Searching for detectable effects induced by anomalous threshold triangle singularity

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Outline

- Earlier study
- Anomalous triangle singularity (ATS)
- Promising processes to detect ATS
- Searching for charmoniumlike states with hidden ssbar
- Production of the pentaquark “Pc”
- Summary

Earlier study in 1960s

- **Connections between kinematic singularities of the S-matrix elements and resonance-like peaks: e.g. Peierls mechanism**

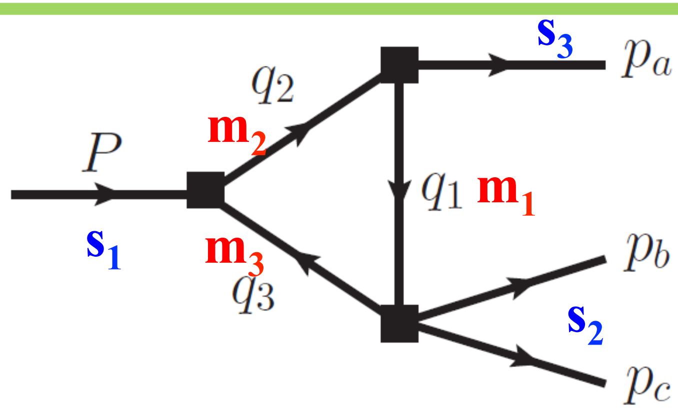
R.F.Peierls, PRL6,641(1961);
R.C.Hwa, PhysRev130,2580(1963);
C.Goebel,PRL13,143(1964);
P.Landshoff&S.Treiman Phys.Rev.127,649(1962);
………

- **Some disadvantages:**

✓ **Few experiments to search for the effects;**
✓ **Low statistics;**
✓ **For the elastic scattering process, singularities of the triangle diagram will be weakened by the corresponding tree diagram without rescattering, according to the so called Schmid theorem**

C.Schmid,Phys.Rev.154,1363(1967);
A.V.Anisovich,PLB345,321(1995)

Anomalous Triangle Singularity



$$P^2 = s_1, \ (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

$$\Gamma_3(s_1, s_2, s_3) = \frac{-1}{16\pi^2} \int_0^1 \int_0^1 \int_0^1 da_1\, da_2\, da_3\, \frac{\delta(1 - a_1 - a_2 - a_3)}{D - i\epsilon}$$

$$D = \sum_{i,j=1}^{3} a_i a_j Y_{ij}, \ \ Y_{ij} = \frac{1}{2}\left[m_i^2 + m_j^2 - (q_i - q_j)^2\right]$$

✓ **Singularity in the complex space**

Necessary conditions (Landau Equation)

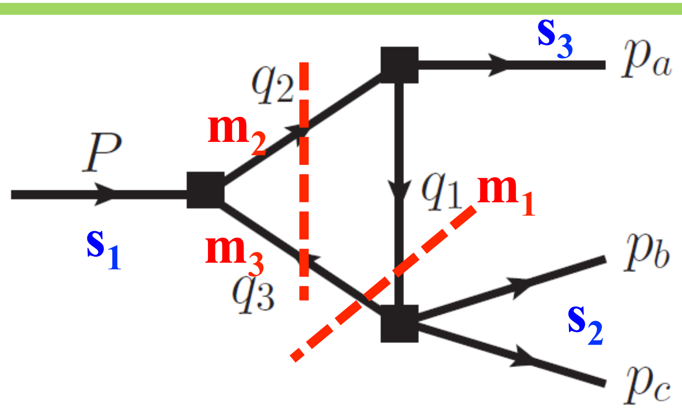
$$D = 0,$$

$$either \ \ a_j = 0 \ \ or \ \boxed{\frac{\partial D}{\partial a_j} = 0.}$$

Leading singularity

Landau, Nucl.Phys.13,181(1959)

Anomalous Triangle Singularity



$$P^2 = s_1, \ (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

✓ **Singularity in the complex space**

The position of the singularity is obtained by solving

$$\det[Y_{ij}] = 0$$

Normal Threshold

s_1, s_3, $m_{1,2,3}$ fixed

$$\boxed{s_2^{\pm}} = \boxed{(m_1+m_3)^2} + \frac{1}{2m_2^2}[(m_1^2+m_2^2-s_3)(s_1-m_2^2-m_3^2) - 4m_2^2 m_1 m_3$$
$$\pm \lambda^{1/2}(s_1, m_2^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)], \quad \lambda(x,y,z) \equiv (x-y-z)^2 - 4yz$$

Anomalous Threshold

$$\boxed{s_1^{\pm}} = (m_2+m_3)^2 + \frac{1}{2m_1^2}[(m_1^2+m_2^2-s_3)(s_2-m_1^2-m_3^2) - 4m_1^2 m_2 m_3$$
$$\pm \lambda^{1/2}(s_2, m_1^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)].$$

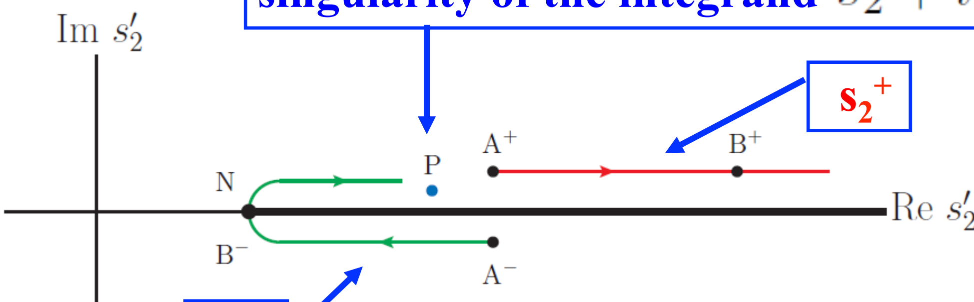
s_2, s_3, $m_{1,2,3}$ fixed

Anomalous Triangle Singularity

By means of single dispersion relation, the locations of $s_2^{\pm}$ in the s_2'-plane can be determined by

$$s_2^{\pm}(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^{\pm}}{\partial s_1}$$

singularity of the integrand $s_2 + i\epsilon$



s_2^+

s_2^-

Trajectory of the branch points s_2^-, s_2^+ with s_1 increases from normal threshold to infinity in the complex s_2'-plane

$A^{\pm}$ s_{1N} $s_1 = (m_2 + m_3)^2$, $B^{\pm}: \; s_1 = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2 - m_1)^2 - s_3]$ $= s_{1C}$

$A^-: \; s_2^- = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3] - i\epsilon$ $= s_{2C}$

$B^-: \; s_2^- = (m_1+m_3)^2$ $= s_{2N}$

s_2^- and P will pinch the integral contour

This pinch singularity is the anomalous triangle singularity (ATS)

Anomalous Triangle Singularity

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$$s_2^{\pm}(s_1 + i\epsilon) = s_2(s_1) + i\epsilon\frac{\partial s_2^{\pm}}{\partial s_1}$$

singularity of the integrand $s_2 + i\epsilon$

Trajectory of the branch

"The kinematic conditions for the existence of singularities on the physical boundary are equivalent to the condition that the relevant Feynman diagram be interpretable as a picture of an energy and momentum-conserving process occurring in space–time, with all internal particles real, on the mass shell and moving forward in time."

