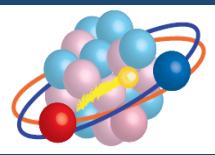


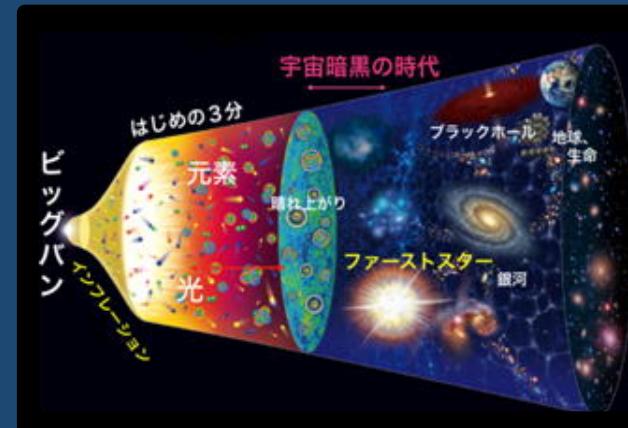
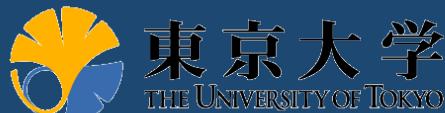
RIKEN 2koma plus seminar  
Wako  
July 1-2, 2015

# *Structure evolutions in exotic nuclei and nuclear forces*

## *- Day 2 -*



Takaharu Otsuka  
University of Tokyo / MSU / KU Leuven



## Addenda to Day 1

renormalization of tensor force  
exotic silicon isotopes

# Response to the renormalization of interactions

## Renormalization processes

- short-range correlations
- in-medium corrections

bare interaction for free space

$$V = V_c + V_{LS} + V_T$$

effective interaction for  
a model space of low-momenta

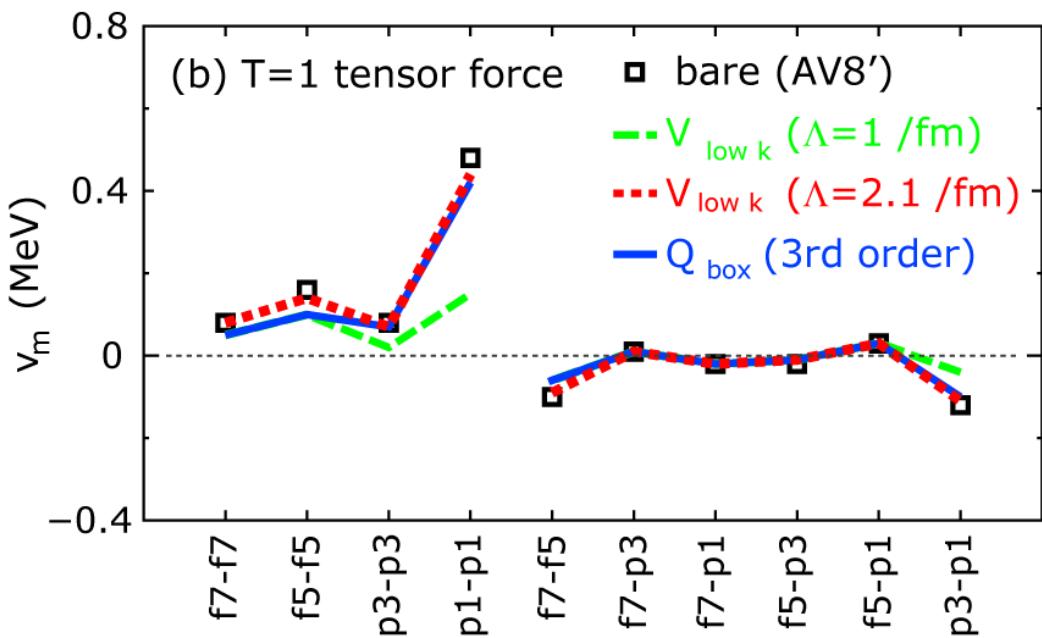
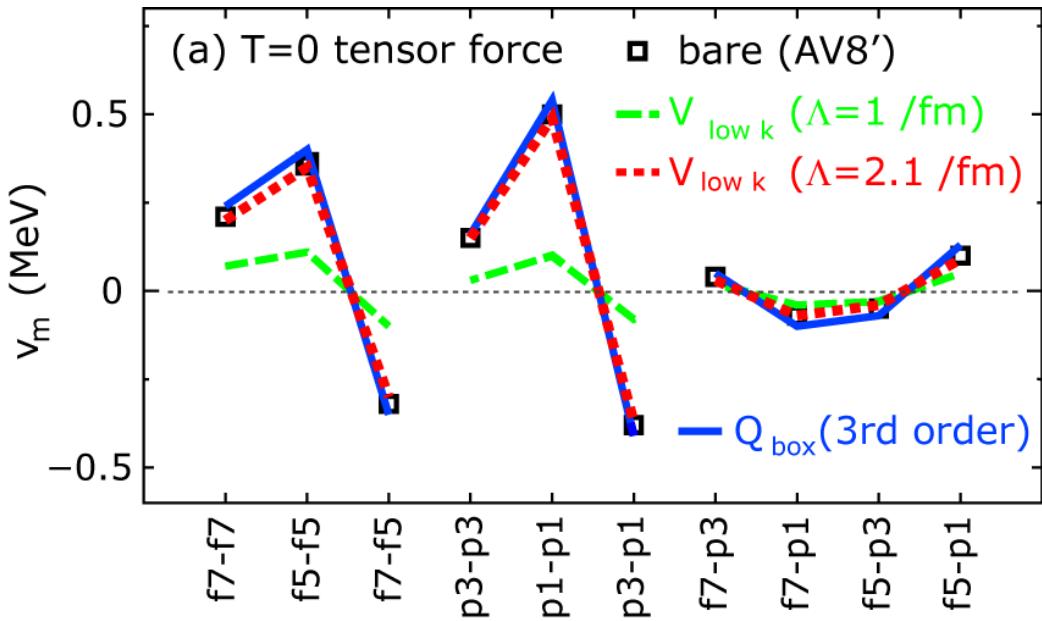
$$V' = V'_c + V'_{LS} + V'_T + V_{NNN} + \dots$$

In general,  $V'_x$  differs from  $V_x$ .

If  $V_x = V'_x$ , Renormalization Persistency holds.

- only good approx. at best, but it makes sense
- new approach to nuclear forces

# Treatment of tensor force by $V_{\text{low } k}$ and Q box (3<sup>rd</sup> order)



Monopole component  
of tensor interactions  
in pf shell

□ Bare (AV8')  
 ↓ short-range correlation  
 by  $V_{\text{low } k}$

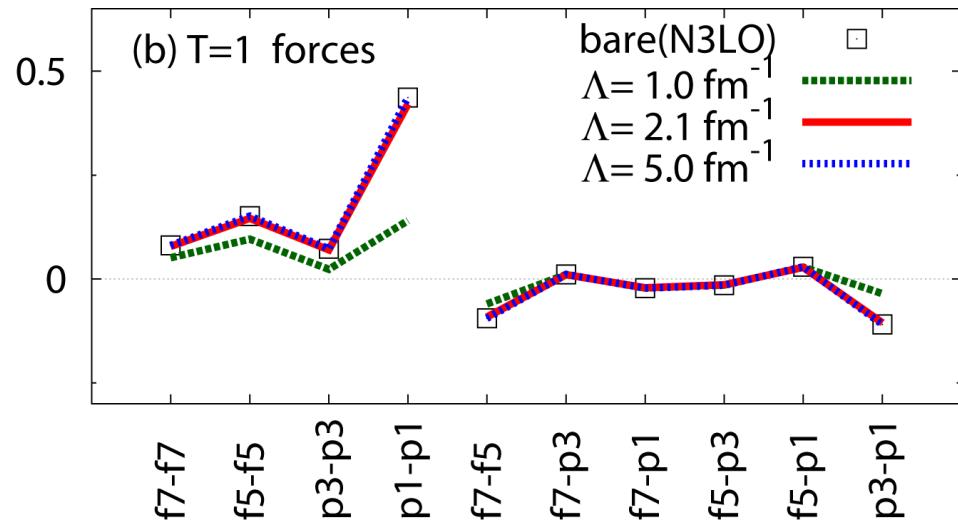
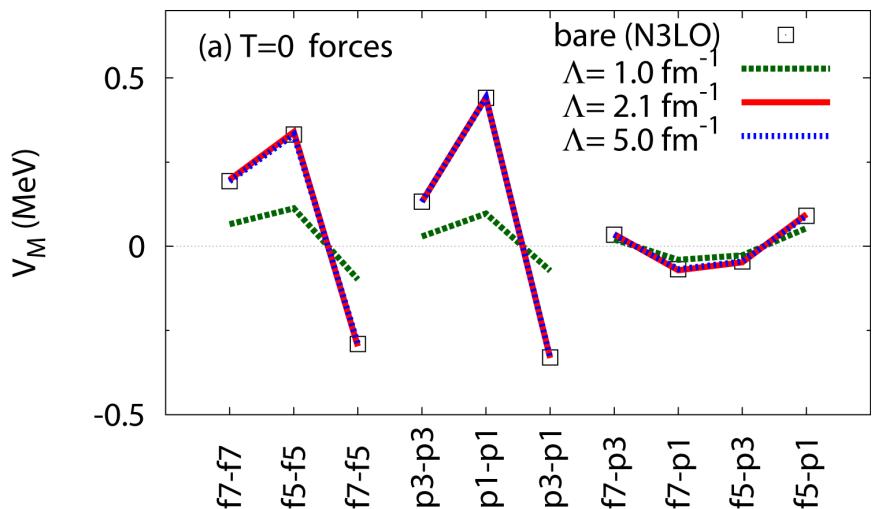
↓ in-medium correction  
 with intermediate states  
 (> 10 hw, 3<sup>rd</sup> order)

$V_{\text{low } k}$  : Bogner, Kuo, Schwenk  
 - - - only for comparison

O, Suzuki, et al.  
 PRL 104, 012501 (2010)

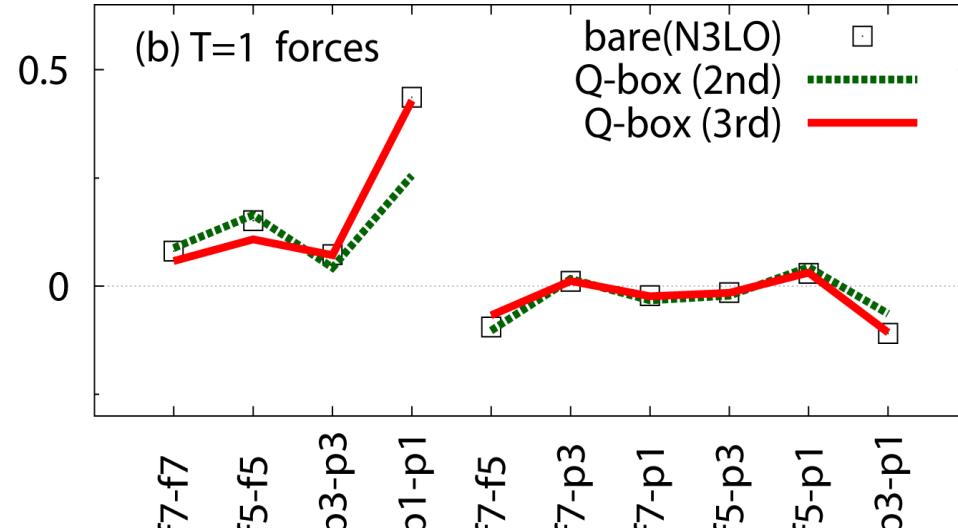
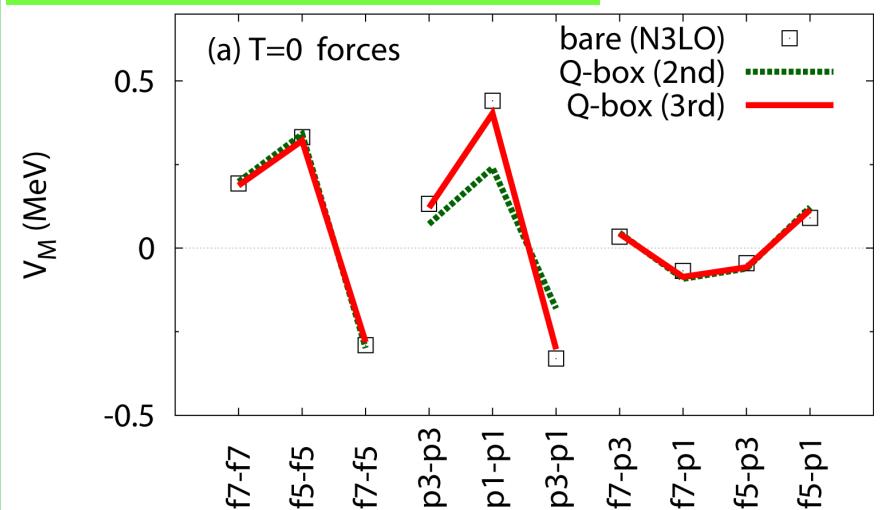
# N3LO (EFT of QCD) for pf-shell

## short-range correl.



## in-medium correc.

$\Lambda = 2.1 \text{ fm}^{-1}$



**Renormalization persistency of the tensor force in nuclei**

N.Tsunoda, T.O., K.Tsukiyama, M.H.-Jensen

Two steps of renormalization from full space to  
low-momentum space relevant to many-nucleon systems

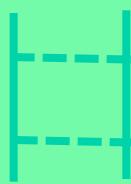
1. Treatment of short range correlations
2. Inclusion of in-medium effects

## Two major components in nuclear force

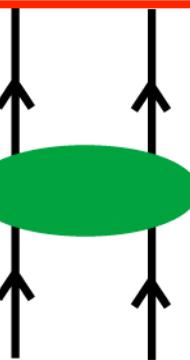
(a) central force :

(strongly renormalized)

$V_{MU}$  =



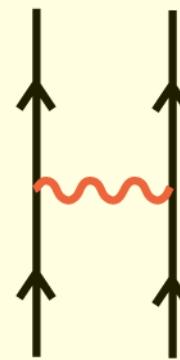
+ ...



+

(b) tensor force :

$\pi + \rho$  meson  
exchange



Renormalization  
Persistency

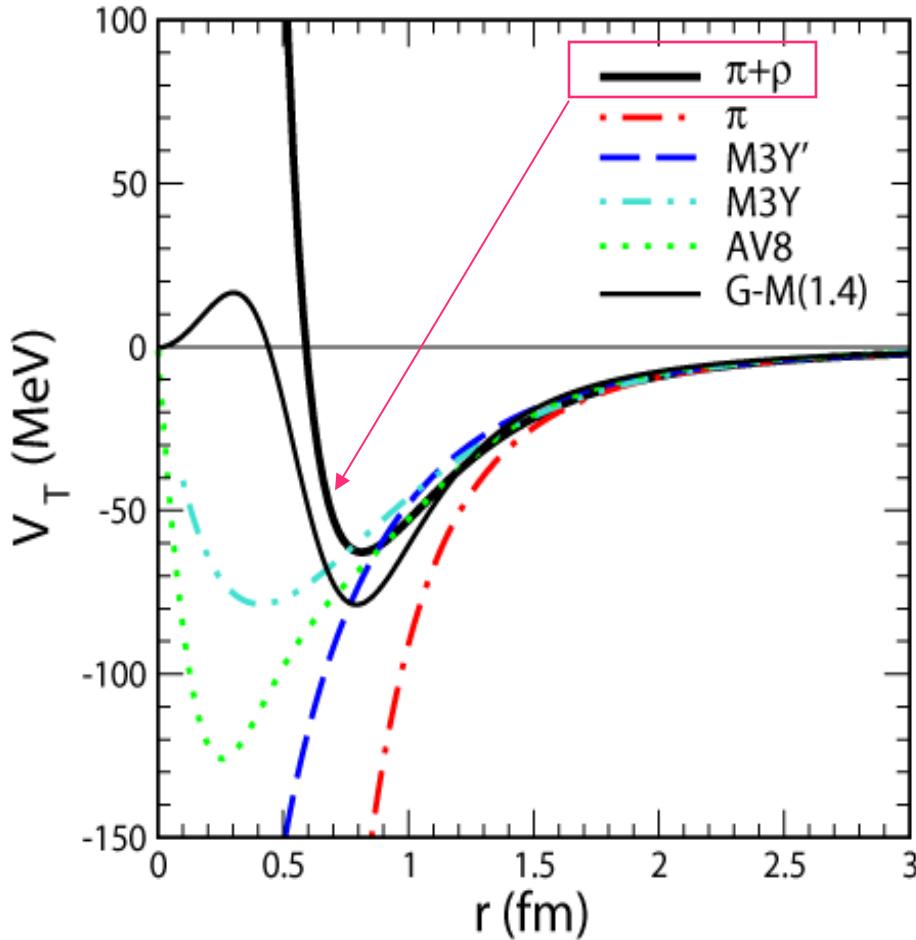
almost equal (no renormalization)

N.Tsunoda, T.O.,  
K.Tsukiyama, M.H.-Jensen,  
PRC84,044322 (2011)

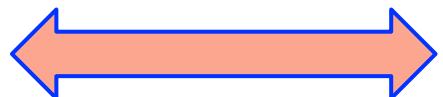
monopole component of  
tensor force **in nuclear medium**

monopole component of  
tensor force **in free space**

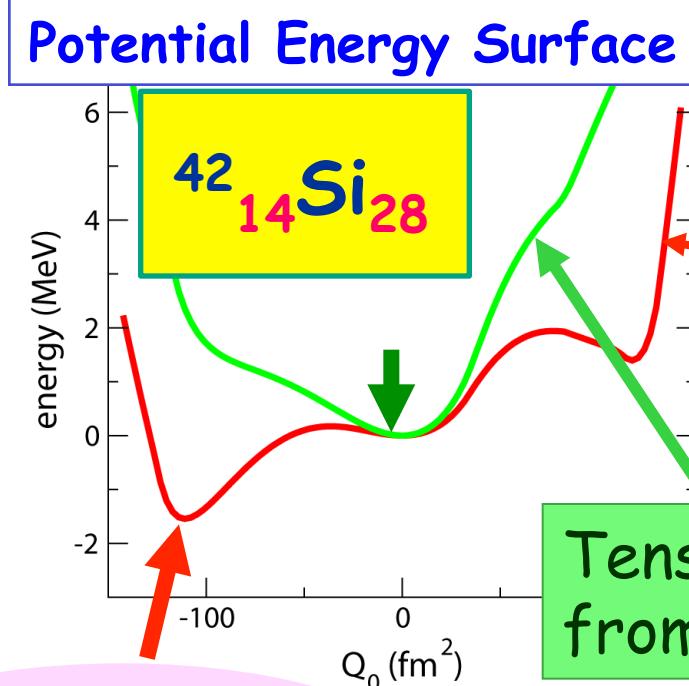
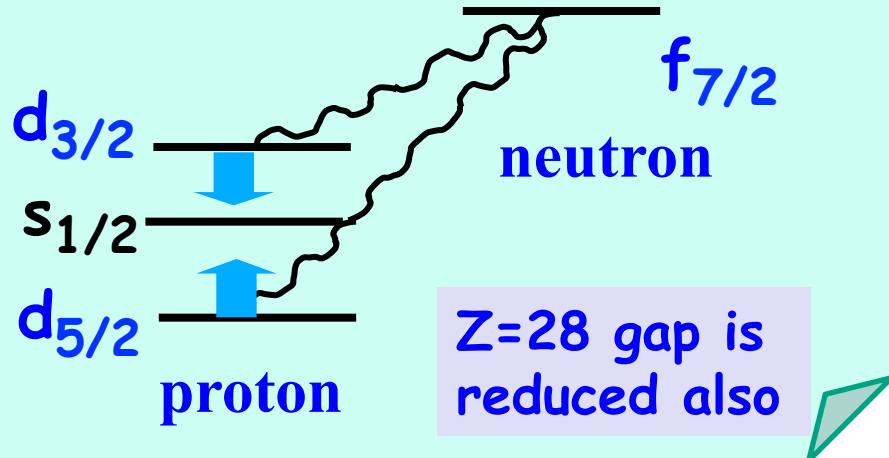
# Tensor potentials



What is the  
*relevant part* of the  
tensor force  
to the present issue.



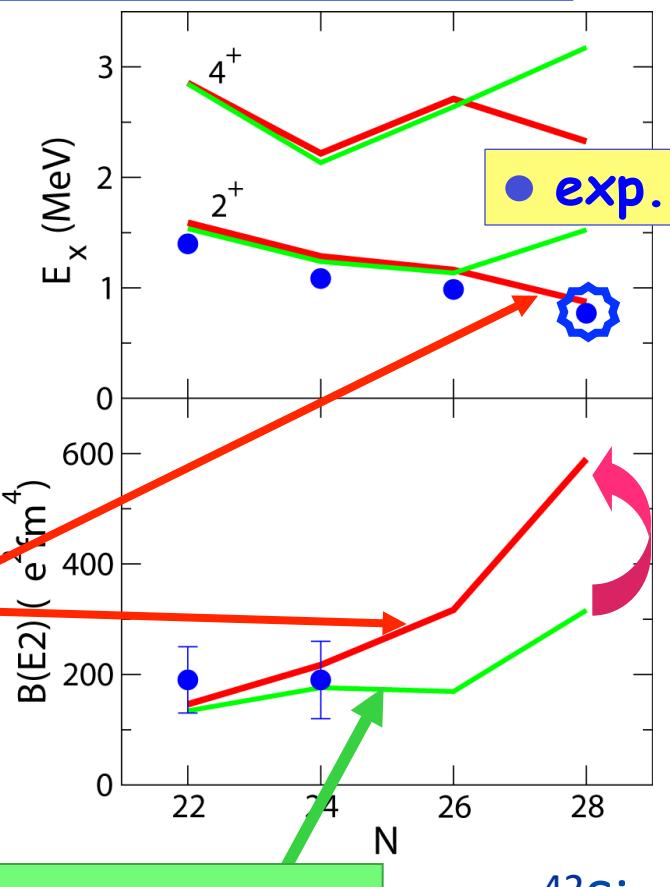
medium-long range



Strong oblate Deformation ?

Other calculations show a variety of shapes.

Si isotopes  
SM calc. by Utsuno et al.



Otsuka, Suzuki and Utsuno,  
Nucl. Phys. A805, 127c (2008)

$^{42}\text{Si}$ : B. Bastin, S. Grévy et al.,  
PRL 99 (2007) 022503

# Tensor force effects in exotic Si isotopes

PRL 109, 182501 (2012)

PHYSICAL REVIEW LETTERS

## Well Developed Deformation in $^{42}\text{Si}$

S. Takeuchi,<sup>1,\*</sup> M. Matsushita,<sup>1,2,†</sup> N. Aoi,<sup>1,‡</sup> P. Doornenbal,<sup>1</sup> K. Li,<sup>1,3</sup> T. Motobayashi,<sup>1</sup> H. Scheit,  
H. Wang,<sup>1,3</sup> H. Baba,<sup>1</sup> D. Bazin,<sup>4</sup> L. Cáceres,<sup>5</sup> H. Crawford,<sup>6</sup> P. Fallon,<sup>6</sup> R. Gernhäuser,<sup>7</sup> J. Gibeli,  
C. Hinke,<sup>7</sup> C. R. Hoffman,<sup>10</sup> R. Hughes,<sup>11</sup> E. Ideguchi,<sup>9,‡</sup> D. Jenkins,<sup>12</sup> N. Kobayashi,<sup>13</sup> Y. Kono,  
T. Le Bleis,<sup>14,15,¶</sup> J. Lee,<sup>1</sup> G. Lee,<sup>13</sup> A. Matta,<sup>16</sup> S. Michimasa,<sup>9</sup> T. Nakamura,<sup>13</sup> S. Ota,<sup>9</sup> M. Petri,<sup>6,§</sup>  
S. Shimoura,<sup>9</sup> K. Steiger,<sup>7</sup> K. Takahashi,<sup>13</sup> M. Takechi,<sup>1,\*\*</sup> Y. Togano,<sup>1,\*\*</sup> R. Winkler,<sup>4,||</sup> and  
<sup>1</sup>RIKEN Nishina Center, Wako, Saitama 351-0198, Japan

PRL 99, 022503 (2007)

PHYSICAL REVIEW LETTERS

## Collapse of the $N = 28$ Shell Closure in $^{42}\text{Si}$

B. Bastin,<sup>2</sup> S. Grévy,<sup>1,\*</sup> D. Sohler,<sup>3</sup> O. Sorlin,<sup>1,4</sup> Zs. Dombrádi,<sup>3</sup> N.L. Achouri,<sup>2</sup> J.C. Angélique,<sup>1</sup>  
D. Baiborodin,<sup>5</sup> R. Borcea,<sup>6</sup> C. Bourgeois,<sup>4</sup> A. Buta,<sup>6</sup> A. Bürger,<sup>7,8</sup> R. Chapman,<sup>9</sup> J. C. Dalouzy,<sup>1</sup> Z. Dlouhý,<sup>10</sup>  
Z. Elekes,<sup>3</sup> S. Frachoo,<sup>4</sup> S. Iacob,<sup>6</sup> B. Laurent,<sup>2</sup> M. Lazar,<sup>6</sup> X. Liang,<sup>9</sup> E. Liénard,<sup>2</sup> J. Mrazek,<sup>5</sup> L. Nalpas,<sup>11</sup> F. Neogoma,<sup>12</sup>  
N. A. Orr,<sup>2</sup> Y. Penionzhkevich,<sup>10</sup> Zs. Podolyák,<sup>11</sup> F. Pougheon,<sup>4</sup> P. Roussel-Chomaz,<sup>1</sup>  
M. G. Saint-Laurent,<sup>1</sup> M. Stanoiu,<sup>4,6</sup> and I. Stefan<sup>1</sup>  
F. Nowacki<sup>12</sup> and A. Poves<sup>13</sup>

The energies of the excited states in very neutron-rich  $^{42}\text{Si}$  and  $^{41,43}\text{P}$  have been measured using in-beam  $\gamma$ -ray spectroscopy from the fragmentation of secondary beams of  $^{42,44}\text{S}$  at 39A MeV. The low  $2^+$  energy of  $^{42}\text{Si}$ , 770(19) keV, together with the level schemes of  $^{41,43}\text{P}$ , provides evidence for the disappearance of the  $Z = 14$  and  $N = 28$  spherical shell closures, which is ascribed mainly to the action of proton-neutron tensor forces. New shell model calculations indicate that  $^{42}\text{Si}$  is best described as a well-deformed oblate rotor.

