

# PHOTO- AND ELECTROPRODUCTION OF HYPERON AND HYPERTRITON

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- Motivation
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- Conclusion



# Motivation, Kaon-Maid is getting Old



## An effective Lagrangian Model for Kaon Photo- and Electroproduction on the Nucleon

[T. Mart \(University of Indonesia\)](#), [C. Bennhold](#) and [H. Haberzettl \(George Washington University\)](#), and [L. Tiator](#)

### References:

- For kaon photoproduction: F.X. Lee, T. Mart, C. Bennhold, H. Haberzettl, L.E. Wright, Nucl. Phys. A695 (2001) 237, or [nucl-th/9907119](#)
- For the missing resonance  $D_{13}(1900)$ : T. Mart, C. Bennhold, [Phys. Rev. C61 \(2000\) 012201](#), or [nucl-th/9906096](#)  
C. Bennhold *et al.*, [nucl-th/0008024](#)
- For kaon electroproduction: C. Bennhold, H. Haberzettl, T. Mart, [nucl-th/9909022](#)

- [Electromagnetic Multipoles](#) ( $E_{J\pm}, M_{J\pm}, L_{J\pm}, S_{J\pm}$ )
- [CGLN and Helicity Amplitudes](#) ( $F_1, \dots, F_6, H_1, \dots, H_6$ )
- [Polarized Response Functions](#) ( $R_T, R_L, R_{LT}, R_{TT}, R_{LT}, R_{TT}$ )
- [Unpolarized 2-fold Diff. Cross Sections](#) ( $L, T, L, T, TT, LT'$ )
- [5-fold Diff. Cross Section](#)
- [Total Cross Sections](#) ( $T, L, LT, LT', TT'$ )
- [Transverse Polarization Observables](#) ( $ds/dW, T, S, P, E, F, \dots$ )
- [Target Polarization](#) ( $P_x, P_y, P_z$ )
- [Recoil Polarization](#) ( $P_x, P_y, P_z$ )

### External services:

[MAID Homepage](#) [MAID2003](#) [MAID2000](#) [DMT2001](#) [ETA-MAID](#)



# Motivation, formulation is not clearly revealed

## APPENDIX C

The contributions to the invariant amplitudes corresponding to the exchange of a  $N^*(5/2^+)$  resonance have the same structure as in the spin-3/2 case. We give only the photoproduction amplitudes

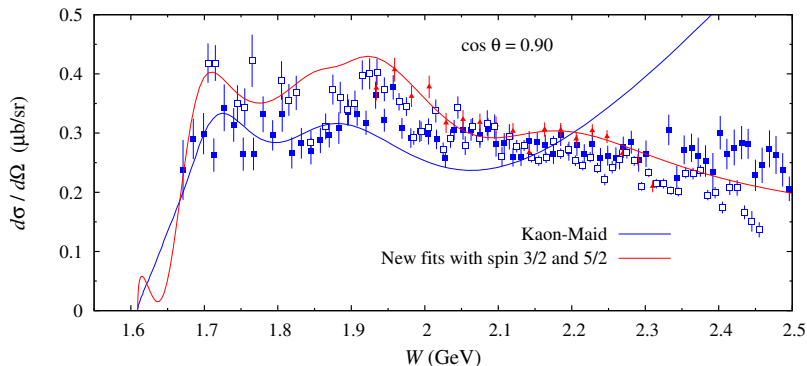
$$\mathcal{A}_j = \frac{1}{10(s - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*})} \sum_{i=1}^2 G_i P_{ij}, \quad j = 1, \dots, 4.$$

Defining  $w = \sqrt{s}$ , the contributions coming from the  $G_1$  coupling are

J.-C. David, *et al.*, Phys. Rev. C **53**, 2613 (1996).



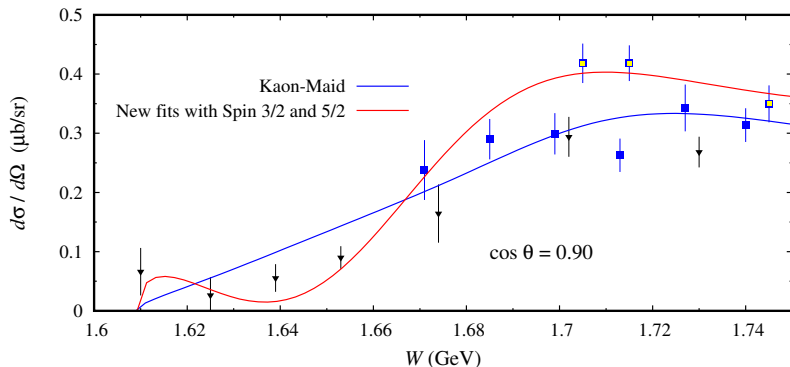
# Motivation, models do not work



- Problem of Data consistency at forward direction
- Even with spin 3/2 and 5/2 [David, *et al.*, PRC **53**, 2613 (1996)] the prediction is far from ideal



# Motivation, models do not work



- ▼ SAPHIR data 1998, not included in the fit
- Data consistency?
- Even with spin 3/2 and 5/2 [David, *et al.*, PRC **53**, 2613 (1996)]

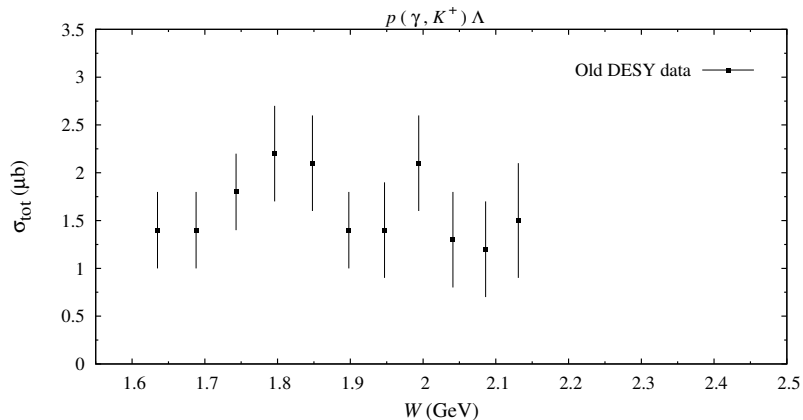


Conclusion:

*“We have to search for other spin 3/2 and spin 5/2 formulations”*

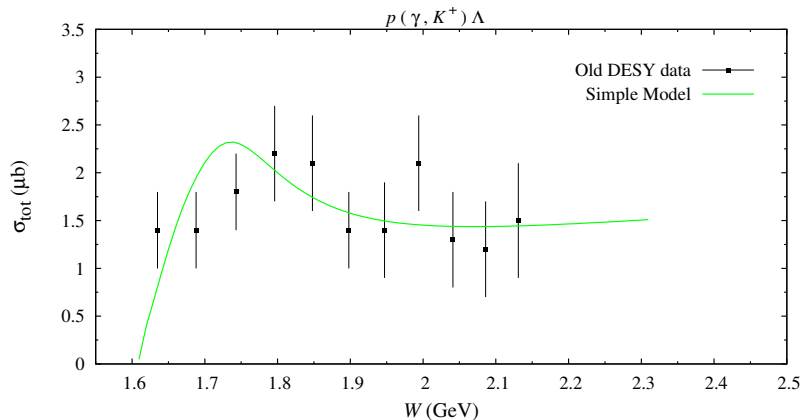


# Progress in Isobar Model, before 1998

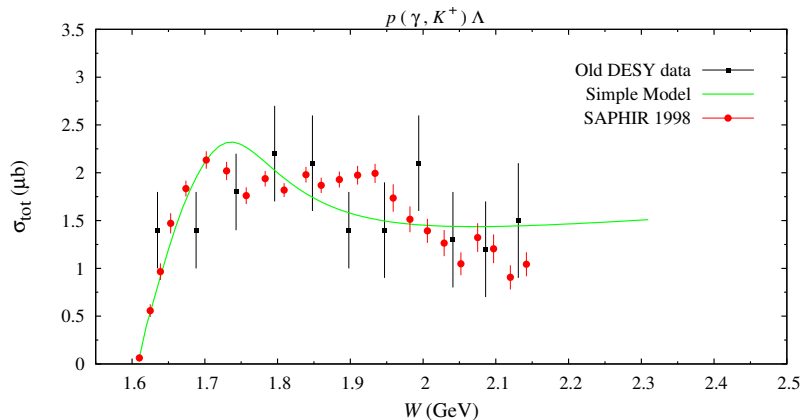




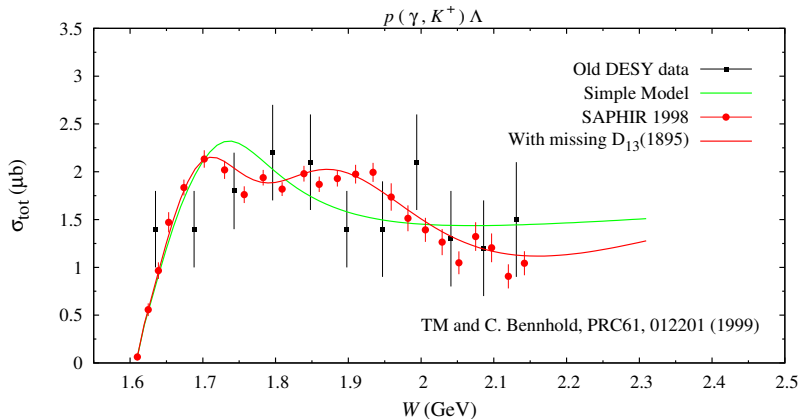
# Progress in Isobar Model, before 1998



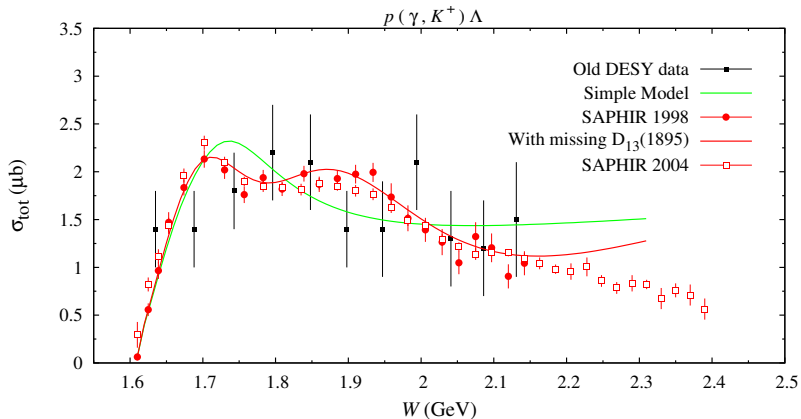
# Progress in Isobar Model, 1998



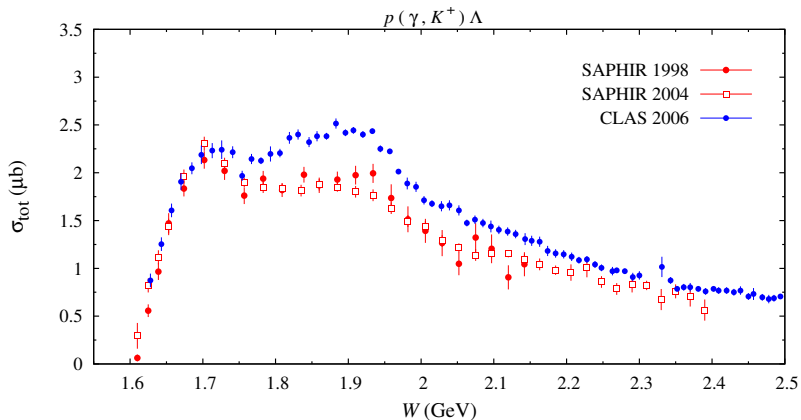
# Progress in Isobar Model, 1998



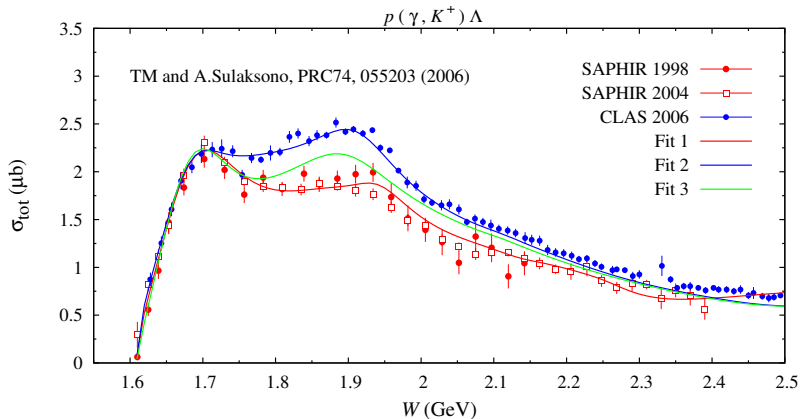
# Progress in Isobar Model, 2006



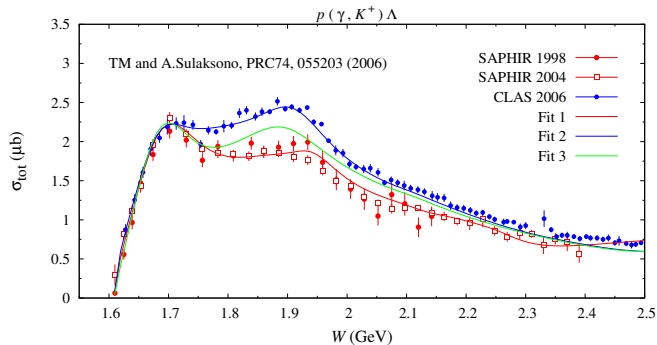
# Progress in Isobar Model, 2006



# Progress in Isobar Model, 2006



# Problems



- Fitting all data simultaneously  $\rightarrow$  democratic model which is not consistent with both data sets
- This problem is simply forgotten, but seems to appear mildly in the new Crystal Ball data
- Unsolved homework!



# Problems with the spin 3/2 and 5/2 formalism

- W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
- P. A. Moldauer and K. M. Case, Phys. Rev. **102**, 279 (1956).
- L. M. Nath, B. Etemadi and J. D. Kimel, Phys. Rev. D **3**, 2153 (1971).
- H. T. Williams, Phys. Rev. C **31**, 2297 (1985).
- M. Benmerrouche, R.M. Davidson and N.C. Mukhopadhyay, Phys. Rev. C **39**, 2339 (1989).
- H. Haberzettl, nucl-th/9812043.
- V. Pascalutsa, Phys. Rev. D **58**, 096002 (1998).
- T. Mizutani, C. Fayard, G. H. Lamot and B. Saghai, Phys. Rev. C **58**, 75 (1998).
- V. Pascalutsa and R. Timmermans, Phys. Rev. C **60**, 042201 (1999).
- V. Pascalutsa, Phys. Lett. B **503**, 85 (2001).
- O. V. Maxwell, Phys. Rev. C **70**, 044612 (2004)
- V. Shklyar, H. Lenske and U. Mosel, Phys. Rev. C **82**, 015203 (2010).
- T. Vrancx, L. De Cruz, J. Ryckebusch, P. Vancraeyveld, Phys. Rev. C **84**, 045201 (2011).
- ...
- ...





# Problems with the spin 3/2 and 5/2 formalism

Finding a consistent propagator without background coming from the lower spin components.

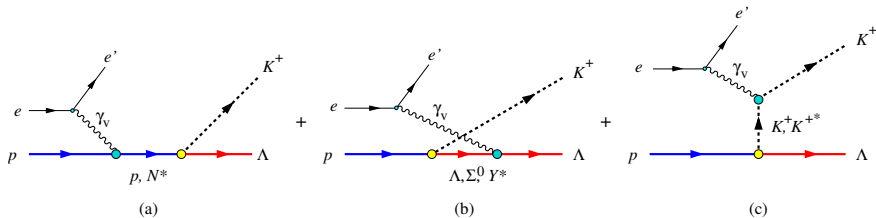
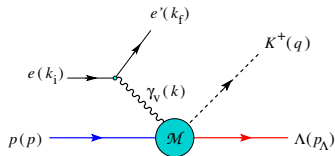
Our solution relies on

Pascalutsca & Timmermans, PRC60, 042201 (1999):

*“Our basic premise is that a consistent interaction should not activate the spurious DOF, and therefore the full interacting theory must obey similar symmetry requirements as the corresponding free theory”*



# Isobar Model



# Isobar Model

From the feynman diagrams we calculate  $\mathcal{M}$  and decompose it to the six gauge- and Lorentz-invariant  $M_i$  matrices

$$\begin{aligned}\mathcal{M} &= \mathcal{M}_p + \mathcal{M}_\Lambda + \mathcal{M}_{\Sigma^0} + \mathcal{M}_{K^*} + \mathcal{M}_{N^*} + \dots \\ &= \bar{u}_\Lambda \sum_{i=1}^6 A_i M_i u_p ,\end{aligned}$$

where

$$\begin{aligned}M_1 &= \frac{1}{2} \gamma_5 (\not{\epsilon} \not{k} - \not{k} \not{\epsilon}) , \\ M_2 &= \gamma_5 [(2q - k) \cdot \epsilon P \cdot k - (2q - k) \cdot k P \cdot \epsilon] , \\ M_3 &= \gamma_5 (q_K \cdot k \not{\epsilon} - q \cdot \epsilon \not{k}) , \\ M_4 &= i \epsilon_{\mu\nu\rho\sigma} \gamma^\mu q^\nu \epsilon^\rho k^\sigma , \\ M_5 &= \gamma_5 (q \cdot \epsilon k^2 - q \cdot k k \cdot \epsilon) , \\ M_6 &= \gamma_5 (k \cdot \epsilon \not{k} - k^2 \not{\epsilon}) ,\end{aligned}$$

we obtain the amplitudes  $A_1, \dots, A_6$  and calculate the observables.



