The $\eta \to 3\pi$ decay in the nuclear medium as a possible probe for chiral restoration

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Introduction -- η→3π decay in free space

✓ Isospin symmetry breaking in QCD

$m_u \neq m_d$

Small contribution from QED

D.G. Sutherland, Phys. Lett. 23(1968)384.

Eigenstate of G parity

$$\eta \neq \eta_8$$

(mixture of the G parity even and odd states)

Physical state

✓ Large effect of final state interaction (FSI)

2π resonances
- σ mode (s channel)
- ρ mode (t channel)

→ Non-perturbative effect of FSI is important

cf.) C. Roiesnel and T. Truong, NPB187(1981)293.,
B. Borasoy and R. Nissler, EPJA26(2005)383.,
A. Abdel-Rehim, *et al.*, PRD67(2003)054001.,
S. Lanz, PoS CD12:007,2013.

$\eta \rightarrow 3\pi$ decay in nuclear medium

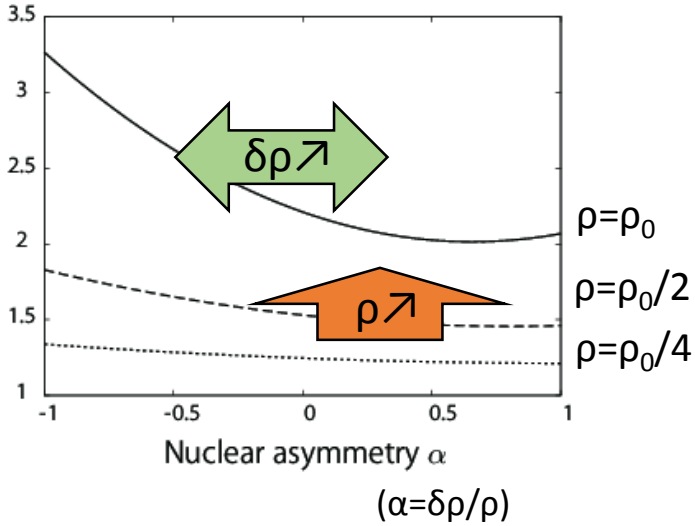
Previous study

The analysis of the nuclear medium effect on the $\eta \rightarrow 3\pi$ decay width with the non-linear σ model

Enhancement of the $\eta \rightarrow 3\pi$ decay width by $\rho=\rho_n+\rho_p$ and $\delta\rho=\rho_n-\rho_p$

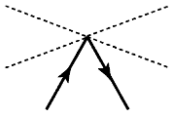
S.S. and T. Kunihiro, PTEP (2015) 013D03, ibid., 089201.

$\eta \rightarrow \pi^+\pi^-\pi^0$ decay width in asymmetric nuclear medium



✓ The enhancement by baryon number density ρ is large (factor 2~3 compared with the value @ρ=0)

※ Significant effect of the 4-meson NN vertex



: Similarity to the enhancement of the ππ cross section in nuclear medium
D. Jido, T. Hatsuda, T. Kunihiro, PRD63(2000)011901.

Chiral restoration is important!

Some relationship with the chiral restoration in nuclear medium?

Purpose of this study

Investigate the significance of the role of the σ meson
and chiral restoration in the η→3π decay in nuclear medium

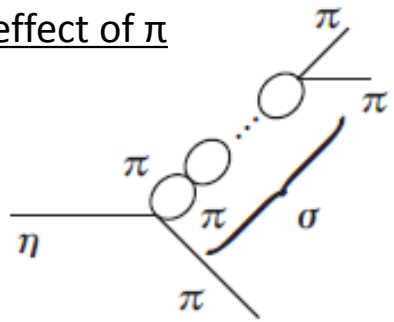
New possible probe for chiral restoration

Analysis with the linear σ model (explicit σ meson degree of freedom)

