Recent progress in Lattice QCD application to parton distribution functions

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核子構造勉強会@ RIKEN

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QCD factorization

Cross sections can be decomposed into parton distribution function(PDF) and perturbative hard part.

$$d\sigma = \underbrace{f(x)}_{\text{PDF}} \otimes \underbrace{\hat{\sigma}(x,Q)}_{\text{parton cross section}} + O\left(\frac{1}{Q}\right)$$

PDF is nonperturbative function.

→ given by the fitting for experimental data

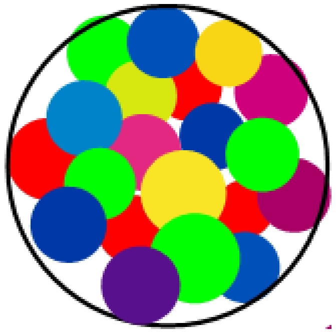
Universality(process independence) of PDF ensures the pQCD predictive power.

Hadron structure

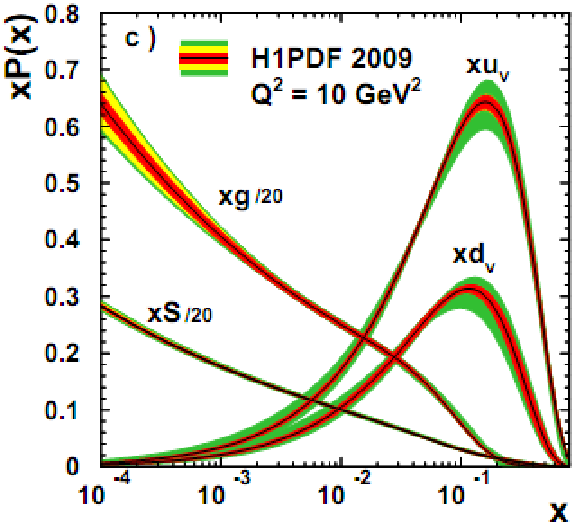
We can obtain the information about the hadron structure from the behavior of the nonperturbative functions.

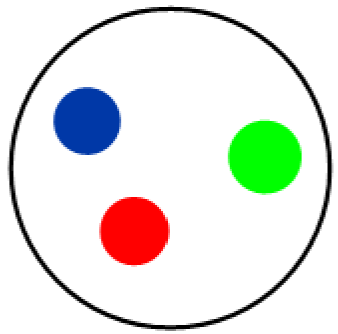
∘ unpolarized PDF··· probability distribution of partons inside the hadron

unpol PDF of proton



small-x





large-x

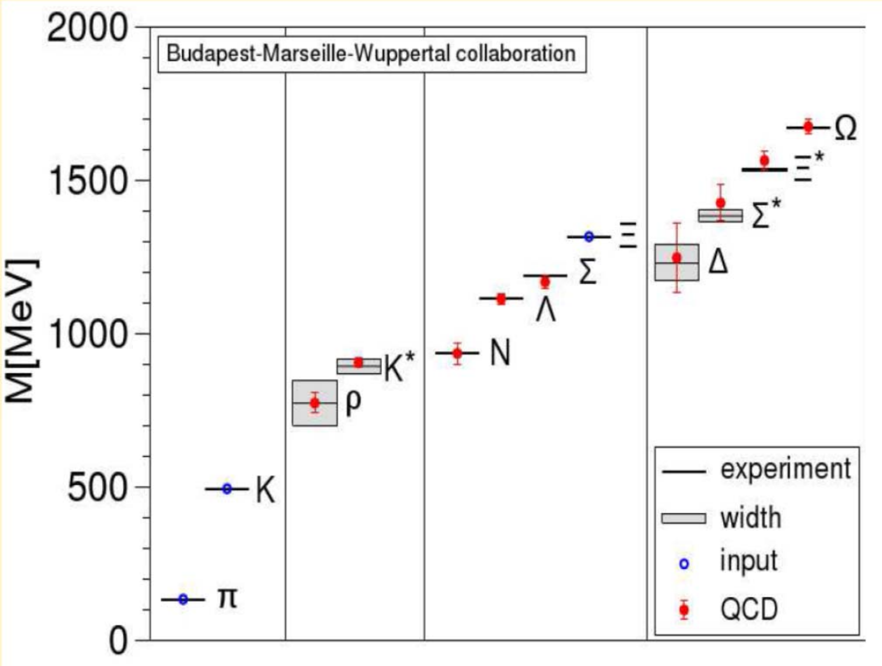
Can we directly calculate PDF by nonperturbative technique ?

Lattice QCD

The most powerful tool to calculate nonperturbative effects.

Hadron mass spectrum

Excellent agreement with experimental data !



○ weak point

Lattice is formulated in Euclidean space($t_E = it_M$).

Real time simulation is difficult.

