Charge Symmetry Breaking in Hypernuclei

Avraham Gal, Hebrew University, Jerusalem

- **Evidence for CSB & recent exp progress on ($^{4}_{\Lambda}$H, $^{4}_{\Lambda}$He) levels from MAMI & J-PARC.**
- **Dalitz (1964): CSB is driven by Λ-Σ^0 mixing.**
- **Bodmer (1966)...Gibson...Akaishi (2000): $\Lambda(N)$-$\Sigma(N)$ SI coupling in s-shell hypernuclei.**
- **Nogga (2002): 1st realistic 4-body calculation.**
- **Millener (2005): apply $\Lambda\Sigma$ coupling in p-shell.**
- **Linking CSB $\Lambda\Sigma$ mixing to SI $\Lambda\Sigma$ coupling in the Akaishi-Millener schematic model. Results for A=4 & p-shell: A. Gal, PLB 744 (2015) 352.**

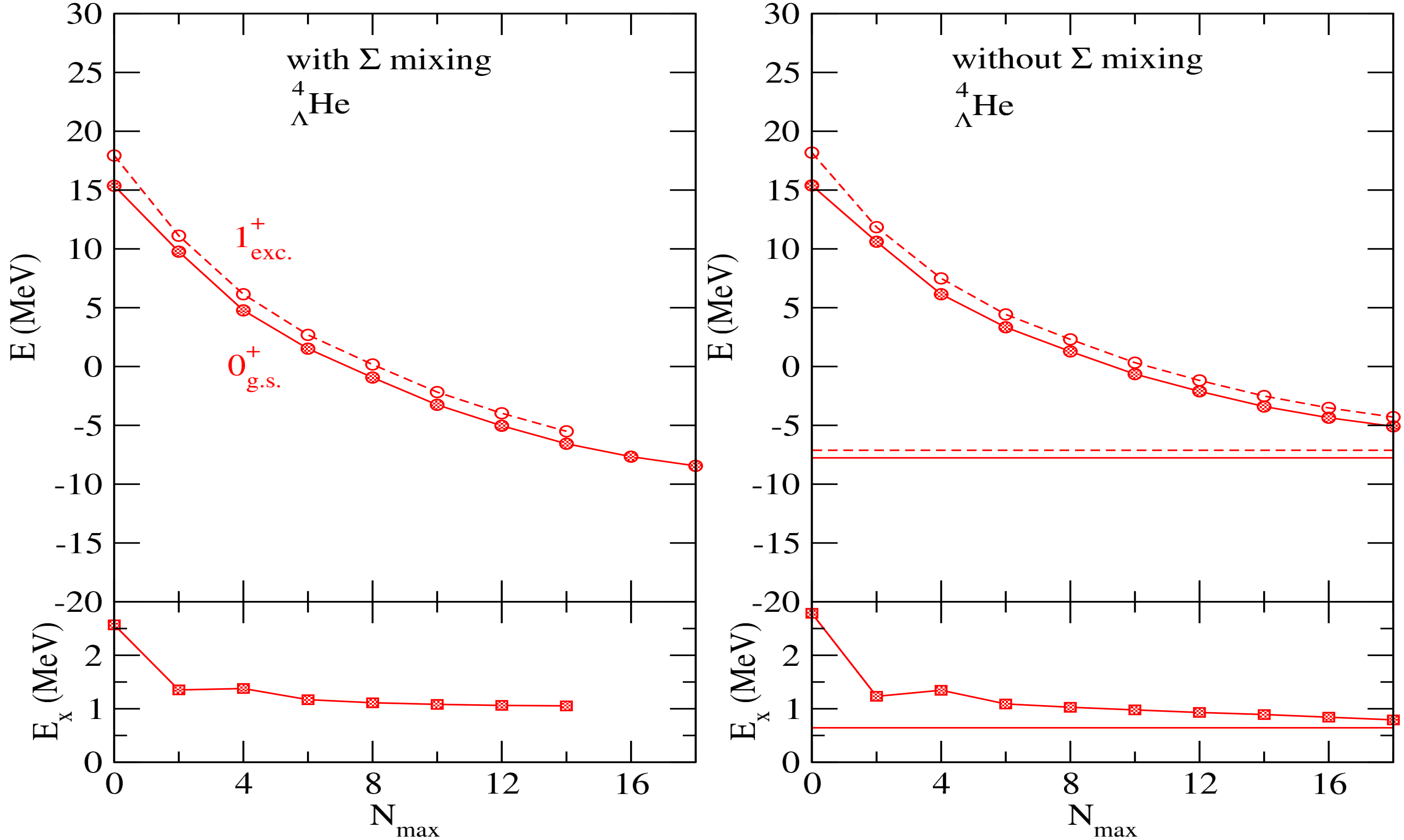
s-shell Λ hypernuclei (updated at HYP2015)

