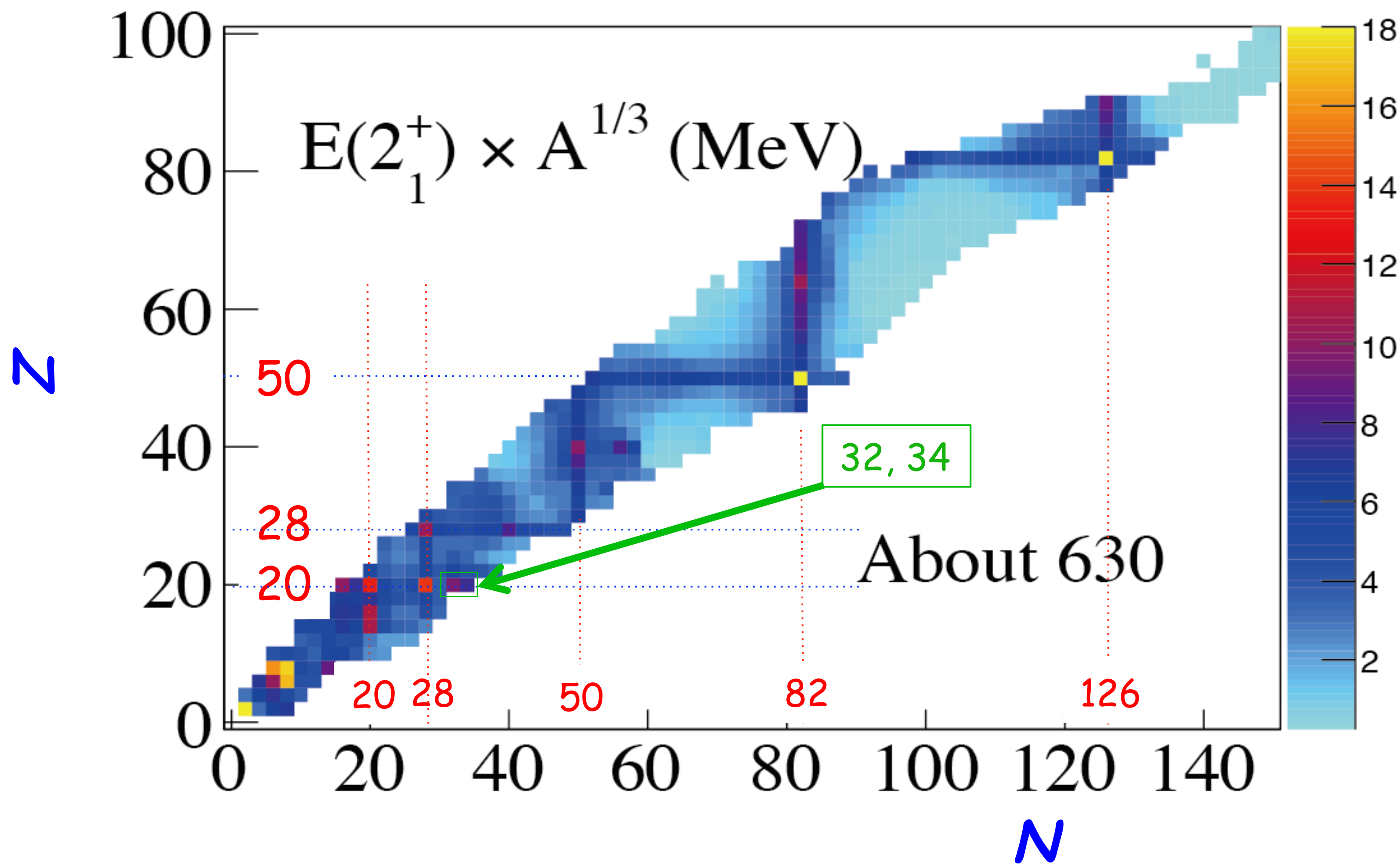
What about ^{52,54}Ca ?