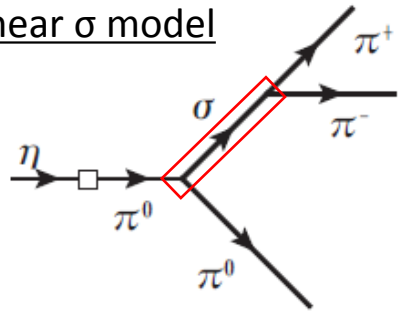
Advantages of linear σ model

- Non-perturbative inclusion of a part of the final state interaction

FSI effect of π



Linear σ model



- Effect of the chiral restoration through the softening of the σ mode

Remarkable effect in the case of the in-medium ππ scattering in linear σ model

Z. Aouissat, et al., PRC61(2000)012202., D. Davesne, et al., PRC62(2000)024604.

※The ρ-meson contribution is ignored for simplicity

Set up

- Linear sigma model with 3 flavor
  - Chiral SU(3) symmetry is respected
  - Isospin symmetry breaking by non-degenerate u and d quarks
  - Explicit σ meson degree of freedom is included

Lagrangian of linear σ model

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}(\partial_\mu M \partial^\mu M^\dagger) - \frac{\mu^2}{2}\mathrm{tr}MM^\dagger - \frac{\lambda}{4}\mathrm{tr}(MM^\dagger)^2 - \frac{\lambda'}{4}\left(\mathrm{tr}MM^\dagger\right)^2 + \frac{B}{2}(\det M + \det M^\dagger) + \frac{A}{2}\mathrm{tr}(\chi M^\dagger + M\chi^\dagger) + \bar{N}\left(i\partial\!\!\!/ - gM_5\right)N$$

M : meson field

N : nucleon field

$$\chi = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$: Isospin symmetry breaking ($m_u \neq m_d \neq m_s$)

$$m_d - m_u = m_{K^\pm}^2 - m_{K^0}^2 - m_{\pi^\pm}^2 + m_{\pi^0}^2$$

$$M = M_{\text{scalar}} + M_{\text{pseudo scalar}} \qquad M_5 = M_{\text{scalar}} + i\gamma_5 M_{\text{pseudo scalar}}$$

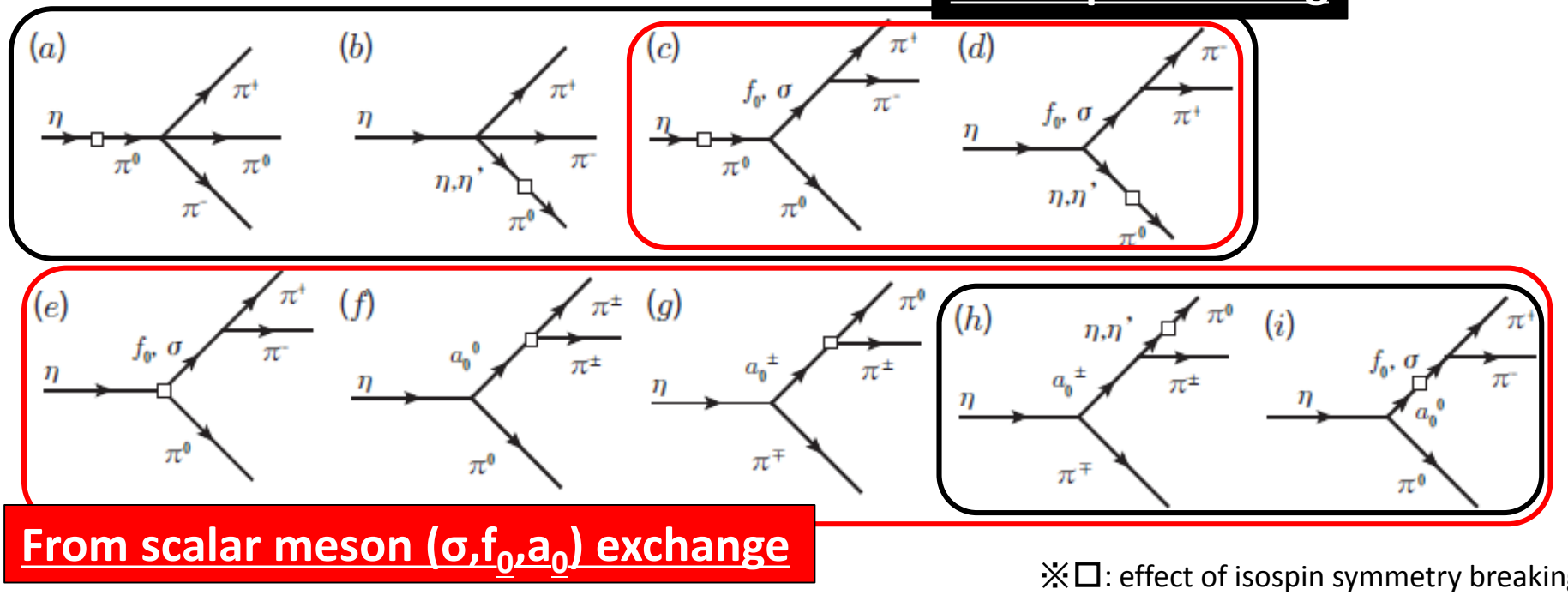
$$M_{\text{scalar}} = \sum_{a=0}^{8} \frac{\sigma_a \lambda_a}{\sqrt{2}} \qquad M_{\text{pseudo scalar}} = \sum_{a=0}^{8} \frac{\pi_a \lambda_a}{\sqrt{2}}$$

