# Double-pole structure on a prototype of kaonic nuclei ``$K^-pp$"


Akinobu Doté (KEK Theory Center, IPNS / J-PARC branch)
Takashi Inoue (Nihon university)
Takayuki Myo (Osaka Institute of Technology)


1. Introduction
2. Method
   - *Feshbach projection of coupled-channel Complex Scaling Method*
3. Result
   - *"$K^-pp$" as a $K^{bar}NN$-$\pi YN$ system with ccCSM+Feshbach method*
4. Double pole of "$K^-pp$"?
5. Summary and future plan


12th International Conference on Hypernuclear and Strange Particle Physics (HYP2015)
07. Sep. '15 @ Tohoku University Centennial Hall (Kawauchi Hagi Hall), Sendai, Japan

# 1. Introduction

# 1. Introduction

## Kaonic nuclei = Exotic system !?

- Strong $K^{bar}N$ attraction ← Λ(1405) ~ quasi-bound state of I=0 $K^{bar}N$
  - Deeply bound (Total B.E. ~100MeV)
  - Quasi-stable (πΣ decay mode closed)
  - Highly dense state … anti-kaon attracts nucleons

Y. Akaishi and T. Yamazaki, PRC65, 044005 (2002)
A. Dote, H. Horiuchi, Y. Akaishi and T. Yamazaki, PRC70, 044313 (2004)

→ "$K^-pp$" = A prototype of kaonic nuclei

## Experimental search for "$K^-pp$"

- FINUDA : $K^-$ stopped on Li, C, Al target
  PRL 94, 212303 (2005)
- DISTO : $p + p \rightarrow p + \Lambda + K^+$ at $T_p$=2.85 GeV
  PRL104, 132502 (2010)
- J-PARC E27 : $d(\pi^+, K^+)$ X at $P_\pi$= 1.69 GeV/c
  PTEP 021D01 (2015)

Signal at ~100 MeV below $K^-pp$ threshold

- J-PARC E15 : $^3He(K^-, n)$ X at $P_K$ = 1 GeV/c
  PTEP 2015, 061D01

Attraction below $K^-pp$ threshold

- LEPS/SPring8 : $d(\gamma, \pi^+K^+)$ X at $E_\gamma$ = 1.5 – 2.4 GeV
  PLB 728, 616 (2014)

No evidence of deeply bound $K^-pp$

## Theoretical studies of "$K^-pp$"

B($K^-pp$) < 100 MeV

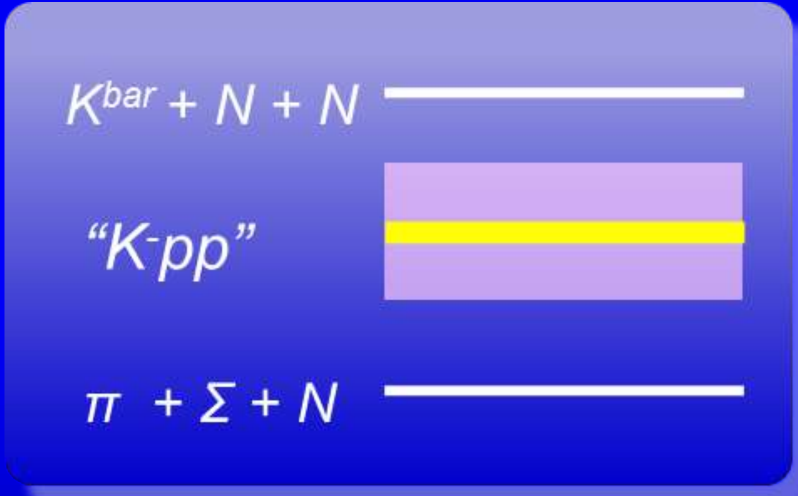
| | Dote-Hyodo-Weise PRC79, 014003(2009) | Akaishi-Yamazaki PRC76, 045201(2007) | Barnea-Gal-Livertz PLB712, 132 (2012) | Ikeda-Kamano-Sato PTP124, 533 (2010) | Shevchenko-Gal-Mares PRC76, 044004(2007) |
|---|---|---|---|---|---|
| B($K^-pp$) | 20±3 | 47 | 16 | 9～16 | 50～70 |
| $\Gamma_{mesonic}$ | 40～70 | 61 | 41 | 34～46 | 90～110 |

- ***Λ(1405) = Resonant state & $K^{bar}N$ coupled with πΣ***

- ***"$K^{-}pp$" … Resonant state of***
  ***$K^{bar}NN$-πYN coupled-channel system***

*Doté, Hyodo, Weise, PRC79, 014003(2009). Akaishi, Yamazaki, PRC76, 045201(2007)*
*Ikeda, Sato, PRC76, 035203(2007). Shevchenko, Gal, Mares, PRC76, 044004(2007)*
*Barnea, Gal, Liverts, PLB712, 132(2012)*

