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Pole of the S-matrix of the $^{4}_{\Sigma}$He hypernucleus on Riemann sheets

T. Harada, Y. Hirabayashi†

Osaka Electro-Communication University, Neyaawa 572-8530, Japan
† *Information Initiative Center, Hokkaido University, Sapporo 060-0811, Japan*

Outline

1. Introduction
2. Properties of Σ-nucleus potentials
3. Calculations Pole search of the *Y-3N* system on Riemann sheets
4. Results and discussion

   S-wave Σ ground state: ${}_{\Sigma}^{4}$He (0^+)

   P-wave Σ excited state: ${}_{\Sigma}^{4}$He (1^-)

   in ^{4}He(K^-,π^-) reactions at 1.5 GeV/c
5. Summary

Our understanding for Σ hypernuclei

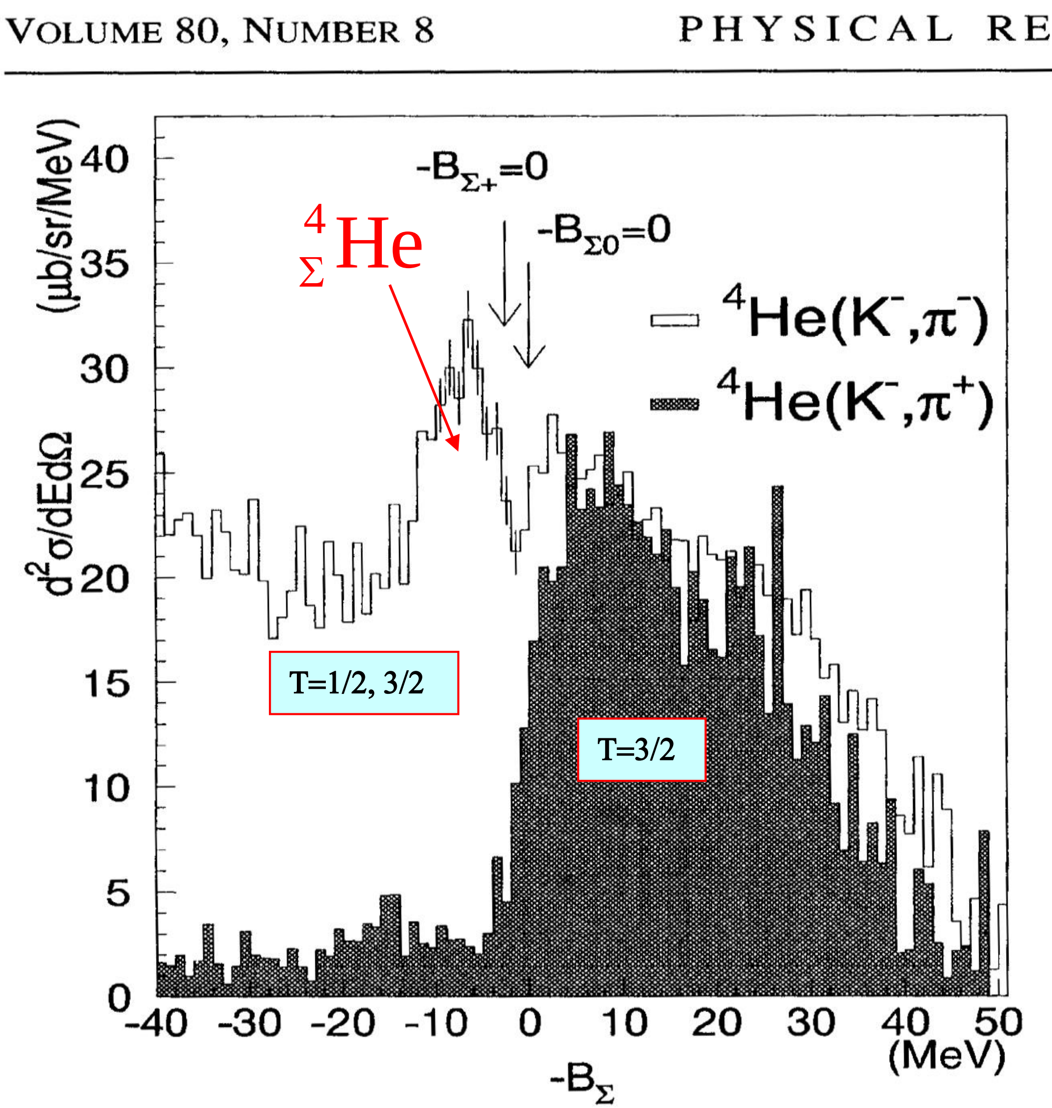
- There is a Σ hypernucler state.
  - $\Sigma N(I = 1/2, {}^3S_1)$ may be a threshold cusp.
    → A strange partner of the deuteron
  - A bound state of ${}_{\Sigma}^{4}He$ with $T = 1/2$, $J^{\pi} = 0^+$ is established.
    → A strange partner of the α particle

- There is a strong isospin dependence of Σ-nucleus potentials in light nuclei, so that no narrow bound state from ${}^6Li$ and ${}^9Be$ targets (A > 4).

- Σ-nucleus potentials may be repulsive.
  - Repulsion inside the nuclear surface and an attraction outside the nucleus.
  - Consistent with (π^-, K^+) spectra and Σ^- atomic x-ray data.

Observation of a ${}^{4}_{\Sigma}$He Bound State at BNL(1995)

Phys. Rev. Lett. 80 (1998) 1605.



T. Nagae, T. Miyachi, T. Fukuda, H.Outa, T. Tamagawa, J. Nakano, R.S.Hayano, H. Tamura, Y. Shimizu, K. Kubota, R. E. Chrien, R. Sutter, A. Rusek, W. J. Briscoe, R. Sawafta, E.V. Hungerford, A. Empl, W. Naing. C. Neerman, K. Johnston, M. Planinic

$$B_{\Sigma^+} = 4.4 \pm 0.3 \text{ MeV}$$

$$\Gamma = 7 \pm 0.7 \text{ MeV}$$

4.6 MeV

7.9 MeV

Theoretical Prediction

$$T \simeq 1/2$$

$$J^{\pi} = 0^{+}$$

T.Harada, S.Shinmura, Y.Akaishi, H.Tanaka, NPA507(1990)715.

Our understanding for Σ hypernuclei

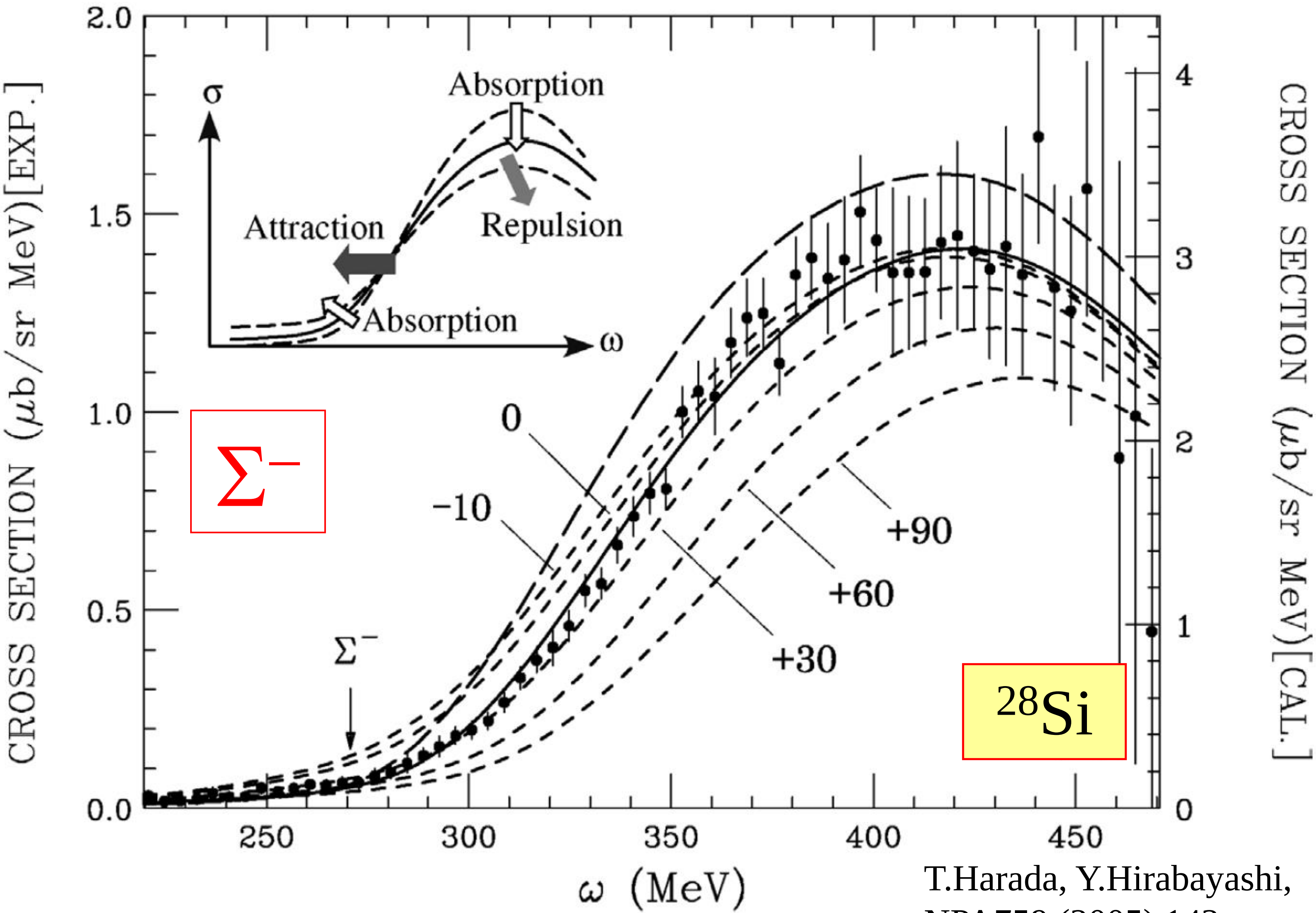
- There is a Σ hypernucler state.
  - $\Sigma N(I=1/2,\, ^3S_1)$ may be a threshold cusp.
    → A strange partner of the deuteron
  - A bound state of ${}_{\Sigma}^{4}He$ with $T=1/2$, $J^{\pi}=0^{+}$ is established.
    → A strange partner of the α particle

- There is a strong isospin dependence of Σ-nucleus potentials in light nuclei, but no narrow bound states are in the reactions from ^6Li and ^9Be targets (A > 4).

- Σ-nucleus potentials may be repulsive in heavier nuclei.
  - Repulsion (caused by $T=3/2$, $^3S_1$) inside the nuclear surface and an attraction outside the nucleus.
  - Consistent with (π^{-},K^{+}) spectra and Σ^{-} atomic x-ray data.

Inclusive spectrum in ^{28}Si(π^-, K$^+$) reaction at 1.2GeV/c

Exp. Data from P.K.Saha, H. Noumi, et al., PRC70(2004)044613



T.Harada, Y.Hirabayashi, NPA759 (2005) 143

- Recently, we demonstrated the inclusive and Σ-Λ conversion spectra in the ^{4}He(K^-,π^-) reaction at $p_{\pi-}$ =1.5GeV/c at J-PARC in order to obtain evidence for *P*-wave Σ resonant states of the $_\Sigma{}^4$He hypernucleus.

[T. Harada, Y. Hirabayashi,PLB740 (2015) 312.]

In this talk,

- We focus on a pole position of the *S* matrix for a $_\Sigma{}^4$He hypernucleus on the Riemann sheets in the complex *E* plane in order to see the structure of *S*- and *P*-wave states in Σ-hypernuclei.