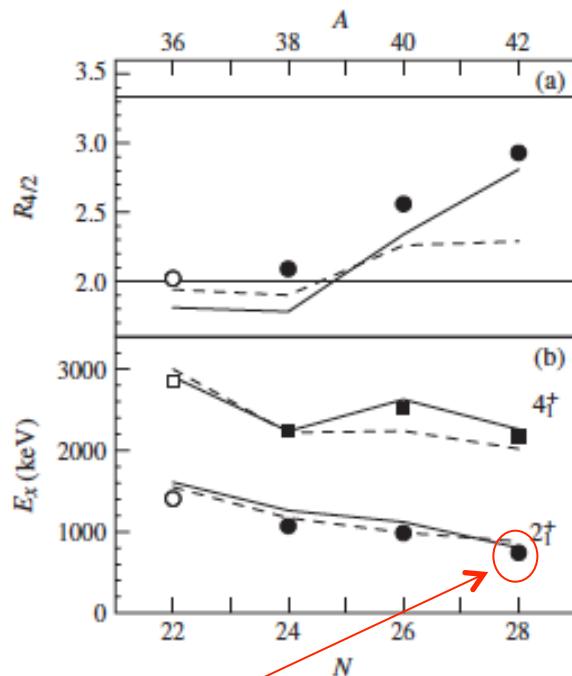


FIG. 3. (a) Ratio between the energies of the  $2_1^+$  and  $4_1^+$  states ( $R_{4/2}$ ) for Si isotopes. The horizontal lines at 2.0 and 3.3 indicate the vibrational and rotational limits, respectively. (b) Excitation energies for  $2_1^+$  and  $4_1^+$  states, which are indicated by circles and squares, respectively. Filled symbols are results of the present study, and solid and dashed lines represent predictions of the SM with SDPF-MU [17] and SM with SDPF-U-MIX [33], respectively (see text for details). The  $2^+$  energies of the  $N = 24, 26, 28$  Si isotopes have been measured in previous works [12,14,16].

# Tensor-force driven Jahn-Teller effect -> shape evolution

PHYSICAL REVIEW C 86, 051301(R) (2012)

C 86, 051301(R) (2012).

Utsuno, TO, Brown, Honma, Mizusaki, Shimizu

RIBF data close to

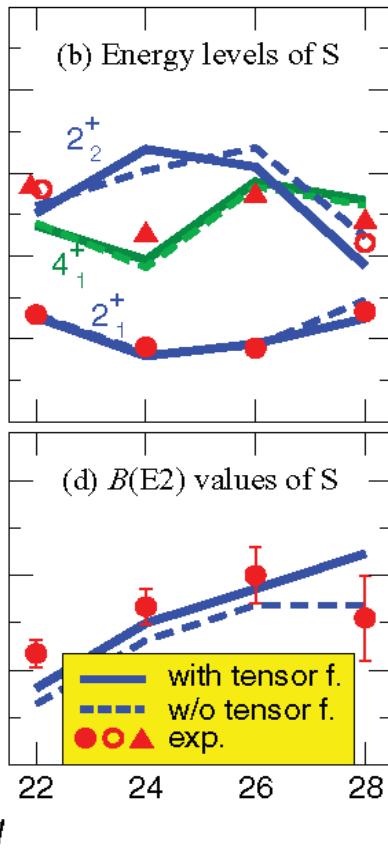
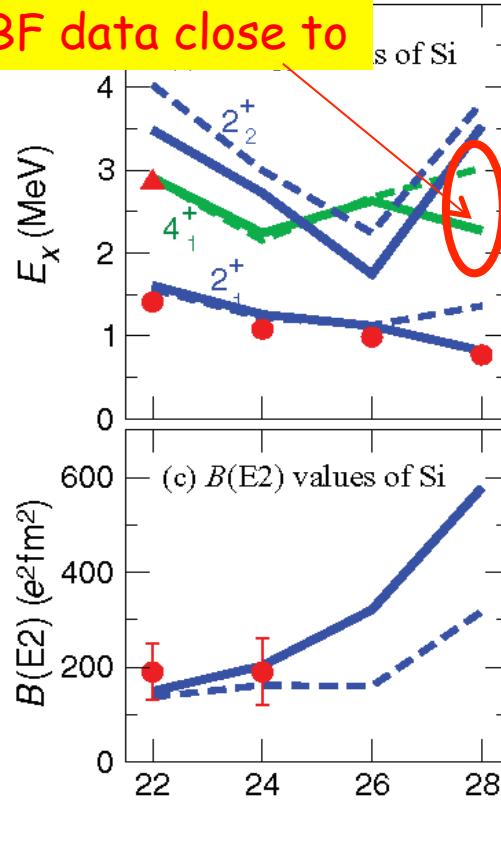
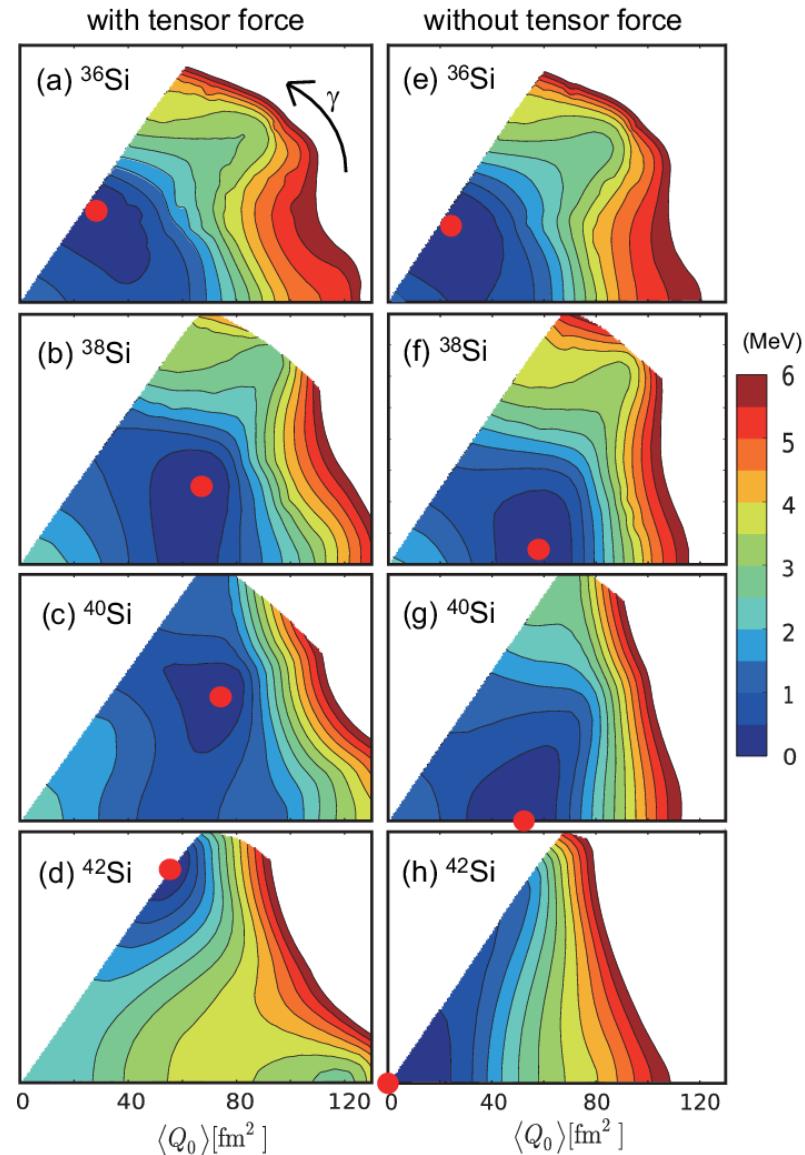
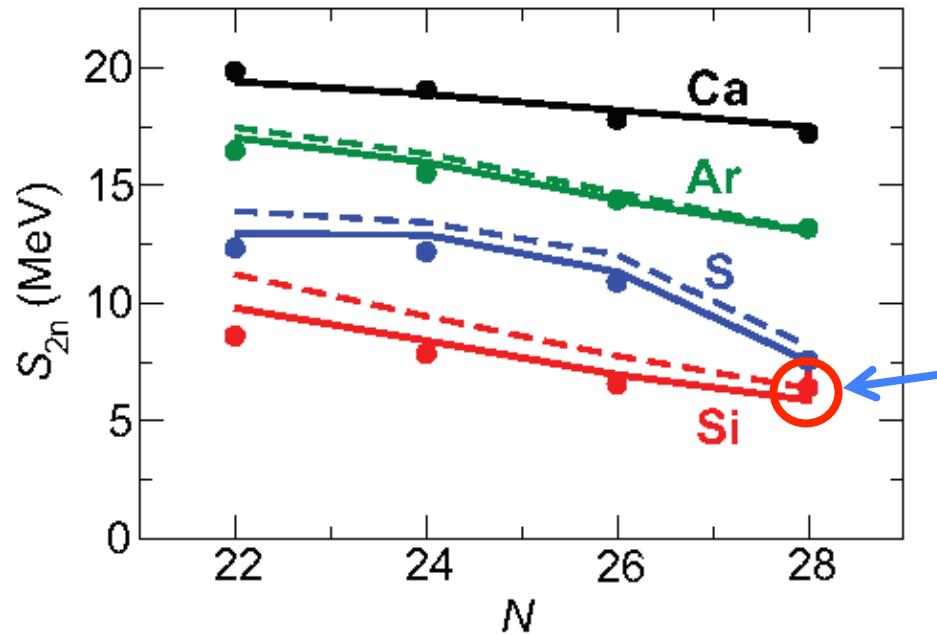


FIG. 3. (Color online) (a) and (b)  $2_{1,2}^+$  (blue lines and red circles) and  $4_1^+$  (green lines and red triangles) energy levels and (c) and (d)  $B(E2; 0_1^+ \rightarrow 2_1^+)$  values of Si and S isotopes for  $N = 22-28$ . Symbols are experimental data [13–19]. Solid (dashed) lines are calculations with (without) the cross-shell tensor force.



# Tensor force effects on binding energy (two-neutron separation energy)



Binding energy  
does not show  
tensor-force  
effect

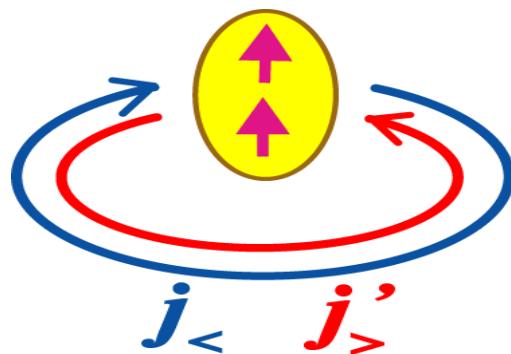
FIG. 6. (Color online) Two-neutron separation energies of Si and S isotopes from  $N = 22$  to 28. Solid (dashed) lines are calculations with (without) the cross-shell tensor force. Points are experimental data [40,41].

## Zero-range spin-momentum tensor coupling term

$$\begin{aligned} v_T = & \frac{1}{2} T \{ [(\sigma_1 \cdot k')(\sigma_2 \cdot k') - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k'^2] \delta(r_1 - r_2) \\ & + \delta(r_1 - r_2) [(\sigma_1 \cdot k)(\sigma_2 \cdot k) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k^2] \} \\ & + U \{ (\sigma_1 \cdot k') \delta(r_1 - r_2) (\sigma_1 \cdot k) \\ & - \frac{1}{3} (\sigma_1 \cdot \sigma_2) [k' \cdot \delta(r_1 - r_2) k] \}, \end{aligned} \quad (1)$$

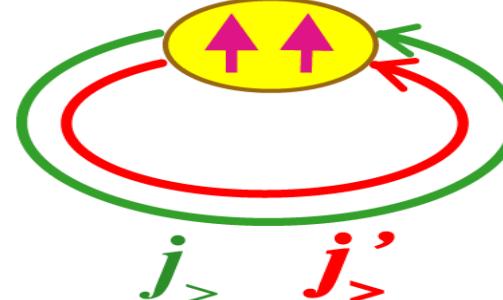
This is not be a good approximation to the tensor force itself, but may simulate the monopole effect of the tensor shown below, picking up differences in relative momenta.

large relative momentum



deuteron  $\Rightarrow$  attractive

small relative momentum



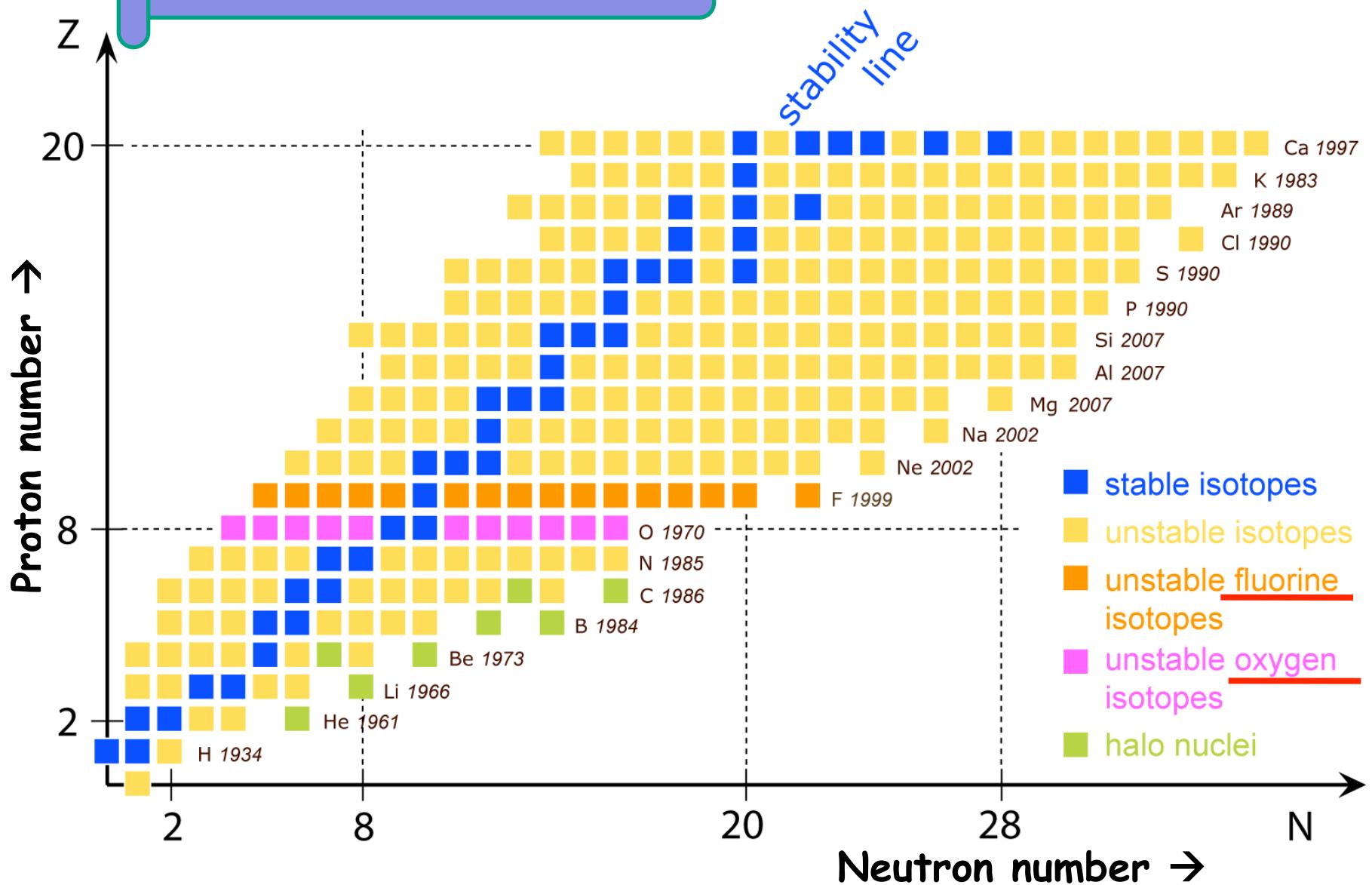
repulsive

# Outline

1. Introduction
2. Shell model and monopole interaction
3. Shell evolution and tensor force
4. Multiple quantum liquid in exotic nuclei
5. Shell evolution and three-nucleon force
6. Monte Carlo Shell Model
7. Summary

# Nuclear Chart - Left Lower Part -

Why is the drip line of Oxygen so near ?



# Single-Particle Energy for Oxygen isotopes

by microscopic eff. int.

G-matrix+ core-pol. : Kuo, Brown

$V_{\text{low-}k}$  : Bogner, Kuo, Schwenk

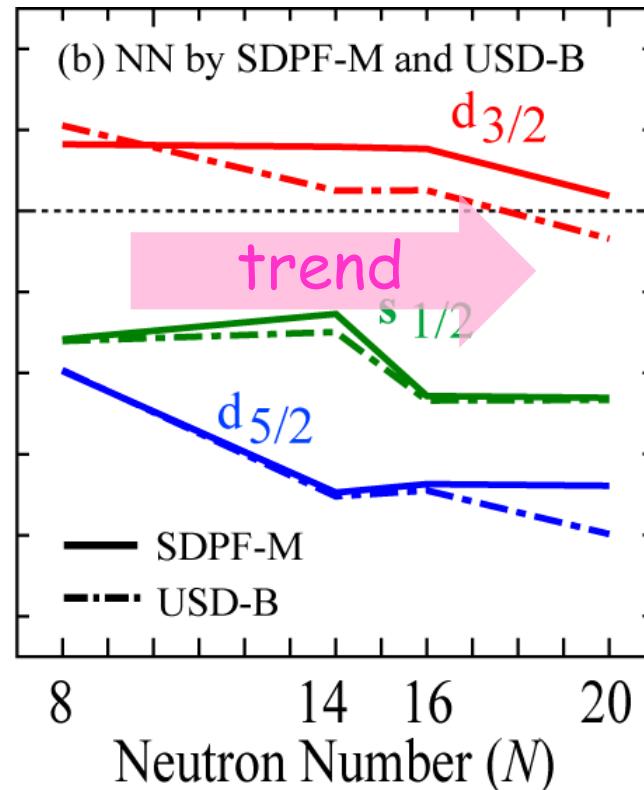
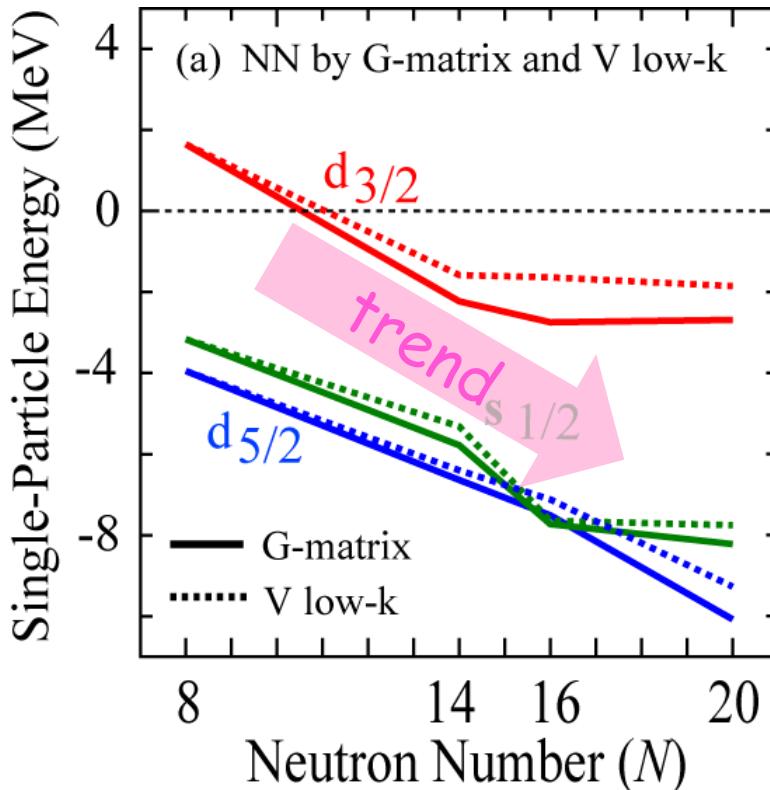
by phenomenological eff. int.  
- G-matrix + fit -

SDPF-M

Utsuno, O., Mizusaki, Honma,  
Phys. Rev. C **60**, 054315 (1999)

USD-B

Brown and Richter,  
Phys. Rev. C **74**, 034315 (2006)



What is the origin of  
the *repulsive modification* of  
 $T=1$  monopole matrix elements ?

The same puzzle as in the pf shell

A solution within *bare* 2-body interaction  
is very unlikely  
(considering efforts made so far)

Zuker, Phys. Rev. Lett. 90, 042502 (2003)

→ 3-body interaction

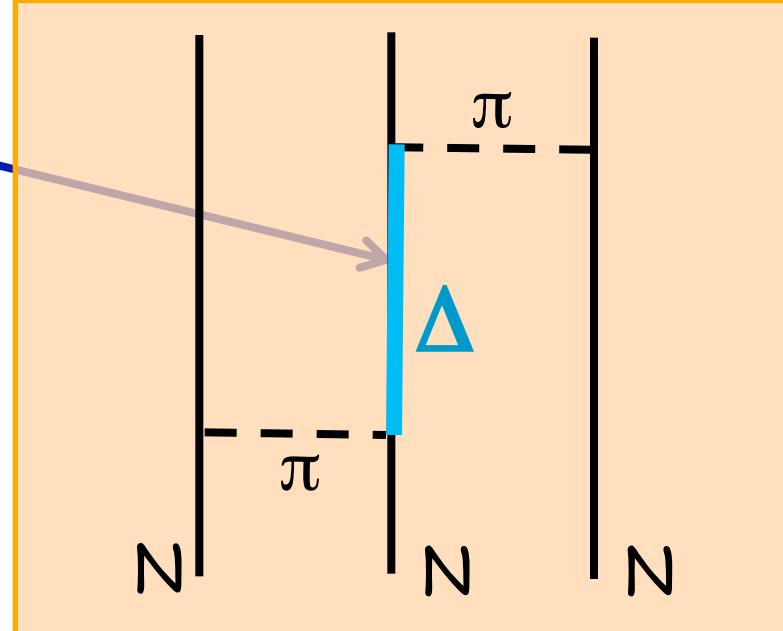
# The clue : Fujita-Miyazawa 3N mechanism ( $\Delta$ -hole excitation)

Progress of Theoretical Physics, Vol. 17, No. 3, March 1957

## Pion Theory of Three-Body Forces

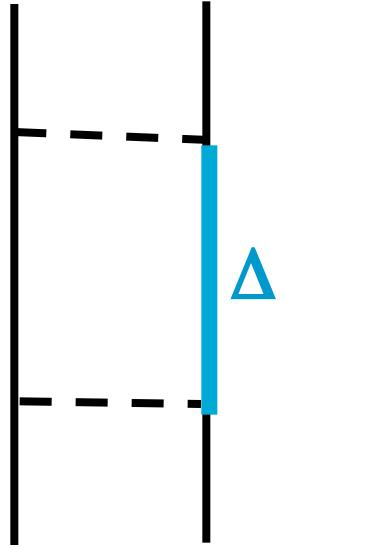
Jun-ichi FUJITA and Hironari MIYAZAWA

$\Delta$  particle  
 $m=1232 \text{ MeV}$   
 $S=3/2, I=3/2$



Oset, Toki and Weise  
Pionic modes of excitation  
Phys. Rep. 83, 281 (1982)

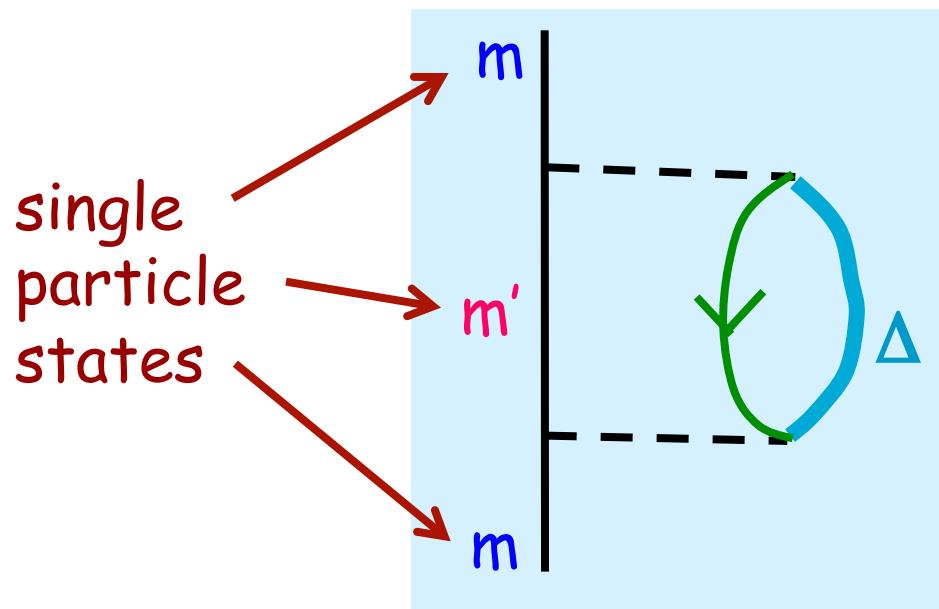
# Renormalization of $NN$ interaction due to $\Delta$ excitation in the intermediate state



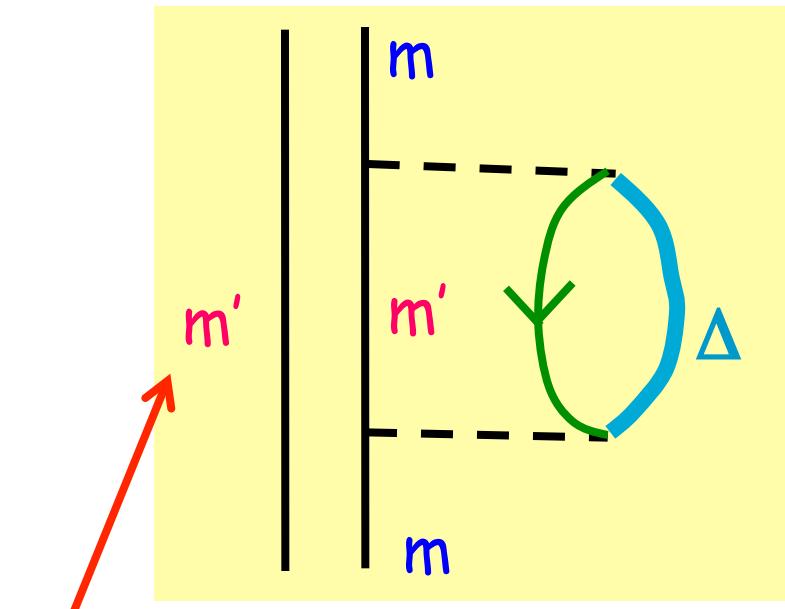
Modification to  
bare  $NN$  interaction  
(for  $NN$  scattering)

