

# Mean Field and Beyond Mean Field Calculations of $\Lambda$ Hypernuclei

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# Formalism & NN interaction

$$H = T_{int} + V + V^{DD}[\rho]$$

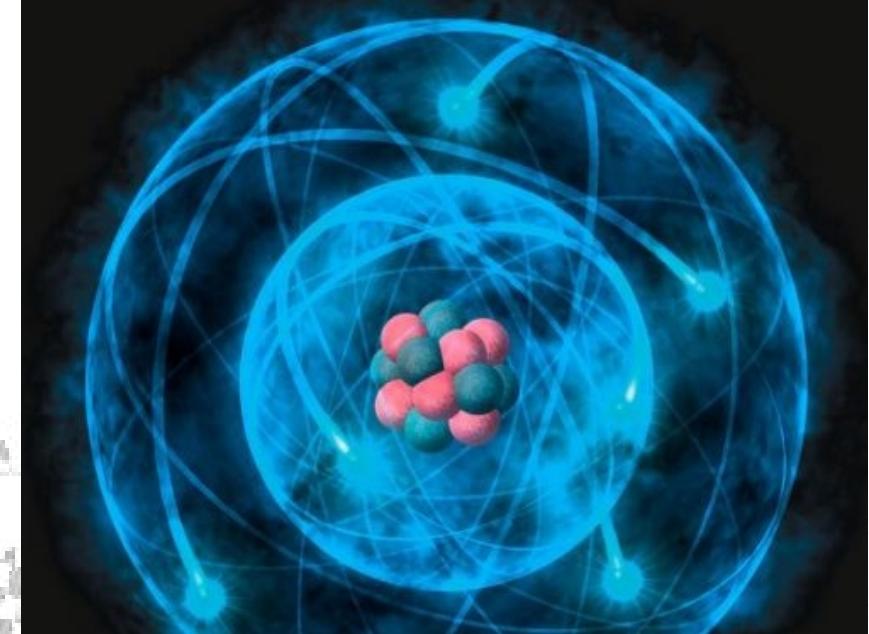
$$T_{int} = \left(1 - \frac{1}{A}\right) \sum_i \frac{\mathbf{p}_i^2}{2m} - \frac{1}{mA} \sum_{i<j} \mathbf{p}_i \cdot \mathbf{p}_j$$

intrinsic kinetic term

2-body density-dependent int.  
contact in S=1, T=0 channel

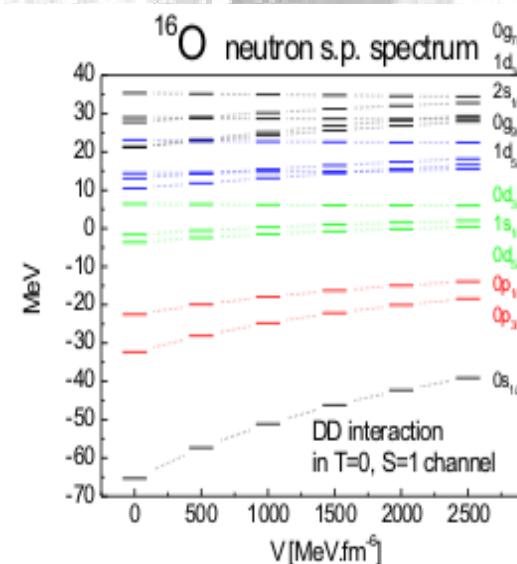
$$V^{DD}[\rho] = \frac{C_{3N}}{6} (1 + P_\sigma) \rho \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2)$$

the same contribution to  $E_{HF}$   
energy as pure contact 3-body  
interaction  $V^{3N} = C_{3N} \delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2) \delta^{(3)}(\mathbf{r}_2 - \mathbf{r}_3)$



2-body density-dependent int.

