A fast algorithm for lattice hyperonic potentials

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Outline

- Introduction
- Brief explanation of effective block algorithm for various baryon-baryon channels
  - See also the poster session
- A benchmark for the implementation of hybrid parallel code
- Very preliminary result of LN potential at the physical point
- Summary

[1] HN, Ishii, Aoki, Hatsuda [PACS-CS Collaboration], Pos LATTICE2008, 156 (2008).
[2] HN [HAL QCD Collaboration and PACS-CS Collaboration], PoS LAT2009, 152 (2009).
[3] HN [HAL QCD Collaboration], PoS LATTICE 2011, 167 (2011).
[4] HN [HAL QCD Collaboration], PoS LATTICE 2013, 426 (2014).

Plan of research

QCD

Baryon interaction

J-PARC
hyperon-nucleon (YN)
scattering

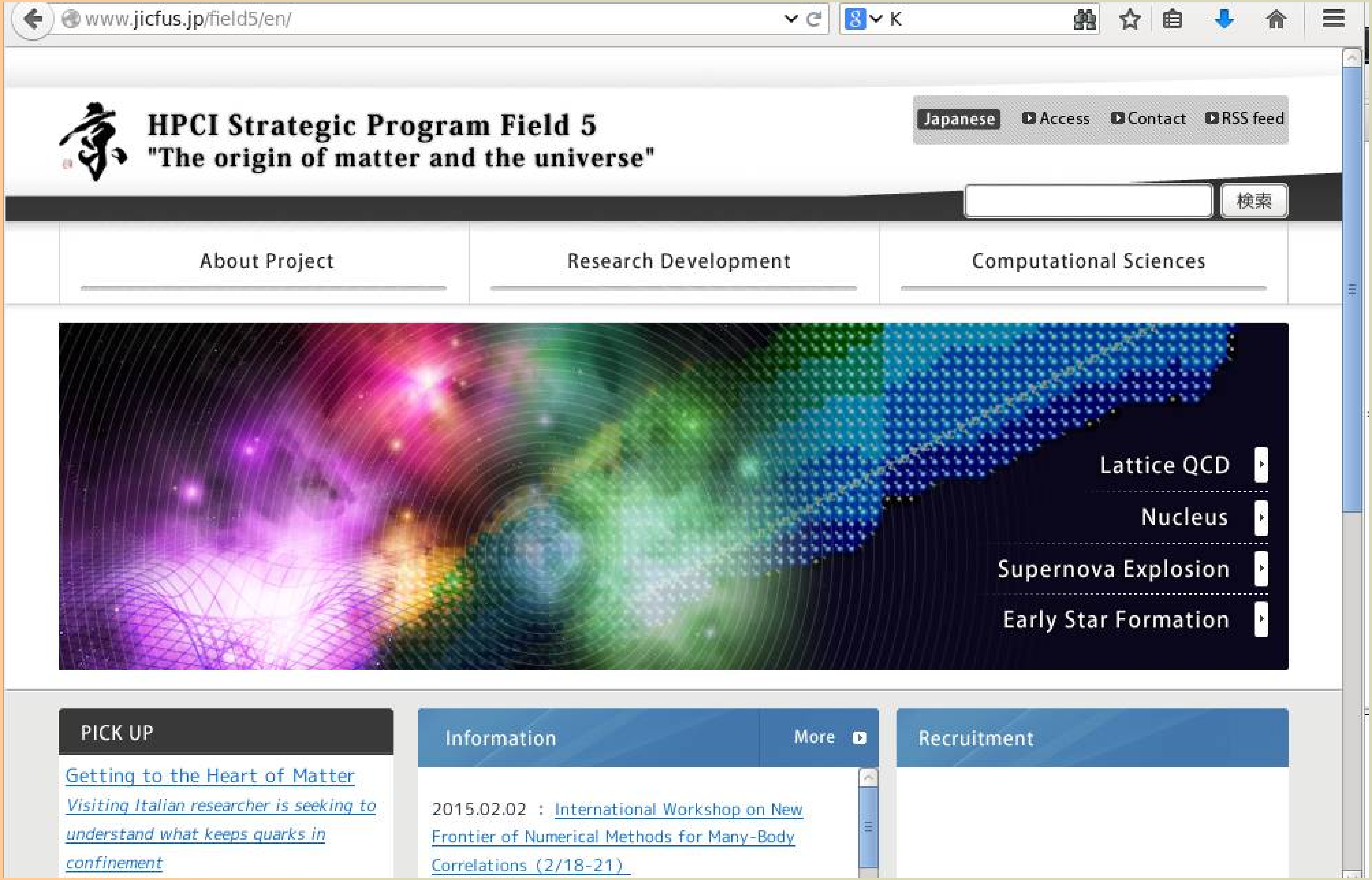
Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

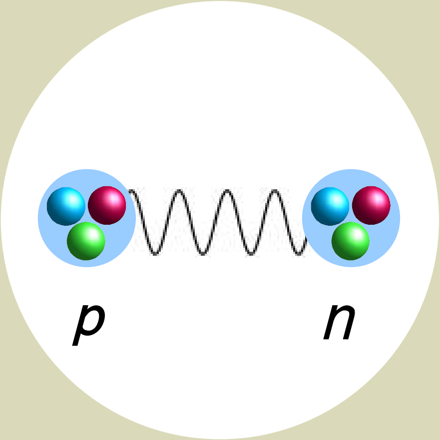
Neutron star and supernova

This talk: A fast algorithm of the lattice calculation, the bench-mark and preliminary LN result

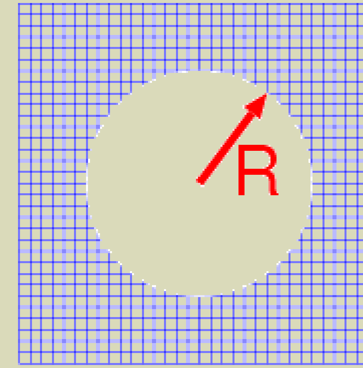
Determination of the baryon-baryon interactions using lattice QCD at the physical point



Lattice QCD calculation



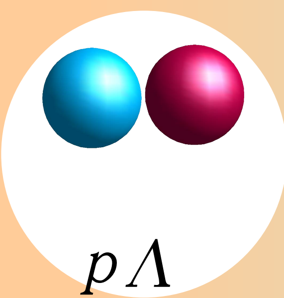
Formulation

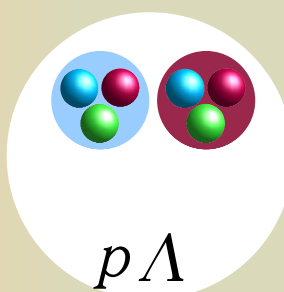
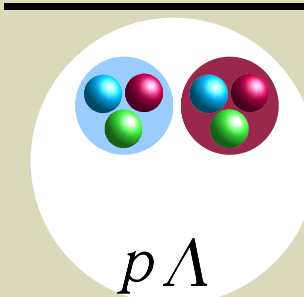


Lattice QCD simulation

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$\begin{aligned}\langle O(\bar{q}, q, U)\rangle &= \int dU\, d\bar{q}\, dq\, \mathrm{e}^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U)\, \mathrm{e}^{-S_U(U)} O(D^{-1}(U)) \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))\end{aligned}$$

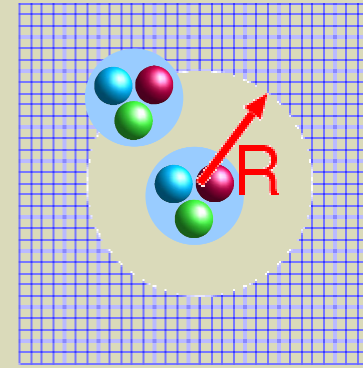


$\rightarrow \langle$  $(t)\ \overline{\text{}}$  $(t_0)\rangle$

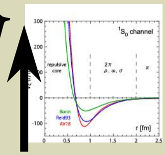
$$= \sum_n A_n \exp(-E_n (t - t_0))$$

Formulation

Lattice QCD simulation



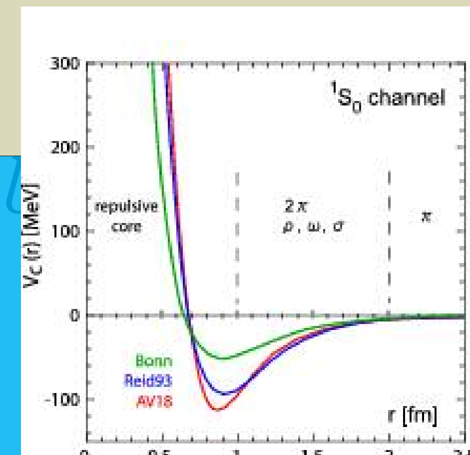
$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$



$$\langle O(\bar{q}, q, U)\rangle = \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q}, q, U)} O(\bar{q}, q$$

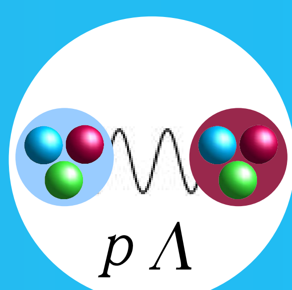
$$= \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U$$

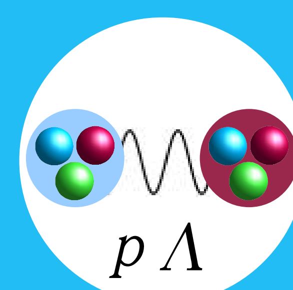
$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$



$$F(\vec{r}, t - t_0)$$

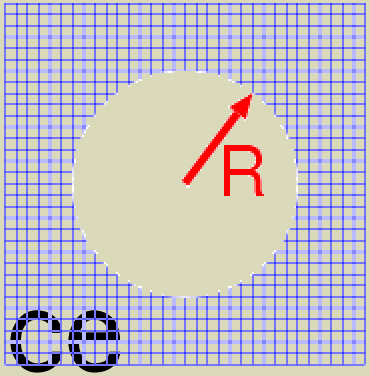
$$\rightarrow \langle \; [p\Lambda] (\vec{r}, t) \; \overline{[p\Lambda]} (t_0) \rangle$$





$$= \sum_n A_n \Psi_n(\vec{r}) \exp(-E_n(t - t_0))$$

HAL formulation



Slogan:

Make better use of the lattice output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., arXiv:0805.2462[hep-ph].

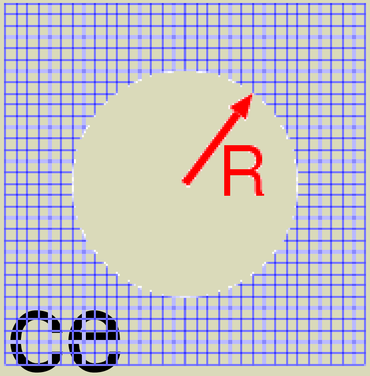
NOTE:

> Potential is not a direct experimental observable.

> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

HAL formulation



Slogan:

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Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., arXiv:0805.2462[hep-ph].