${}^{A}_{\Lambda}$Z	T	$J^{\pi}_{\rm g.s.}$	B_{Λ} (MeV)	$J^{\pi}_{\rm exc.}$	E$_x$ (MeV)
${}^{3}_{\Lambda}$H	0	$1/2^+$	0.13(5)		
${}^{4}_{\Lambda}$H–${}^{4}_{\Lambda}$He	1/2	0^+	2.04(4)–2.39(3)	1^+	1.09(2)–1.406(3)
${}^{5}_{\Lambda}$He	0	$1/2^+$	3.12(2)		

- **No Λnn bound state is expected.**
- **$\Delta B_{\Lambda}({}^{4}_{\Lambda}\mathrm{He}–{}^{4}_{\Lambda}\mathrm{H})$=0.35(5) MeV: very large CSB.**

Recent $A = 3, 4$ few-body calculations

- **A. Nogga, NPA 914 (2013) 140**
  Faddeev & Faddeev-Yakubovsky (chiral LO & NLO).
- **E. Hiyama et al., PRC 89 (2014) 061302(R)**
  Jacobi-coordinates Gaussian basis (Nijmegen soft-core).
- **R. Wirth et al., PRL 113 (2014) 192502.**
  ab-initio Jacobi-NCSM (chiral LO).



ΛN-ΣN ($\Lambda\Sigma$) coupling provides ≈1/3 of E_x

LO chiral potentials, Daniel Gazda (2014) at HYP2015

D. Gazda et al., FBS 55, 857; R. Wirth et al., PRL 113, 192502.

$\Lambda\Sigma$ coupling considerations

Schematic $\Lambda\Sigma$ coupling model

($1s_\Lambda \to 1s_\Sigma$ & same nucleon orbital wavefunction)

- $\Lambda\Sigma$ coupling: $\sqrt{4/3}\, t_N \cdot t_{\Lambda\Sigma}\, (V_{\Lambda\Sigma} + s_N \cdot s_Y\, \Delta_{\Lambda\Sigma})$
  leading to Fermi (F) & Gamow-Teller (GT) nuclear transition matrix elements.
- **The important $\Lambda\Sigma$ coupling matrix elements involve Σ and Λ hyperons coupled to the same nuclear core, and nuclear states connected by a large GT matrix element to the dominant core state.**
- **Sizable $\Lambda\Sigma$ matrix elements arise in realistic models,** see Millener, Lect. Notes. Phys. 724 (2007) 31.
- $V_{\Lambda\Sigma} = 2.96\ (3.35), \quad \Delta_{\Lambda\Sigma} = 5.09\ (5.76)\ \text{MeV}$,
  for s-shell baryons in simulated models NSC97e(f).

Schematic model A=4 matrix elements

$$|{}^4_\Lambda\mathrm{Z}(\mathrm{T}=1/2)\rangle = \alpha s^3 s_\Lambda + \beta s^3 s_\Sigma$$

$$V_{\Lambda\Sigma} = \sqrt{4/3}\,\vec{t}_N\cdot\vec{t}_{\Lambda\Sigma}\,(V_{\Lambda\Sigma} + \Delta_{\Lambda\Sigma}\,\vec{s}_N\cdot\vec{s}_Y)$$

$$v(0^+_{\mathrm{g.s.}}) = V_{\Lambda\Sigma} + \frac{3}{4}\Delta_{\Lambda\Sigma}, \quad v(1^+_{\mathrm{exc}}) = V_{\Lambda\Sigma} - \frac{1}{4}\Delta_{\Lambda\Sigma}$$

$$\delta E_\downarrow(J^+) = v^2(J^+)/(M_\Sigma - M_\Lambda)$$

NSC97	$V_{\Lambda\Sigma}$	$\Delta_{\Lambda\Sigma}$	$\delta E_\downarrow(0^+_{\mathrm{g.s.}})$	$\delta E_\downarrow(1^+_{\mathrm{exc}})$	$\Delta E_{\Lambda\Sigma}$	$\Delta E(0^+_{\mathrm{g.s.}} - 1^+_{\mathrm{exc}})$		
model	as calculated in models $(\Lambda\Sigma)_{\mathrm{e,f}}$					NKG	AHSM	NAS
$\mathrm{NSC97_e}$	2.96	5.09	0.574	0.036	0.539	0.75	0.89	1.13
$\mathrm{NSC97_f}$	3.35	5.76	0.735	0.046	0.689	1.10	1.48	1.51