# Model 1, Propagator

## Spin 3/2 resonance<sup>1</sup>

$$P_{\mu\nu}^{3/2} = \frac{\not{p} + \not{k} + \sqrt{s}}{3(s - m_{N^*}^2 + im_{N^*}\Gamma_{N^*})} \left[ g_{\mu\nu} + \gamma_\nu \gamma_\mu - \frac{2}{s} (p+k)_\mu (p+k)_\nu - \frac{1}{\sqrt{s}} \left\{ \gamma_\mu (p+k)_\nu - \gamma_\nu (p+k)_\mu \right\} \right]$$

## Spin 5/2 resonance<sup>1,2</sup>

$$P_{\mu\nu\alpha\beta}^{5/2} = \frac{\not{p} + \not{k} + \sqrt{s}}{10(s - m_{N^*}^2 + im_{N^*}\Gamma_{N^*})} \left[ 5P_{\mu\alpha} P_{\nu\beta} - 2P_{\mu\nu} P_{\alpha\beta} + 5P_{\mu\beta} P_{\nu\alpha} + P_{\mu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\alpha} P_{\nu\beta} \right. \\ \left. + P_{\nu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\beta} P_{\mu\alpha} + P_{\mu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\beta} P_{\nu\alpha} + P_{\nu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\alpha} P_{\mu\beta} \right],$$

with

$$P_{\mu\nu} = -g_{\mu\nu} + \frac{1}{s} (p+k)_\mu (p+k)_\nu.$$

1. J.-C. David, *et al.*, *Phys. Rev. C* **53**, 2613 (1996).

2. V. Shklyar, H. Lenske and U. Mosel, *Phys. Rev. C* **82**, 015203 (2010).



# Model 1, vertex factors

For spin 3/2 resonances the electromagnetic vertex is<sup>†</sup>

$$\Gamma_{N^* p \gamma}^{v(\pm)} = \left\{ g_{N^* p \gamma}^a \left( \varepsilon^v - \frac{\not{\epsilon} k^v}{\sqrt{s} \pm m_p} \right) + g_{N^* p \gamma}^b \frac{p \cdot \varepsilon k^v - p \cdot k \varepsilon^v}{(\sqrt{s} \pm m_p)^2} + g_{N^* p \gamma}^c \frac{k \cdot \varepsilon k^v - k^2 \varepsilon^v}{(\sqrt{s} \pm m_p)^2} \right\} \Gamma_{\pm},$$

where  $\Gamma_+ = i\gamma_5$  and  $\Gamma_- = 1$ .  
and the hadronic vertex reads

$$\Gamma_{K \Lambda N^*}^{\mu(\pm)} = \frac{g_{K \Lambda N^*}}{m_{N^*}} p_{\Lambda}^{\mu} \Gamma_{\mp}.$$

The vertex factors for spin 5/2 resonances are analogous.

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<sup>†</sup>J.-C. David, *et al.*, Phys. Rev. C **53**, 2613 (1996).



# Model 2, Propagator

## Spin 3/2 resonance<sup>1</sup>

$$P_{\mu\nu}^{3/2} = \frac{\not{p} + \not{k} + m_{N^*}}{3(s - m_{N^*}^2 + im_{N^*}\Gamma_{N^*})} \left[ -3g_{\mu\nu} + \gamma_\mu \gamma_\nu + \frac{1}{s} \left\{ (\not{p} + \not{k}) \gamma_\mu (\not{p} + \not{k})_\nu + (\not{p} + \not{k})_\mu \gamma_\nu (\not{p} + \not{k}) \right\} \right]$$

## Spin 5/2 resonance<sup>2</sup>

$$P_{\mu\nu\alpha\beta}^{5/2} = \frac{\not{p} + \not{k} + \sqrt{s}}{10(s - m_{N^*}^2 + im_{N^*}\Gamma_{N^*})} \left[ 5P_{\mu\alpha} P_{\nu\beta} - 2P_{\mu\nu} P_{\alpha\beta} + 5P_{\mu\beta} P_{\nu\alpha} + P_{\mu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\alpha} P_{\nu\beta} \right. \\ \left. + P_{\nu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\beta} P_{\mu\alpha} + P_{\mu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\beta} P_{\nu\alpha} + P_{\nu\rho} \gamma^\rho \gamma^\sigma P_{\sigma\alpha} P_{\mu\beta} \right],$$

with

$$P_{\mu\nu} = -g_{\mu\nu} + \frac{1}{s} (\not{p} + \not{k})_\mu (\not{p} + \not{k})_\nu.$$

1. V. Pascalutsa, *Phys. Lett. B* **503**, 85 (2001).

2. J.-C. David, *et al.*, *PRC* **53**, 2613 (1996), V. Shklyar, *et al.*, *PRC* **82**, 015203 (2010), T. Vrancx, *et al.*, *PRC* **84**, 045201 (2011)



## Model 2, vertex factors

For spin 3/2 resonances the electromagnetic vertex is<sup>†</sup>

$$\Gamma_{N^* p \gamma}^{\nu(\pm)} = -\frac{i}{m_{N^*}^2} \left[ g^{(1)} (\varepsilon^\nu \not{k} - k^\nu \not{\varepsilon}) \not{p} + g^{(2)} (k^\nu p \cdot \varepsilon - \varepsilon^\nu p \cdot k) + g^{(3)} p^\nu (\not{\varepsilon} \not{k} - \not{k} \not{\varepsilon}) \right. \\ \left. + g^{(4)} \gamma^\nu (\not{k} \not{\varepsilon} - \not{\varepsilon} \not{k}) \not{p} + g^{(5)} \gamma^\nu (p \cdot k \not{\varepsilon} - p \cdot \varepsilon \not{k}) \right] \Gamma_\pm,$$

where  $\Gamma_+ = i\gamma_5$  and  $\Gamma_- = 1$ .  
and the hadronic vertex reads

$$\Gamma_{K\Lambda N^*}^{\mu(\pm)} = \frac{g_{K\Lambda N^*}}{m_{N^*}^2} \Gamma_\mp i\varepsilon^{\mu\nu\rho\sigma} p_{\Lambda\nu} \gamma_5 \gamma_\rho q_\sigma.$$

The vertex factors for spin 5/2 resonances are analogous.

<sup>†</sup>V. Pascalutsa and R. Timmermans, Phys. Rev. C **60**, 042201 (1999).



# Spin 3/2 Model 1

e.g.  $A_1, \dots, A_4$  for Spin 3/2

$$\begin{aligned}
 A_1 &= \pm \left[ (m_p + m_\Lambda) \left( \frac{3}{2} - \frac{1}{s} c_\Lambda \pm \frac{1}{2\sqrt{s}} m_\Lambda \right) \mp c_\pm \{ s - m_K^2 - 3b_\Lambda + m_\Lambda (\pm 2\sqrt{s} + m_p) \} \mp \frac{2}{s} c_\Lambda c_\pm c_k \pm \frac{1}{\sqrt{s}} c_\pm \right. \\
 &\quad \times (m_\Lambda c_k + m_\Lambda c_\Lambda - m_p c_\Lambda) \left. \right] G_1 \mp \left[ \frac{1}{2} (m_p + m_\Lambda) \{ b_p - 3b_\Lambda + 2c_\Lambda + m_\Lambda (\pm \sqrt{s} - m_p) + \frac{2}{s} k^2 c_\Lambda \right. \\
 &\quad \left. \mp \frac{1}{\sqrt{s}} [k^2 m_\Lambda \pm c_\Lambda (\sqrt{s} \pm m_p)] \right\} + b_p (m_p \mp \frac{1}{\sqrt{s}} c_\Lambda) \left. \right] G_2 \pm k^2 \left[ \frac{1}{2s} (m_p + m_\Lambda) (2c_\Lambda - s \mp m_\Lambda \sqrt{s}) - m_p \pm \frac{1}{\sqrt{s}} c_\Lambda \right] G_3, \\
 A_2 &= \frac{1}{t - m_K^2} \left[ \left\{ 3c_\pm (c_4 - 2b_\Lambda) \pm 2m_\Lambda + \frac{2}{s} c_\pm c_\Lambda (2c_k - s + m_p^2) \mp \frac{1}{\sqrt{s}} m_\Lambda c_\pm [(\sqrt{s} \pm m_p)^2 + 2c_k] \right\} G_1 \right. \\
 &\quad + \left\{ 3(b_\Lambda - b_p)(\sqrt{s} \mp m_p) + k^2 [\pm m_\Lambda - \frac{2}{s} c_\Lambda c_\pm (s - m_p^2)] - \frac{1}{\sqrt{s}} c_\Lambda \mp \frac{1}{\sqrt{s}} m_\Lambda (\sqrt{s} \pm m_p) \right\} G_2 \\
 &\quad \left. + k^2 \left\{ -3(\sqrt{s} \mp m_p) \mp 2m_\Lambda + \frac{2}{s} c_\pm c_\Lambda (s - m_p^2) \pm \frac{1}{\sqrt{s}} m_\Lambda (\sqrt{s} \pm m_p) \right\} G_3 \right], \\
 A_3 &= \mp \frac{1}{2} \left\{ 3 \mp 2c_\pm m_\Lambda + \frac{2}{s} c_\Lambda \mp \frac{1}{\sqrt{s}} (m_\Lambda \pm 2c_\pm c_\Lambda) \right\} G_1 \pm \frac{1}{2} \left[ 3(b_p + b_\Lambda) - 2c_\Lambda \mp c_\pm m_\Lambda (s - m_p^2) - \frac{2}{s} k^2 c_\Lambda \right. \\
 &\quad \left. \pm \frac{1}{\sqrt{s}} \{ k^2 m_\Lambda \pm c_\Lambda (\sqrt{s} \pm m_p) \} \right] G_2 \pm \frac{1}{2} k^2 \left( 3 + \frac{2}{s} c_\Lambda \mp \frac{1}{\sqrt{s}} m_\Lambda \right) G_3, \\
 A_4 &= \pm \frac{1}{2} \left\{ 3 \pm 2c_\pm m_\Lambda - \frac{2}{s} c_\Lambda \pm \frac{1}{\sqrt{s}} (m_\Lambda \pm 2c_\pm c_\Lambda) \right\} G_1 \mp \frac{1}{2} \left[ 3(b_p - b_\Lambda) + 2c_\Lambda \pm c_\pm m_\Lambda (s - m_p^2) + \frac{2}{s} k^2 c_\Lambda \right. \\
 &\quad \left. \mp \frac{1}{\sqrt{s}} \{ k^2 m_\Lambda \pm c_\Lambda (\sqrt{s} \pm m_p) \} \right] G_2 \mp \frac{1}{2} k^2 \left( 3 - \frac{2}{s} c_\Lambda \pm \frac{1}{\sqrt{s}} m_\Lambda \right) G_3,
 \end{aligned}$$