Coleman&Norton, Nuovo Cimento 38,5018 (1965)

Fronsdal&Norton,J.Math.Phys. 5, 100(1964)

………

Anomalous Triangle Singularity

Normal threshold and critical point

s_1 $$s_{1N} = (m_2+m_3)^2,\ s_{1C} = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2-m_1)^2 - s_3],$$

ATS s_2^- $$s_{2N} = (m_1+m_3)^2,\ s_{2C} = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3],$$

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Discrepancy between anomalous and normal threshold

When $s_2=s_{2N}$

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$

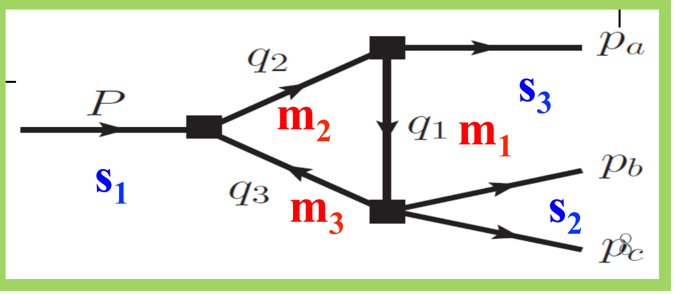
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \boxed{\frac{m_3}{2m_2(m_1+m_3)}}[(m_2-m_1)^2 - s_3].$$

When $s_1=s_{1N}$

Largest discrepancy

Enlarge

How to amplify the discrepancy between normal and anomalous threshold?



Anomalous Triangle Singularity

Normal threshold and critical point

$$s_{1N} = (m_2+m_3)^2,\ s_{1C} = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2-m_1)^2 - s_3],$$
$$s_{2N} = (m_1+m_3)^2,\ s_{2C} = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3],$$

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$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Discrepancy between anomalous and normal threshold

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$
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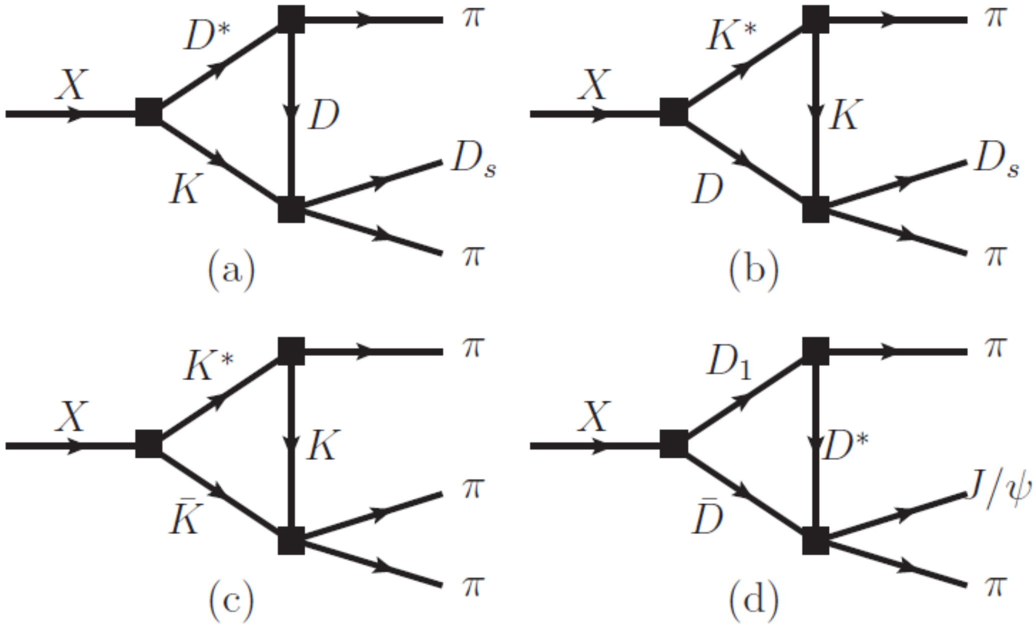
Largest discrepancy

How to amplify the discrepancy between normal and anomalous threshold?

If the discrepancy is larger, maybe it could be used to distinguish the kinematic singularities from genuine particles.

Possible rescatterings to detect ATS

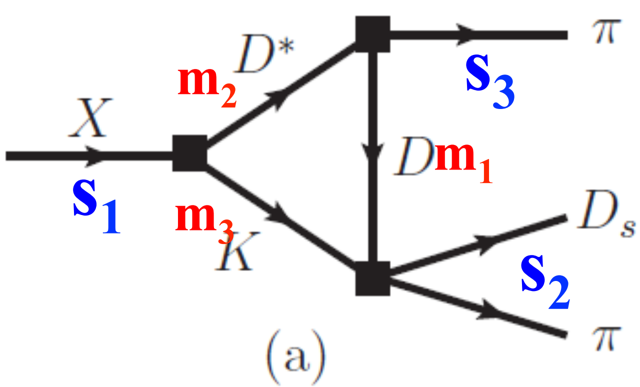


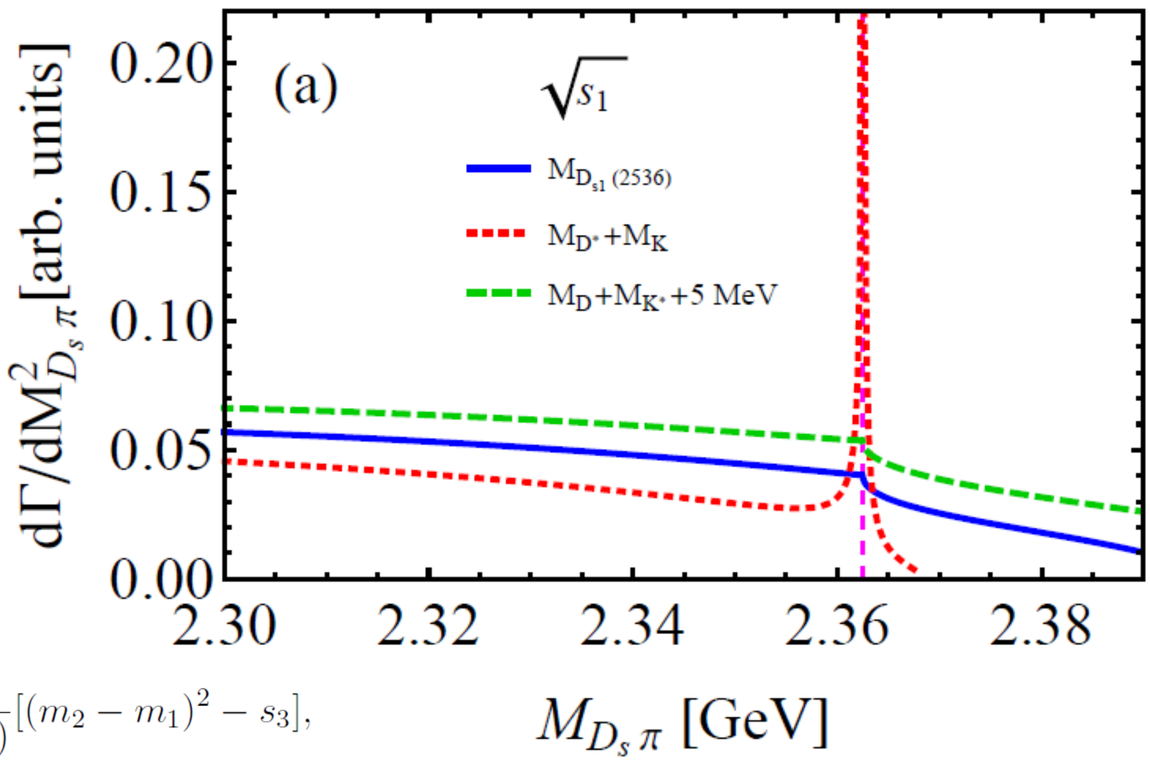
Kinematic region of ATS

[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s_1}^{\max}$	0.089	96	49	16
$\Delta_{s_2}^{\max}$	0.087	62	38	15