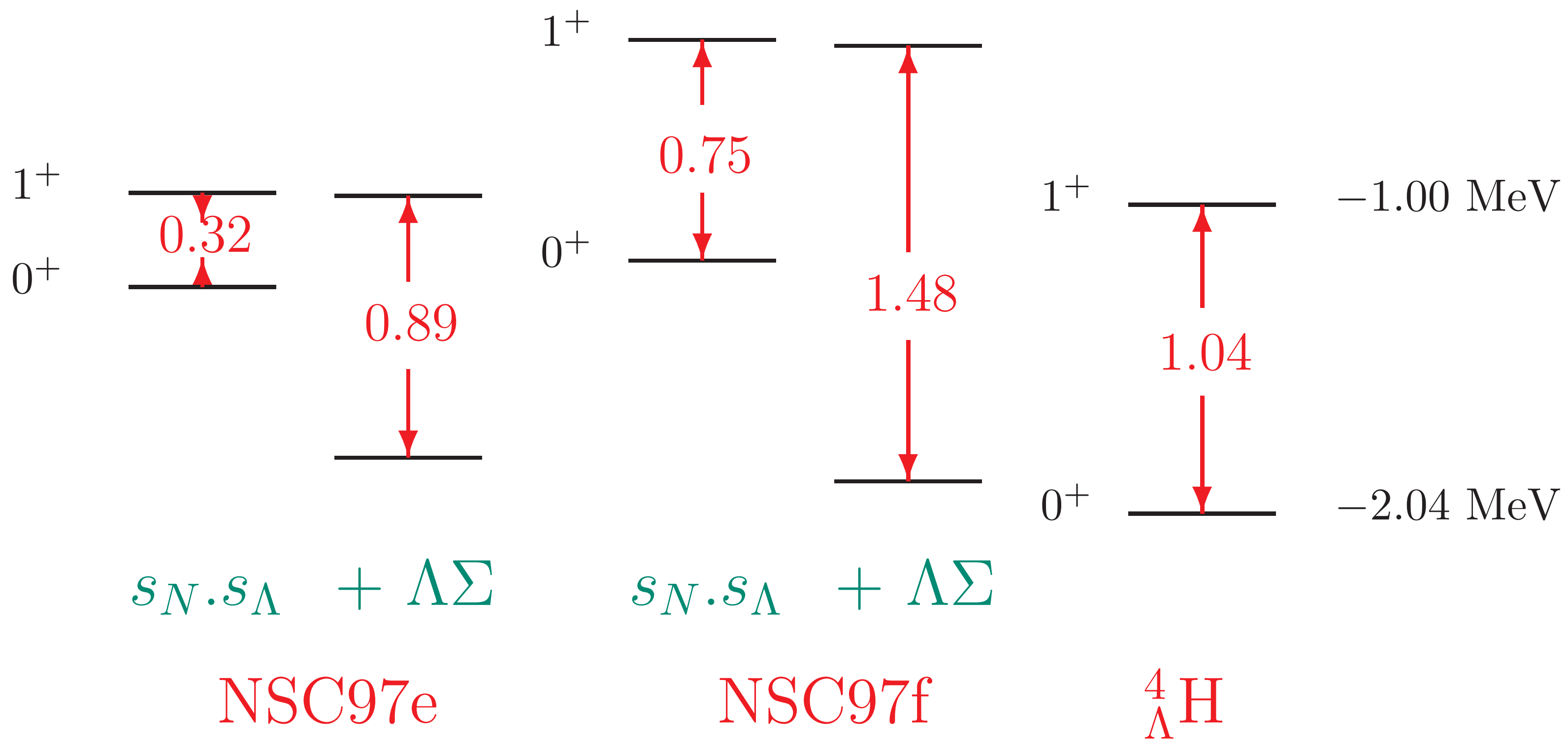
Nogga, Kamada, Gloeckle, PRL 88 (2002) 172501.

Akaishi, Harada, Shinmura, Myint, PRL 84 (2000) 3539.

Nemura, Akaishi, Suzuki, PRL 89 (2002) 142504.

ΛN-ΣN coupling in NSC97 models

Akaishi et al., PRL 84 (2000) 3539



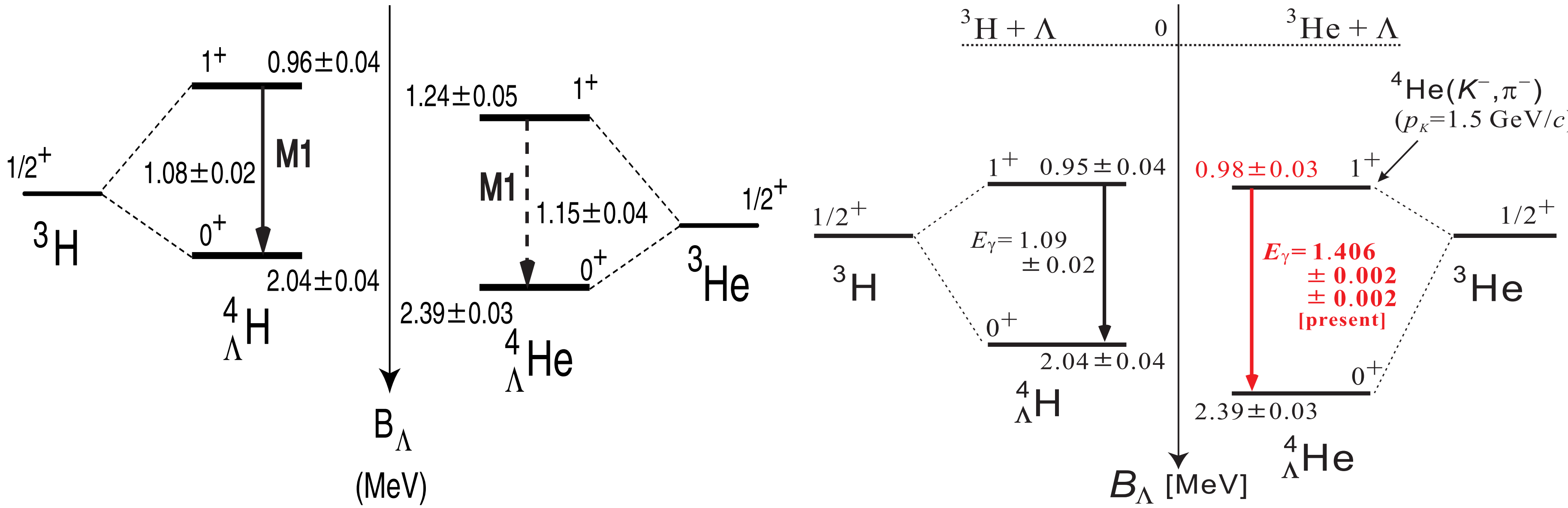
$\Lambda\Sigma$ coupling affects dominantly the 0^+ g.s.