- > Phase shift
- > Nuclear many-body problems

$$R^{(J,M)}_{\alpha\beta}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{\mathcal{J}^{(J,M)}_{B_3 B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$p = \varepsilon_{abc} \left(u_a C\gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc} \left(u_a C\gamma_5 d_b\right) d_c, \tag{2}$$

$$\Sigma^+ = -\varepsilon_{abc} \left(u_a C\gamma_5 s_b\right) u_c, \qquad \Sigma^- = -\varepsilon_{abc} \left(d_a C\gamma_5 s_b\right) d_c, \tag{3}$$

$$\Sigma^0 = \frac{1}{\sqrt{2}} \left(X_u - X_d\right), \qquad \Lambda = \frac{1}{\sqrt{6}} \left(X_u + X_d - 2X_s\right), \tag{4}$$

$$\Xi^0 = \varepsilon_{abc} \left(u_a C\gamma_5 s_b\right) s_c, \qquad \Xi^- = -\varepsilon_{abc} \left(d_a C\gamma_5 s_b\right) s_c, \tag{5}$$

where

$$X_u = \varepsilon_{abc} \left(d_a C\gamma_5 s_b\right) u_c, \quad X_d = \varepsilon_{abc} \left(s_a C\gamma_5 u_b\right) d_c, \quad X_s = \varepsilon_{abc} \left(u_a C\gamma_5 d_b\right) s_c, \tag{6}$$

A simplified (historical) version of the potential

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' \, U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{\rm LO}(\vec{r}) R(\vec{r}, t) + \cdots \tag{8}$$

See Ishii's talk for more precise (modern) formulation of the potential

Effective block algorithm for various baryon-baryon calculations

Consider the proton-Lambda system as a specific example.

$$p = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) d_c, \qquad (2)$$

$$\Sigma^+ = \varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) u_c, \qquad \Sigma^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) d_c, \qquad (3)$$

$$\Sigma^0 = \frac{1}{\sqrt{2}}\left(X_u - X_d\right), \qquad \Lambda = \frac{1}{\sqrt{6}}\left(X_u + X_d - 2X_s\right), \qquad (4)$$

$$\Xi^0 = \varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) s_c, \qquad \Xi^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) s_c, \qquad (5)$$

where

$$X_u = \varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) u_c, \quad X_d = \varepsilon_{abc}\left(s_a C\gamma_5 u_b\right) d_c, \quad X_s = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) s_c,$$

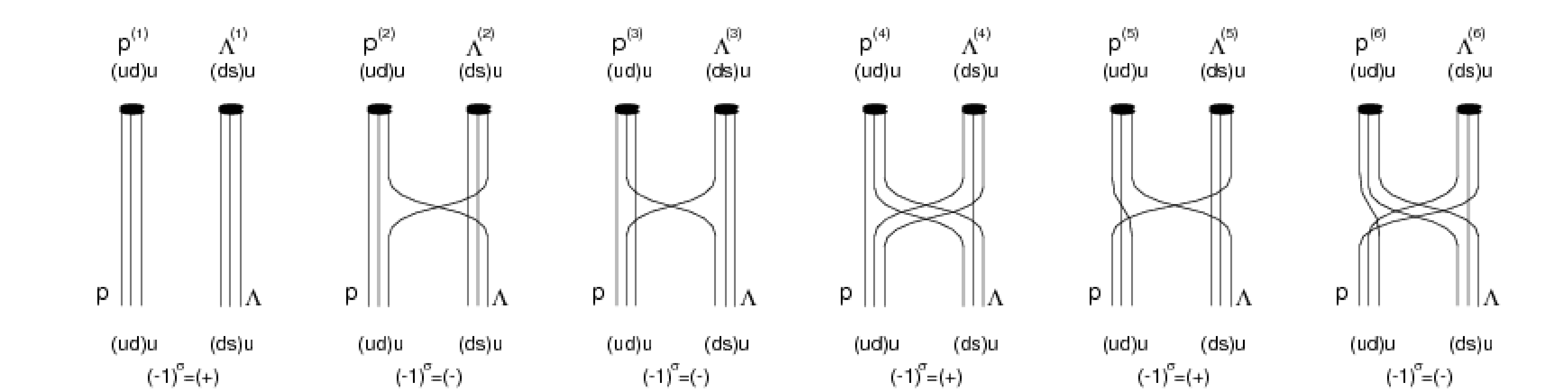
$$\begin{aligned} p_\alpha(x) &= \varepsilon(c_1, c_2, c_3)(C\gamma_5)(\alpha_1, \alpha_2)\delta(\alpha, \alpha_3)u(\xi_1)d(\xi_2)u(\xi_3), \qquad (\xi_i = x_i\alpha_i c_i) \\ &= \varepsilon(1,2,3)(C\gamma_5)(1,2)\delta(\alpha,3)u(1)d(2)u(3). \end{aligned} \qquad (11)$$

$$\begin{aligned} &\sum_{\vec{X}} \left\langle 0 \left| p_\alpha(\vec{X}+\vec{r},t)\Lambda_\beta(\vec{X},t)\overline{\mathcal{J}_{p_{\alpha'}\Lambda_{\beta'}}(t_0)} \right| 0 \right\rangle \\ = \; &\sum_{\vec{X}} \tfrac{1}{6}\varepsilon(1,4,2)\varepsilon(5,6,3)\varepsilon(1',4',2')\varepsilon(5',6',3')(C\gamma_5)(1,4)\delta(\alpha,2)(C\gamma_5)(1',4')\delta(\alpha',2') \\ &\times\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\} \\ &\times\{(C\gamma_5)(5',6')\delta(\beta',3') + (C\gamma_5)(6',3')\delta(\beta',5') - 2(C\gamma_5)(3',5')\delta(\beta',6')\} \\ &\times\langle u(1)d(4)u(2)d(5)s(6)u(3)\bar{u}(3')\bar{s}(6')\bar{d}(5')\bar{u}(2')\bar{d}(4')\bar{u}(1')\rangle. \end{aligned} \qquad (12)$$

$$\sum_{c_1,\ldots,c_6}\;\sum_{\alpha_1,\ldots,\alpha_6}\;\sum_{c_1',\ldots,c_6'}\;\sum_{\alpha_1',\ldots,\alpha_6'}$$

E.g., 3981312
for Lambda-Nucleon →

$$(N_c!\,N_\alpha)^{2B} \times N_u!\,N_d!\,N_s!$$



$$\begin{aligned} p_\alpha(x) &= \varepsilon(c_1,c_2,c_3)(C\gamma_5)(\alpha_1,\alpha_2)\delta(\alpha,\alpha_3)u(\xi_1)d(\xi_2)u(\xi_3), \qquad (\xi_i = x_i\alpha_i c_i) \\ &= \varepsilon(1,2,3)(C\gamma_5)(1,2)\delta(\alpha,3)u(1)d(2)u(3). \end{aligned} \tag{11}$$

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$$\sum_{c_1,\ldots,c_6}\ \sum_{\alpha_1,\ldots,\alpha_6}\ \sum_{c_1',\ldots,c_6'}\ \sum_{\alpha_1',\ldots,\alpha_6'}$$

E.g., 3981312
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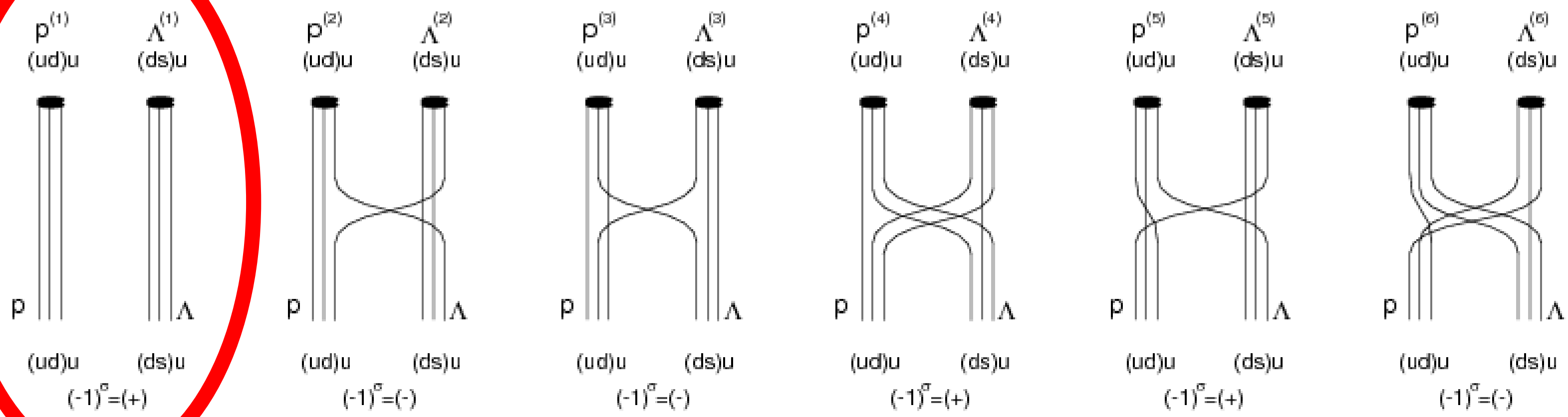
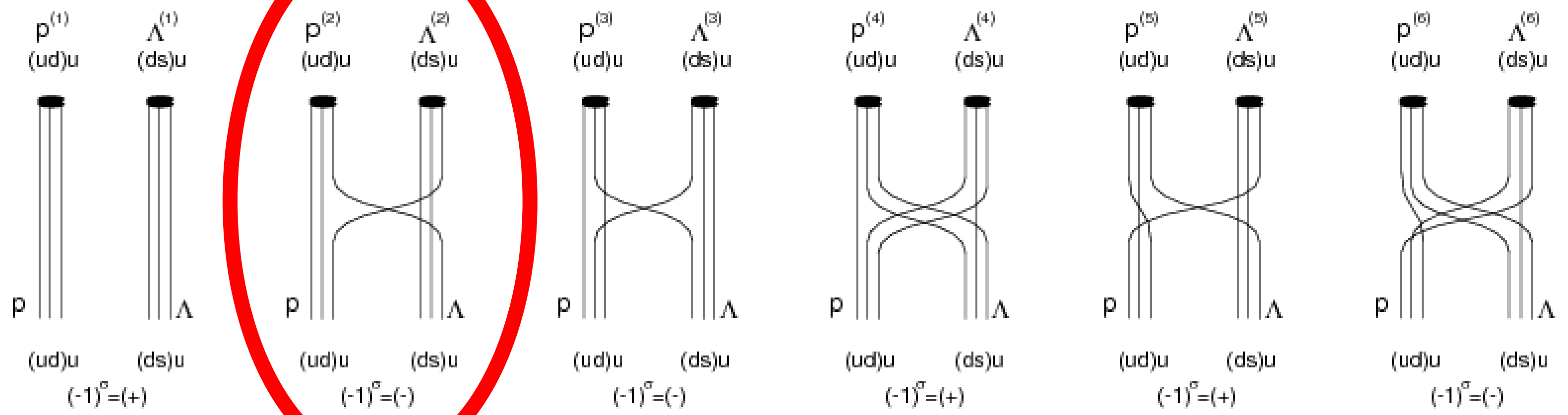
$$(N_c!\,N_\alpha)^{2B}\times N_u!\,N_d!\,N_s!$$



$$
\begin{aligned}
[p^{(1)}_{\alpha\alpha'}](\vec{x}) &= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \\
&\times \det \begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(2')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(2')\rangle \end{vmatrix} \langle d(4)\bar{d}(4')\rangle,
\end{aligned}
$$

$$
\begin{aligned}
[\Lambda^{(1)}_{\beta\beta'}](\vec{y}) &= \langle u(3)\bar{u}(3')\rangle\langle d(5)\bar{d}(5')\rangle\langle s(6)\bar{s}(6')\rangle \\
&\times\varepsilon(5,6,3)\left\{(C\gamma_5)(5,6)\delta(\beta,3)+(C\gamma_5)(6,3)\delta(\beta,5)-2(C\gamma_5)(3,5)\delta(\beta,6)\right\} \\
&\times\varepsilon(5',6',3')\left\{(C\gamma_5)(5',6')\delta(\beta',3')\right\}. \qquad (18)
\end{aligned}
$$