Courtesy from Pieter Doornenbal

2+ levels x A 1/3

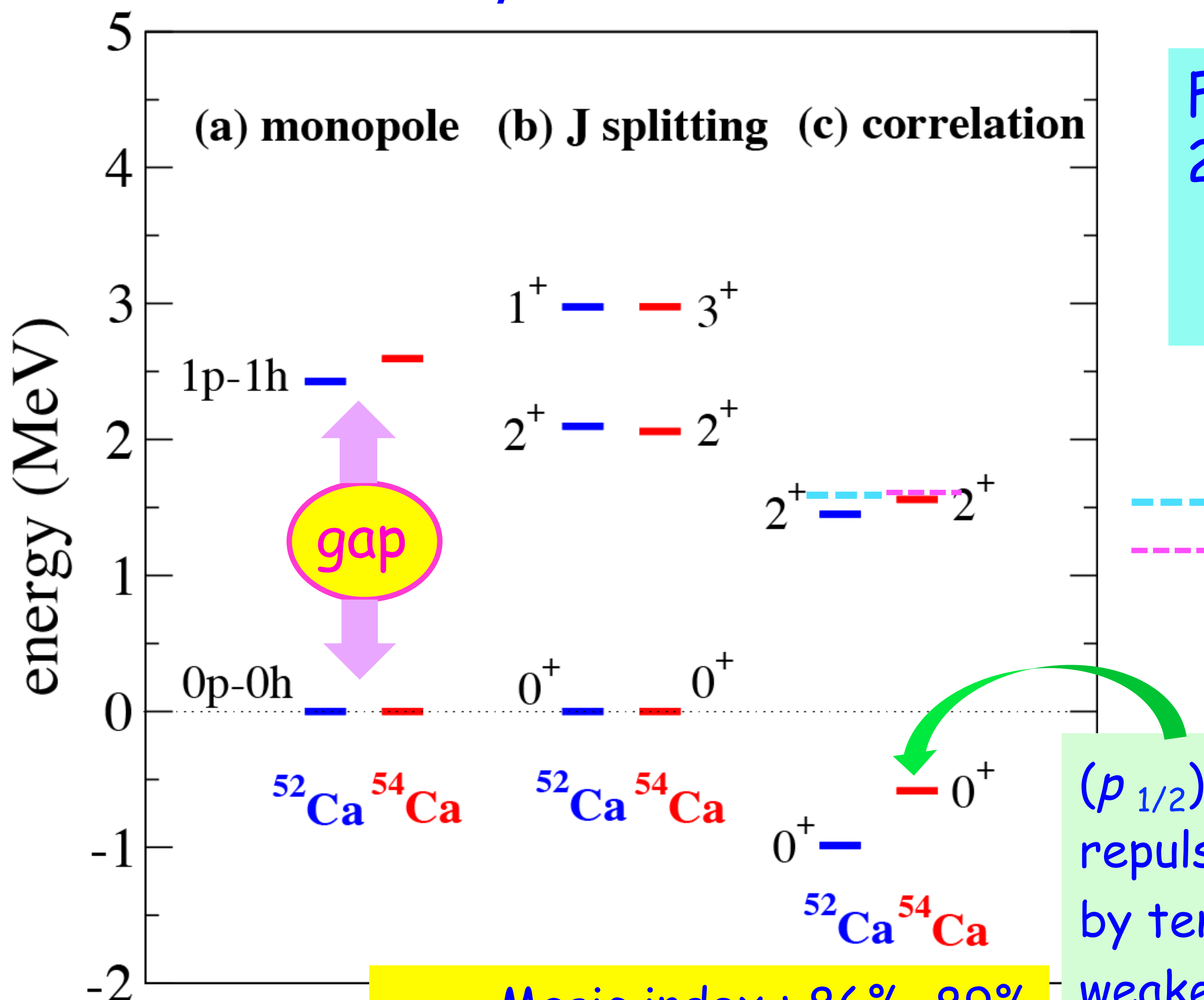
Z, N even numbers only

Red numbers : Conventional magic numbers



2^+ energy level v.s. shell gap

Calculation by GXPF1Br interaction

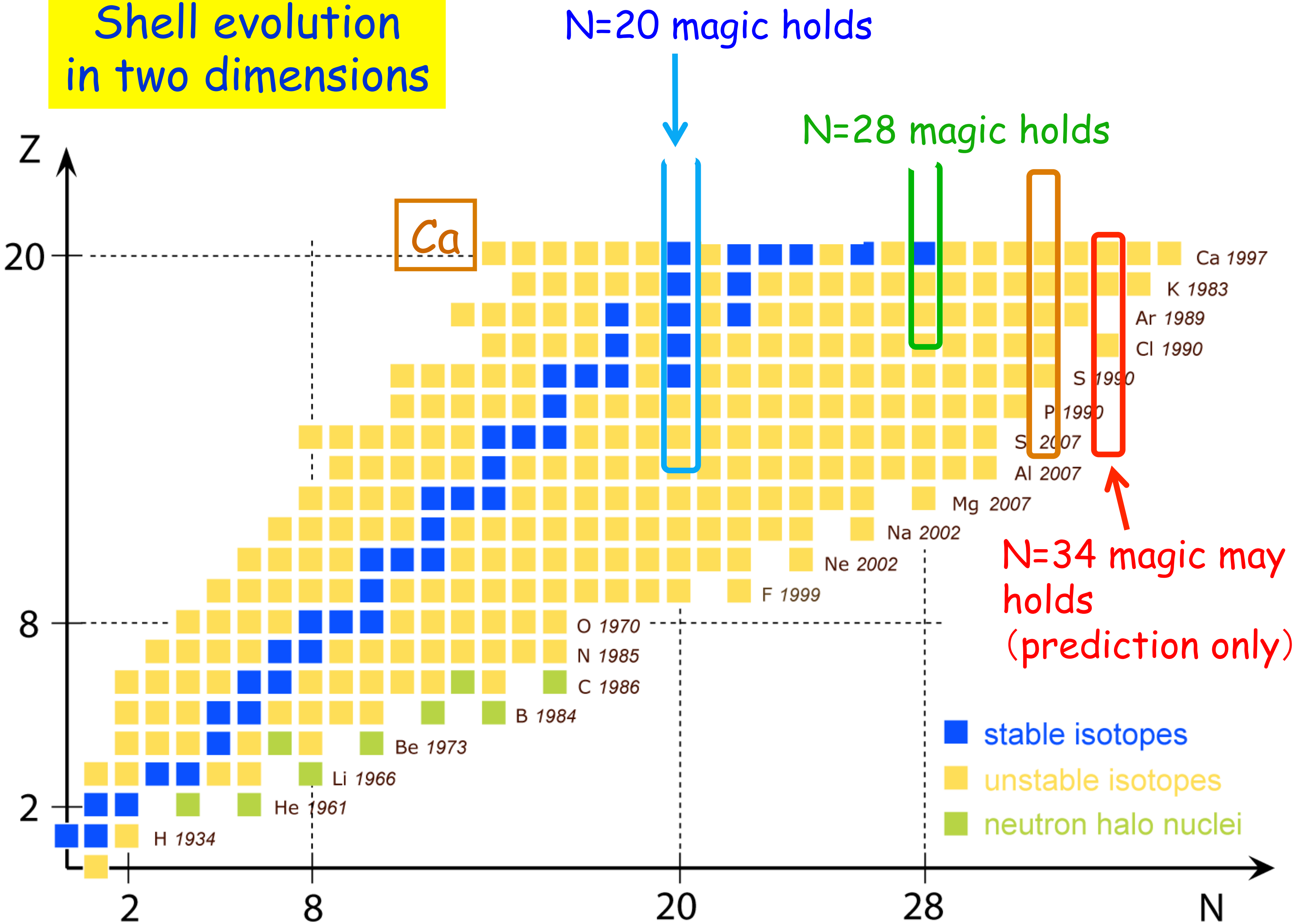


For ^{54}Ca,
2^+ excitation energy
< gap energy

$(p_{1/2})^2$ pairing repulsive (+0.5 MeV) by tensor force, weakening total pairing

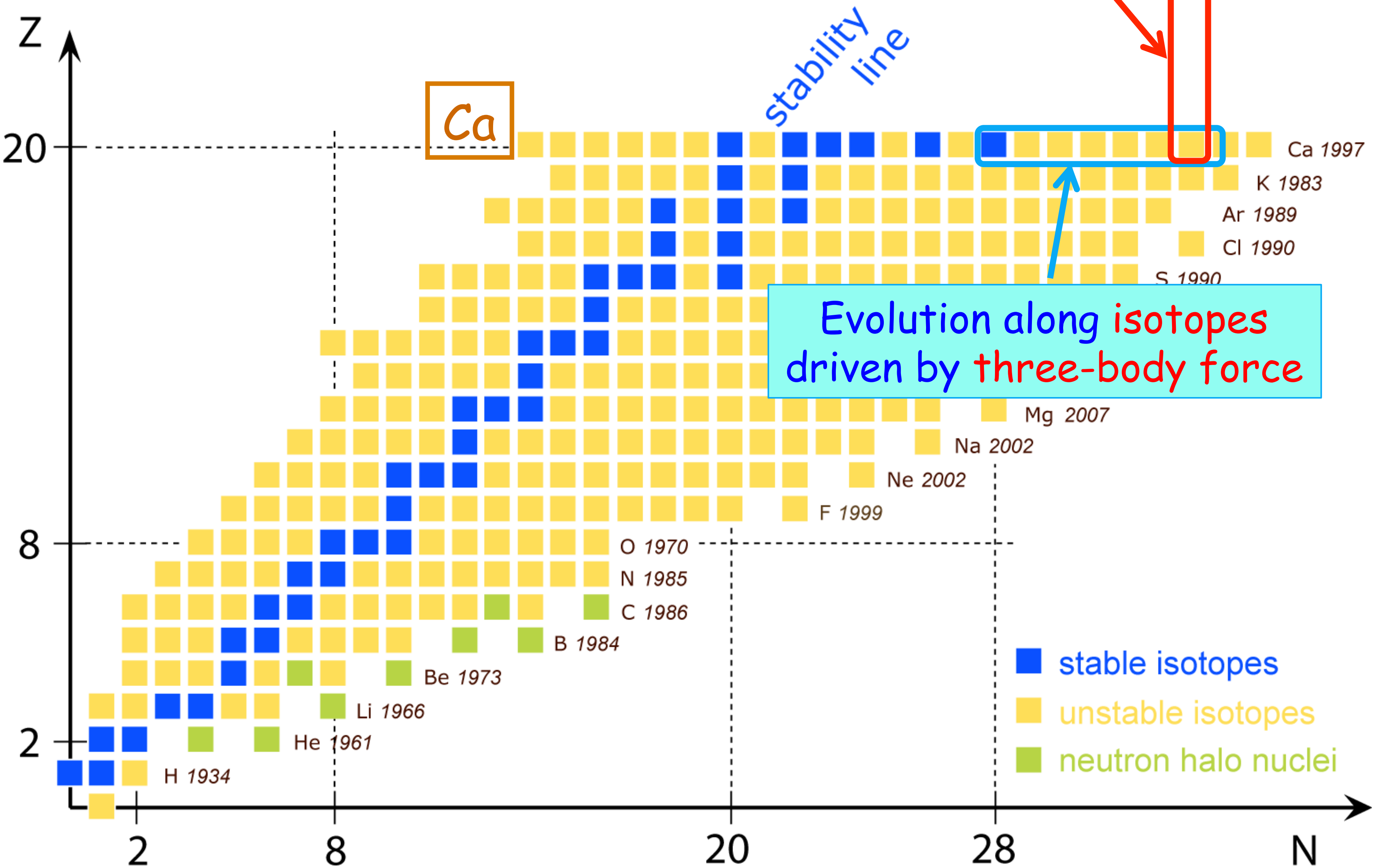
Magic index : 86% 89%
(proton part not considered)