- ***Resonant state***
- ***Coupled-channel system***




⇒ ***"coupled-channel Complex Scaling Method"***

S. Aoyama, T. Myo, K. Kato, K. Ikeda, PTP116, 1 (2006)
T. Myo, Y. Kikuchi, H. Masui, K. Kato, PPNP79, 1 (2014)

# Complex Scaling Method

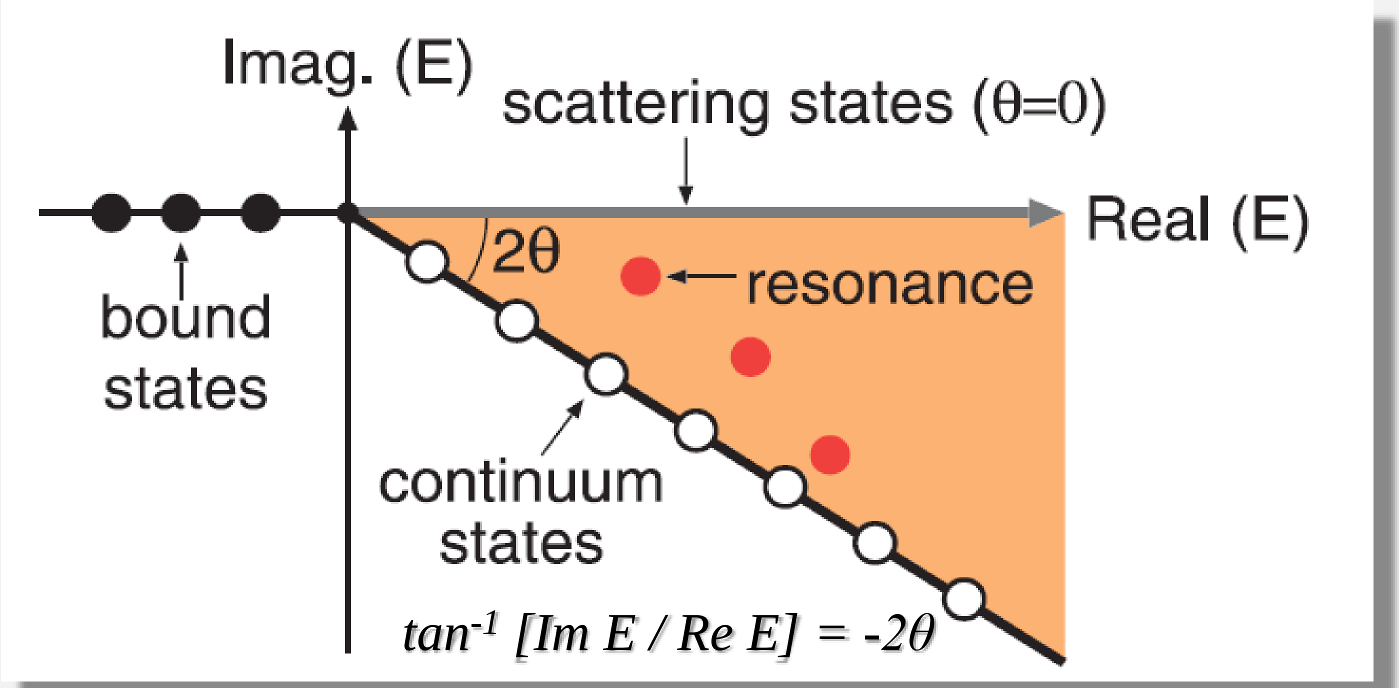
*… Powerful tool for resonance study of many-body system*

*Complex rotation (Complex scaling) of coordinate*

*Resonance wave function → $L^2$ integrable*

$$U(\theta):\quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

*Diagonalize $H_\theta = U(\theta)\, H\, U^{-1}(\theta)$ with Gaussian base,*




- *Continuum state appears on 2ϑ line.*
- *Resonance pole is off from 2ϑ line, and independent of ϑ. (ABC theorem)*

# *Chiral SU(3) potential with a Gaussian form*

A. D., T. Inoue, T. Myo, Nucl. Phys. A 912, 66 (2013)

- **Anti-kaon = Nambu-Goldstone boson**

**⇒ *Chiral SU(3)-based $K^{bar}N$ potential***

- *Weinberg-Tomozawa term of effective chiral Lagrangian*
- *Gaussian form in r-space*
- *Semi-rela. / Non-rela.*
- *Based on Chiral SU(3) theory*

**→ *Energy dependence***

*A non-relativistic potential (NRv2c)*

$$V_{ij}^{(I=0,1)}(r) = -\frac{C_{ij}^{(I=0,1)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{1}{m_i m_j}}\, g_{ij}(r)$$

$$g_{ij}(r) = \frac{1}{\pi^{3/2} d_{ij}^3} \exp\left[-\left(r/d_{ij}\right)^2\right]$$ *: Gaussian form*