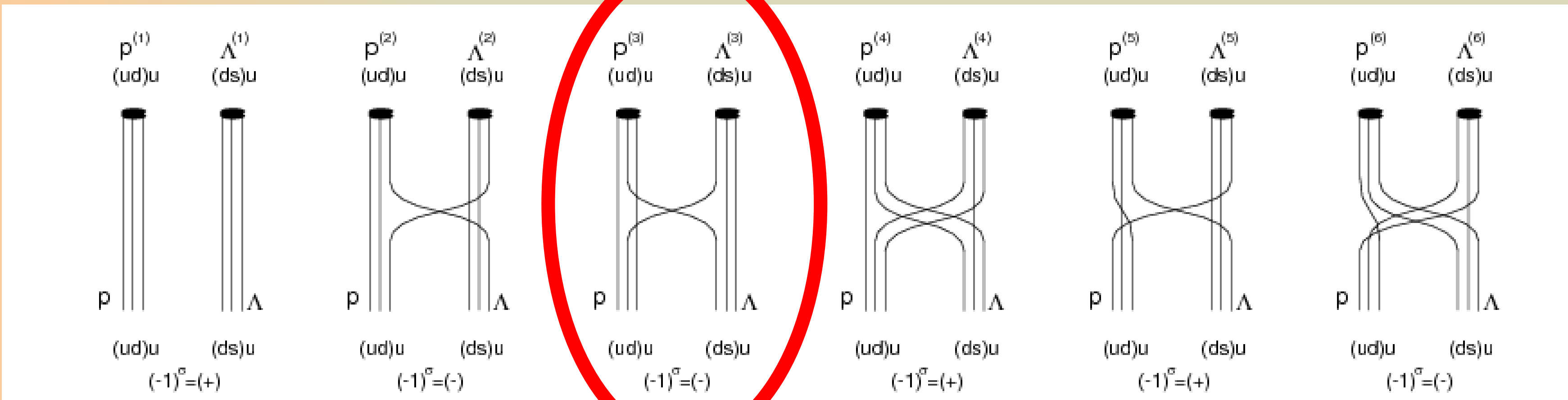
This fact significantly slashes in the computational cost: The reduction factor at the first diagram is $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/1 = 1152.$



$$
\begin{aligned}
[p^{(2)}_{\alpha\beta'}](\vec{x};c'_2,c'_3) &= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\times\delta(\beta',3') \\
&\times \det\begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix} \langle d(4)\bar{d}(4')\rangle,
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
[\Lambda^{(2)}_{\beta;\alpha'}](\vec{y};c'_2,c'_3) &= \langle u(3)\bar{u}(2')\rangle\langle d(5)\bar{d}(5')\rangle\langle s(6)\bar{s}(6')\rangle\times\delta(\alpha',2') \\
&\times\varepsilon(5,6,3)\{(C\gamma_5)(5,6)\delta(\beta,3)+(C\gamma_5)(6,3)\delta(\beta,5)-2(C\gamma_5)(3,5)\delta(\beta,6)\} \\
&\times\varepsilon(5',6',3')\{(C\gamma_5)(5',6')\}\,.
\end{aligned} \tag{22}
$$

are crossed as $[p^{(2)}_{\alpha\beta'}]$ and $[\Lambda^{(2)}_{\beta\alpha'}]$. Performed these manipulations, the number of explicit summations of indices reduces to only two colors which makes the reduction factor $(N_c!N_\alpha)^2\times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2)=128$.



$$[p^{(3)}_{\alpha;\alpha'}](\vec{x}; c'_4, c'_5, \alpha'_4, \alpha'_5) \tag{25}$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \tag{26}$$

$$\times \det \begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(2')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(2')\rangle \end{vmatrix} \langle d(4)\bar{d}(5')\rangle, \tag{27}$$

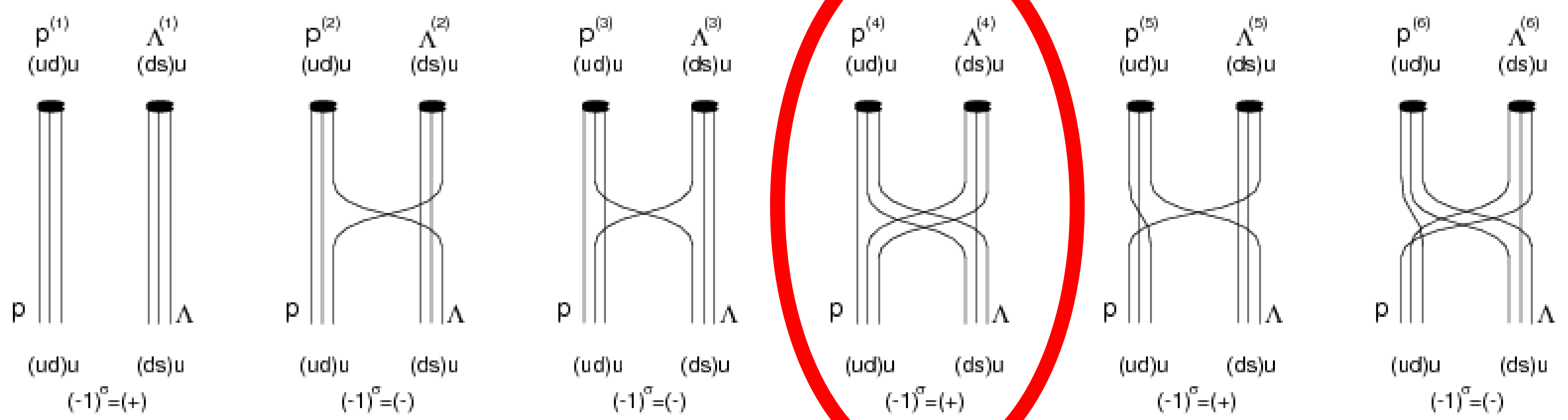
$$[\Lambda^{(3)}_{\beta;\beta'}](\vec{y}; c'_4, c'_5, \alpha'_4, \alpha'_5) \tag{28}$$

$$= \varepsilon(5,6,3)\left\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\right\} \tag{29}$$

$$\times\varepsilon(5',6',3')\left\{(C\gamma_5)(5',6')\delta(\beta',3')\right\}$$

$$\times\langle u(3)\bar{u}(3')\rangle\langle d(5)\bar{d}(4')\rangle\langle s(6)\bar{s}(6')\rangle. \tag{30}$$

The number of explicit summations of indices reduces to two colors and two spinors which makes the reduction factor $(N_c! N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2 N_\alpha^2) = 8$.



$$[p^{(4)}_{\alpha;\beta'}](\vec{x}; c'_1, c'_6, \alpha'_1, \alpha'_6) \tag{33}$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2) \tag{34}$$

$$\times\varepsilon(5',6',3')\,\{(C\gamma_5)(5',6')\delta(\beta',3')\}$$

$$\times\,\det\begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix}\langle d(4)\bar{d}(5')\rangle, \tag{35}$$

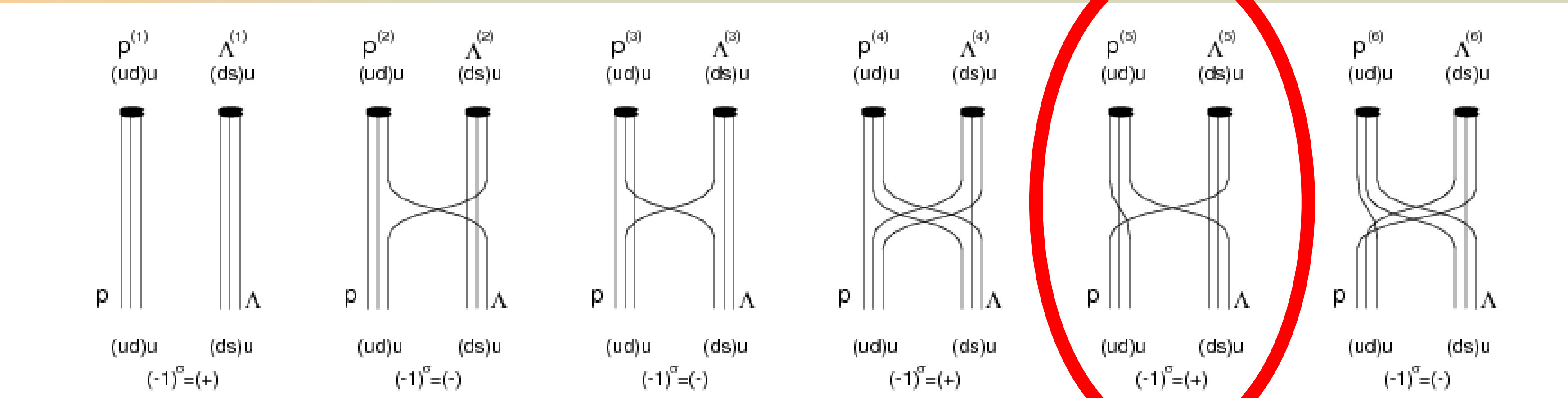
$$[\Lambda^{(4)}_{\beta;\alpha'}](\vec{y}; c'_1, c'_6, \alpha'_1, \alpha'_6) \tag{36}$$

$$= \varepsilon(5,6,3)\,\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\} \tag{37}$$

$$\times\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \tag{38}$$

$$\times\langle u(3)\bar{u}(2')\rangle\langle d(5)\bar{d}(4')\rangle\langle s(6)\bar{s}(6')\rangle. \tag{39}$$

exchanged, too. The number of explicit summations of indices reduces to two colors and two spinors which makes the reduction factor $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2 N_\alpha^2) = 8$.



$$[p^{(5)}_{\alpha\alpha'\beta'}](\vec{x}; c'_1, c'_3, \alpha'_1) \tag{42}$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \times \delta(\beta',3') \tag{43}$$

$$\times \det \begin{vmatrix} \langle u(1)\bar{u}(2')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(2')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix} \langle d(4)\bar{d}(4')\rangle, \tag{44}$$

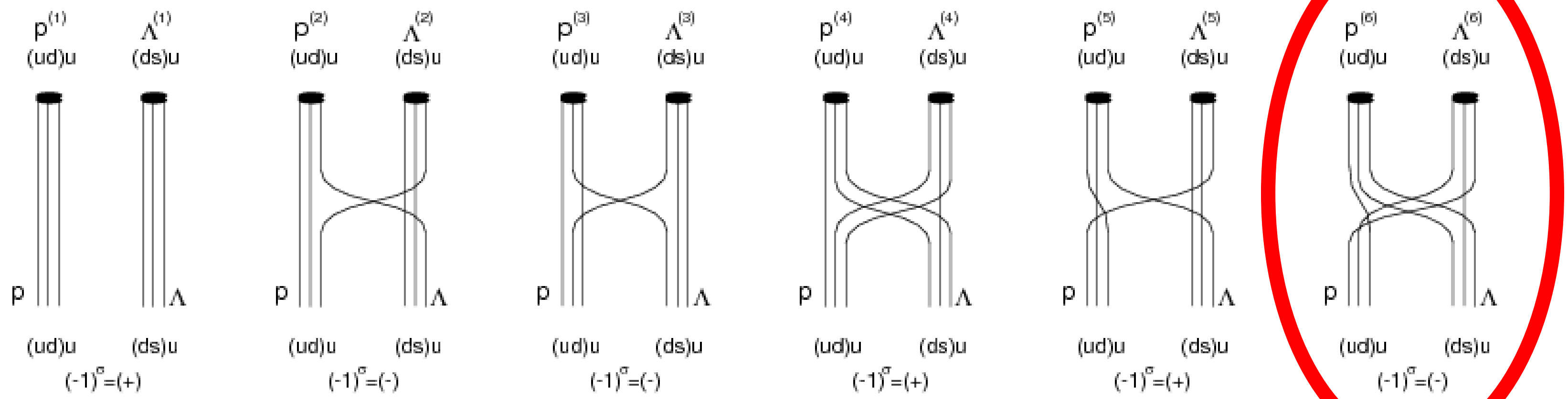
$$[\Lambda^{(5)}_{\beta}](\vec{y}; c'_1, c'_3, \alpha'_1) \tag{45}$$

$$= \varepsilon(5,6,3)\,\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\} \tag{46}$$

$$\times\varepsilon(5',6',3')\,\{(C\gamma_5)(5',6')\}$$

$$\times\langle u(3)\bar{u}(1')\rangle\langle d(5)\bar{d}(5')\rangle\langle s(6)\bar{s}(6')\rangle. \tag{47}$$

accompanied in the $[p^{(5)}_{\alpha\alpha'\beta'}]$. The number of explicit summations of indices reduces to two colors and one spinor which makes the reduction factor $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2 N_\alpha) = 32$.