Parton distribution function

○ definition

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ \exp\Big(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\Big)\psi(0)|P\rangle$$

light-cone coordinate $a^\pm = \frac{1}{\sqrt{2}}(a^t \pm a^z)$

PDF depends on the real-time in Minkowski space

Moment of PDF $\int dx\ x^n q(x,\mu^2)$ gives local matrix element

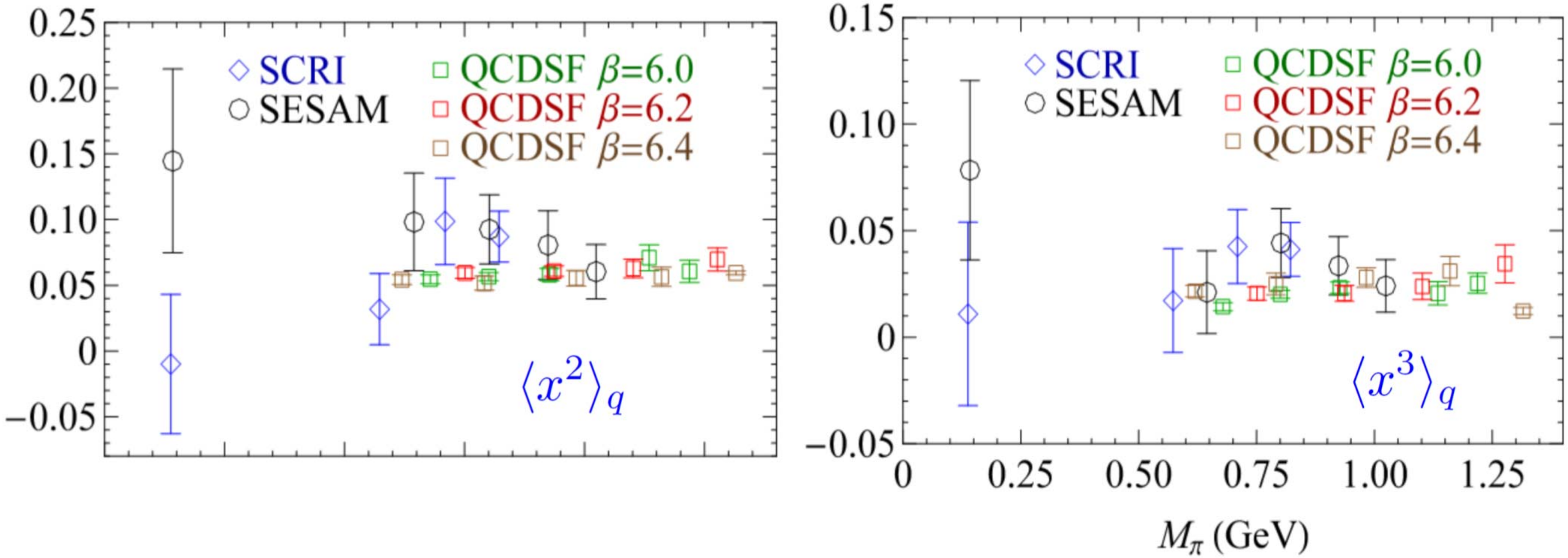
⟶ Calculable by lattice QCD

$$\begin{aligned}
\int dx\ x^n q(x,\mu^2) &= \int dx\ x^n \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ \exp\Big(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\Big)\psi(0)|P\rangle \\
&= (\frac{1}{iP^+})^{n+1} \int \frac{d\xi^-}{4\pi} (\frac{\partial}{\partial\xi^-})^n \delta(\xi^-) \langle P|\bar{\psi}(\xi^-)\gamma^+ \exp\Big(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\Big)\psi(0)|P\rangle \\
&= \frac{1}{4\pi}(\frac{1}{iP^+})^{n+1} \langle P|\bar{\psi}(0)\gamma^+ (D^+(0))^n \psi(0)|P\rangle
\end{aligned}$$

Moments Calculation

Moments of PDF give local matrix elements

$$\langle x^n(\mu^2)\rangle_q = \int_{-1}^{1} dx x^n q(x,\mu^2)$$



D. Dolgov et al., PRD66 (2002) 034506 M. Göckeler et al., PRD71 (2005) 114511

Limited moments, difficult to determine x-dependence

Ji's approach

X. Ji, PRL110 (2013) 262002

X. Xiong et al., PRD90 (2014) 014051

- Quasi-PDF

$$\tilde{f}(x,\mu^2,P_z) = \int \frac{dz}{4\pi} e^{ixP_z z} \langle P|\bar{\psi}(z)\gamma^z \exp\Big(-ig\int_0^z dz' A^z(z')\Big)\psi(0)|P\rangle$$

  - Quasi-PDF does not depend on real-time in Minkowski space and it's measurable on the Euclidean lattice
  - Consistent with light-cone distribution when $P_z \to \infty$

- Matching with light-cone PDF $q(x,\mu^2)$

$$\tilde{f}(x,\mu^2,P_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\Big(\frac{x}{y},\frac{\mu}{P_z}\Big) q(y,\mu^2) + O\Big(\frac{\Lambda^2_{QCD}}{P_z^2}\Big)$$

  - $O(\frac{1}{p_z^2})$ corrections
  - Factorization based on high-P_z effective field theory

Lattice QCD simulation gives independent data points for the PDF fitting

One-loop result

X. Xiong et al., PRD90 (2014) 014051

$$\tilde{f}(x,\mu^2,P_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\Big(\frac{x}{y},\frac{\mu}{P_z}\Big) q(y,\mu^2) + O\Big(\frac{\Lambda_{QCD}^2}{P_z^2}\Big)$$

Matching coefficient $Z(\frac{x}{y},\frac{\mu}{P_z})$ has been calculated up to one-loop order.