Charge Symmetry Breaking

experiment, phenomenology & theory

${}^4_\Lambda$H–${}^4_\Lambda$He levels before and after J-PARC E13 exp.

T. O. Yamamoto et al., arXiv:1508.00376, submitted to PRL



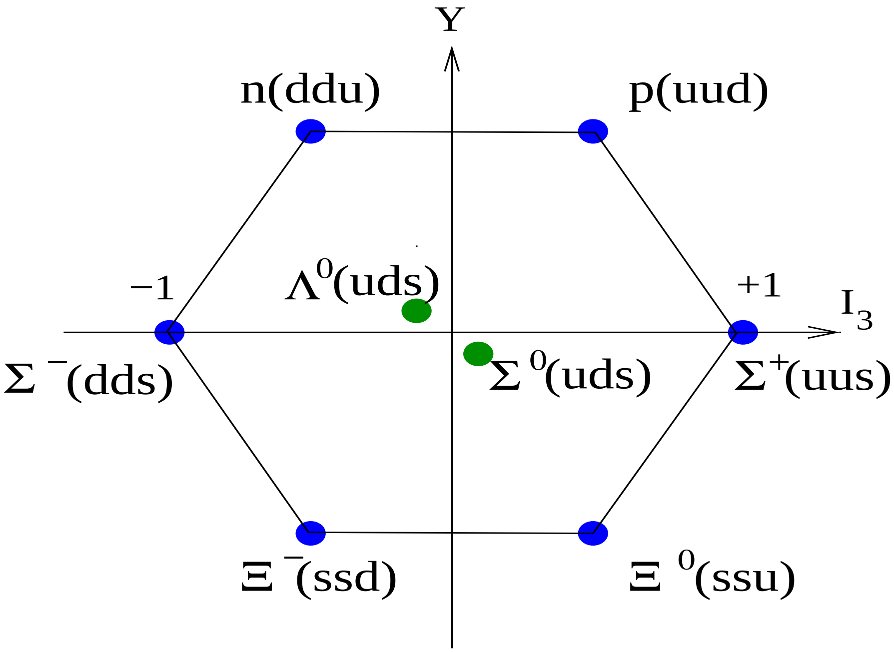
MAMI's new value $B_\Lambda({}^4_\Lambda\mathrm{H})$=2.12±0.01±0.09 MeV, PRL 114 (2015) 232501, consistent with emulsion data.

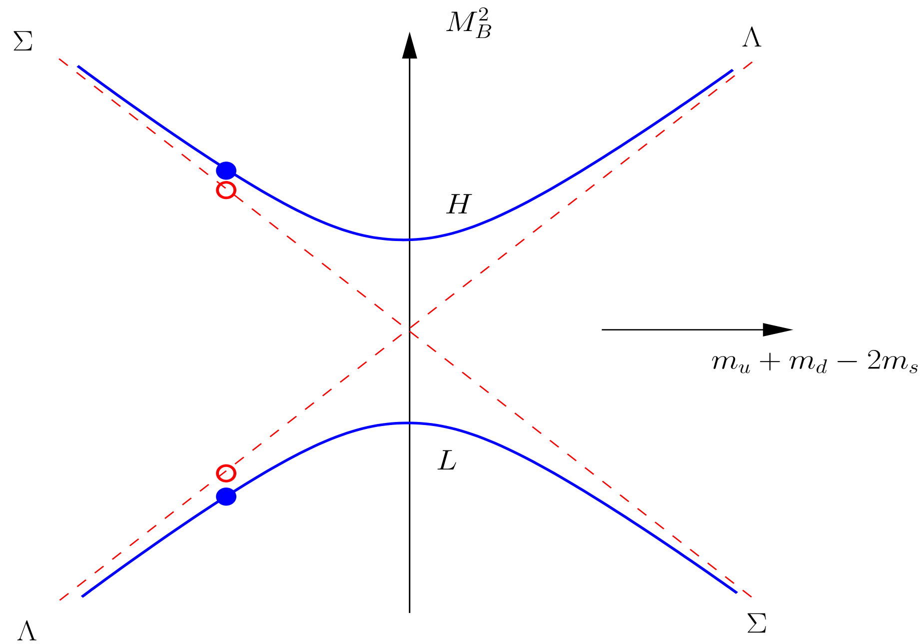
CSB is strongly spin dependent, dominantly in $0^+_{\mathrm{g.s.}}$

350±60 keV in ${}^4_\Lambda$H-${}^4_\Lambda$He vs. ≈−70 keV in ^{3}H-^{3}He.