$_\Sigma{}^4$He

This state is identified as a *S*-wave Σ quasibound (or unstable bound) state with $J = 0+$, $T \simeq 1/2$.
How about *P*-waves ? (no information)

Why do we study Σ hypernuclear states ?

- The advantage of Σ hypernuclear study in the (K^-, π^-) reaction at 1.5GeV/c in the J-PARC facilities

i. Large cross sections for Σ production

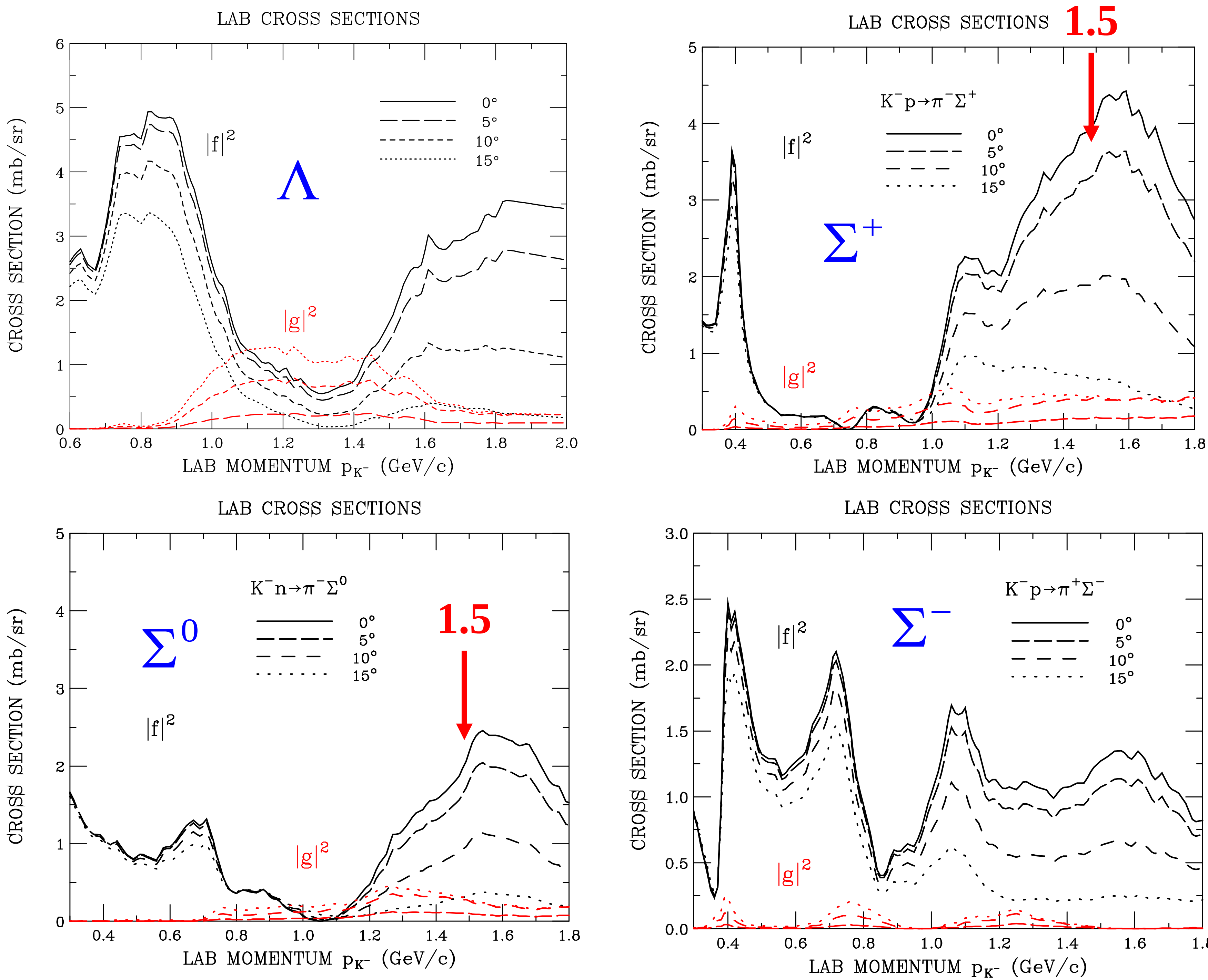
- $d\sigma(\Sigma^+)$ at 1.5GeV/c $\cong 10 \times d\sigma(\Sigma^+)$ at 0.6GeV/c

ii. Momentum transfer that we controlled

- depending on angular distributions
- *P*-wave Σ states are rather excited.

iii. Strange partner of the α particle

- Nuclear SU(4) supermultiplet: $4 \otimes 4 = 1 \oplus 15$
- …



LAB CROSS SECTIONS
CROSS SECTION (mb/sr)
LAB MOMENTUM p_{K^-} (GeV/c)
0°
5°
10°
15°
$|f|^2$
$|g|^2$
Λ
LAB CROSS SECTIONS
1.5
$K^-p \to \pi^-\Sigma^+$
Σ^+
LAB CROSS SECTIONS
$K^-n \to \pi^-\Sigma^0$
Σ^0
1.5
LAB CROSS SECTIONS
$K^-p \to \pi^+\Sigma^-$
Σ^-

Why do we study Σ hypernuclear states ?

- The advantage of Σ hypernuclear study in the (K^-, π^-) reaction at 1.5GeV/c in the J-PARC facilities

i. Large cross sections for Σ production

- $d\sigma/d\Omega(1.5\mathrm{GeV/c}) \cong 10 \times d\sigma/d\Omega(0.6\mathrm{GeV/c})$

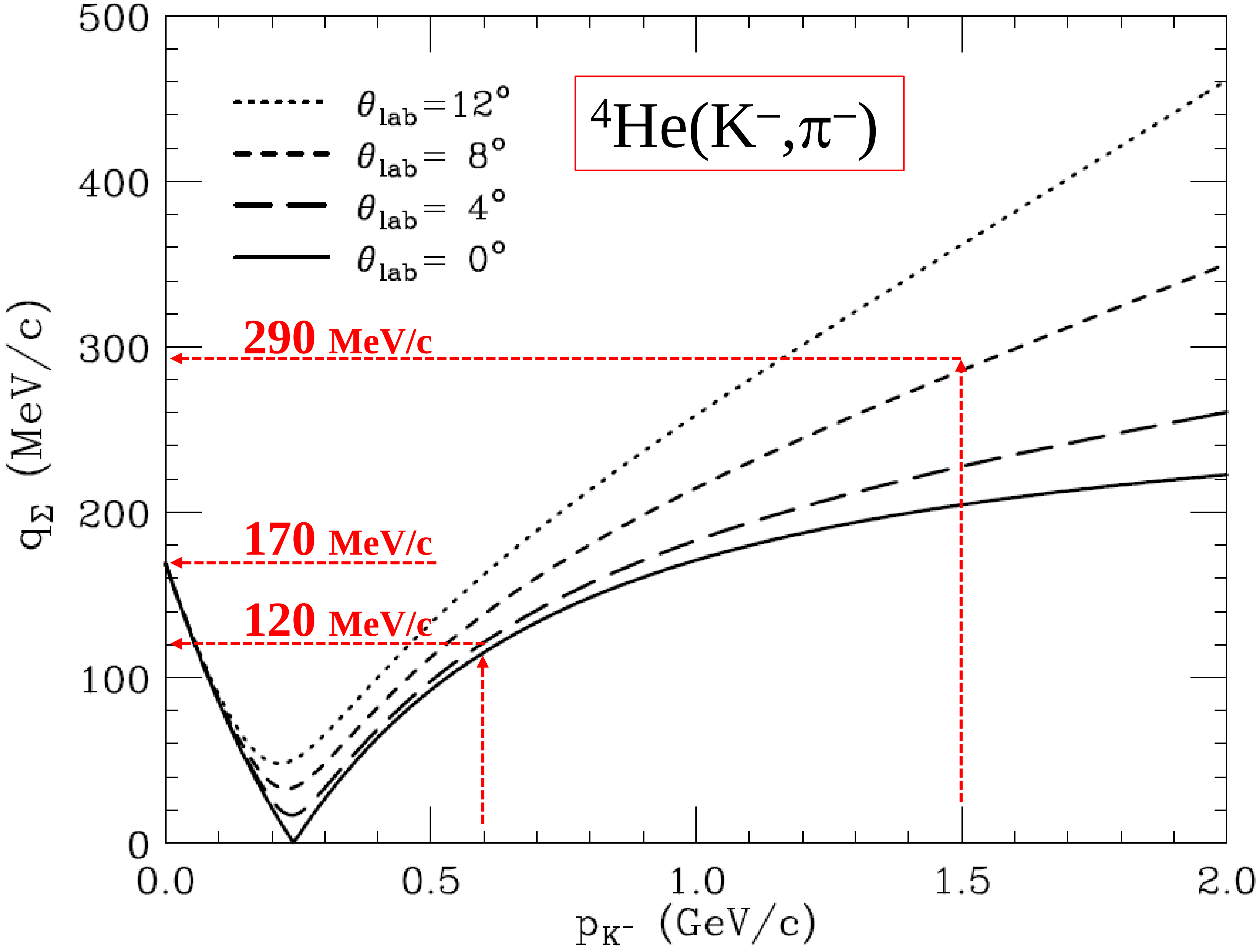
ii. Momentum transfer that we controlled

- depending on angular distributions
- *P*-wave Σ states are rather excited.

iii. Strange partner of the α particle

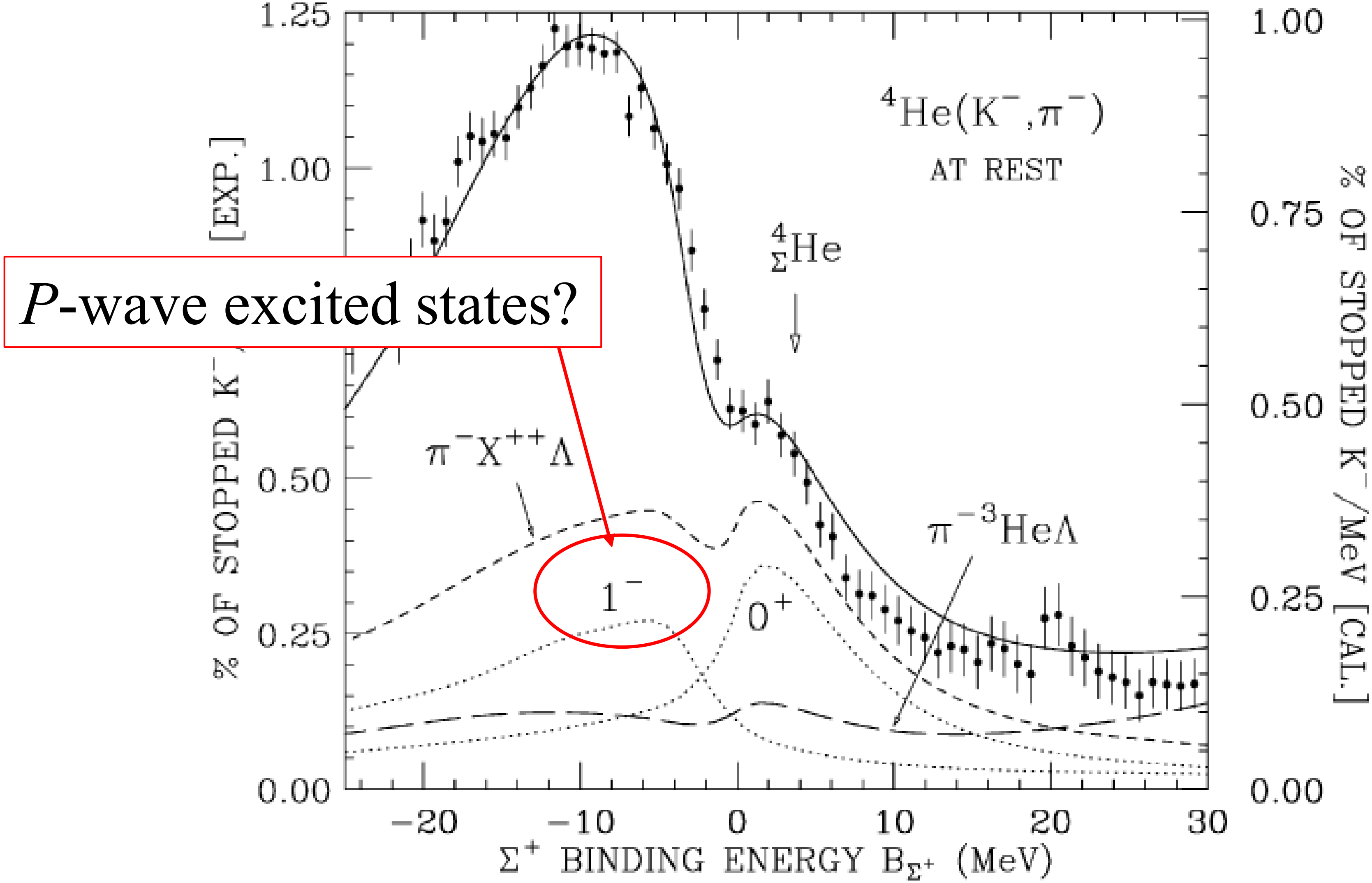
- Nuclear SU(4) supermultiplet: $4 \otimes 4 = 1 \oplus 15$
- …