$$[p^{(6)}_{\alpha\alpha'\beta'}](\vec{x}; c'_2, c'_6, \alpha'_6) \quad (50)$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2) \times \delta(\alpha',2') \quad (51)$$

$$\times\varepsilon(5',6',3')\,\{(C\gamma_5)(5',6')\delta(\beta',3')\}$$

$$\times \det \begin{vmatrix} \langle u(1)\bar{u}(2')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(2')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix} \langle d(4)\bar{d}(5')\rangle, \quad (52)$$

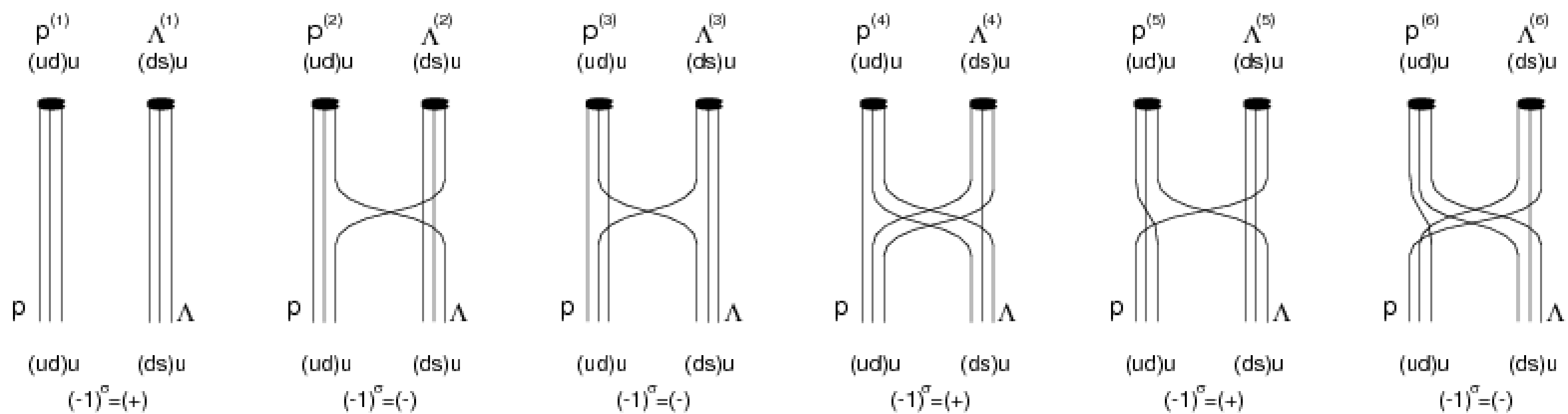
$$[\Lambda^{(6)}_{\beta}](\vec{y}; c'_2, c'_6, \alpha'_6) \quad (53)$$

$$= \varepsilon(5,6,3)\,\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\} \quad (54)$$

$$\times\varepsilon(1',4',2')(C\gamma_5)(1',4') \quad (55)$$

$$\times\langle u(3)\bar{u}(1')\rangle\langle d(5)\bar{d}(4')\rangle\langle s(6)\bar{s}(6')\rangle. \quad (56)$$

are exchanged between $[p^{(6)}_{\alpha;\alpha'\beta'}]$ and $[\Lambda^{(6)}_{\beta}]$ while $\delta(\alpha', 2')$ is kept in $[p^{(6)}_{\alpha\alpha'\beta'}]$. The number of explicit summations of indices reduces to two colors and one spinor which makes the reduction factor $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2 N_\alpha) = 32$.



Performed these manipulations based on the diagrammatic classification, most of the summations can be carried out prior to evaluating the FFT so that the number of iterations significantly reduces; The numbers of iteration are $\{1, 9, 144, 144, 36, 36\}$ for the baryon blocks $\{([p_\alpha^{(i)}]\times[\Lambda_\beta^{(i)}]); i=1,\cdots,6\}$. Therefore only 370 iterations should be explicitly performed to obtain the four-point correlation function of the $p\Lambda$ system when we take the operator $\overline{X}_u$ in $\overline{\Lambda}_{\beta'}$ in the source. For the sake of completeness, the total number of iterations does not change when we take the operator $\overline{X}_s$ in $\overline{\Lambda}_{\beta'}$ in the source whereas the numbers of iteration are $\{1, 36, 36, 144, 144, 36\}$ when we consider the contribution from the operator $\overline{X}_d$ in $\overline{\Lambda}_{\beta'}$ in the source which slightly differ from the former cases and the total number of iterations is 397.

Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{array}{lll} \langle p\Lambda\overline{p\Lambda}\rangle, & \langle p\Lambda\overline{\Sigma^+ n}\rangle, & \langle p\Lambda\overline{\Sigma^0 p}\rangle, \\ \langle \Sigma^+ n\overline{p\Lambda}\rangle, & \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, & \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle, \\ \langle \Sigma^0 p\overline{p\Lambda}\rangle, & \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, & \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle, \end{array} \tag{4.2}$$

$$\begin{array}{llllll} \langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, & \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, & \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, & \\ \langle p\Xi^-\overline{\Lambda\Lambda}\rangle, & \langle p\Xi^-\overline{p\Xi^-}\rangle, & \langle p\Xi^-\overline{n\Xi^0}\rangle, & \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle, \\ \langle n\Xi^0\overline{\Lambda\Lambda}\rangle, & \langle n\Xi^0\overline{p\Xi^-}\rangle, & \langle n\Xi^0\overline{n\Xi^0}\rangle, & \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle, \\ \langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle, \\ \langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, & \\ & \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & & \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle, \end{array} \tag{4.3}$$

$$\begin{array}{lll} \langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle, \\ \langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle, \\ \langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle, \end{array} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Make better use of the computing resources!

Generalization to the various baryon-baryon channels strangeness S=0 and -1 systems

channel	# of diagrams	$\{(\#\text{ of iterations})^{\text{sign}}\}$	# of total iterations
$\langle pn\overline{pn}\rangle$	9	$\{1^+, 36^-, 144^-, 36^+, 36^+, 144^-, 144^+, 9^-, 36^+\}$	586
$\langle p\Lambda\overline{p\Lambda_{X_{u,s}}}\rangle$	6	$\{1^+, 9^-, 144^-, 144^+, 36^+, 36^-\}$	370
$\langle p\Lambda\overline{p\Lambda_{X_d}}\rangle$	6	$\{1^+, 36^-, 36^-, 144^+, 144^+, 36^-\}$	397
$\langle p\Lambda\overline{\Sigma^+ n}\rangle$	6	$\{144^-, 36^+, 36^+, 144^-, 9^-, 36^+\}$	405
$\langle p\Lambda\overline{\Sigma^0_{X_u} p}\rangle$	6	$\{144^+, 36^-, 9^-, 36^+, 144^+, 1^-\}$	370
$\langle p\Lambda\overline{\Sigma^0_{X_d} p}\rangle$	6	$\{144^-, 36^+, 36^+, 144^-, 36^-, 1^+\}$	397
$\langle \Sigma^+ n\overline{p\Lambda_{X_u}}\rangle$	3	$\{144^-, 144^+, 36^-\}$	324
$\langle \Sigma^+ n\overline{p\Lambda_{X_d}}\rangle$	3	$\{144^-, 36^+, 9^-\}$	189
$\langle \Sigma^+ n\overline{p\Lambda_{X_s}}\rangle$	3	$\{36^-, 144^+, 36^-\}$	216
$\langle \Sigma^+ n\overline{\Sigma^+ n}\rangle$	3	$\{1^+, 36^-, 144^+\}$	181
$\langle \Sigma^+ n\overline{\Sigma^0_{X_u} p}\rangle$	3	$\{144^-, 36^+, 144^-\}$	324
$\langle \Sigma^+ n\overline{\Sigma^0_{X_d} p}\rangle$	3	$\{36^+, 9^-, 144^+\}$	189
$\langle \Sigma^0 p\overline{p\Lambda_{X_{u,s}}}\rangle$	6	$\{36^+, 144^-, 144^+, 36^-, 9^+, 1^-\}$	370
$\langle \Sigma^0 p\overline{p\Lambda_{X_d}}\rangle$	6	$\{36^+, 144^-, 36^+, 144^-, 36^+, 1^-\}$	397
$\langle \Sigma^0 p\overline{\Sigma^+ n}\rangle$	6	$\{36^-, 144^+, 36^-, 9^+, 36^-, 144^+\}$	405
$\langle \Sigma^0 p\overline{\Sigma^0_{X_u} p}\rangle$	6	$\{1^+, 36^-, 9^+, 144^-, 36^+, 144^-\}$	370
$\langle \Sigma^0 p\overline{\Sigma^0_{X_d} p}\rangle$	6	$\{1^-, 144^+, 36^-, 36^+, 36^-, 144^+\}$	397

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=-2 systems

channel	# of diagrams	$\{(\text{\# of iterations})^{\text{sign}}\}$	# of total iterations
$\langle \Lambda\Lambda\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Lambda\Lambda\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^+,36^-,144^-,36^+,36^-,144^+,36^+,1^-\}$	434
$\langle \Lambda\Lambda\overline{p\Xi^-}\rangle$	8	$\{36^+,144^-,9^-,36^+,36^-,9^+,144^+,36^-\}$	450
$\langle \Lambda\Lambda\overline{n\Xi^0}\rangle$	8	$\{36^+,36^-,9^-,144^+,144^-,9^+,36^+,36^-\}$	450
$\langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-,144^+,36^+,9^-,9^+,36^-,144^-,36^+\}$	450
$\langle \Lambda\Lambda\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Lambda\Lambda\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-,36^+,144^+,36^-,36^+,144^-,36^-,1^+\}$	434
$\langle p\Xi^-\overline{\Lambda_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,s)$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(s,d),(d,s))$	2	$\{36^+,144^-\}$	180
$\langle p\Xi^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,u),(u,s))$	2	$\{9^+,144^-\}$	153
$\langle p\Xi^-\overline{\Lambda_{X_d}\Lambda_{X_d}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle p\Xi^-\overline{p\Xi^-}\rangle$	2	$\{1^+,144^-\}$	145
$\langle p\Xi^-\overline{n\Xi^0}\rangle$	2	$\{36^+,144^-\}$	180
$\langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle$	2	$\{144^-,36^+\}$	180
$\langle p\Xi^-\overline{\Sigma^0_{X_u}\Sigma^0_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^-\overline{\Sigma^0_{X_d}\Sigma^0_{X_d}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle p\Xi^-\overline{\Sigma^0_{X_u}\Lambda_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^-\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(d,s))$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^-\overline{\Sigma^0_{X_d}\Lambda_{X_d}}\rangle$	2	$\{144^-,144^+\}$	288
$\langle p\Xi^-\overline{\Sigma^0_{X_u}\Lambda_{X_s}}\rangle$	2	$\{144^+,9^-\}$	153

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=-2 systems (cont' d)