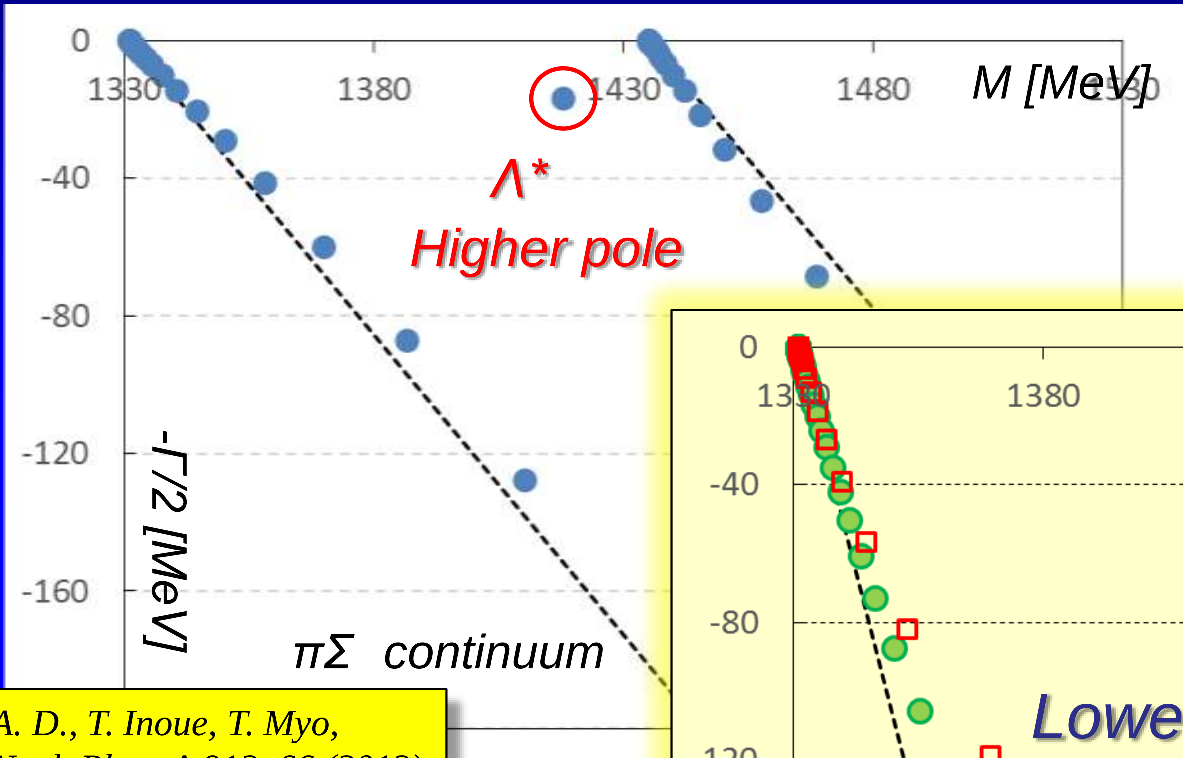
***$\omega_i$: meson energy***

*Constrained by $K^{bar}N$ scattering length*

$a_{KN(I=0)}$ = -1.70+i0.67fm, $a_{KN(I=1)}$ = 0.37+i0.60fm

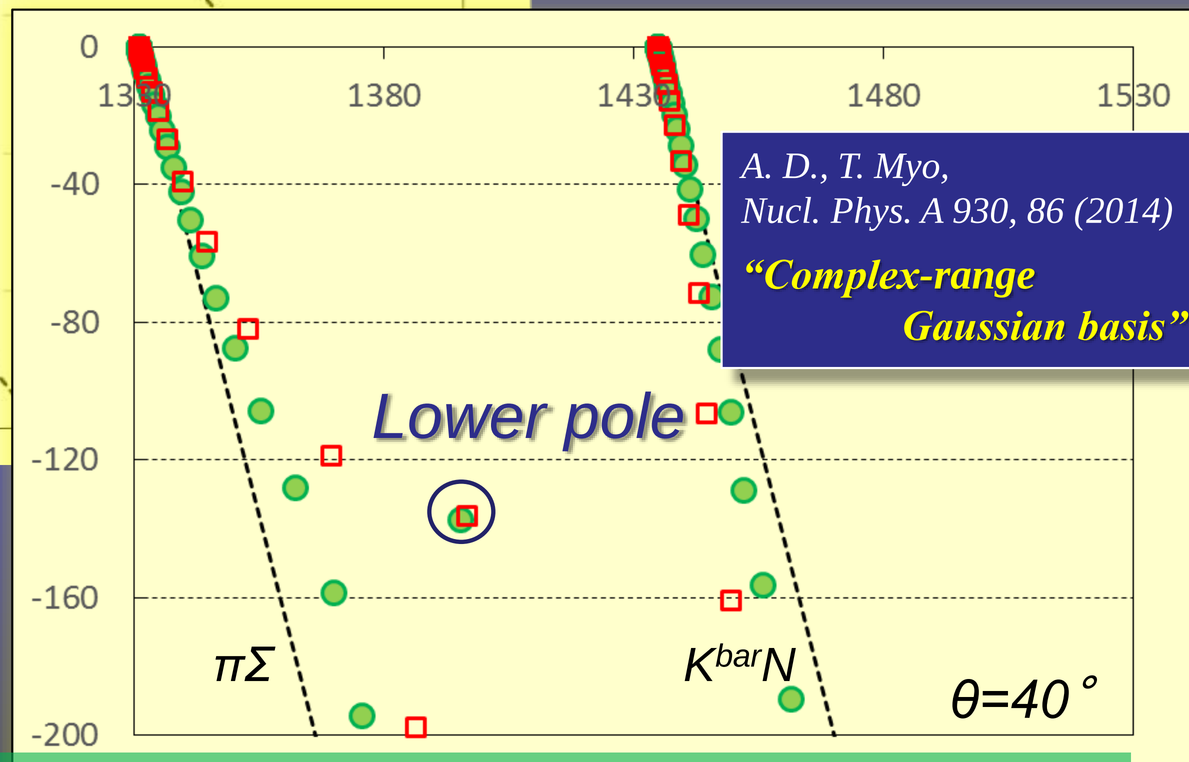
A. D. Martin, NPB179, 33(1979)

# Λ(1405) on coupled-channel Complex Scaling Method




A. D., T. Inoue, T. Myo,
Nucl. Phys. A 912, 66 (2013)

$K^{bar}N$ potential:
a chiral SU(3) potential
(NRv2, $f_\pi$=110)




A. D., T. Myo,
Nucl. Phys. A 930, 86 (2014)
**"Complex-range Gaussian basis"**

## Double-pole structure of Λ(1405)

D. Jido, J.A. Oller, E. Oset, A. Ramos, U.-G. Meißner, Nucl. Phys. A 725 (2003) 181

# 2. Method

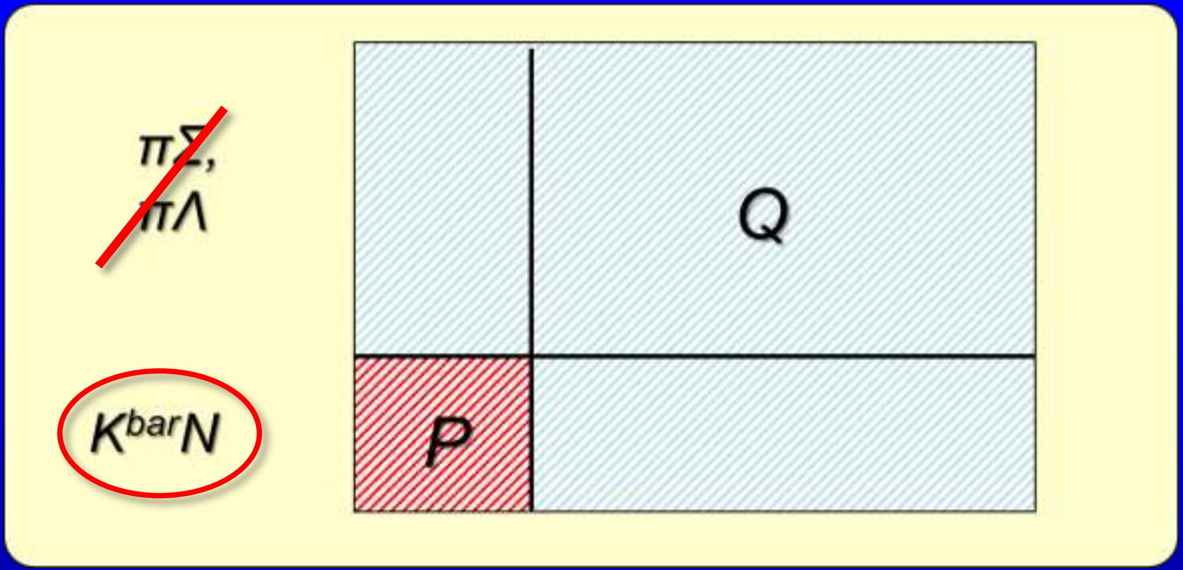
- ***Feshbach projection on coupled-channel Complex Scaling Method***

***"ccCSM+Feshbach method"***

*A. D., T. Inoue, T. Myo, PTEP 2015, 043D02 (2015)*

# ccCSM+Feshbach method

- $\Lambda(1405)$ = two-body system of $K^{bar}N$-$\pi\Sigma$
  → Explicitly treat coupled-channel problem
- "$K^{-}pp$" = three-body system of $K^{bar}NN$-$\pi YN$
  … High computational cost




For economical treatment of "$K^{-}pp$", we construct an effective $K^{bar}N$ single-channel potential by means of Feshbach projection on CSM.