Calculation in free space

J. Schechter and Y. Ueda, PRD4(1971)733., W. Hudnall and J. Schechter, PRD9(1974)2111., S.Raby, PRD13(1976)2594.

Diagrams

From $\eta^{(\prime)}\pi^0$ mixing



From scalar meson (σ,f_0,a_0) exchange

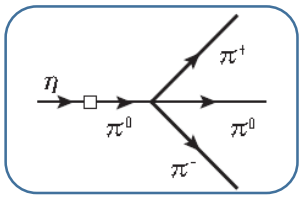
※□: effect of isospin symmetry breaking

Matrix element of $\eta \to 3\pi$ decay with linear σ model

$$\mathcal{M}^{\mathrm{L}\sigma\mathrm{M}}_{\eta\to\pi^+\pi^-\pi^0} = \mathcal{M}^{\mathrm{contact}}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{M}^{\mathrm{isoscalar}}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{M}^{\mathrm{isovector}}_{\eta\to\pi^+\pi^-\pi^0}$$

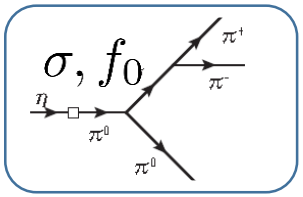
・Contribution purely from meson 4pt. vertex

$$\mathcal{M}^{\mathrm{contact}}_{\eta\to\pi^+\pi^-\pi^0} = 2(-\sin\theta_{\eta\pi^0}) g_{\pi_3\pi_3\pi^+\pi^-} + 2\sin\theta_{\eta\pi^0} g_{\eta\eta\pi^+\pi^-} + \sin\theta_{\eta'\pi^0} g_{\eta\eta'\pi^+\pi^-}$$



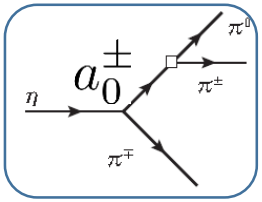
・Contribution from scalar meson (σ, f_0) exchange

$$\mathcal{M}^{\mathrm{isoscalar}}_{\eta\to\pi^+\pi^-\pi^0} = -\, g_{\sigma\eta\pi^0}\frac{1}{s-m_\sigma^2} g_{\sigma\pi^+\pi^-} - g_{f_0\eta\pi^0}\frac{1}{s-m_{f_0}^2} g_{f_0\pi^+\pi^-}$$



・Contribution from isovector meson (a_0) exchange

$$\mathcal{M}^{\mathrm{isovector}}_{\eta\to\pi^+\pi^-\pi^0} = -\, g_{\eta\pi^- a_0^0}\frac{1}{s-m_{a_0^0}^2} g_{a_0^0\pi^0\pi^0} - g_{\eta\pi^- a_0^+}\frac{1}{t-m_{a_0^+}^2} g_{a_0^+\pi^-\pi^0} - g_{\eta\pi^- a_0^+}\frac{1}{u-m_{a_0^-}^2} g_{a_0^-\pi^+\pi^0}$$