Momentum transfer q_{Σ} for Σ production



Is there P-wave Σ excited states ?

^{4}He(stopped K$^-$, π$^-$) reactions



T.Harada, NPA672(2000)181

Properties of Σ-nucleus potentials

Remarks

- Properties of the Σ-nucleus potentials

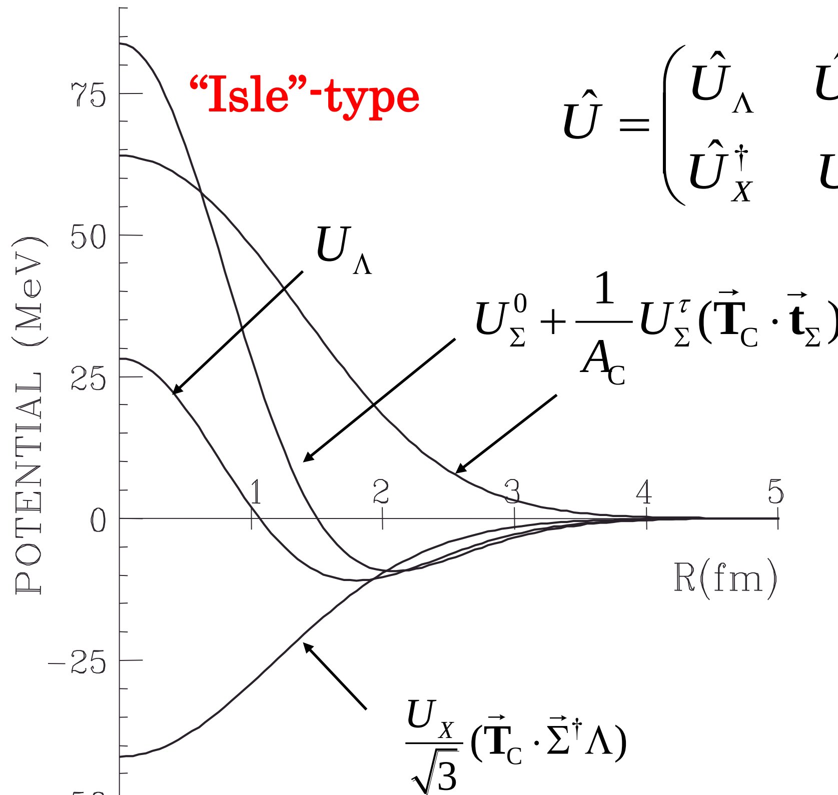
$$U_\Sigma(\boldsymbol{r}) = U^0_\Sigma(\boldsymbol{r}) + \frac{1}{A_{\rm core}} U^\tau_\Sigma(\boldsymbol{r})(\vec{\boldsymbol{T}}_{\rm core}\cdot\vec{\boldsymbol{t}}_\Sigma)$$

"repulsion inside the nuclear surface"

"shallow attraction outside the nucleus"

"strong isospin-dependence"

$$\hat{U} = \begin{pmatrix} \hat{U}_\Lambda & \hat{U}_X \\ \hat{U}_X^\dagger & \hat{U}_\Sigma \end{pmatrix}$$

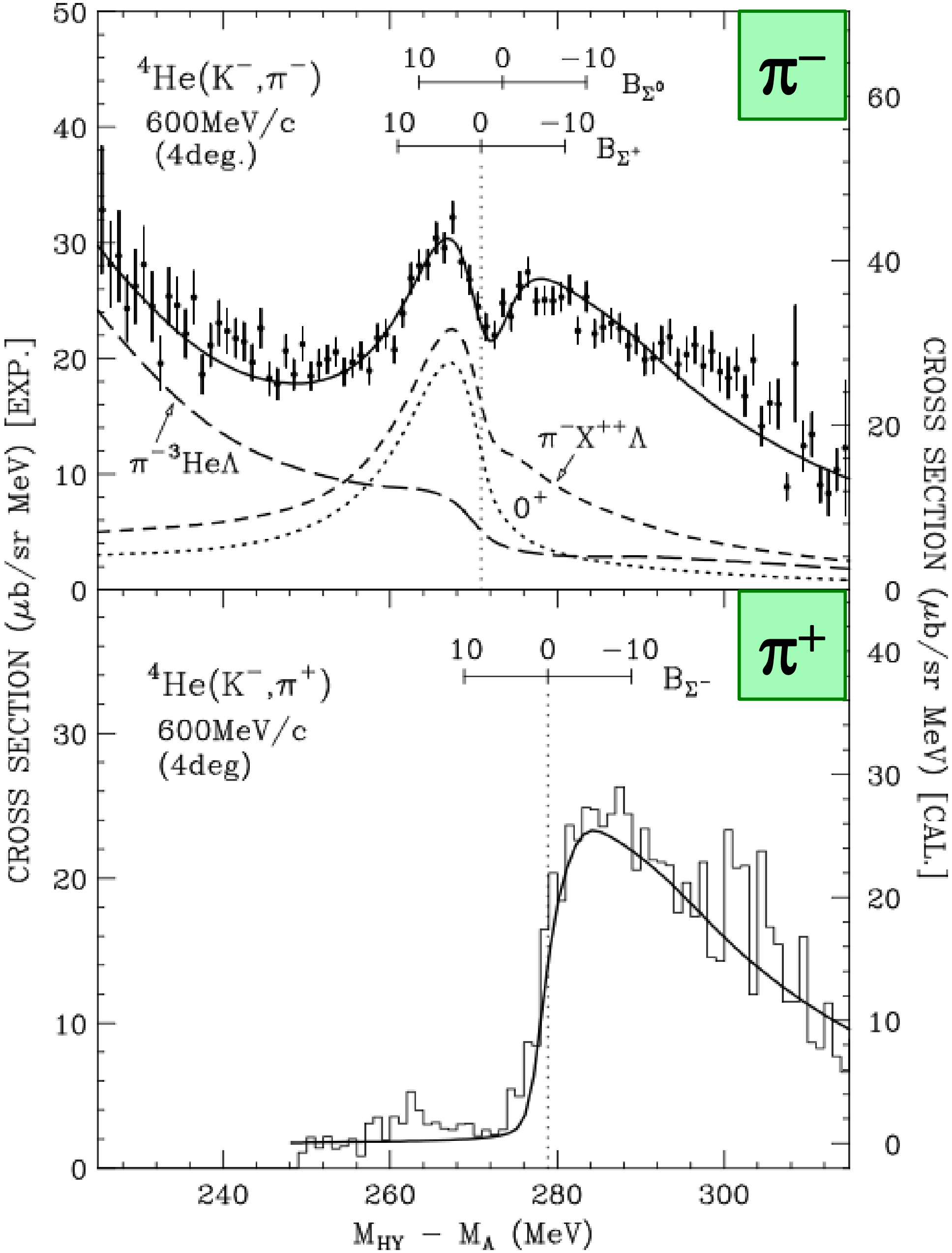


Σ-3N potential:
the ${}_\Sigma^4$He bound state
with T=1/2, $J^\pi=0^+$

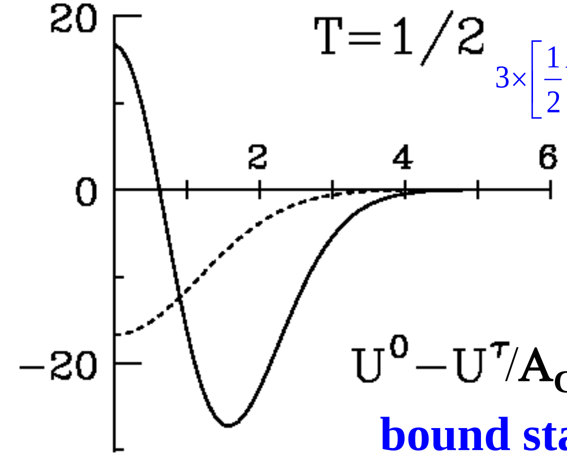
Strong Lane (isospin-dependent) potential
and Coherent Λ-Σ coupling

Isospin dependence of the *3N*-Σ potentials

T.Harada, PRL81(1998)5287.



T.Harada, NPA507 (1990) 715.

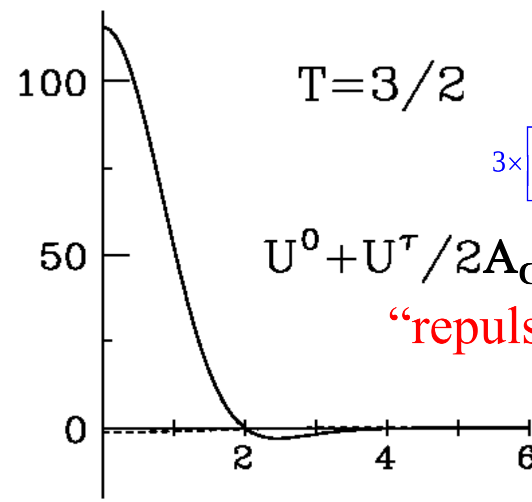


$3\times\left[\frac{1}{2}V(10^*)+\frac{1}{2}V(27)\right]$

bound state !!