$$Z^{(0)}\Big(\frac{x}{y},\frac{\mu}{P_z}\Big) = \delta\Big(\frac{x}{y}-1\Big)$$

$$Z^{(1)}\Big(\frac{x}{y},\frac{\mu}{P_z}\Big) = \boxed{\tilde{f}^{(1)}\Big(\frac{x}{y},\mu^2,P_z\Big) - q^{(1)}\Big(\frac{x}{y},\mu^2\Big)}$$

infrared divergences are exactly cancelled

$Z^{(1)}\Big(\frac{x}{y},\frac{\mu}{P_z}\Big) \to 0$ when $P_z \to \infty$

$$\tilde{f}(x,\mu^2,P_z\to\infty) = \int_{-1}^{1} \frac{dy}{|y|} Z\Big(\frac{x}{y}-1\Big) q(y,\mu^2) = q(x,\mu^2)$$

Ji's conjecture has no inconsistency at one-loop order.

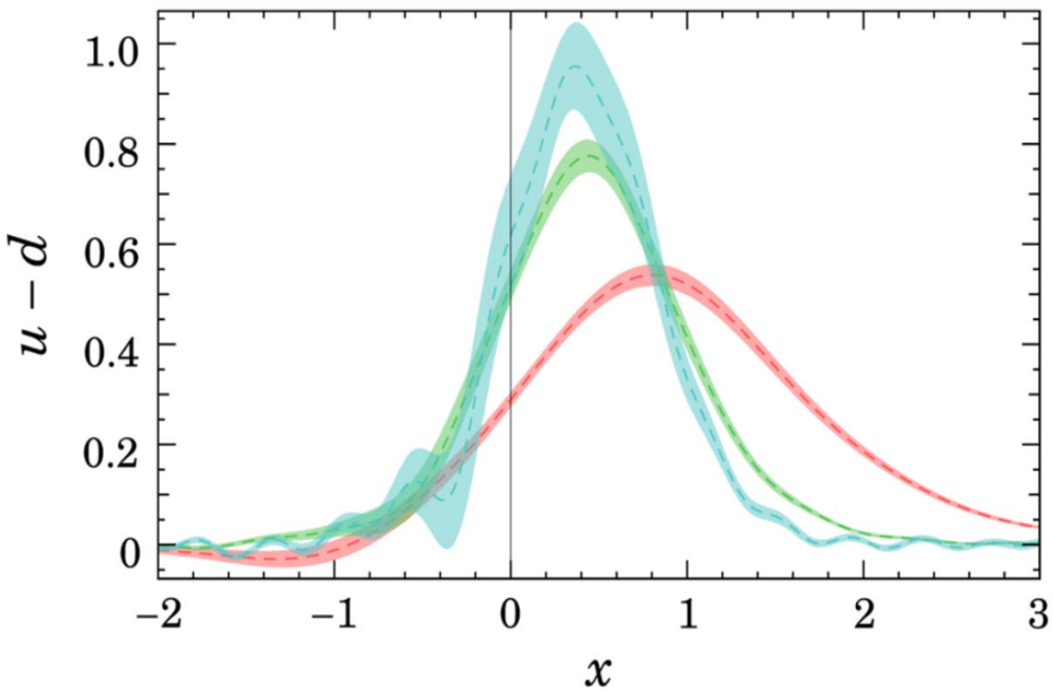
First result

H. -W. Lin et al., PRD91 (2015) 054510

Improved staggered fermion $N_f = 2+1+1$

lattice size: $24^3 \times 64$ $M_\pi \simeq 310\text{MeV}$ $L \simeq 4\text{fm}$

$P_z \simeq 0.3,\ 0.6,\ 0.9\text{GeV}$



still need more improvement

- ○ high-P_z ○ matching ○ other lattice fermions ○ physical mass
- ○ anisotropic lattice

Applicability of Ji's method

$$\tilde{f}(x,\mu^2,P_z) = \int \frac{dz}{4\pi} e^{ixP_z z} \langle P|\bar{\psi}(z)\gamma^z \exp\Big(-ig\int_0^z dz' A^z(z')\Big)\psi(0)|P\rangle$$

The factor $e^{ixP_z z}$ rapidly oscillates at large-x region

→ limited range of z (small lattice size)

At small-x region, the oscillation becomes slower

→ wide range of z (large lattice)

Lattice QCD simulation at $x = 10^{-2}$ needs 100times more lattice sites than the simulation at $x = 1$.

Ji's method is available at large-x region.