$\langle p\Xi^- \overline{\Sigma^0_{X_u}\Sigma^0_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^- \overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^- \overline{\Sigma^0_{X_d}\Sigma^0_{X_d}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle p\Xi^- \overline{\Sigma^0_{X_u}\Lambda_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^- \overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(d,s))$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^- \overline{\Sigma^0_{X_d}\Lambda_{X_d}}\rangle$	2	$\{144^-,144^+\}$	288
$\langle p\Xi^- \overline{\Sigma^0_{X_u}\Lambda_{X_s}}\rangle$	2	$\{144^+,9^-\}$	153
$\langle n\Xi^0 \overline{\Lambda_{X_u}\Lambda_{X_u}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle n\Xi^0 \overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(s,u),(u,s))$	2	$\{144^+,36^-\}$	180
$\langle n\Xi^0 \overline{\Lambda_{X_q}\Lambda_{X_q}}\rangle$ $(q=d,s)$	2	$\{36^+,36^-\}$	72
$\langle n\Xi^0 \overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,d),(d,s))$	2	$\{9^+,144^-\}$	153
$\langle n\Xi^0 \overline{p\Xi^-}\rangle$	2	$\{36^+,144^-\}$	180
$\langle n\Xi^0 \overline{n\Xi^0}\rangle$	2	$\{1^+,144^-\}$	145
$\langle n\Xi^0 \overline{\Sigma^+\Sigma^-}\rangle$	2	$\{144^-,36^+\}$	180
$\langle n\Xi^0 \overline{\Sigma^0_{X_u}\Sigma^0_{X_u}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle n\Xi^0 \overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{144^-,36^+\}$	180
$\langle n\Xi^0 \overline{\Sigma^0_{X_d}\Sigma^0_{X_d}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle n\Xi^0 \overline{\Sigma^0_{X_u}\Lambda_{X_u}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle n\Xi^0 \overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(u,s))$	2	$\{144^-,36^+\}$	180
$\langle n\Xi^0 \overline{\Sigma^0_{X_d}\Lambda_{X_d}}\rangle$	2	$\{36^-,36^+\}$	72
$\langle n\Xi^0 \overline{\Sigma^0_{X_d}\Lambda_{X_s}}\rangle$	2	$\{144^-,9^+\}$	153

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=-2 systems (cont' d)

channel	# of diagrams	$\{(\#\text{ of iterations})^{\text{sign}}\}$	# of total iterations
$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-, 36^+\}$	72
$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d))$	2	$\{9^-, 144^+\}$	153
$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,u),(s,d),(u,s),(d,s))$	2	$\{36^-, 144^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_s}\Lambda_{X_s}}\rangle$	2	$\{144^-, 144^+\}$	288
$\langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle$	2	$\{144^-, 36^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle$	2	$\{36^-, 144^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle$	2	$\{1^+, 144^-\}$	145
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-, 36^+\}$	72
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{9^+, 144^-\}$	153
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-, 36^+\}$	72
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d))$	2	$\{9^+, 144^-\}$	153
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_s}}\rangle$ $(q=u,d)$	2	$\{144^-, 36^+\}$	180
$\langle \Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+, 9^-, 144^-, 144^+, 144^-, 144^+, 9^+, 1^-\}$	596
$\langle \Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^+, 36^-, 144^-, 36^+, 36^-, 144^+, 36^+, 1^-\}$	434
$\langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle$	8	$\{36^+, 144^-, 9^-, 36^+, 36^-, 9^+, 144^+, 36^-\}$	450
$\langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle$	8	$\{36^+, 36^-, 9^-, 144^+, 144^-, 9^+, 36^+, 36^-\}$	450
$\langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-, 144^+, 36^+, 9^-, 9^+, 36^-, 144^-, 36^+\}$	450
$\langle \Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+, 9^-, 144^-, 144^+, 144^-, 144^+, 9^+, 1^-\}$	596
$\langle \Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-, 36^+, 144^+, 36^-, 36^+, 144^-, 36^-, 1^+\}$	434
$\langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle$	8	$\{36^+, 144^-, 9^-, 36^+, 36^-, 9^+, 144^+, 36^-\}$	450
$\langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle$	8	$\{36^+, 36^-, 9^-, 144^+, 144^-, 9^+, 36^+, 36^-\}$	450

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=−2 systems (cont’ d)

$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,u),(s,d),(u,s),(d,s))$	2	$\{36^-,144^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_s}\Lambda_{X_s}}\rangle$	2	$\{144^-,144^+\}$	288
$\langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle$	2	$\{144^-,36^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle$	2	$\{36^-,144^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle$	2	$\{1^+,144^-\}$	145
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-,36^+\}$	72
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{9^+,144^-\}$	153
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-,36^+\}$	72
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d))$	2	$\{9^+,144^-\}$	153
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_s}}\rangle$ $(q=u,d)$	2	$\{144^-,36^+\}$	180
$\langle \Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^+,36^-,144^-,36^+,36^-,144^+,36^+,1^-\}$	434
$\langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle$	8	$\{36^+,144^-,9^-,36^+,36^-,9^+,144^+,36^-\}$	450
$\langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle$	8	$\{36^+,36^-,9^-,144^+,144^-,9^+,36^+,36^-\}$	450
$\langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-,144^+,36^+,9^-,9^+,36^-,144^-,36^+\}$	450
$\langle \Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-,36^+,144^+,36^-,36^+,144^-,36^-,1^+\}$	434
$\langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle$	8	$\{36^+,144^-,9^-,36^+,36^-,9^+,144^+,36^-\}$	450
$\langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle$	8	$\{36^+,36^-,9^-,144^+,144^-,9^+,36^+,36^-\}$	450
$\langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-,144^+,36^+,9^-,9^+,36^-,144^-,36^+\}$	450
$\langle \Sigma^0\Lambda\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Sigma^0\Lambda\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-,36^+,144^+,36^-,36^+,144^-,36^-,1^+\}$	434

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=-3 and -4 systems

channel	# of diagrams	$\{(\#\text{ of iterations})^{\text{sign}}\}$	# of total iterations
$\langle \Xi^-\Lambda\overline{\Xi^-\Lambda_{X_{u,s}}}\rangle$	6	$\{1^+, 36^-, 144^+, 144^-, 36^+, 9^-\}$	370
$\langle \Xi^-\Lambda\overline{\Xi^-\Lambda_{X_d}}\rangle$	6	$\{1^+, 36^-, 144^+, 36^-, 144^+, 36^-\}$	397
$\langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle$	6	$\{36^-, 9^+, 144^-, 144^+, 36^-, 36^+\}$	405
$\langle \Xi^-\Lambda\overline{\Sigma^0_{X_u}\Xi^-}\rangle$	6	$\{36^+, 9^-, 144^+, 36^-, 144^+, 1^-\}$	370
$\langle \Xi^-\Lambda\overline{\Sigma^0_{X_d}\Xi^-}\rangle$	6	$\{144^-, 36^+, 36^-, 36^+, 144^-, 1^+\}$	397
$\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda_{X_u}}\rangle$	3	$\{36^-, 144^+, 36^-\}$	216
$\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda_{X_d}}\rangle$	3	$\{9^-, 36^+, 144^-\}$	189
$\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda_{X_s}}\rangle$	3	$\{36^-, 144^+, 144^-\}$	324
$\langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle$	3	$\{1^+, 144^-, 36^+\}$	181
$\langle \Sigma^-\Xi^0\overline{\Sigma^0_{X_u}\Xi^-}\rangle$	3	$\{36^-, 36^+, 144^-\}$	216
$\langle \Sigma^-\Xi^0\overline{\Sigma^0_{X_d}\Xi^-}\rangle$	3	$\{144^+, 9^-, 36^+\}$	189
$\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda_{X_{u,s}}}\rangle$	6	$\{9^+, 36^-, 144^+, 144^-, 36^+, 1^-\}$	370
$\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda_{X_d}}\rangle$	6	$\{36^+, 144^-, 36^+, 144^-, 36^+, 1^-\}$	397
$\langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle$	6	$\{36^-, 36^+, 144^-, 144^+, 9^-, 36^+\}$	405
$\langle \Sigma^0\Xi^-\overline{\Sigma^0_{X_u}\Xi^-}\rangle$	6	$\{1^+, 144^-, 36^+, 144^-, 9^+, 36^-\}$	370
$\langle \Sigma^0\Xi^-\overline{\Sigma^0_{X_d}\Xi^-}\rangle$	6	$\{1^-, 144^+, 36^-, 36^+, 36^-, 144^+\}$	397
$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle$	6	$\{1^+, 36^-, 9^+, 144^+, 36^-, 144^+\}$	370

Each number of iterations is less than 600

Effective block algorithm to calculate the 52 channels of 4pt correlator

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{array}{lll}
\langle p\Lambda\overline{p\Lambda}\rangle, & \langle p\Lambda\overline{\Sigma^+ n}\rangle, & \langle p\Lambda\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^+ n\overline{p\Lambda}\rangle, & \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, & \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^0 p\overline{p\Lambda}\rangle, & \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, & \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{array} \tag{4.2}$$

$$\begin{array}{llllll}
\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, & \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, & \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, & \\
\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, & \langle p\Xi^-\overline{p\Xi^-}\rangle, & \langle p\Xi^-\overline{n\Xi^0}\rangle, & \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, & \langle n\Xi^0\overline{p\Xi^-}\rangle, & \langle n\Xi^0\overline{n\Xi^0}\rangle, & \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, & \\
 & \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & & \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{array} \tag{4.3}$$

$$\begin{array}{lll}
\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{array} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

- Elapse times to calculate the 52 matrix correlators (MPI+OpenMP)
- [tasks_per_node] x [OMP_NUM_THREADS]

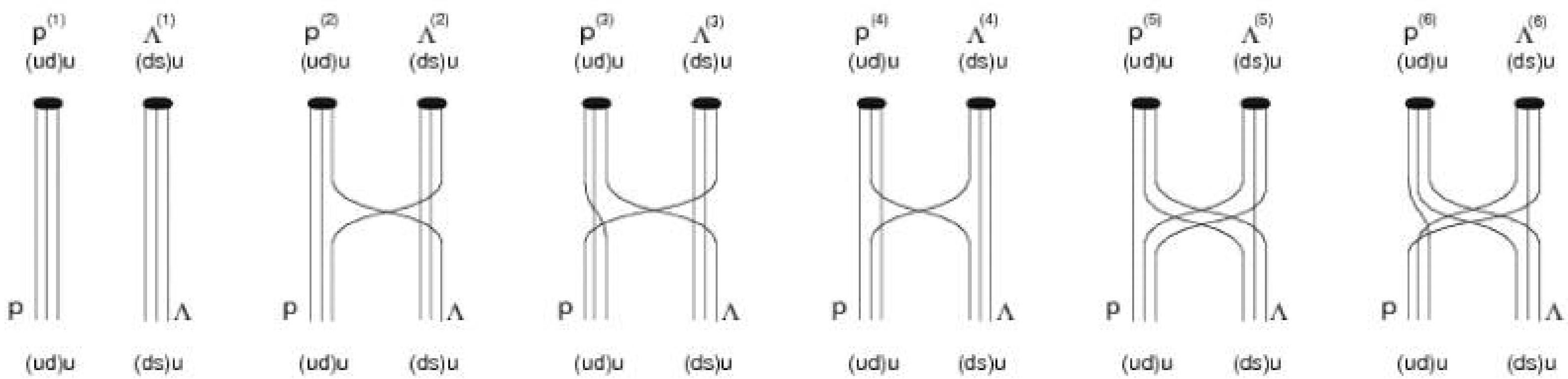
	64x1	32x2	16x4	8x4	4x8	2x16	1x32
Step-1	0:14	0:16	0:09	0:09	0:07	0:06	0:06
Step-2	0:10	0:11	0:12	0:12	0:12	0:13	0:14

HN, PoS(LATTICE2013) 426.