Is Λ-Σ^0 CSB mixing correlated with ΛΣ SI coupling?

CSB main contribution: Λ-Σ^0 mixing





Dalitz–von Hippel, PL 10 (1964) 153:

$\langle \Sigma^0 | \delta M | \Lambda \rangle = \frac{1}{\sqrt{3}}[(M_{\Sigma^0} - M_{\Sigma^+}) - (M_n - M_p)]$**=1.14(05) MeV,**

and mixing angle $\tan\alpha = \langle \Sigma^0 | \delta M | \Lambda \rangle / (M_{\Sigma^0} - M_\Lambda)$**=0.015(1).**

LQCD gives half of it; see Gal, PRD 92 (2015) 018501.

- **$\Lambda - \Sigma^0$ mixing implies a nonzero $g_{\Lambda\Lambda\pi^0}$**

$$g_{\Lambda\Lambda\pi^0} = -2\,\frac{\langle\Sigma^0|\delta M|\Lambda\rangle}{M_{\Sigma^0}-M_\Lambda} g_{\Lambda\Sigma^0\pi^0} = -0.0297 g_{\Lambda\Sigma^0\pi^0},$$

leading to a ΛN OPE potential $V_{\Lambda N}^{\rm CSB}({\rm OPE})$

$$= -0.0242\,\tau_{Nz}\frac{f^2_{NN\pi}}{4\pi}\frac{m_\pi}{3}[\vec{\sigma}_\Lambda\cdot\vec{\sigma}_N + T(r)S(\hat{r};\,\vec{\sigma}_\Lambda,\vec{\sigma}_N)]Y(r),$$

giving $\geq$100 keV for the A=4 $0^+_{\rm g.s.}$ levels.

- **Other isovector meson exchanges, notably ρ exchange, augment the π's central contribution and cancel largely its tensor contribution.**

- **However, ΣNNN admixtures to ΛNNN require a dynamical treatment, perhaps abused by going too soon to use $V_{\Lambda N}^{\rm CSB}({\rm OPE})$. CSB and SI are linked for $I_{NY} = 1/2$ matrix elements by**

$$\langle N\Lambda|V_{\Lambda N}^{\rm CSB}|N\Lambda\rangle = -0.0297\,\tau_{Nz}\,\frac{1}{\sqrt{3}}\,\langle N\Sigma|V^{\rm SI}|N\Lambda\rangle.$$

CSB contributions (keV) in A=4 hypernuclei in several model calculations

${}^{4}_{\Lambda}$He–${}^{4}_{\Lambda}$H model	$P_{\Sigma}(\%)$ $0^+_{\rm g.s.}$	ΔT_{YN} $0^+_{\rm g.s.}$	ΔV_C $0^+_{\rm g.s.}$	ΔV_{YN} $0^+_{\rm g.s.}$	ΔB^4_{Λ} $0^+_{\rm g.s.}$	ΔB^4_{Λ} $1^+_{\rm exc}$
ΛNNN [CHCG]	–	–	−42	91	49	−61
NSC97$_{\rm e}$ [NKG]	1.6	47	−16	44	75	−10
NSC97$_{\rm f}$ [HMNP]	1.8				100	−10
NLO chiral [N]	2.1	55	−9	–	46	
$(\Lambda\Sigma)_{\rm e}$ [G]	0.72	39	−45	232	226	30
$(\Lambda\Sigma)_{\rm f}$ [G]	0.92	49	−46	263	266	39

Coon, Han, Carlson, Gibson, arXiv:nucl-th/9903034.