Note:  $A_5$  and  $A_6$  have been also calculated  $\rightarrow$  only for electroproduction





# Spin 5/2 Model 1

e.g.  $A_1, \dots, A_3$  for Spin 5/2

$$A_1 = \mp \left[ 5c_1 \{ (m_p + m_\Lambda) c_s \mp 2c_\mp c_1 \} \mp 2c_2 \{ c_\mp c_3 \mp \frac{1}{2s} (m_p + m_\Lambda) c_k \} - d_\mp \left\{ 2c_1 \sqrt{s} [1 + 2c_\mp (\sqrt{s} \pm m_\Lambda)] - 4c_1 c_\mp c_k \pm (m_p + m_\Lambda) [c_1 - c_s \sqrt{s} (\sqrt{s} \pm m_p) + c_s c_k] \right\} \right] G_1 \pm \left[ (m_p + m_\Lambda) \left\{ 5c_1 \times \left( \frac{1}{s} k^2 c_\Lambda + b_p - b_\Lambda \right) - k^2 c_2 \left( \frac{1}{s} c_k - 1 \right) \right\} + d_\mp \left\{ -c_1 (2b_p \sqrt{s} + (m_p + m_\Lambda) [\sqrt{s} (m_p \pm \sqrt{s}) \mp k^2]) \mp (m_p + m_\Lambda) (k^2 c_s - b_q) [\sqrt{s} (\sqrt{s} \pm m_p) - c_k] \right\} \right] G_2 \pm k^2 \left[ (m_p + m_\Lambda) (5c_1 c_s + \frac{1}{s} c_k c_2) + d_\mp \left\{ -2c_1 \sqrt{s} \mp c_1 (m_p + m_\Lambda) \pm c_s (m_p + m_\Lambda) [\sqrt{s} (\sqrt{s} \pm m_p) - c_k] \right\} \right] G_3 ,$$

$$A_2 = \frac{2}{t - m_K^2} \left( \left[ 5c_1 \{ (\sqrt{s} \pm m_p) c_s - 2c_\mp c_1 \} + k^2 c_\mp c_2 (2 - \frac{1}{s} c_k) \pm d_\mp \left\{ c_1 c_\mp (k^2 + 2c_k) - c_s (b_p \sqrt{s} \pm m_p c_k) \right\} \right] G_1 + \left[ (\sqrt{s} \pm m_p) \times \left\{ 5c_1 (k^2 c_s - b_q) + k^2 c_2 \left( \frac{1}{s} c_k - 1 \right) \right\} - d_\mp \times \left\{ c_1 k^2 m_p \pm (k^2 c_s - b_q) [b_p \sqrt{s} \pm c_k m_p] \right\} \right] G_2 - k^2 \left[ (\sqrt{s} \pm m_p) (5c_1 c_s + \frac{1}{s} c_2 c_k) \mp d_\mp \{ c_1 (\sqrt{s} \pm m_p) + c_s (b_p \sqrt{s} \pm c_k m_p) \} \right] G_3 \right),$$

$$A_3 = \pm \left[ 5c_1 \left( 1 + \frac{1}{s} c_\Lambda \right) - \frac{1}{s} c_k c_2 \pm d_\mp \left\{ c_1 (1 + 4c_\mp \sqrt{s}) + \left( 1 + \frac{1}{s} c_\Lambda \right) [b_p \pm m_p (\sqrt{s} \pm m_p)] \right\} \right] G_1 \mp \left[ 5c_1 (2b_p - b_q + k^2 c_s) + k^2 c_2 \left( \frac{1}{s} c_k - 1 \right) \mp d_\mp \left\{ c_1 [k^2 - \sqrt{s} (\sqrt{s} \pm m_p)] + (2b_p - b_q + k^2 c_s) [c_k - \sqrt{s} (\sqrt{s} \pm m_p)] \right\} \right] G_2 \mp k^2 \left[ 5c_1 \left( 1 + \frac{1}{s} c_\Lambda \right) - \frac{1}{s} c_k c_2 \pm d_\mp \left\{ c_1 + \left( 1 + \frac{1}{s} c_\Lambda \right) [\sqrt{s} (\sqrt{s} \pm m_p) - c_k] \right\} \right] G_3 ,$$

Note:  $A_4, \dots, A_6$  have been also calculated.



# Spin 3/2 Model 2

e.g.  $A_1, \dots, A_4$  for Spin 3/2

$$\begin{aligned}A_1 &= \pm m_p \sqrt{s} \left[ d_{\pm} \left\{ -2\sqrt{s}(m_{\Lambda} \mp \sqrt{s}) \mp (c_k + \frac{1}{2}(m_p + m_{\Lambda})(m_p \mp \sqrt{s})) \right\} - 3 \left\{ c_1 + \frac{1}{2} c_s (m_p + m_{\Lambda}) \right. \right. \\ &\quad \left. \left. \times (m_p \pm \sqrt{s}) \right\} \right] G_1 \pm \frac{1}{2} \sqrt{s} \left[ d_{\pm} \left\{ 2b_p \sqrt{s} + (m_p + m_{\Lambda})(m_p \mp \sqrt{s}) \sqrt{s} \pm k^2 (m_p + m_{\Lambda}) \right\} + 3(m_p + m_{\Lambda})(c_1 - b_p c_s) \right] G_2 \\ &\quad \pm 2\sqrt{s} \left[ d_{\pm} \left\{ 2b_p \sqrt{s} + (m_p + m_{\Lambda})(m_p \mp \sqrt{s}) \sqrt{s} - (m_p \sqrt{s} \mp c_p)(m_{\Lambda} \mp \sqrt{s}) \right\} - 3c_1 (m_{\Lambda} \pm \sqrt{s}) \right] G_3, \\ A_2 &= \pm \frac{1}{t - m_k^2} \left[ m_p \sqrt{s} \left\{ \pm k^2 d_{\pm} + 3(k^2 c_s - 2b_q) \right\} G_1 + \sqrt{s} \left\{ \mp d_{\pm} m_p k^2 - 3(m_p \mp \sqrt{s})(c_1 - b_p c_s) \right\} G_2 + 4sk^2 d_{\pm} G_3 \right], \\ A_3 &= \pm m_p \sqrt{s} \left[ d_{\pm} \left\{ -2\sqrt{s} \mp \frac{1}{2}(m_p \mp \sqrt{s}) \right\} + \frac{3}{2}(m_p \pm \sqrt{s})(1 + \frac{1}{s} c_{\Lambda}) \right] G_1 \pm \frac{1}{2} \sqrt{s} \left[ d_{\pm} \left\{ (m_p \mp \sqrt{s}) \sqrt{s} \pm k^2 \right\} \right. \\ &\quad \left. + 3 \left\{ c_1 + b_p (1 + \frac{1}{s} c_{\Lambda}) \right\} \right] G_2 \mp 2\sqrt{s} (\pm c_k d_{\pm} + 3c_1) G_3 \\ A_4 &= \pm m_p \sqrt{s} \left[ d_{\pm} \left\{ -2\sqrt{s} \mp \frac{1}{2}(m_p \mp \sqrt{s}) \right\} - \frac{3}{2}(m_p \pm \sqrt{s}) c_s \right] G_1 \pm \frac{1}{2} \sqrt{s} \left[ d_{\pm} \left\{ (m_p \mp \sqrt{s}) \sqrt{s} \pm k^2 \right\} \right. \\ &\quad \left. + 3 \left\{ c_1 - b_p c_s \right\} \right] G_2 \mp 2\sqrt{s} (\pm c_k d_{\pm} + 3c_1) G_3\end{aligned}$$