4pt correlator through the FFT

- Cuda implementations for various parts:

1 $[B]_\alpha(r,\alpha_1,c_1,\alpha_2,c_2,\alpha_3,c_3)$ $\quad [B]=[N,\Sigma,\Xi,\Lambda(dsu),\Lambda(sud),\Lambda(uds)]$



$$\phi_{\alpha\beta}(r)=\sum_i F^{(i)}\phi^{(i)}_{\alpha\beta}(r) =\sum_i F^{(i)}\sum_x p^{(i)}_\alpha(X+\vec{r})\Lambda^{(i)}_\beta(X)$$

$$=\sum_i F^{(i)}\sum_q \tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)e^{iqr} =\sum_q\left(\sum_i F^{(i)}\tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)\right)e^{iqr}$$

3 $\tilde{p}^{(i)}_\alpha(q)$ $\quad$ 3 $\tilde{\Lambda}^{(i)}_\beta(-q)$ $\quad$ 5 $\phi_{\alpha\beta}(r)$

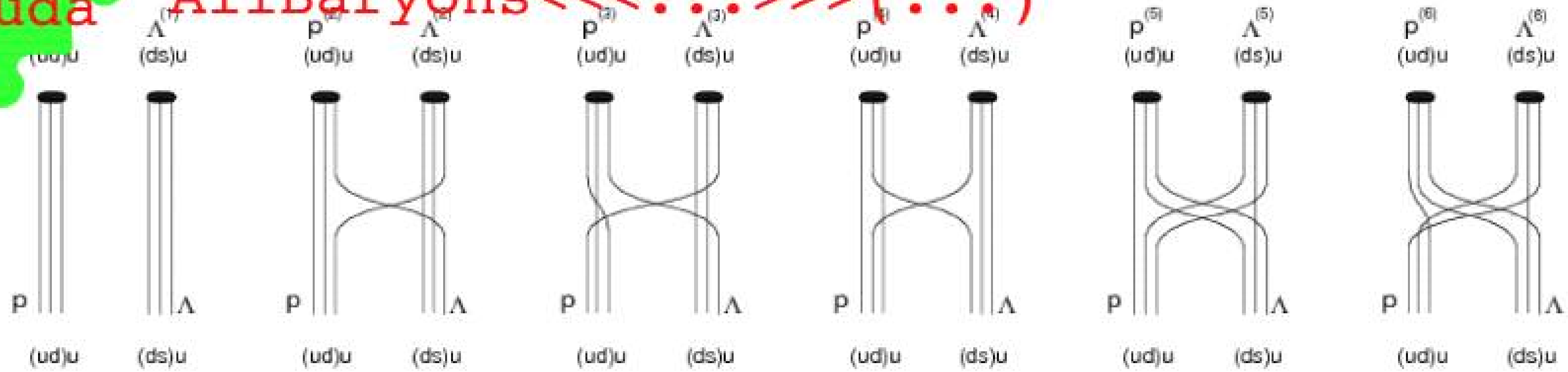
2 $p^{(i)}_\alpha(X+r)$ $\quad$ 2 $\Lambda^{(i)}_\beta(X)$ $\quad$ 4 $\left(\sum_i F^{(i)}\tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)\right)$

4pt correlator through the FFT

- Cuda implementations for various parts:

$[B]_\alpha(r,\alpha_1,c_1,\alpha_2,c_2,\alpha_3,c_3)$ $\quad [B]=[N,\Sigma,\Xi,\Lambda(dsu),\Lambda(sud),\Lambda(uds)]$

cuda AllBaryons<<<...>>>(...)



$$\phi_{\alpha\beta}(r)=\sum_i F^{(i)}\phi^{(i)}_{\alpha\beta}(r) = \sum_i F^{(i)}\sum_x p^{(i)}_\alpha(X+r)\Lambda^{(i)}_\beta(X)$$

$$=\sum_i F^{(i)}\sum_q \tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)e^{iqr} = \sum_q\left(\sum_i F^{(i)}\tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)\right)e^{iqr}$$

$\tilde{p}^{(i)}_\alpha(q)$ $\quad \tilde{\Lambda}^{(i)}_\beta(-q)$ $\quad \phi_{\alpha\beta}(r)$

cuda $p^{(i)}_\alpha(X+r)$ $\quad$ cuda $\Lambda^{(i)}_\beta(X)$ $\quad$ cuda $\left(\sum_i F^{(i)}\tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)\right)$

A<<<...>>>(1,...) A<<<...>>>(2,...) B<<<...>>>(...)

HA-PACS

- BASE
  - Intel E5-2670 (16core) + NVIDIA M2090 (x4)

    332.8 GFlops　　665 GFlops (x4)

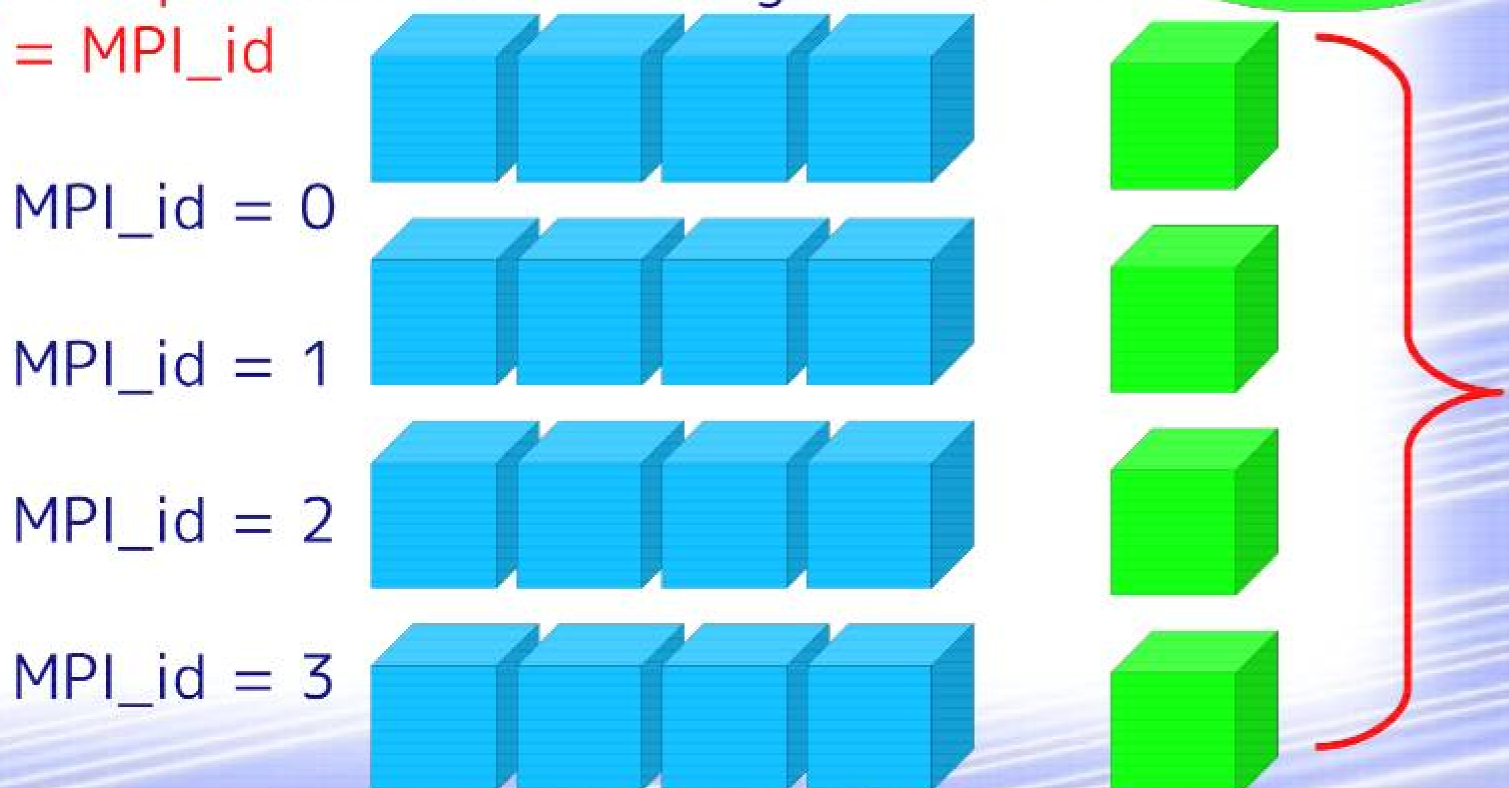
    128 GBytes　　6 GBytes (x4)
- TCA
  - Intel E5-2680v2 (20core) + NVIDIA K20X (x4)

    448 GFlops　　1310 GFlops (x4)

    128 GBytes　　6 GBytes (x4)

MPI+OpenMP + CUDA

- For HA-PACS, 1PE has 16 CPU cores and 4 GPUs:
  - cudaSetDevice(GPU_id); specifies the GPU
  - GPUid is determined by MPI_id or thread_id
  - We take "4mpi * 4threads" configuration and
  - GPU_id = MPI_id



Detailed elapsed time

	GPU	CPU-1	CPU-2	CPU-3	
0	(0.0e+00)	0.0e+00	0.0e+00	0.0e+00	Start
...					
3	(1.2e-02)	2.3e-01	2.1e-01	2.3e-01	End A(1)
7	(5.8e-02)	4.7e-01	5.1e-01	4.7e-01	End A(2)
53	(1.7e+00)	4.2e+00	4.3e+00	4.2e+00	End B
...					
55	(1.7e+00)	4.4e+00	4.5e+00	4.4e+00	End A(1)
57	(1.7e+00)	4.6e+00	4.7e+00	4.6e+00	End A(2)
86	(1.9e+00)	7.2e+00	7.0e+00	7.2e+00	End B
...					
88	(1.9e+00)	*	*	*	End A(1)
90	(1.9e+00)	*	*	*	End A(2)
119	(2.1e+00)	*	*	*	End B
...					
121	(2.1e+00)	*	*	*	End A(1)
123	(2.2e+00)	*	*	*	End A(2)
152	(2.3e+00)	*	*	*	End B
...					
847	(6.9e+00)	*	*	*	End A(1)
849	(6.9e+00)	*	*	*	End A(2)
878	(7.1e+00)	*	*	*	End B

GPU performs a lot of job!