"repulsive core

+attractive pocket"



$3\times\left[\frac{1}{2}V(10)+\frac{1}{2}V(27)\right]$

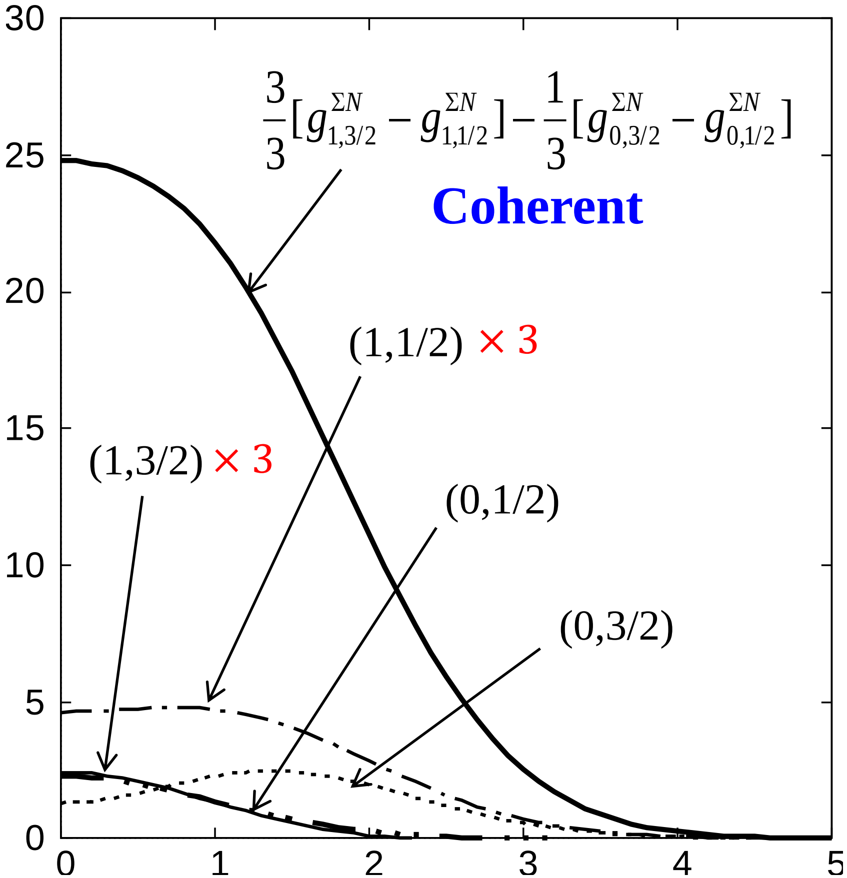
"repulsive"

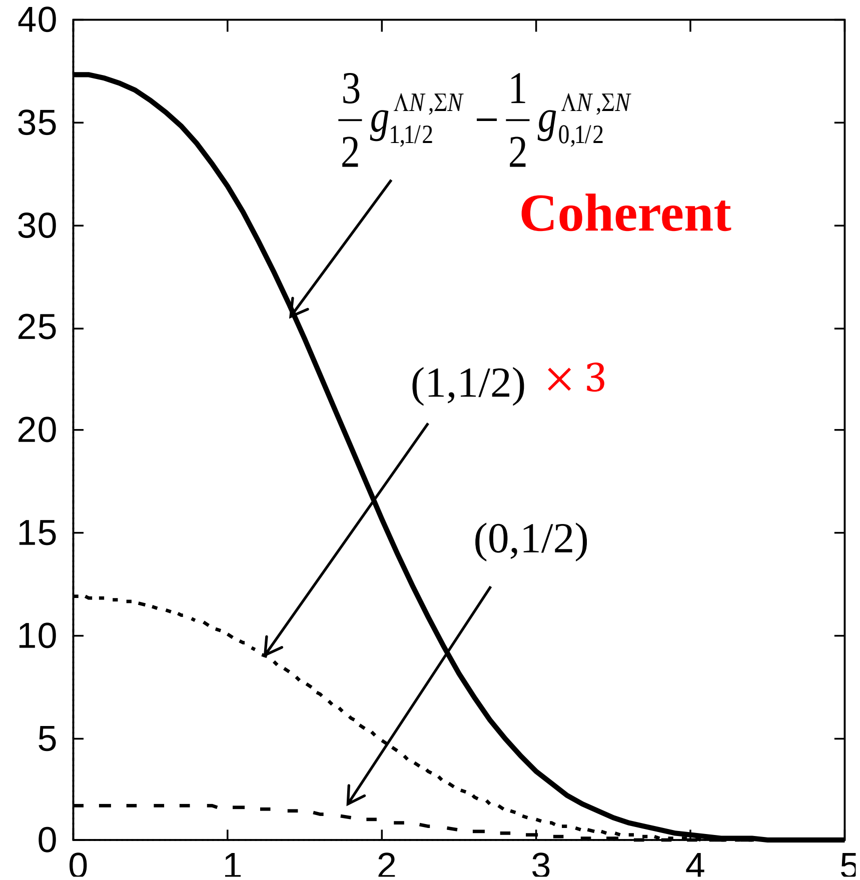
Lane and Λ-Σ coupling terms in the *3N-Y* potential

Microscopic folding model based by g-matrix D2'g: $g^{\Sigma N}_{S,T}$ $g^{\Lambda N,\Sigma N}_{S,1/2}$

Lane term $\hat{U}^{\tau}_{\Sigma}/A_{\text{core}}$

Λ-Σ coupling $\hat{U}_X$





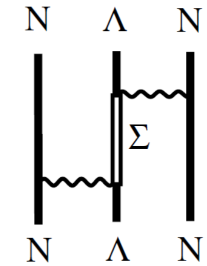
→ Strong isospin-dependence

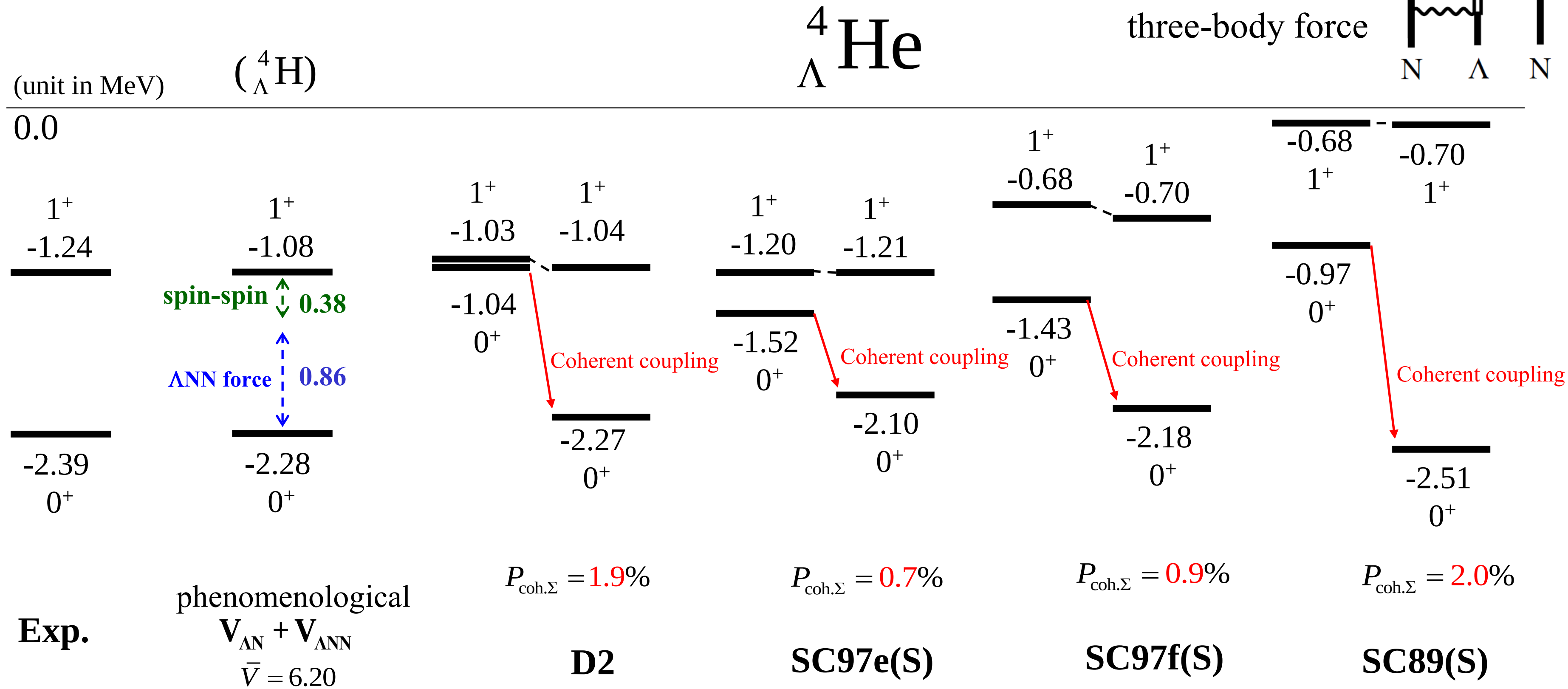
→ Coherent Λ-Σ mixing

Coherent Λ-Σ coupling in s-shell Λ hypernuclei

"The 0^+-1^+ difference is not a measure of ΛN spin-spin interaction."
by B.F. Gibson

ΛNN
three-body force





VMC

R. Sinha, Q.N.Usmani, NPA684(2001)586c

Breuckner-Hartree-Fock

Y. Akaishi, T.Harada, S.Shinmura, Khun Swe Myint, PRL84(2000)3539

Calculations

for pole search on multichannel Riemann sheets

Solving the multichannel Lippmann-Schwinger equation

Lippmann-Schwinger equation

$$|\Psi^{(+)}\rangle = |\phi_{\boldsymbol{k}}\rangle + \frac{1}{E-H_0+i\varepsilon}U|\Psi^{(+)}\rangle$$

Partial wave expansion

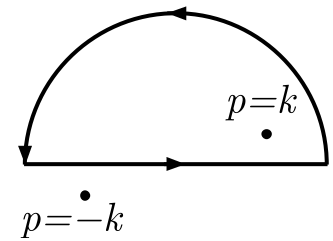
$$R^{\ell}_{\beta\alpha}(k_\beta, r) = k_\alpha r j_\ell(k_\alpha r)\delta_{\beta\alpha} + \sum_{\gamma}\int_0^\infty dr' g^{(+)}_{\beta,\ell}(k_\beta; r, r')U_{\beta\gamma}(r')R^{\ell}_{\gamma\alpha}(k_\gamma, r')$$

Green's function for β-channel with boundary conditions

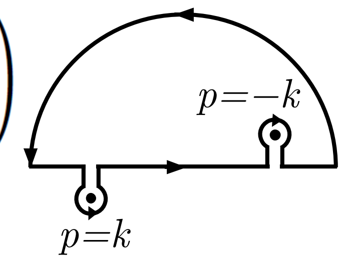
Pearce, Gibson, PRC40(1989)902

$$g^{(+)}_{\beta,\ell}(k_\beta; r, r') = \begin{cases} \dfrac{2\mu_\beta}{\hbar^2}\dfrac{2}{\pi}rr'\displaystyle\int_0^\infty \frac{p^2 j_\ell(pr)j_\ell(pr')}{k_\beta^2-p^2+i\varepsilon}dp \\ \dfrac{2\mu_\beta}{\hbar^2}\dfrac{2}{\pi}rr'\left(\displaystyle\int_0^\infty \frac{p^2 j_\ell(pr)j_\ell(pr')}{k_\beta^2-p^2+i\varepsilon}dp - 2\pi i \mathrm{Res}|_{p=-k}\right) \end{cases}$$

(+) sheet



(–) sheet



Multichannel T matrix (or S matrix)

