

$\Xi(1690)$ as a $\bar{K}\Sigma$ molecular state

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1. Introduction
 2. Formulation
 3. Results and discussions
 4. Summary

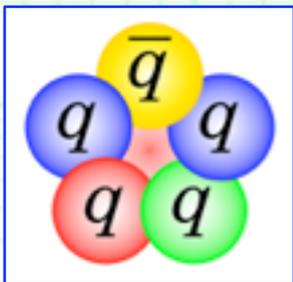
[1] T. S., *Prog. Theor. Exp. Phys. Letters*, in press [arXiv:1505.02849 [hep-ph]].



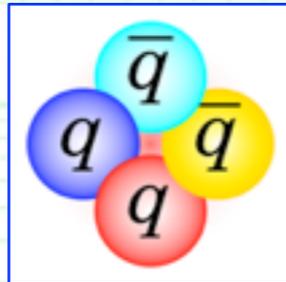
1. Introduction

++ $\Xi(1690)$ as an exotic hadron ? ++

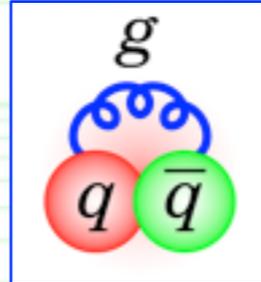
- **Exotic hadrons** --- not same quark component as ordinary hadrons = not qqq nor $q\bar{q}$. <-- Do exotic hadrons really exist ???



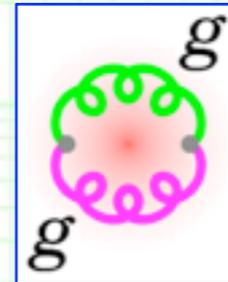
Penta-quarks



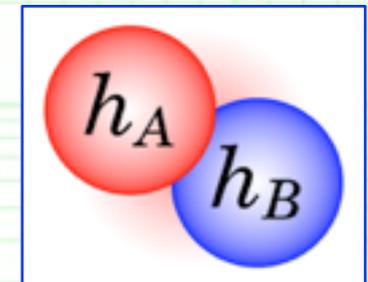
Tetra-quarks



Hybrids



Glueballs



Hadronic molecules

- **The $\Xi(1690)$ resonance** may be an exotic hadron.

Citation: K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: <http://pdg.lbl.gov>)

$\Xi(1690)$

$$I(J^P) = \frac{1}{2}(??) \quad \text{Status: } ***$$

AUBERT 08AK, in a study of $\Lambda_c^+ \rightarrow \Xi^- \pi^+ K^+$, finds some evidence that the $\Xi(1690)$ has $J^P = 1/2^-$.

DIONISI 78 sees a threshold enhancement in both the neutral and negatively charged $\Sigma \bar{K}$ mass spectra in $K^- p \rightarrow (\Sigma \bar{K}) K \pi$ at 4.2 GeV/c. The data from the $\Sigma \bar{K}$ channels alone cannot distinguish between a resonance and a large scattering length. Weaker evidence at the same mass is seen in the corresponding $\Lambda \bar{K}$ channels, and a coupled-channel analysis yields results consistent with a new Ξ .

BIAGI 81 sees an enhancement at 1700 MeV in the diffractively

- Mass: 1690 ± 10 MeV.
- Width: < 30 MeV,
but a relatively narrow width has been reported.

Particle Data Group.

, 2015)

1. Introduction

++ Experiments of the $\Xi(1690)$ resonance ++

- Historically $\Xi(1690)$ was discovered as a **threshold enhancement** in both the neutral and charged $\bar{K}\Sigma$ mass spectra in the $K^- p \rightarrow (\bar{K}\Sigma) K \pi$ reaction at 4.2 GeV/c.

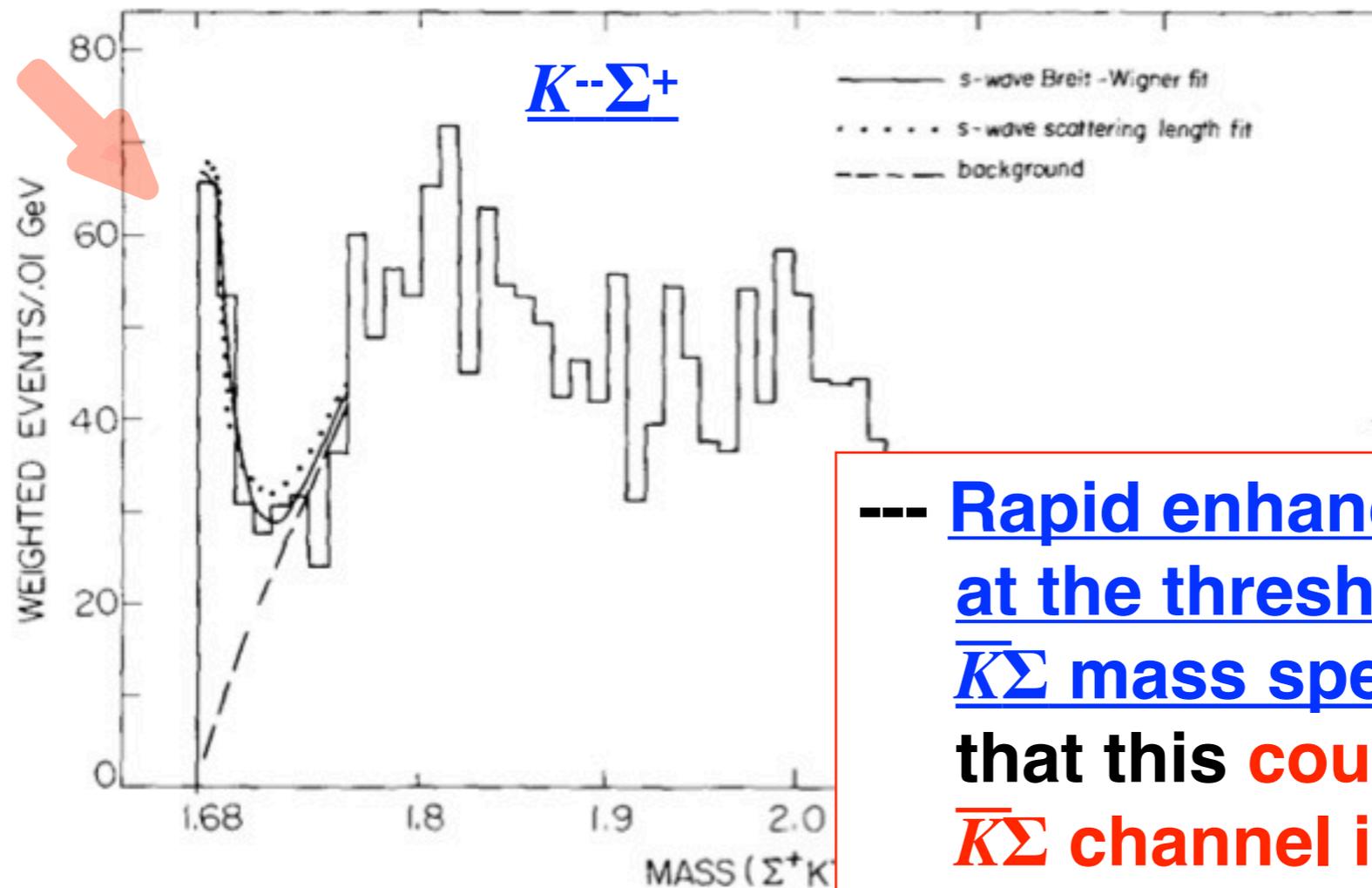


Fig. 1. The Σ^+K^- mass spectrum for the reaction $K^- p \rightarrow \Sigma^+ K^- K^+ \pi^-$ after background subtraction. The origin of the curves is indicated.