Liu, Oka, Zhao, arXiv:1507.01674

Possible rescatterings to detect ATS





$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$

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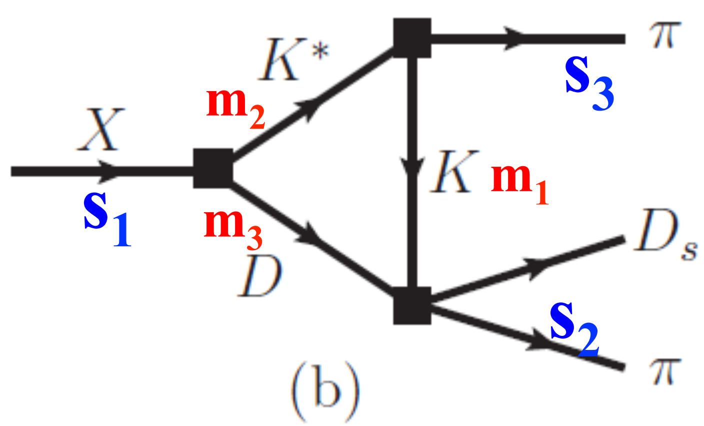

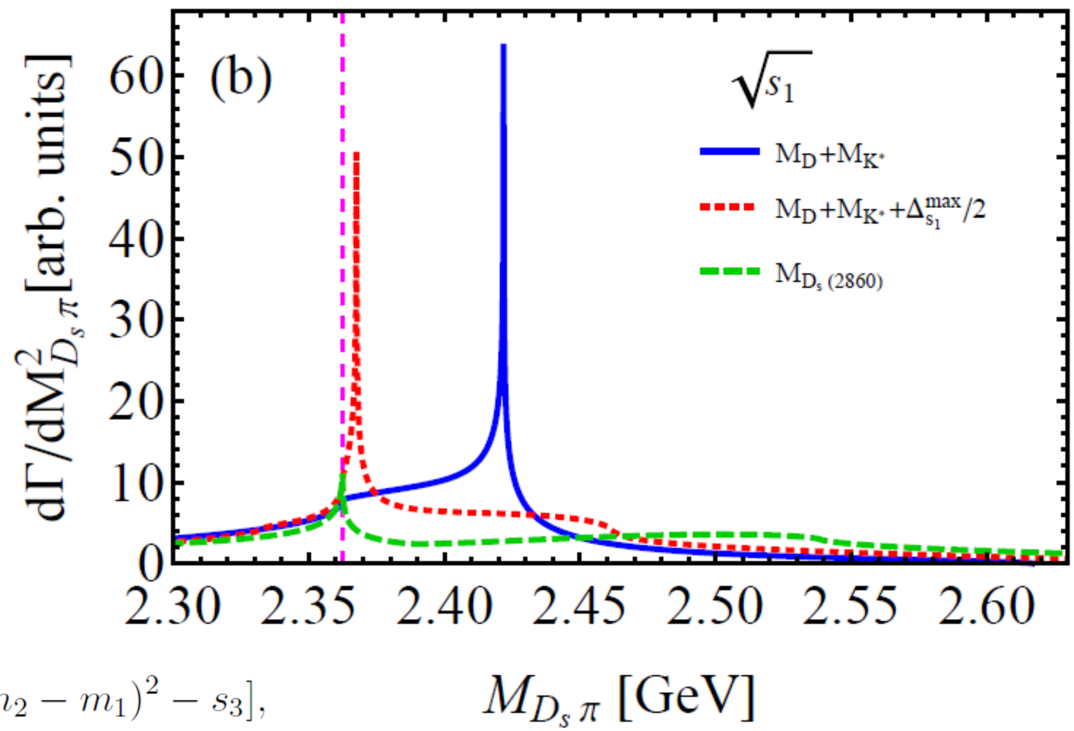

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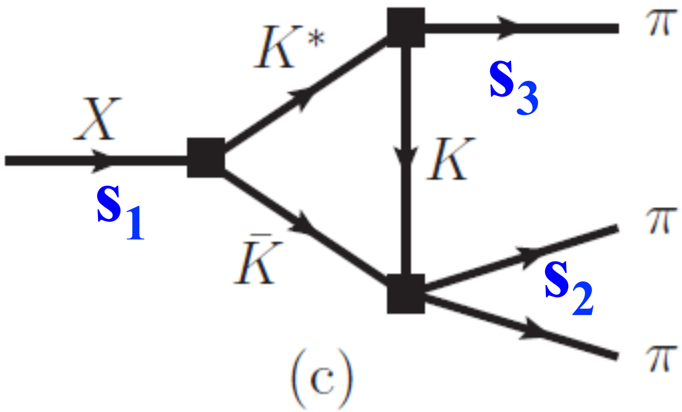
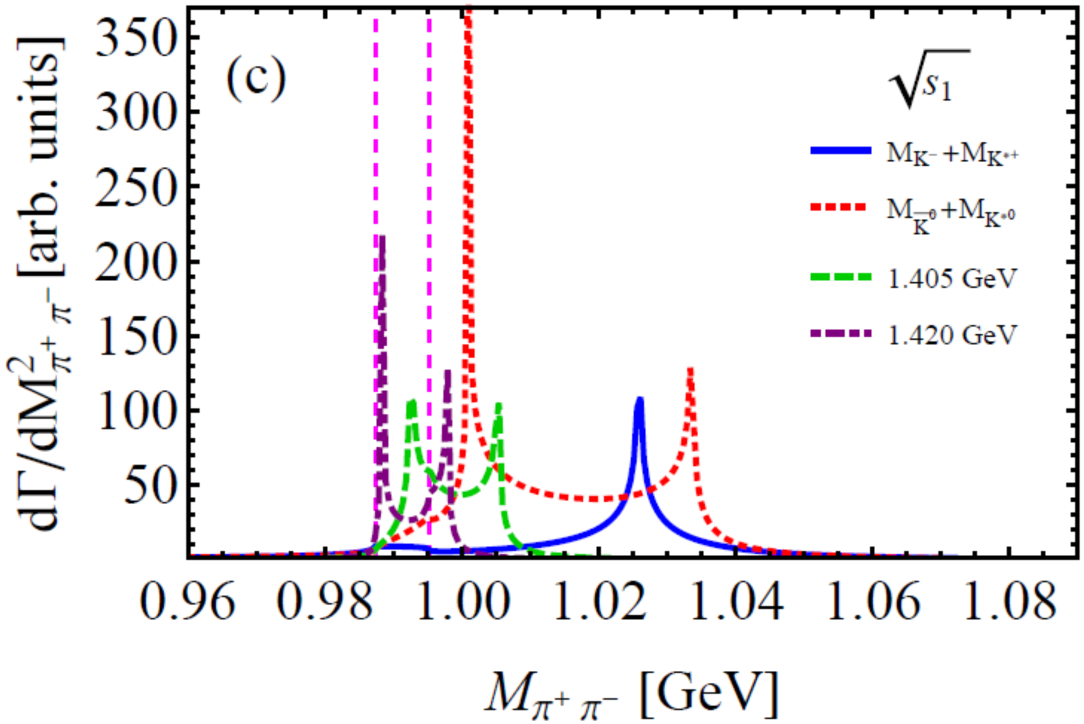