# Spin 5/2 Model 2

e.g.  $A_1, \dots, A_4$  for Spin 5/2

$$\begin{aligned}
 A_1 &= \pm 2m_p \sqrt{s} \left[ d_{\mp} \left\{ -3c_1 \sqrt{s} (m_{\Lambda} \pm \sqrt{s}) - \frac{1}{2} c_s k^2 (m_p + m_{\Lambda}) \sqrt{s} \pm [2c_1 c_k + \frac{1}{2} (m_p + m_{\Lambda}) (m_p \pm \sqrt{s}) (c_1 + c_k c_s)] \right\} \right. \\
 &\quad + \left. \left\{ -(5c_1^2 + c_2 c_3) - \frac{1}{2} (m_p + m_{\Lambda}) (m_p \mp \sqrt{s}) (5c_1 c_s + \frac{1}{s} c_2 c_k) \right\} G_1 \pm \sqrt{s} \left[ d_{\mp} \left\{ 2b_p c_1 \sqrt{s} + (m_p + m_{\Lambda}) (m_p \pm \sqrt{s}) \right. \right. \right. \\
 &\quad \times \left. \left. \sqrt{s} (2c_1 - b_p c_s) \pm (m_p + m_{\Lambda}) (b_p b_q - 2c_1 k^2) \right\} + (m_p + m_{\Lambda}) \left\{ 5c_1 (c_1 - b_p c_s) - \frac{1}{s} c_2 c_p k^2 \right\} \right] G_2 \\
 &\quad \pm 2\sqrt{s} \left[ 4\sqrt{s} c_1 d_{\mp} (b_p \pm \frac{1}{\sqrt{s}} c_k m_{\Lambda}) - (m_{\Lambda} \mp \sqrt{s}) \times (10c_1^2 + 2c_2 c_3) \right] G_3, \\
 A_2 &= \pm \frac{1}{t - m_k^2} \left( 2m_p \sqrt{s} \left[ d_{\mp} \left\{ c_s k^2 \sqrt{s} (m_p \mp \sqrt{s}) \mp [(c_1 + c_k c_s) k^2 - 2b_q c_k] \right\} + \left\{ 5c_1 (k^2 c_s - 2b_q) - c_2 k^2 (2 - \frac{1}{s} c_k) \right\} \right] G_1 \right. \\
 &\quad + 2\sqrt{s} \left[ d_{\mp} \left\{ (2c_1 k^2 + b_p c_s k^2 - 2b_p b_q) \sqrt{s} \pm (m_p \mp \sqrt{s}) (2c_1 k^2 - b_p b_q) \right\} + (m_p \pm \sqrt{s}) \left\{ 5c_1 (b_p c_s - c_1) + \frac{1}{s} c_p c_2 k^2 \right\} \right] \\
 &\quad \times G_2 + 16sk^2 c_1 d_{\mp} G_3), \\
 A_3 &= \pm \sqrt{s} \left( 2m_p \left[ d_{\mp} \sqrt{s} \left\{ -3c_1 + \frac{1}{2} k^2 (1 + \frac{1}{s} c_{\Lambda}) \pm \frac{1}{2\sqrt{s}} (m_p \pm \sqrt{s}) [c_1 - (1 + \frac{1}{s} c_{\Lambda}) c_k] \right\} + \frac{1}{2} (m_p \mp \sqrt{s}) \left\{ 5c_1 (1 + \frac{1}{s} c_{\Lambda}) \right. \right. \right. \\
 &\quad \left. \left. - \frac{1}{s} c_2 c_k \right\} \right] G_1 + \left[ d_{\mp} \left\{ (m_p \pm \sqrt{s}) \sqrt{s} [2c_1 + b_p (1 + \frac{1}{s} c_{\Lambda})] \pm (b_p b_q - 2c_1 k^2 - 2b_p c_k) \right\} \right. \\
 &\quad \left. + \left\{ 5c_1 [c_1 + b_p (1 + \frac{1}{s} c_{\Lambda})] - \frac{1}{s} c_2 c_p k^2 \right\} \right] G_2 + 2 \left[ \pm 4c_1 c_k d_{\mp} - (10c_1^2 + 2c_2 c_3) \right] G_3, \\
 A_4 &= \pm \sqrt{s} \left( 2m_p \left[ d_{\mp} \sqrt{s} \left\{ -(3c_1 + \frac{1}{2} c_s k^2) \pm \frac{1}{2\sqrt{s}} (m_p \pm \sqrt{s}) (c_1 + c_k c_s) \right\} - \frac{1}{2} (m_p \mp \sqrt{s}) \left\{ 5c_1 c_s \right. \right. \right. \\
 &\quad \left. \left. + \frac{1}{s} c_2 c_k \right\} \right] G_1 + \left[ d_{\mp} \left\{ (m_p \pm \sqrt{s}) \sqrt{s} [2c_1 - b_p c_s] \pm (b_p b_q - 2c_1 k^2) \right\} + \left\{ 5c_1 [c_1 - b_p c_s] \right. \right. \\
 &\quad \left. \left. - \frac{1}{s} c_2 c_p k^2 \right\} \right] G_2 + 4 \left\{ \pm 2c_1 c_k d_{\mp} - (5c_1^2 + c_2 c_3) \right\} \times G_3),
 \end{aligned}$$



# Four Models Analyzed

From the two formulations of spin 3/2 and 5/2 resonances used in the present analysis we obtain four combinations (models)

Model	Spin 3/2	Spin 5/2
Model A	Model 1	Model 1
Model B	Model 2	Model 1
Model C	Model 1	Model 2
<b>Model D</b>	Model 2	Model 2



Observables	$N_{\text{data}}$
$d\sigma/d\Omega$	4745
$P_{\Lambda}$	2006
$\Sigma$	100
$T$	66
$C_x$	159
$C_z$	160
$O_x$	66
$O_z$	66
Total	7433



## Nucleon resonances used in our calculation (PDG2014)

Resonance	Status	Mass (MeV)	Width (MeV)
$N(1440)P_{11}$	****	$1430 \pm 20$	$350 \pm 100$
$N(1520)D_{13}$	****	$1515 \pm 5$	$115^{+10}_{-15}$
$N(1535)S_{11}$	****	$1535^{+20}_{-10}$	$150 \pm 25$
$N(1650)S_{11}$	****	$1655^{+15}_{-10}$	$140 \pm 30$
$N(1675)D_{15}$	****	$1675 \pm 5$	$150^{+15}_{-20}$
$N(1680)F_{15}$	****	$1685 \pm 5$	$130 \pm 10$
$N(1700)D_{13}$	***	$1700 \pm 50$	$150^{+100}_{-50}$
$N(1710)P_{11}$	***	$1710 \pm 30$	$100^{+150}_{-50}$
$N(1720)P_{13}$	****	$1720^{+30}_{-20}$	$250^{+150}_{-100}$
$N(1860)F_{15}$	**	$1860^{+100}_{-40}$	$270^{+140}_{-50}$
$N(1875)D_{13}$	***	$1875^{+45}_{-55}$	$200 \pm 25$
$N(1880)P_{11}$	**	$1870 \pm 35$	$235 \pm 65$
$N(1895)S_{11}$	**	$1895 \pm 15$	$90^{+30}_{-15}$
$N(1900)P_{13}$	***	1900	250
$N(2000)F_{15}$	**	$2050 \pm 100$	$198 \pm 2$
$N(2060)D_{15}$	**	2060	$375 \pm 25$
$N(2120)D_{13}$	**	2120	$330 \pm 45$

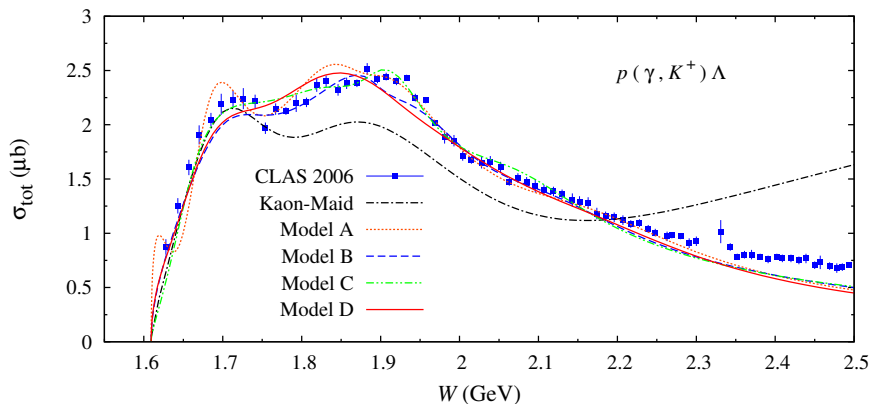


# Extracted background parameters and the resonance hadronic cutoff $\Lambda_R$

Parameters	A	B	C	D
$g_{K\Lambda N}/\sqrt{4\pi}$	-3.37	-3.00	-3.00	-3.00
$g_{K\Sigma N}/\sqrt{4\pi}$	0.90	0.90	1.30	1.27
$G_{K^*}^V/4\pi$	-0.25	0.12	-0.37	0.15
$G_{K^*}^T/4\pi$	0.17	-0.08	0.72	0.26
$G_{K_1}^V/4\pi$	0.42	0.43	0.23	1.46
$G_{K_1}^T/4\pi$	-0.72	-0.08	-0.91	0.07
$G_{\Lambda(1600)}/4\pi$	-6.30	-9.00	5.12	8.41
$G_{\Lambda(1810)}/4\pi$	10.00	10.00	-4.48	-9.61
$\Lambda_B$ (GeV)	0.72	0.89	0.70	0.70
$\Lambda_R$ (GeV)	2.00	2.0	2.00	1.31
$\theta_{\text{had.}}$ (deg)	180	122	56	130
$\phi_{\text{had.}}$ (deg)	72	180	180	177
$\chi^2$	15736	13192	14679	11724
$N_{\text{data}}$	7433	7433	7433	7433
$N_{\text{par.}}$	74	86	84	96
$\chi^2/N_{\text{d.o.f}}$	2.14	1.77	1.97	<b>1.58</b>