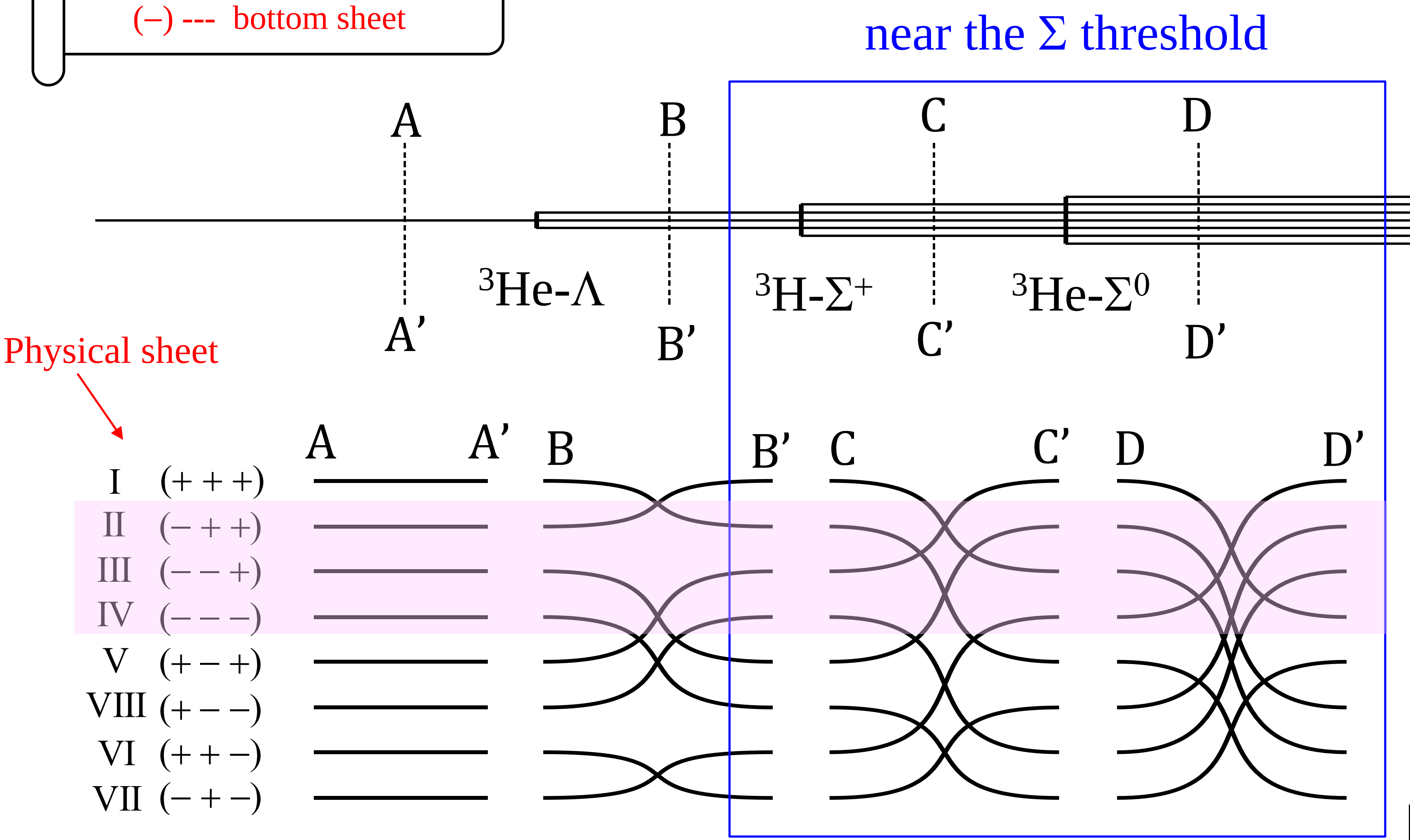
$$T^{\ell}_{\beta\alpha}(E) = -\frac{2\mu_\beta}{\hbar^2}\sum_{\gamma}\int_0^\infty r'^2 dr' j_\ell(k_\beta r')U_{\beta\gamma}(r')\frac{R^{\ell}_{\beta\alpha}(k_\gamma, r')}{k_\gamma r'}$$

Interconnection of the sheets on crossing the real *E* axis

Badalyan, Kok, Polikarpov, Simonov,
Phys. Rep. 82(1982)31

$(\mathrm{Im}\, p_{\Lambda}, \mathrm{Im}\, p_{\Sigma^+}, \mathrm{Im}\, p_{\Sigma^0})$

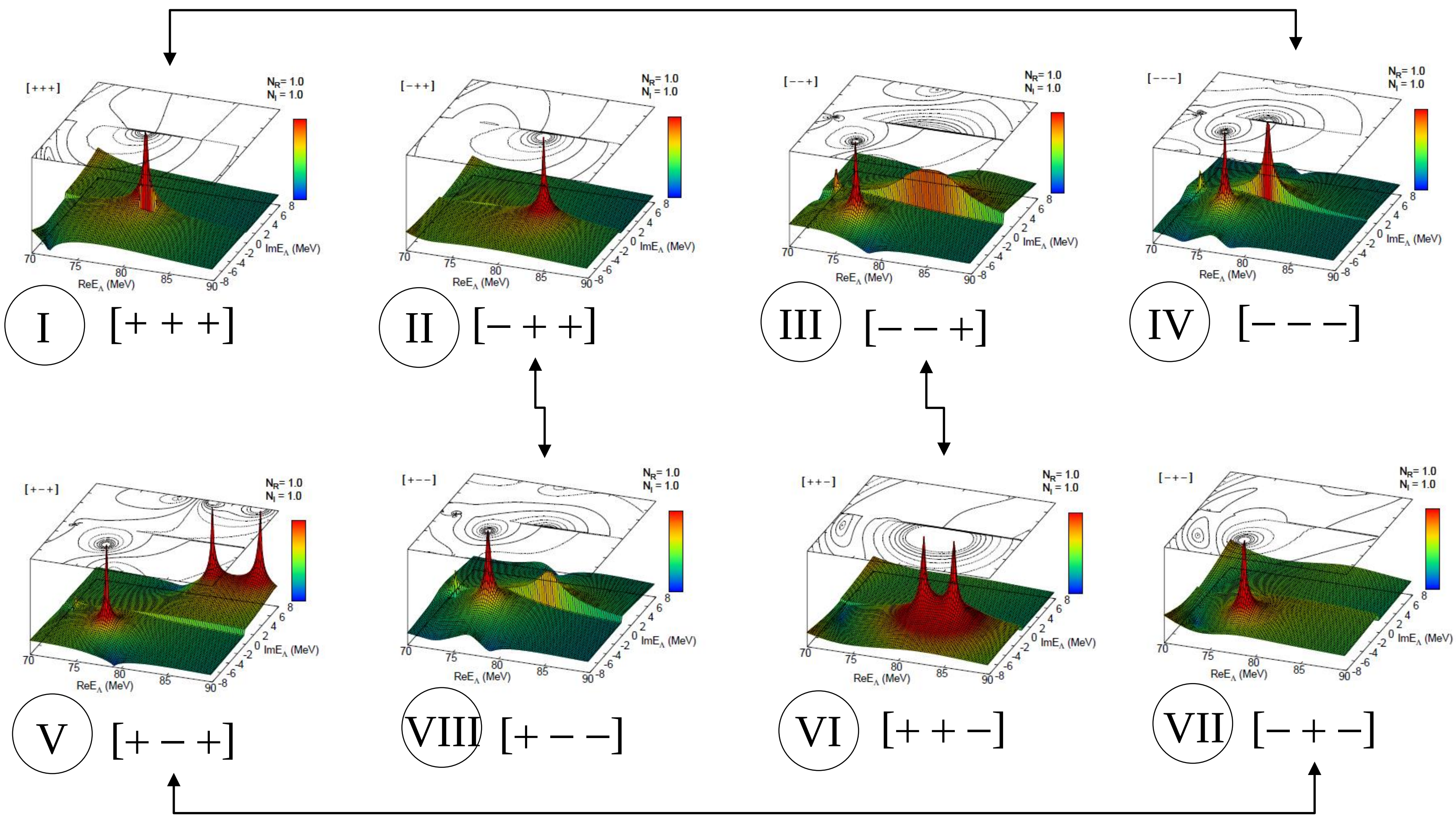
(+) --- top sheet
(−) --- bottom sheet



Poles of the S matrix of $_{\Sigma}^{4}$He on Riemann Sheets

$J^{\pi}=1^{-}$ (p-wave)