Wu, Liu, Zhao & Zou, PRL108,081803(2012)



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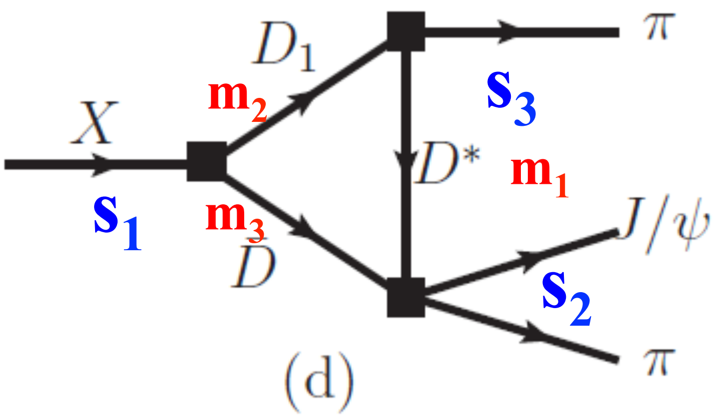

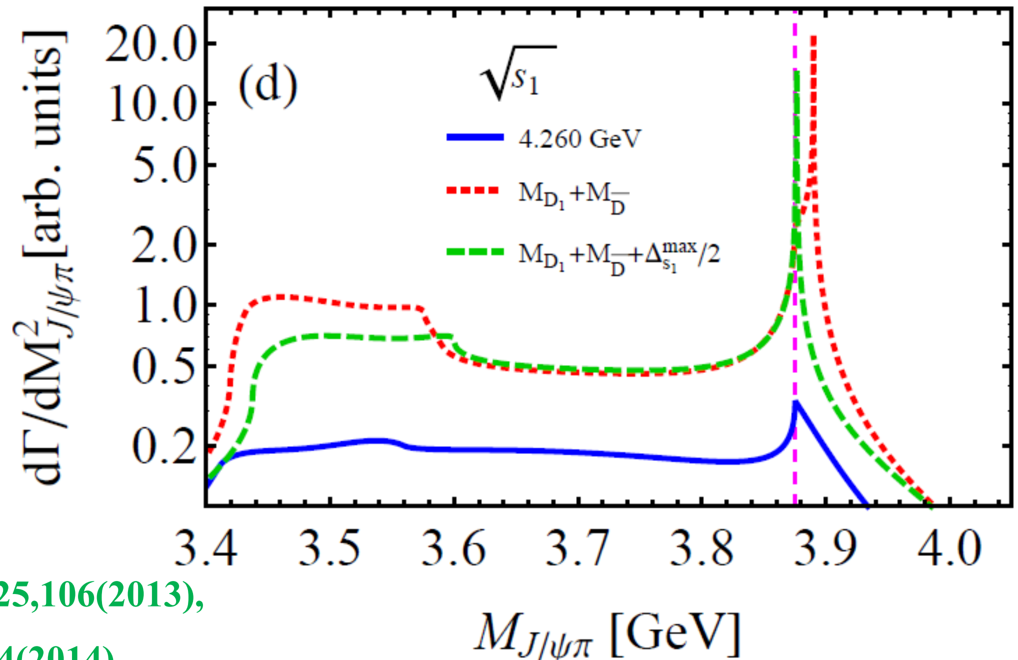

Wang,Hanhart,Zhao,PRL111,132003(2013);PLB725,106(2013),

X.H.Liu,G.Li, PRD88,014013(2013);PRD90,074004(2014)

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$

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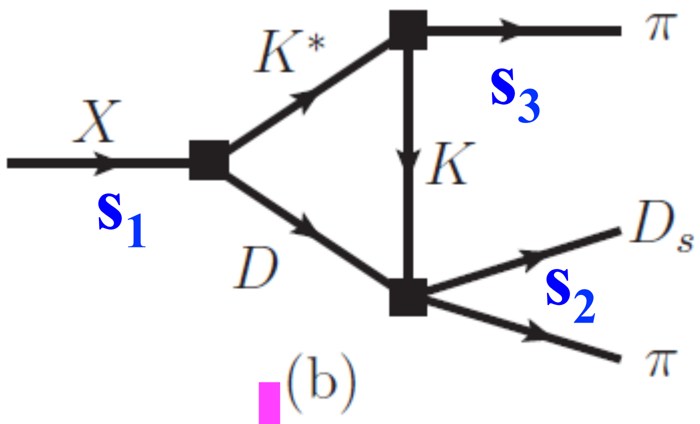