--- **Rapid enhancement at the threshold of the $\bar{K}\Sigma$ mass spectra** implies that this **couples to the $\bar{K}\Sigma$ channel in s wave.**
 $\Leftrightarrow J^P = 1/2^-$ is implied.

C. Dionisi *et al.*, *Phys. Lett.* **B80** (1978) 145.

1. Introduction

++ Experiments of the $\Xi(1690)$ resonance ++

- $\Xi(1690)$ has been observed and investigated in several experiments.
- Especially **small total decay width** and **tiny branching fraction to the $\pi\Xi$ state** have been reported, for instance:

$$\Gamma = 10 \pm 6 \text{ MeV}$$

M. I. Adamovich *et al.* [WA89 Collab.],
*Eur. Phys. J. C*5 (1998) 621.

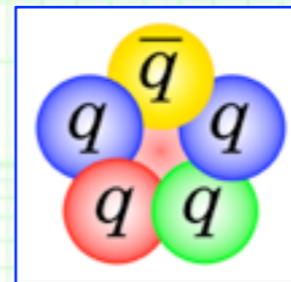
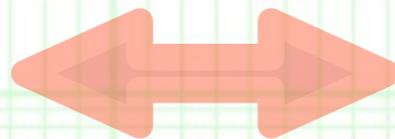
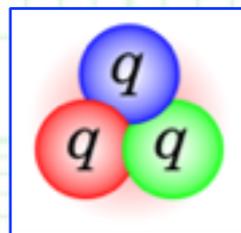
$$\Gamma(\pi\Xi)/\Gamma(\bar{K}\Sigma) < 0.09$$

Particle Data Group.

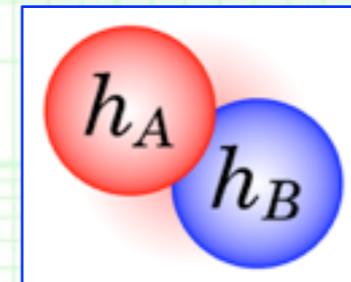
- However, **the small decay width and tiny BR fraction to $\pi\Xi$ bring a difficulty**, assuming $J^P = 1/2^-$ for $\Xi(1690)$.

--- Naive quark models inevitably predict decay to $\pi\Xi$ to some extent

--> $\Xi(1690)$ might have a **non-trivial structure** than usual qqq state ?



or



--- But its properties and structure are **still unclear**.

1. Introduction

++ Theories of the $\Xi(1690)$ resonance ++

- $\Xi(1690)$ and other Ξ^* resonances has been [investigated in several theoretical frameworks](#) as well, for instance:
 - **Quark models.**
 - K. T. Chao, N. Isgur and G. Karl, *Phys. Rev.* D23 (1981) 155;
 - S. Capstick and N. Isgur, *Phys. Rev.* D34 (1986) 2809;
 - M. Pervin and W. Roberts, *Phys. Rev.* C77 (2008) 025202;
 - L. Y. Xiao and X. H. Zhong, *Phys. Rev.* D87 (2013) 094002;
 - N. Sharma, A. Martinez Torres, K. P. Khemchandani and H. Dahiya, *Eur. Phys. J.* A49 (2013) 11;
 - ...
 - **Skyrme model.**
 - Y. Oh, *Phys. Rev.* D75 (2007) 074002.
 - **Chiral unitary approach.**
 - A. Ramos, E. Oset and C. Bennhold, *Phys. Rev. Lett.* 89 (2002) 252001;
 - C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* B582 (2004) 49;
 - D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* D84 (2011) 056017.



1. Introduction

++ Ξ^* resonances in chiral unitary approach ++

- Ξ^* resonances in chiral unitary approach.

--- Based on the combination of the chiral perturbation theory and the unitarization of the scattering amplitude.

$(\frac{1}{2}, -2)$		$[\pi \Xi]$	7.5	5.6	seen	2.6
$\Xi(1620)^*$		$[\bar{K}\Lambda]$	5.2	2.8	seen	-1.5
$M \approx 1620$	1565	$[\bar{K}\Sigma]$	0.7	2.6	0	-0.8
$\Gamma = 23$	247	$[\eta \Xi]$	0.3	4.9	0	0.3
$(\frac{1}{2}, -2)$		$[\pi \Xi]$	0.02	0.1	seen	-0.1
$\Xi(1690)^{***}$		$[\bar{K}\Lambda]$	0.16	6.0	seen	0.9
$M = 1690 \pm 10$	1663	$[\bar{K}\Sigma]$	5.15	3.1	seen	-2.5
$\Gamma = 10 \pm 6$	4	$[\eta \Xi]$	2.28	3.2	0	-1.7

- **Systematic studies were done for several Ξ^* states together with many other resonances.**

C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* **B582** (2004) 49.

8 (1134)	2037-24i	0.6	0.6	0.3	0.2	0.3	↑0.5	1.5	0.6	1.8	2.4	1.1	0.2	1.0	2.1	
10 (70)	1729-46i	0.6	1.4	0.4	↑1.6	1.4	2.1	1.0	0.4	3.3	1.5	0.4	0.2	1.6	1.0	$\Xi(1950)$ ***
8 (70)	1651-2i	0.2	0.3	↑2.2	1.3	1.0	2.6	0.2	0.6	0.9	0.4	0.2	1.7	0.4	0.2	$\Xi(1690)$ ***
8 (56)	1577-139i	2.6	↑1.7	0.5	0.1	0.8	1.0	0.7	0.1	0.6	1.3	0.3	0.1	0.2	1.2	$\Xi(1620)$ *

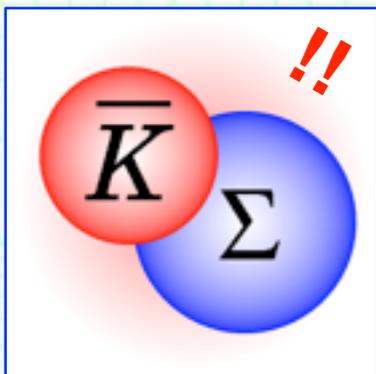
--- Narrow width for $\Xi(1690)$!
But its mass is lower than Exp. value.

D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* **D84** (2011) 056017.

1. Introduction

++ In this study ... ++

- In this study we **concentrate on the phenomena near the $\bar{K}\Sigma$ threshold** and **on the $\Xi(1690)$ resonance**.
- By using **the chiral unitary approach** and adjusting parameters, we show **the narrow $\Xi(1690)$ state**, which was studied in the previous studies, **can exist near the $\bar{K}\Sigma$ threshold with $J^P = 1/2^-$** , and it **reproduces experimental mass spectra qualitatively well**.
- We **investigate and clarify properties of the $\Xi(1690)$ state**, including **its small decay width**, molecular structure, etc.
- We especially show that **the $\Xi(1690)$ resonance can be indeed an s -wave $\bar{K}\Sigma$ molecular state** in terms of the **compositeness**.



Hyodo-Jido-Hosaka (2012), Aceti-Oset (2012), Nagahiro-Hosaka (2014), ...