# Formalism of ccCSM + Feshbach method

## Elimination of channels by Feshbash method

Schrödinger eq.
in model space "P" and out of model space "Q"

$$\begin{pmatrix} T_P + v_P & V_{PQ} \\ V_{QP} & T_Q + v_Q \end{pmatrix} \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix} = E \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix}$$

Schrödinger eq. in P-space :

$$\left( T_P + U_P^{Eff}(E) \right) \Phi_P = E \Phi_P$$

## Effective potential for P-space

$$U_P^{Eff}(E) = v_P + V_{PQ} G_Q(E) V_{QP}$$

Q-space Green function:

$$G_Q(E) = \frac{1}{E - H_{QQ}}$$

## Extended Closure Relation in Complex Scaling Method

$$H_{QQ}^{\theta} \left| \chi_n^{\theta} \right\rangle = \varepsilon_n^{\theta} \left| \chi_n^{\theta} \right\rangle$$

$$H_{QQ}^{\theta} = U(\theta) H_{QQ} U^{-1}(\theta)$$

$$\int_C \sum_{R+B} \left| \chi_n^{\theta} \right\rangle \left\langle \chi_n^{\theta} \right| = 1$$

Diagonalize $H^{\theta}_{QQ}$ with **Gaussian base**,

$$\sum_n \left| \chi_n^{\theta} \right\rangle \left\langle \chi_n^{\theta} \right| \approx 1$$

Well approximated

T. Myo, A. Ohnishi and K. Kato, PTP 99, 801 (1998)
R. Suzuki, T. Myo and K. Kato, PTP 113, 1273 (2005)

## Express the $G_Q(E)$ with **Gaussian base** using ECR

$$G_Q^{\theta}(E) = \frac{1}{E - H_{QQ}^{\theta}} \approx \sum_n \left| \chi_n^{\theta} \right\rangle \frac{1}{E - \varepsilon_n^{\theta}} \left\langle \chi_n^{\theta} \right|$$

$$U_P^{Eff}(E) = v_P + V_{PQ} \underbrace{U^{-1}(\theta) G_Q^{\theta}(E) U(\theta)}_{G_Q(E)} V_{QP}$$

$\left\{ \left| \chi_n^{\theta} \right\rangle \right\}$ : expanded with Gaussian base.

# Apply ccCSM + Feshbach method to $K^-pp$

## "$K^-pp$" … $K^{bar}NN$ - $\pi\Sigma N$ - $\pi\Lambda N$ ($J^\pi=0^-$, $T=1/2$)

For the two-body system, P = $K^{bar}N$, Q = $\pi Y$

$$V\left(K^{bar}N-\pi Y; I=0,1\right)$$
$$V\left(\pi Y-\pi Y' \quad ; I=0,1\right)$$

→ **Feshbach + ccCSM** →

$$U^{Eff}_{K^{bar}N(I=0,1)}\left(E\right)$$

- Schrödinger eq. for $K^{bar}NN$ channel :

$$\left(T_{K^{bar}NN}+V_{NN}+\sum_{i=1,2}U^{Eff}_{K^{bar}N_i(I)}\left(E_{K^{bar}N}\right)\right)\Phi_{K^{bar}NN} = E\,\Phi_{K^{bar}NN}$$

- Trial wave function

$$\left|"K^- pp"\right\rangle = \sum_a C_a^{(KNN,1)}\left\{G_a^{(KNN,1)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)+G_a^{(KNN,1)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_1\right]_{T=1/2}\right\rangle$$

Ch. 1: $K^{bar}NN$, NN:[1]E

$$+\sum_a C_a^{(KNN,2)}\left\{G_a^{(KNN,2)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)-G_a^{(KNN,2)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_0\right]_{T=1/2}\right\rangle$$

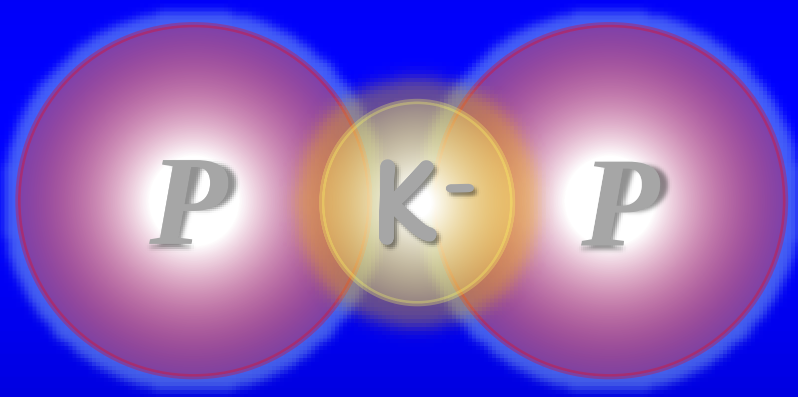
Ch. 2: $K^{bar}NN$, NN:[1]O

- Basis function ＝ Correlated Gaussian
  …including 3-types Jacobi-coordinates

$$G_a^{(KNN,i)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)=N_a^{(KNN,i)}\exp\left[-\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)A_a^{(KNN,i)}\begin{pmatrix}\mathbf{x}_1^{(3)}\\ \mathbf{x}_2^{(3)}\end{pmatrix}\right]$$