PDFs at large-x

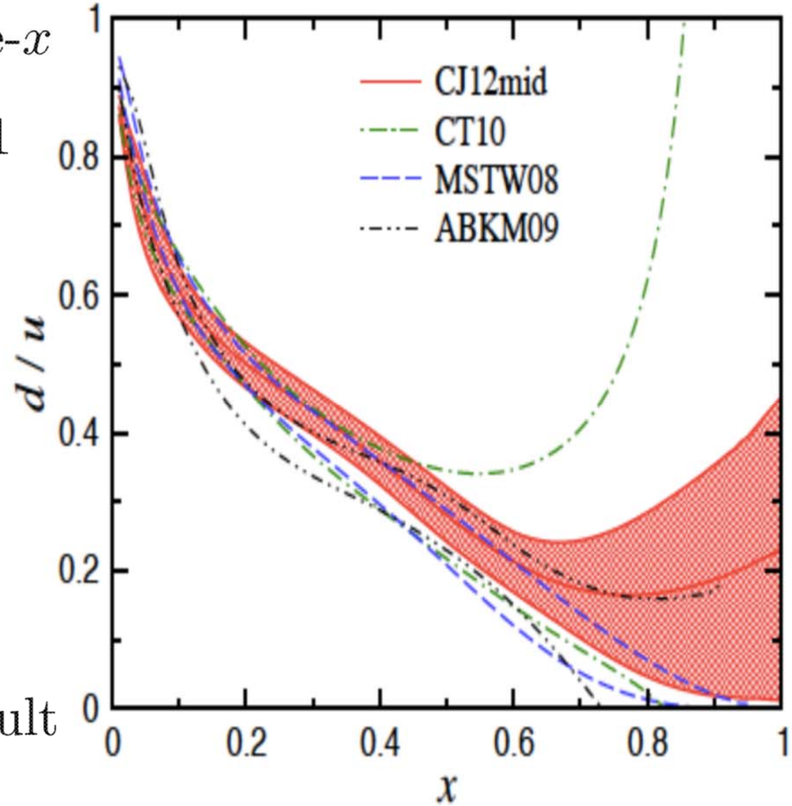
◦ Proton

Proton PDFs are not well determined at large-x

Some models predict different values at $x \to 1$

$d/u \to 1/2$	SU(6) spin-flavor symmetry
$d/u \to 0$	scalar diquark dominance
$d/u \to 1/5$	pQCD power counting
	etc.

Lattice QCD can give model indenpendent result



◦ Pion

Valence quark distribution should behave as $(1-x)^2$ at $x \to 1$

On the lattice, pion matrix element is the simplest case

ΔG on the Lattice

- helicity distribution of gluons

$$\Delta G \frac{S^+}{P^+} = \int dx \Big\{ \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS| F^{\alpha +}(\xi^-) W_{[\xi^-,0]} \tilde{F}_\alpha{}^+(0)|PS\rangle \Big\}$$

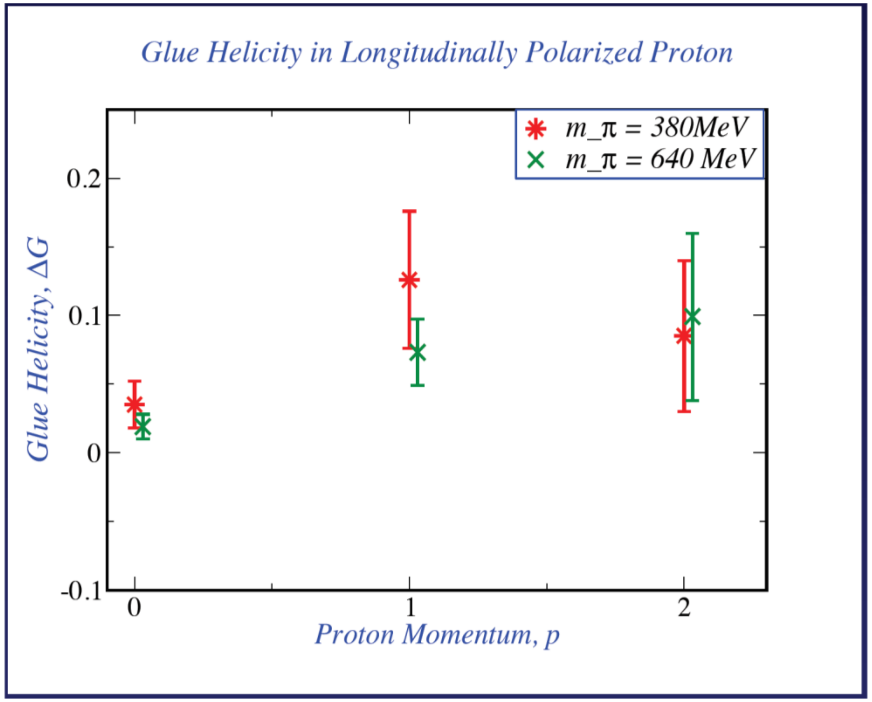
$$= \frac{1}{2P^+} \langle PS|\epsilon^{ij} F^{i+}(0) A^j_{\rm phys}(0)|PS\rangle$$

$$\Delta\tilde{G}(P_z) = \frac{1}{2P^0} \langle PS|\epsilon^{ij} F^{i0}(0) A^j(0)|PS\rangle$$

$$\Delta\tilde{G}(P_z,\mu)$$

$$= Z_{gg}(P_z/\mu)\Delta G(\mu) + Z_{gq}(P_z/\mu)\Delta q(\mu)$$

Y. Hatta et al., PRD89 (2014) 085030



Kei-Fei's talk in SPIN2014

Matching of the quasi-PDFs between continuum and lattice

based on the work with : Tomomi Ishikawa (RBRC)
Jianwei Qiu (BNL)

Two matching procedures

- PDF has the renormalization scheme dependence
- Precise determination of quasi-PDF requires the matching between different schemes

X. Xiong et al., PRD90 (2014) 014051

Y. -Q. Ma and J. Qiu, arXiv:1404.6860

$$\tilde{f}_{\mathrm{cont}}(x, \mu^2, P_z) = \int \frac{dy}{y} \sum_f C_f\left(\frac{x}{y}\right) \tilde{f}_{\mathrm{lat}}(y, \frac{1}{a}, P_z)$$