See Hyodo, *Int. J. Mod. Phys. A* **28** (2013) 1330045;

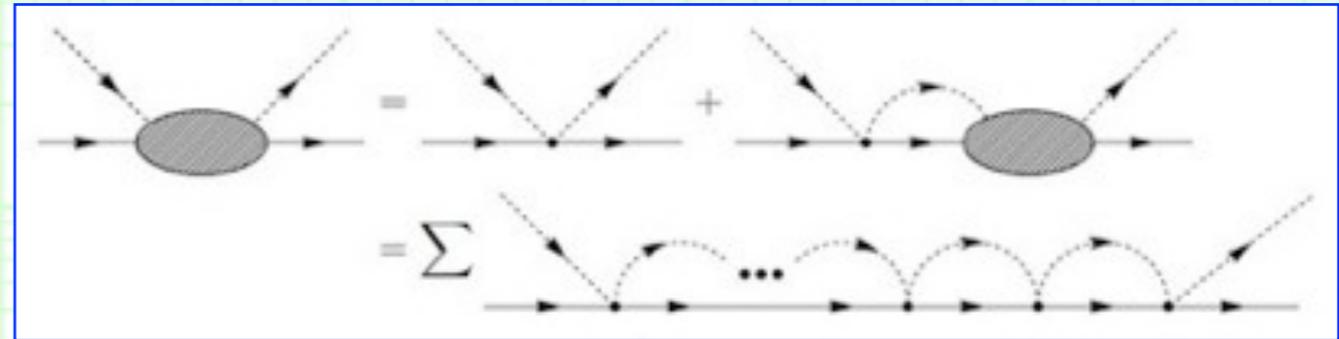
also T. S., Hyodo and Jido, *PTEP* (2015) 063D04.

2. Formulation

++ Chiral unitary approach ++

- We employ **the chiral unitary approach** for the s -wave $\bar{K}\Sigma$ - $\bar{K}\Lambda$ - πE - ηE coupled-channels scattering.

$$T_{jk}(w) = V_{jk}(w) + \sum_l V_{jl}(w)G_l(w)T_{lk}(w)$$



- The chiral unitary approach is **most successful in the $\bar{K}N$ interaction and $\Lambda(1405)$.**

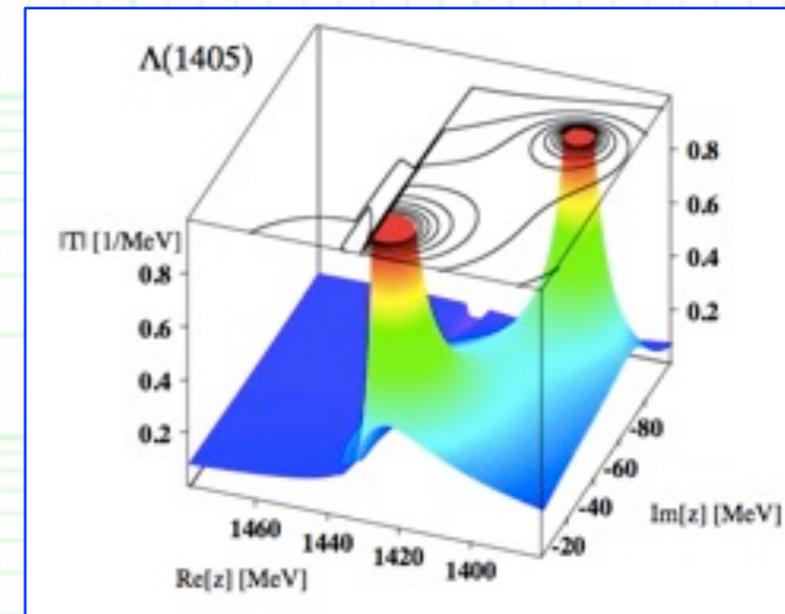
Kaiser-Siegel-Weise (1995), Oset-Ramos (1998),
Oller-Meissner (2001), Lutz-Kolomeitsev (2002),
Jido *et al.* (2003),

- In this study we use **the Weinberg-Tomozawa interaction** for the interaction kernel V .

- The leading order term of ChPT in s wave:

$$V_{jk}(w) = -\frac{C_{jk}}{4f_j f_k} (2w - M_j - M_k) \sqrt{\frac{E_j + M_j}{2M_j}} \sqrt{\frac{E_k + M_k}{2M_k}}$$

← We have **no free parameters in the interaction kernel.**



Hyodo and Jido (2012).

2. Formulation

++ Chiral unitary approach ++

- For the loop function G we take **a covariant expression**:

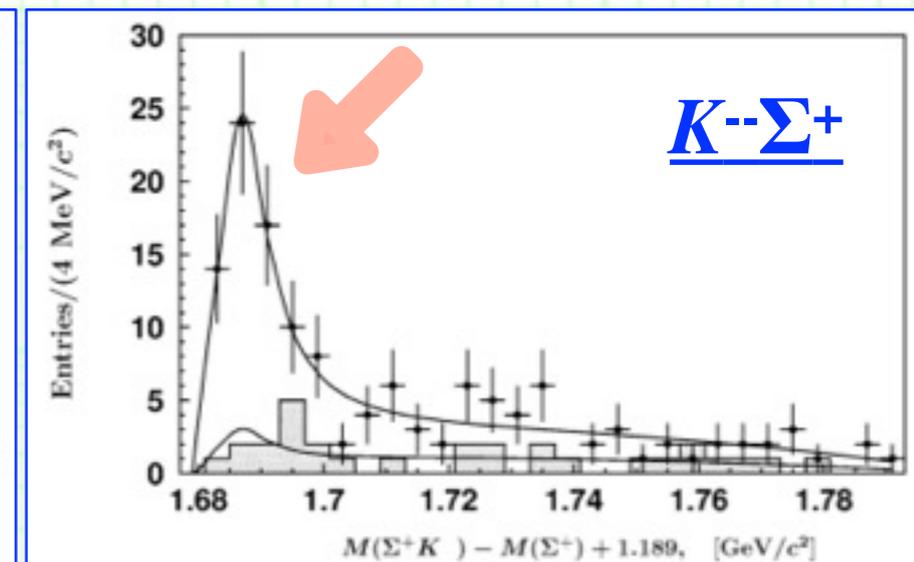
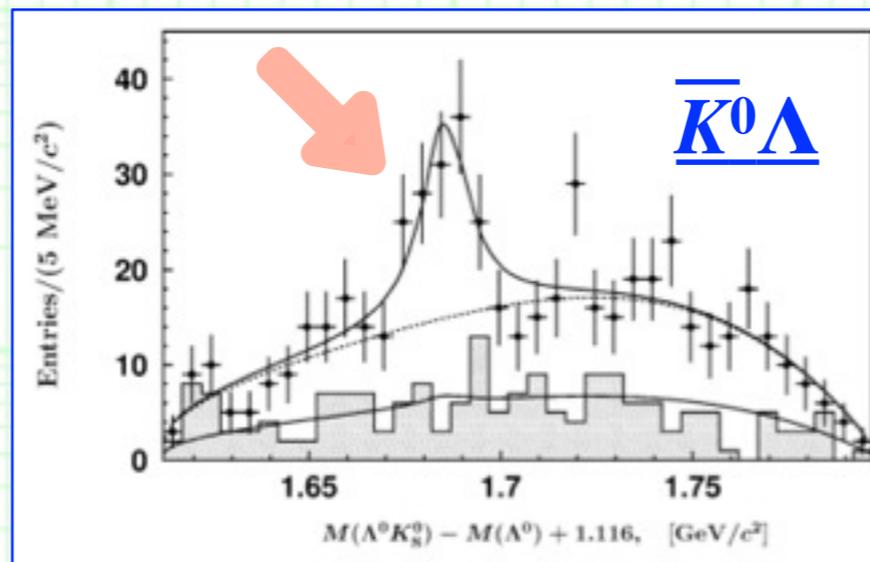
$$G_j(w) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P/2 + q)^2 - m_j^2 + i0} \frac{2M_j}{(P/2 - q)^2 - M_j^2 + i0}$$

- The integral is calculated with the dimensional regularization, and an infinite constant is replaced with a subtraction constant in each channel.
- > **Subtraction constants are free parameters.**
- We assume the isospin symmetry for the subtraction constants, so we have **4 free parameters** ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$), which are fixed so as to reproduce the mass spectra by Belle.