52 channel calculation with 16^3x32 lattice

- Without GPU, elapsed time is 2:22
- With GPU (M2090), 1:45

$$\langle p n \overline{p n} \rangle, \tag{4.1}$$

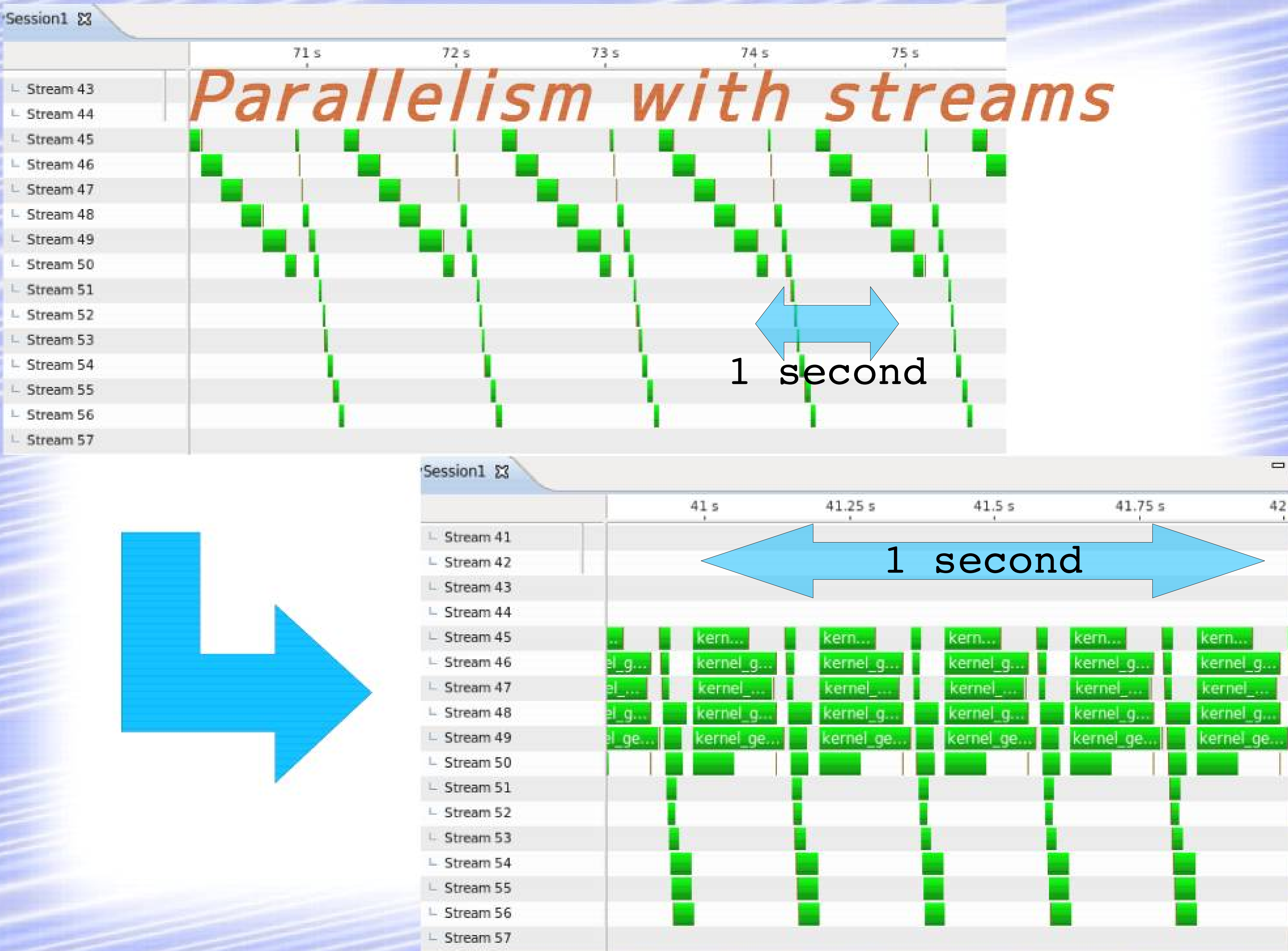
$$\begin{aligned}
&\langle p\Lambda \overline{p\Lambda} \rangle, \quad \langle p\Lambda \overline{\Sigma^+ n} \rangle, \quad \langle p\Lambda \overline{\Sigma^0 p} \rangle, \\
&\langle \Sigma^+ n \overline{p\Lambda} \rangle, \quad \langle \Sigma^+ n \overline{\Sigma^+ n} \rangle, \quad \langle \Sigma^+ n \overline{\Sigma^0 p} \rangle, \\
&\langle \Sigma^0 p \overline{p\Lambda} \rangle, \quad \langle \Sigma^0 p \overline{\Sigma^+ n} \rangle, \quad \langle \Sigma^0 p \overline{\Sigma^0 p} \rangle,
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
&\langle \Lambda\Lambda \overline{\Lambda\Lambda} \rangle, \quad \langle \Lambda\Lambda \overline{p\Xi^-} \rangle, \quad \langle \Lambda\Lambda \overline{n\Xi^0} \rangle, \quad \langle \Lambda\Lambda \overline{\Sigma^+\Sigma^-} \rangle, \quad \langle \Lambda\Lambda \overline{\Sigma^0\Sigma^0} \rangle, \\
&\langle p\Xi^- \overline{\Lambda\Lambda} \rangle, \quad \langle p\Xi^- \overline{p\Xi^-} \rangle, \quad \langle p\Xi^- \overline{n\Xi^0} \rangle, \quad \langle p\Xi^- \overline{\Sigma^+\Sigma^-} \rangle, \quad \langle p\Xi^- \overline{\Sigma^0\Sigma^0} \rangle, \quad \langle p\Xi^- \overline{\Sigma^0\Lambda} \rangle, \\
&\langle n\Xi^0 \overline{\Lambda\Lambda} \rangle, \quad \langle n\Xi^0 \overline{p\Xi^-} \rangle, \quad \langle n\Xi^0 \overline{n\Xi^0} \rangle, \quad \langle n\Xi^0 \overline{\Sigma^+\Sigma^-} \rangle, \quad \langle n\Xi^0 \overline{\Sigma^0\Sigma^0} \rangle, \quad \langle n\Xi^0 \overline{\Sigma^0\Lambda} \rangle, \\
&\langle \Sigma^+\Sigma^- \overline{\Lambda\Lambda} \rangle, \quad \langle \Sigma^+\Sigma^- \overline{p\Xi^-} \rangle, \quad \langle \Sigma^+\Sigma^- \overline{n\Xi^0} \rangle, \quad \langle \Sigma^+\Sigma^- \overline{\Sigma^+\Sigma^-} \rangle, \quad \langle \Sigma^+\Sigma^- \overline{\Sigma^0\Sigma^0} \rangle, \quad \langle \Sigma^+\Sigma^- \overline{\Sigma^0\Lambda} \rangle, \\
&\langle \Sigma^0\Sigma^0 \overline{\Lambda\Lambda} \rangle, \quad \langle \Sigma^0\Sigma^0 \overline{p\Xi^-} \rangle, \quad \langle \Sigma^0\Sigma^0 \overline{n\Xi^0} \rangle, \quad \langle \Sigma^0\Sigma^0 \overline{\Sigma^+\Sigma^-} \rangle, \quad \langle \Sigma^0\Sigma^0 \overline{\Sigma^0\Sigma^0} \rangle, \\
&\quad \langle \Sigma^0\Lambda \overline{p\Xi^-} \rangle, \quad \langle \Sigma^0\Lambda \overline{n\Xi^0} \rangle, \quad \langle \Sigma^0\Lambda \overline{\Sigma^+\Sigma^-} \rangle, \quad \langle \Sigma^0\Lambda \overline{\Sigma^0\Lambda} \rangle,
\end{aligned} \tag{4.3}$$

$$\begin{aligned}
&\langle \Xi^-\Lambda \overline{\Xi^-\Lambda} \rangle, \quad \langle \Xi^-\Lambda \overline{\Sigma^-\Xi^0} \rangle, \quad \langle \Xi^-\Lambda \overline{\Sigma^0\Xi^-} \rangle, \\
&\langle \Sigma^-\Xi^0 \overline{\Xi^-\Lambda} \rangle, \quad \langle \Sigma^-\Xi^0 \overline{\Sigma^-\Xi^0} \rangle, \quad \langle \Sigma^-\Xi^0 \overline{\Sigma^0\Xi^-} \rangle, \\
&\langle \Sigma^0\Xi^- \overline{\Xi^-\Lambda} \rangle, \quad \langle \Sigma^0\Xi^- \overline{\Sigma^-\Xi^0} \rangle, \quad \langle \Sigma^0\Xi^- \overline{\Sigma^0\Xi^-} \rangle,
\end{aligned} \tag{4.4}$$

$$\langle \Xi^-\Xi^0 \overline{\Xi^-\Xi^0} \rangle. \tag{4.5}$$

Parallelism with streams



Results

⋆ HA-PACS:

M2090 (665 GFlops)	K20X (1310 GFlops)	
20 GFlops	27 GFlops	`AllBaryons<<<...>>>(...)`
1.6 GFlops	1.1 GFlops	`A<<<...>>>(...)`
5.4 GFlops	6.1 GFlops	`B<<<...>>>(...)`
4.7 GFlops	26 GFlops	`B<<<...>>>(...)` [using streams]

- Lattice size: 16^3 x 32

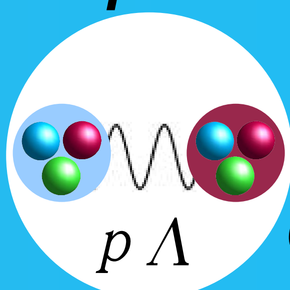
Summary on main part

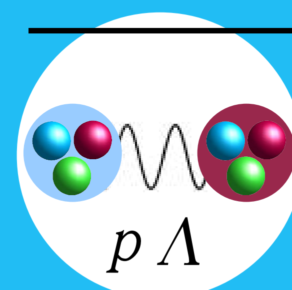
(1) We present a fast algorithm to calculate the 4pt correlation function of Lambda-Nucleon system, which was used to study the hyperonic nuclear forces from lattice QCD.
(2) Generalize the target system to various baryon-baryon channels. (E.g., 52 channel NBS wave functions can be obtained at the same time from one computing job for the 2+1 lattice QCD.)
(3) In this approach, the number of iterations to obtain the four-point correlation function is remarkably smaller than the numbers given in the unified contraction algorithm[2].
(4) A hybrid parallel (C++) and multi-GPU (CUDA) program has been implemented with MPI and OpenMP, working on supercomputer (HA-PACS); Concurrent kernel executions with streams improve the computing performance for K20X (TCA part of HA-PACS).

[1] H.N. PoS(LAT2013)426;(LAT2008)156;(LAT2009)152;(LAT2011)167;
(LAT2013)426.
[2] Doi and Endres, Comput. Phys. Commun. 184, 117 (2013).

Benchmark

$$F_{\alpha\beta\alpha'\beta'}(\vec{r},\, t - t_0) \quad ,$$

$$\rightarrow \langle$$ 

$$_{\alpha\beta}(\vec{r},\, t)$$ 

$$\overline{}_{\alpha'\beta'}(t_0)\rangle$$

Benchmark of the hybrid parallel C++ code implementation

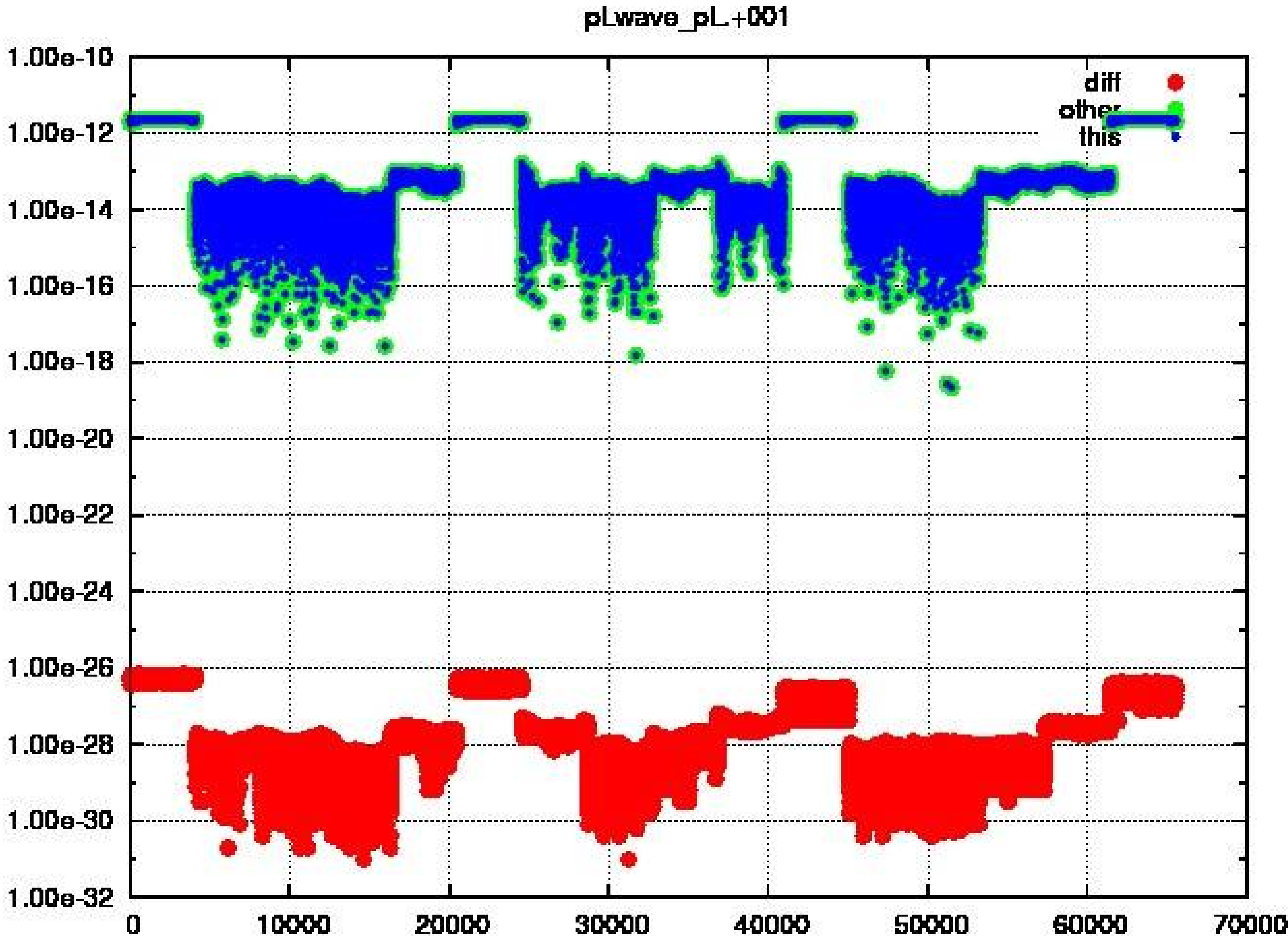
Comparisons have been made for all 52 channels over 31 time-slices, 16*16*16 points for spatial, and 2*2*2*2 points for the spin degrees of freedom.