$T=1$   
attraction  
between  $NN$   
effectively

# Pauli blocking effect on the renormalization of single-particle energy



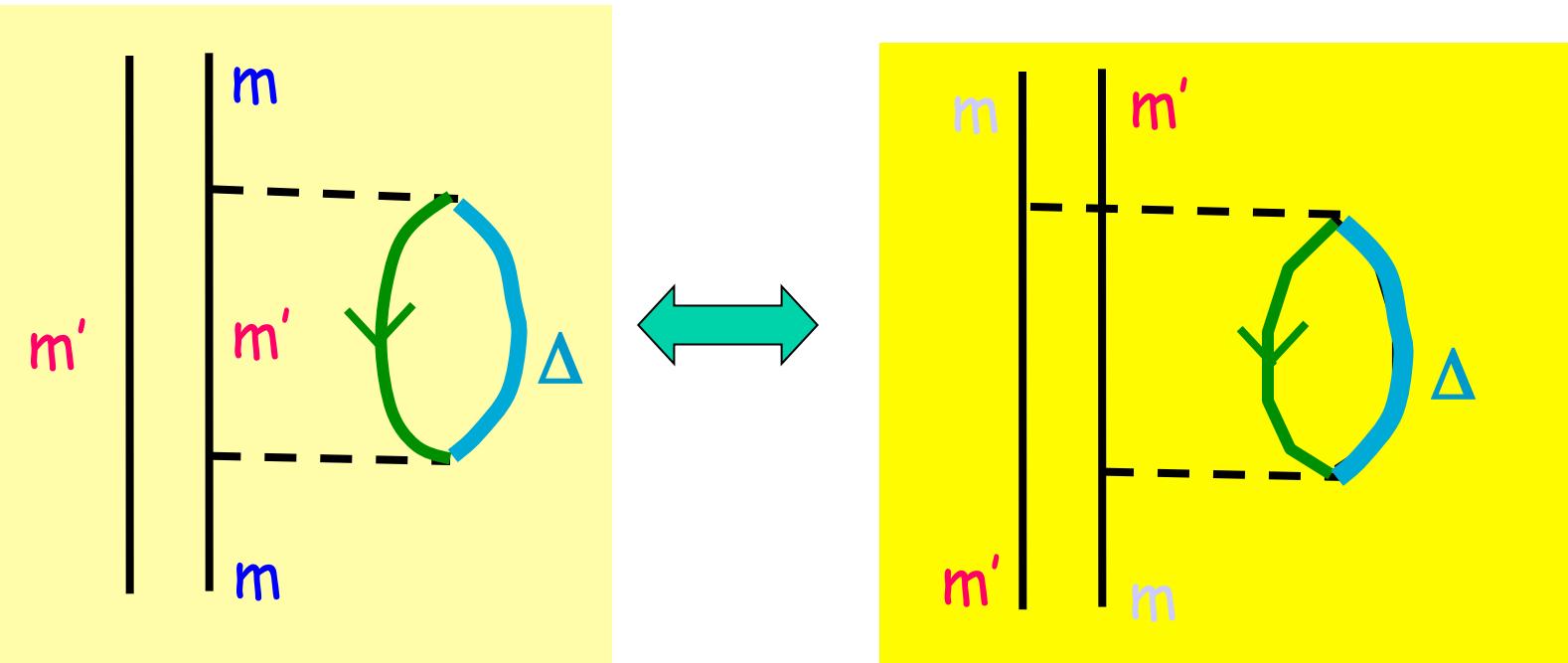
Renormalization of  
single particle energy  
due to  
 $\Delta$ -hole excitation  
→ more binding (attractive)



Another valence  
particle in state  $m'$

Pauli Forbidden  
→ The effect is  
suppressed

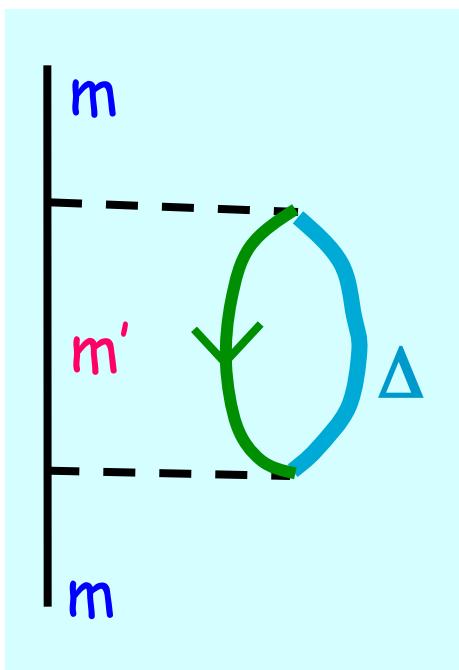
# Inclusion of Pauli blocking



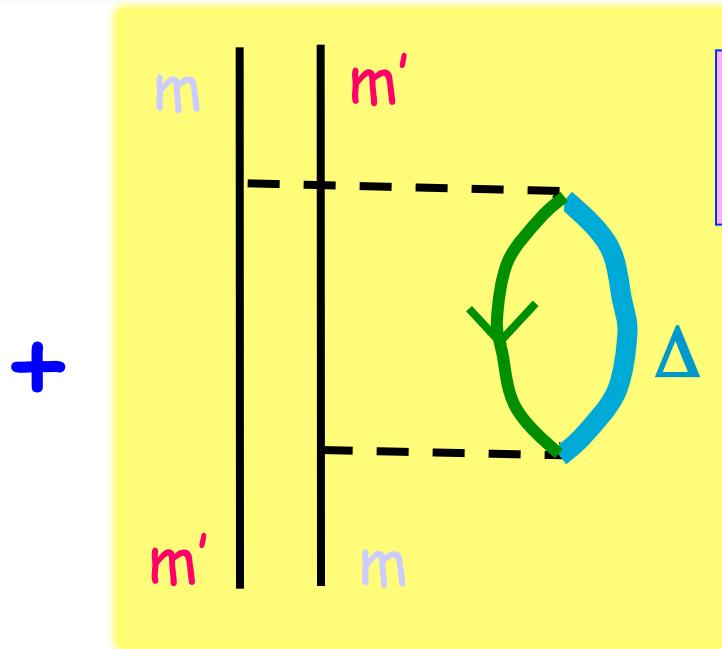
Pauli forbidden  
(from previous page)

This Pauli effect is  
included automatically  
by the exchange term.

# Most important message with Fujita-Miyazawa 3NF



Renormalization  
of single particle  
energy



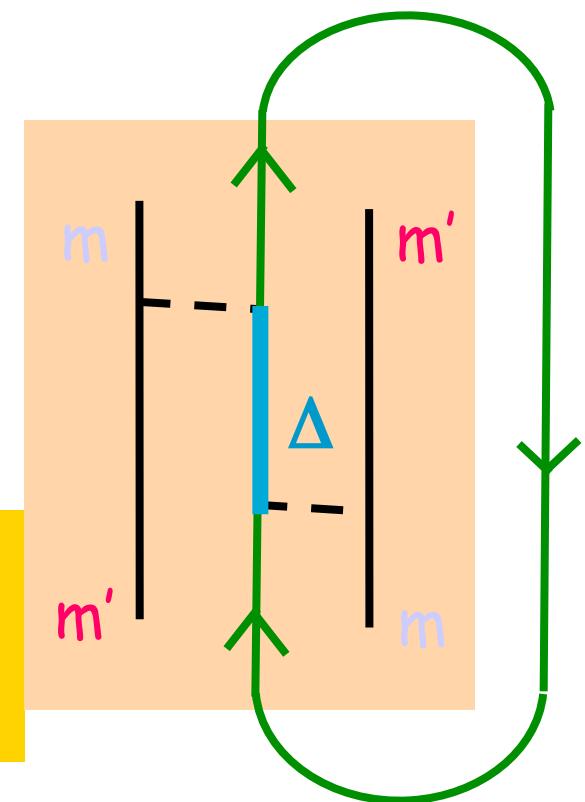
Pauli blocking

Monopole part of  
Fujita-Miyazawa  
3-body force

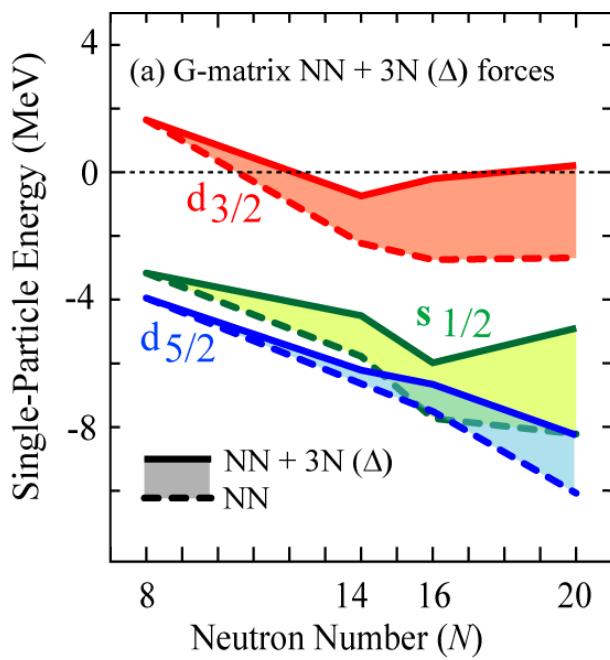
Effective monopole  
repulsive interaction



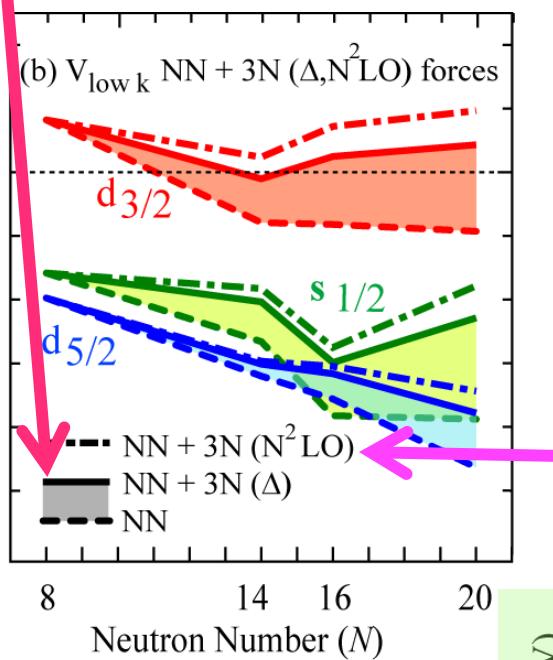
same



(i)  $\Delta$ -hole excitation in a conventional way

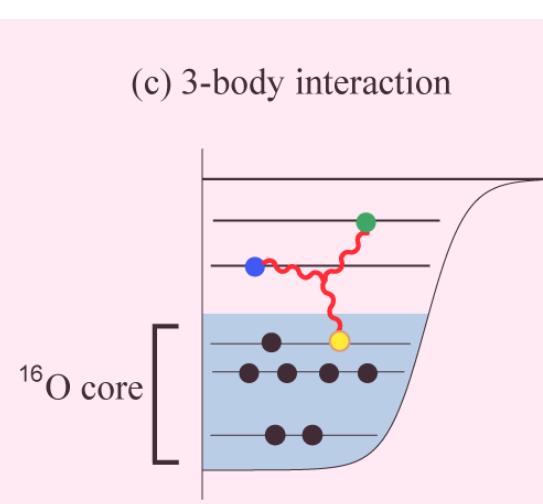


(ii) EFT with  $\Delta$

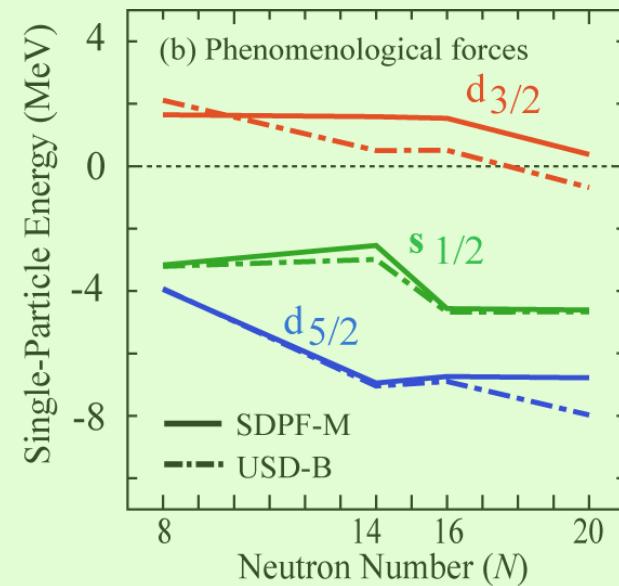
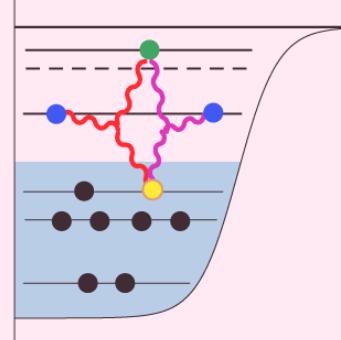


$\Delta$ -hole dominant role in determining oxygen drip line

(iii) EFT incl. contact terms ( $N^2\text{LO}$ )

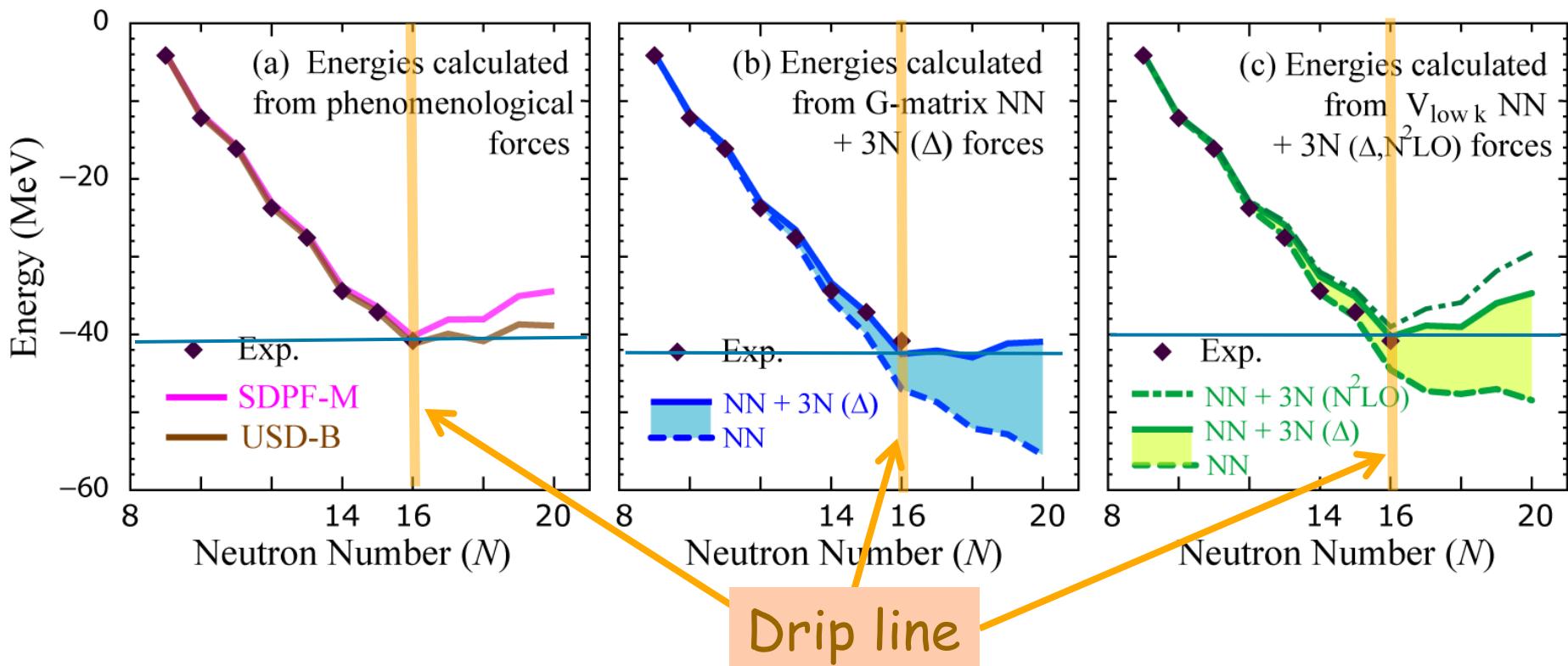
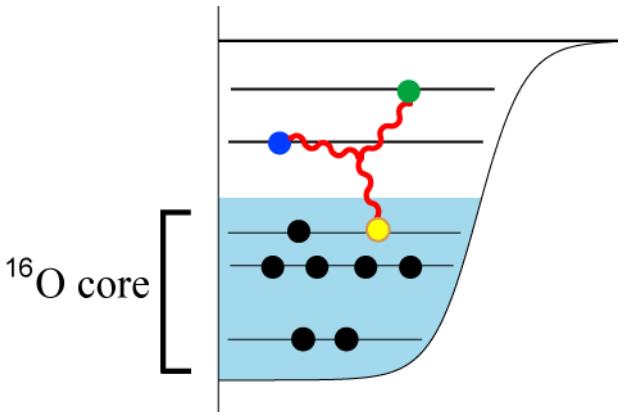


(d) 3-body interaction with one more neutron added to (c)



# Ground-state energies of oxygen isotopes

NN force + 3N-induced NN force  
(Fujita-Miyazawa force)



# Conventional calculation with $\pi N\Delta$ coupling

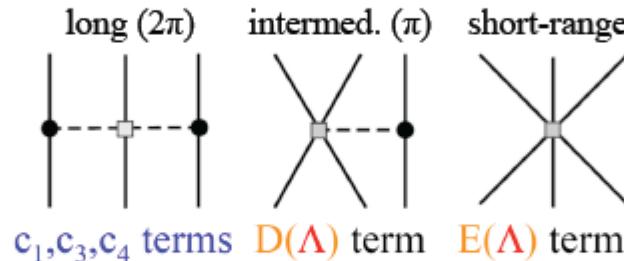
$\pi$  exchange with radial cut-off at 0.5 fm ,  $\Delta E = 293$  MeV

$$f_{\{\pi N\Delta\}}/f_{\{\pi NN\}} = \sqrt{9/2}$$

A.M. Green, Rep. Prog. Phys. 39, 1109 (1976)

## Low-momentum 3N interactions

from leading N<sup>2</sup>LO chiral EFT  $\sim (Q/\Lambda)^3$  van Kolck (1994), Epelbaum et al. (2002)



$c_i$  from  $\pi N$ , consistent with NN  
Meissner (2007)

$$c_1 = -0.9^{+0.2}_{-0.5}, c_3 = -4.7^{+1.2}_{-1.0}, c_4 = 3.5^{+0.5}_{-0.2}$$

$c_3, c_4$  important for structure, large uncertainties at present

NN for smooth cutoff  $V_{\text{low } k}$  ( $n_{\text{exp}}=4$ ) from N<sup>3</sup>LO(500)

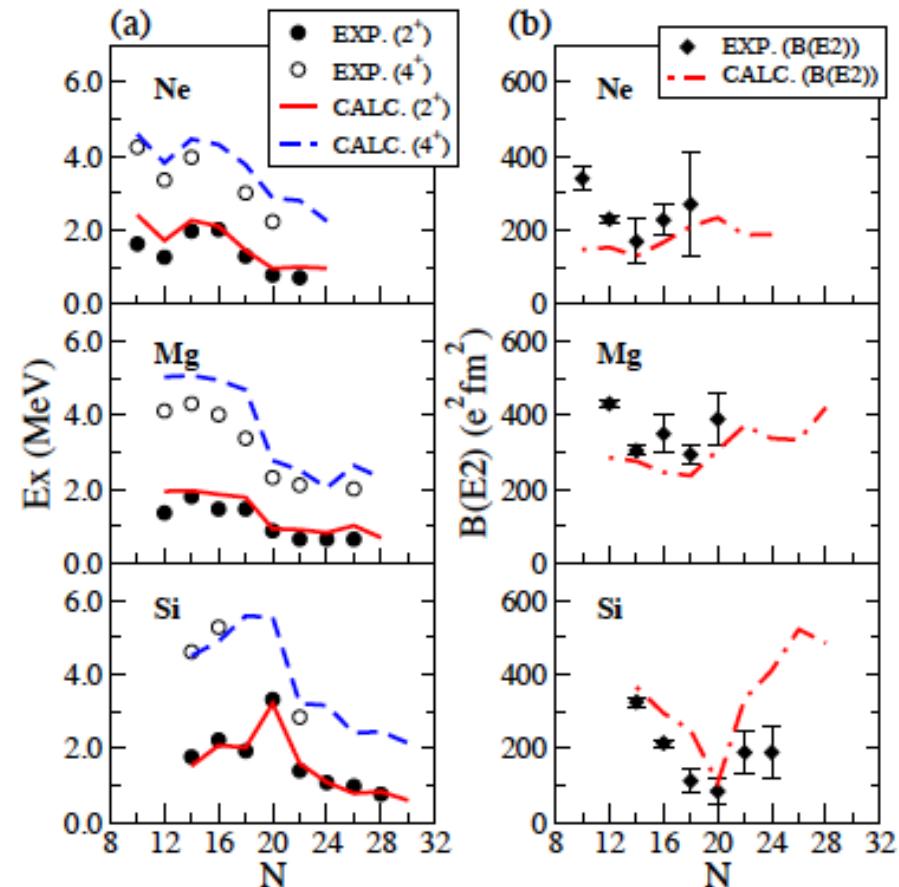
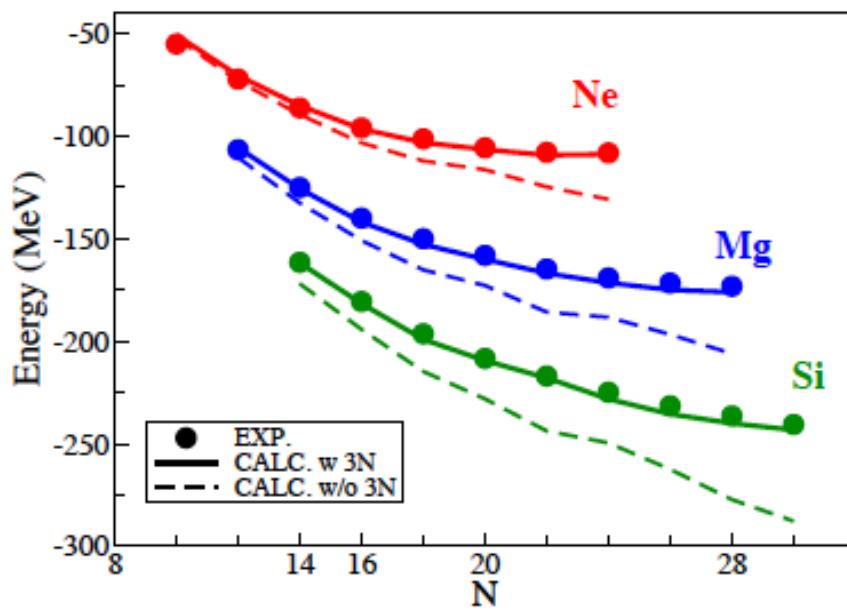
D, E terms fitted to E(3H) and radius(4He)

# Ne-Mg-Si in the sd-pf shell

## Prototype of future shell-model calculation

New & preliminary

Chiral N3LO NN interaction  
+ EKK (Extended Kuo-Krenciglowa) method  
+ three-body Fujita-Miyazawa force

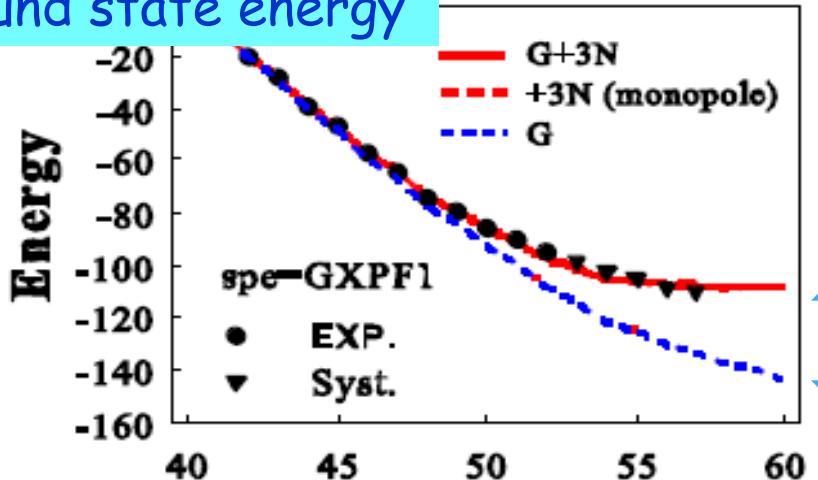


In preparation, Tsunoda, Shimizu, Otsuka,  
H.-Jensen, Takayanagi, and Suzuki

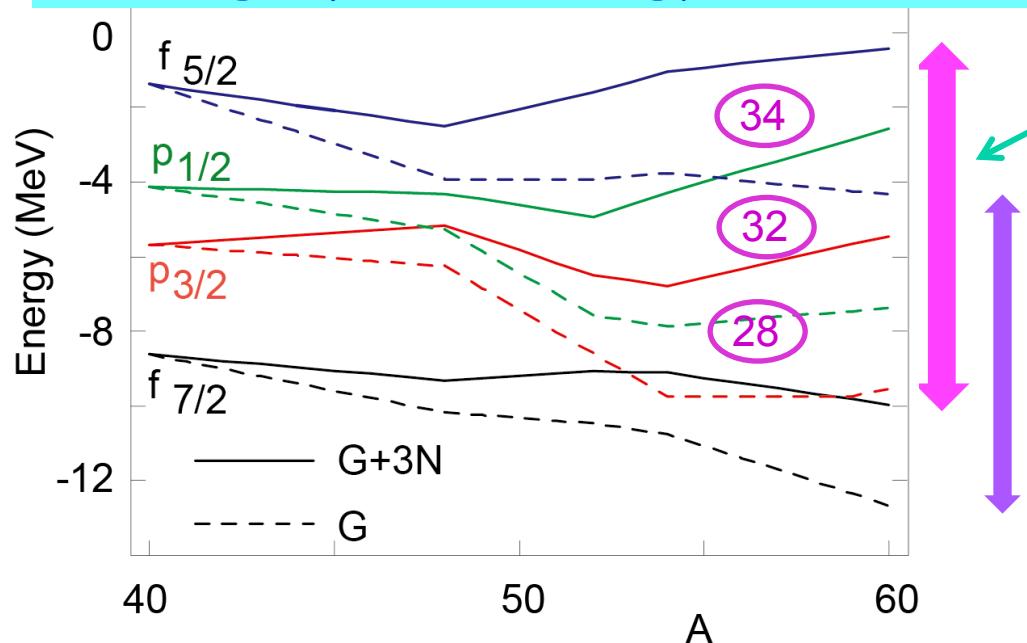
# Ca isotopes

Nuclear Structure and Dynamics 2012  
Proceedings, Suzuki, Otsuka and Honma

ground state energy



eff. single-particle energy of neutrons



G-matrix (revised)