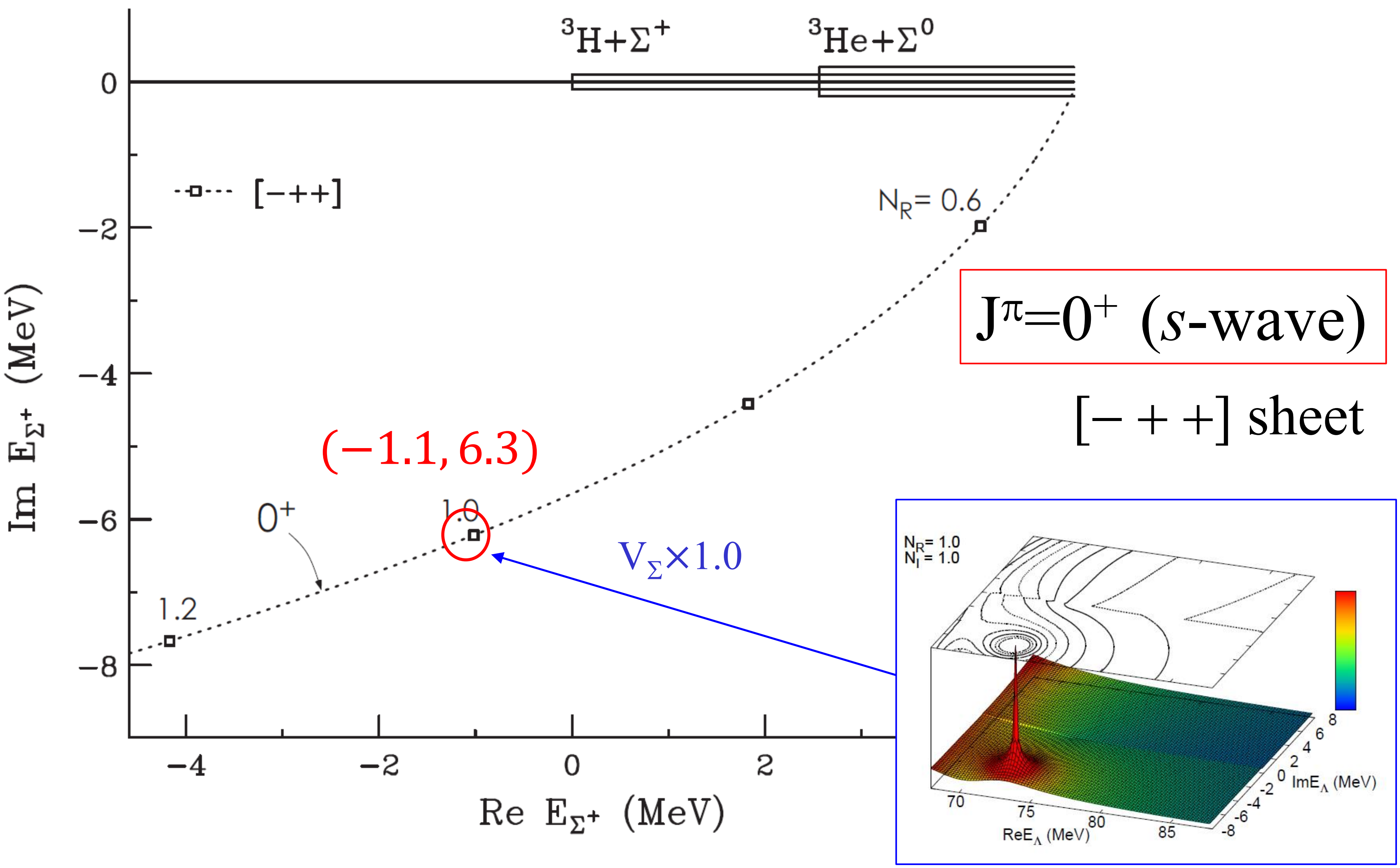
$N_R=1.0$, $N_I=1.0$



Results and discussion

Pole behavior on Riemann sheets

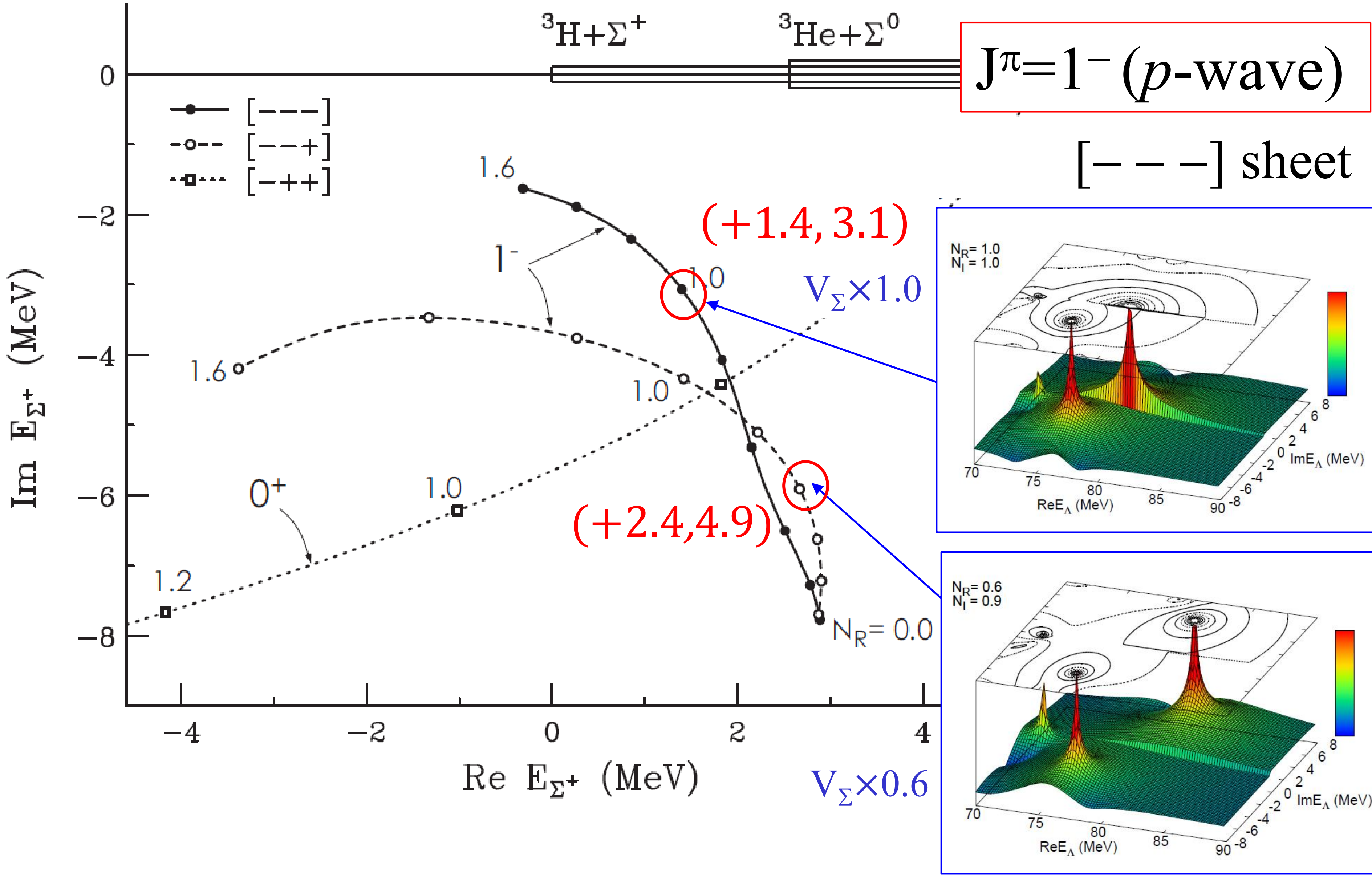
Poles trajectory of the S matrix of $_{\Sigma}^{4}He$ on Riemann Sheets



$J^\pi=0^+$ (s-wave)

[− + +] sheet

Poles trajectory of the S matrix of $_{\Sigma}^{4}He$ on Riemann Sheets



Poles positions of the S matrix of $_{\Sigma}^{4}$He

Case	N_R	N_I	Sheet $[--+]$			Sheet $[---]$		
			E_{Σ^+} (MeV)	E_{Σ^0} (MeV)	$\frac{1}{2}\Gamma_\Sigma$ (MeV)	E_{Σ^+} (MeV)	E_{Σ^0} (MeV)	$\frac{1}{2}\Gamma_\Sigma$ (MeV)
A	0.6	0.9	+2.8	+0.0	5.9	+2.3	−0.4	5.3
A′	0.6	0.0	+3.8	+1.1	5.7	+3.9	+1.1	4.9
B	0.0	0.9	+2.9	+0.2	7.7	+2.9	+0.2	7.8
C	1.0	1.0	+1.4	−1.3	4.3	+1.4	−1.3	3.1

Probability of the channel β component

$$P_\beta = \int_0^{a_c} |\phi_\beta(r)|^2 r^2 dr$$

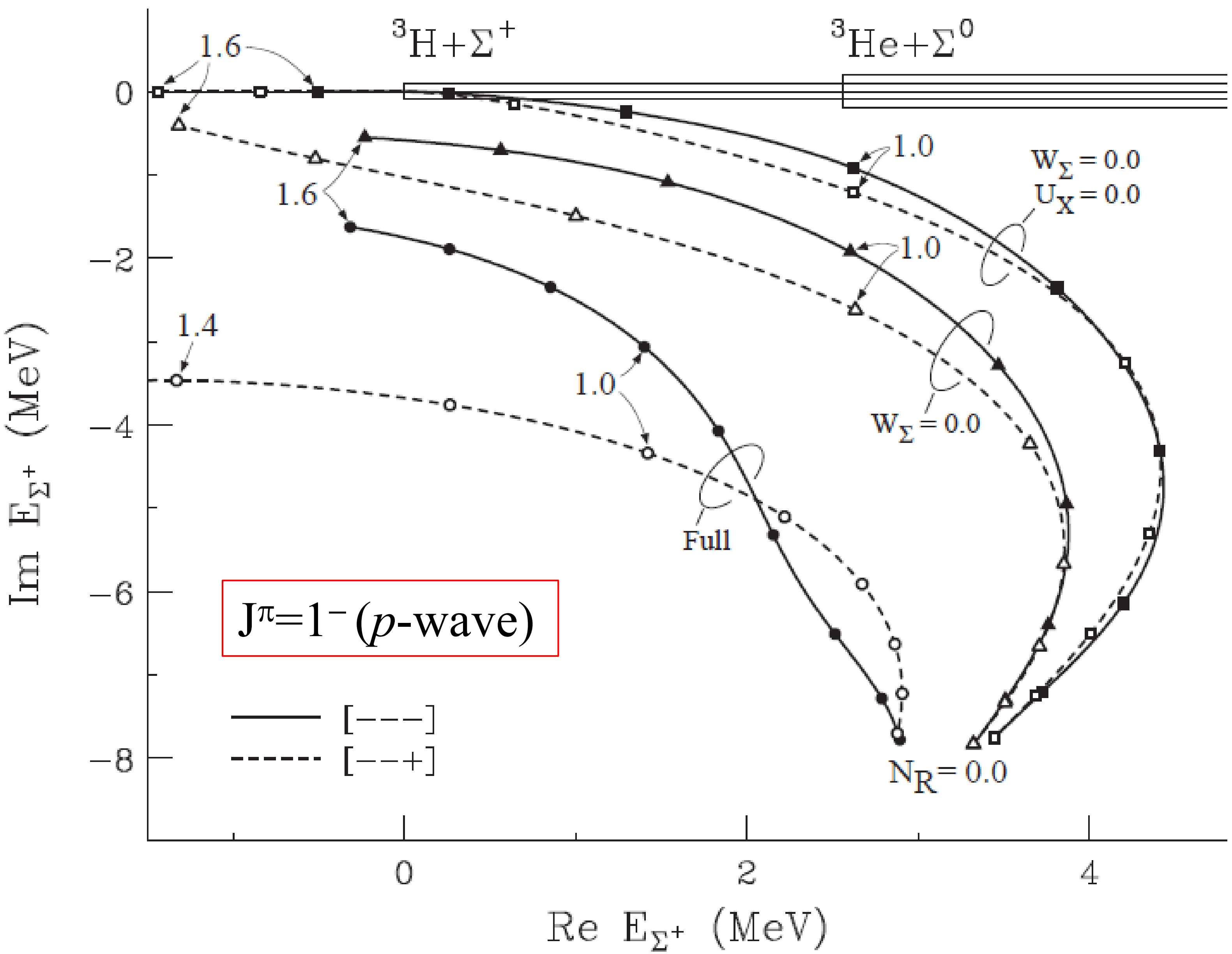
$$\sum_\beta \int_0^{a_c} |\phi_\beta(r)|^2 r^2 dr = 1$$

Σ resonant state

β =	3He-Λ,	^{3}H-Σ^+,	^{3}He-Σ^0
$[--+]$	23%	68%	9%
$[---]$	25%	49%	26%

$a_c = 1.65$ fm

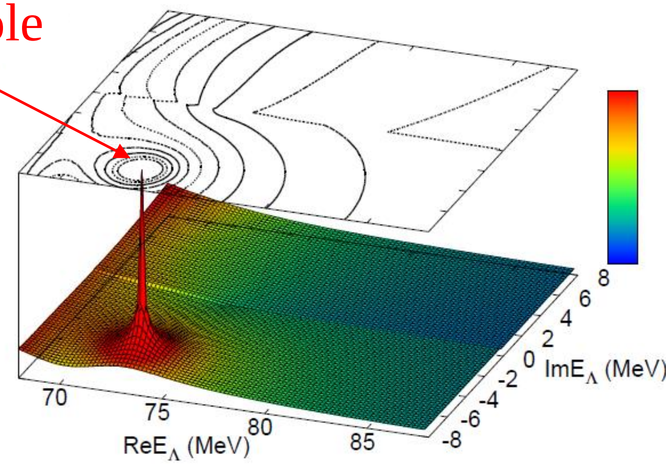
Poles trajectory of the S matrix of $_{\Sigma}{}^{4}He$ on Riemann Sheets