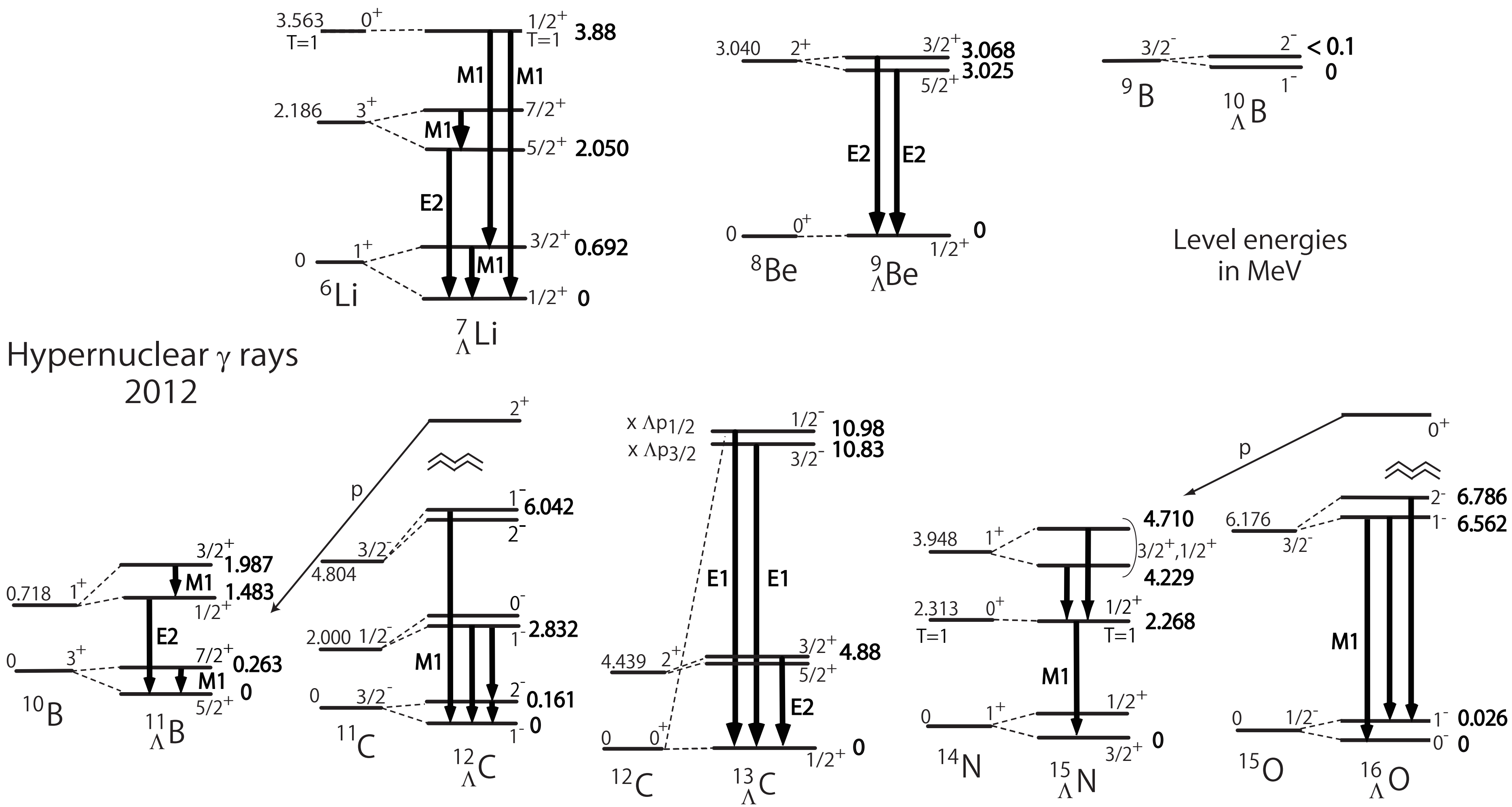
Nogga, Kamada, Gloeckle, PRL 88 (2002) 172501.

Haidenbauer, Meiβner, Nogga, Polinder, LNP 724 (2007) 113.

Nogga, NPA 914 (2013) 140. Gal, PLB 744 (2015) 352.

CSB(0^+)−CSB(1^+) $\approx$ 100 keV, 210 keV, 320 keV from γ rays.

$\Lambda\Sigma$ coupling & CSB in the p shell



Level schemes of Λ hypernuclei from γ-ray measurements

H. Tamura et al., Nucl. Phys. A 835 (2010) 3 [HYP09], updated at HYP12

Λ spin-orbit splitting: 150 keV in ${}^{13}_{\Lambda}$C & related 43 keV in ${}^{9}_{\Lambda}$Be

p-shell Λ hypernuclei

$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\, s_N \cdot s_\Lambda + V_{LS}(r)\, l_{N\Lambda} \cdot (s_\Lambda + s_N) + V_{ALS}(r)\, l_{N\Lambda} \cdot (s_\Lambda - s_N) + V_T(r)\, S_{12}$$

For $p_N s_Y$: $$V_{\Lambda N} = \bar{V} + \Delta\, s_N \cdot s_\Lambda + S_\Lambda\, l_N \cdot s_\Lambda + S_N\, l_N \cdot s_N + T\, S_{12}$$

R.H Dalitz, A. Gal, Ann. Phys. 116 (1978) 167

D.J. Millener, A. Gal, C.B. Dover, R.H. Dalitz, PRC 31 (1985) 499

$N\Lambda$-$N\Lambda$	$\bar{V}$	Δ	S_Λ	S_N	T	from
$A = 7-9$	(-1.32)	0.430	-0.015	-0.390	0.030	fit
$A = 11-16$	(-1.32)	0.330	-0.015	-0.350	0.024	fit
$N\Lambda$-$N\Sigma$	1.45	3.04	-0.085	-0.085	0.157	input