The η→3π decay: ~200eV

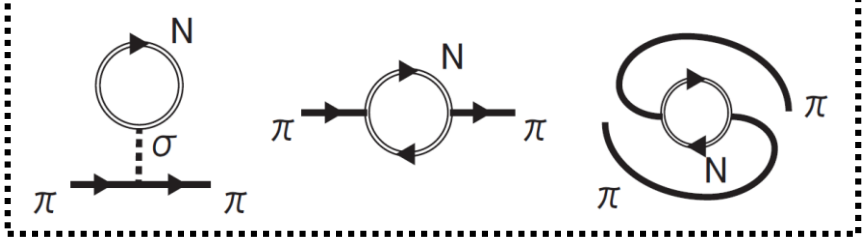
※1. About 70eV w/o scalar meson contribution
← Large effect of scalar meson exchange

※2. A certain dependence on the σ meson mass (180~250eV)
(The σ meson mass is an input.)

Medium effect on $\eta \rightarrow 3\pi$ decay

※1. Nucleon field (double solid line) ← Mean field approximation
※2. Inclusion of nucleon 1-loop ← Fermi momentum k_f: small

Self energy

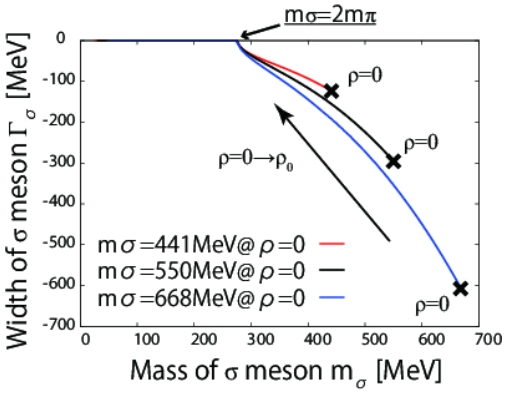


※ Mass modification of the ps mesons (π, η): relatively small
(30~40MeV enhancement @ $\rho=\rho_0$)

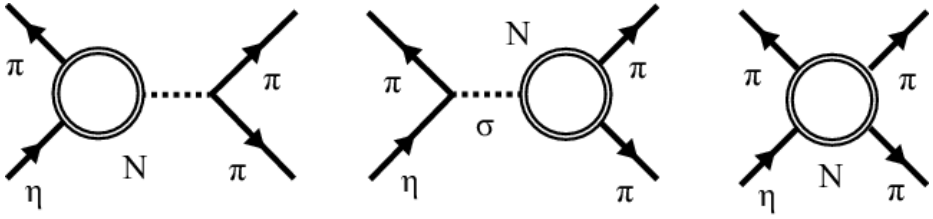
⇔ Large reduction of mass and width of σ meson
(several 100MeV reduction @$\rho=\rho_0$)