There are 16*16*16*2*2*2*2 = 65536 points per time-slice per channel.

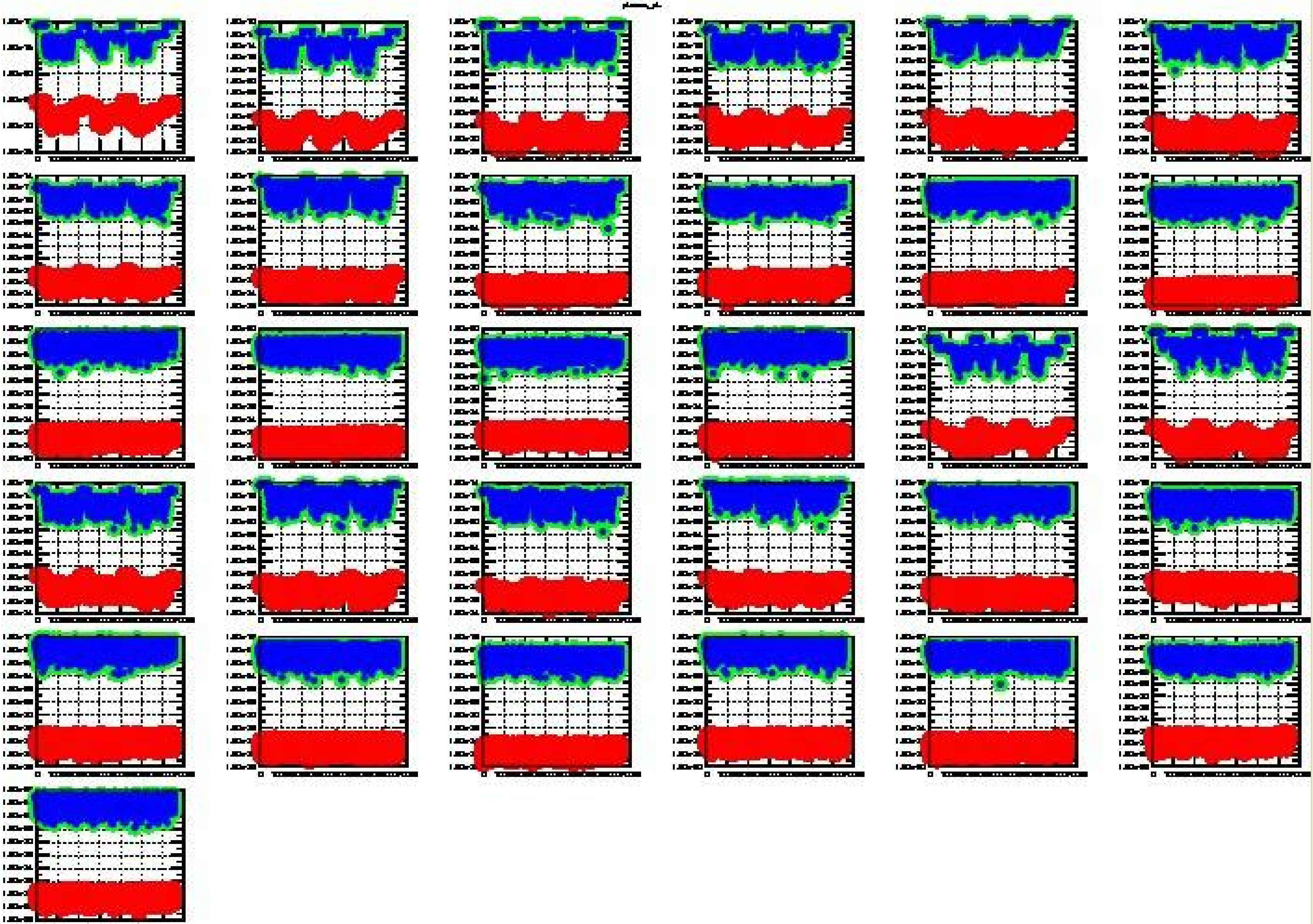
For example:

>>>	a	b	aP	bP	x	y	z	ratio	From_other_work	From_this_work	diff (=From_other-From_this)
>>>	0	1	0	1	0	0	0	1.000000000000e+00	4.755520154185e-20	4.755520154185e-20	-8.907425992791e-34
>>>	0	1	0	1	1	0	0	1.000000000000e+00	1.064483789589e-19	1.064483789589e-19	-4.935195482492e-34
>>>	0	1	0	1	2	0	0	1.000000000000e+00	1.034021677632e-19	1.034021677632e-19	-8.185202263646e-34
>>>	0	1	0	1	3	0	0	1.000000000000e+00	9.903890629525e-20	9.903890629525e-20	-9.750020343460e-34
>>>	0	1	0	1	4	0	0	1.000000000000e+00	9.480310911656e-20	9.480310911656e-20	-1.095372655870e-33
>>>	0	1	0	1	5	0	0	1.000000000000e+00	1.030125191701e-19	1.030125191701e-19	-9.388908478888e-34
>>>	0	1	0	1	6	0	0	1.000000000000e+00	8.655174217981e-20	8.655174217981e-20	-6.500013562307e-34
>>>	0	1	0	1	7	0	0	1.000000000000e+00	2.229267218351e-20	2.229267218351e-20	-9.087981925077e-34
>>>	0	1	0	1	8	0	0	9.999999999996e-01	2.330235858756e-21	2.330235858757e-21	-8.911187574714e-34
>>>	0	1	0	1	9	0	0	1.000000000000e+00	2.034855576332e-20	2.034855576332e-20	-7.011588703785e-34
>>>	0	1	0	1	10	0	0	1.000000000000e+00	3.822483016345e-20	3.822483016345e-20	-6.981496048404e-34
>>>	0	1	0	1	11	0	0	1.000000000000e+00	6.506309796602e-20	6.506309796602e-20	-4.453712996395e-34
>>>	0	1	0	1	12	0	0	1.000000000000e+00	8.118512223003e-20	8.118512223003e-20	-6.981496048404e-34
>>>	0	1	0	1	13	0	0	1.000000000000e+00	5.202429085187e-20	5.202429085187e-20	-1.017131751880e-33
>>>	0	1	0	1	14	0	0	1.000000000000e+00	5.725220267276e-20	5.725220267276e-20	-7.342607912976e-34
>>>	0	1	0	1	15	0	0	1.000000000000e+00	4.331313772187e-20	4.331313772187e-20	-8.185202263646e-34

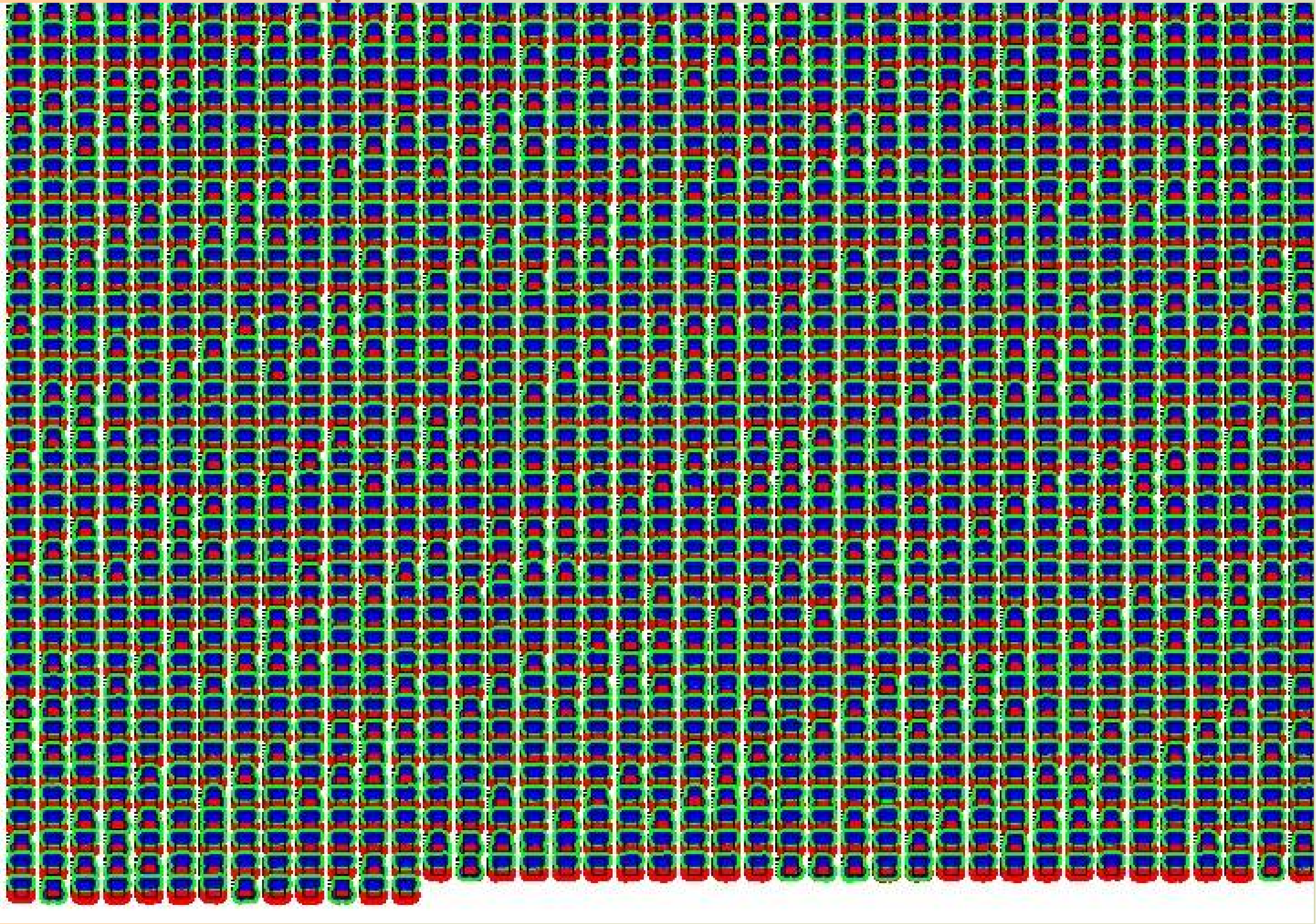
Benchmark (only LN-LN channel at $t-t_0$=+001)



Benchmark (at LN-LN channel)



Benchmark (from NN to XiXi channels)



Summary

(I-1) Lattice QCD calculation for hyperon potentials toward the physical point calculation. (Lambda-N, Sigma-N: central, tensor)
(I-2) Effective hadron block algorithm for the various baron-baryon interaction

A hybrid parallel (C++) and multi-GPU (CUDA) program is implemented (MPI + OpenMP).
Reasonable performances at various hybrid parallel execution on the supercomputers (BlueGene/Q and HA-PACS)

(II-1) Benchmark calculation for the 52 channel of NBS WFs.
(II-2) Very preliminary LN potential at physical point.