Shell evolution in two dimensions



Shell evolution in two dimensions

Evolution along isotones driven by tensor force



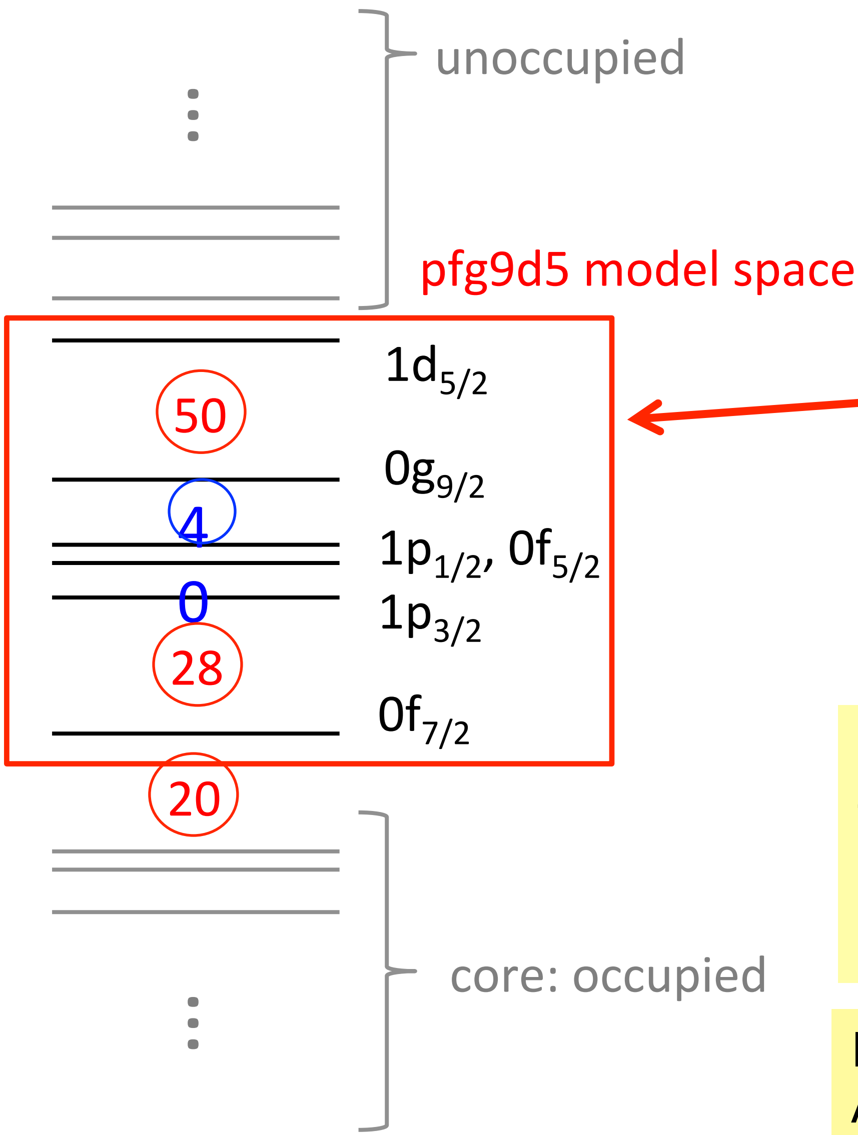
Evolution along isotopes driven by three-body force

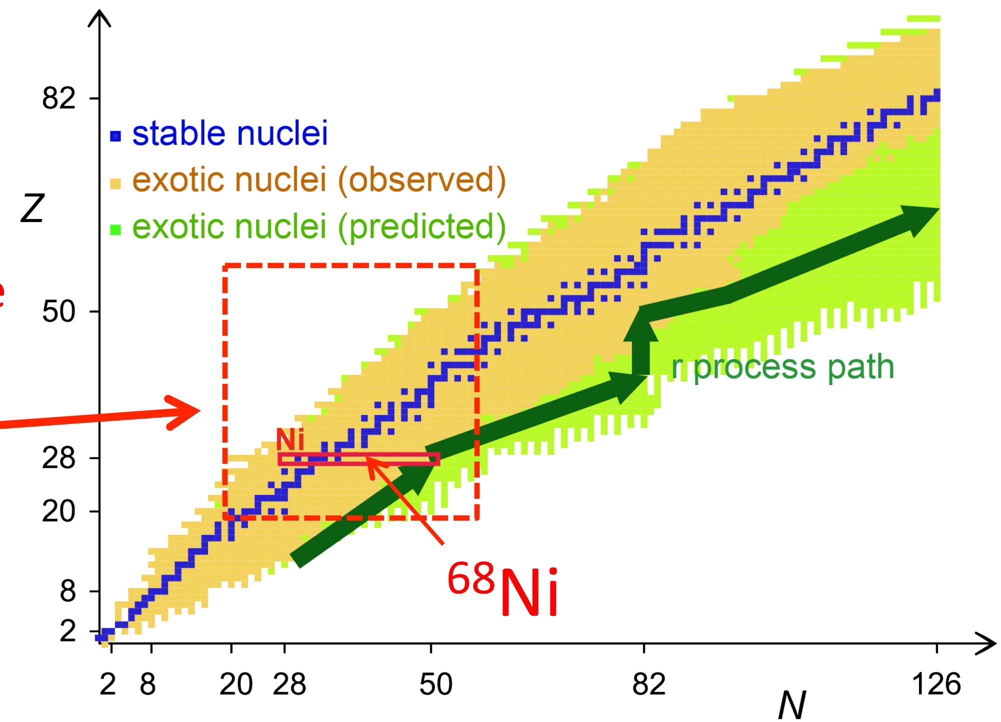
Outline

1. Introduction
2. Shell model and monopole interaction
3. Shell evolution and tensor force
4. Multiple quantum liquid in exotic nuclei
5. Shell evolution and three-nucleon force
6. Summary

MCSM calculation on Ni isotopes

Y. Tsunoda *et*





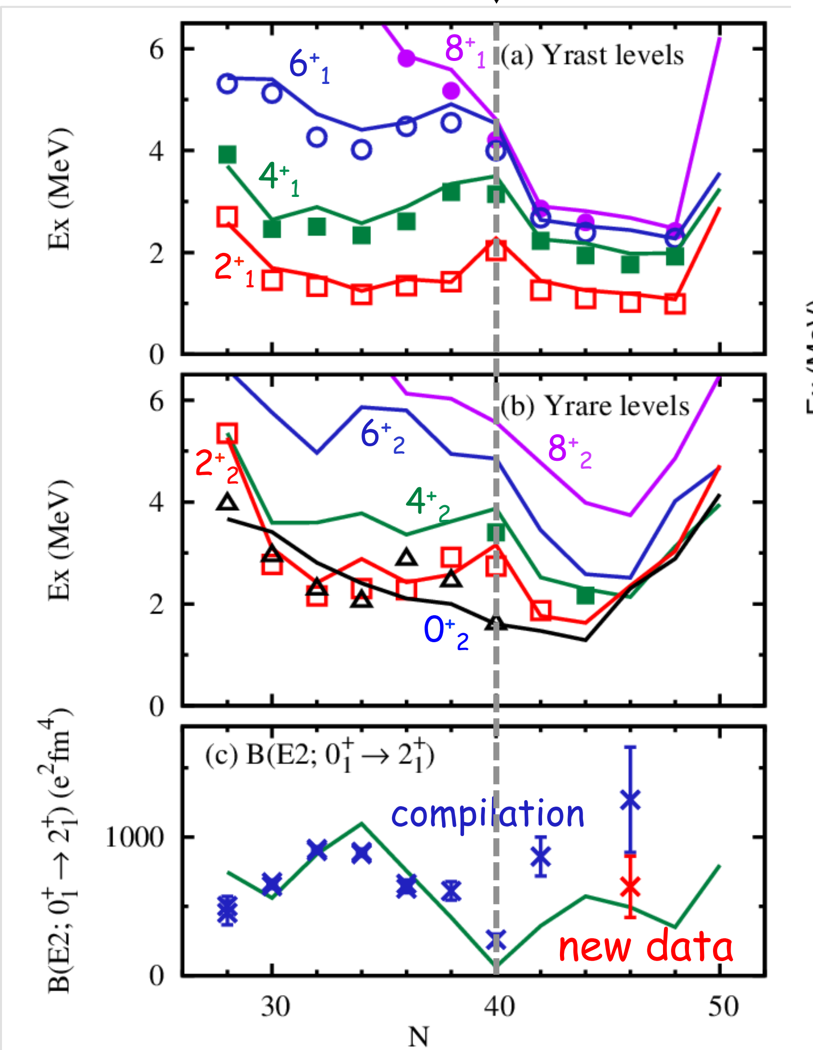
This model space is wide enough to discuss how magic numbers 28, 50 and semi-magic number 40 are visible or smeared out.