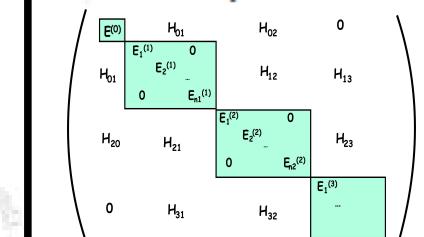
2-body “realistic” NN interaction



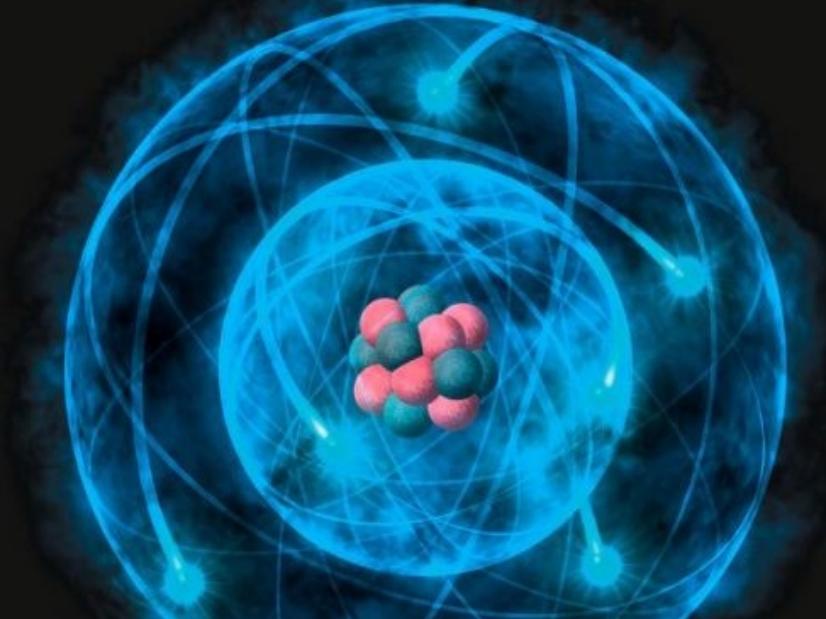
spectrum becomes more realistic

TDA phonons

$$O_\lambda^\dagger = \sum_{ph} c_{ph}^\lambda a_p^\dagger a_{\bar{h}}^\dagger$$



# Formalism & NΛ interaction



several choices for  $V_{N\Lambda}$ :

$\Lambda N$  int. =  $\chi$ PT LO (H. Polinder, J. Haidenbauer, U. Meissner, **Nucl. Phys. A** 779 (2006) 244)

contact momentum dependent force  
(analogy to Skyrme)      H.-J. Schulze,  
E. Hiyama, **Phys. Rev. C** 90, 047301 (2014)

$$V_{\Lambda N}(\mathbf{r}_\Lambda - \mathbf{r}_N) = t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) + \frac{1}{2} t_1 [\mathbf{k}'^2 \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \\ + \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \mathbf{k}'^2] + t_2 \mathbf{k}' \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \cdot \mathbf{k}$$

Density dependent part

$$V_{\Lambda N}(\mathbf{r}_\Lambda, \mathbf{r}_N, \rho) = \frac{3}{8} t_3 (1 + x_3 P_\sigma) \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \rho^\gamma \left( \frac{\mathbf{r}_\Lambda + \mathbf{r}_N}{2} \right)$$