# 3. Result

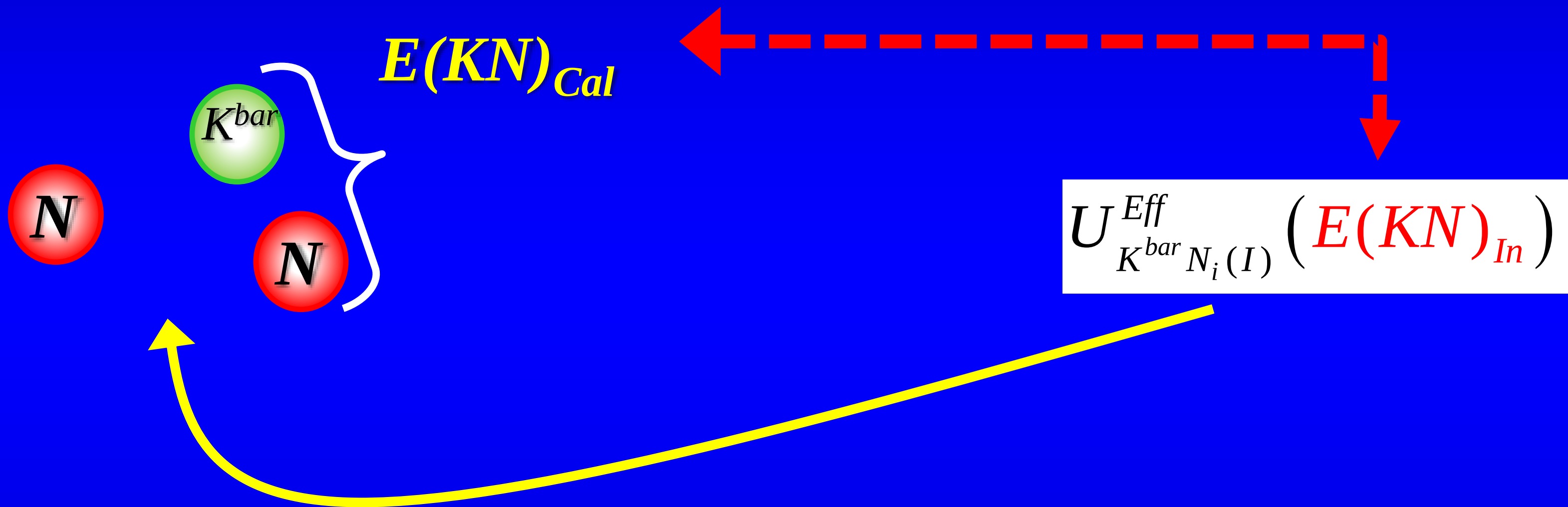
*Three-body "$K^-pp$" resonance*
*on ccCSM+Feshbach projection*




"$K^-pp$" =
$K^{bar}NN - \pi\Sigma N - \pi\Lambda N$ ($J^\pi = 0^-$, $T=1/2$)

# Self-consistency for complex $K^{bar}N$ energy

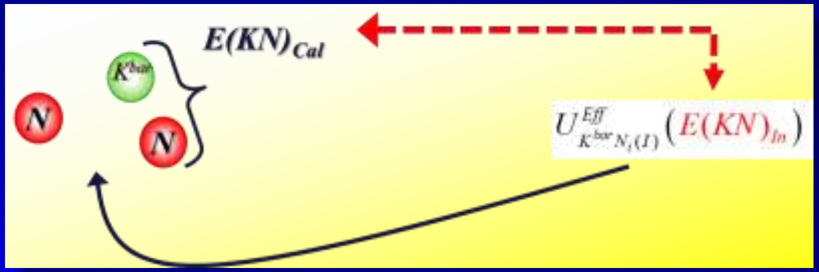
*Effective $K^{bar}N$ potential has energy dependence…*




- $E(KN)_{In}$ : assumed in the $K^{bar}N$ potential
- $E(KN)_{Cal}$ : calculated with the obtained $K^{-}pp$

***When $E(KN)_{In} = E(KN)_{Cal}$, a self-consistent solution is obtained.***

# Self-consistency for complex $K^{bar}N$ energy



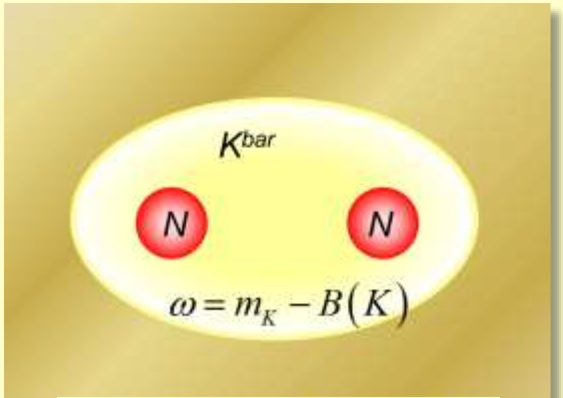

## How to determine the two-body energy in the three-body system?

A. D., T. Hyodo, W. Weise, PRC79, 014003 (2009)

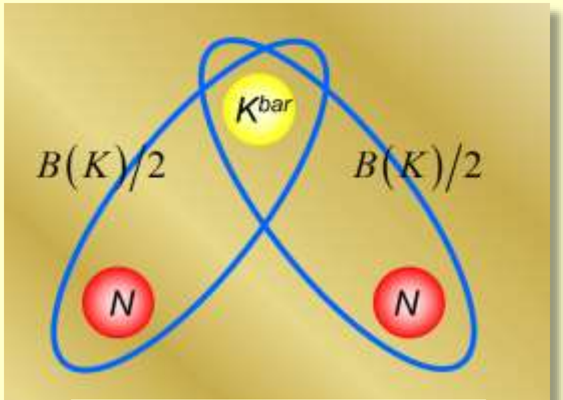
1. Kaon's binding energy: $B(K) \equiv -\left\{ \langle H \rangle - \langle H_{NN} \rangle \right\}$  $H_{NN}$ : Hamiltonian of two nucleons

2. Define a $K^{bar}$N-bond energy in two ways

$$E_{KN} = M_N + \omega = \begin{cases} M_N + m_K - B(K) & \text{: Field picture} \\ M_N + m_K - B(K)/2 & \text{: Particle picture} \end{cases}$$