--- **Neutral $\Xi(1690)$.**

K. Abe et al. [Belle Collab.],

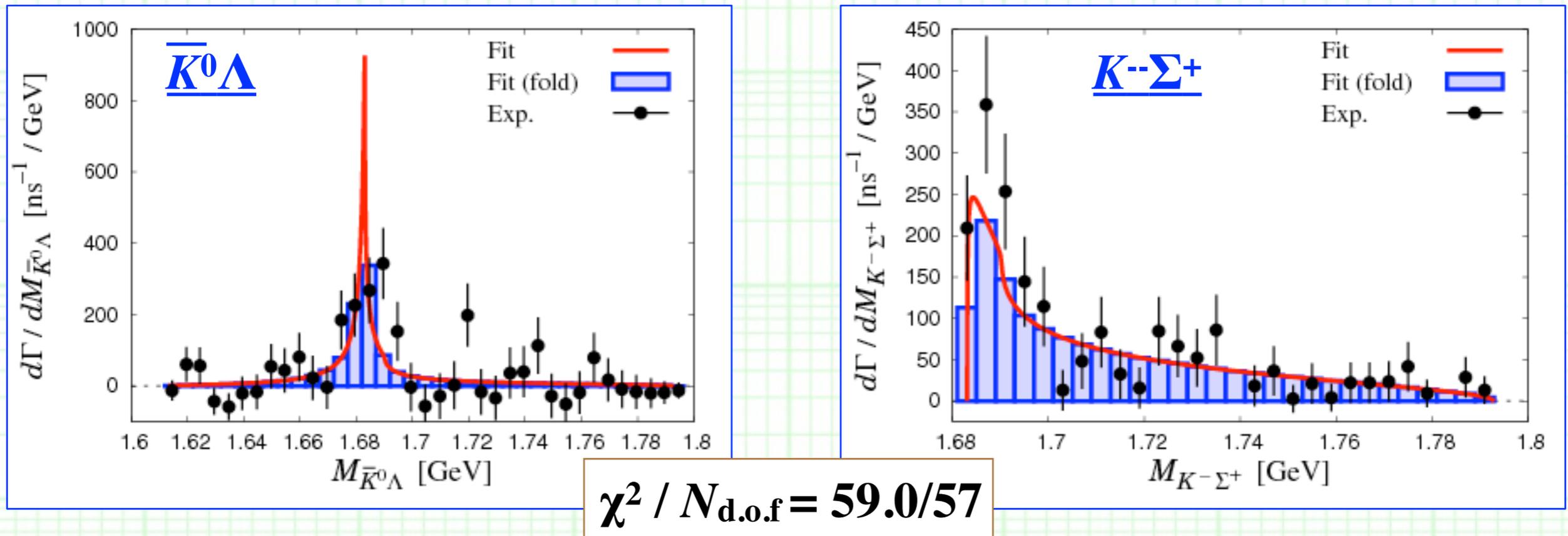
Phys. Lett. B524 (2002) 33.



3. Results and discussions

++ Fitting to the Belle data ++

- We fix 4 free parameters ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$) so as to reproduce the mass spectra by Belle. The result of the best fit is:



- Background of the Belle data is subtracted.
- Relative scale between $\bar{K}^0\Lambda$ and $K^-\Sigma^+$ is fixed with the branching fractions:

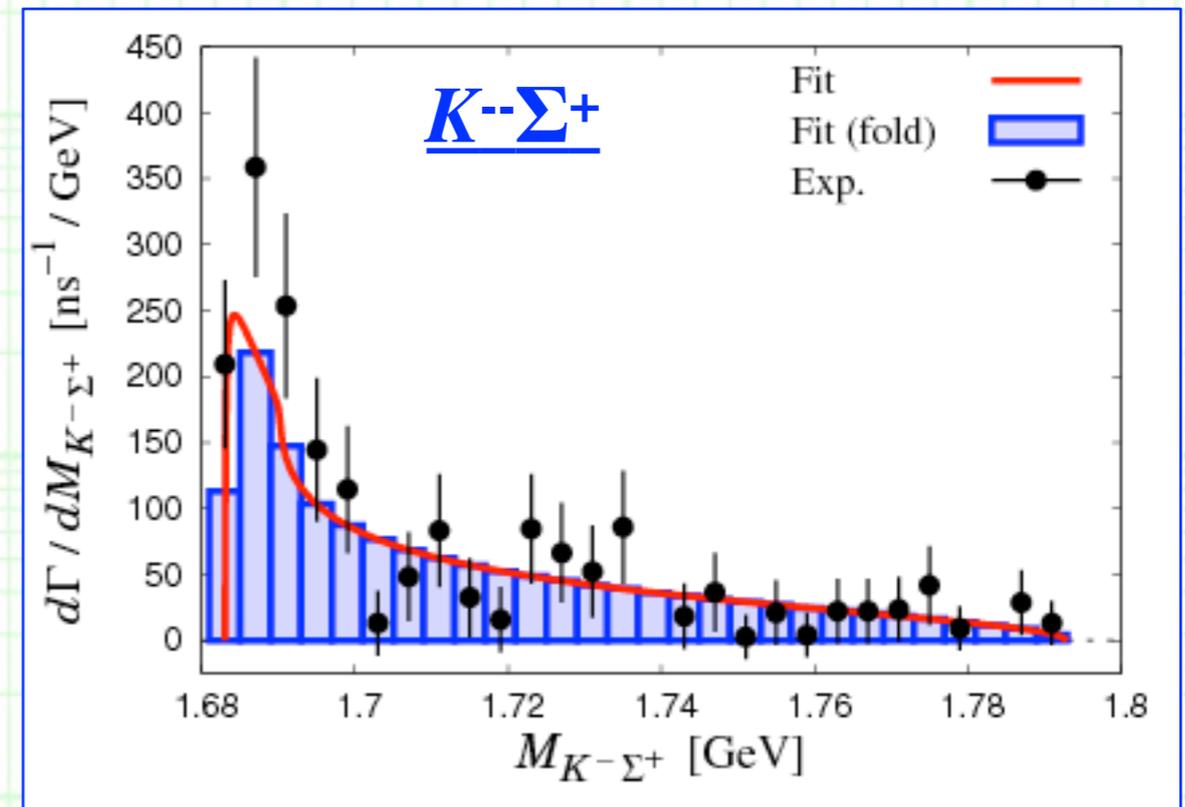
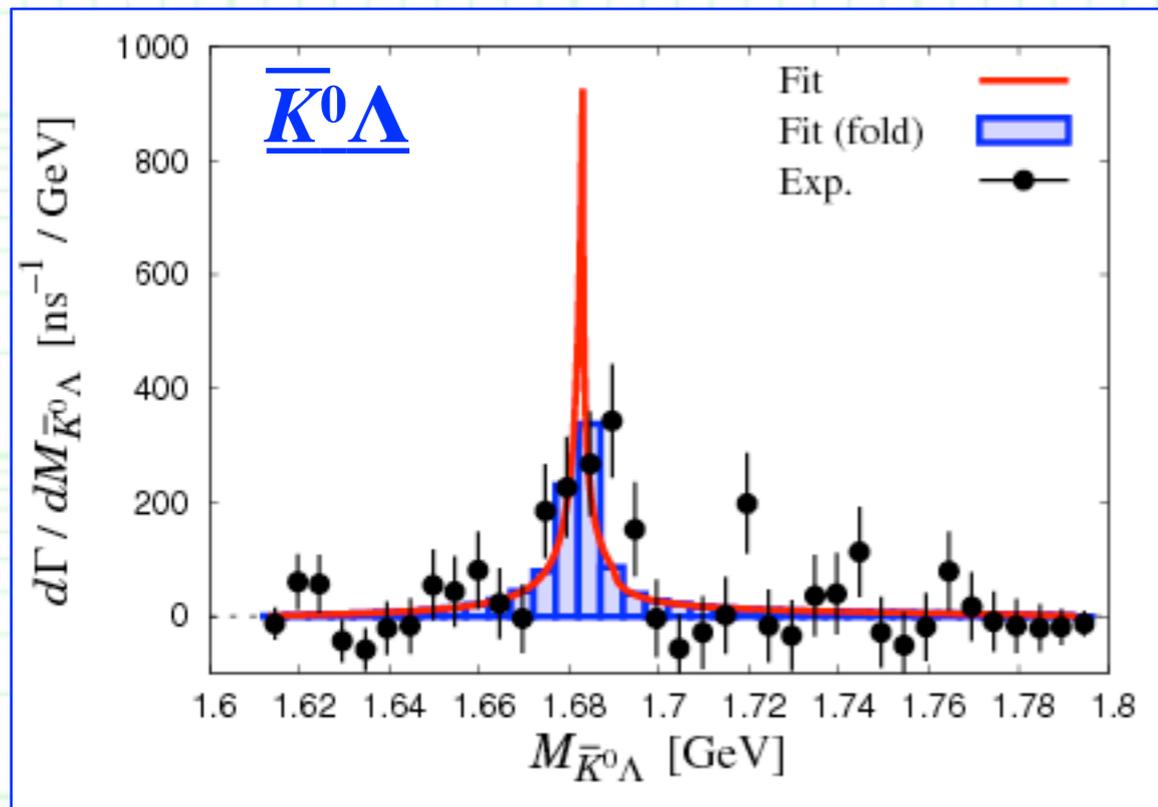
$$\mathcal{B}[\Lambda_c^+ \rightarrow \Xi(1690)^0 K^+ \rightarrow (K^-\Sigma^+)K^+] = (1.3 \pm 0.5) \times 10^{-3}$$