**Nijmegen+Gmatrix** interaction **ESC08c**

M. M. Nagel, Th. A. Rijken, Y. Yamamoto,  
**ArXiv:1501.06636v1**

the dependence on  $k_F$  parameter

symmetric and anti-symmetric LS term

$$V = V_C + V_{SLS} + V_{ALS}$$

$$V_C = \sum_{i=1}^3 (a_i + b_i k_F + c_i k_F^2) \exp(-r^2/b_i^2)$$

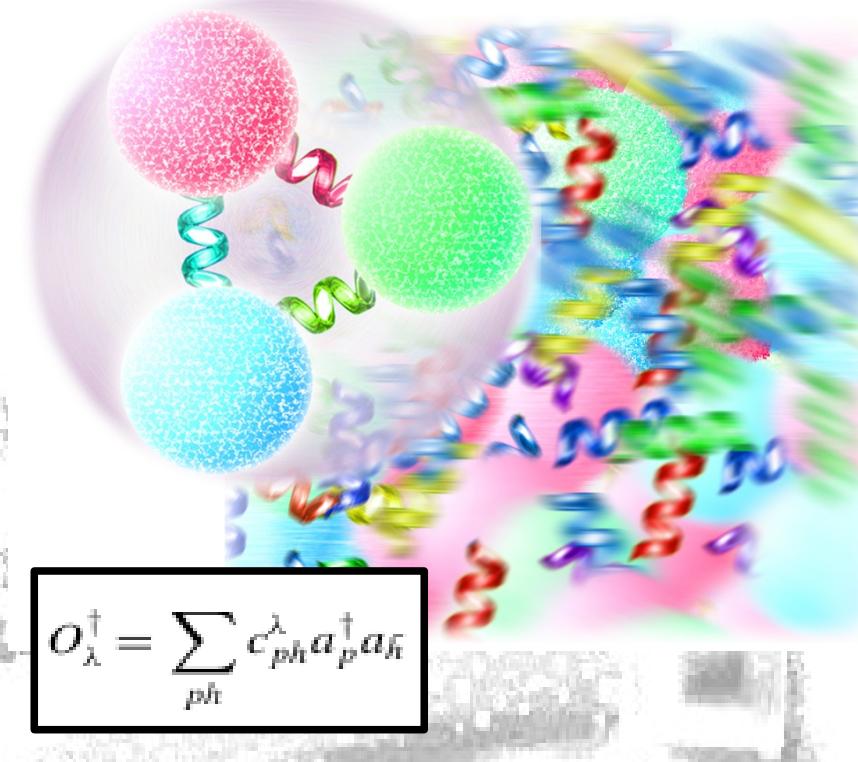
$$V_{SLS} = v_{SLS}(r, k_F) \vec{L} \cdot (\vec{s}_1 + \vec{s}_2)$$

$$V_{ALS} = v_{ALS}(r, k_F) \vec{L} \cdot (\vec{s}_1 - \vec{s}_2)$$

# Single $\Lambda$ hypernuclei (s.p. spectra)

Hilbert space

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \dots \oplus \mathcal{H}_n$$



even-even core +  $\Lambda$ :

$$\mathcal{H}_0 = \{ c_m^+ | HF \rangle \}$$

$$\mathcal{H}_1 = \{ c_m^+ Q_\lambda^+ | HF \rangle \}$$

$$\mathcal{H}_2 = \{ c_m^+ Q_\lambda^+ Q_\lambda^+ | HF \rangle \}$$

even-odd core +  $\Lambda$ :

$$\mathcal{H}_0 = \{ R_\alpha^+ | HF \rangle \}$$

$$\mathcal{H}_1 = \{ R_\alpha^+ Q_\lambda^+ | HF \rangle \}$$

$$\mathcal{H}_2 = \{ R_\alpha^+ Q_\lambda^+ Q_\lambda^+ | HF \rangle \}$$

$$O_\lambda^\dagger = \sum_{ph} c_{ph}^\lambda a_p^\dagger a_h$$

$c_m^+$  ... creation operator of Lambda

$$R_\alpha^+ = \sum_{mh} r_{mh}^\alpha c_m^+ a_h$$

$Q_\lambda^+$  ... phonon excitation operator (in nucleus)

$$\langle m | \hat{H} | n \rangle = \langle HF | c_m \hat{H} c_n^\dagger | HF \rangle =$$

$$= \delta_{mn} E_{HF} + \left[ \sum_{ij} \bar{\sigma}_{imjn}^{p\Lambda} \beta_{ji}^p + \sum_{ij} \bar{\sigma}_{imjn}^{n\Lambda} \beta_{ji}^n \right] =$$

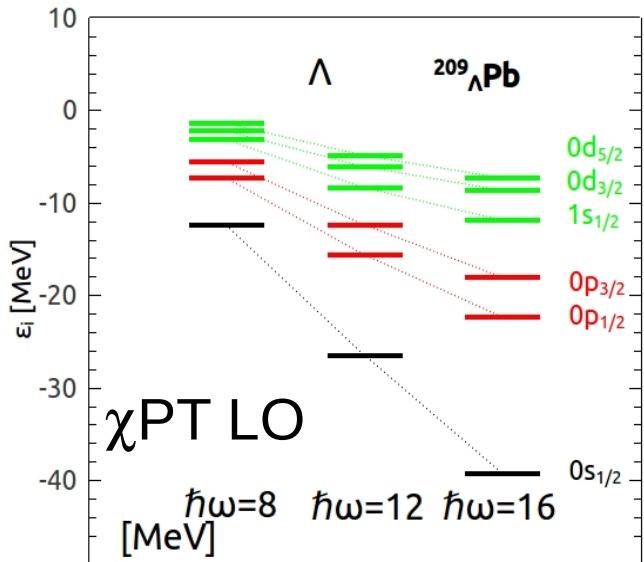
$$= \delta_{mn} E_{HF} + e_m^\Lambda \delta_{mn}$$