3<sup>rd</sup> order Q-box

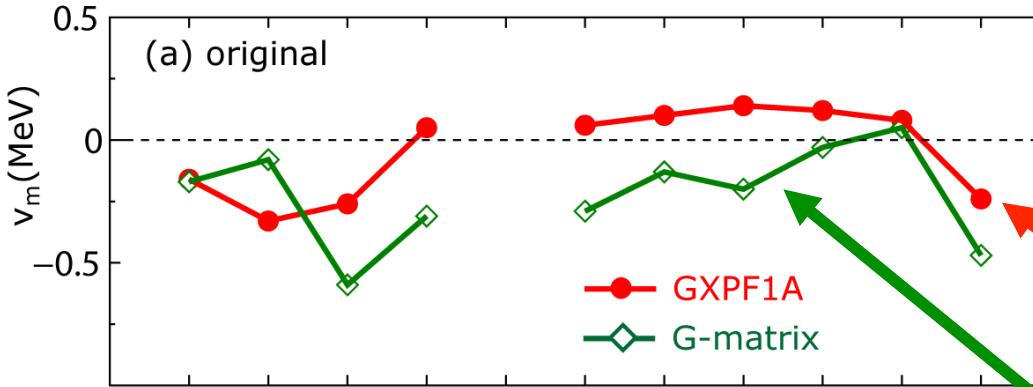
24 hw int. states

3NF : Fujita-Miyazawa

3-5 MeV rising  
of SPEs

=> ~40 MeV shift  
of B.E. of  $^{60}\text{Ca}$   
from 2NF result

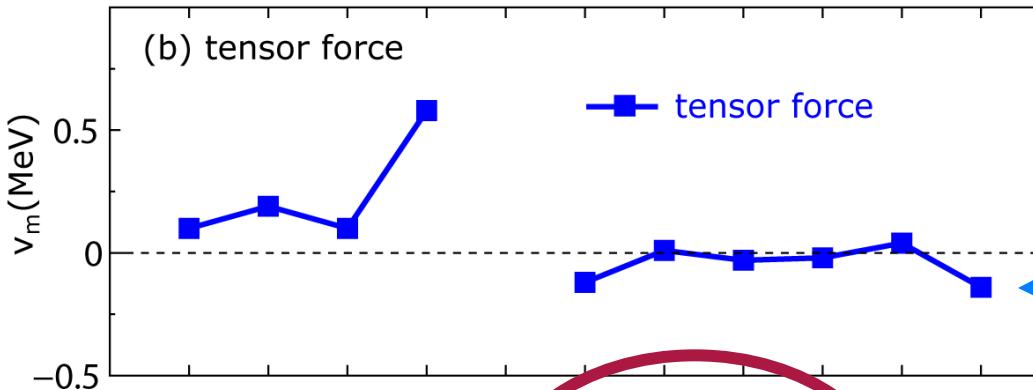
2 MeV enlargement of  
full shell height  
(anti-quenching)



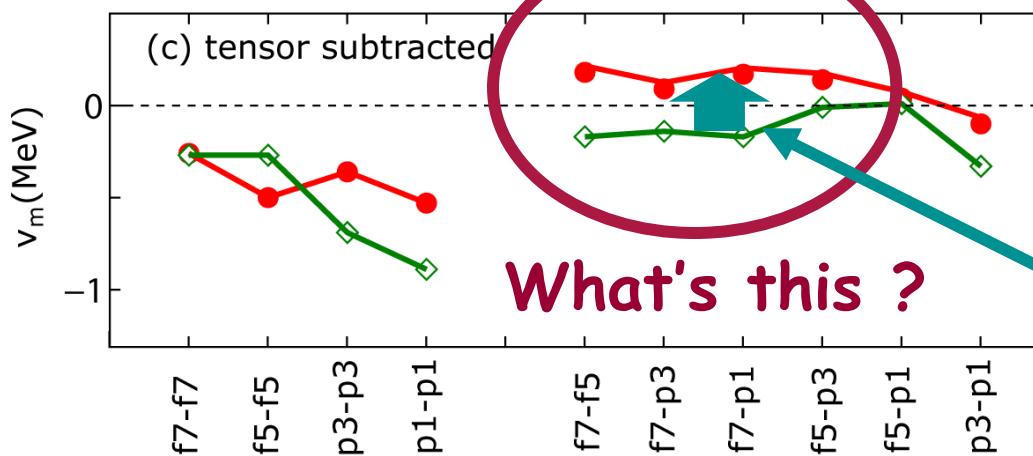
$T=1$  monopole interactions in the pf shell

GXPFI1A

G-matrix  
(H.-Jensen)



Tensor force  
( $\pi+\rho$  exchange)



Basic scale  
~ 1/10 of  $T=0$

Repulsive corrections to G-matrix

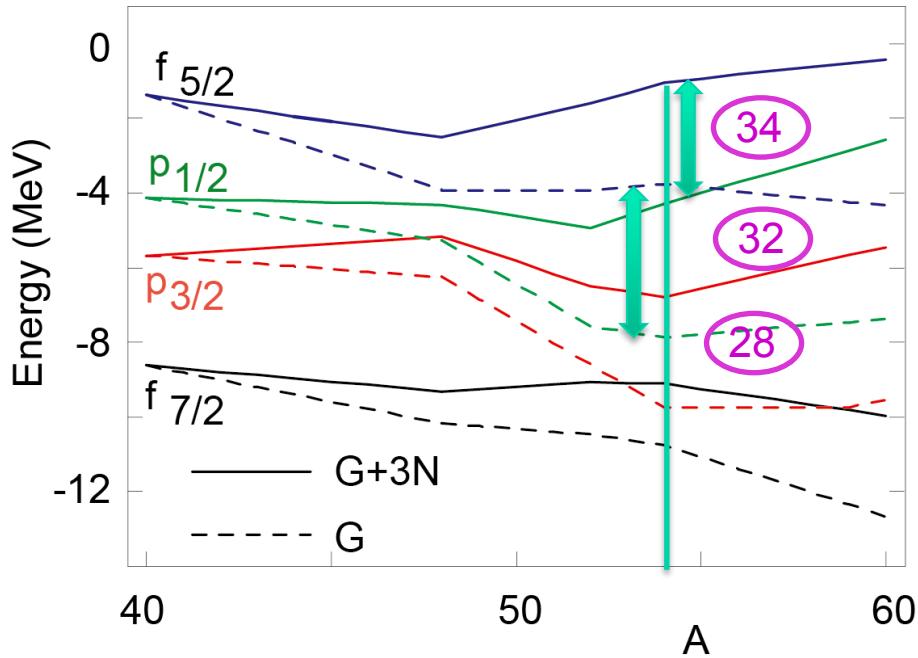
$j = j'$

$j \neq j'$

# 3-body forces does produce another shell evolution

## Ca isotopes

Neutron single-particle energy of Ca isotopes



Three-body force  
reduces  
 $f_{5/2} - p_{1/2}$  gap  
by pushing up  $p_{1/2}$

G-matrix  
3<sup>rd</sup> order Q-box  
24 hw int. states  
3NF : Fujita-Miyazawa

# Three-body forces and shell structure in calcium isotopes

Jason D Holt<sup>1,2</sup>, Takaharu Otsuka<sup>3,4</sup>, Achim Schwenk<sup>5,6</sup>  
and Toshio Suzuki<sup>7</sup>

## Ground-state energy of Ca isotopes

exp + extrap.

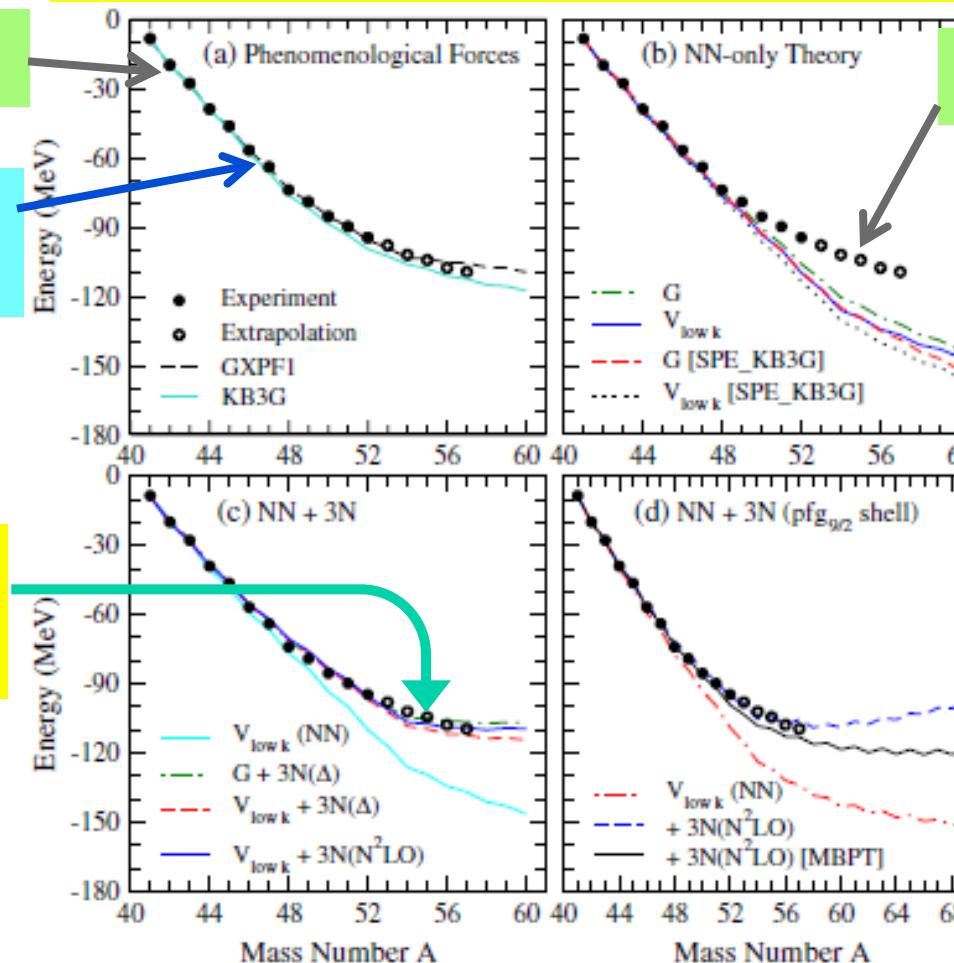
Phenomenological  
NN

Microscopic  
NN + 3NF

exp + extrap.

Microscopic  
NN

+  $g_{9/2}$



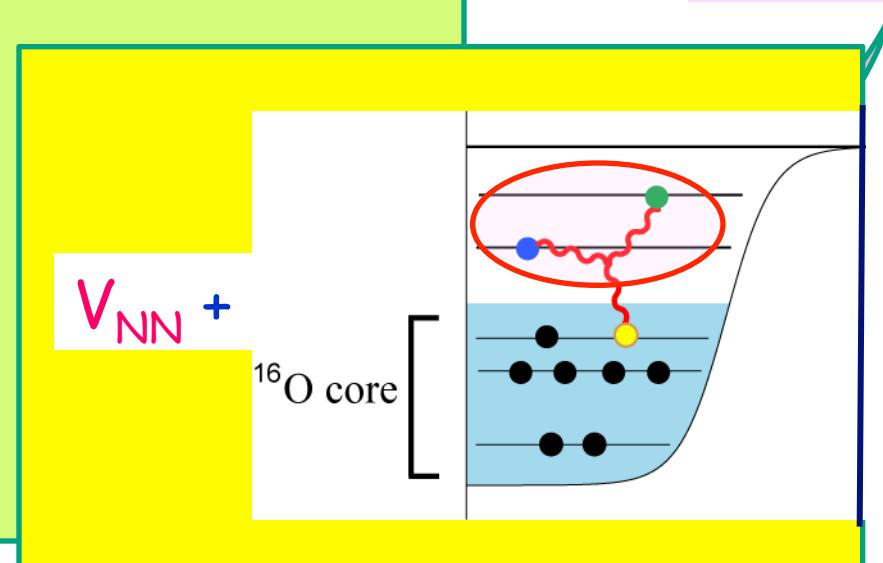
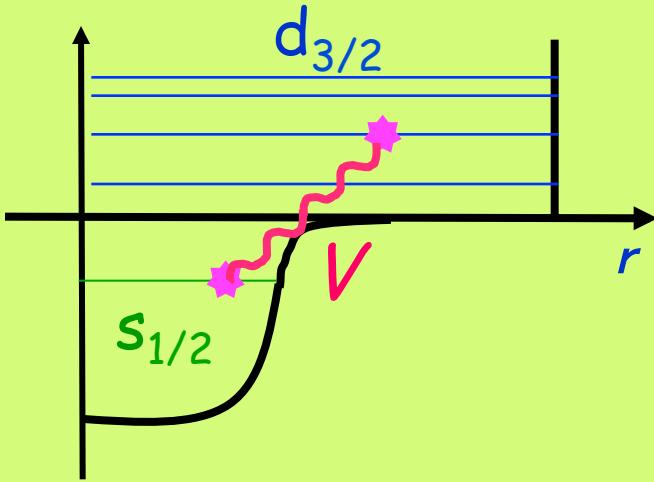
# Continuum-coupled shell model (CCSM)

Hamiltonian :  $H = H_0 + \hat{V} = \sum_j \tilde{\epsilon}_j n_j + \hat{V}$

$H_0 = T + U_{WS} + V_{wall} = \sum_j \tilde{\epsilon}_j n_j$

approximated  
by Gaussian

basis state-vector (denoted by  $j$ ):  
bound states + discretized continuum states  
wall very far (3000 fm, ~3000 basis states)



included

$$\hat{V}(r) = \sum_{i=1,2} g_i (1 + a_i \sigma \cdot \sigma) e^{-r^2/d_i^2}$$

$$d_{1,2}=1.4, 0.7 \text{ fm}$$

SDPF-M TBME = TBME of this  $V(r)$   
for HO wave functions

$$\langle 1s_{1/2} 0d_{3/2} | V | 1s_{1/2} 0d_{3/2} \rangle_{J=1,2}$$

$$\langle 0d_{3/2} 0d_{3/2} | V | 0d_{3/2} 0d_{3/2} \rangle_{J=0,2}$$

under the assumption that 3-body force effect  
is included in SDPF-M interaction effectively

$V(r)$  is fixed only by interaction

$^{24}\text{O} = ^{22}\text{O} + 2\text{n}$  in the space

ground state :  $2\text{n}$  in  $1s_{1/2}$

excited states of  $1^+$  and  $2^+$ :

$$|iJ^+\rangle = |1s_{1/2} \otimes id_{3/2}; J^+\rangle$$



discretized continuum  $id_{3/2}$  ( $i = 1, 2, \dots$ )

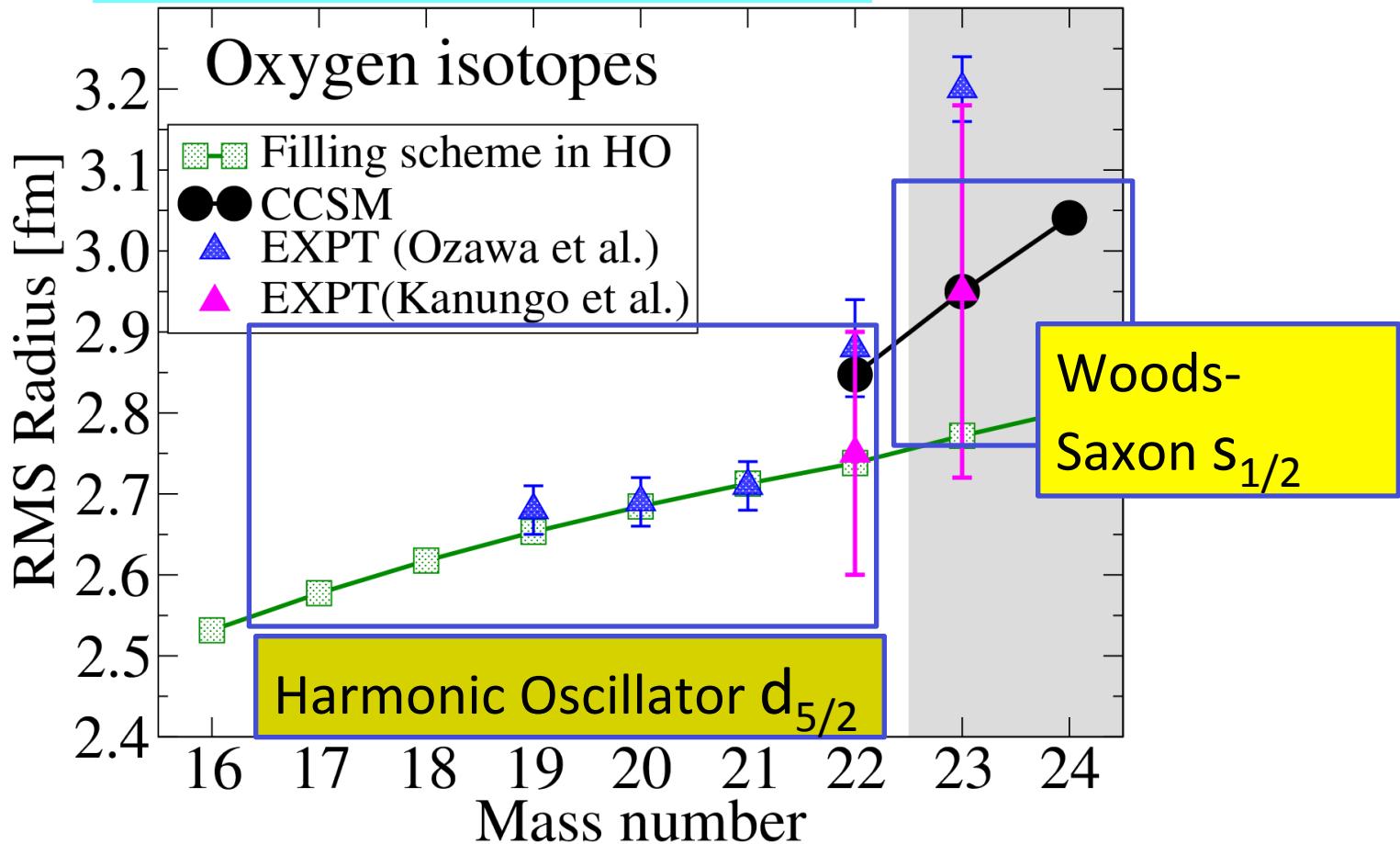
$1s_{1/2}$  : solution of Woods-Saxon potential with observed  $S_n$

diagonalize  $H$

Eigenfunction :

$$|J_k^+\rangle = \sum_i c_i^{(J,k)} |iJ^+\rangle.$$

# RMS Radius: $^{16-24}\text{O}$



Exp: Ozawa et al., Nucl. Phys. A693, 32 (2001)

Kanungo et al., Phys. Rev. C84, 061304 (2011)

# Removal of one proton and one neutron from $^{26}\text{F}$

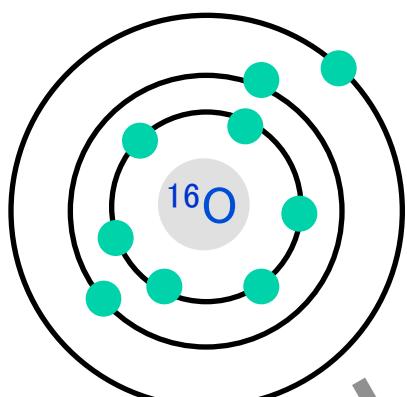
$^9\text{Be}(^{26}\text{F}, ^{24}\text{O})X$

C. Hoffman,  
M. Thoennessen et al.

knockout reaction @MSU (2009)

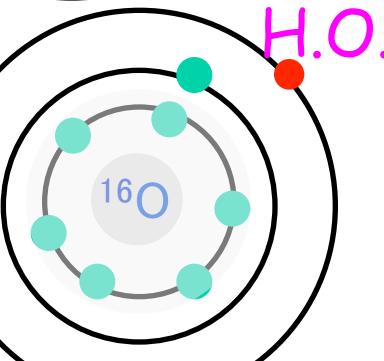
less probable

<= large  $s_{1/2}$ - $d_{3/2}$  neutron gap



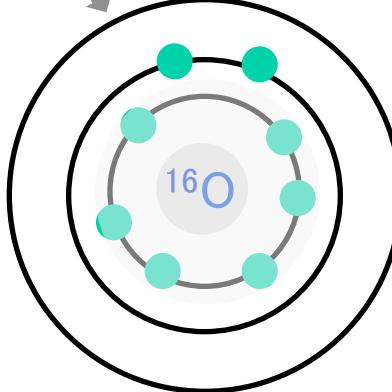
bound nucleus  
 $^{26}\text{F}$

-p  
-n

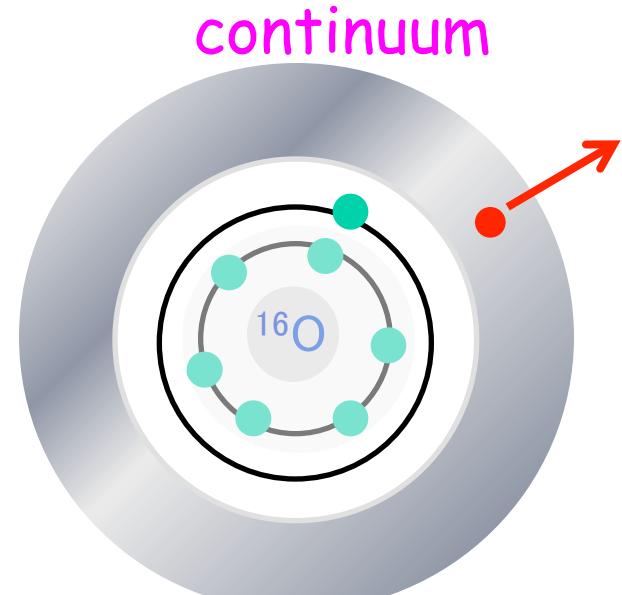


doorway state

$$|1s_{1/2}0d_{3/2}; J_k^+\rangle$$



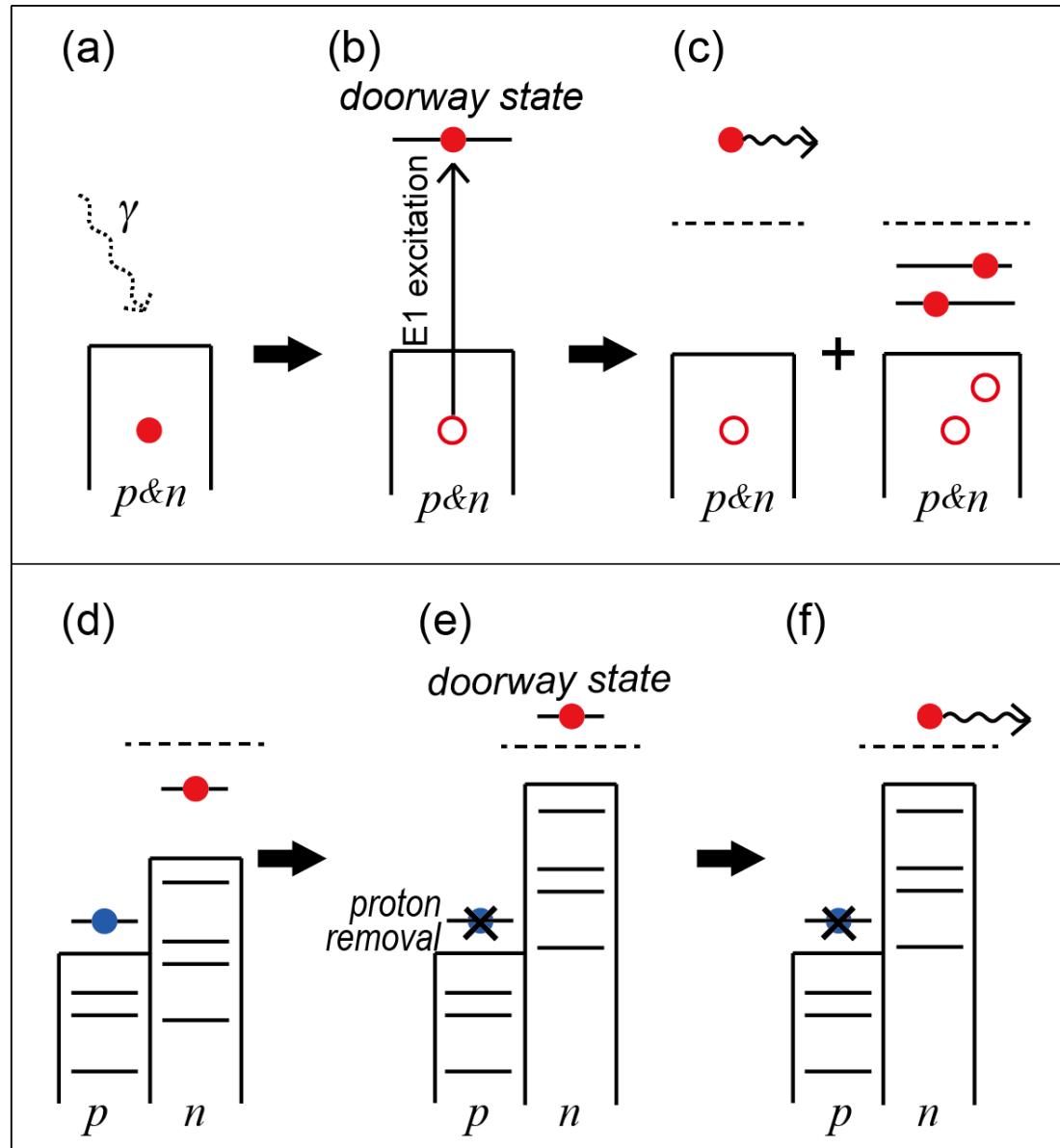
ground state  
 $1s_{1/2}$  is bound.  
Kanungo et al. (2009)



excited states in  $^{24}\text{O}$

$$H^{\text{CCSM}} |J_k^+\rangle = E_k |J_k^+\rangle$$

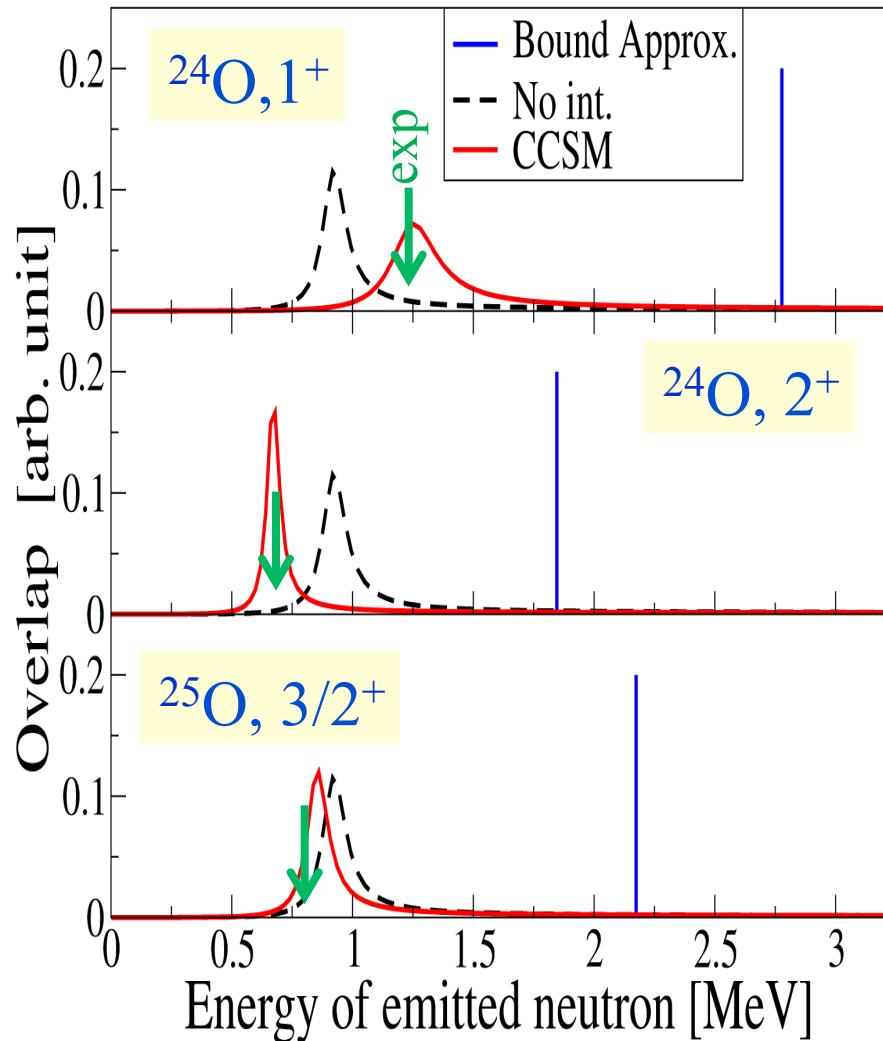
# Doorway state $\rightarrow$ doorway-state resonance