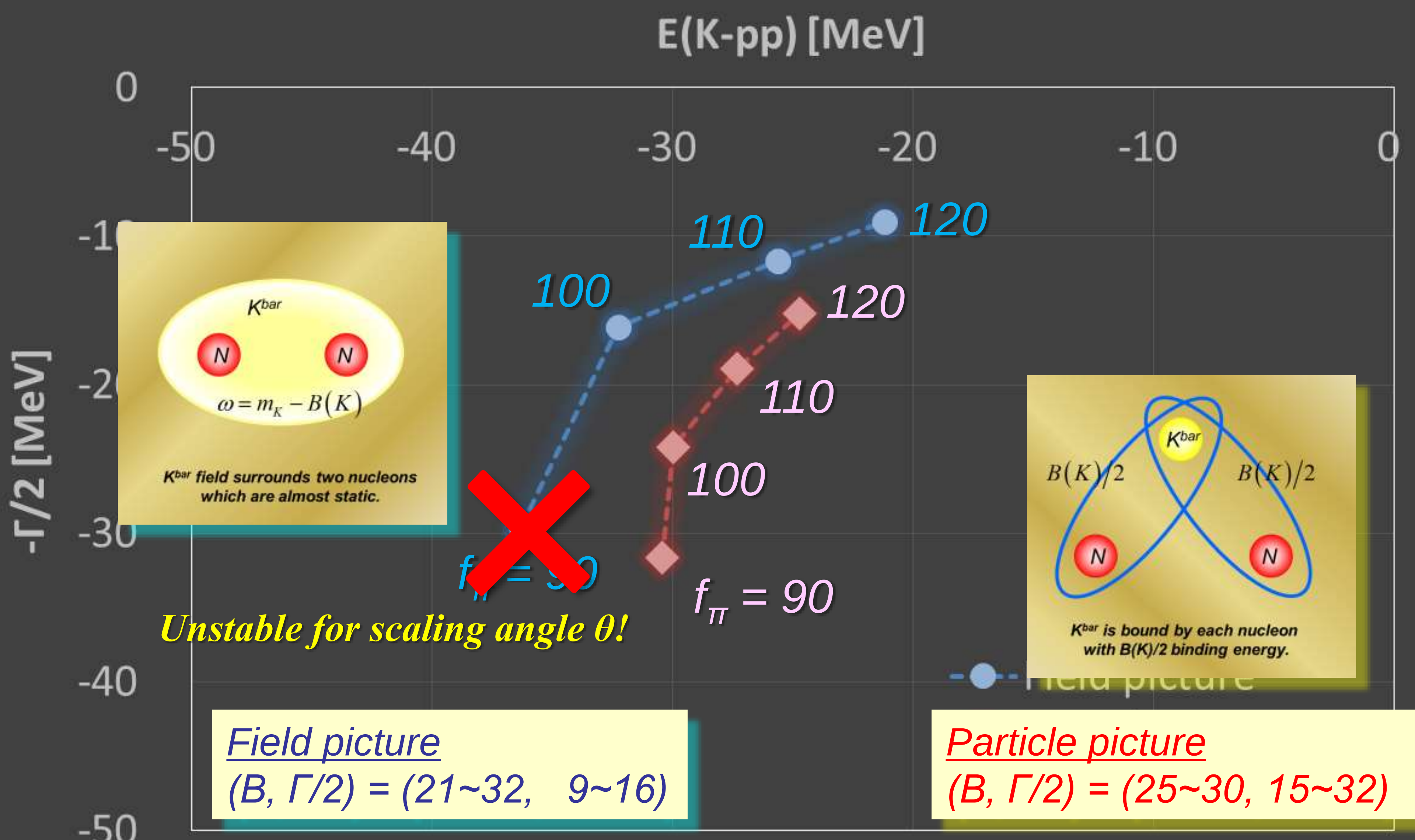

**Field picture**




**Particle picture**

# Self-consistent results
# $f_\pi$=90~120MeV

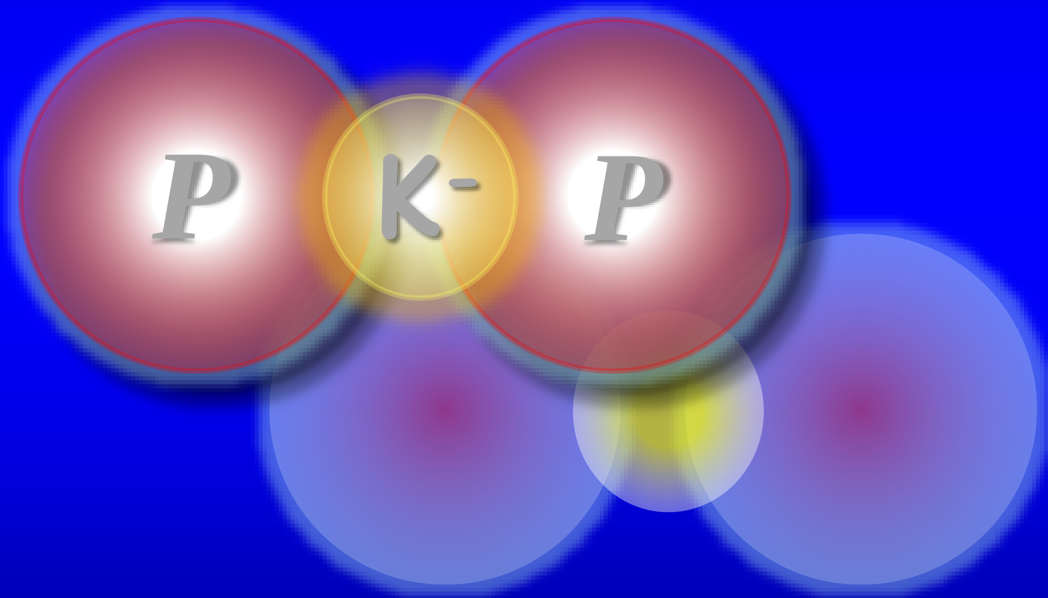
NN pot. : Av18 (Central)
$K^{bar}$N pot. : NRv2c potential
($f_\pi$=90 - 120MeV)




Field picture
(B, Γ/2) = (21~32, 9~16)

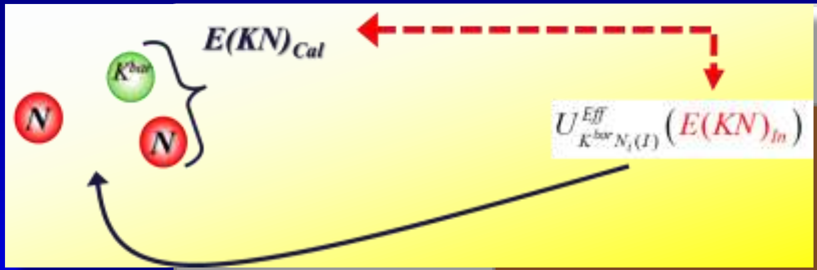
Particle picture
(B, Γ/2) = (25~30, 15~32)