$$\begin{aligned} \hat{H} = & \sum_{ij} t_{ij}^p a_i^\dagger a_j + \frac{1}{4} \sum_{ijse} V_{ijse}^{pp} a_i^\dagger a_j^\dagger a_e a_s \\ & + \sum_{ij} t_{ij}^n b_i^\dagger b_j + \frac{1}{4} \sum_{ijse} V_{ijse}^{nn} b_i^\dagger b_j^\dagger b_e b_s \\ & + \sum_{ij} t_{ij}^\Lambda c_i^\dagger c_j \\ & + \sum_{ij} \bar{\sigma}_{ijse}^{pn} a_i^\dagger b_j^\dagger b_e a_s + \sum_{ijse} \bar{\sigma}_{ijse}^{p\Lambda} a_i^\dagger c_j^\dagger c_e a_s + \sum_{ijse} \bar{\sigma}_{ijse}^{n\Lambda} b_i^\dagger c_j^\dagger c_e b_s \end{aligned}$$

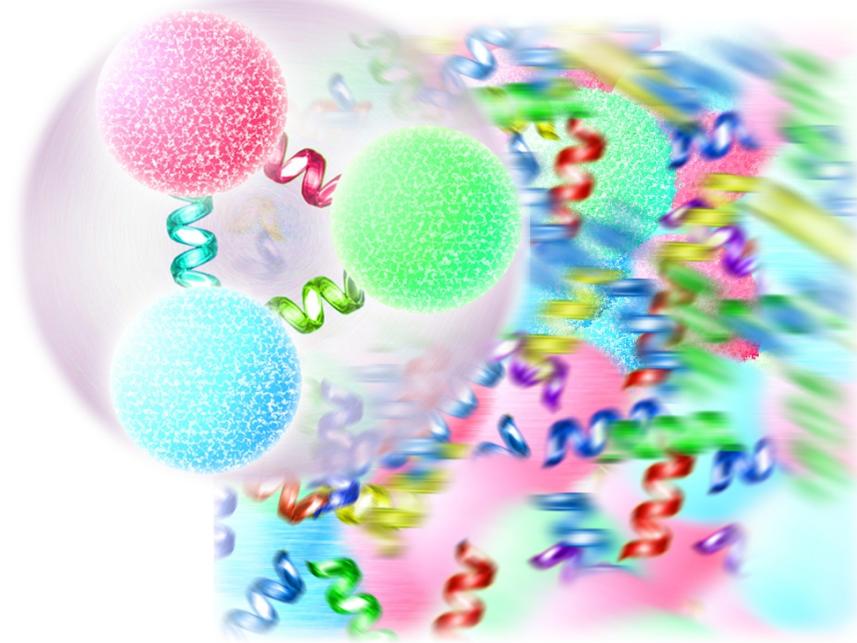
solution in the Hilbert subspace  $\mathcal{H}_0$