$E1$  excitation

nucleon transfer  
(proton removal)

# Low-lying Continuum Spectra in $^{24,25}\text{O}$



doorway-state resonance

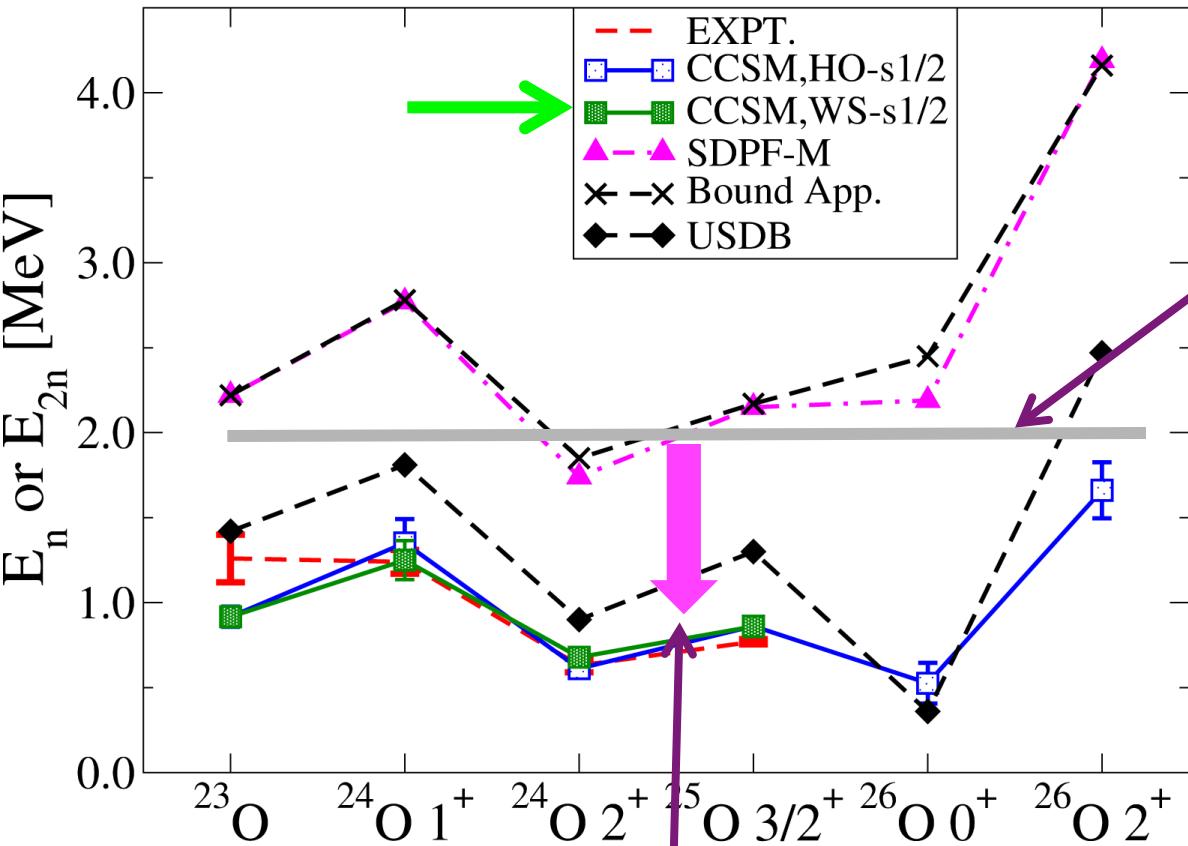
Doorway state ==> continuum states in  $^{24}\text{O}$

$$p_k^J = |\langle J_k^+ | \Phi_{\text{doorway}} \rangle|^2 = \left| \sum_i C_i^{(k)} \langle i d_{3/2} | 0 d_{3/2} \rangle \right|^2$$

- **bound approximation:** Normal shell model with the same Hamiltonian : NO continuum effect
- **CCSM :** With continuum effect  
*incl. residual interaction*
- **no int.** : With continuum effect but  
*no residual interaction.*

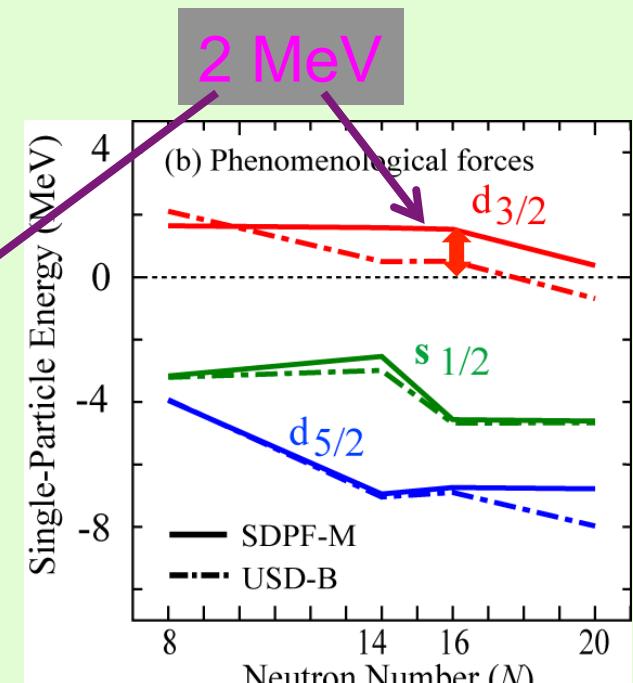
- Continuum effect is about 1 MeV
- No bound excited state.
- $1^+$ - $2^+$  splitting by 2-body interaction
- $1^+$ - $2^+$  splitting is in good agreement with experiments.

## Peak Energies of neutron emission



Lowering due to continuum effect

## SPE as bound state



Exp. : MSU (Hoffman et al),  
RIKEN (Elekes et al)

Continuum spectra are consistent with the shell evolution

Oxygen isotopes



Fluorine isotopes

# Neutron single-particle energies at N=20 for Z=8~20

solid line : full (central + tensor)

dashed line : central only

A proton in  $d_{5/2}$  moves neutron orbits by

$d_{3/2}$  -2.0 MeV

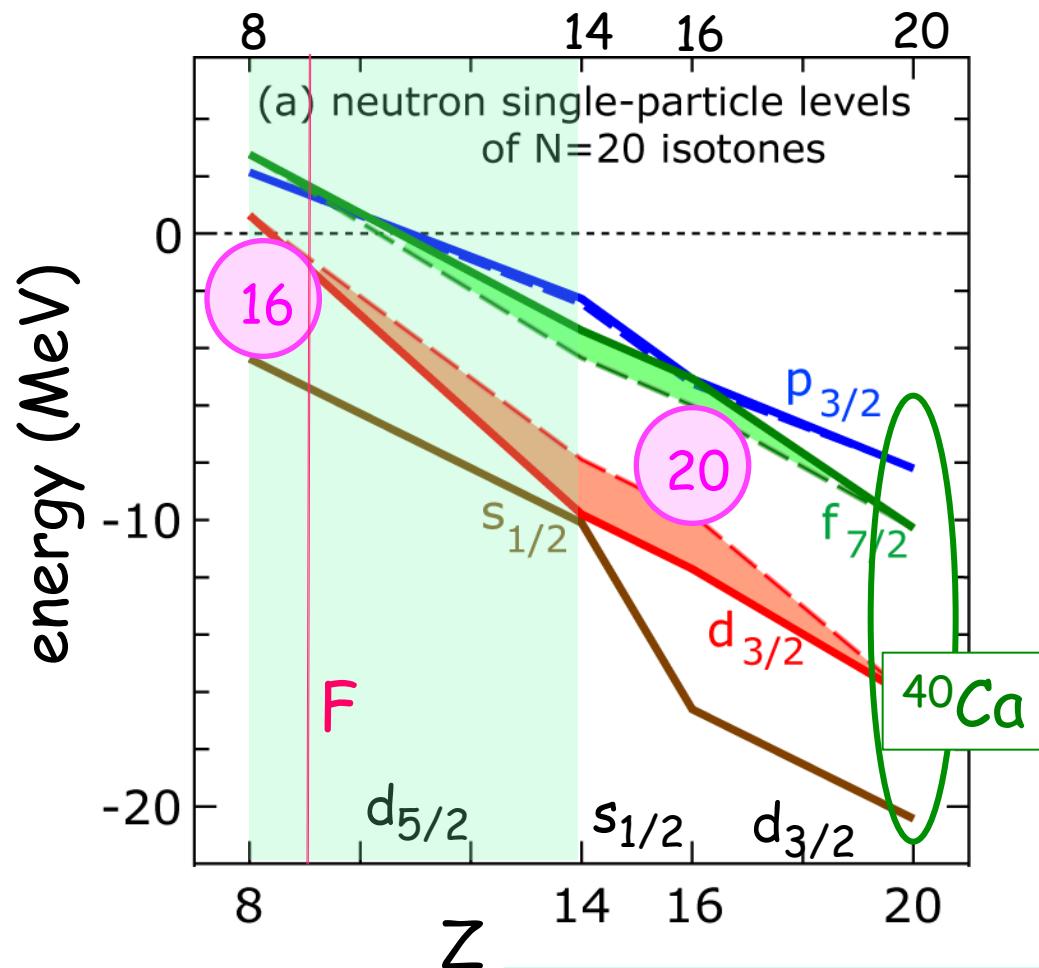
$s_{1/2}$  -1.1 MeV

$d_{5/2}$  -1.6 MeV



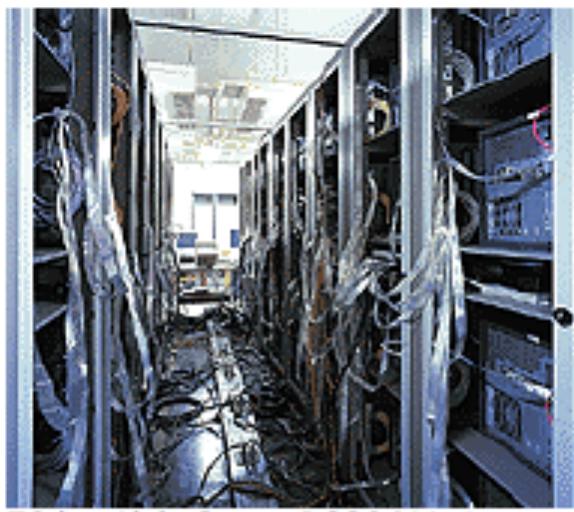
$^{29}\text{F}$  well bound already by s. p. e.

$^{31}\dots\text{F}$  bound through mixing with pf shell



# Outline

1. Introduction
2. Shell model and monopole interaction
3. Shell evolution and tensor force
4. Multiple quantum liquid in exotic nuclei
5. Shell evolution and three-nucleon force
6. Monte Carlo Shell Model
7. Summary



70台のAlphaServer DS20をMyrinetで  
高速に並列結合し、最大のパラレル処理  
パフォーマンスを発揮するAlphleet

理化学研究所



クラスタ化した70台(140CPU)のAlphaServerとCompaq Tru64 UNIXが高い並列処理パフォーマンスを実現し、ピックパン以降の原子核の構造解明を推進

## Major outcome

PHYSICAL REVIEW C, VOLUME 60, 054315

1999

### Varying shell gap and deformation in $N \sim 20$ unstable nuclei studied by the Monte Carlo shell model

Yutaka Utsuno,<sup>1,2</sup> Takaharu Otsuka,<sup>1,2</sup> Takahiro Mizusaki,<sup>1</sup> and Michio Honma<sup>3</sup>

Computations have been carried out partly by the Alphleet  
computer system of RIKEN.



Physics Letters B 346 (1995) 9–14

### Large deformation of the very neutron-rich nucleus $^{32}\text{Mg}$ from intermediate-energy Coulomb excitation

T. Motobayashi<sup>a,1</sup>, Y. Ikeda<sup>a</sup>, Y. Ando<sup>a</sup>, K. Ieki<sup>a</sup>, M. Inoue<sup>a</sup>, N. Iwasa<sup>a</sup>, T. Kikuchi<sup>a</sup>,  
M. Kurokawa<sup>a</sup>, S. Moriya<sup>a</sup>, S. Ogawa<sup>a</sup>, H. Murakami<sup>a</sup>, S. Shimoura<sup>a</sup>, Y. Yanagisawa<sup>a</sup>,  
T. Nakamura<sup>b</sup>, Y. Watanabe<sup>b</sup>, M. Ishihara<sup>b,c</sup>, T. Teranishi<sup>c</sup>, H. Okuno<sup>c</sup>, R.F. Casten<sup>d</sup>

Advanced  
MCSM

## Present status

PTEP

Prog. Theor. Exp. Phys. 2012, 01A205 (27 pages)  
DOI: 10.1093/ptep/pts012

### New-generation Monte Carlo shell model for the K computer era

Noritaka Shimizu,<sup>1,\*</sup>, Takashi Abe<sup>1</sup>, Yusuke Tsunoda<sup>2</sup>, Yutaka Utsuno<sup>3</sup>, Tooru Yoshida<sup>1</sup>,  
Takahiro Mizusaki<sup>4</sup>, Michio Honma<sup>5</sup>, and Takaharu Otsuka<sup>1,2,6</sup>

Project

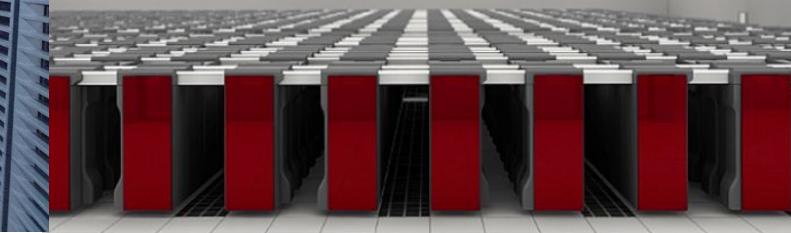
HPCI Strategic Programs for Innovative Research (SPIRE)  
Field 5 “The origin of matter and the universe”

Computer



Present Alphleet system

Large-scale  
calculations



# Outline

- **Methodology:** advanced Monte Carlo shell model (MCSM)
  - intrinsic shape can be the objectives
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- **Shape coexistence** and Quantum Liquid picture
  - exotic Ni isotopes (+ Co, Cu)
- **Shape evolution** (from seniority to rotor)
  - Xe and Ba isotopes
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  - E1 excitation, GDR and PDR in Ca and Sr isotopes
  - Level density with shell model Hamiltonian

# Advanced Monte Carlo shell model (MCSM)

Superposition of the projected Slater determinants  
+ Extrapolation by energy variance

$$|\Psi\rangle = \sum_{k=1}^{N_{MCSM}} f_k P^{J,\pi} |\phi_k\rangle$$

$$|\phi_k\rangle = \prod_{\alpha=1}^N \left( \sum_{i=1}^{N_{sp}} c_i^\dagger D_{i\alpha}^{(k)} \right) |-\rangle$$

MCSM basis, deformed Slater det.

>10<sup>10</sup> basis vectors

$$\mathbf{H} = \begin{pmatrix} * & * & * & * & * & \cdots \\ * & * & * & * & \cdot & \cdots \\ * & * & * & \cdot & \cdot & \cdots \\ * & * & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \end{pmatrix} \xrightarrow{\text{diagonalization}} \begin{pmatrix} \varepsilon_1 & & & & 0 \\ \varepsilon_2 & & & & \\ \varepsilon_3 & & & & \\ \cdot & & & & \\ 0 & & & & \ddots \end{pmatrix}$$

**Conventional Shell Model**  
all Slater determinants

$$\mathbf{H} \approx \begin{pmatrix} * & * & * & \cdot \\ * & * & * & \cdot \\ * & * & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \xrightarrow{\text{diagonalization}} \begin{pmatrix} \varepsilon'_1 & & 0 \\ \varepsilon'_2 & & \\ \cdot & & \\ 0 & & \ddots \end{pmatrix}$$

**Monte Carlo Shell Model**  
bases important for a specific eigenstate

~10<sup>2</sup> basis vectors

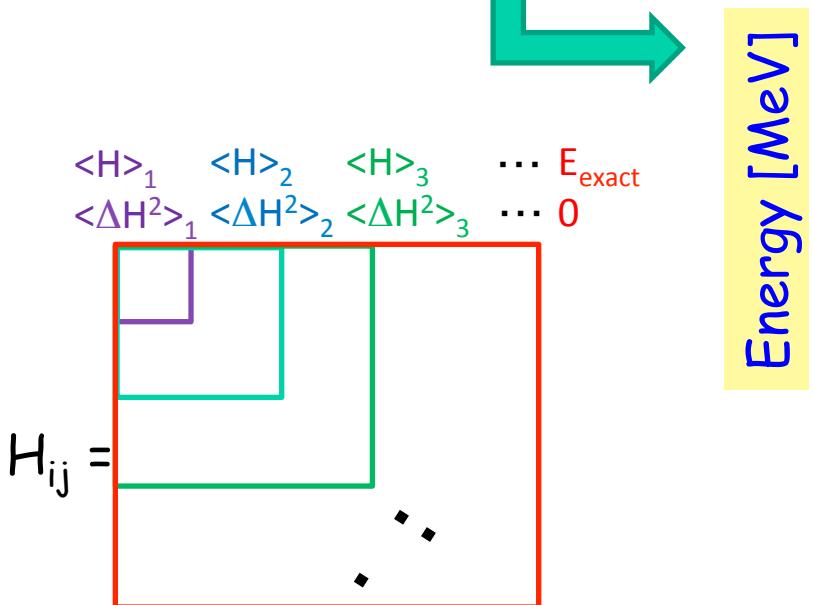
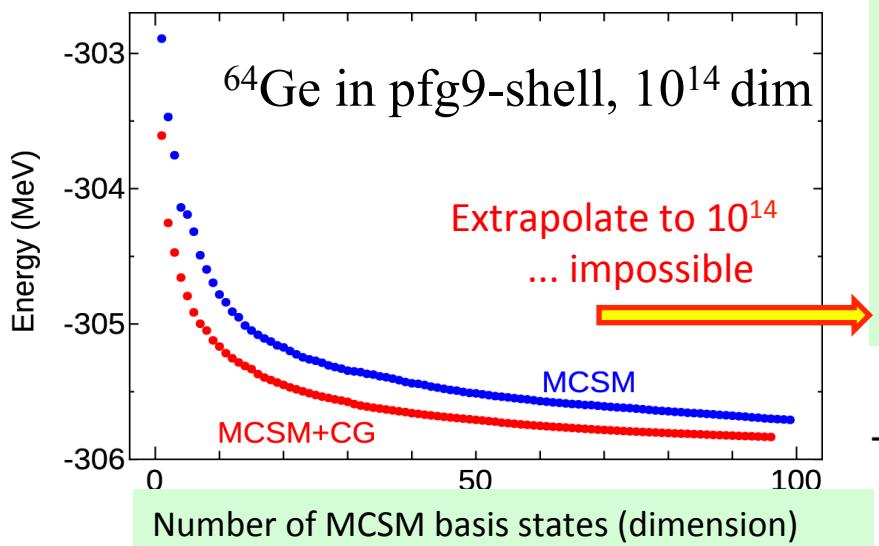
## Step 1

- Select  $D$  stochastically from many candidates generated by the auxiliary-field Monte Carlo technique

## Step 2

- optimize  $D$  variationally by the conjugate gradient method so as to minimize the energy eigenvalue of this small matrix

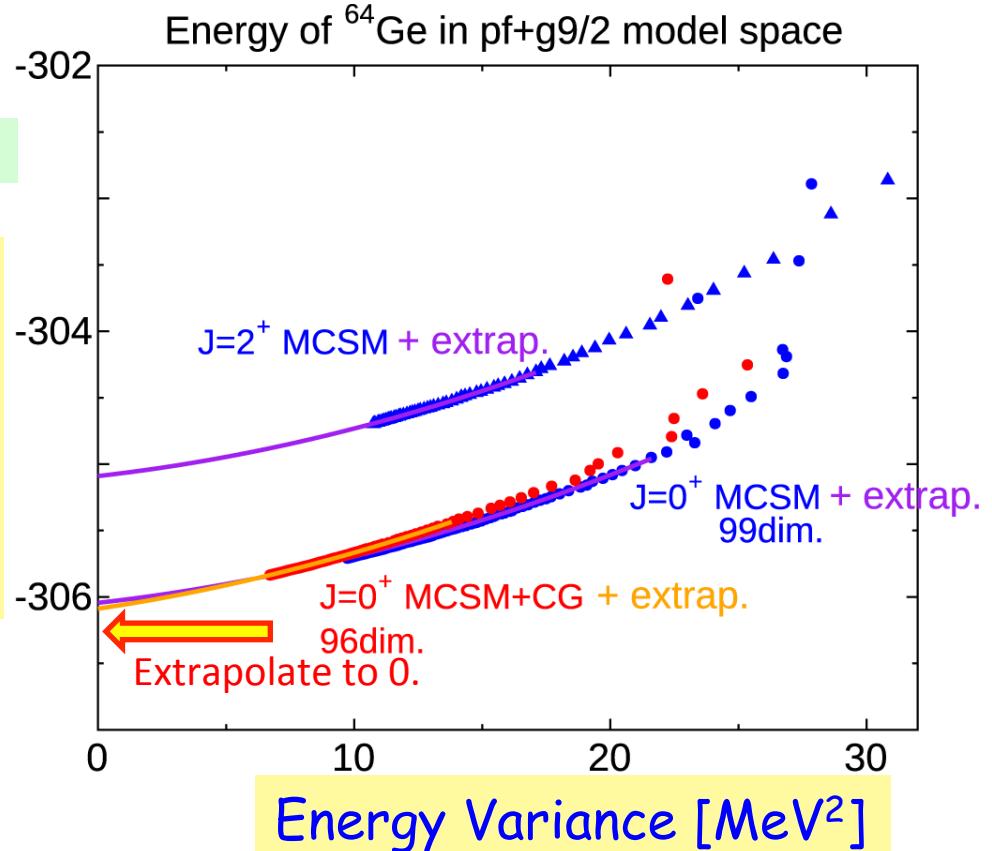
## Step 3 : Energy variance extrapolation



Energy variance:  $\langle \Delta H^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2$

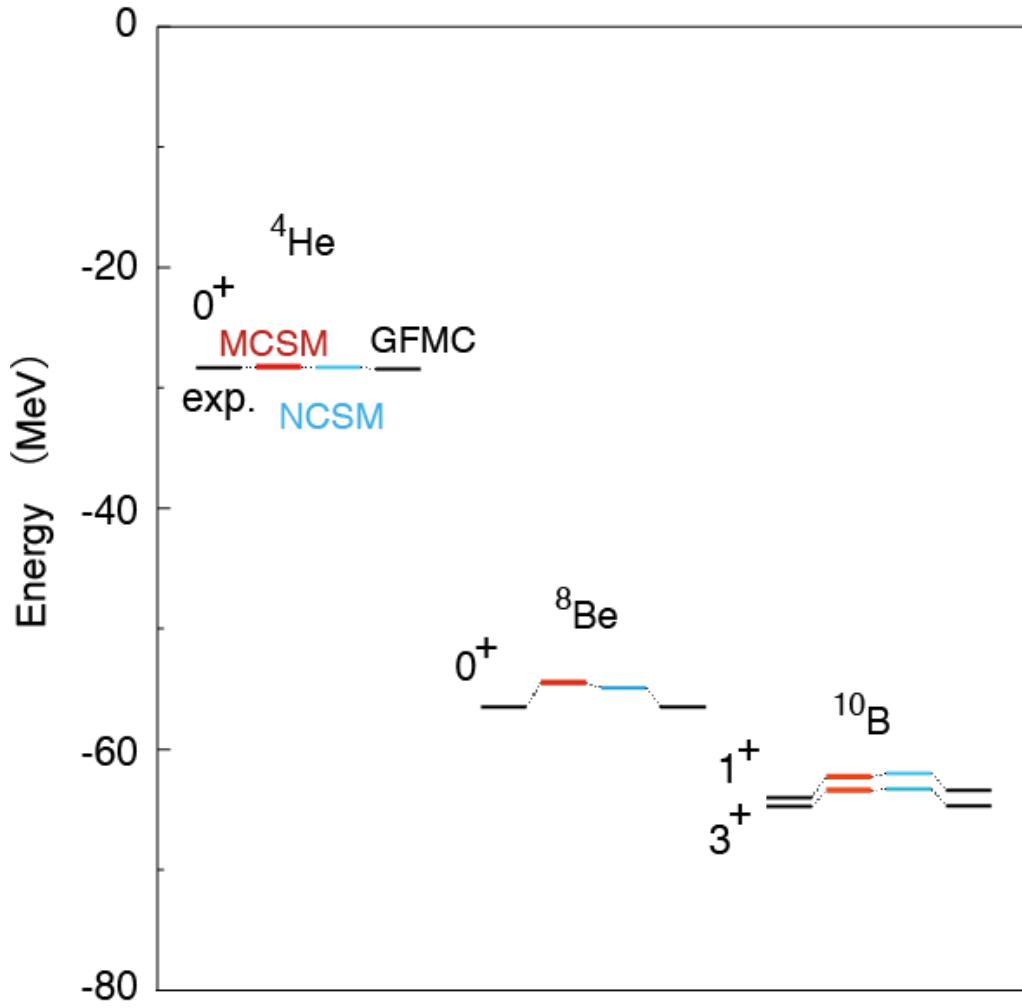
As the number of basis vectors increases, the approximated w.f. approaches the exact one and the energy variance approaches zero.