## Comparison with experimental data, total cross section

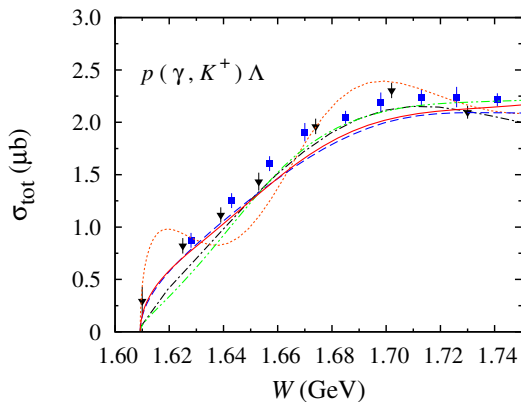


Total CS data are not used in the fit database





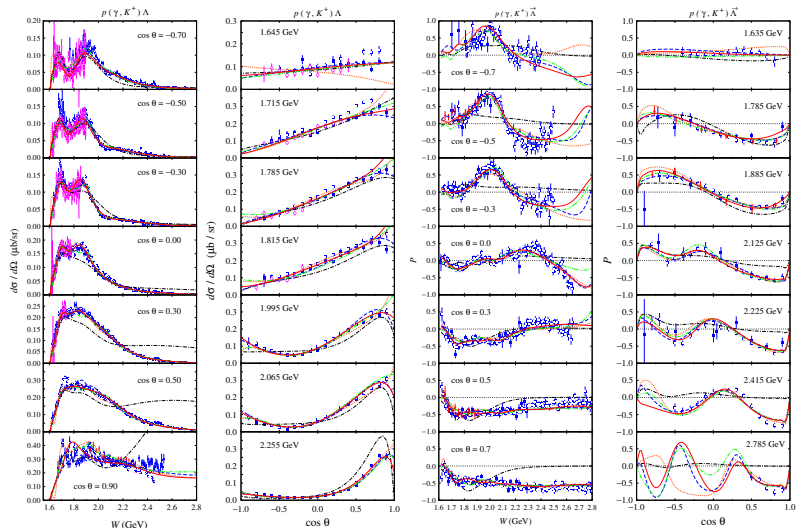
## Comparison with experimental data, total cross section near threshold



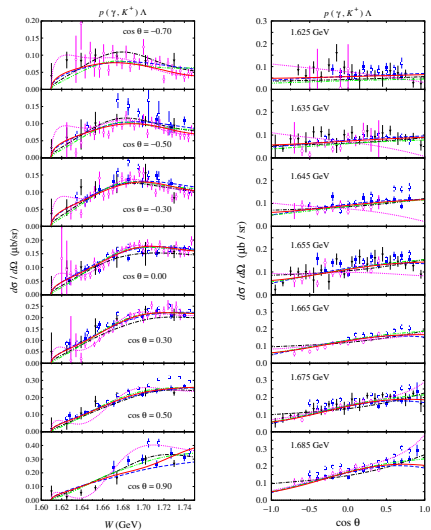
Oscillation in Model A originates from diff. CS



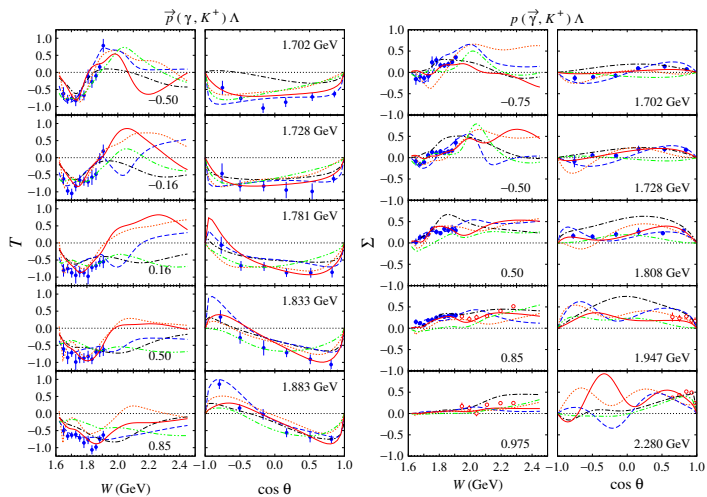
# Comparison with experimental data, diff. cross section & recoil pol.



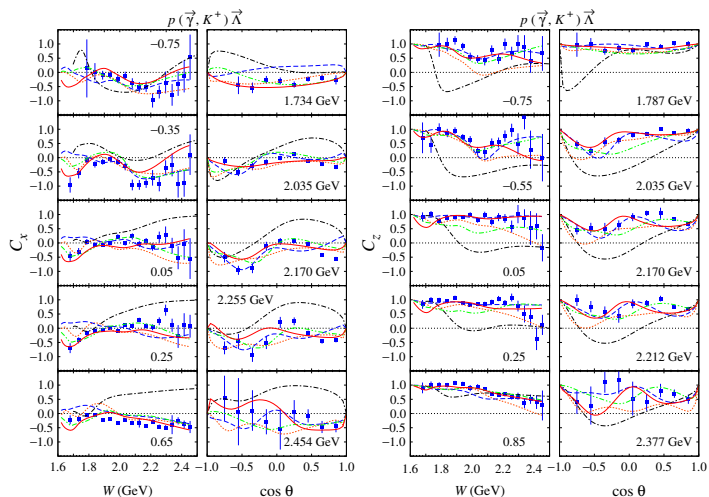
# Comparison with experimental data, diff. cross section & recoil pol.



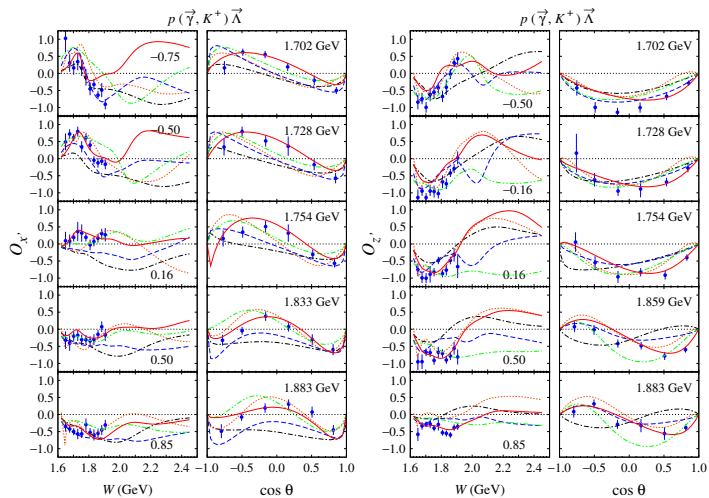
# Comparison with experimental data, target & photon polarization



# Comparison with experimental data, photon-recoil double polarization



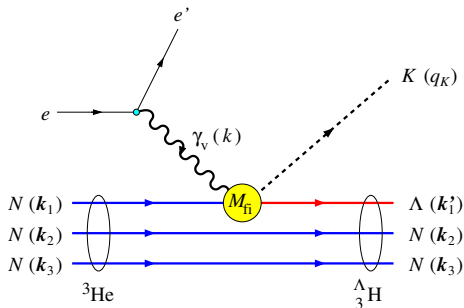
# Comparison with experimental data, photon-recoil double polarization



# Photo and electroproduction of the hypertriton

To calculate the cross section we need the transition current

$$\langle f | J^\mu | i \rangle = \sqrt{3} \int d^3 p d^3 q \Psi_f^*(p, q) J^\mu(k^0, k, k_1^0, k_1, k_1^0, k_1', q_K^0, q_K) \Psi_i(p, q),$$



where the three-body momentum coordinates

$$p = \frac{1}{2}(k_2 - k_3), \quad q = k_1,$$

and the hyperon momentum in the hypertriton

$$q' \equiv k'_1 = k_1 + \frac{m_2 + m_3}{m_1 + m_2 + m_3} Q,$$

with  $Q = k - q_K$ .

The transition amplitude  $M_{fi}$  is obtained from the isobar model.



# Three-body wave functions

The initial wave function  $\Psi_i$  (Similar for the final  $\Psi_f$ )

$$\begin{aligned} \Psi_i(p, q) &= \sum_{\alpha=(LSJlJT)} \phi_{\alpha}(p, q) \left| \{(LS)J, (I\frac{1}{2})j\} \frac{1}{2} M_i \right\rangle \left| (T\frac{1}{2}) \frac{1}{2} M_t \right\rangle \\ &= \sum_{\alpha=(LSJlJT)} \sum_{m_L m_S m_l m_s m_J m_j} \phi_{\alpha}(p, q) (L m_L S m_S | J m_J) \\ &\quad (I m_l \frac{1}{2} m_s | j m_j) (J m_J j m_j | \frac{1}{2} M_i) Y_{m_L}^L(\hat{p}) Y_{m_l}^I(\hat{q}) \chi_{m_S}^S \chi_{m_s}^{\frac{1}{2}} \left| (T\frac{1}{2}) \frac{1}{2} M_t \right\rangle, \end{aligned}$$

The transition current

$$\begin{aligned} \langle f | J^{\mu} | i \rangle &= \sqrt{6} \sum_{\alpha, \alpha'} \sum_{m, m'} \sum_{n, m_n} (L m_L S m_S | J m_J) (L m_L S m_S | J' m_{J'}) (I m_l \frac{1}{2} m_s | j m_j) \\ &\quad \times (I' m_{l'} \frac{1}{2} m_{s'} | j' m_{j'}) (J m_J j m_j | \frac{1}{2} M_i) (J' m_{J'} j' m_{j'} | \frac{1}{2} M_f) \left( \frac{1}{2} - m_{s'} \frac{1}{2} m_s | n m_n \right) \\ &\quad \times (-1)^{n - \frac{1}{2} - m_{s'}} \delta_{LL'} \delta_{m_L m_{L'}} \delta_{SS'} \delta_{m_S m_{S'}} \delta_{T0} \\ &\quad \times \int p^2 dp d^3 q \phi_{\alpha'}(p, q') \phi_{\alpha}(p, q) Y_{m_{l'}}^{I'}(\hat{q}') Y_{m_l}^I(\hat{q}) [j^{\mu}]_{m_n}^{(n)}. \end{aligned}$$

The  $[j^{\mu}]_{m_n}^{(n)} \rightarrow$  elementary operator  $\propto \varepsilon_{\mu} j^{\mu}$ .