(in MeV) **D.J. Millener, Nucl. Phys. A 804 (2008) 84**

Doublet spacings in p-shell hypernuclei (in keV)

D.J. Millener, NPA 881 (2012) 298

	J_u^π	J_l^π	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE^{th}	ΔE^{exp}
${}^{7}_{\Lambda}$Li	$3/2^+$	$1/2^+$	72	628	−1	−4	−9	693	692
${}^{7}_{\Lambda}$Li	$7/2^+$	$5/2^+$	74	557	−32	−8	−71	494	471
${}^{8}_{\Lambda}$Li	2^-	1^-	151	396	−14	−16	−24	450	(442)
${}^{9}_{\Lambda}$Be	$3/2^+$	$5/2^+$	−8	−14	37	0	28	44	43
${}^{11}_{\Lambda}$B	$7/2^+$	$5/2^+$	56	339	−37	−10	−80	267	264
${}^{11}_{\Lambda}$B	$3/2^+$	$1/2^+$	61	424	−3	−44	−10	475	505
${}^{12}_{\Lambda}$C	2^-	1^-	61	175	−22	−13	−42	153	161
${}^{15}_{\Lambda}$N	$3/2^+_2$	$1/2^+_2$	65	451	−2	−16	−10	507	481
${}^{16}_{\Lambda}$O	1^-	0^-	−33	−123	−20	1	188	23	26
${}^{16}_{\Lambda}$O	2^-	1^-_2	92	207	−21	1	−41	248	224

$\Lambda\Sigma$ coupling contributions normally are below 100 keV

YN interaction contributions to g.s. binding energies

	${}^{7}_{\Lambda}$Li	${}^{8}_{\Lambda}$Li	${}^{9}_{\Lambda}$Li	${}^{10}_{\Lambda}$B	${}^{11}_{\Lambda}$B	${}^{11}_{\Lambda}$Be	${}^{12}_{\Lambda}$B	${}^{13}_{\Lambda}$C	${}^{15}_{\Lambda}$N	${}^{16}_{\Lambda}$N
keV	$1/2^+$	1^-	$3/2^+$	1^-	$5/2^+$	$1/2^+$	1^-	$1/2^+$	$3/2^+$	1^-
$\Lambda\Sigma$	78	160	183	35	66	99	103	28	59	62
Δ	419	288	350	125	203	2	108	-4	40	94
S_Λ	0	-6	-10	-13	-20	0	-14	0	12	6
S_N	94	192	434	386	652	540	704	841	630	349
T	-2	-9	-6	-15	-43	0	-29	-1	-69	-45
sum	589	625	952	518	858	641	869	864	726	412
Exp	5.58	6.80	8.50	8.89	10.24		11.37	11.69		13.76
MeV		6.84	8.29	9.11						
$\bar{V}$	-0.94	-1.02	-1.06	-1.05	-1.04		-1.05	-0.96		-0.93

$B_\Lambda^{\rm exp}({\rm g.s.}) = [B_\Lambda^{\rm exp}({}^{5}_{\Lambda}{\rm He}) = 3.12 \pm 0.02 \text{ MeV}] - (A-5)\bar{V} + \text{‘sum'}$

Improve fit by adding a spin-independent ΛNN term,

see Millener-Gal-Dover-Dalitz, PRC **31** (1985) 499

$\Lambda\Sigma$ matrix elements & contributions to $B_\Lambda^{\text{g.s.}}$ (in MeV) across the periodic table

A. Gal & D.J. Millener PLB 725 (2013) 445

$N-Z$	${}^A_\Lambda Z$	$V_{\Lambda\Sigma}$	$\Lambda\Sigma(V)$	$\Delta_{\Lambda\Sigma}$	$\Lambda\Sigma(\Delta)$	$\Delta B_\Lambda^{\text{g.s.}}(\Lambda\Sigma)$
4	${}^9_\Lambda$He	1.194	0.143	4.070	0.104	0.246
8	${}^{49}_\Lambda$Ca	0.175	0.010	0.946	0.014	0.024
22	${}^{209}_\Lambda$Pb	0.0788	0.052	0.132	0.001	0.053