# 4. Double pole of "K⁻pp"?



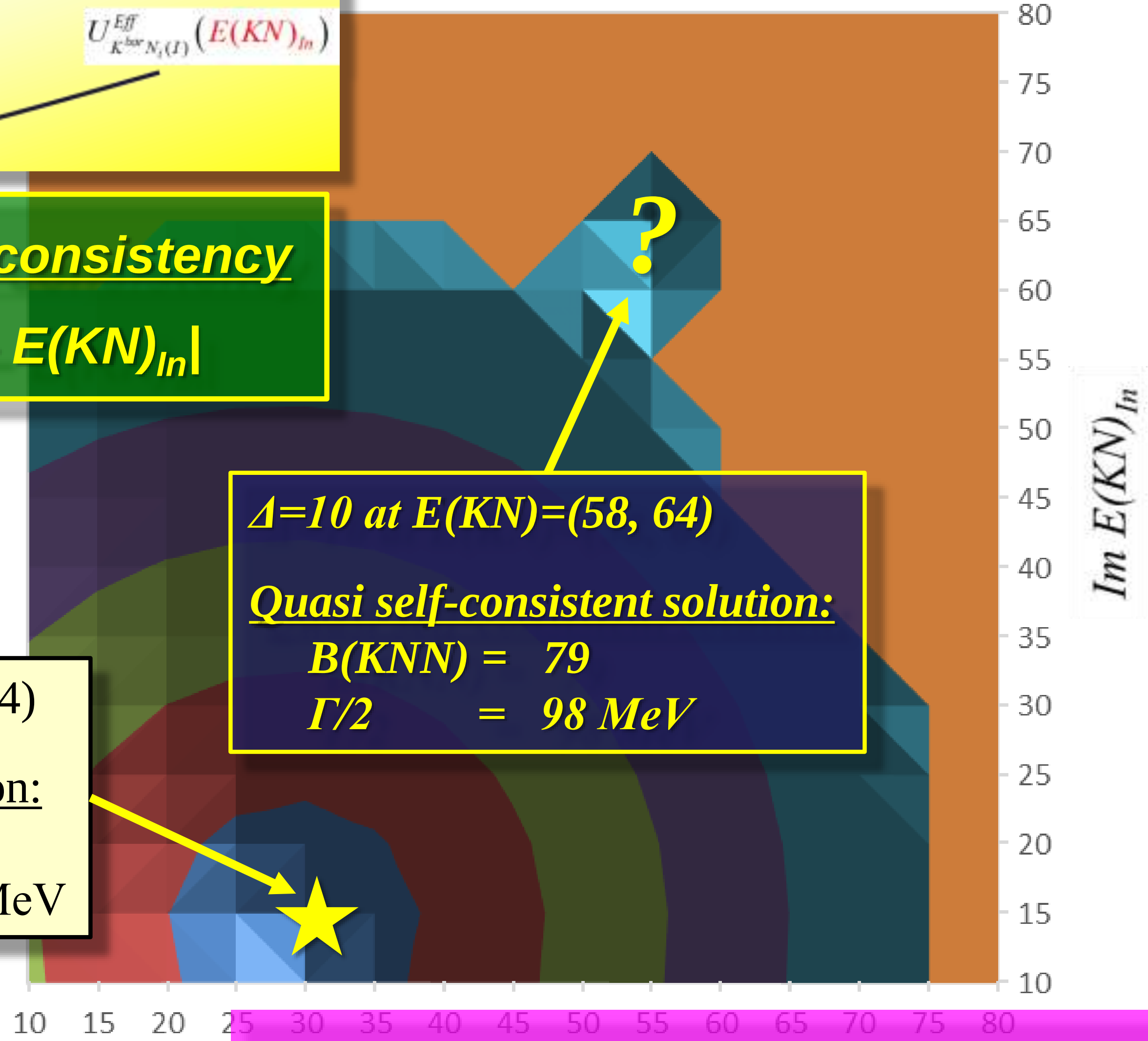
# Quasi self-consistent solution

NRv2c ($f_\pi$=110 MeV)
Particle picture




**Indicator of self-consistency**

$\Delta = |E(KN)_{Cal} - E(KN)_{In}|$




*Δ=10 at E(KN)=(58, 64)*

*Quasi self-consistent solution:*
*B(KNN) = 79*
*Γ/2 = 98 MeV*

Δ=0 at E(KN)=(29, 14)

Self-consistent solution:
B(KNN) = 27.3
Γ/2 = 18.9 MeV

## "Double pole of $K^-pp$" ?

# Double-pole structure in "K⁻pp"?

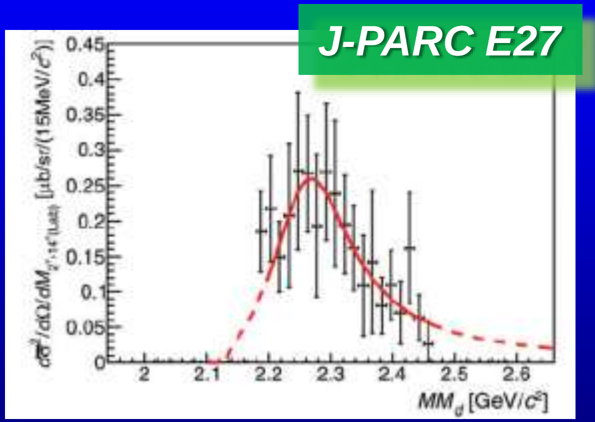
- ✓ *Quasi self-consistent solution is obtained …*
  *(B(KNN), Γ/2) = (62～79, 74～104) MeV for $f_\pi$=90～120 MeV*
  *with Particle picture*
- ✓ *Such solutions are not obtained with Field picture.*

- *A Faddeev-AGS calc. has predicted the double-pole structure of "K⁻pp".*

  *Lower pole : (B(KNN), Γ/2) = (67～89, 122～160) MeV*
  *Higher pole : (B(KNN), Γ/2) = (9～16, 17～23) MeV*

  *Y. Ikeda, H. Kamano, and T. Sato, PTP 124, 533 (2010)*

- *Relation to signals observed by J-PARC E27, DISTO?*




*Signal at ~100 MeV below $K^{bar}$NN thr.*

*Lower pole of "K⁻pp" ($J^\pi$=0⁻, I=1/2)*
*… "K⁻pp" has two poles similarly to Λ(1405).*
*The lower pole appears.*