Poles of the S matrix of $_{\Sigma}{}^{4}$He on Riemann Sheets

Σ quasibound state pole

$J^{\pi}=0^{+}$ [− + +] sheet

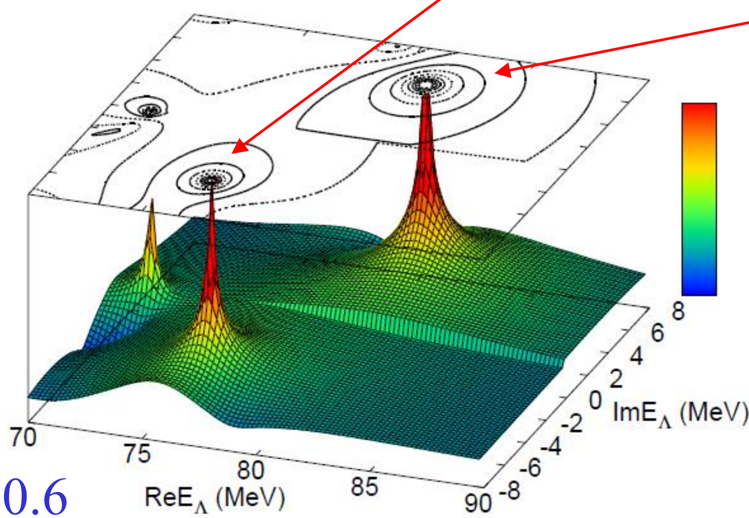


$E_{\Sigma+} = -1.1 - i6.3$ MeV

$J^{\pi}=1^{-}$ [− − −] sheet

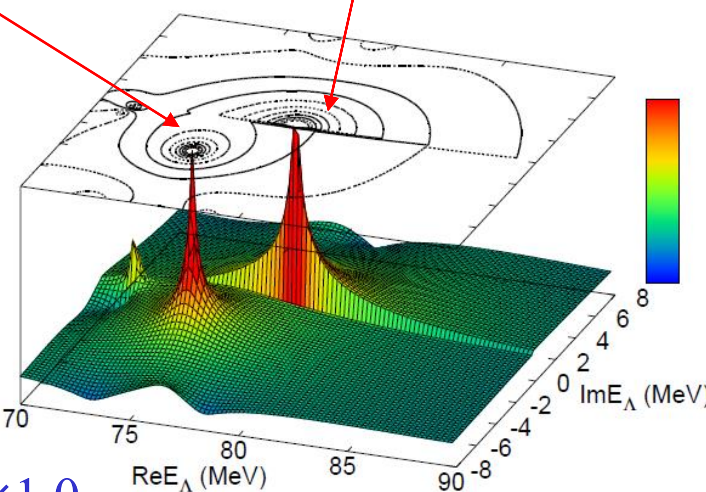
Σ resonance pole

Σ anti-resonance pole
(not observed)



$V_{\Sigma}\times0.6$

$E_{\Sigma+} = +2.4 - i4.9$ MeV



$V_{\Sigma}\times1.0$

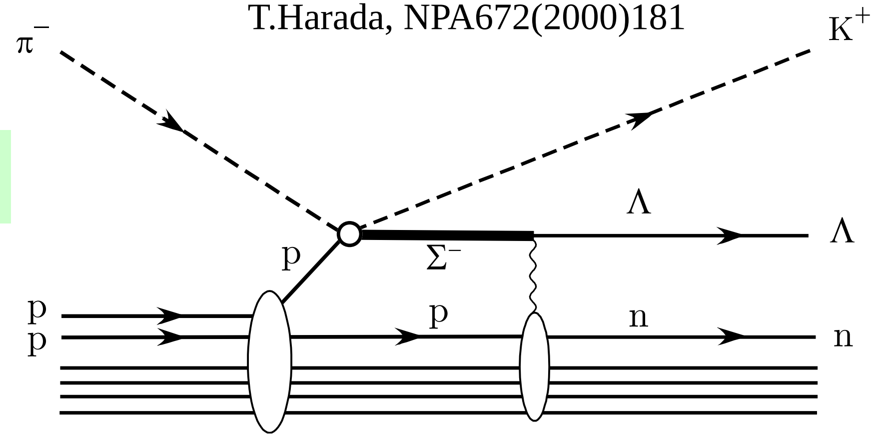
$E_{\Sigma+} = +1.4 - i3.1$ MeV

Σ-production spectra

$^{4}He(K^{-},\pi^{-})$ reactions at 1.5 GeV/c

Coupled-channel calculations in one-step process

T.Harada, NPA672(2000)181



Coupled-channel Green's function

$$\hat{\mathbf{G}}(\omega) = \hat{\mathbf{G}}^{(0)}(\omega) + \hat{\mathbf{G}}^{(0)}(\omega)\hat{\mathbf{U}}\hat{\mathbf{G}}(\omega)$$

$$\hat{\mathbf{G}}^{(0)}(\omega) = \begin{bmatrix} G_\Lambda^{(0)} & \\ & G_{\Sigma^-}^{(0)} \end{bmatrix} \quad \hat{\mathbf{U}} = \begin{bmatrix} U_\Lambda & U_X \\ U_X & U_\Sigma \end{bmatrix}$$

$$\mathrm{Im}\,\hat{G} = \underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\,\hat{G}_\Lambda^{(0)}\}\hat{\Omega}^{(-)}}_{\Lambda\text{ escape}} + \underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\,\hat{G}_{\Sigma^-}^{(0)}\}\hat{\Omega}^{(-)}}_{\Sigma^-\text{ escape}} + \underbrace{\hat{G}^{\dagger}\{W_{Y,T}\}\hat{G}}_{\text{Spreading}}$$

Λ escape

Σ⁻ escape

Spreading (nuclear-core breakup)
= Complicated excited states

Strength function — *Green's function method* — **Morimatsu, Yazaki, NPA483(1988)493**

$$S(\omega) = \sum_f |< f\,|\hat{O}|\,i >|^2\,\delta(\omega + E_K - E_\pi) = -\frac{1}{\pi}\mathrm{Im}\int \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}' F^{\dagger}(\mathbf{r})\, G(\omega + i\varepsilon; \mathbf{r}, \mathbf{r}')\, F(\mathbf{r}')$$

Green's function

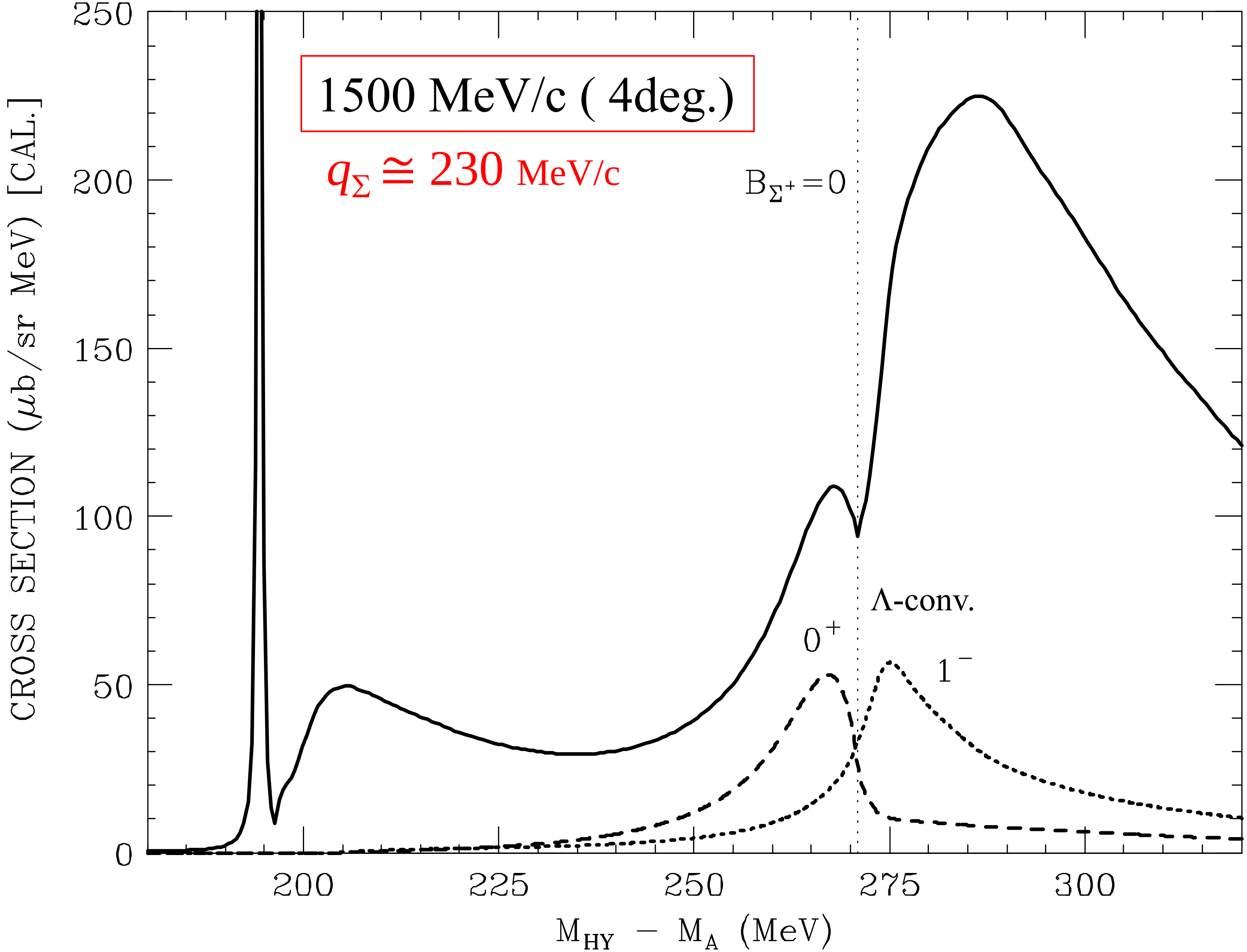
Distorted waves for mesons

Eikonal distortion: $\bar{\sigma} = (\sigma_\pi + \sigma_K)/2 = 20$ mb, $\alpha_\pi = \alpha_K = 0$

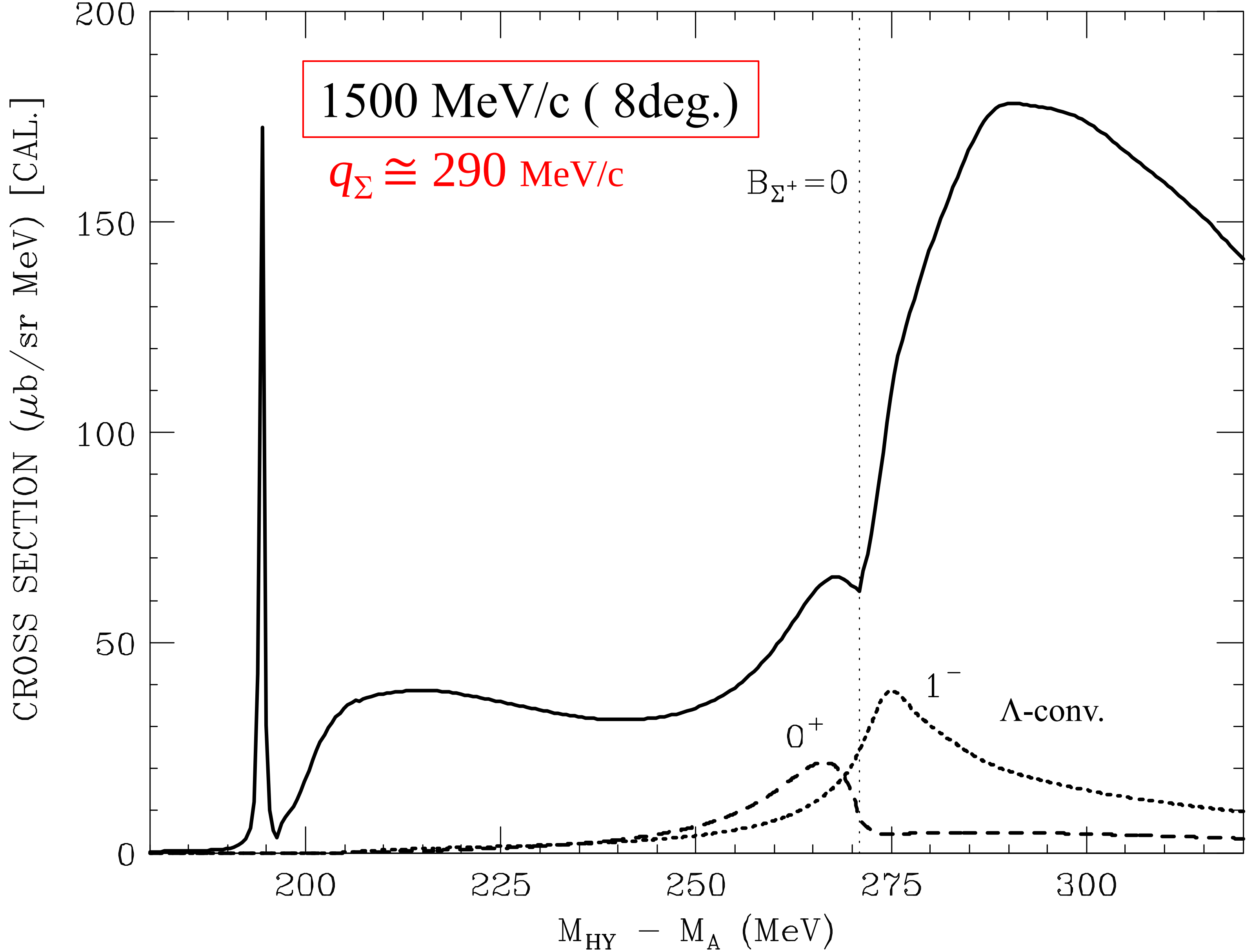
Elementary cross section $\pi^- p \to K^+ \Sigma^-$