Extrapolate towards  $\langle \Delta H^2 \rangle \rightarrow 0$



Systematic calculations in terms of  
*ab initio Monte Carlo Shell Model*  
with JISP-16 interaction

Extrapolation to infinite  
model space is included.



MCSM: same as present

NCSM:  
No-core shell model  
with JISP-16

T. Abe, P. Maris, et al.  
PRC 86, 054301 (2012)

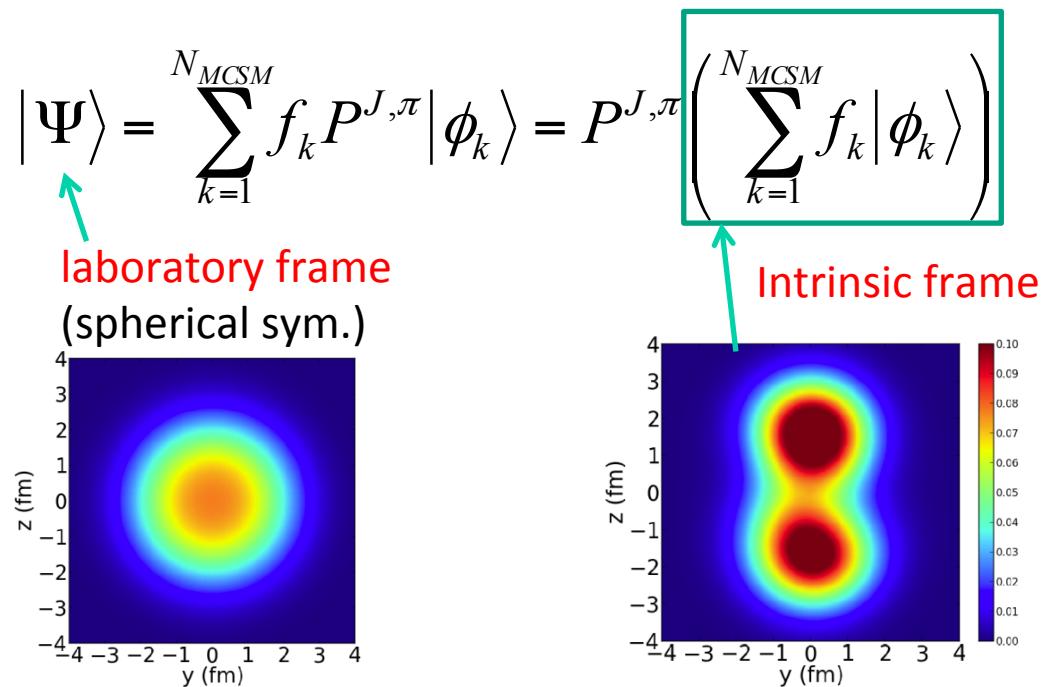
GFMC  
AV18 + IL7  
J. Carlson, et al.,  
arXiv:1412.3081 (2014).

# Ab initio (no-core) MCSM : Be isotopes

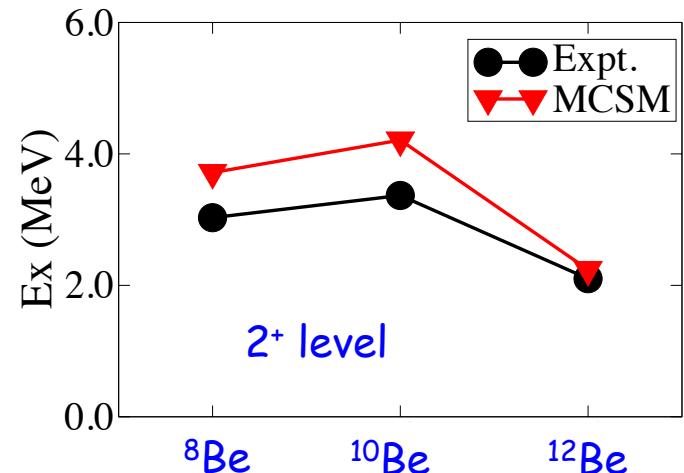
T. Yoshida, T. Abe *et al.*

JISP-16 interaction used

Matter density of the ground state  $J^\pi = 0^+$  of  ${}^8\text{Be}$



Matter density of Be isotopes



Cluster structure emerges spontaneously in  ${}^8\text{Be}$ , and fades away as N increases

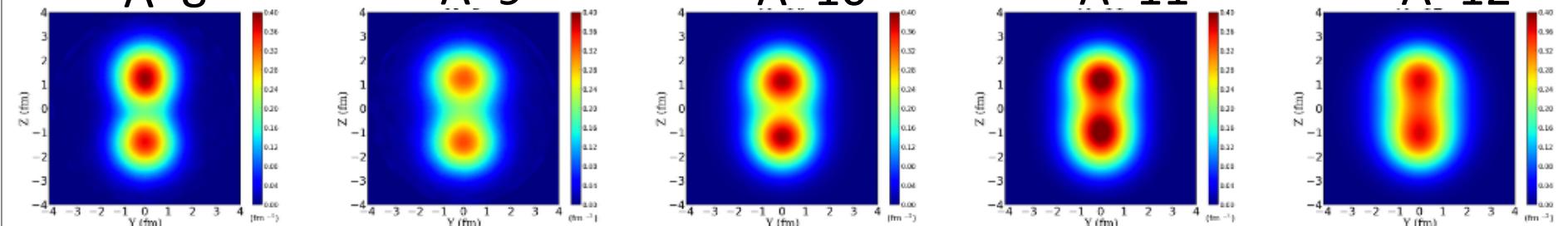
$A=8$

$A=9$

$A=10$

$A=11$

$A=12$



# Alignment of each basis state (Slater determinant)

Before the alignment; orientations are random

$$|\Psi_{B.A.}(D)\rangle = |c_1 \text{ [random orientation]} + c_2 \text{ [random orientation]} + c_3 \text{ [random orientation]} + \dots + c_{98} \text{ [random orientation]} + c_{99} \text{ [random orientation]} + c_{100} \text{ [random orientation]} \rangle$$

All basis states are aligned → “intrinsic state”

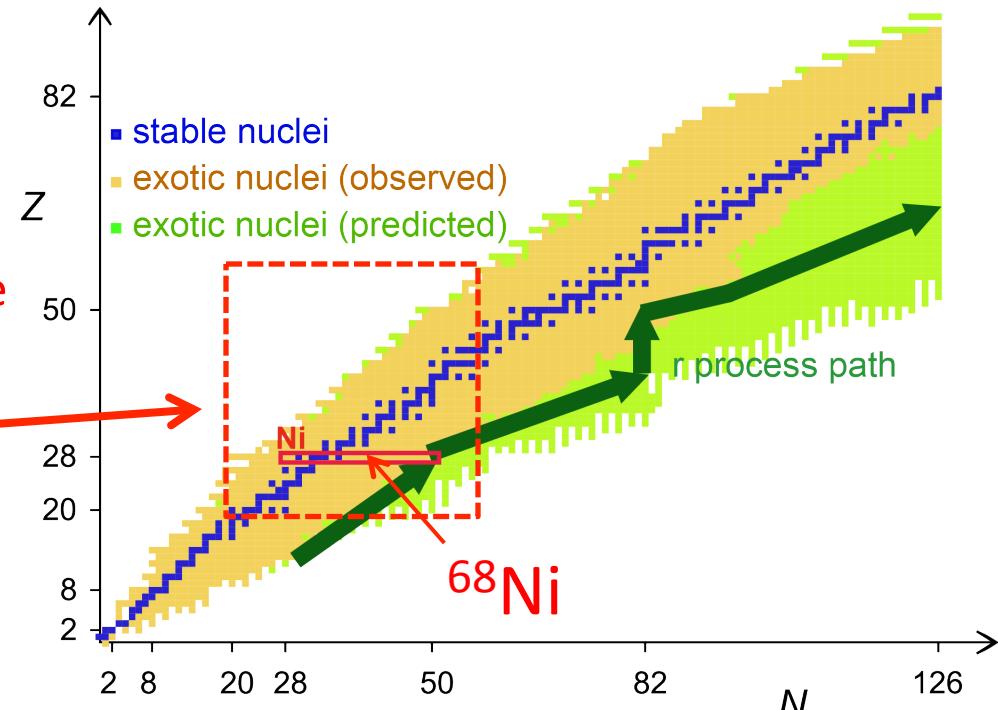
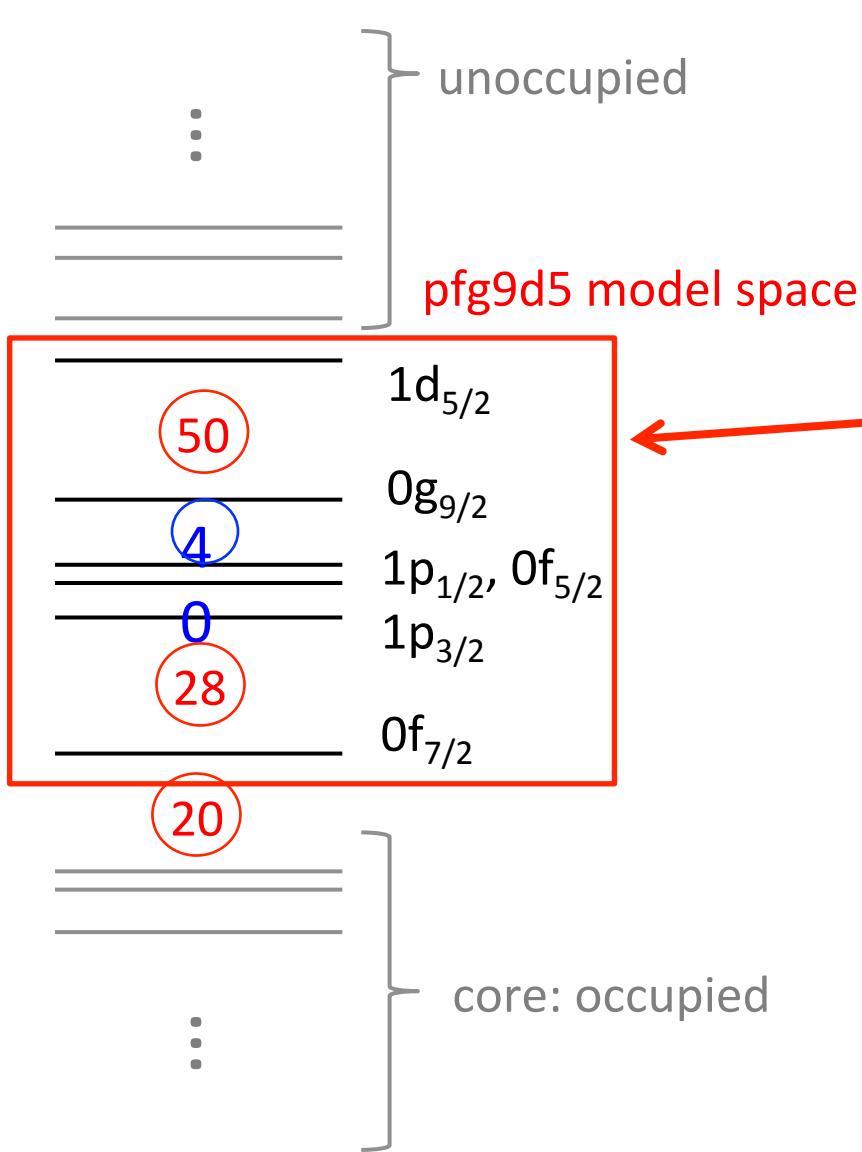
$$|\Psi_{intr}(D)\rangle = |c_1 \text{ [aligned orientation]} + c_2 \text{ [aligned orientation]} + c_3 \text{ [aligned orientation]} + \dots + c_{98} \text{ [aligned orientation]} + c_{99} \text{ [aligned orientation]} + c_{100} \text{ [aligned orientation]} \rangle$$

# Outline

- **Methodology:** advanced Monte Carlo shell model (MCSM)
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  - Level density with shell model Hamiltonian

# MCSM calculation on Ni isotopes

Y. Tsunoda et

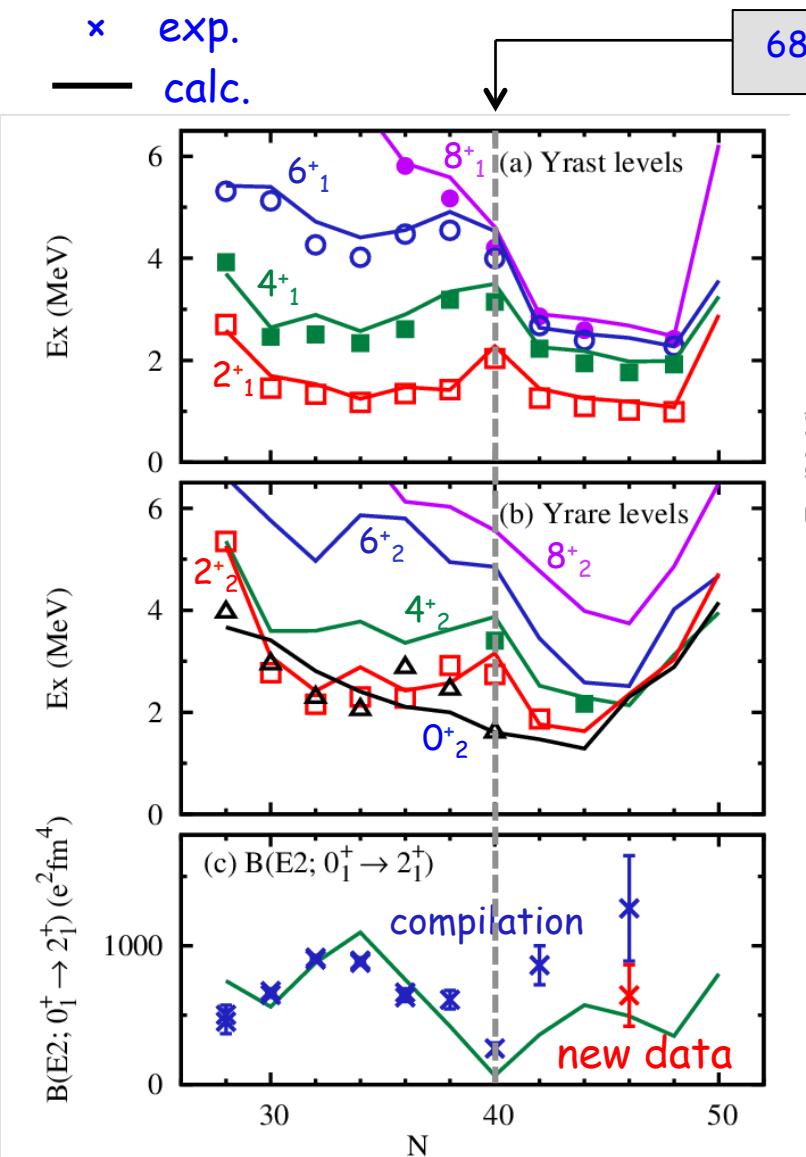


This model space is wide enough to discuss how **magic numbers 28, 50** and **semi-magic number 40** are visible or smeared out.

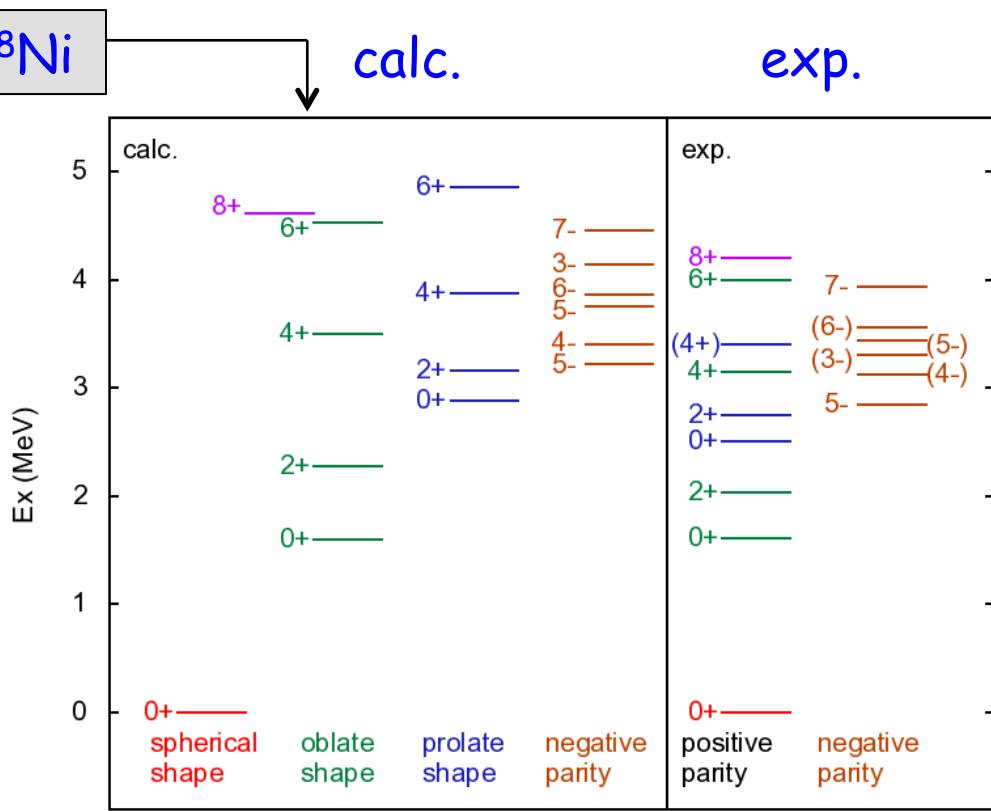
Interaction:  
A3DA interaction is used with minor corrections

# Energy levels and B(E2) values of Ni isotopes

Description by the same Hamiltonian



Shape coexistence in  $^{68}\text{Ni}$



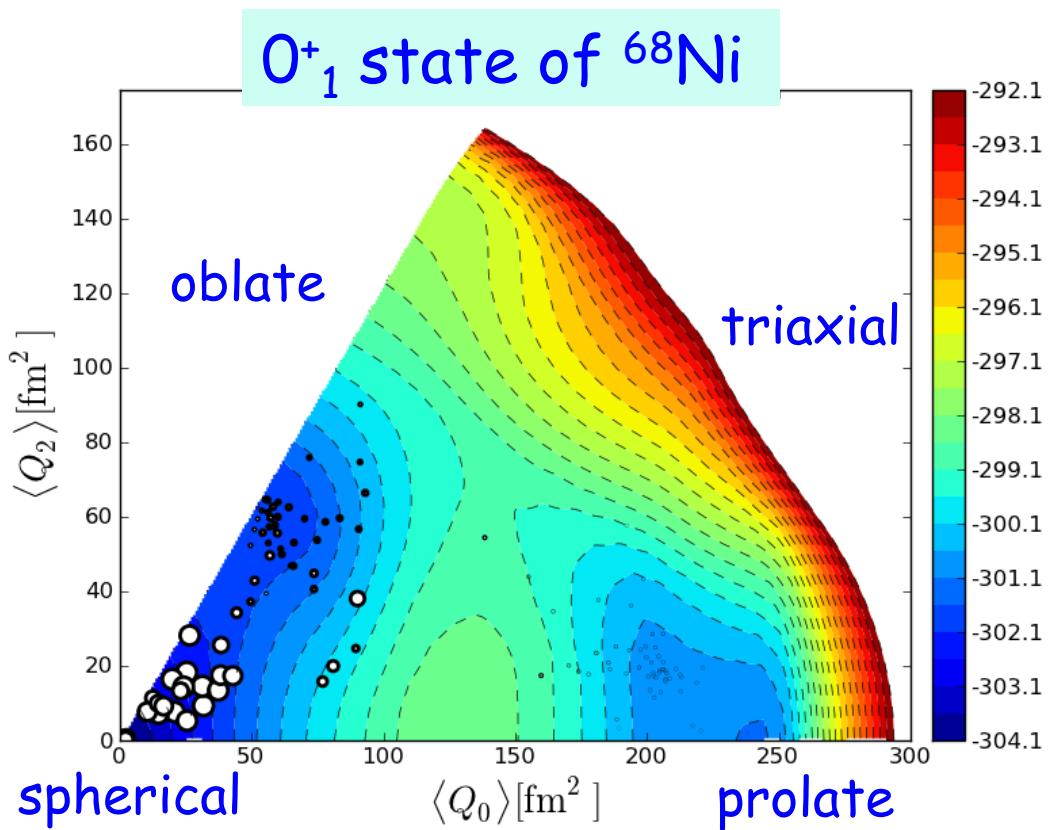
Y. Tsunoda, TO, Shimizu, Honma and Utsuno,  
PRC 89, 031301 (R) (2014)

# MCSM basis vectors on Potential Energy Surface

$$\text{eigenstate } \Psi = \sum_i c_i P[J^\pi] \Phi_i$$

Slater determinant → intrinsic shape

- PES is calculated by CHF for the shell-model Hamiltonian
- Location of circle : quadrupole deformation of unprojected MCSM basis vectors
- Area of circle : overlap probability between each projected basis and eigen wave function



Called *T-plot* in reference to

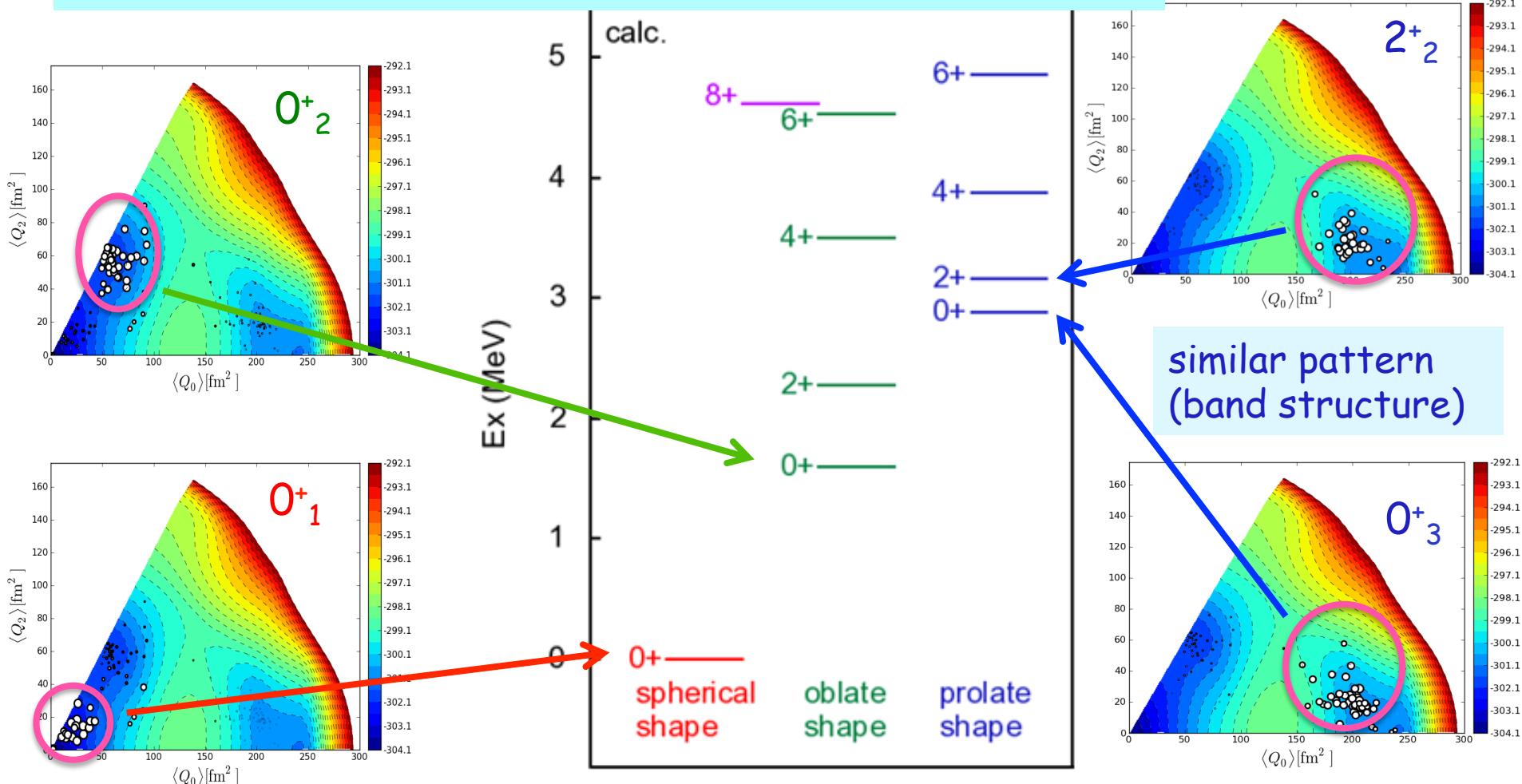
Y. Tsunoda, TO, Shimizu, Honma and Utsuno,  
PRC 89, 031301 (R) (2014)

## General properties of T-plot :

Certain number of large circles in a small region of PES  
↔ pairing correlations

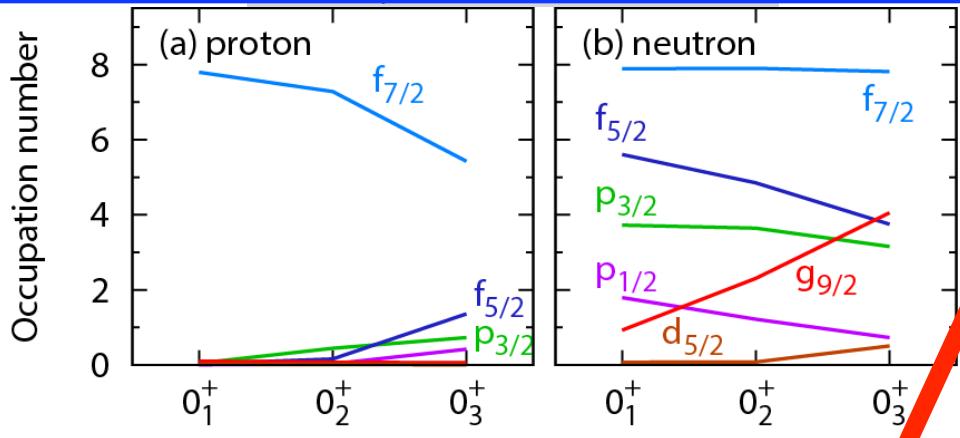
Spreading beyond this can be due to shape fluctuation