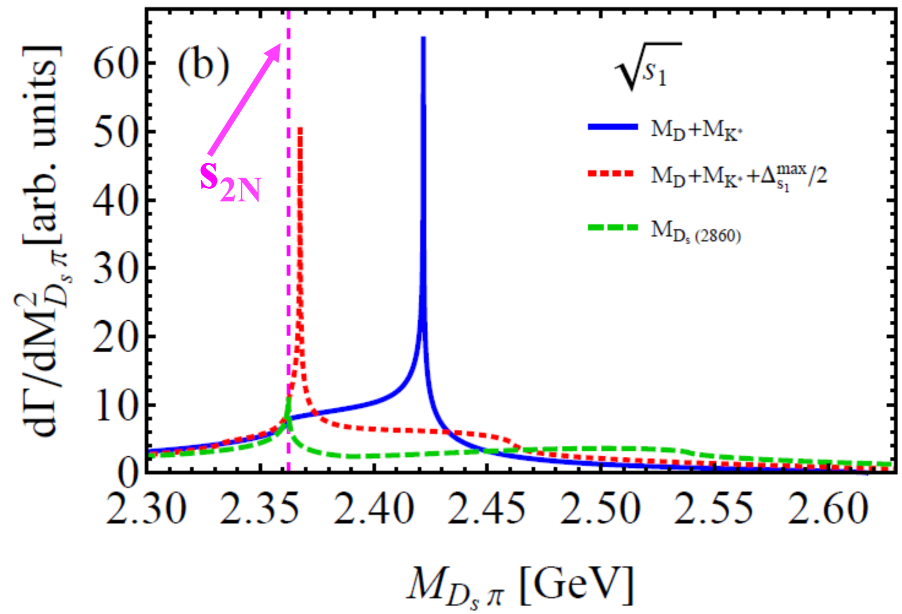

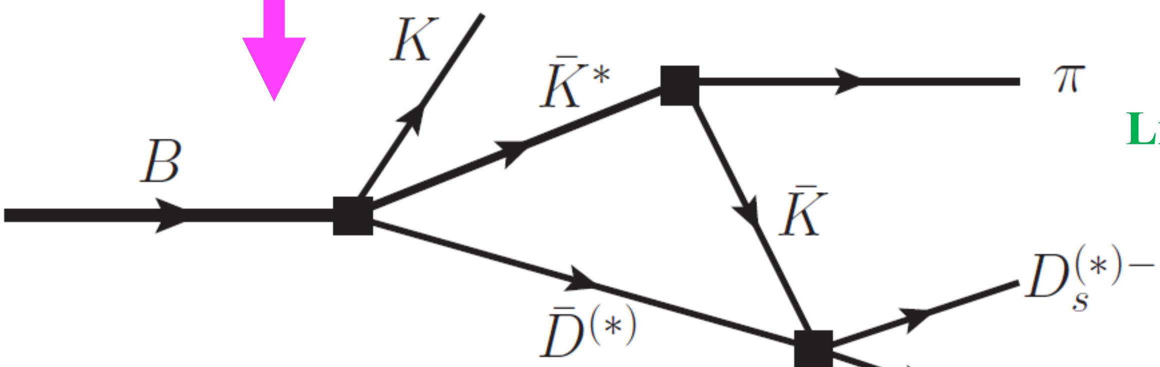

Liu, Oka, Zhao, arXiv:1507.01674

Br($B \to K\bar{K}^*\bar{D}^{(*)}$ **) $10^{-4} \sim 10^{-3}$**

Br($B \to K D_s^{(*)-}\pi\pi$ **) $\sim 10^{-4}$**

[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta^{max}_{s_1}$	0.089	96	49	16
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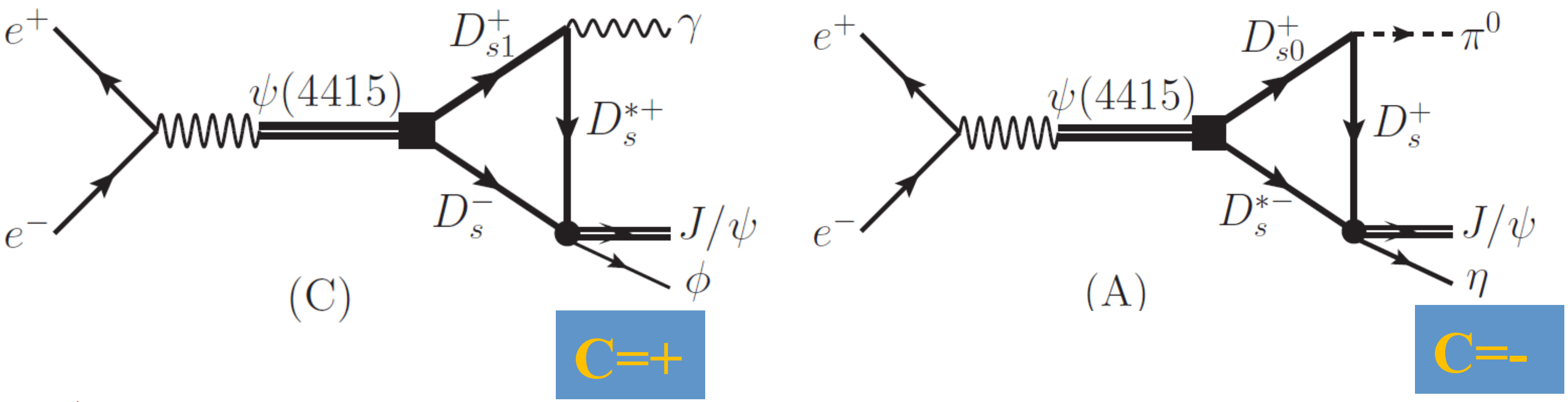
Searching for charmoniumlike states with hidden $s\bar{s}$



Advantages:

- ✓ $D_{s0}(2317)$ and $D_{s1}(2436)$ are very narrow , (Γ <3.8 MeV), influence of ATS will be obvious;
- ✓ Heavy quark spin symmetry is conserved at each vertex, leading order contribution;
- ✓ Thresholds of $D_{s0}(2317)$ D_s^* , $D_{s1}(2436)$ D_s are very close to $\psi(4415)$

Disadvantage: The discrepancy will not be very large (pure charmed meson loops)

higher energy resolution is necessary to observe the discrepancy;

Searching for charmoniumlike states with hidden $s\bar{s}$

Charmoniumlike states with hidden $s\bar{s}$

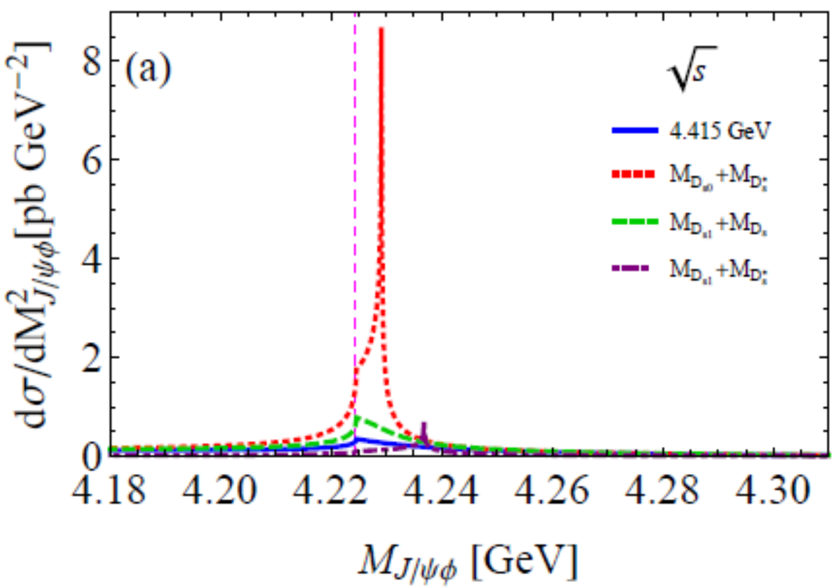
Y(4140),Y(4274),Y(4350):

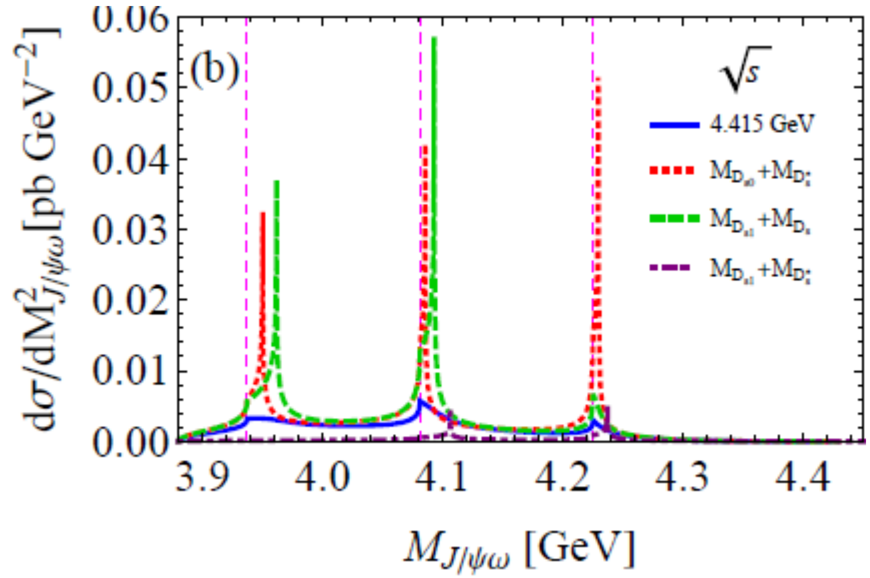
- ✓ Observed in $J/\psi\varphi$ invariant mass distribution; very narrow (Γ ~20,30 MeV);
- ✓ Close to $D_s^{(*)} D_s^{(*)}$, $D_{s0} D_s^{(*)}$ thresholds;
- ✓ Existence need to be confirmed.