$$\mathcal{B}[\Lambda_c^+ \rightarrow \Xi(1690)^0 K^+ \rightarrow (\bar{K}^0\Lambda)K^+] = (8.1 \pm 3.0) \times 10^{-4}$$

3. Results and discussions

++ Fitting to the Belle data ++

- We fix 4 free parameters ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$) so as to reproduce the mass spectra by Belle. The result of the best fit is:



1. The Belle data on $\Xi(1690)$ are reproduced qualitatively well with very small width ~ 1 MeV.

--- We can calculate the ratio

$$R \equiv \frac{\mathcal{B}[\Lambda_c^+ \rightarrow \Xi(1690)^0 K^+ \rightarrow (K^-\Sigma^+)K^+]}{\mathcal{B}[\Lambda_c^+ \rightarrow \Xi(1690)^0 K^+ \rightarrow (\bar{K}^0\Lambda)K^+]}$$

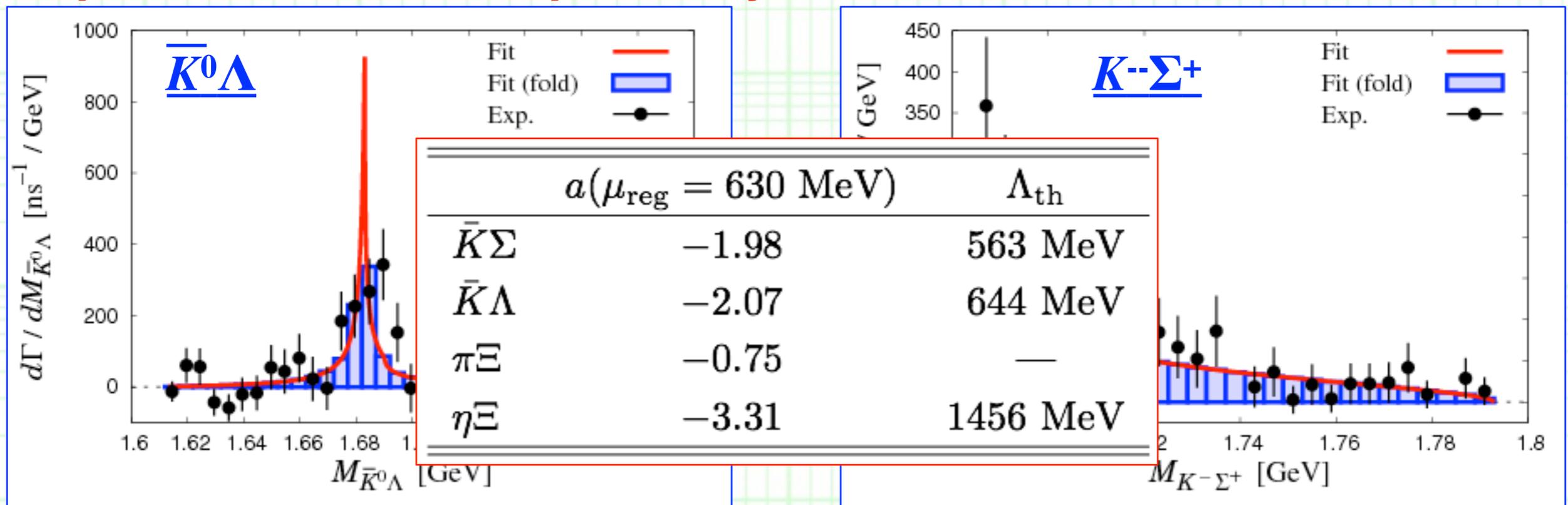
$$R_{\text{th}} = 1.06 \Leftrightarrow R_{\text{exp}} = 0.62 \pm 0.33. \quad \text{--- In } 2\sigma \text{ errors.}$$



3. Results and discussions

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- We fix 4 free parameters ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$) so as to reproduce the mass spectra by Belle. The result of the best fit is:



1. The Belle data on $\Xi(1690)$ are reproduced qualitatively well.

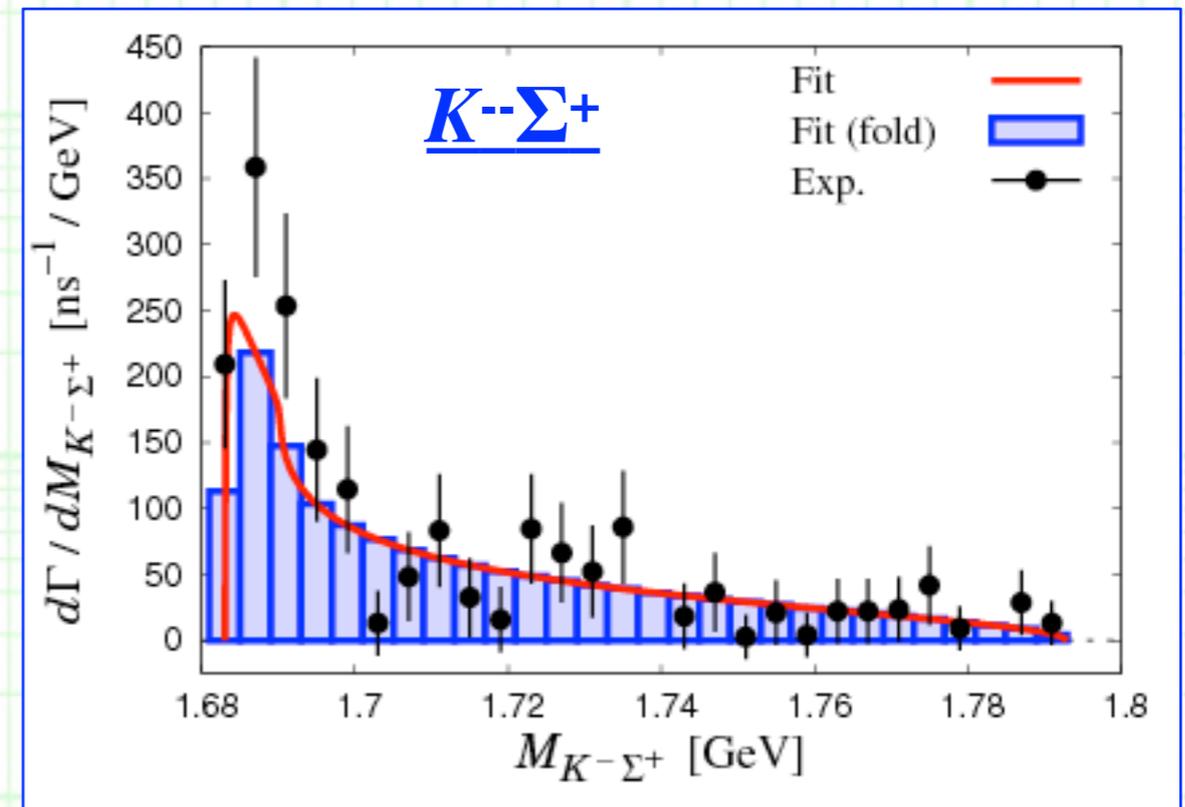
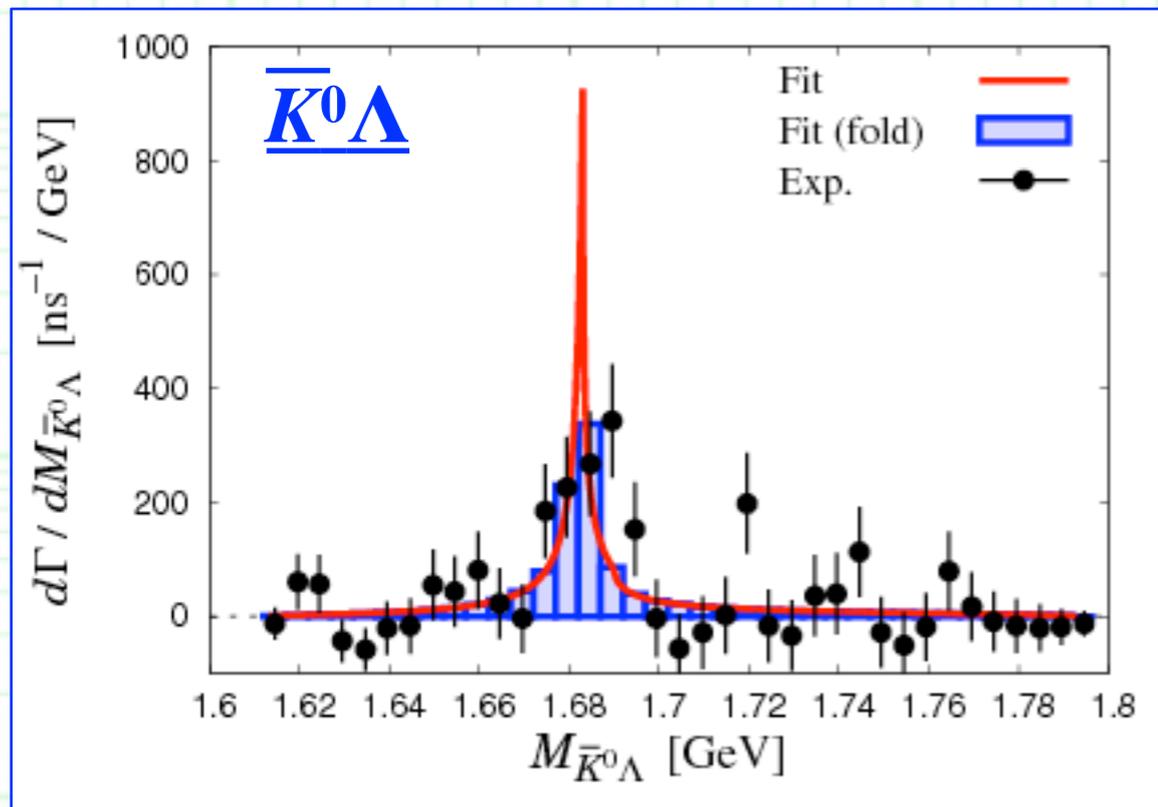
2. Subtraction constants are “natural” (except for $a_{\pi\Xi}$), as the values of the corresponding three-dimensional cut-off at the threshold, Λ_{th} , is about 500 - 1500 MeV.

--- The $\pi\Xi$ channel negligibly contributes to $\Xi(1690)$.

3. Results and discussions

++ Fitting to the Belle data ++

- We fix 4 free parameters ($a_{K\Sigma}$, $a_{K\Lambda}$, $a_{\pi\Xi}$, and $a_{\eta\Xi}$) so as to reproduce the mass spectra by Belle. The result of the best fit is:



1. The Belle data on $\Xi(1690)$ are reproduced qualitatively well.
2. Subtraction constants are “natural” (except for $a_{\pi\Xi}$).
3. The $\Xi(1690)$ pole is dynamically generated at $1684.3 - 0.5 i$ MeV, whose real part is between the $K^-\Sigma^+$ and the $\bar{K}^0\Sigma^0$ thresholds.
--- This pole exists in the first Riemann sheet of both $\bar{K}\Sigma$ channels.

3. Results and discussions

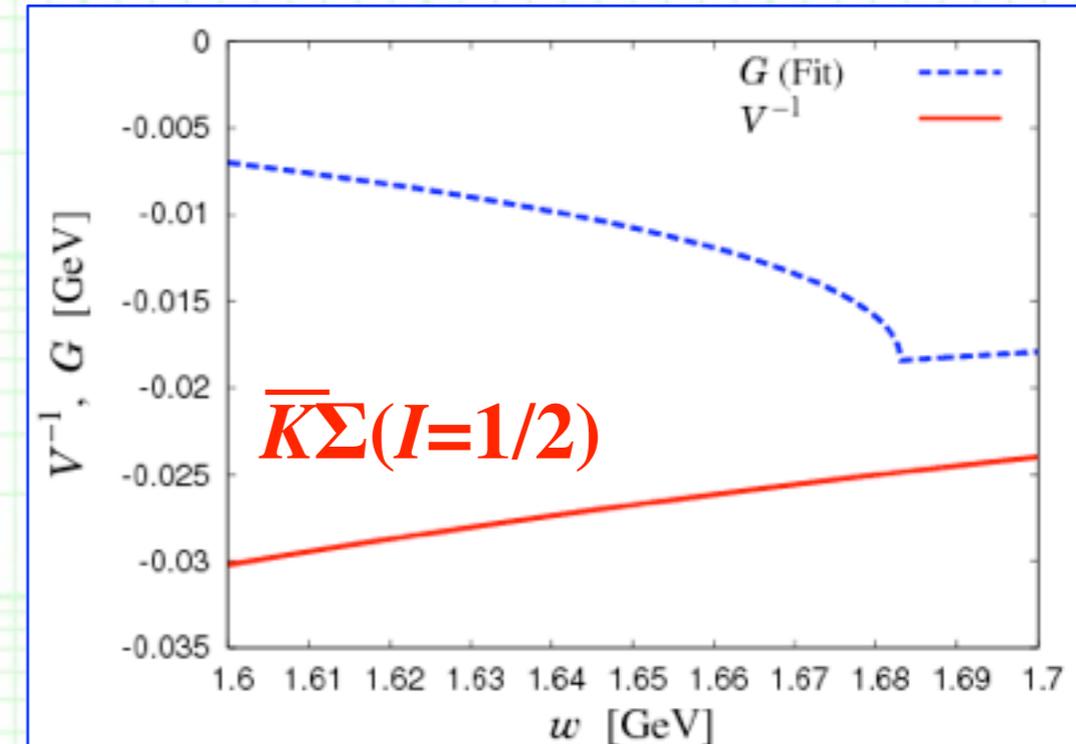
++ Origin of $\Xi(1690)$ ++

- We naively expect that **the $\Xi(1690)^0$ (pole at 1684.0 -- 0.6 i MeV) would originate from the $\bar{K}\Sigma(I=1/2)$ bound state generated by the strongly attractive interaction between $\bar{K}\Sigma(I=1/2)$.**

--- *cf.* The strongly attractive $\bar{K}N(I=0)$ interaction for $\Lambda(1405)$.