Interaction:
A3DA interaction is used with minor corrections

Energy levels and B(E2) values of Ni isotopes

Description by the same Hamiltonian

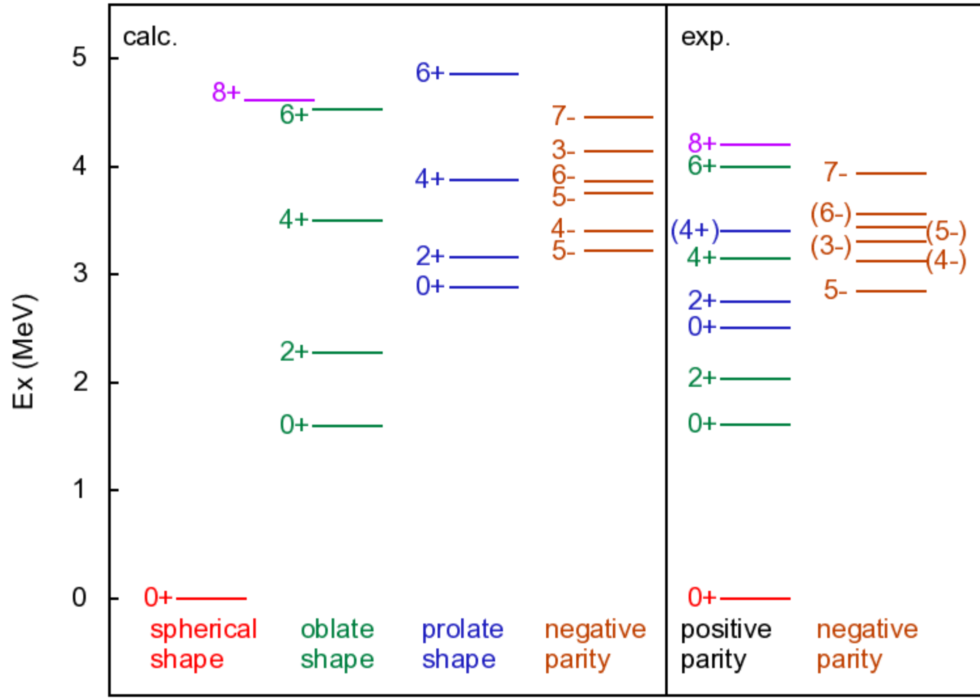
× exp.
— calc.



^{68}Ni

Shape coexistence in ^{68}Ni

calc. exp.



Y. Tsunoda, TO, Shimizu, Honma and Utsuno, PRC 89, 031301 (R) (2014)

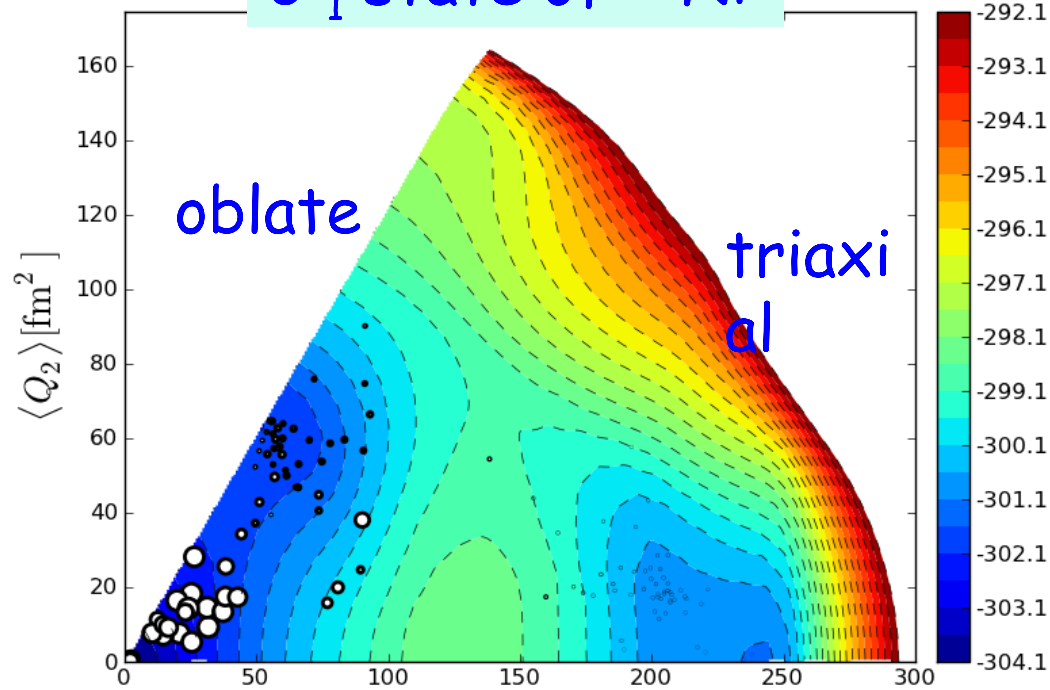
MCSM basis vectors on Potential Energy Surface

eigenstate $\Psi = \sum_i c_i P[J^\pi] \Phi_i$

Slater determinant → intrinsic shape

- PES is calculated by CHF for the shell-model Hamiltonian
- Location of circle : quadrupole deformation of unprojected MCSM basis vectors
- Area of circle : overlap probability between each projected basis and eigen wave function

0^+_1 state of ^{68}Ni



Called *T-plot* in reference to

Y. Tsunoda, TO, Shimizu, Honma and Utsuno, PRC 89, 031301 (R) (2014)

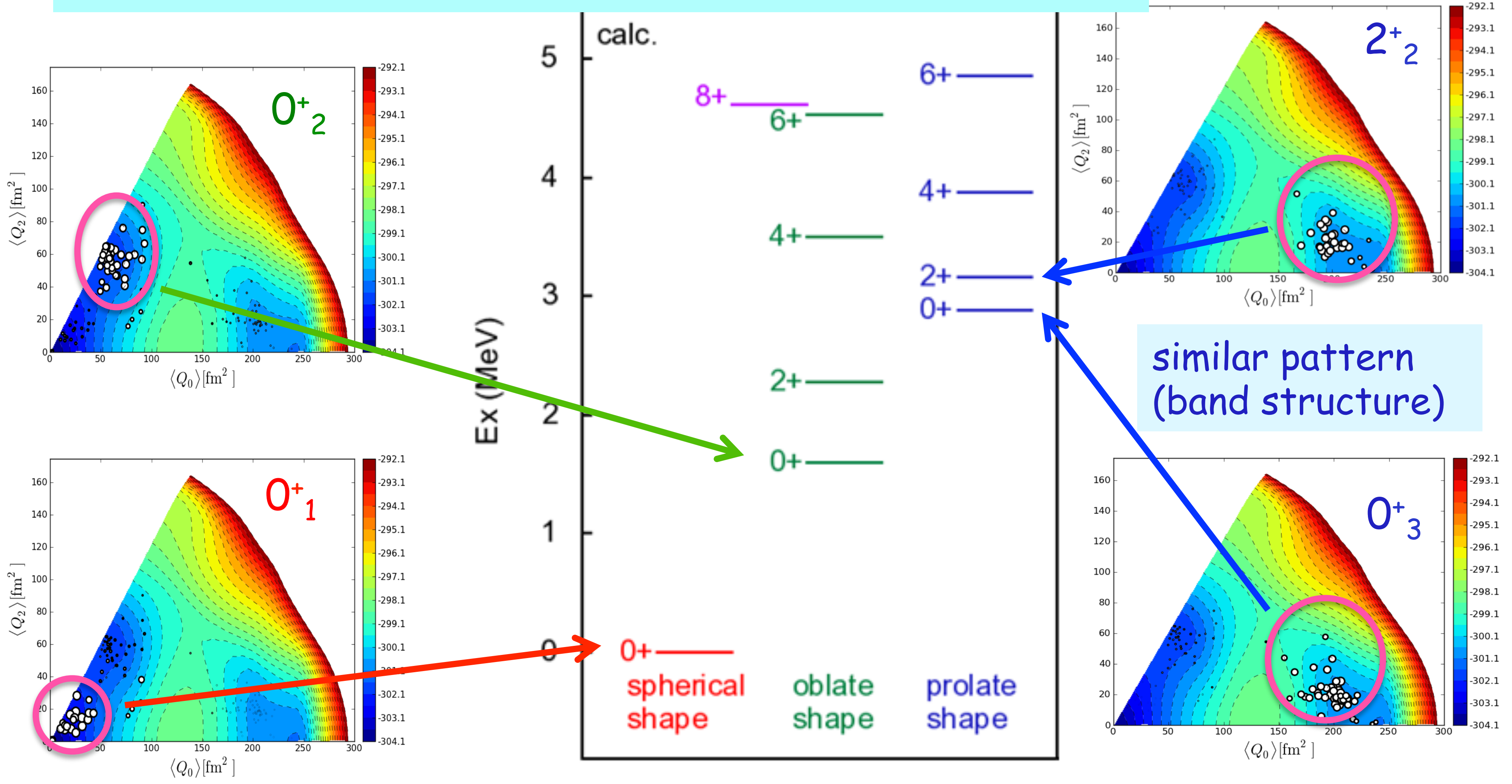
General properties of T-plot :