Minkowski space

$$C_f^{(1)}(x) = \tilde{f}^{(1)}_{\mathrm{cont}}(x) - \tilde{f}^{(1)}_{\mathrm{lat}}(x)$$

$q(x, \mu^2)$ $\qquad$ $\tilde{f}_{\mathrm{cont}}(x, \mu^2, P_z)$ $\qquad$ $\tilde{f}_{\mathrm{lat}}(x, \frac{1}{a}, P_z)$

Euclidean space

This talk

Lattice QCD

Lattice QCD is formulated in the descretized Euclidean space

$$S^f = a^4 \sum_x \Big[\frac{1}{2a} \sum_\mu [\bar{\psi}(x)\gamma_\mu U_\mu(x)\psi(x+a\hat{\mu}) - \bar{\psi}(x+a\hat{\mu})\gamma_\mu U_\mu^\dagger(x)\psi(x)] + m\bar{\psi}(x)\psi(x) \Big]$$

$$S^g = \frac{1}{g_0^2} a^4 \sum_{x,\mu\nu} \Big[N_c - \mathrm{ReTr}[U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)] \Big]$$

$$U_\mu(x) = e^{-igaT^a A_\mu^a(x+\frac{1}{2})}$$

Boundary condition is imposed on each field in finite volume system

Momemtum space is restricted in finite Brillouin zone $\{-\frac{\pi}{a}, \frac{\pi}{a}\}$

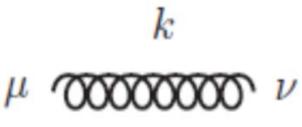
UV finite theory

Lattice action is not unique, above action is the simplest one

Many improvements have been proposed to reduce discretization error

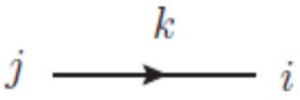
Feynman rules of lattice PT (Feynman gauge)

○gluon propagator



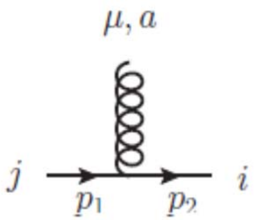
$$\frac{1}{k^2}\delta_{\mu\nu} \longrightarrow a^2\frac{1}{4\sum_\mu \sin^2(\frac{ak_\mu}{2})}\delta_{\mu\nu}$$

○quark propagator



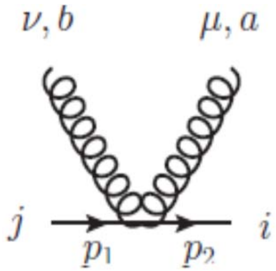
$$-i\frac{\not{k}}{k^2}\delta_{ij} \longrightarrow -ia\frac{\sum_\mu \gamma_\mu \sin(ak_\mu)}{\sum_\mu \sin^2(ak_\mu)}\delta_{ij}$$

○quark-gluon vertex



$$-ig(T^a)_{ij}\gamma_\mu \longrightarrow -ig(T^a)_{ij}\gamma_\mu \cos(\frac{a(p_1+p_2)_\mu}{2})$$

○quark-gluon vertex 2



$$\frac{a}{2}g(T^aT^b)_{ij}i\gamma_\mu \sin(\frac{a(p_1+p_2)_\mu}{2})\delta_{\mu\nu}$$

Doubling problem

Naive discretized fermion shown in previous slide is useless for numerical simulation because of the doubling problem.

quark propagator

$$-ia\frac{\sum_\mu \gamma_\mu \sin(ak_\mu)}{\sum_\mu \sin^2(ak_\mu)}\delta_{ij}$$

$$\sum_\mu \sin^2(ak_\mu) = 0 \text{ at } (k_0, k_1, k_2, k_3) = (0,0,0,0), (\pi,0,0,0) \cdots (\pi,\pi,\pi,\pi)$$

Pole of the propagator gives dispersion relation of the particle.

$\longrightarrow$ 16 fermions !

Some lattice fermions have been proposed to avoid the problem.

◦ Wilson ◦ domain wall ◦ overlap

But we first use naive discretized fermion to demonstrate the cancelation of the infrared divergence in the matching.

Discretized quasi-PDF

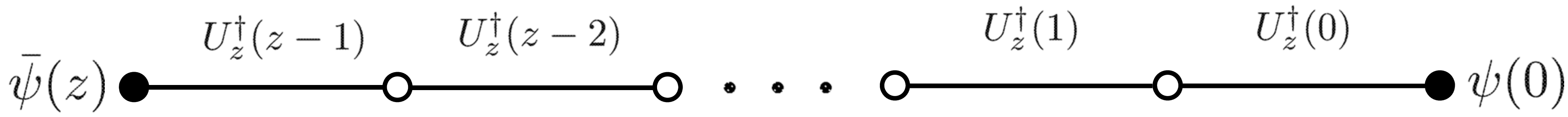
$$\tilde{f}(x,\mu^2,P_z) = \int \frac{dz}{4\pi} e^{ixP_z z} \langle P|\bar{\psi}(z)\gamma^z \exp\Big(-ig\int_0^z dz'\,A^z(z')\Big)\psi(0)|P\rangle$$