# Three-body wave functions

$\alpha$	$L$	$S$	$J$	$I$	$2j$	$2T$	$P(^3\text{He})$	$P(^3_\Lambda\text{H})$
1	0	0	0	0	1	1	44.580	-
2	0	1	1	0	1	0	44.899	93.491
3	2	1	1	0	1	0	2.848	5.794
4	0	1	1	2	3	0	0.960	0.034
5	2	1	1	2	3	0	0.189	0.027
6	1	0	1	1	1	0	0.089	0.004
7	1	0	1	1	3	0	0.198	0.008
8	1	1	0	1	1	1	1.107	-
9	1	1	1	1	1	1	1.113	-
10	1	1	1	1	3	1	0.439	-
11	1	1	2	1	3	1	0.064	-
12	3	1	2	1	3	1	0.306	-
13	1	1	2	3	5	1	1.018	-
14	3	1	2	3	5	1	0.024	-
15	2	0	2	2	3	1	0.274	-
16	2	0	2	2	5	1	0.425	-
17	2	1	2	2	3	0	0.122	0.024
18	2	1	2	2	5	0	0.095	0.018
19	2	1	3	2	5	0	0.205	0.053
20	4	1	3	2	5	0	0.053	0.006
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
33	4	1	4	4	7	0	0.011	0.004
34	4	1	4	4	9	0	0.009	0.003



# Nuclear cross section

Calculate

$$W^{\mu\nu} = \frac{1}{2} \sum_{M_i M_f} \langle f | J^\mu | i \rangle \langle f | J^\nu | i \rangle^*,$$

related to the nuclear structure functions by

$$W_T = \frac{1}{4\pi} (W_{xx} + W_{yy}),$$

$$W_L = \frac{1}{4\pi} W_{00},$$

$$W_{TT} = \frac{1}{4\pi} (W_{xx} - W_{yy}),$$

$$W_{LT} = \frac{1}{4\pi} (W_{0x} + W_{x0}).$$

where

$$\frac{d\sigma_V}{d\Omega_K} = \frac{d\sigma_T}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \varepsilon \frac{d\sigma_{TT}}{d\Omega_K} \cos 2\phi_K + \sqrt{2\varepsilon_L(1+\varepsilon)} \frac{d\sigma_{LT}}{d\Omega_K} \cos \phi_K,$$

and obtain the cross sections

$$\frac{d\sigma_T}{d\Omega_K^{c.m.}} = \alpha_e \frac{q_K^{c.m.}}{K_L} \frac{M_{\Lambda H}^3}{2W} W_T^{c.m.},$$

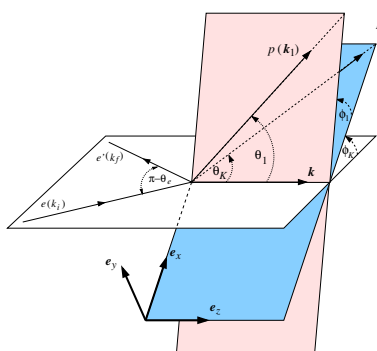
$$\frac{d\sigma_L}{d\Omega_K^{c.m.}} = \alpha_e \frac{q_K^{c.m.}}{K_L} \frac{M_{\Lambda H}^3}{W} W_L^{c.m.},$$

$$\frac{d\sigma_{TT}}{d\Omega_K^{c.m.}} = \alpha_e \frac{q_K^{c.m.}}{K_L} \frac{M_{\Lambda H}^3}{2W} W_{TT}^{c.m.},$$

$$\frac{d\sigma_{LT}}{d\Omega_K^{c.m.}} = -\alpha_e \frac{q_K^{c.m.}}{K_L} \frac{M_{\Lambda H}^3}{2W} W_{LT}^{c.m.},$$



Required for technical calculation, e.g.,



$$\begin{aligned}
 k_1 &= k_1 (\sin \theta_1 \cos \phi_1 e_x + \sin \theta_1 \sin \phi_1 e_y + \cos \theta_1 e_z), \\
 q_K &= q_K (\sin \theta_K e_x + \cos \theta_K e_z), \\
 k'_1 &= k_1 + \frac{2}{3} (k - q_K) \\
 &= (k_1 \sin \theta_1 \cos \phi_1 - \frac{2}{3} q_K \sin \theta_K) e_x + (k_1 \sin \theta_1 \sin \phi_1) e_y \\
 &\quad + (k_1 \cos \theta_1 + \frac{2}{3} k - \frac{2}{3} q_K \cos \theta_K) e_z \\
 &\equiv k'_1 (\sin \theta'_1 \cos \phi'_1 e_x + \sin \theta'_1 \sin \phi'_1 e_y + \cos \theta'_1 e_z)
 \end{aligned}$$

$$q_K \cdot k = q_K k \cos \theta_K,$$

$$k_1 \cdot k = k_1 k \cos \theta_1,$$

$$q_K \cdot k_1 = q_K k_1 (\sin \theta_K \sin \theta_1 \cos \phi_1 + \cos \theta_K \cos \theta_1),$$

$$k \times q_K = k q_K \sin \theta_K e_y,$$

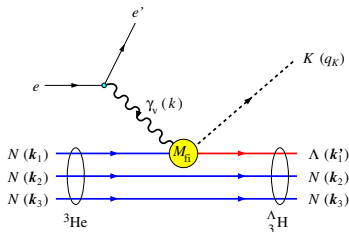
$$k_1 \times k = k_1 k \sin \theta_1 (\sin \phi_1 e_x - \cos \phi_1 e_y),$$

$$k'_1 \times k_1 = \frac{2}{3} (k \times k_1 - q_K \times k_1),$$

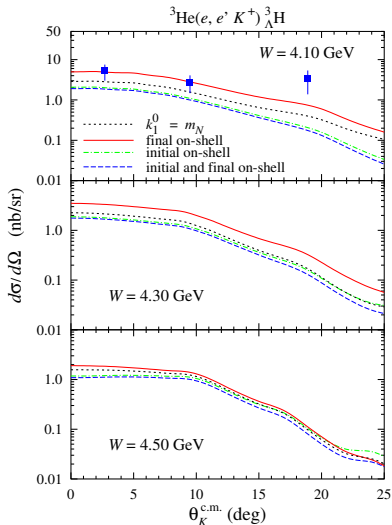


# Investigate the Off-Shell Assumptions

- both initial and final baryons are on-shell  $[k_1^0 = (m_N^2 + k_1^2)^{1/2}, k_1'^0 = (m_Y^2 + k_1'^2)^{1/2}]$ ,
- the initial nucleon is on-shell and the final hyperon is off-shell  $[k_1^0 = (m_N^2 + k_1^2)^{1/2}, k_1'^0 = k_1^0 + k_0 - E_K]$ ,
- the initial nucleon is off-shell and the final hyperon is on-shell,  $[k_1^0 = k_1'^0 + E_K - k_0, k_1'^0 = (m_Y^2 + k_1'^2)^{1/2}]$ ,
- both initial and final baryons are off-shell. In this case the static approximation  $k_1^0 = m_N$  is used for the initial nucleon, while  $k_1'^0 = k_1^0 + k_0 - E_K$ .



# Result



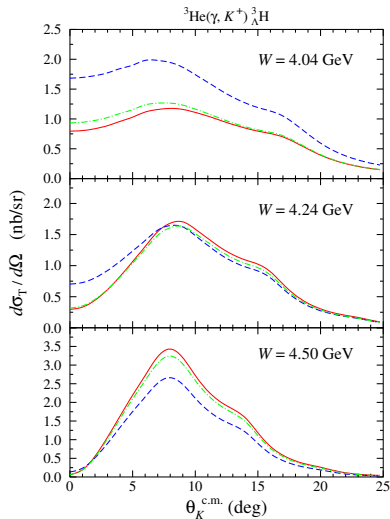
“final on-shell” is closer to exp. data  
→ Hyperon in the final state is  
weakly bound  
→ 130 keV

Data from: F. Dohrmann *et al.*, Phys. Rev. Lett. **93**, 242501 (2004)

T.M. and B. Van Der Ventel, Phys. Rev. C **78**, 014004 (2008)



# Effects of Fermi motion

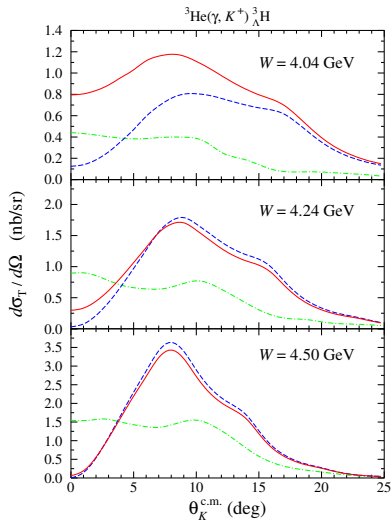


- **dashed curve** → “frozen nucleon” approximation  $\langle k_1 \rangle = 0$
- **dash-dotted curve** → average momentum approximation  $\langle k_1 \rangle = -\frac{1}{3}Q$
- **solid curve** → exact treatment of Fermi motion