Certain number of large circles in a small region of PES

⇔ pairing correlations

Spreading beyond this can be due to shape fluctuation

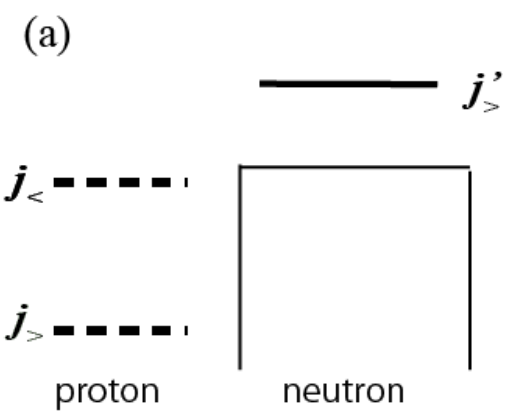
Example : shape assignment to various 0^+ states of ^{68}Ni



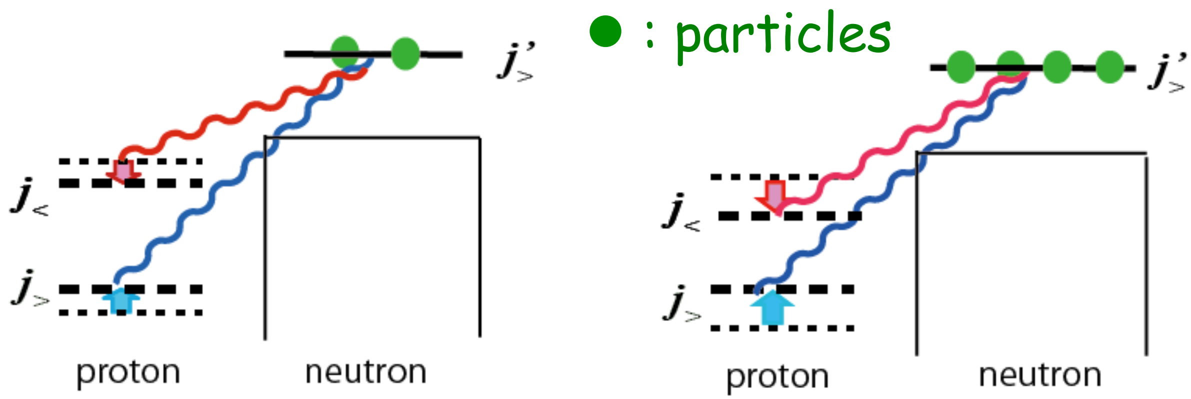
similar pattern (band structure)

Underlying mechanism of the appearance of low-lying deformed states : Type II Shell Evolution

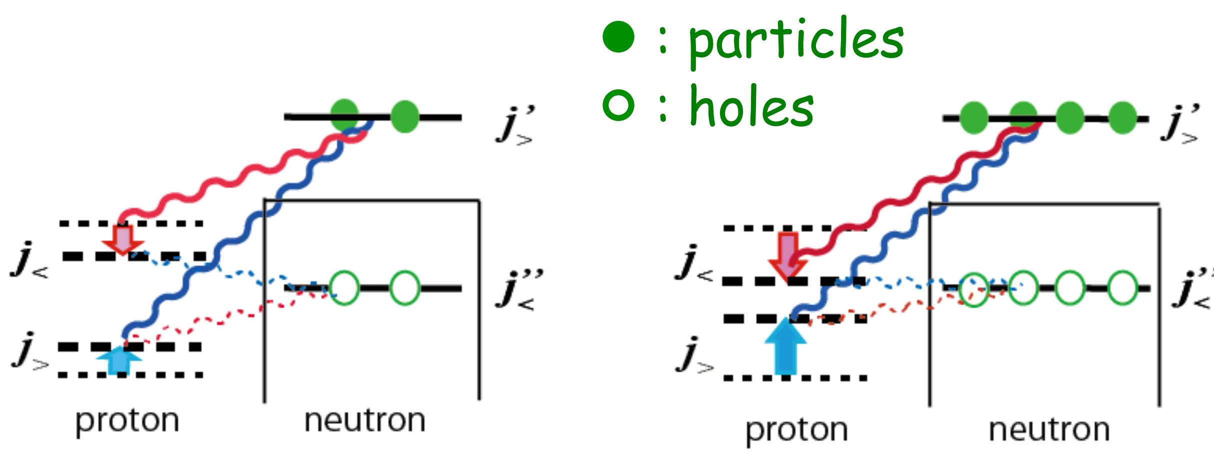
Monopole effects on the shell structure from the tensor interaction



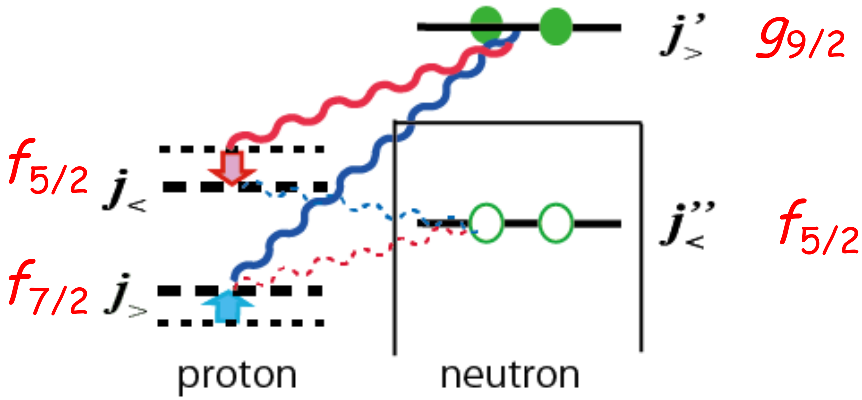
Type I Shell Evolution : different isotopes