Example : shape assignment to various  $0^+$  states of  $^{68}\text{Ni}$



# $^{68}\text{Ni}$ $0^+$ states

## occupation numbers



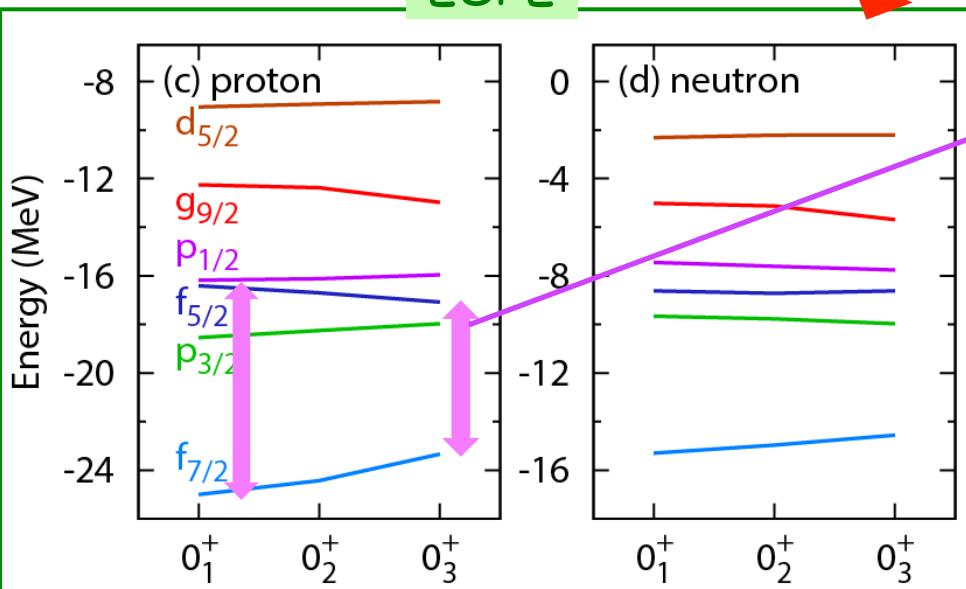
effective single-particle energies (ESPE) for correlated eigenstate

$$\epsilon_j = \langle \frac{\partial H_m}{\partial n_j} \rangle$$

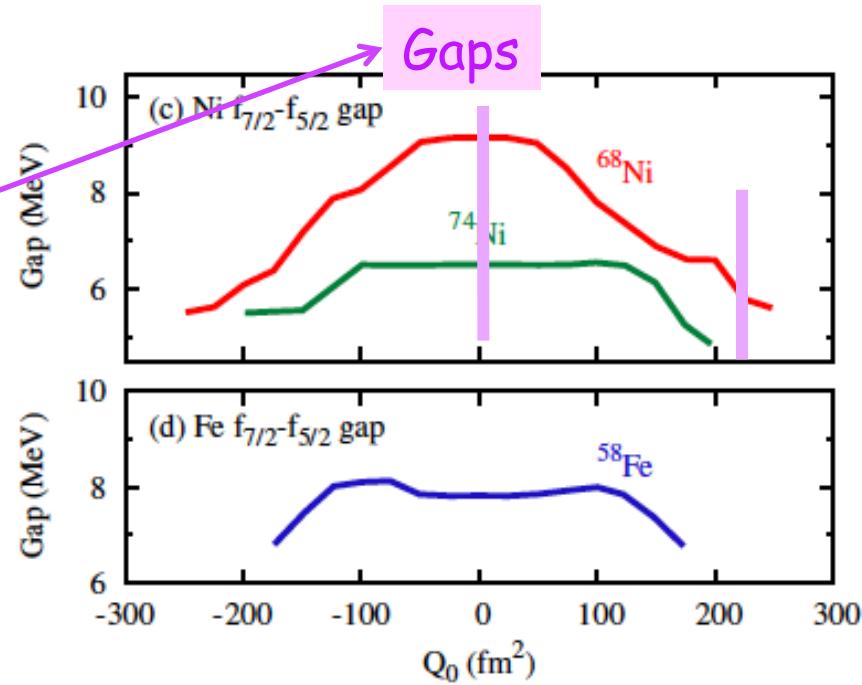
$H_m$  monopole part of  $H$

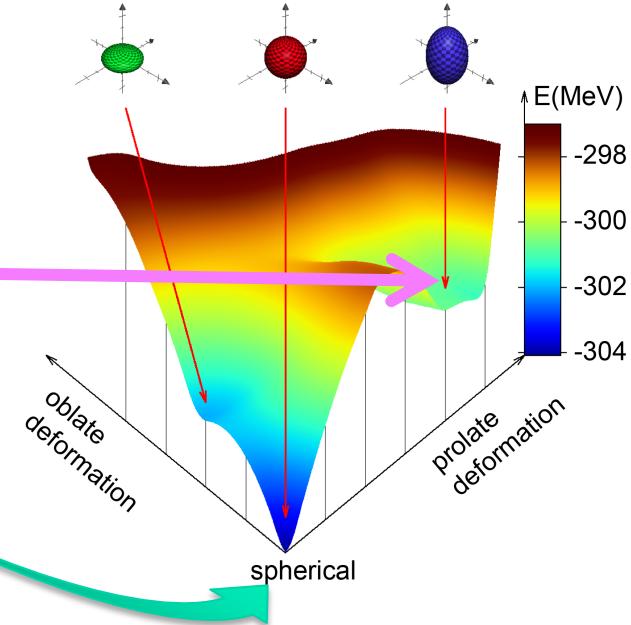
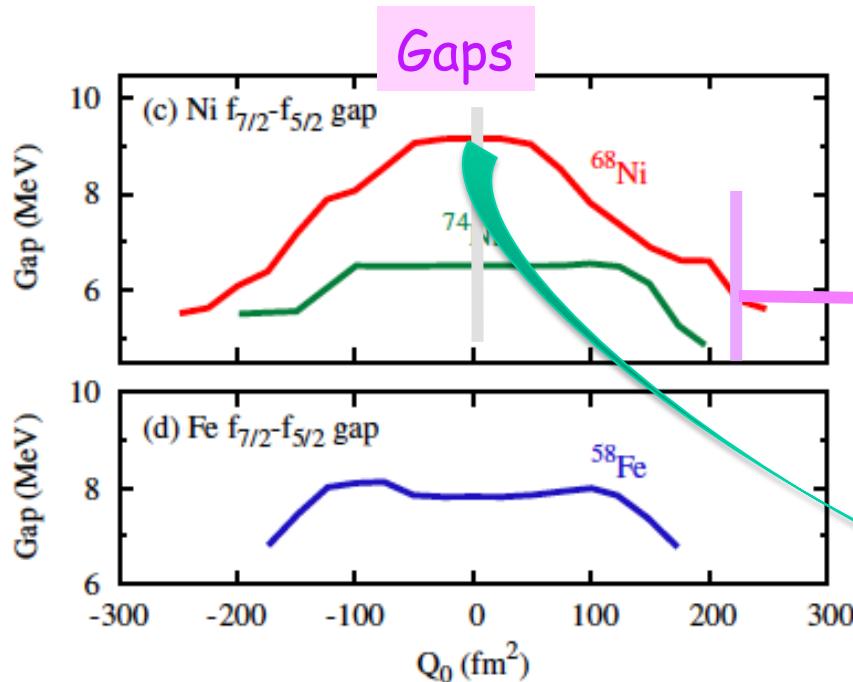
$\langle \rangle$ : by actual occup. numbers

## ESPE



## Gaps



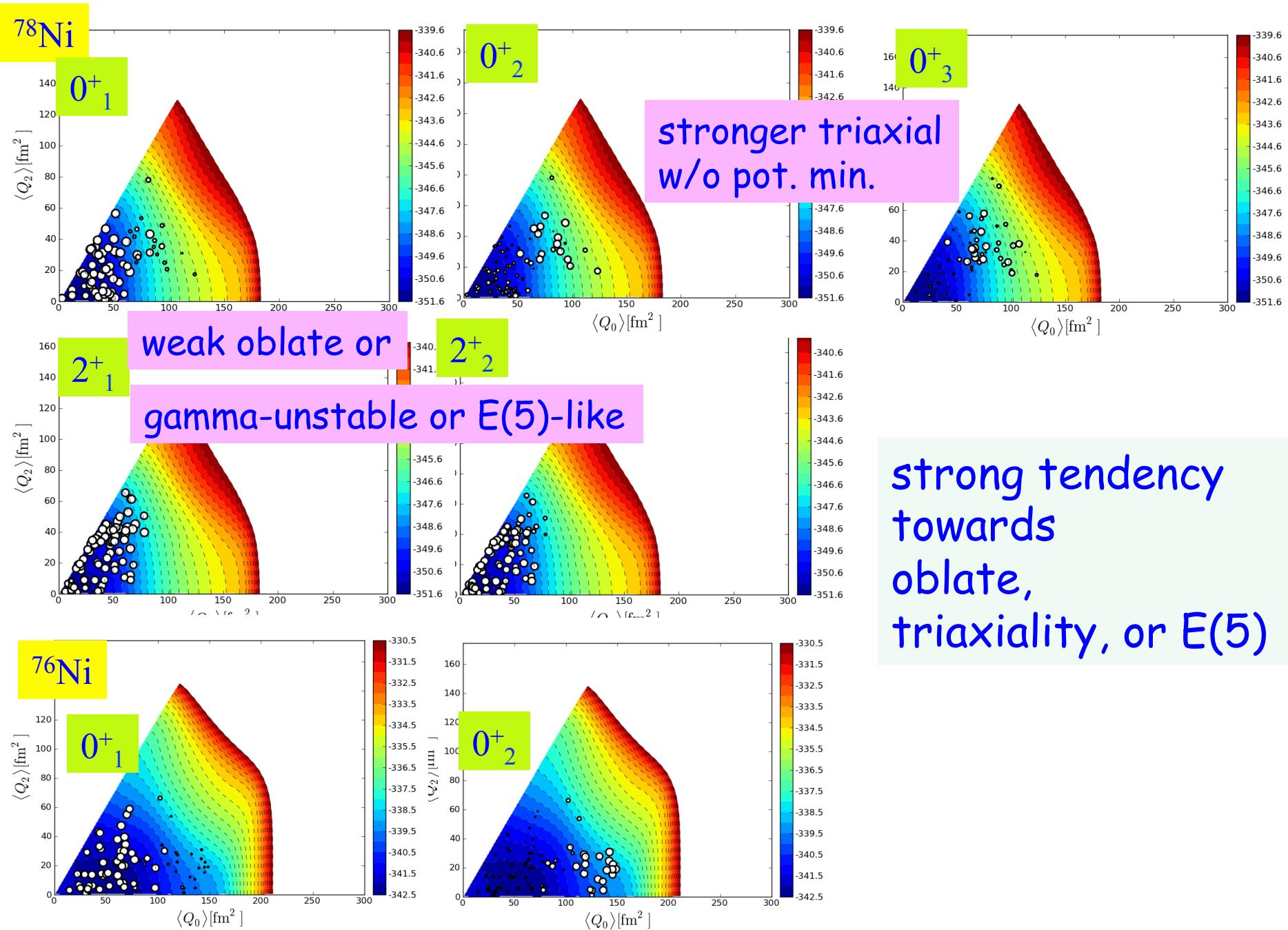


shell gap (spherical)  
configurations = shapes

determined self-consistently  
and non-linearly

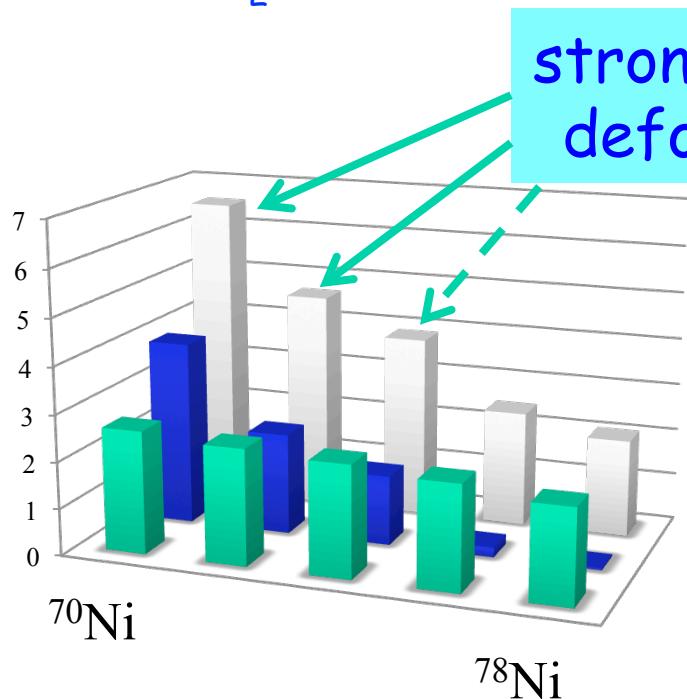
*Type II Shell Evolution*  
shell evolution within the same nucleus  
driven by the tensor force

Spherical shell gap is not changed in Nilsson model

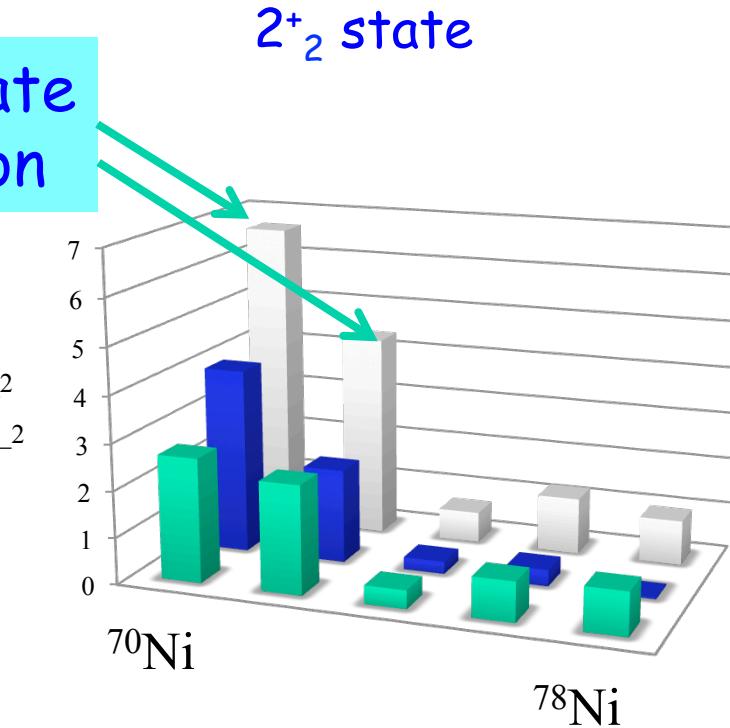


## Number of protons and neutrons excited across Z=28 or N=40 magic numbers

$O^+_2$  state



$2^+_2$  state



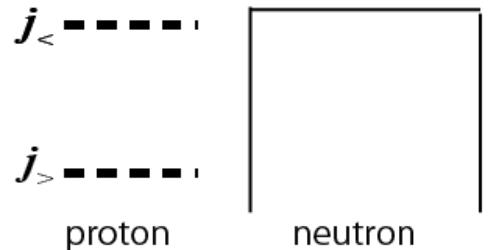
strong prolate  
deformation

- proton  $2^+_2$
- neutron  $2^+_2$
- total  $2^+_2$

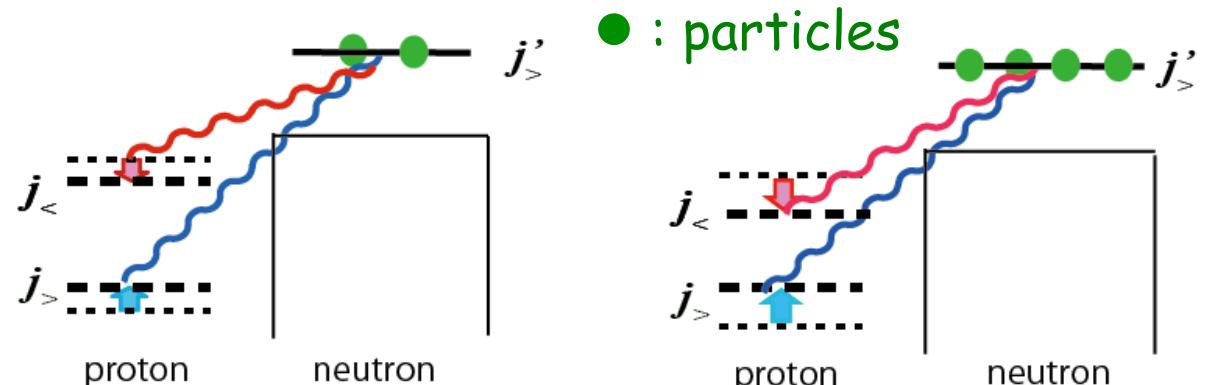
# Underlying mechanism of the appearance of low-lying deformed states : Type II Shell Evolution

Monopole effects on  
the shell structure  
from the tensor  
interaction

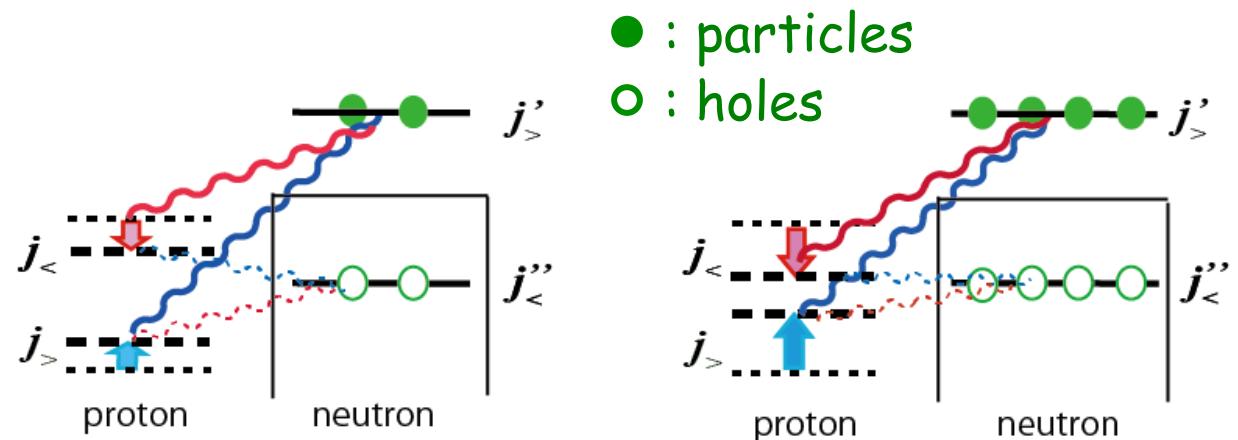
(a)



Type I Shell Evolution : different isotopes



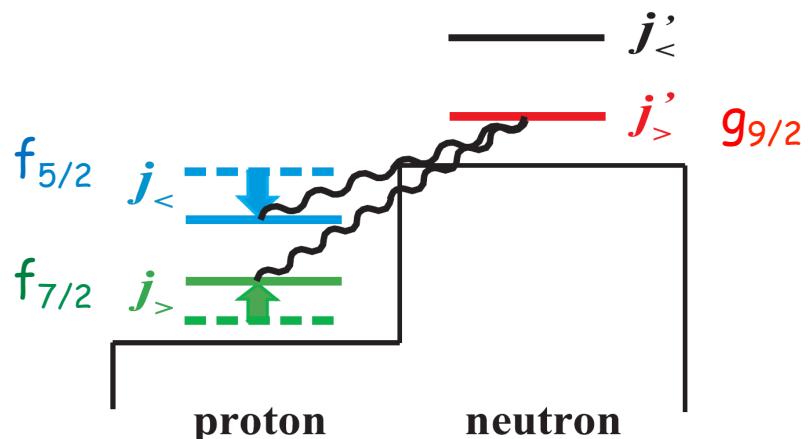
Type II Shell Evolution : within the same nucleus



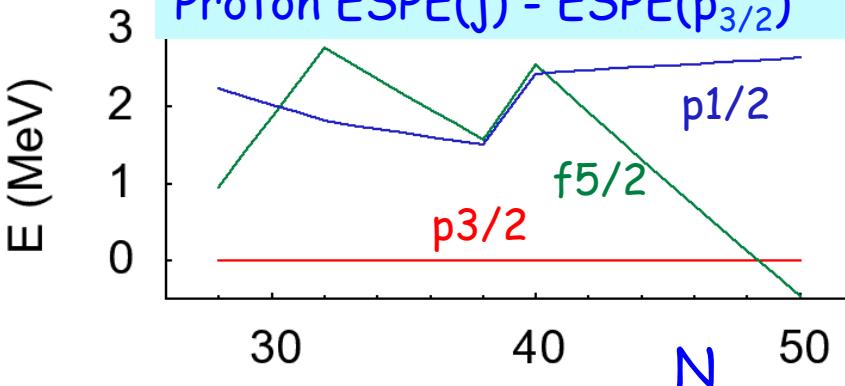
# Cu isotopes

- proton  $p_{3/2}$ - $f_{5/2}$  level crossing from  $N = 40$  to  $N = 50$   
**(type I shell evolution)**
- Calculated states show agreement with experiments, although they are not pure single-particle states.

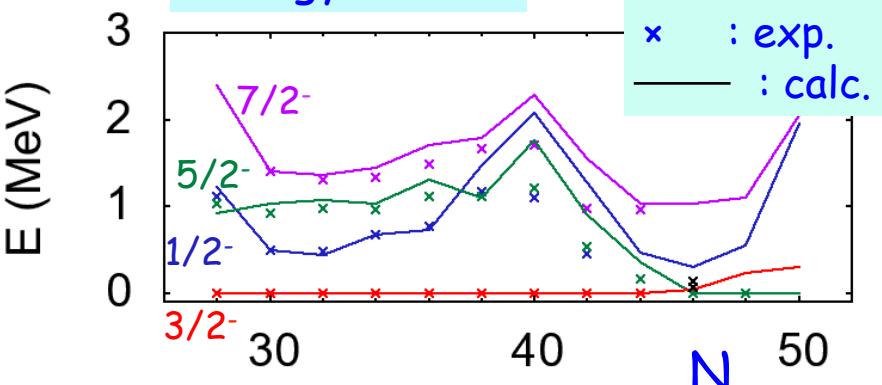
## Monopole effect of tensor force



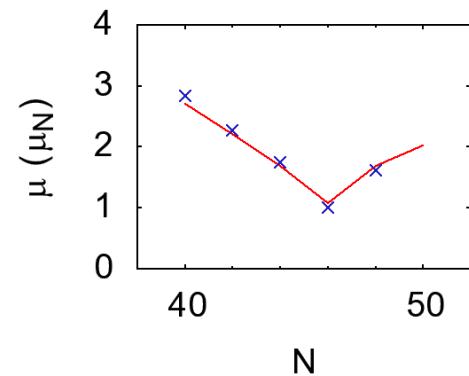
## Proton ESPE(j) - ESPE( $p_{3/2}$ )



## Energy Levels



magnetic moment of ground state



# Shape coexistence of $^{70}\text{Co}$ ( $Z=27$ , $N=43$ )

g.s. and an isomer in  $^{70}\text{Co}$  are known experimentally (PRC 61, 054308 (2000))

**High-spin state ( $6^-$ ,  $7^-$ )**

$$\pi f_{7/2}^{-1} \nu g_{9/2}^{+3}$$

and **Low-spin state ( $3^+$ )**

$$\pi f_{7/2}^{-1} \nu p_{1/2}^{-1} \nu g_{9/2}^{+4}$$

were suggested

From our calculations,

**High-spin state ( $7^-$ ) is near-spherical**

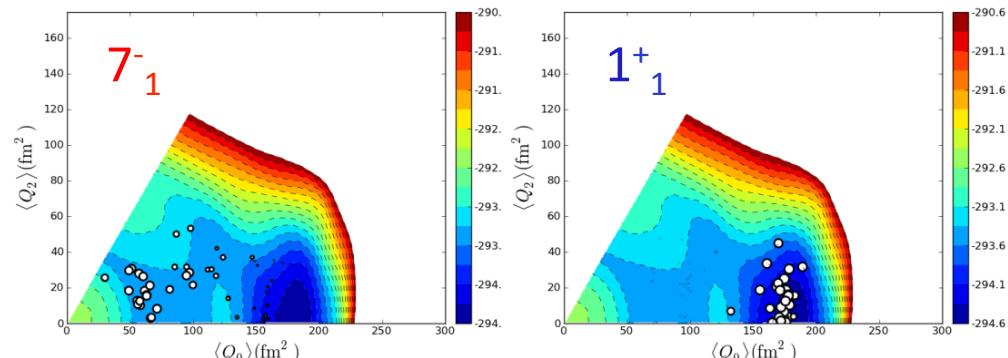
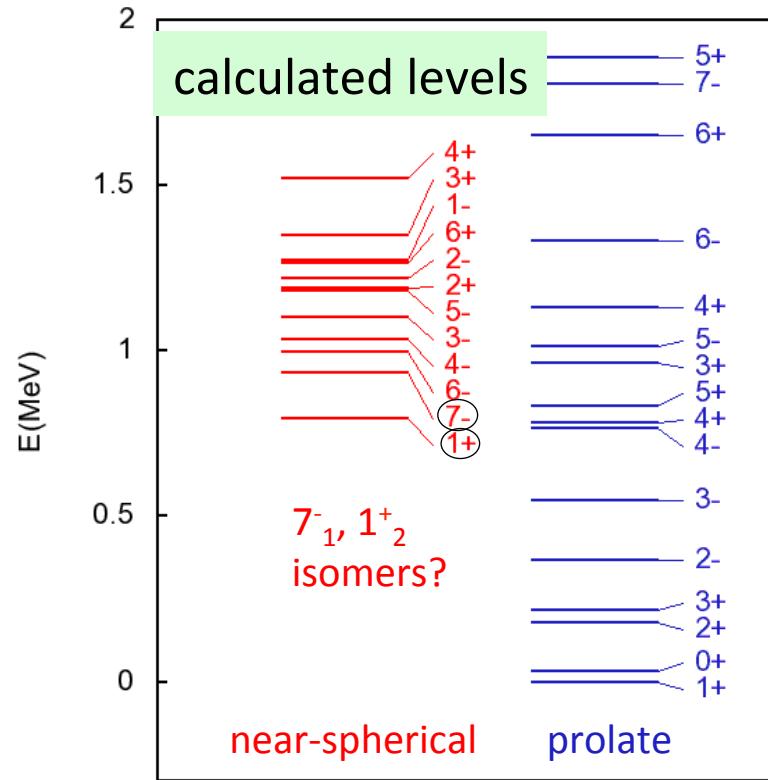
**Low-spin state ( $1^+$ ) is prolate deformed**

In the **prolate state  $1^+_1$** ,

many nucleons are excited

~2.7 protons above  $Z=28$  gap

~3.1 neutron holes below  $N=40$  gap

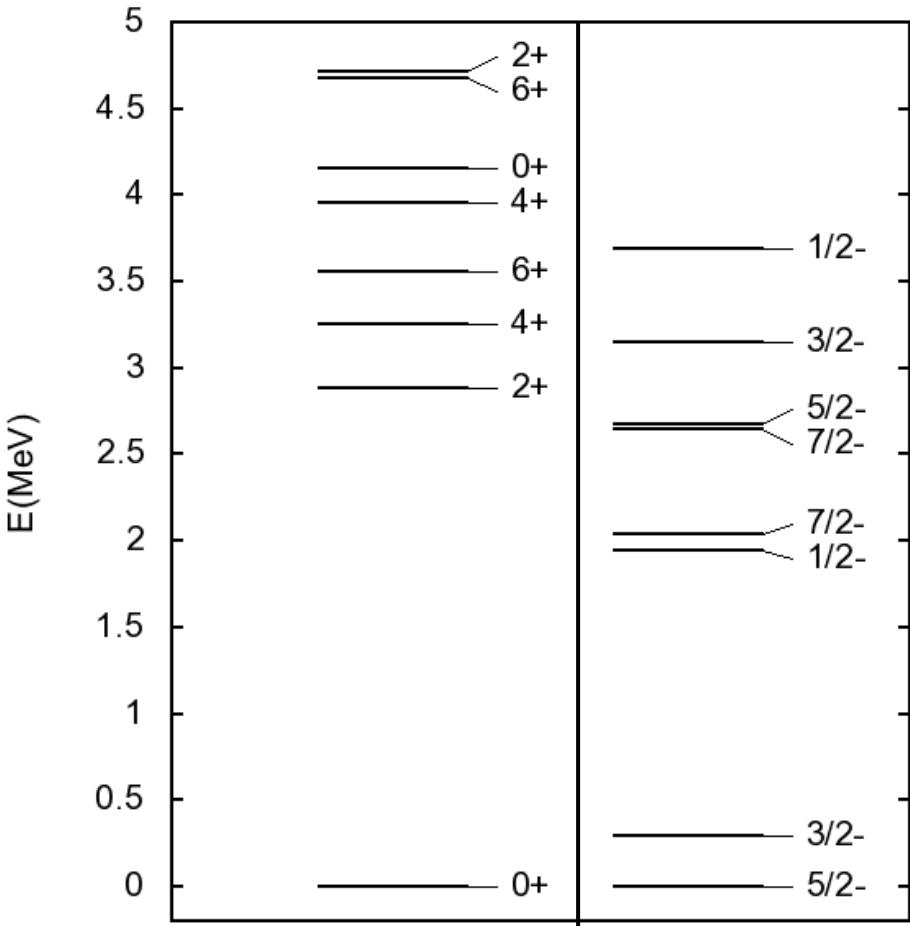


# $^{68}\text{Ni} - ^{69}\text{Cu}$ v.s. $^{78}\text{Ni} - ^{79}\text{Cu}$

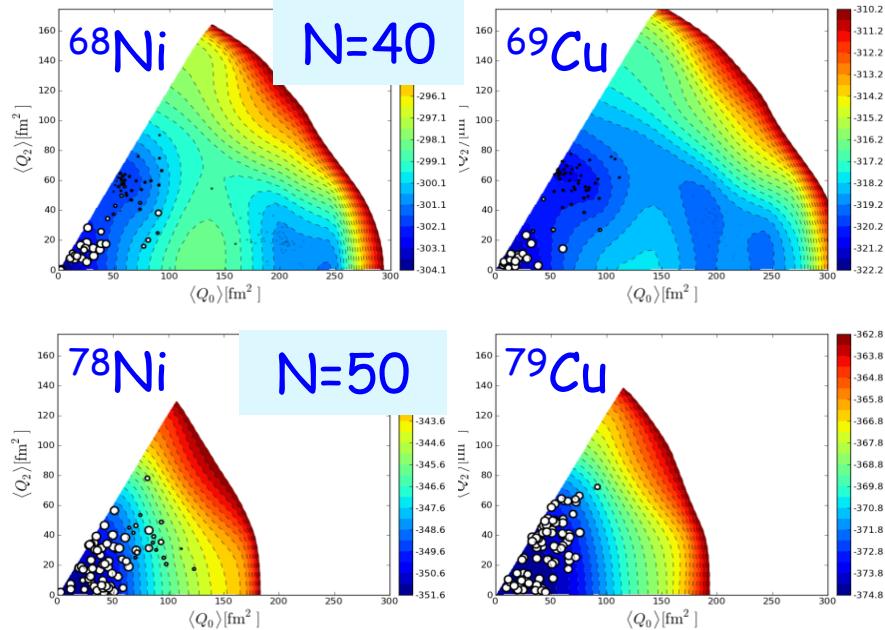
$^{78}\text{N}$

calc.