- Discretized operator is not unique
- We use the simplest discretized operator

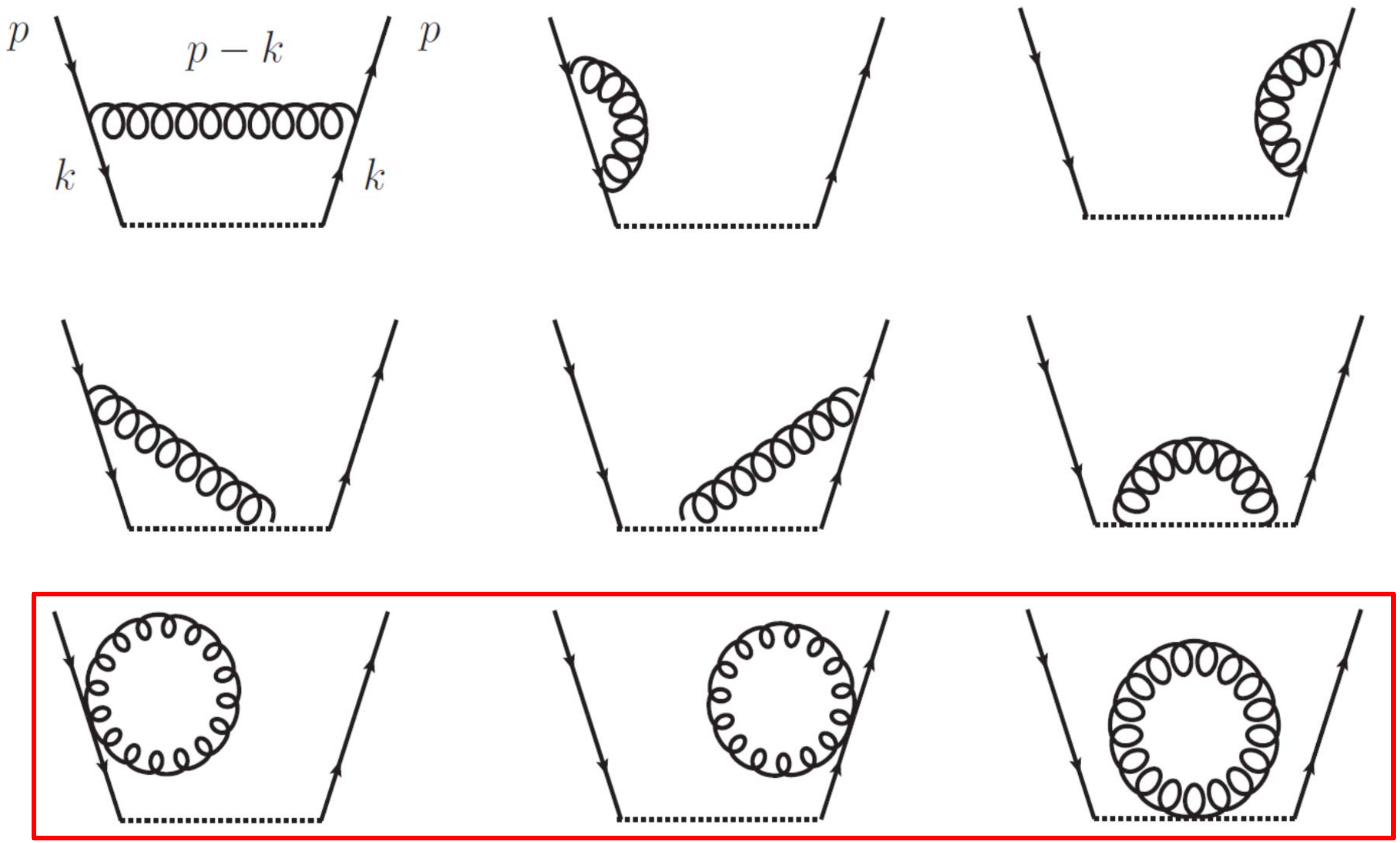
$$\bar{\psi}(z)\exp\Big(-ig\int_0^z dz'\,A^z(z')\Big)\psi(0)$$

$$\Downarrow$$

$$\bar{\psi}(z)U_z^\dagger(z-1)U_z^\dagger(z-2)\cdots U_z^\dagger(1)U_z^\dagger(0)\psi(0)$$

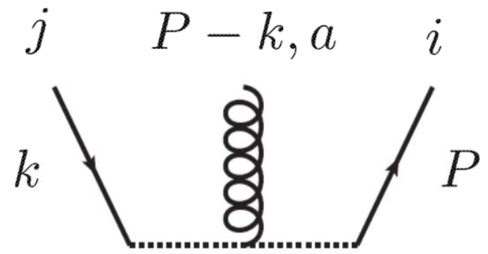


Diagrams in Feynman gauge



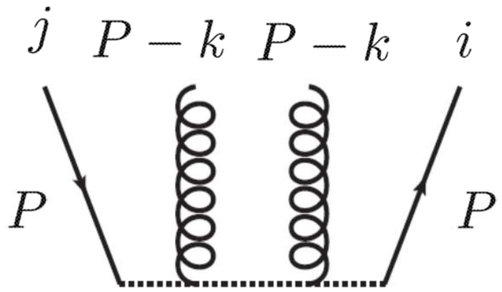
Only appear in LPT(Tadpole diagrams)