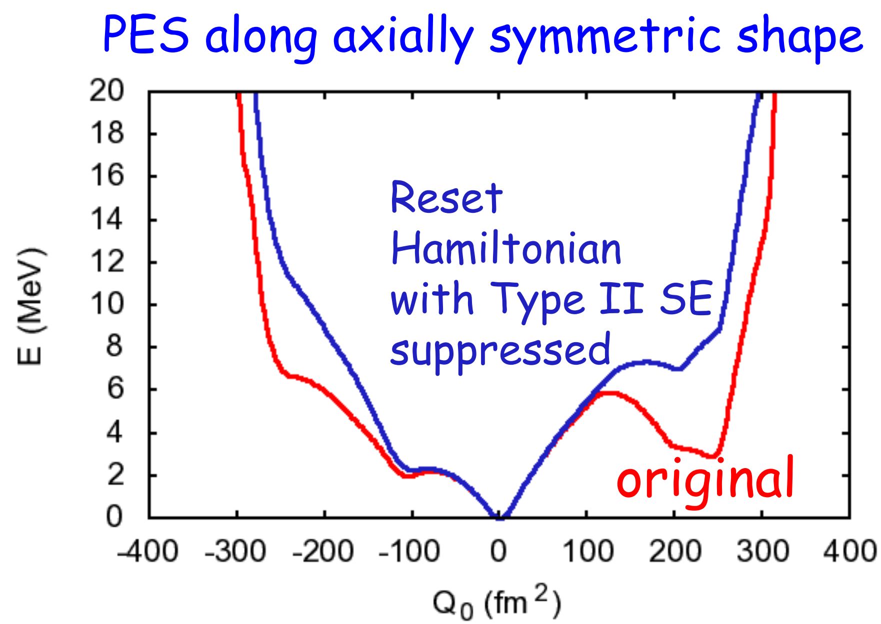
Type II Shell Evolution : within the same nucleus



Type II Shell Evolution in ^{68}Ni (Z=28, N=40)



Spin-orbit splitting works against quadrupole deformation (*cf. Elliott's SU(3)*).



weakening of spin-orbit splitting

Type II shell evolution

stronger deformation of protons
→ more neutron p-h excitation

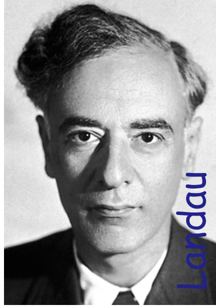
Type II shell evolution is suppressed by resetting monopole interactions as

$$\pi f_{7/2} - \nu g_{9/2} = \pi f_{5/2} - \nu g_{9/2}$$
$$\pi f_{7/2} - \nu f_{5/2} = \pi f_{5/2} - \nu f_{5/2}$$

The local minima become much less pronounced.

Shape coexistence is enhanced by type II shell evolution as the same quadrupole interaction works more efficiently.

Nucleus is a quantum liquid



Dual quantum liquids in the same nucleus

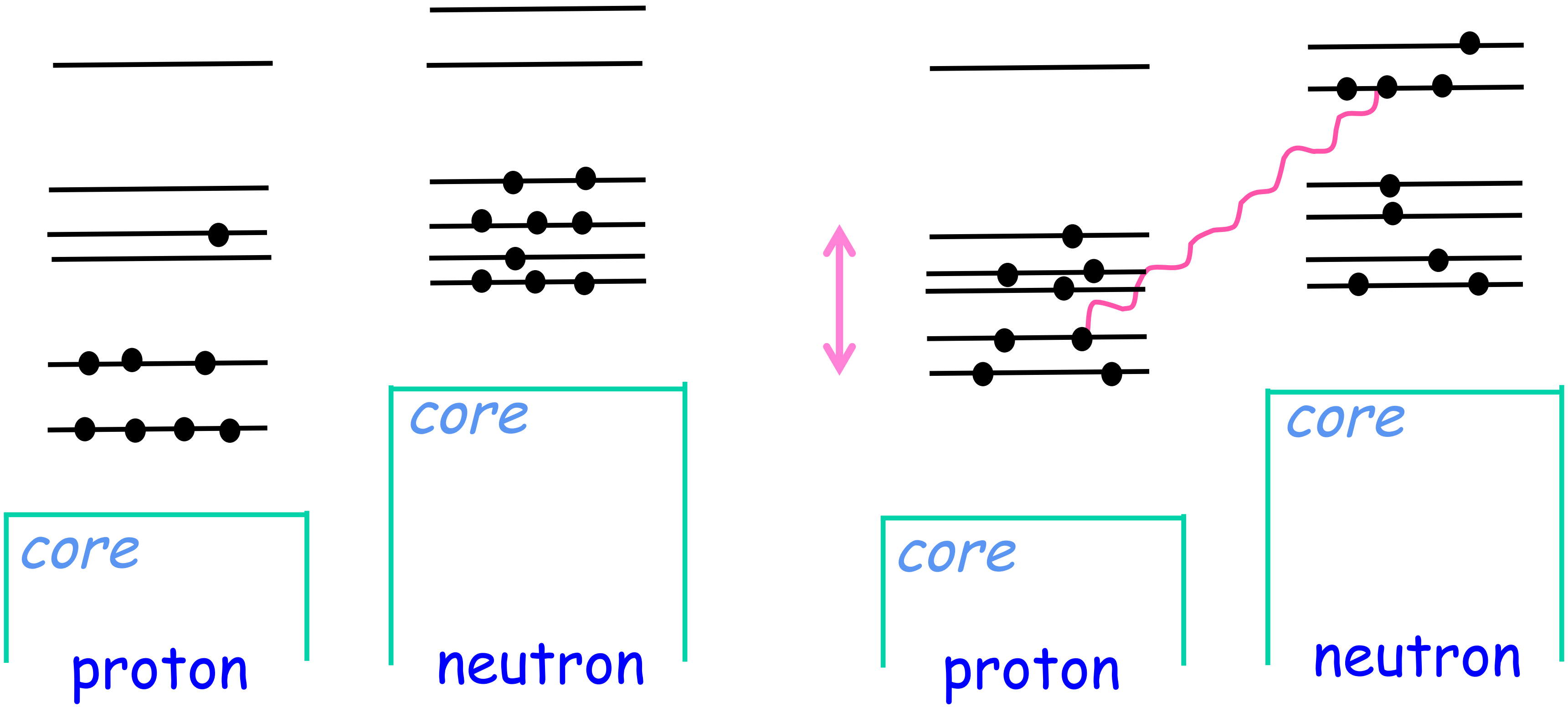
Certain configurations produce different shell structures
owing to (i) tensor force and (ii) proton-neutron compositions

Liquid 1 (~constant spherical SPE)

relevant to normal states in general

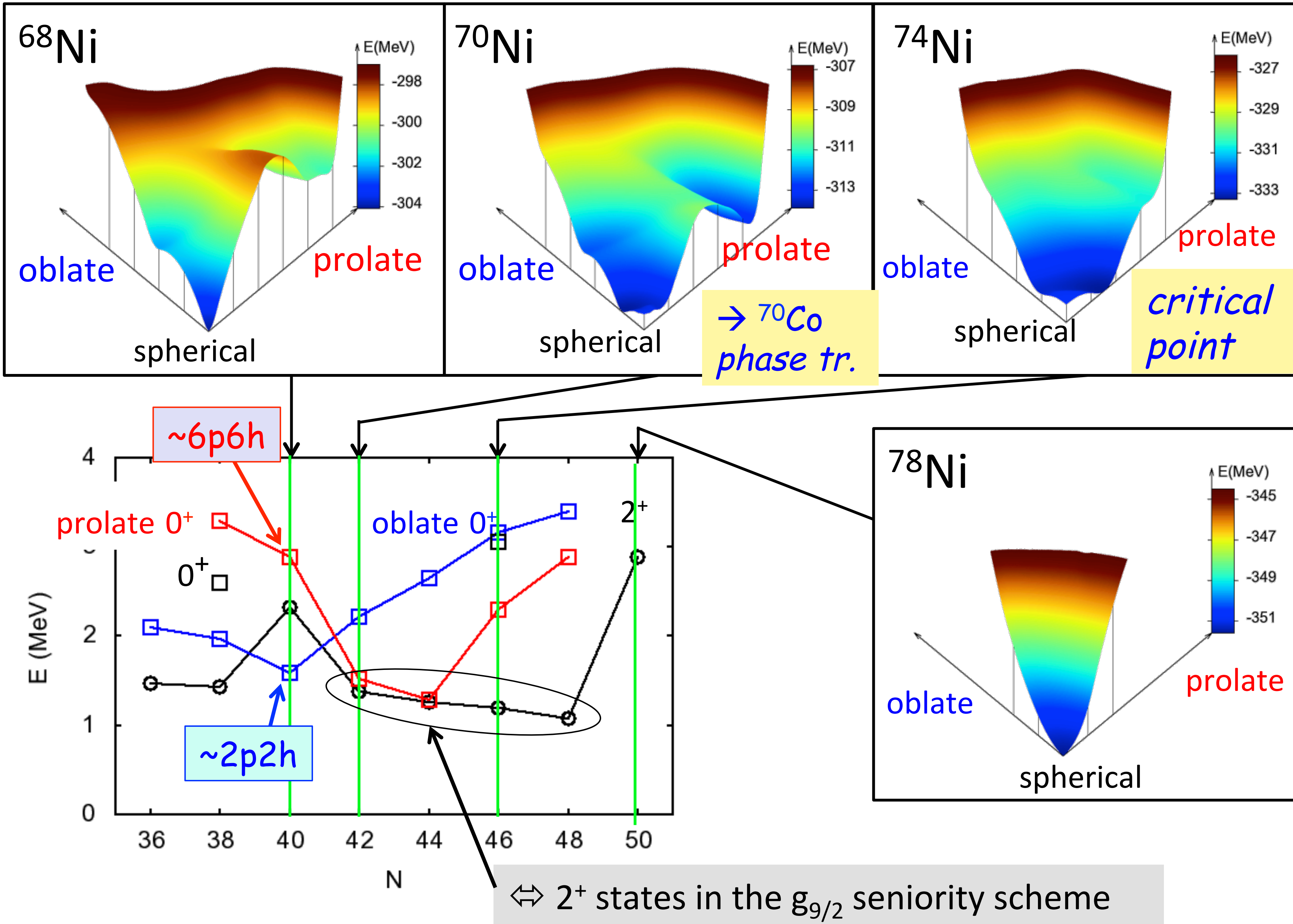
Liquid 2 (varying spherical SPE)