$^{79}\text{Cu}$



T-plot of ground state



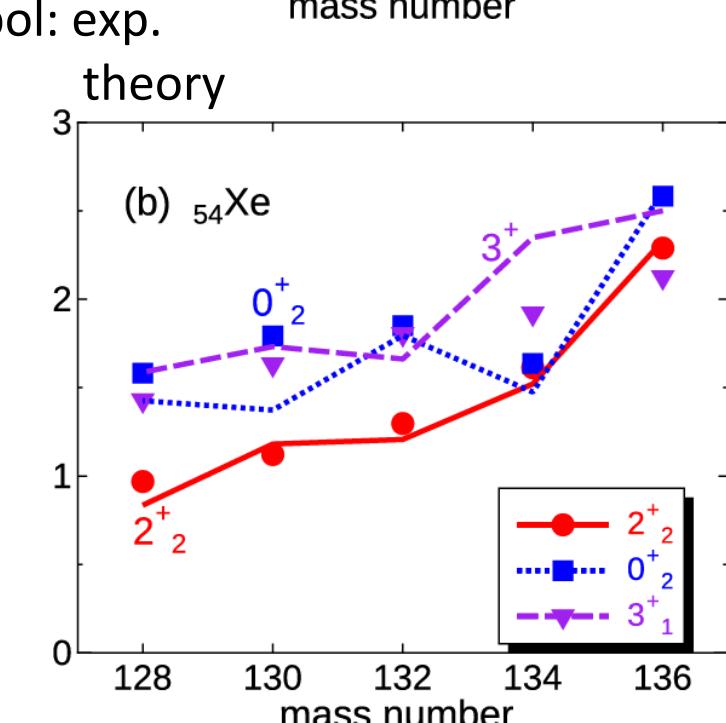
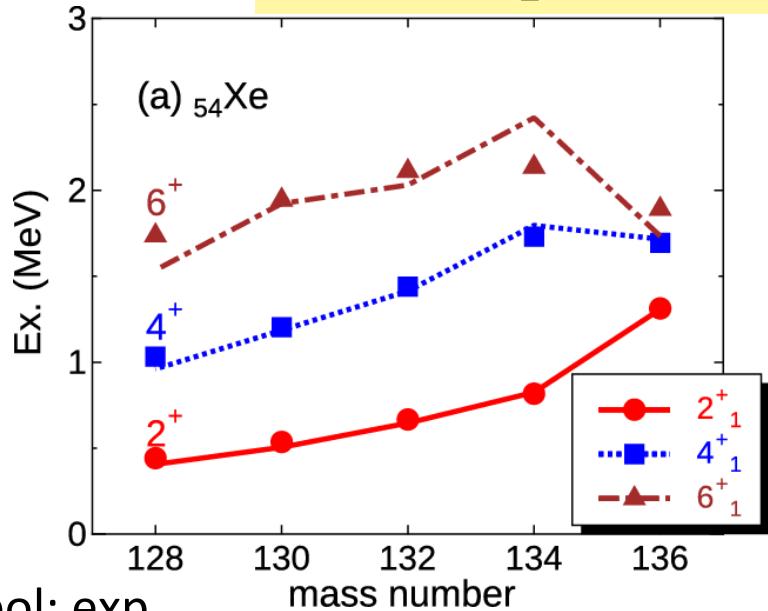
- Similar distribution patterns between Ni and Cu, while Cu is somewhat more deformed
- Shape fluctuations are larger in N=50 isotones

# Outline

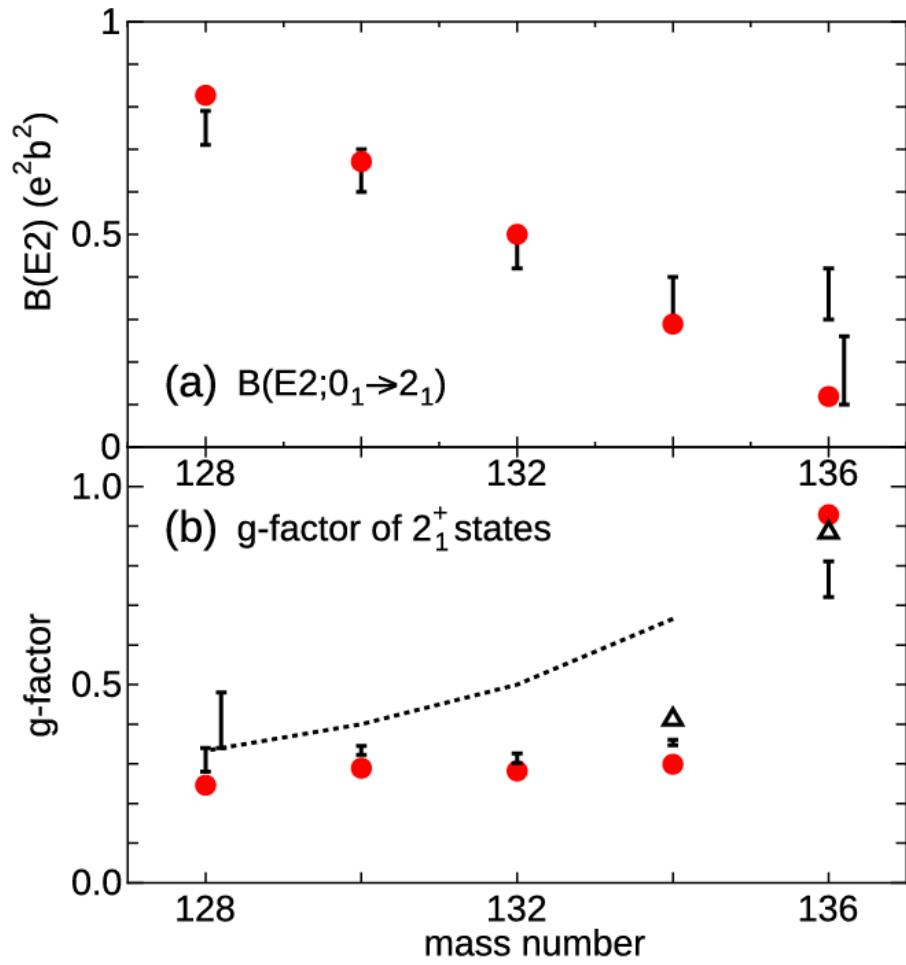
- **Methodology:** advanced Monte Carlo shell model (MCSM)
  - intrinsic shape can be the objectives
  - ab initio (no core) MCSM and clustering in Be isotopes
- **Shape coexistence** and Quantum Liquid picture
  - exotic Ni isotopes (+ Co, Cu)

- **Shape evolution** (from seniority to rotor)
  - Xe and Ba isotopes
- **Extension:** Spectra to high-lying collective states
  - E1 excitation, GDR and PDR in Ca and Sr isotopes
  - Level density with shell model Hamiltonian

# Shape evolution of Xe isotopes

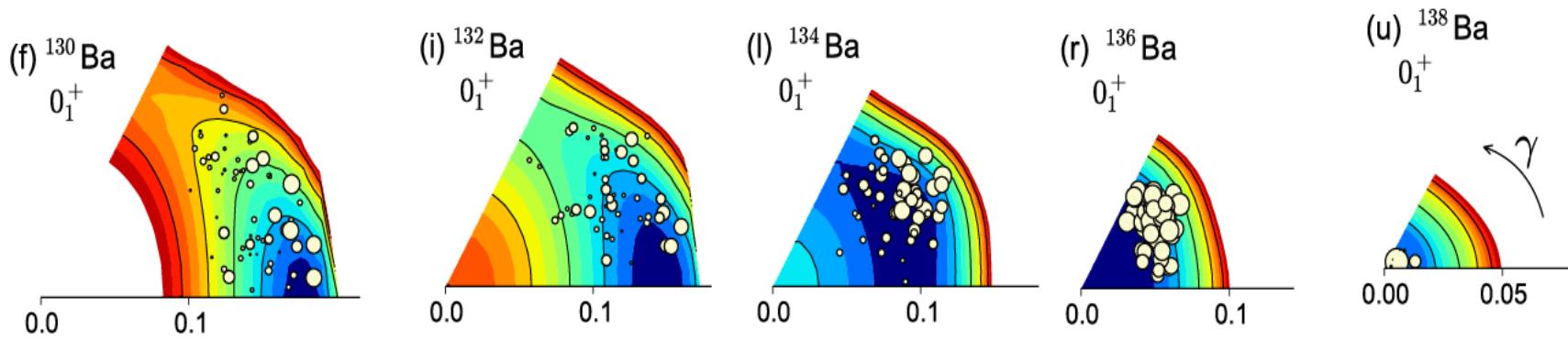
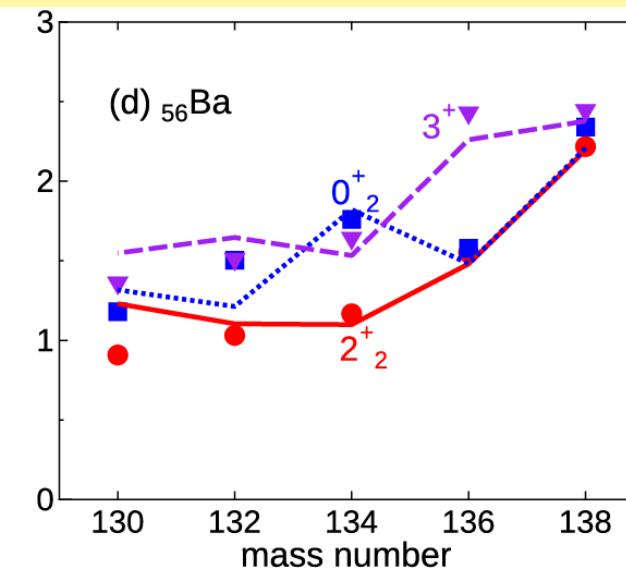
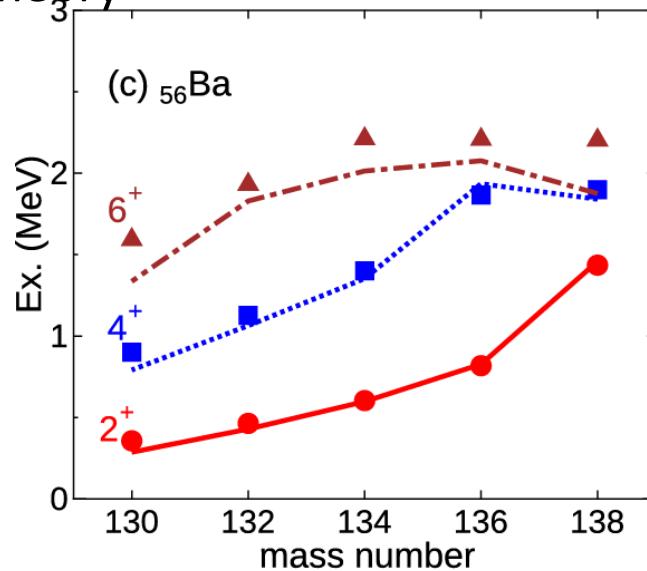


P+QQ int.  
model space:  $50 < N, Z < 82$



symbol: exp.  
line : theory

# Shape evolution of Ba isotopes



MCSM w.f.

$$|\Psi\rangle = \sum_{k=1}^{N_{\text{MCSM}} \sim 100} f_k P^{J,\pi} |\phi_k\rangle$$

PES by Q-constraint HFB calc. for the SM Hamiltonian

Location of circle : quadrupole deformation of  $|\phi_k\rangle$

Area of circle : overlap probability with the eigenstate

P+QQ int.  
model  
space:  
 $50 < N, Z < 82$

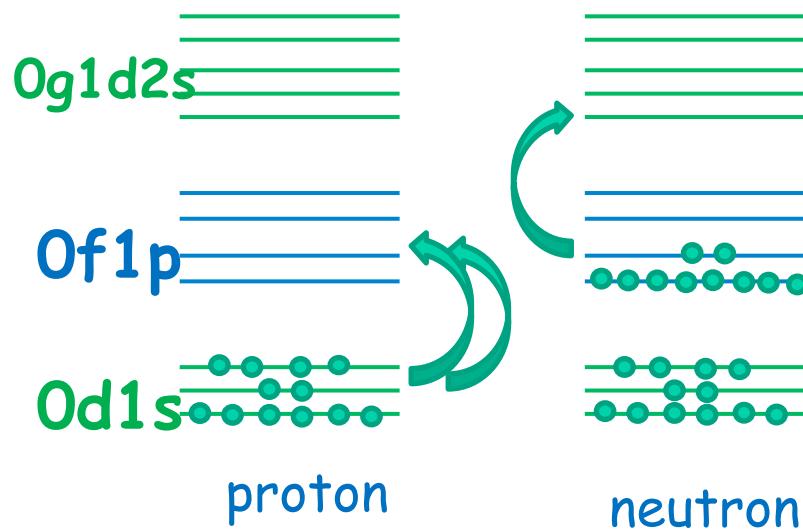
# Outline

- **Methodology:** advanced Monte Carlo shell model (MCSM)
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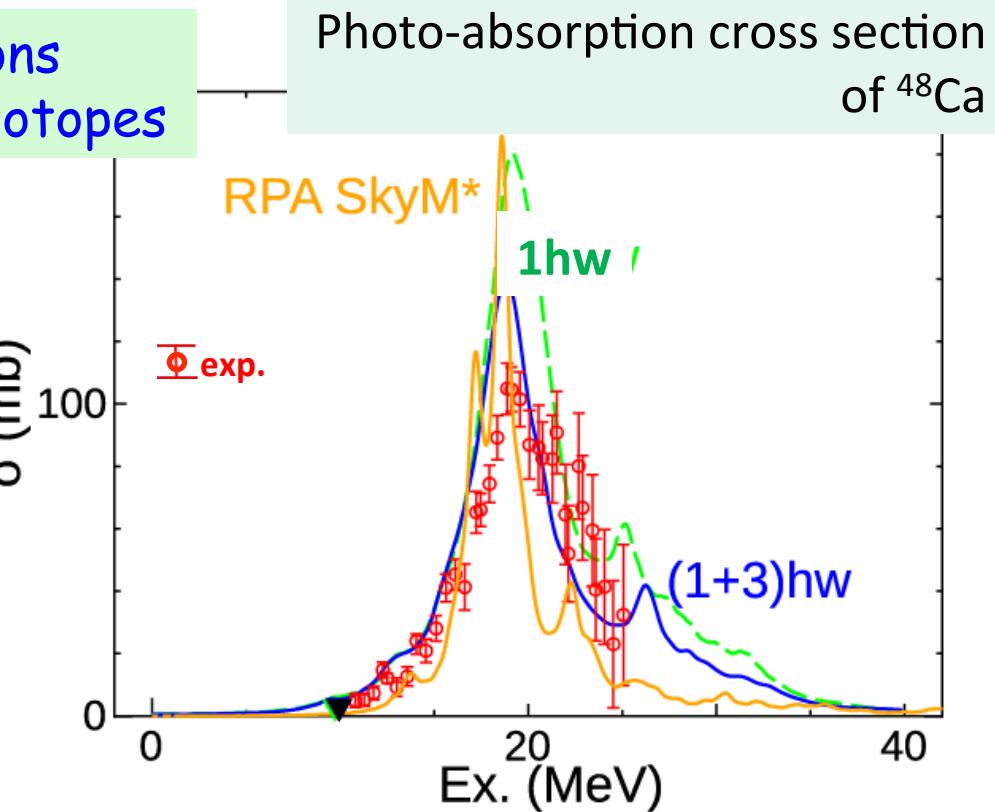
# E1 excitation of Ca isotopes in (conventional) LSSM

NS, Y. Utsuno, S. Ebata, T. Otsuka, M. Honma and T. Mizusaki, in preparation

1hw/3hw sd-pf-sdg shell calculations  
for negative-parity states of Ca isotopes



M-scheme shell-model code  
"KSHELL"  
by Lanczos method  
on massive parallel computer



1hw : upto 1hw excitation in sd-pf-sdg shell  
 $4.1 \times 10^6$  M-scheme dim. at PC

(1+3)hw: up to 3hw excitation in sd-pf-sdg shell  
 $1.2 \times 10^{10}$  M-scheme dim. at supercomputer

B(E1) sum rule  
by Monte Carlo shell model  
 $N_{\max} \hbar\omega$  configuration

E1 sum rule from fully correlated ground state wave function

- $^{48}\text{Ca}$       B(E1) sum rule ( $e^2\text{fm}^2$ )
  - $1\hbar\omega$            ... **16.5**
  - $(1+3)\hbar\omega$    ... **13.6**
  - MCSM 50dim. ... **10.1**

-18%  
-38%
- $^{51}\text{V}$       B(E1) sum rule ( $e^2\text{fm}^2$ )
  - $1\hbar\omega$            ... **18.1**
  - $(1+3)\hbar\omega$    ... **NA**
  - MCSM 50dim ... **12.4**

-32%

Many-body correlations  
reduces B(E1) sum rule

# E1 excitation spectrum can be calculated by MCSM

Ground state:

$$|\Psi\rangle = \sum_{k=1}^{N_{MCSM}} f_k P^{J,\pi} |\varphi_k\rangle$$

Basis vector  
of the  
ground state

Basis vectors for E1 spectrum ( $a, b, c, d, \dots$  : orbits)

$$\exp(i\varepsilon \cdot E1(a \rightarrow b)) |\varphi_k\rangle, \exp(i\varepsilon \cdot E1(c \rightarrow d)) |\varphi_k\rangle, \dots (k = 1, 2, \dots)$$

These are still Slater determinants

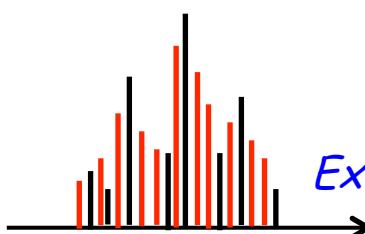
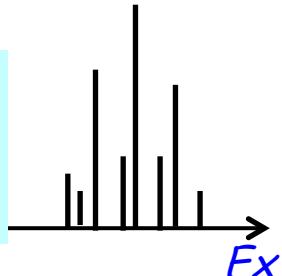
+

Additional bases for fine tuning :  
variation for energy average by the conjugate gradient

$$|\varphi_k(E1(a \rightarrow b))^{Var}\rangle, |\varphi_k(E1(c \rightarrow d))^{Var}\rangle, \dots$$

The global feature of excitation spectrum can be calculated.

Diagonalization with  
these basis vectors  
after projection to 1-

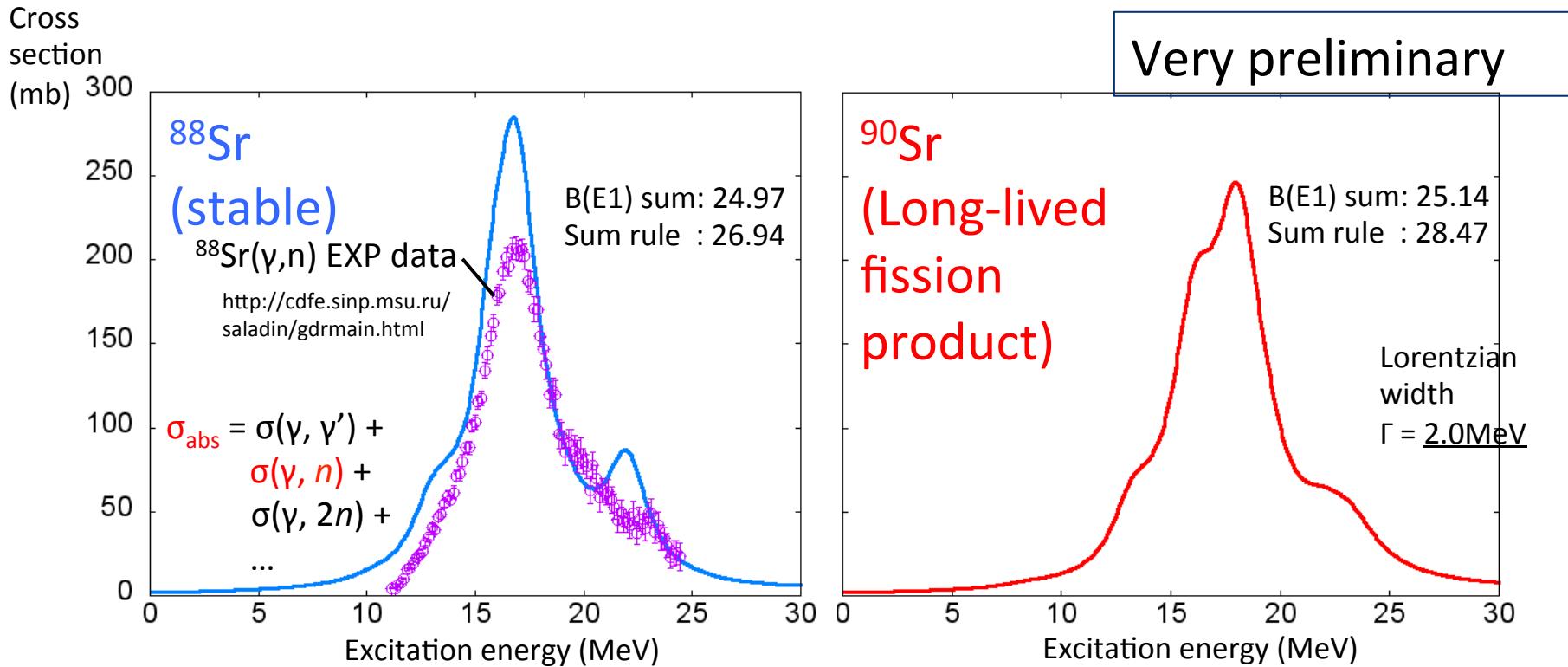


E1 spectrum with ~3000 levels  
connected somehow to the g.s.  
(confirmed by E1 sum rule)

# E1 excitation spectrum by MCSM

T. Togashi, T. Otsuka *et al.*

## Photoabsorption cross section of $^{88}\text{Sr}$ , $^{90}\text{Sr}$



Application to the nuclear transmutation  
for the purpose of nuclear waste processing (ex.  $^{90}\text{Sr}$  LLFP)  
as basic data for nuclear technology

## Perspectives

1. Ab initio at the level of shell-model Hamiltonian based on EKK method (for multi-shells)  
tensor + 3NF inclusive
2. Variety of magic nuclei (magic index)
3. Quantum liquid - single or dual - implications ?
4. Shell evolution in continuum and doorway-state resonance - reminiscence of bound states -
5. Shapes and clusters by MCSM
6. Excitation spectrum by MCSM and their application to astrophysics, nuclear energy, etc.

# Collaborators

## HPCI project, Field 5

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- Takahiro Mizusaki (Senshu Univ.)
- Noritaka Shimuzu (CNS, Tokyo)
- Tomoaki Togashi (CNS, Tokyo)
- Naofumi Tsunoda (CNS, Tokyo)
- Yusuke Tsunoda (CNS, Tokyo)
- Yutaka Utsuno (JAEA)
- Tooru Yoshida (CNS, Tokyo)
  
- James Vary (Iowa)
- Pieter Maris (Iowa)

END

ご清聴ありがとうございました