*Partial restoration of chiral symmetry*
*… $K^{bar}$N potential is enhanced by 17%.*
*S. Maeda, Y. Akaishi, T. Yamazaki, Proc. Jpn. Acad., Ser. B 89, 418 (2013)*

*Pion assisted dibaryon "Y = πΣN-πΛN ($J^\pi$=2⁺, I=3/2)"*
*A. Gal, arXiv:1412.0198 (Proceeding of EXA2014)*

# 5. Summary and future plans

# 5. Summary

*A prototype of $K^{bar}$ nuclei "$K^{-}pp$" = Resonance state of $K^{bar}NN$-$\pi YN$ coupled system*

*"coupled-channel Complex Scaling Method + Feshbach projection"*

*… Represent the Q-space Green function with the Extended Complete Set well approximated by Gaussian base*

*⇒ Eliminate $\pi Y$ channels to reduce the problem to a $K^{bar}NN$ single channel problem.*

## *$K^{-}pp$ studied with ccCSM+Feshbch method*

- *Used a Chiral SU(3)-based potential (Gaussian form in r-space)*
- *Self-consistency for $K^{bar}N$ complex energy (Field and Particle pictures)*

*$K^{-}pp$ ($J^{\pi}=0^{-}$, $T=1/2$) …. ($B$, $\Gamma/2$) ≒ (20~30, 10~30) MeV*

- *Quasi self-consistent solution in case of Particle picture*

  *… Deeper binding and larger decay width*

*$K^{-}pp$ ($J^{\pi}=0^{-}$, $T=1/2$) …. ($B$, $\Gamma/2$) ≒ (60~80, 75~105) MeV*

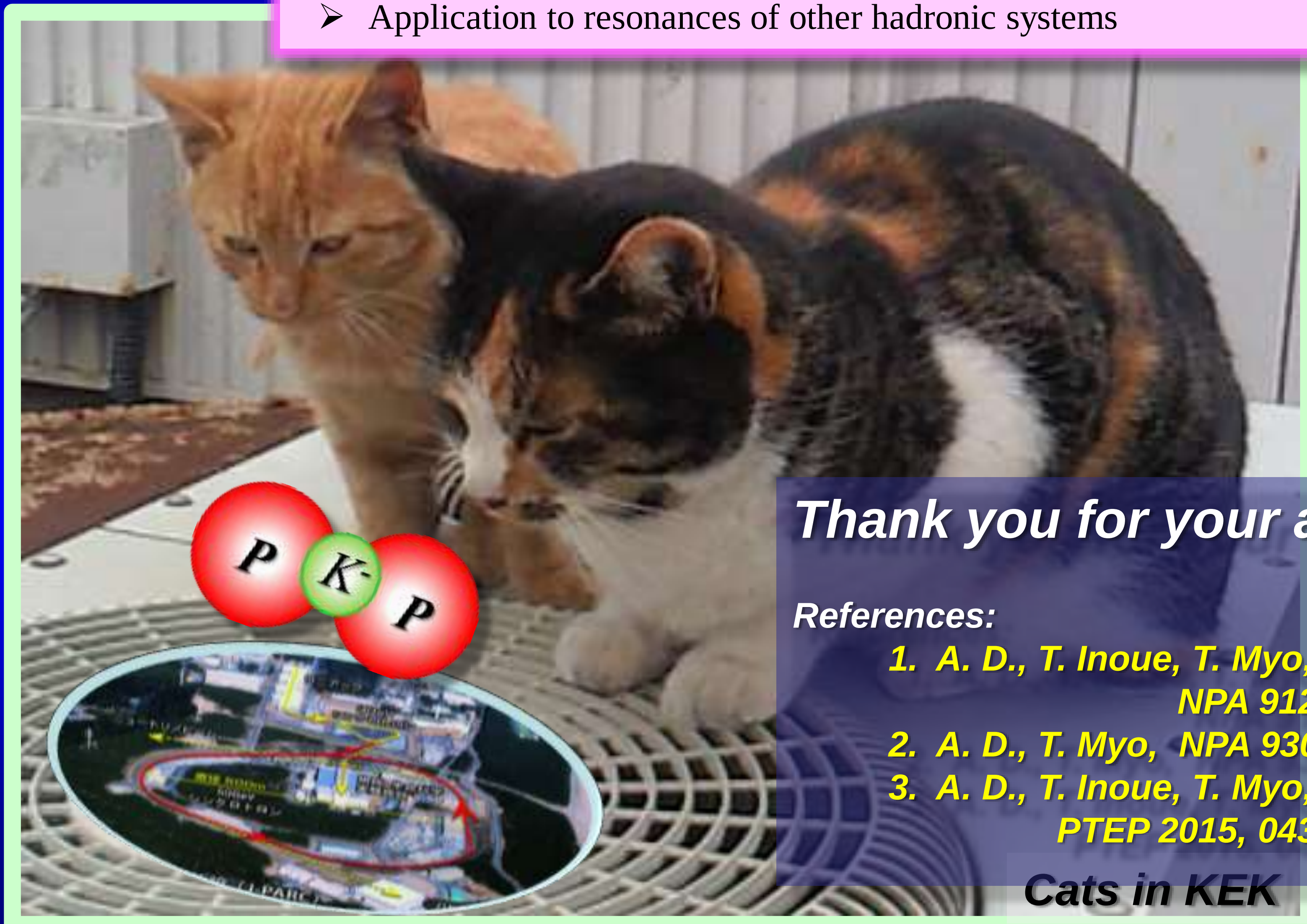
*"$K^{-}pp$" has a double-pole structure similarly to $\Lambda(1405)$?*

- *Relation to the $K^{-}pp$ search experiments*

  *The signal observed in J-PARC E27 is considered to correspond to the lower pole of "$K^{-}pp$"??*
  *J-PARC E15 may pick up the higher pole of "$K^{-}pp$"???*

# 5. Future plans

- Full-coupled channel calculation of $K^-pp$
  … *Detailed study for the double pole structure of $K^-pp$*
- Application to resonances of other hadronic systems




***Thank you for your attention!***

***References:***


1. A. D., T. Inoue, T. Myo, NPA 912, 66 (2013)
2. A. D., T. Myo, NPA 930, 86 (2014)
3. A. D., T. Inoue, T. Myo, PTEP 2015, 043D02 (2015)