relevant to specific intruder states



Note : Despite almost the same density, different single-particle energies appear

Shape evolution and phase properties of Ni isotopes



Shape coexistence of ^{70}Co (Z=27, N=43)

g.s. and an isomer in ^{70}Co are known experimentally (PRC **61**, 054308 (2000))

High-spin state (6^-,7^-)

$$\pi f_{7/2}^{-1} \nu g_{9/2}^{+3}$$

and Low-spin state (3^+)

$$\pi f_{7/2}^{-1} \nu p_{1/2}^{-1} \nu g_{9/2}^{+4}$$

were suggested

From our calculations,

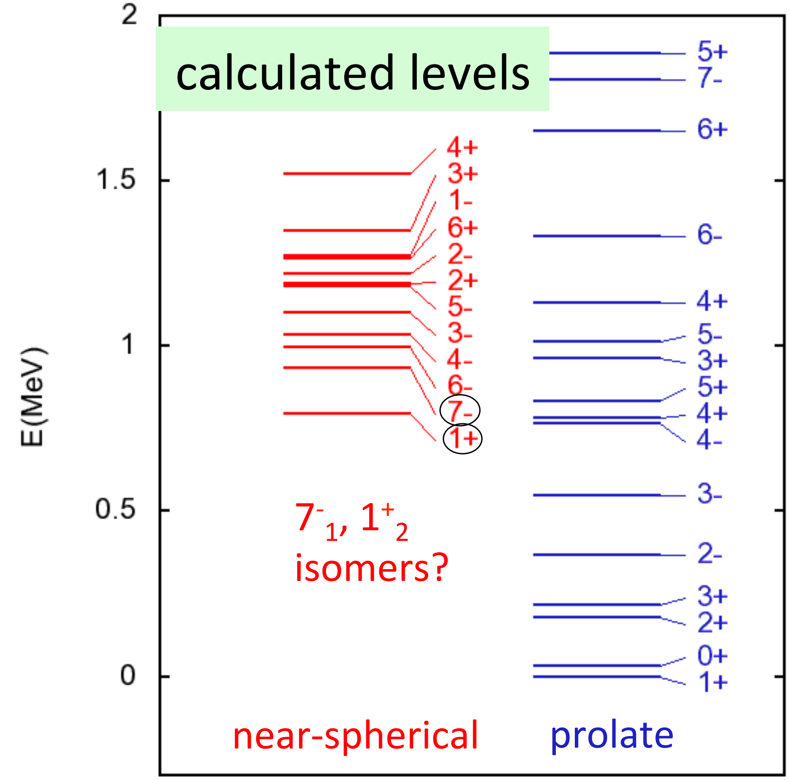
High-spin state (7^-) is near-spherical

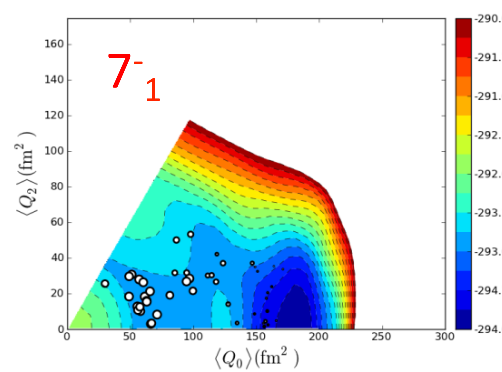
Low-spin state (1^+) is prolate deformed

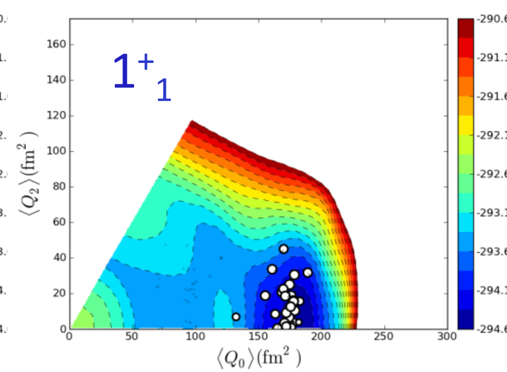
In the prolate state 1^+_1,
many nucleons are excited