- **$\Lambda\Sigma$ from Halderson, following NSC97f.**
- **$V_{\Lambda\Sigma}$ & $\Delta_{\Lambda\Sigma}$ decrease drastically as overlap between 0s hyperon and high-ℓ excess neutrons becomes poorer with A ($0f_{7/2}$ in ${}^{49}_\Lambda$Ca, $0h_{9/2}$ & $0i_{13/2}$ in ${}^{209}_\Lambda$Pb).**
- **Conclusion: $\Lambda\Sigma$ contributes less than 100 keV to binding of medium & heavy n-rich hypernuclei.**

p-shell CSB contributions in $(\Lambda\Sigma)_{\rm e}$ coupling model

${}^{A}_{\Lambda}$Z$_{>}$–${}^{A}_{\Lambda}$Z$_{<}$ pairs	I_C, J^{π}_C	P_{Σ} (%)	ΔT_{YN} (keV)	ΔV_C (keV)	ΔV_{YN} (keV)	$\Delta B^{\rm calc}_{\Lambda}$ (keV)	$\Delta B^{\rm exp}_{\Lambda}$ [D] (keV)
${}^{4}_{\Lambda}$He–${}^{4}_{\Lambda}$H	$\frac{1}{2}, 0^+$	0.72	39	−45	232	226	+350±60
${}^{7}_{\Lambda}$Be–${}^{7}_{\Lambda}$Li*	$1, \frac{1}{2}^+$	0.12	3	−70 [HYMK]	50	−17	−100±90
${}^{8}_{\Lambda}$Be–${}^{8}_{\Lambda}$Li	$\frac{1}{2}, 1^-$	0.20	11	−81 [HKMYY]	119	+49	+40±60
${}^{9}_{\Lambda}$B–${}^{9}_{\Lambda}$Li	$1, \frac{3}{2}^+$	0.23	10	−145 [M]	81	−54	−210±220
${}^{10}_{\Lambda}$B–${}^{10}_{\Lambda}$Be	$\frac{1}{2}, 1^-$	0.053	3	−156 [M]	17	−136	−220±250

Davis, NPA 754 (2005) 3c. Millener, unpublished (2015).

Hiyama Yamamoto Motoba Kamimura PRC 80 (2009) 054321.

Hiyama Kamimura Motoba Yamada Yamamoto, PRC 66 (2002) 024007.

Negligible ΔT_{YN} no longer cancels ΔV_C. ΔV_{YN} is smaller than in s shell and, except for A=8, is dominated by the larger-size negative ΔV_C, so CSB becomes negative in the p shell as suggested by the data. However, $B_{\Lambda}({}^{7}_{\Lambda}\text{He})$ is missed by 1-2 σ.

g.s. doublet splittings (keV) in several p-shell hypernuclei

${}^{A}_{\Lambda}$Z	$J^{\pi}_{\rm exc}$	$J^{\pi}_{\rm g.s.}$	$\Delta E^{\rm exp}$	$\Delta E^{\rm CS}$ [M]	$\Delta E^{\rm CS}_{\Lambda\Sigma}$ [M]	$\Delta E^{\rm CSB}_{\Lambda\Sigma}$
${}^{8}_{\Lambda}$Li	2^-	1^-	442±2 [C]	445	149	−53.2
${}^{9}_{\Lambda}$Li	$\frac{5}{2}^+$	$\frac{3}{2}^+$	570±120 [U]	590	116	+12.5
${}^{10}_{\Lambda}$B	2^-	1^-	<100 [C,T]	110	−10	−26.5

Millener, NPA 881 (2012) 298. Chrien et al., PRC 41 (1990) 1062.

Urciuoli et al. (JLab Hall A), PRC 91 (2015) 034308.

Tamura et al., NPA 754 (2005) 58c.

- **CSB contributions to doublet splittings are not negligible on the scale of shell-model CS fits to γ ray spectra.**
- **$\Delta E^{\rm CS}({}^{10}_{\Lambda}{\rm B})$ gets reduced significantly by CSB, so that the $2^-_{\rm exc}$ level most likely decays weakly, thereby explaining why experiments [C,T] failed to observe a $2^-_{\rm exc} \to 1^-_{\rm g.s.}$ γ ray.**

Summary and Outlook

- **CSB is particularly strong in Λ hypernuclei owing to no OPE in a single-channel ΛN SI description. $\Lambda\Sigma$ coupling brings in OPE and relates CSB to SI.**

- **The A=4 hypernuclei, with two bound states each, offer a good test-ground for CSB, highlighted by the γ rays constraint CSB(0^+)−CSB(1^+)≈320 keV.**

- **CSB(0^+)−CSB(1^+)≃100 keV for SI-acceptable NSC97. CSB(0^+)−CSB(1^+)≈300 keV for EFT(LO) with cutoff 600 MeV/c, but the cutoff dependence is appreciable (Gazda-Gal, work in progress).**

- **The Akaishi-Millener $\Lambda\Sigma$ schematic model produces CSB(0^+)$-$CSB(1^+)$\approx$200 keV. Can it be readjusted to get the necessary $\approx$300 keV?**
- **The Akaishi-Millener $\Lambda\Sigma$ schematic model explains the CSB sign reversal in the p shell. Its magnitude is poorly constrained by the old emulsion values known for a few p-shell isomultiplets. Can any of these be remeasured systematically in a modern facility?**
- **The recently determined $B_\Lambda(^{7}_{\Lambda}\mathrm{He})$ cannot be related by CSB to the emulsion-based B_Λ values of $^{7}_{\Lambda}\mathrm{Li}$ and $^{7}_{\Lambda}\mathrm{Be}$.**

- **Special thanks to Daniel Gazda and John Millener for their contributions to this work.**