- However, **the chiral $\bar{K}\Sigma(I=1/2)$ interaction is attractive but not strong enough to generate a bound state in a single channel case.**

--- In contrast to the $\bar{K}N(I=0)$ Int. , which can solely generate a bound state for $\Lambda(1405)$.



C_{jk}	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\pi\Sigma$	$\eta\Sigma$
$\bar{K}\Sigma$	2	0	-1/2	3/2
$\bar{K}\Lambda$	0	0	-3/2	-3/2
$\pi\Sigma$	-1/2	-3/2	2	0
$\eta\Sigma$	3/2	-3/2	0	0

- This fact implies that **the multiple scatterings**, such as $\bar{K}\Sigma \rightarrow \eta\Sigma \rightarrow \bar{K}\Sigma$, **assist the $\bar{K}\Sigma$ interaction** for $\Xi(1690)$.

3. Results and discussions

++ Small decay width ++

- In addition, **the structure of the interaction strength** qualitatively explains **a remarkable property of $\Xi(1690)^0$, its very small width:**

$$\Gamma = -2 \operatorname{Im}(w_{\text{pole}}) \sim 1 \text{ MeV.}$$

1. Transition of $\bar{K}\Sigma \leftrightarrow \bar{K}\Lambda$ is forbidden at the leading order ($C_{jk} = 0$), so the $\bar{K}\Sigma \rightarrow \bar{K}\Lambda \rightarrow \bar{K}\Sigma$ multiple process gives **zero**.

2. $\bar{K}\Sigma \leftrightarrow \pi\Sigma$ is not strong compared to, e.g., $\bar{K}N(I=0) \leftrightarrow \pi\Sigma$.
--- $C_{jk} = 0.5$ vs. $\sqrt{1.5} = 1.22 \dots$

3. $\bar{K}\Sigma \leftrightarrow \eta\Sigma$ is the strongest.

- > As a consequence, the $\eta\Sigma$ channel is most important in the multiple scatterings for $\bar{K}\Sigma$ to dynamically generate $\Xi(1690)$ which cannot couple strongly to $\bar{K}\Lambda$ nor $\pi\Sigma$.
--- This reproduces small decay width and tiny BR fraction to $\pi\Sigma$.

C_{jk}	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\pi\Sigma$	$\eta\Sigma$
$\bar{K}\Sigma$	2	0	-1/2	3/2
$\bar{K}\Lambda$	0	0	-3/2	-3/2
$\pi\Sigma$	-1/2	-3/2	2	0
$\eta\Sigma$	3/2	-3/2	0	0

($I = 1/2$, isospin basis)

C_{jk}	$\bar{K}N$	$\pi\Sigma$	$\eta\Sigma$	$K\Sigma$
$\bar{K}N$	3	$-\sqrt{3/2}$	$3/\sqrt{2}$	0
$\pi\Sigma$	$-\sqrt{3/2}$	4	0	$\sqrt{3/2}$
$\eta\Sigma$	$3/\sqrt{2}$	0	0	$-3/\sqrt{2}$
$K\Sigma$	0	$\sqrt{3/2}$	$-3/\sqrt{2}$	3



3. Results and discussions

++ Compositeness for $\Xi(1690)$ ++

- **Our $\Xi(1690)$ pole** exists at $1684.3 - 0.5 i$ MeV, whose real part is **very close to the $K^- \Sigma^+$ threshold** (= 1863.1 MeV).
- The pole exists in the first Riemann sheet of the $K^- \Sigma^+$ channel.

- **“Theorem” (single channel, s wave):**

The bound state with **the field renormalization const. $Z \sim 0$** naturally appears when the state exists **near the threshold**, and especially **Z vanishes in the limit $B \rightarrow 0$** .

T. Hyodo (2014);

C. Hanhart, J. R. Pelaez and G. Rios, (2014).

--> The state should be **genuinely composite**.

- **Therefore, we expect that our $\Xi(1690)$ state should be genuinely $\bar{K}\Sigma$ composite ! (coupled-channels version)**

- **Indeed, the result of the compositeness X strongly indicates that $\Xi(1690)$ is a $\bar{K}\Sigma$ molecular state.**

$$\langle \Psi^* | \Psi \rangle = \sum_j X_j + Z = 1$$

Hyodo (2013); T. S., Hyodo and Jido (2015).

$$X_j = -g_j^2 \left[\frac{dG_j}{dw} \right]_{w=w_{\text{pole}}}, \quad Z = - \sum_{j,k} g_k g_j \left[G_j \frac{dV_{jk}}{dw} G_k \right]_{w=w_{\text{pole}}}$$

$X_{K^- \Sigma^+}$	$0.84 - 0.27i$
$X_{\bar{K}^0 \Sigma^0}$	$0.11 + 0.15i$
$X_{\bar{K}^0 \Lambda}$	$-0.01 + 0.01i$
$X_{\pi^+ \Xi^-}$	$0.00 + 0.00i$
$X_{\pi^0 \Xi^0}$	$0.00 + 0.00i$
$X_{\eta \Xi^0}$	$0.01 + 0.02i$
Z	$0.06 + 0.09i$



4. Summary

++ Summary ++

- We have investigated **dynamics of $\bar{K}\Sigma$ and its coupled channels in the chiral unitary approach.**
 - We employ the simplest interaction: Weinberg-Tomozawa term.
 - Subtraction constants as free parameters are fixed by fitting the $\bar{K}^0\Lambda$ and $K^-\Sigma^+$ mass spectra to the experimental data.
- As a result, we have found that:
 - The obtained scattering amplitude can qualitatively reproduce the experimental data of the $\bar{K}^0\Lambda$ and $K^-\Sigma^+$ mass spectra.
 - **Dynamically generates a Ξ^* pole near the $\bar{K}\Sigma$ threshold as a $\bar{K}\Sigma$ molecule**, which can be identified with **the $\Xi(1690)^0$ resonance.**
 - However, the $\bar{K}\Sigma$ interaction alone is slightly insufficient to bring a $\bar{K}\Sigma$ bound state, so multiple scattering is important for $\Xi(1690)$.
 - The small or vanishing couplings of the $\bar{K}\Sigma$ channel to others can naturally explain small decay width of $\Xi(1690)$.



**Thank you very much
for your kind attention !**



Appendix



Appendix

++ Comparison with previous ChUA calculations ++

- The discussion on [the \$\bar{K}\Sigma\$ interaction](#) can be further utilized for **comparison of our result on $\Xi(1690)$ (pole at $1684.3 - 0.5 i$ MeV) with previous ones in chiral unitary approach.**

$(\frac{1}{2}, -2)$		$[\pi \Xi]$ 7.5	5.6	seen	2.6
$\Xi(1620)^*$		$[\bar{K}\Lambda]$ 5.2	2.8	seen	-1.5
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$(\frac{1}{2}, -2)$		$[\pi \Xi]$ 0.02	0.1	seen	-0.1
$\Xi(1690)^{***}$		$[\bar{K}\Lambda]$ 0.16	6.0	seen	0.9
$M = 1690 \pm 10$	1663	$[\bar{K}\Sigma]$ 5.15	3.1	seen	-2.5
$\Gamma = 10 \pm 6$	4	$[\eta \Xi]$ 2.28	3.2	0	-1.7

\leftrightarrow [Qualitatively similar](#), but **the mass (= real part of the pole position) of our result is 20 - 30 MeV larger than others.**

C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* **B582** (2004) 49.