T.M. and B. Van Der Ventel, Phys. Rev. C **78**, 014004 (2008)



# Different elementary models



- **dashed curve** → Kaon-Maid without missing resonance
- **dash-dotted curve** → Very simple model
- **solid curve** → Kaon-Maid with missing resonance

T.M. and B. Van Der Ventel, Phys. Rev. C **78**, 014004 (2008)



# Limiting the number of partial waves

The integrations requires heavy numerical calculation. To obtain one point in the plot of observables integrations using  $34 \times 16 \times 34 \times 20 \times 30 \times 10 = 110,976,000$  grid points is necessary

→ limit the number of partial waves

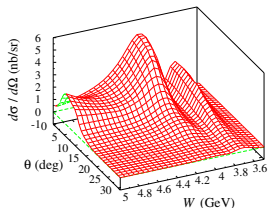
$\alpha$	$L$	$S$	$J$	$l$	$2j$	$2T$	$P(^3\text{He})$	$P(^3_\lambda\text{H})$
1	0	0	0	0	1	1	44.580	-
2	0	1	1	0	1	0	44.899	93.491
3	2	1	1	0	1	0	2.848	5.794
4	0	1	1	2	3	0	0.960	0.034
5	2	1	1	2	3	0	0.189	0.027
6	1	0	1	1	1	0	0.089	0.004
7	1	0	1	1	3	0	0.198	0.008
8	1	1	0	1	1	1	1.107	-
9	1	1	1	1	1	1	1.113	-
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
33	4	1	4	4	7	0	0.011	0.004
34	4	1	4	4	9	0	0.009	0.003



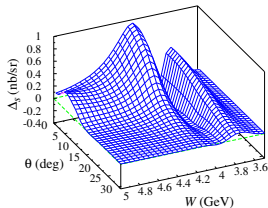
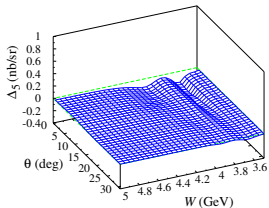
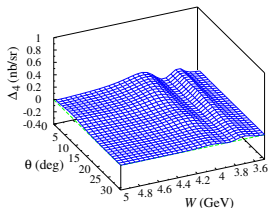


# Limiting the number of partial waves

full calculation



full  $-\alpha \leq 5$



full  $-\alpha \leq 4$

full  $-s$  wave only

T.M. Nucl. Phys. A 815, 18 (2009)



# New calculation

In this approach the  $^3\text{He}$  wave functions reads

$$\begin{aligned}\Psi_{J_b M_b}(\mathbf{r}_b, \mathbf{R}_b) &= \sum_{b=1}^3 \sum_{t_a, s_b, \Sigma_b} \sum_{\ell_b, L_b, I_b} \left[ [\eta_{1/2}(N_i) \eta_{1/2}(N_j)]_{t_a} \eta_{1/2}(N_k) \right]_{T, T_z} \\ &\times \left[ [\chi_{1/2}(N_i) \chi_{1/2}(N_j)]_{s_b} \chi_{1/2}(N_k) \right]_{\Sigma_b} \left[ \phi_{\ell_b}(\mathbf{r}_b) \phi_{L_b}(\mathbf{R}_b) \right]_{I_b} \Big]_{J_b M_b},\end{aligned}$$

whereas the hypertriton wave function can be written as

$$\begin{aligned}\Psi_{J_a M_a}(\mathbf{r}_a, \mathbf{R}_a) &= \sum_{a=1}^3 \sum_{s_a, \Sigma_a} \sum_{\ell_a, L_a, I_a} \left[ \eta_{1/2}(N_1) \eta_{1/2}(N_2) \right]_{t=0, t_z=0} \\ &\times \left[ [\chi_{1/2}(N_1) \chi_{1/2}(N_2)]_{s_a} \chi_{1/2}(\Lambda) \right]_{\Sigma_a} \left[ \phi_{\ell_a}(\mathbf{r}_a) \phi_{L_a}(\mathbf{R}_a) \right]_{I_a} \Big]_{J_a M_a}.\end{aligned}$$



# Jacoby coordinate system for three-body states

The relation between the possible configurations is expressed by

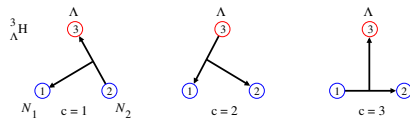
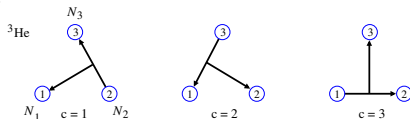
$$\chi_{1/2}^{(b)}(s_b, 1/2) = \sum_{\mathbf{s}} U_{1/2}^{(\pm,0)}(s_b, \mathbf{s}) \chi_{1/2}^{(3)}(\mathbf{s}, 1/2),$$

with the “transportation” coefficients

$$U_{1/2}^{(+)} = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix},$$

$$U_{1/2}^{(-)} = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix},$$

$$U_{1/2}^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$



Jacobi coordinates for the possible  ${}^3\text{He}$  and  ${}^3_{\Lambda}\text{H}$  configurations



# The transition matrix

By writing the elementary operator as

$$\mathcal{O} = \mathcal{O}_I (L + i\sigma \cdot \mathbf{K}) e^{i\mathbf{t} \cdot \mathbf{r}_p},$$

where  $L$  and  $\mathbf{K}$  are obtained from the isobar model, the nuclear transition matrix element in the laboratory frame can be written as

$$\begin{aligned} & \left\langle \Psi_{J_a M_a}(\Lambda^3\text{H}) | \mathcal{O} | \Psi_{J_b M_b}({}^3\text{He}) \right\rangle = \sum_{a,b} \sum_t U_T^{b \rightarrow 3}(t_b, t) \\ & \times \left\langle \left[ [\eta_{1/2}(N_1) \eta_{1/2}(N_2)]_t \eta_0(\Lambda) \right]_{T=0} | \mathcal{O}_I | \left[ [\eta_{1/2}(N_i) \eta_{1/2}(N_j)]_{t_a} \eta_{1/2}(N_k) \right]_{T=1/2} \right\rangle \\ & \times \sum_{\Sigma_a \Sigma_b} \sum_{s_a s_b} \sum_{l_a l_b} \sum_s U_{\Sigma_b}^{b \rightarrow 3}(s_b, s) \left\langle \left[ [\chi_{1/2}(N_1) \chi_{1/2}(N_2)]_{s_a} \chi_{1/2}(N_3) \right]_{\Sigma_a m_{\Sigma_a}} | L + i\sigma \cdot \mathbf{K} | \right. \\ & \times \left. \left[ [\chi_{1/2}(N_1) \chi_{1/2}(N_2)]_{s_b} \chi_{1/2}(N_3) \right]_{\Sigma_b m_{\Sigma_b}} \right\rangle \\ & \times \left\langle [\phi_{\ell_a}(\mathbf{r}_a) \varphi_{L_a}(\mathbf{R}_a)]_{l_a M_a} | e^{i\mathbf{t} \cdot \mathbf{r}_p} | [\phi_{\ell_b}(\mathbf{r}_b) \varphi_{L_b}(\mathbf{R}_b)]_{l_b M_b} \right\rangle, \end{aligned}$$



# The transition matrix

The last bracket can be written as

$$\begin{aligned} F_{\alpha\beta}(\mathbf{t}) &= \left\langle [\phi_{\ell_a}(\mathbf{r}_a) \varphi_{L_a}(\mathbf{R}_a)]_{I_a M_a} | e^{i\mathbf{t}\cdot\mathbf{r}_p} | [\phi_{\ell_b}(\mathbf{r}_b) \varphi_{L_b}(\mathbf{R}_b)]_{I_b M_b} \right\rangle \\ &= \sum_{\lambda, \mu} i^\lambda f_{I_a I_b}^{(\lambda)}(t) Y_{\lambda\mu}(\hat{\mathbf{x}}) (I_b M_b \lambda \mu | I_a M_a) , \end{aligned}$$

with

$$f_{I_a I_b}^{(\lambda)}(t) = \sum_n A_n(\lambda, I_a, I_b) \left( \frac{\pi}{u_n} \right)^{\frac{3}{2}} \left( \frac{t}{2u_n} \right)^\lambda e^{-t^2/4u_n} ,$$

where  $A_n(\lambda, I_a, I_b)$  is obtained from the Gaussian expansion method:

E. Hiyama, Y. Kino and M. Kamimura, *Prog. Part. Nucl. Phys.* **51**, 223 (2003)

$$\frac{d\sigma}{d\Omega} \propto \left| \left\langle \Psi_{J_a M_a}({}^3\text{H}) | \mathcal{O} | \Psi_{J_b M_b}({}^3\text{He}) \right\rangle \right|^2$$



# Conclusion

- New model for kaon photoproduction which includes spin  $3/2$  and  $5/2$  nucleon resonances has been proposed
- The consistent gauge-invariant formulation of the spin  $3/2$  and  $5/2$  interactions leads to a better agreement with experimental data
- A reliable elementary model is required for accurate prediction of the hypernuclear photo- and electroproduction
- A new framework has been established to calculate the hypernuclear photo- and electroproduction



THANK YOU FOR YOUR PATIENCE