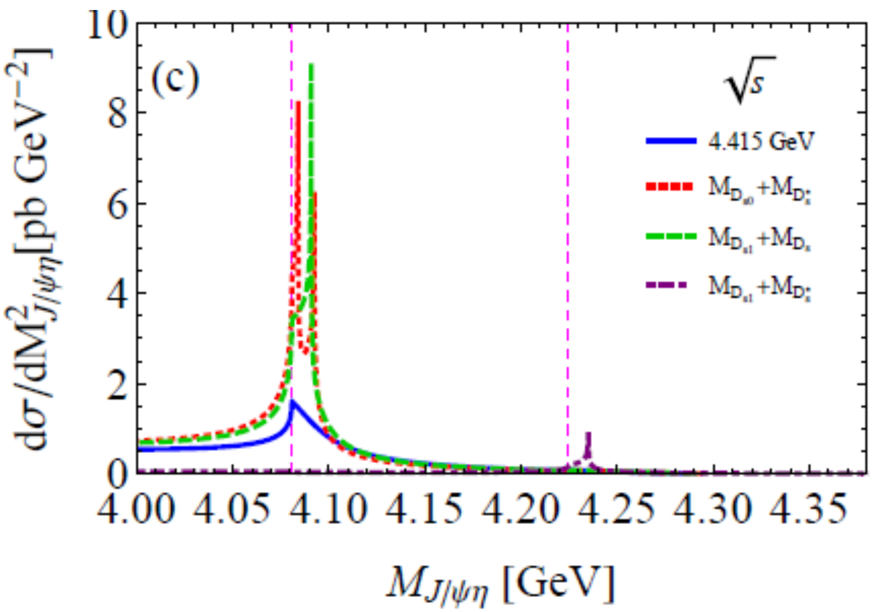
X(3915):

- ✓ Observed in $J/\psi\omega$ invariant mass distribution; very narrow (Γ ~20 MeV);
- ✓ Some problems if identify it as $\chi_{c0}(2P)$: not observed in DDbar; fine splitting $M[\chi_{c2}(2P)] – M[\chi_{c0}(2P)]$ is too small; ……
- ✓ Close to $D_s D_s$ threshold; the $s\bar{s}$ component in ω is not very small, may be connected.

Searching for charmoniumlike states with hidden $s\bar{s}$





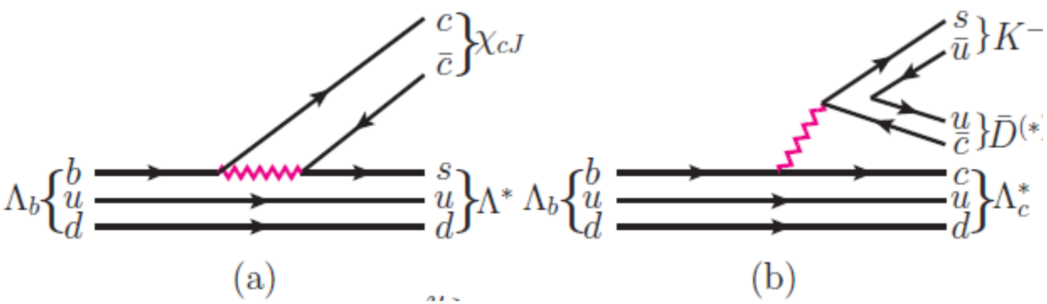


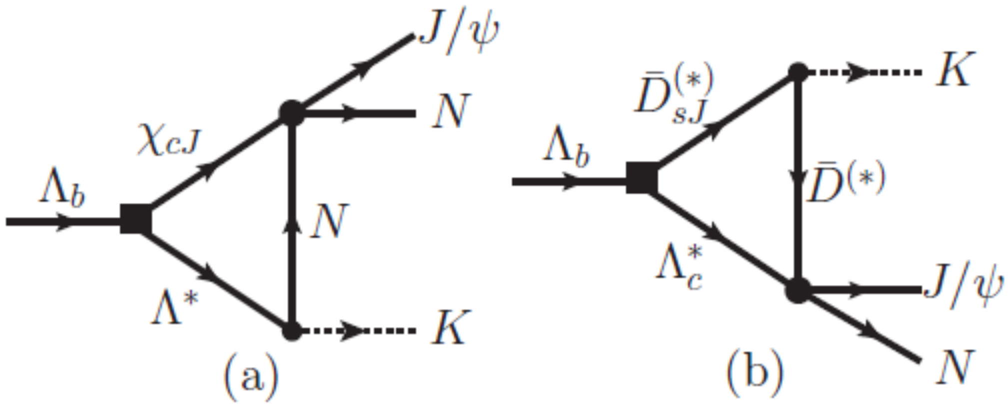
Possible connections between ATS and these charmoniumlike states with hidden $\bar{s}s$

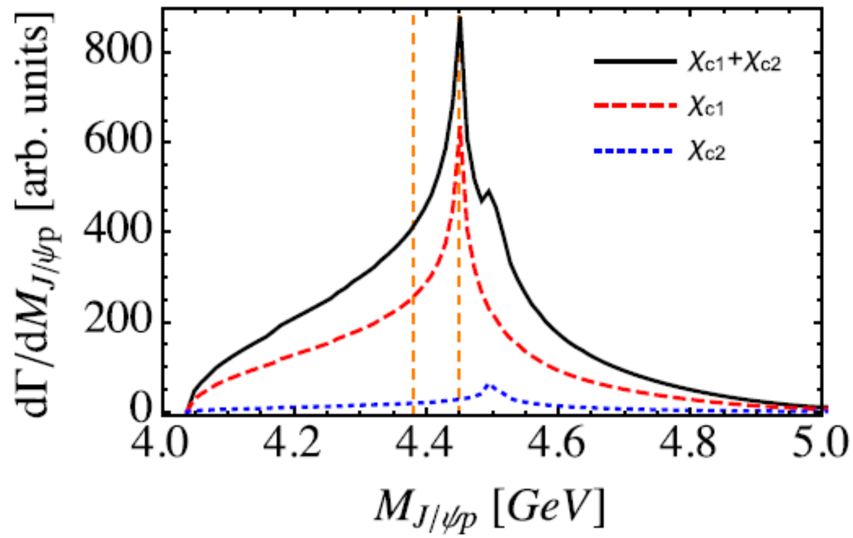
[MeV]	Fig. 1(A)	Fig. 1(B)	Fig. 1(C)	Fig. 1(D)	Fig. 1(E)
$\Delta_{s_1}^{\max}$	4.8	27	13	28	13
$\Delta_{s_2}^{\max}$	4.6	24	12	25	12