~2.7 protons above Z=28 gap

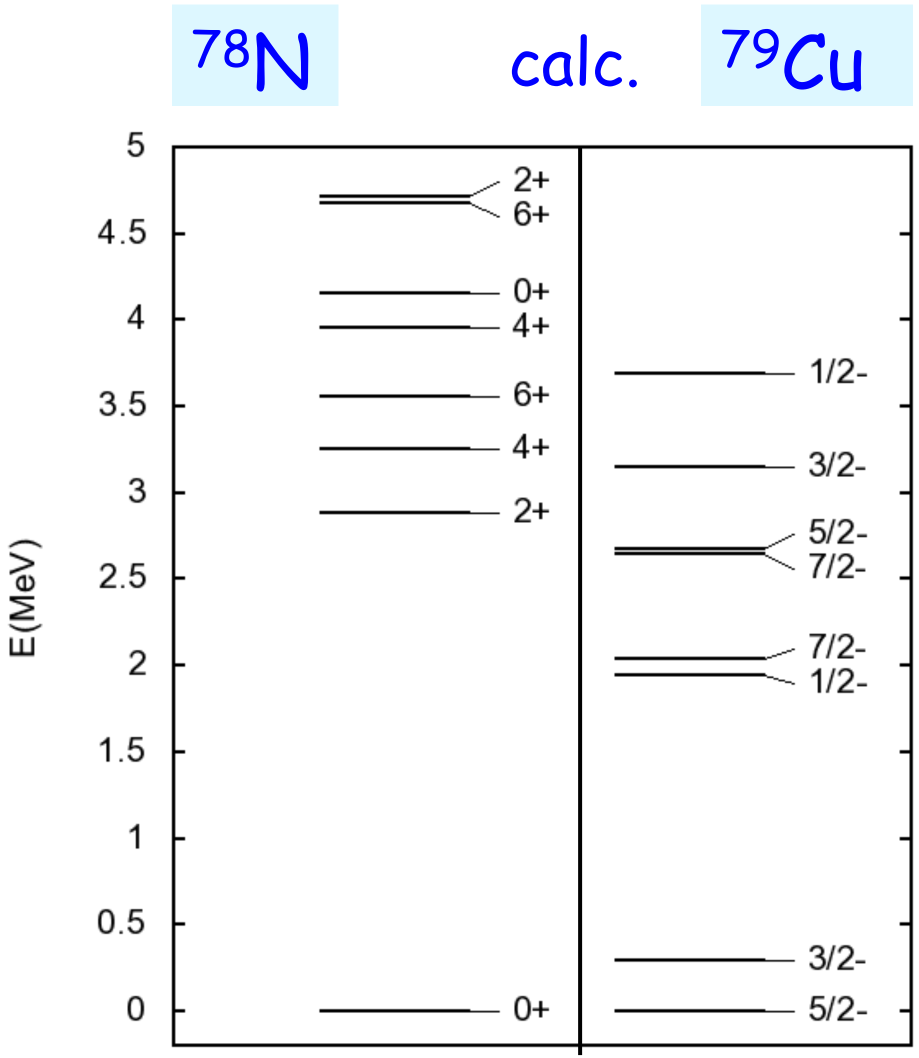
~3.1 neutron holes below N=40 gap



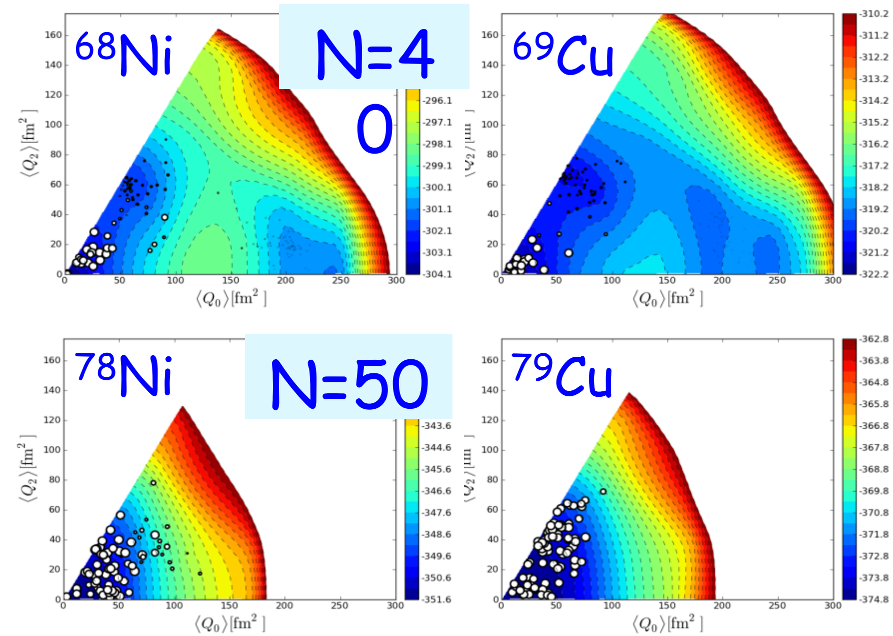




^{68}Ni - ^{69}Cu v.s. ^{78}Ni - ^{79}Cu

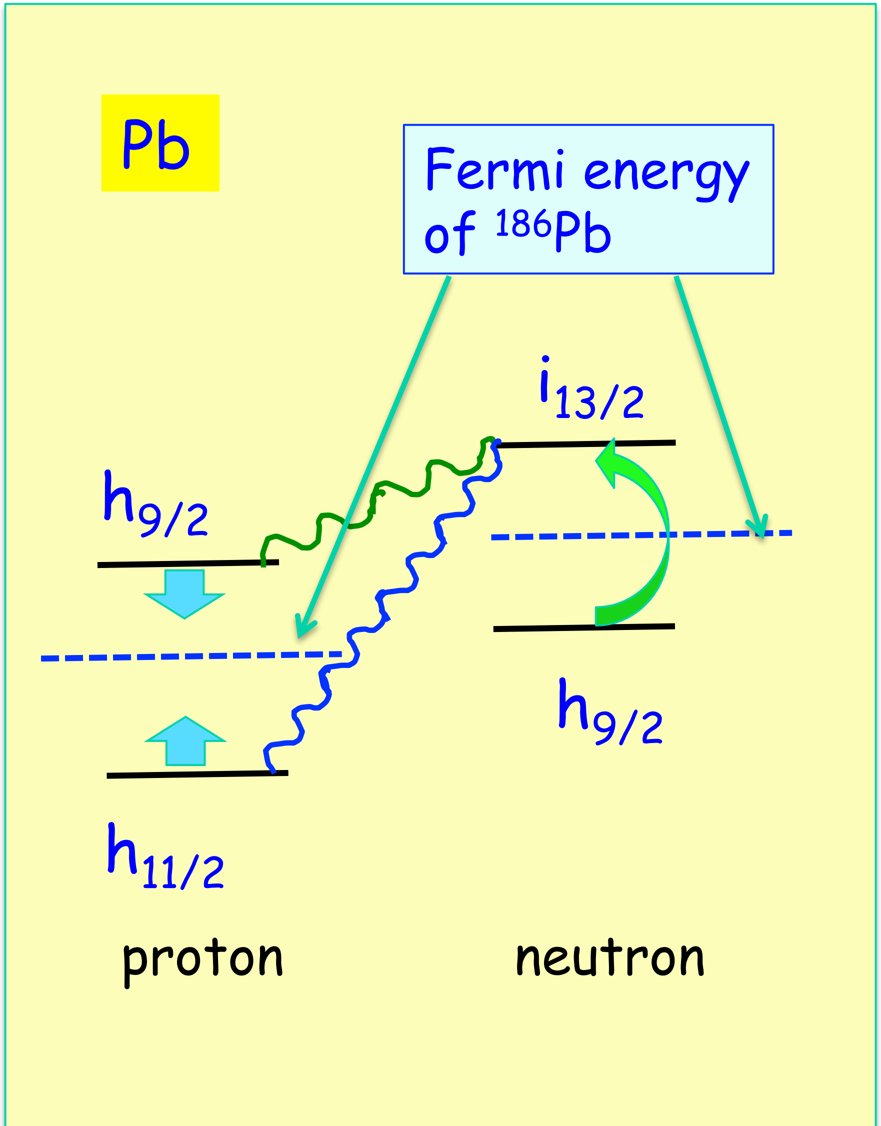


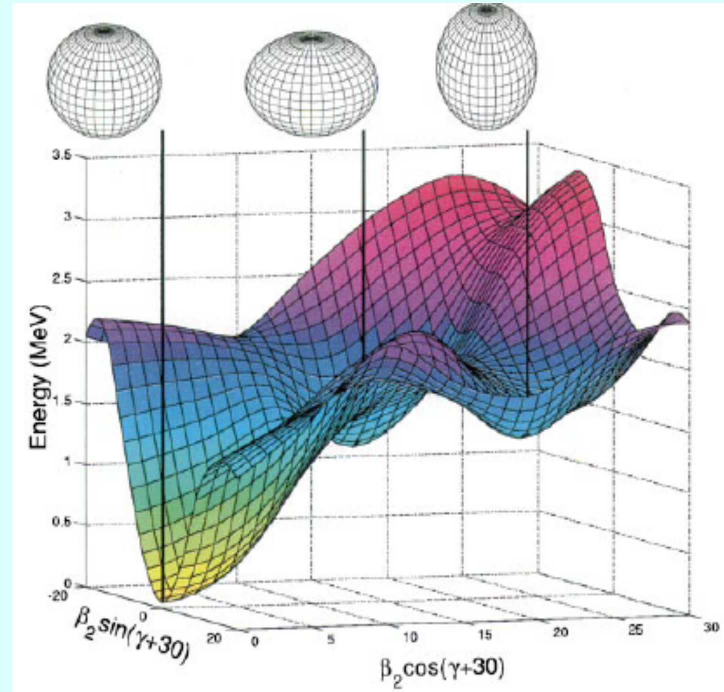
T-plot of ground state



- Similar distribution patterns between Ni and Cu, while Cu is somewhat more deformed
- Shape fluctuations are larger in N=50 isotones

Other cases ….. just an example





^{186}Pb Andreyev *et al.*,
Nature **405**, 430 (2000)

Let's call it a day

Thank you for your listening

See you tomorrow