✓ Width of the σ meson: tree level

$$\Gamma_\sigma = \int d\Pi \left| \sigma \text{—} \pi\pi \right|^2$$



Correction of vertices

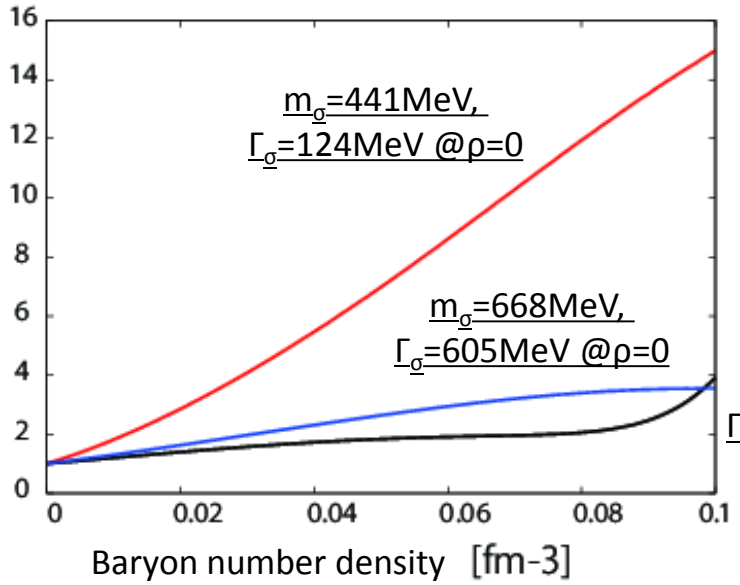


- m_σ=441MeV @ρ=0 (Γ_σ=124MeV)
  from the analysis of the ππ cross section
  I. Caprini, *et al.*, PRL96(2006)132001.
- m_σ=550MeV @ρ=0 (Γ_σ=296MeV)
  used in A. Abdel-Rehim, *et al.*(2003)
- m_σ=668MeV @ρ=0 (Γ_σ=605MeV)
  from the 3 flavor NJL
  H. Hatsuda, T. Kunihiro, PR247(1994)221.

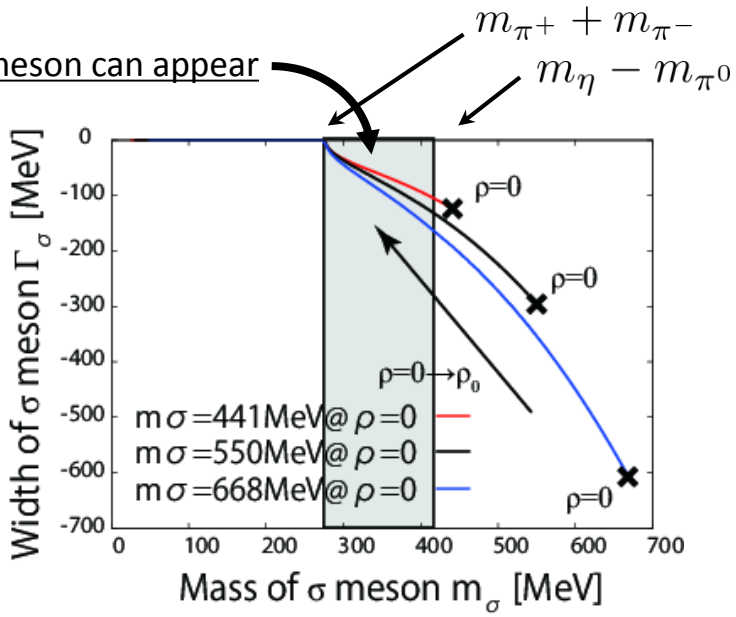
(The σ meson mass @ρ=0 is an input.)

Results

$\eta \to \pi^+\pi^-\pi^0$ decay width in nuclear medium
(width is normalized by the value @ ρ=0)



On-shell σ meson can appear



- $\eta \to \pi^+\pi^-\pi^0$ decay width is enhanced by the nuclear medium in the wide range of σ meson mass
- Large dependence of the enhancement on the σ meson mass @ρ=0
  - The enhancement is factor 2~15 ← m_σ=441~668MeV
  - The enhancement in the case of m_σ=441MeV (light σ meson mass) is especially large

Summary

- The σ meson in linear σ model plays

  important role in $\eta \rightarrow 3\pi$ decay even in free space

  (about 200eV with the σ meson exchange)

- $\eta \rightarrow \pi^+\pi^-\pi^0$ decay width is enhanced

  by the effect of nuclear medium

- The enhancement largely depends on the σ meson mass @ρ=0

  (2~15 times larger than the value @ρ=0 from m_σ=441~668MeV)

Future prospects

- Effect of the asymmetric nuclear medium (δρ≠0)
- More reasonable treatment of the final state interaction
  - The ππ composite component of the σ mode
  - Contribution from the ρ meson
- Contribution of the excited baryons (N^*, Δ) in nuclear medium