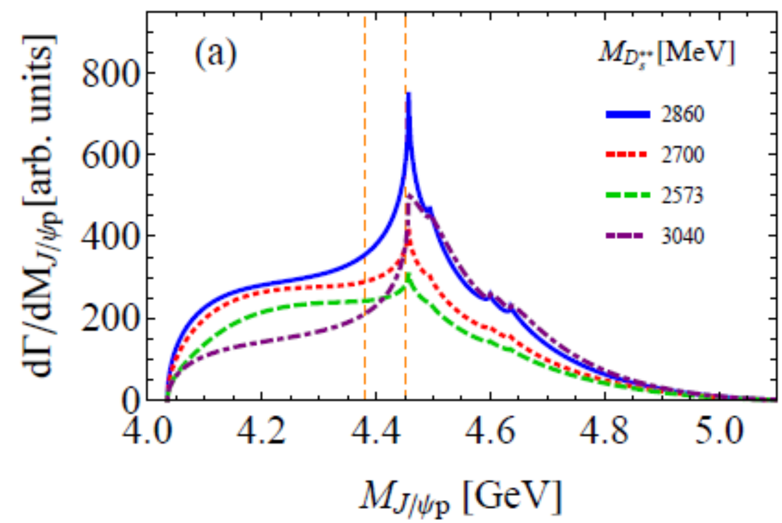
[MeV]	Fig. 2(A)	Fig. 2(B)	Fig. 2(C)
$\Delta_{s_1}^{\max}$	13	11	11
$\Delta_{s_2}^{\max}$	12	10	10

Explanation of pentaquark Pc









Thresholds [GeV]	$\chi_{c0}(1P)\ 0^+$	$\chi_{c1}(1P)\ 1^+$	$\chi_{c2}(1P)\ 2^+$
$p\ 1/2^+$	4.353	4.449	4.494

Thresholds [GeV]	$\Lambda_c(2286)\ 1/2^+$	$\Lambda_c(2595)\ 1/2^-$	$\Lambda_c(2625)\ 3/2^-$
$\bar{D}(1865)\ 0^-$	4.151	4.457	4.493
$\bar{D}^*(2007)\ 1^-$	4.293	4.599	4.635

Liu,Wang,Zhao, arXiv:1507.05359

Guo et al, arXiv:1507.04950

Summary

- Kinematic singularities (ATS) of the rescattering amplitude will behave themselves as peaks in the invariant mass distribution, which may imply that non-resonance interpretations for some resonance-like structures (XYZ particles) is possible.
- Criterion to distinguish kinematic singularities from genuine particles: peak positions (or "masses") of the resonance-like peaks will depend on their production modes; we can make this dependence become sensitive to the kinematics in some special processes; For a genuine particle, its measured mass will not be changed too much in different production modes.
- It is promising to search for the charmoniumlike structures in the processes $e^+e^- \to \gamma J/\psi\phi,\ \gamma J/\psi\omega,\ J/\psi\eta\pi^0$ at the center of mass energy around $D_{s0}(2317)\ D_s^*$, $D_{s1}(2436)\ D_s$ thresholds.
- One possible mechanism for the production of the pentaquark Pc.

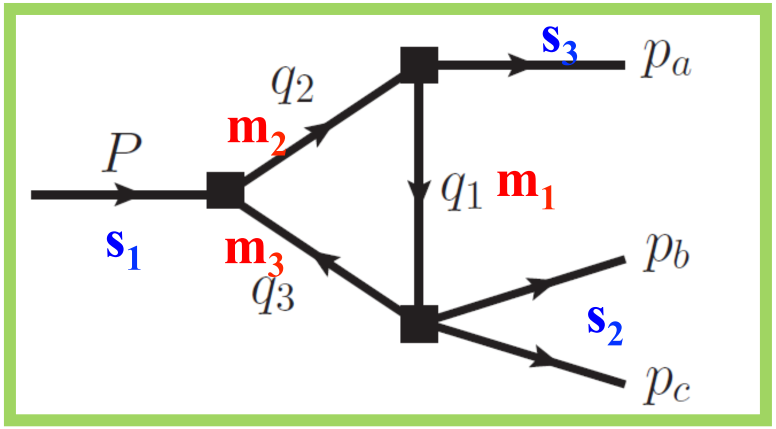
Backup

Anomalous Triangle Singularity

Single dispersion relation

$$\Gamma_3(s_1,s_2,s_3)=\frac{1}{\pi}\int_{(m_1+m_3)^2}^{\infty}\frac{ds_2'}{s_2'-s_2-i\epsilon}\sigma(s_1,s_2',s_3)$$

$$\sigma(s_1,s_2,s_3)\;=\;\sigma_+-\sigma_-$$



$$\sigma_\pm(s_1,s_2,s_3)\;=\;\frac{-1}{16\pi\lambda^{1/2}(s_1,s_2,s_3)}\log[-s_2(s_1+s_3-s_2+m_1^2+m_3^2-2m_2^2)$$
$$-\;(s_1-s_3)(m_1^2-m_3^2)\pm\lambda^{1/2}(s_1,s_2,s_3)\lambda^{1/2}(s_2,m_1^2,m_3^2)].$$

Work in the kinematical region

$$s_1\;\le\;(m_2+m_3)^2,\;s_3\;\le\;(m_2-m_1)^2\qquad 0<s_2<(m_1+m_3)^2$$

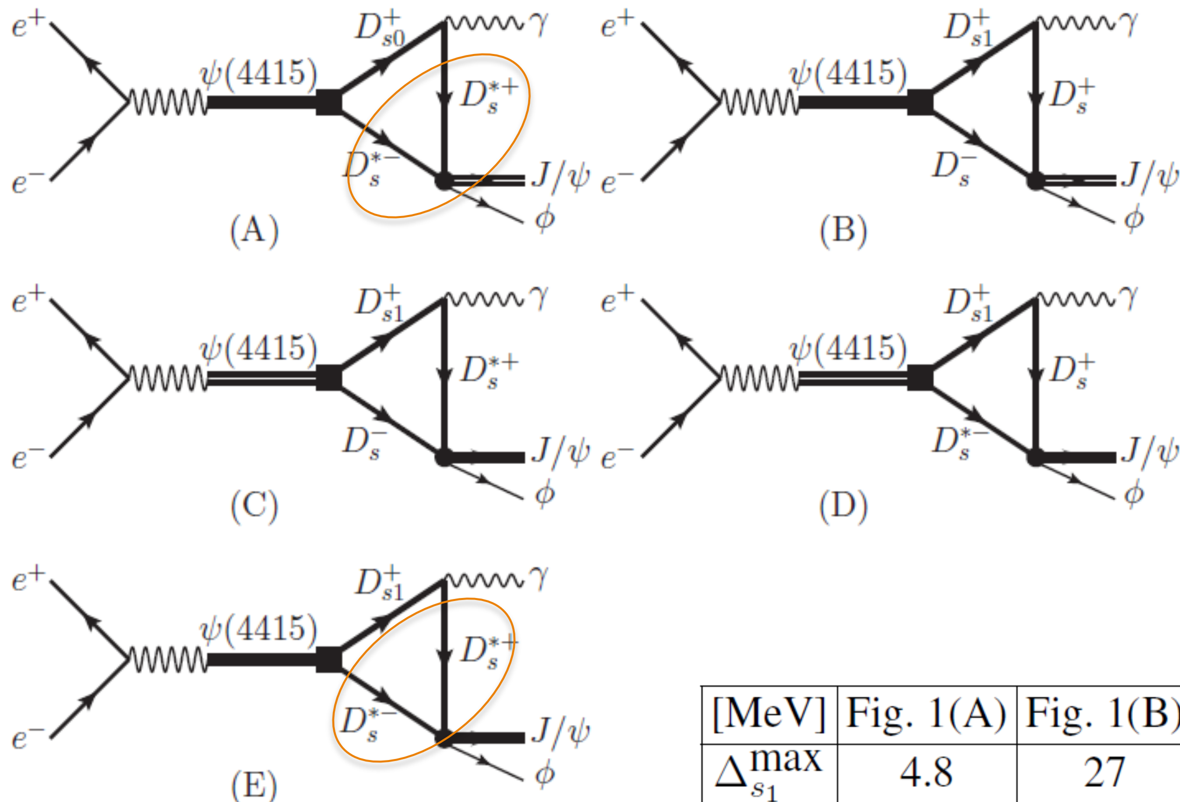
By analytic continuation, it can be extended into the over threshold reigon Fronsdal&Norton,J.Math.Phys. 5,100(1964)