Feynman rule for Wilson line



$$ga(T^a)_{ij}(\gamma^z)_{ij}\frac{e^{-iP_z z}-e^{-ik_z z}}{2\sin(\frac{a(P_z-k_z)}{2})}$$

$$\xrightarrow[a\to 0]{} g(T^a)_{ij}(\gamma^z)_{ij}\boxed{\frac{e^{-iP_z z}-e^{-ik_z z}}{(P_z-k_z)}}$$



$$-g^2 C_F\delta_{ij}(\gamma^z)_{ij}\left(a^2\frac{e^{-iP_z z}-e^{-ik_z z}}{4\sin^2(\frac{a(P_z-k_z)}{2})}-a\frac{ze^{-iP_z z}}{1-e^{-i(P_z-k_z)a}}\right)$$

$$\xrightarrow[a\to 0]{} -g^2 C_F\delta_{ij}(\gamma^z)_{ij}\boxed{\left(\frac{e^{-iP_z z}-e^{-ik_z z}}{(P_z-k_z)^2}+i\frac{ze^{-iP_z z}}{(P_z-k_z)}\right)}$$

No singularity in $k_z \to P_z$!

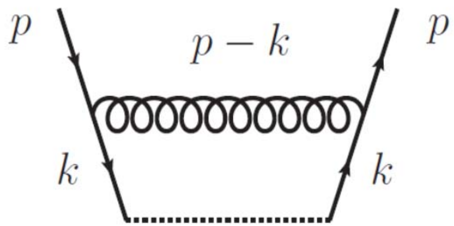
$$-\frac{g^2 a}{2}C_F\delta_{ij}(\gamma^z)_{ij}ze^{-iP_z z}\xrightarrow[a\to 0]{} 0$$

Cancelation of the soft divergence is manifested in Feynman gauge

Calculation technique

H. Kawai, NPB189 (1981) 40

S. Capitani, Phys. Rept. 382 (2003) 113



$$\Gamma_\mu = \sin(k_\mu) \quad C_\mu = \cos\left(\frac{a(P+k)_\mu}{2}\right)$$

$$2W = 4\sum_\mu \sin^2\left(\frac{a(P-k)_\mu}{2}\right)$$

$$a^4 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{\Gamma} \gamma_\mu \not{P} \gamma_\mu \not{\Gamma}] \frac{C_\mu^2}{2W(\Gamma^2)^2}$$

$$= a^4 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{\Gamma} \gamma_\mu \not{P} \gamma_\mu \not{\Gamma}] \Bigg(\underbrace{\frac{C_\mu^2}{2W(\Gamma^2)^2} - \frac{C_\mu^2}{2W(\Gamma^2)^2}\bigg|_{P=0}}_{\text{UV finite}} + \frac{C_\mu^2}{2W(\Gamma^2)^2}\bigg|_{P=0} \Bigg)$$

$$= a^4 \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{k} \gamma_\mu \not{P} \gamma_\mu \not{k}] \left(\frac{1}{(P-k)^2 (k^2)^2} - \frac{1}{(k^2)^3} \right) + O(a)$$

- analytically calculable
- easy to identify collinear divergence

$$+ a^4 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4} \delta\left(\tilde{x} - \frac{k_z}{P_z}\right) \frac{1}{4P_z} \mathrm{Tr}[\gamma^z \not{\Gamma} \gamma_\mu \not{P} \gamma_\mu \not{\Gamma}] \frac{C_\mu^2}{2W(\Gamma^2)^2}\bigg|_{P=0}$$

- no P-dependence

One-loop result of continuum quasi-PDF

UV-cutoff

$$d^4k = \int_{-\infty}^{\infty} dk_0 \int_{-\infty}^{\infty} dk_3 \int_{\lambda^2}^{\mu^2} dk_\perp^2 \int_0^{2\pi} d\theta$$

IR-cutoff

$\tilde{f}^{(1)}_{\rm cont}(x)$

collinear divergence

$$= C_F \frac{\alpha_s}{2\pi} \int dy \Big\{ \delta(x-y) - \delta(x-1) \Big\} \Big\{ \frac{1+y^2}{1-y} \log \frac{-(1-y) + \Lambda_{\rm IR}(1-y)}{y + \Lambda_{\rm IR}(y)} \Big\}$$

$$+ C_F \frac{\alpha_s}{2\pi} \int dy \frac{1}{y^2} \Big(\delta(x-1+y) - \delta(1-x) - y \frac{\partial}{\partial x} \delta(1-x) \Big) \Big\{ \frac{\mu}{p_z} \Big\}$$

linear UV divergence

+finite terms

$$\Lambda_{\rm IR}(y) = \sqrt{(\frac{\lambda}{p_z})^2 + y^2}$$

One-loop result of lattice quasi-PDF

$\tilde{f}^{(1)}_{\rm lat}(x)$

same collinear divergence

$$= C_F \frac{\alpha_s}{2\pi} \int dy \Big\{ \delta(x-y) - \delta(x-1) \Big\} \Big\{ \frac{1+y^2}{1-y} \log \frac{-(1-y) + \Lambda_{\rm IR}(1-y)}{y + \Lambda_{\rm IR}(y)} \Big\}$$

$$+ \ (4\pi\alpha_s) a^4 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk_3 \Big\{ \frac{1}{(2\sin(\frac{ak_3}{2}))^2} \{ \delta(x - 1 + \frac{k_3}{p_z}) - \delta(1-x) \} - \frac{\cos(\frac{ak_3}{2})}{2a\sin(\frac{ak_3}{2})} \frac{\partial}{\partial x} \delta(x-1) \Big\}$$

$$\times \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^3k}{(2\pi)^4} \frac{1}{4\sum_\mu \sin^2(\frac{ak_\mu}{2})} + 2\pi\alpha_s Z_0 \delta(\tilde{x} - 1)$$

gives $\frac{1}{aP_z}$

+finite terms

The one-loop matching factor $C^{(1)}_f(x) = \tilde{f}^{(1)}_{\rm cont}(x) - \tilde{f}^{(1)}_{\rm lat}(x)$ is completely IR-safe