$$|m\rangle = c_m^+ |HF\rangle$$

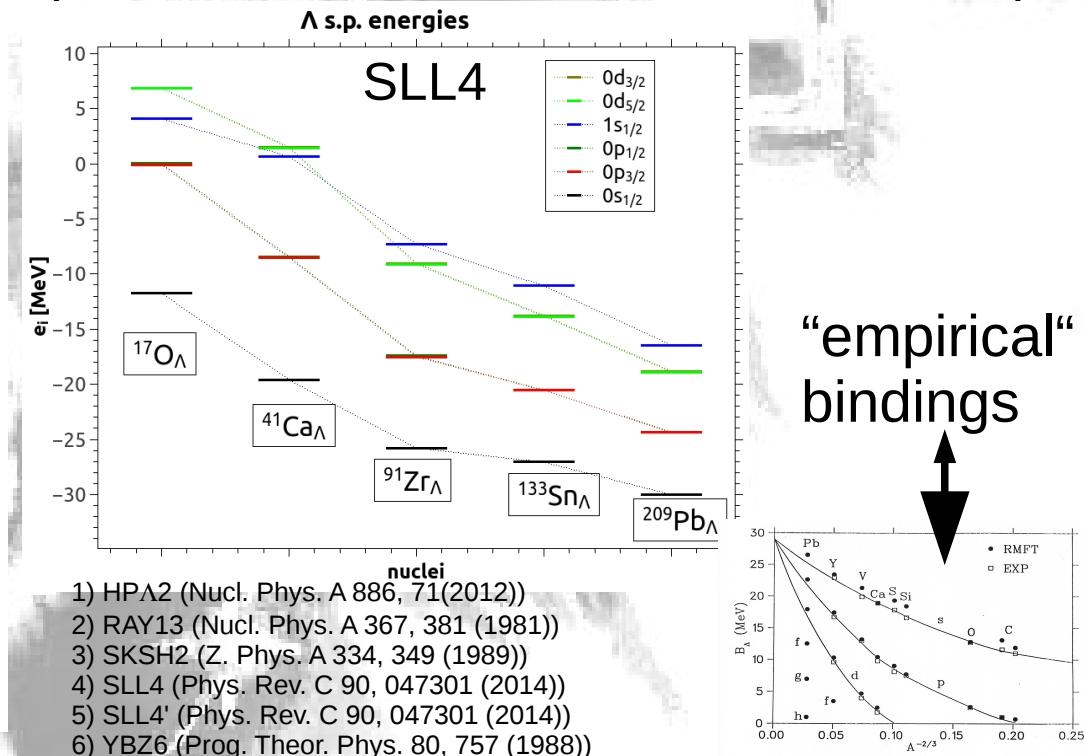
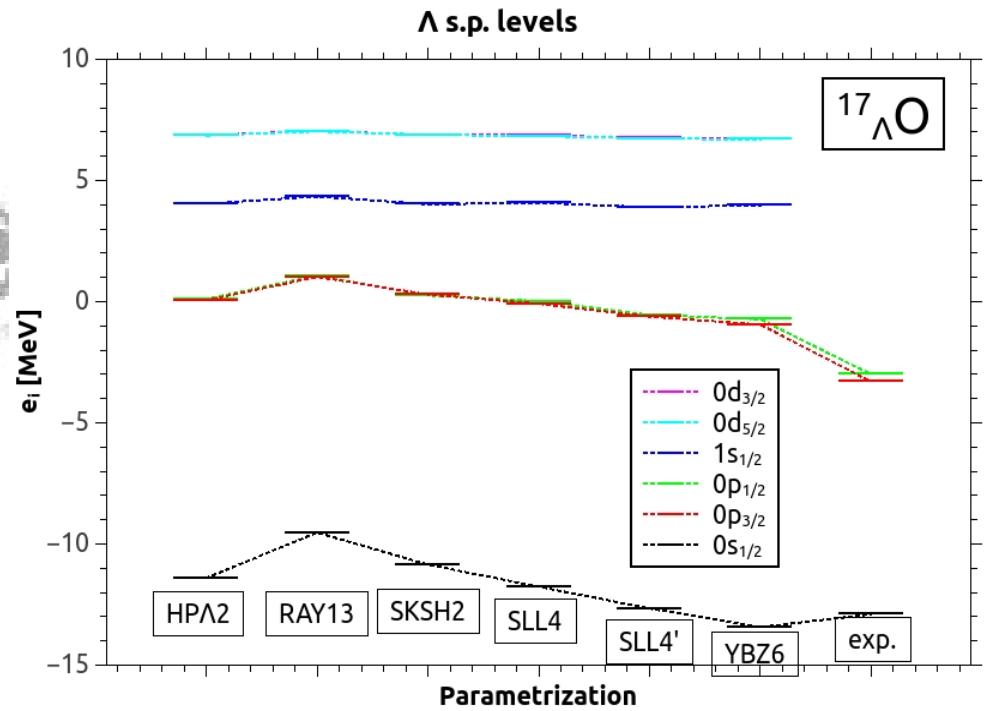
# Single $\Lambda$ hypernuclei (s.p. spectra)