$$s_1\ge(m_2+m_3)^2,\;(m_1+m_3)^2\le s_2\le(\sqrt{s_1}-\sqrt{s_3})^2,\;0\le\sqrt{s_3}\le m_2-m_1$$

Brach points of the log function $s_2^{\pm}$

Searching for charmoniumlike states with hidden $s\bar{s}$

Possible connections between ATS and these charmoniumlike states with hidden $s\bar{s}$



J/ ψφ C=+

φ→ω via mixing

[MeV]	Fig. 1(A)	Fig. 1(B)	Fig. 1(C)	Fig. 1(D)	Fig. 1(E)
$\Delta_{s_1}^{\max}$	4.8	27	13	28	13
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Taking data around the center of mass energy: **4.430~4.435 GeV or 4.572~4.585 GeV**

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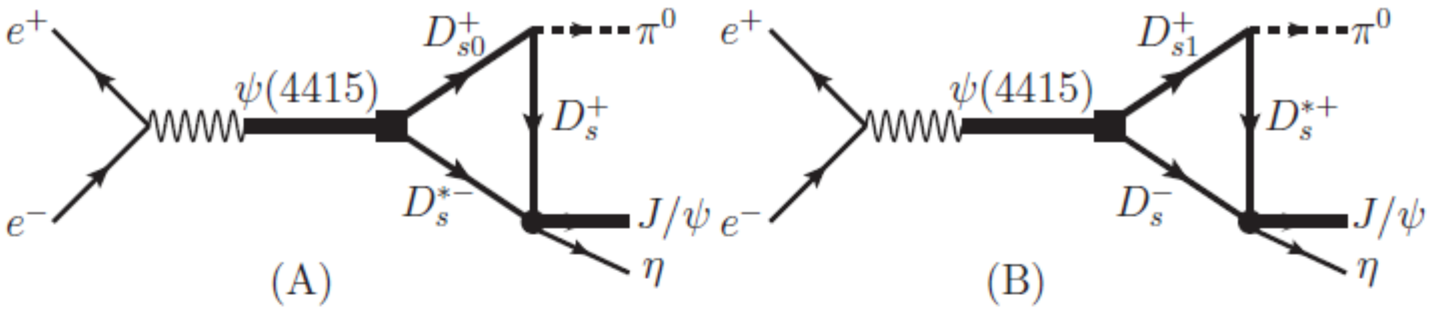
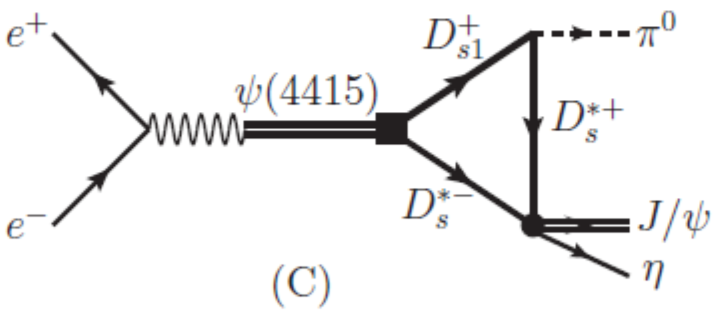




J/ ψη C=-

[MeV]	Fig. 2(A)	Fig. 2(B)	Fig. 2(C)
$\Delta_{s_1}^{\max}$	13	11	11
$\Delta_{s_2}^{\max}$	12	10	10

Taking data around the center of mass energy: 4.428~4.443 GeV or 4.572~4.583 GeV

Searching for charmoniumlike states with hidden $s\bar{s}$

Our Model is constructed in the framework of HHChPT

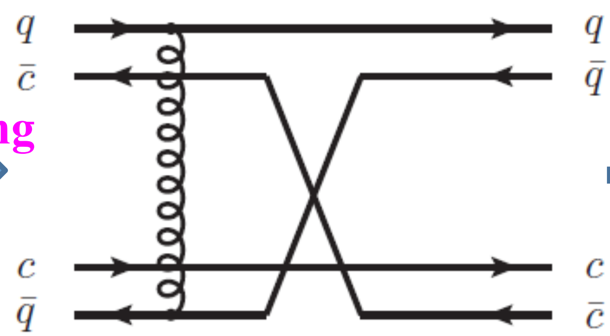
$$\mathcal{L}_{eff} = g_S < J\bar{S}_{2a}\bar{H}_{1a} + J\bar{H}_{2a}\bar{S}_{1a} > + C_S < J\bar{H}_{2b}\gamma_\mu\gamma_5\bar{H}_{1a}\mathcal{A}^\mu_{ba} > + C_V < J\bar{H}_{2b}\gamma_\mu\bar{H}_{1a}\rho^\mu_{ba} >$$
$$+ ih < \bar{H}_{1a}S_{1b}\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba} > + \frac{e\tilde{\beta}}{4} < \bar{H}_{1a}S_{1b}\sigma^{\mu\nu}F_{\mu\nu}Q_{ba} > + \cdots \quad (19)$$

$$\mathcal{L}_{mix} = \theta_{\eta\pi}\eta\pi^0 + \theta_{\omega\phi}\phi\omega$$
$$\theta_{\eta\pi} \simeq 0.01, \quad \theta_{\omega\phi} \simeq 0.065$$

Estimate coupling constants: matching the amplitude with that obtained from quark interchange model

$$D_s^{(*)+}D_s^{(*)-} \to J/\psi\phi$$
$$D_s^{(*)+}D_s^{(*)-} \to J/\psi\eta$$

matching



C1-prior

$$C_S \approx 1.73\ \mathrm{GeV}^{-3/2}$$
$$C_V \approx 46\ \mathrm{GeV}^{-3/2}$$

Barnes&Swanson,PRD46,4868(1992), X.H.Liu et al., PRD90,074020(2014)

$$\psi(4S) \to D_{s0}D_s^*$$

matching

Barnes et al.,PRD72,054026(2005)

$^3P_0$ decay model

$$g_S \approx 1.51\ \mathrm{GeV}^{-1/2}$$

This estimation is model dependent, but will make sense at the order of magnitude.