Wilson fermion

Introduce the Wilson term to avoid the doubling problem

$$S_W = ra^4 \sum_{x,\mu} \frac{1}{a}\left[\bar{\psi}(x)U_\mu(x)\psi(x+a\hat{\mu}) + \bar{\psi}(x+a\hat{\mu})U_\mu^\dagger(x)\psi(x) - 2\bar{\psi}(x)\psi(x)\right]$$

r: Wilson parameter

$$\xrightarrow[a \to 0]{} \quad ar \int d^4x\, \bar{\psi}(x)D^2\psi(x)$$

$O(a)$ breaking of chiral symmetry

quark propagator with Wilson fermion

$$a\frac{\sum_\mu -i\gamma_\mu \sin(ak_\mu) + 2r\sum_\mu \sin^2(\frac{ak_\mu}{2})}{\sum_\mu \sin^2(ak_\mu) + (2r\sum_\mu \sin^2(\frac{ak_\mu}{2}))^2}\delta_{ij}$$

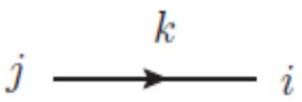
Quark mass

$$ma = \left(2r\sum_\mu \sin^2(\frac{ak_\mu}{2})\right) \begin{cases} m = 0 \text{ at } k_\mu = (0,0,0,0) \\ m \propto \frac{2r}{a} \text{ at other 15 poles} \end{cases}$$

Doublers decouple at small lattice spacing a

Feynman rules with Wilson fermion

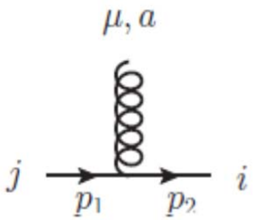
○quark propagator



$$-ia\frac{\sum_\mu \gamma_\mu \sin(ak_\mu)}{\sum_\mu \sin^2(ak_\mu)}\delta_{ij}$$

$$\longrightarrow \quad a\frac{\sum_\mu -i\gamma_\mu \sin(ak_\mu) + 2r\sum_\mu \sin^2(\frac{ak_\mu}{2})}{\sum_\mu \sin^2(ak_\mu) + (2r\sum_\mu \sin^2(\frac{ak_\mu}{2}))^2}\delta_{ij}$$

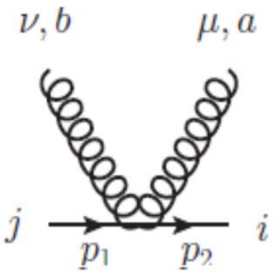
○quark-gluon vertex



$$-ig(T^a)_{ij}\gamma_\mu \cos(\frac{a(p_1+p_2)_\mu}{2})$$

$$\longrightarrow \quad g(T^a)_{ij}\Big(\gamma_\mu \cos(\frac{a(p_1+p_2)_\mu}{2}) + r\sin(\frac{a(p_1+p_2)_\mu}{2})\Big)$$

○quark-gluon vertex 2



$$\frac{a}{2}g(T^aT^b)_{ij}i\gamma_\mu \sin(\frac{a(p_1+p_2)_\mu}{2})\delta_{\mu\nu}$$

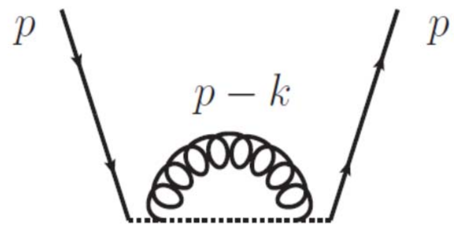
$$\longrightarrow \quad \frac{a}{2}g(T^aT^b)_{ij}\Big(i\gamma_\mu \sin(\frac{a(p_1+p_2)_\mu}{2}) - r\cos((\frac{a(p_1+p_2)_\mu}{2})\Big)\delta_{\mu\nu}$$

One-loop calculation with Wilson fermion

Lattice fermions give $O(a)$ correction to naive fermion in continuum limit.

⟶ collinear divergent part does not change

At one-loop order, UV divergence comes from the Wilson line self-energy diagram.



No quark-gluon vertex, no quark propagator

⟶ UV divergent part does not change

Lattice fermions just give finite correction to the matching factor

Summary

- We discussed the calculation of discretized quasi-PDF in momemtum space based on the lattice perturbation theory and demonstrated how the infrared divergences are canceled through the matching.

- Matching factor is conventionally calculated in the coordinate space because direct observable on the lattice is matrix element itself.

  Tomomi has done.

Future work

- Consistency between the results in coordinate space and momentum space

- Numerical simulation for proton PDF at large-x

- Application to other PDFs

  ○ pion PDF ○ polarized proton PDFs