$\beta\,[\mathrm{d}\sigma/\mathrm{d}\Omega]$ Optimal Fermi-averaging ～10-20 μb/sr (p_π=1.2GeV/c)

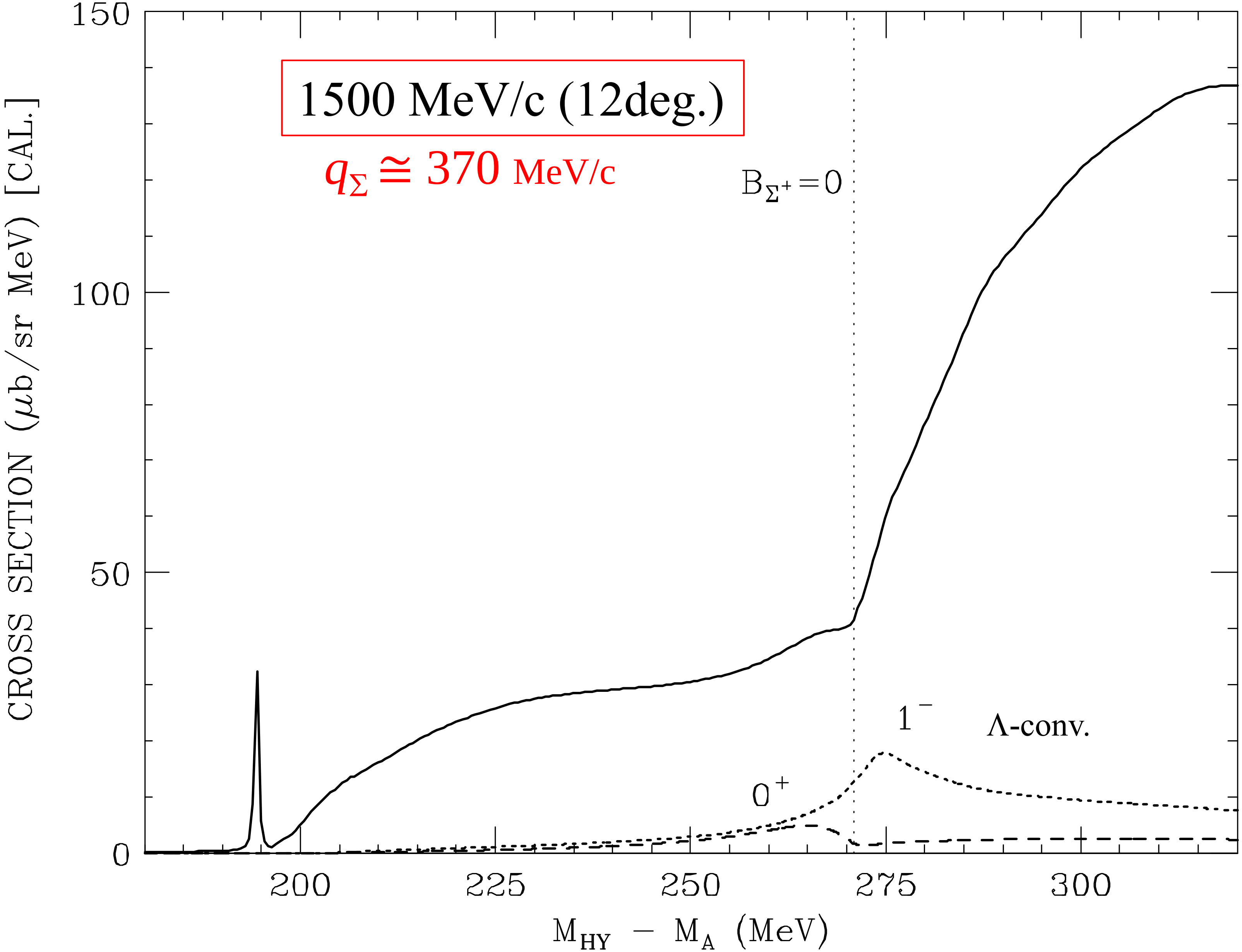
Inclusive spectrum in $^{4}He(K^{-}, \pi^{-})$ reaction at 1.5GeV/c



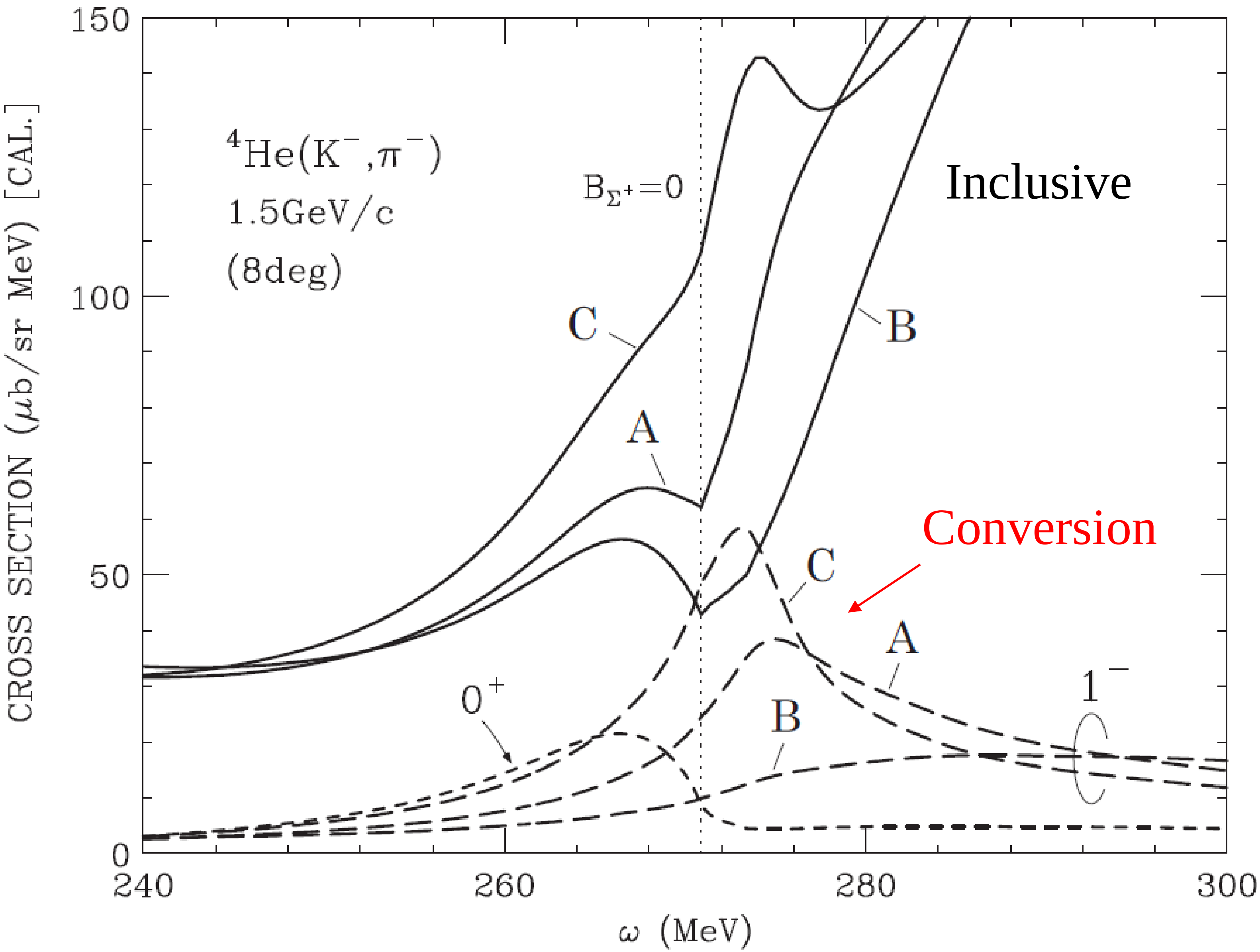
Inclusive spectrum in $^4He(K^-, \pi^-)$ reaction at 1.5GeV/c



Inclusive spectrum in $^{4}He(K^-, \pi^-)$ reaction at 1.5GeV/c



Conversion spectrum in $^4He(K^-, \pi^-)$ reaction at 1.5GeV/c



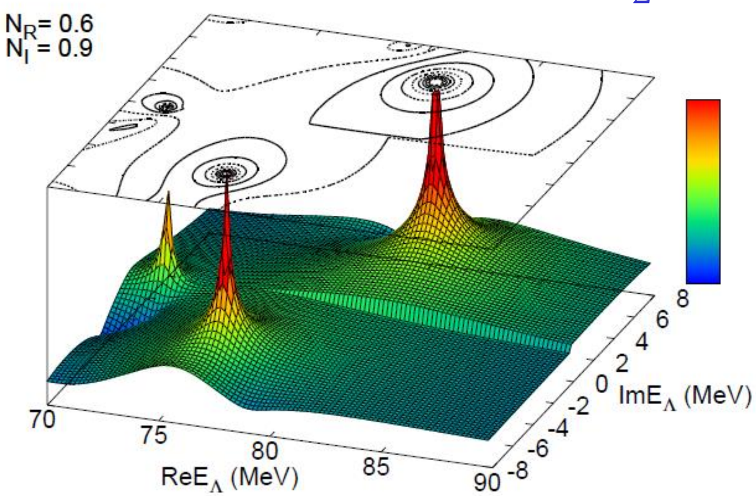
Remarks

We have indicated the existence of the *p*-wave Σ resonant state in ${}_{\Sigma}^{4}$He with $J = 1^{-}$, $T \simeq 1/2$, of which the poles are located near

$J^{\pi}=1^{-}$

$[- - -]$ sheet

$V_{\Sigma}\times 0.6$



$E_{\Sigma+} = +2.4 - i4.9$ MeV

$E_{\Sigma+} = +2.8 - i\,5.9$ MeV
on the *third* Riemann sheet
$[- - +]$

and

$E_{\Sigma+} = +2.3 - i\,5.3$ MeV
on the *fourth* Riemann sheet
$[- - -]$

in the complex *E* plane.

Summary

- We have shown that a promising signal of the ${}_{\Sigma}^{4}$He resonant state could be clearly observed above the Σ threshold in the conversion spectrum by the tagged-Λ measurements.
- The angular distributions of the inclusive and conversion spectra provide significant information on the nature of the *p*-wave resonant state and the Σ-nucleus potential.
- This work is the first attempt to search for a *p*-wave Σ resonant state. We expect that such ^{4}He(K^{-}, π^{-}) experiments are carried out at J-PARC facilities in the near future.

Thank you very much.