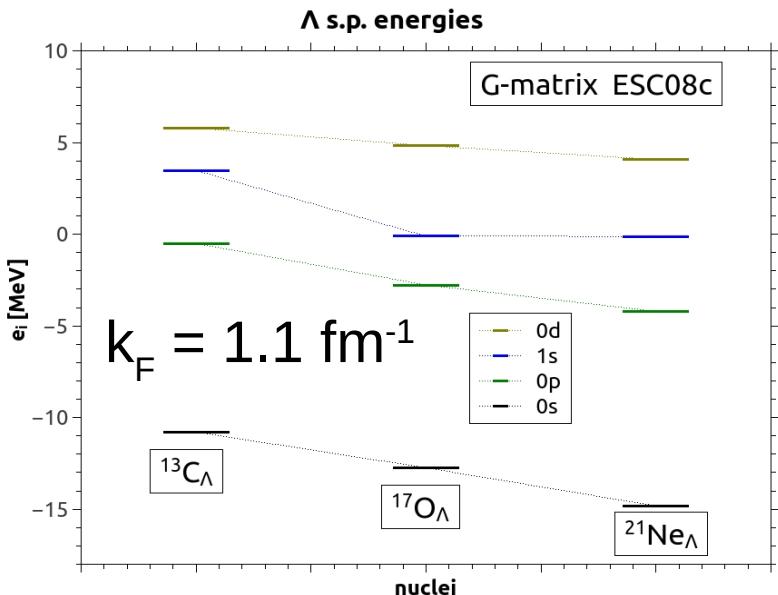
very unstable  
for  $\hbar\Omega$   
moreover –  
problems with  
convergence  
wrong s.o.  
splittings



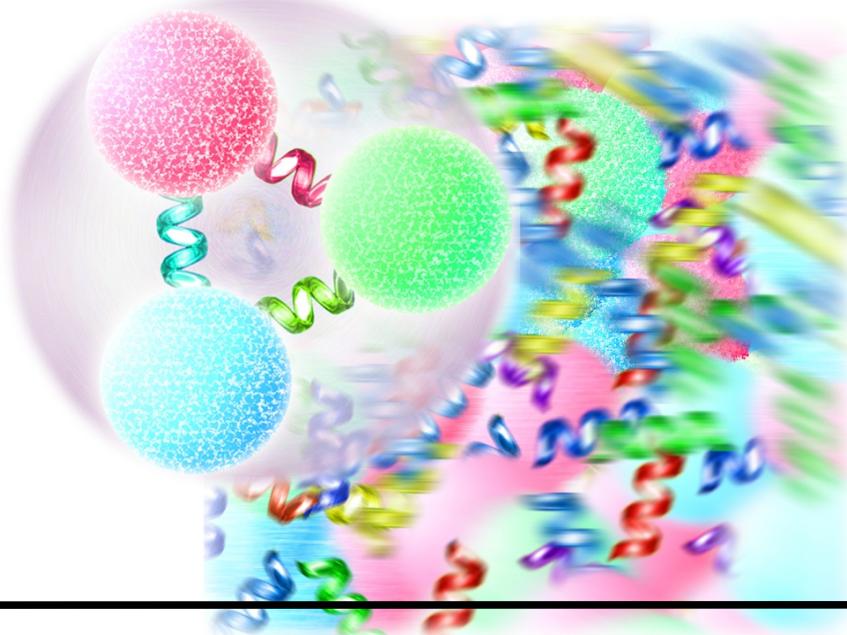
contact momentum dependent force (HPL2, RAY13, SKSH2, SLL4, SLL4', YBZ6)



# Single $\Lambda$ hypernuclei (s.p. spectra)

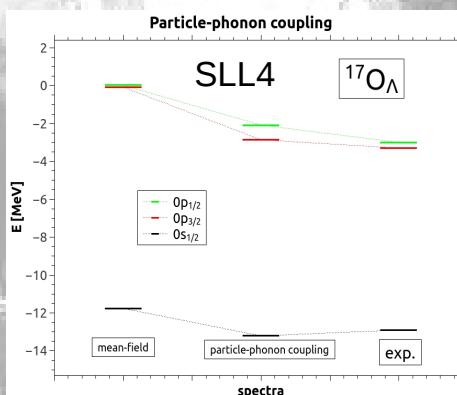


realistic gaps  
between s & d  
shells



## Outline:

- tensor force &  $\Lambda$ - $\Sigma$  coupling & NN $\Lambda$  force
- adding Lambda-Phonon coupling
- even-odd+ $\Lambda$  systems
- odd-odd+ $\Lambda$  systems
- deformed code for Mean Field



... quite a lot  
of work!

## Open questions:

- behavior of s.o. splitting (the role of SLS and ALS terms in Hamiltonian must be examined)
- too big gap between  $1s_{1/2}$  &  $0d$  levels (the role of tensor force &  $\Lambda$ - $\Sigma$  coupling must be examined)

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