8 (1134)	2037-24i	0.6	0.6	0.3	0.2	0.3	↑0.5	1.5	0.6	1.8	2.4	1.1	0.2	1.0	2.1	
10 (70)	1729-46i	0.6	1.4	0.4	↑1.6	1.4	2.1	1.0	0.4	3.3	1.5	0.4	0.2	1.6	1.0	$\Xi(1950)$ ***
8 (70)	1651-2i	0.2	0.3	↑2.2	1.3	1.0	2.6	0.2	0.6	0.9	0.4	0.2	1.7	0.4	0.2	$\Xi(1690)$ ***
8 (56)	1577-139i	2.6	↑1.7	0.5	0.1	0.8	1.0	0.7	0.1	0.6	1.3	0.3	0.1	0.2	1.2	$\Xi(1620)$ *

D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* **D84** (2011) 056017.



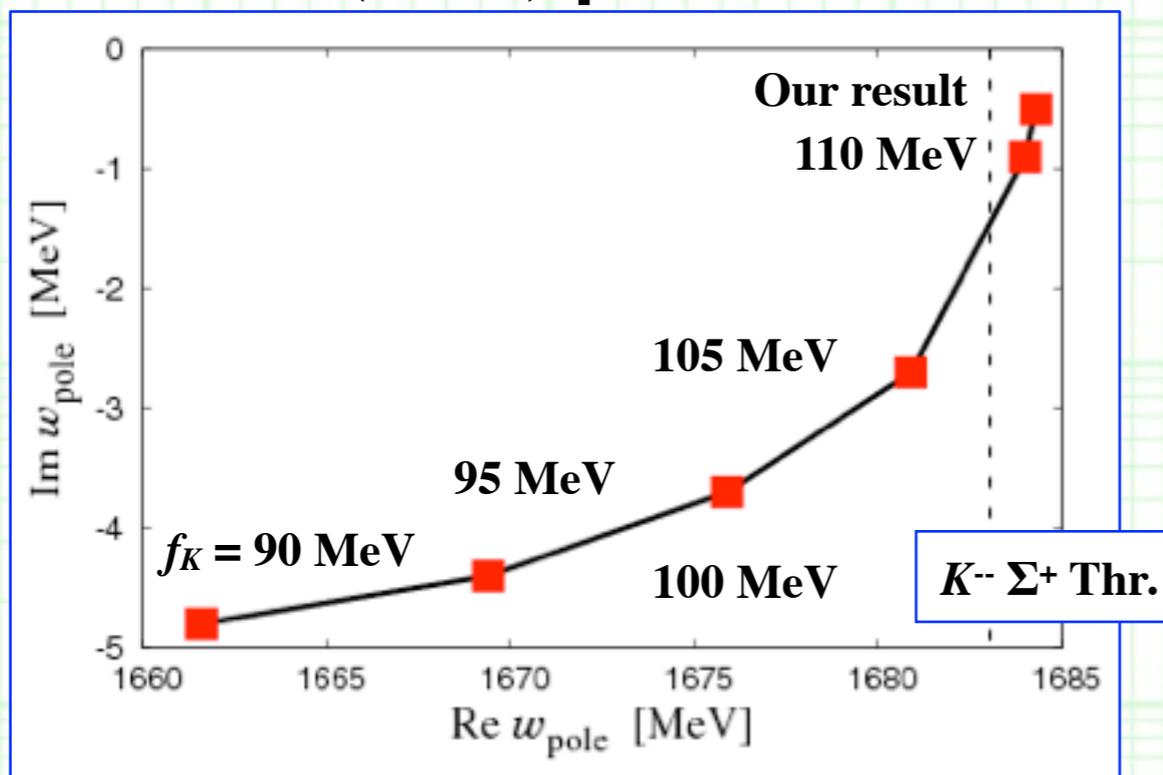
Appendix

++ Comparison with previous ChUA calculations ++

- The discussion on [the \$\bar{K}\Sigma\$ interaction](#) can be further utilized for **comparison of our result on $\Xi(1690)$ (pole at $1684.3 - 0.5 i$ MeV) with previous ones in chiral unitary approach.**

- In Ref. [1] they used the meson decay constant $f = 90$ MeV in [all channels](#), while we use their physical values ($f_K = 110.64$ MeV).

-> The $\Xi(1690)$ pole moves as:



- In Ref. [2] they [introduced channels with vector mesons](#), which **would assist more** the $\bar{K}\Sigma$ interaction, and hence the mass of $\Xi(1690)$ shifted to lower energies.

[1] C. Garcia-Recio, M. F. M. Lutz and J. Nieves, *Phys. Lett.* **B582** (2004) 49.

[2] D. Gamermann, C. Garcia-Recio, J. Nieves and L. L. Salcedo, *Phys. Rev.* **D84** (2011) 056017.

Appendix

++ Charged $\Xi(1690)$ ++

- Finally we consider **the charged $\Xi(1690)$** in the same parameter set as the neutral one. As a result, we obtain the $\Xi(1690)^-$ pole as:

w_{pole}	$1693.4 - 10.5i$ MeV
$X_{\bar{K}^0\Sigma^-}$	$0.86 - 0.50i$
$X_{K^-\Sigma^0}$	$-0.27 + 0.31i$
$X_{K^-\Lambda}$	$-0.02 + 0.04i$
$X_{\pi^-\Xi^0}$	$0.00 + 0.00i$
$X_{\pi^0\Xi^-}$	$0.00 + 0.00i$
$X_{\eta\Xi^-}$	$0.07 + 0.03i$
Z	$0.36 + 0.12i$

- The $\Xi(1690)^-$ pole is located between the $K^-\Sigma^0$ and $\bar{K}^0\Sigma^-$ thresholds; The pole is in the first Riemann sheet of the $\bar{K}^0\Sigma^-$ and $\eta\Xi^-$ channels and **in the second Riemann sheet of the $K^-\Lambda$, $K^-\Sigma^0$, $\pi^-\Xi^0$, and $\pi^0\Xi^-$ channels**.

- The pole position has **a larger imaginary part ~ 10 MeV** compared to the neutral case, since it exists above the $\bar{K}^0\Sigma^-$ threshold in its second Riemann sheet and hence **the decay to $\bar{K}^0\Sigma^-$ is allowed**.
- Although both $X_{\bar{K}^0\Sigma^-}$ and $X_{K^-\Sigma^0}$ have large imaginary part, sum of them is the dominant contribution with its small imaginary part, which implies that **the $\Xi(1690)^-$ state is also a $\bar{K}\Sigma$ molecular state**.

Appendix

++ Outlook ++

- **Theoretical study:**
 - Propose reactions which can clarify properties of the $\Xi(1690)$ resonance in experiments, both neutral and charged states.
 - Predict the $\Xi(1690)$ production cross section.
 - Improvement of model by, *e.g.*, introducing s - and u -channel Born terms.
- **Experimental study:**
 - **Determine J^P of the $\Xi(1690)^0$ resonance.**
 - Measure the $\bar{K}\Lambda$ and $\bar{K}\Sigma$ mass spectra and ratio of their branching fractions.
 - Furthermore, precise determination of **its pole position** should be important to discuss the internal structure of $\Xi(1690)$.
 - Flatte parameterization may be necessary since it exists near the $\bar{K}\Sigma$ threshold.

