Coupled channel baryon-baryon interactions on the lattice

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for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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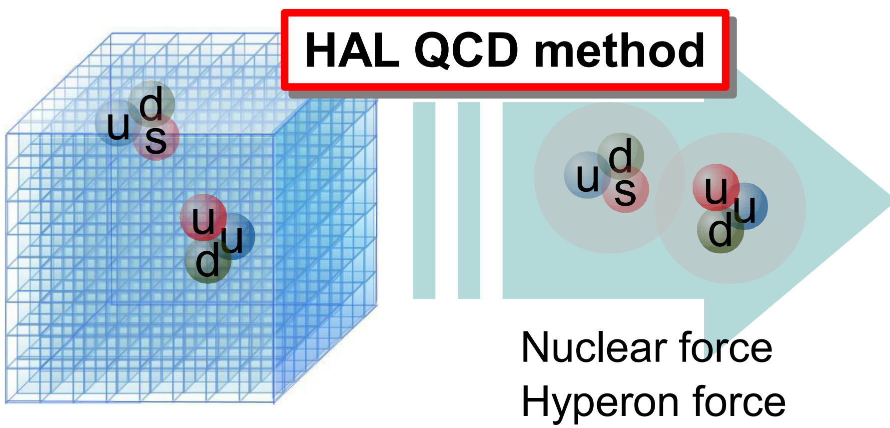
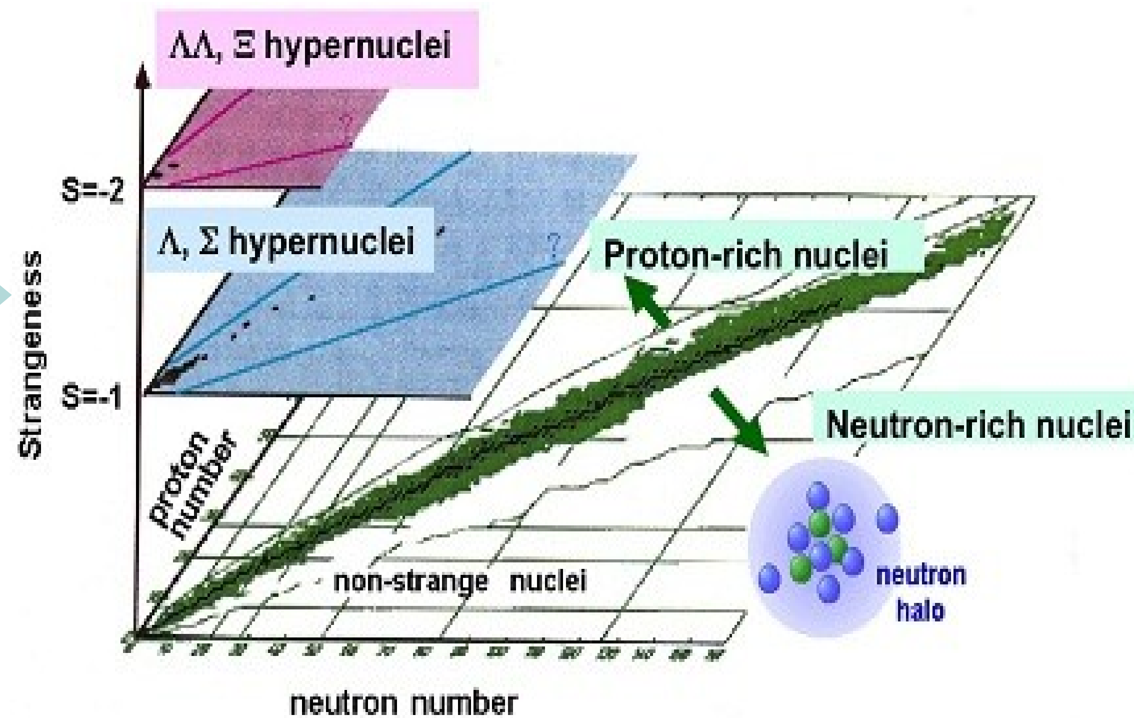
Introduction

Introduction

<u>BB interactions are inputs to investigate the nuclear structure</u>





Technical improvements

- **Unified Contraction Algorithm,**
- **Time dependent method,**
- **Higher partial waves,**
- **Finite volume method vs potential**

Extensions of the method

- **Generalized BB interaction**
- **Charmed baryon system**
- **Meson-meson,meson-baryon system**
- **Three-body interaction**

S=-2 BB interaction

Interests of S=-2 multi-baryon system

H-dibaryon

- The flavor singlet state with J=0 predicted by R.L. Jaffe.
  - Strongly attractive color magnetic interaction.
  - No quark Pauli principle for flavor singlet state.

Double-Λ hypernucleus

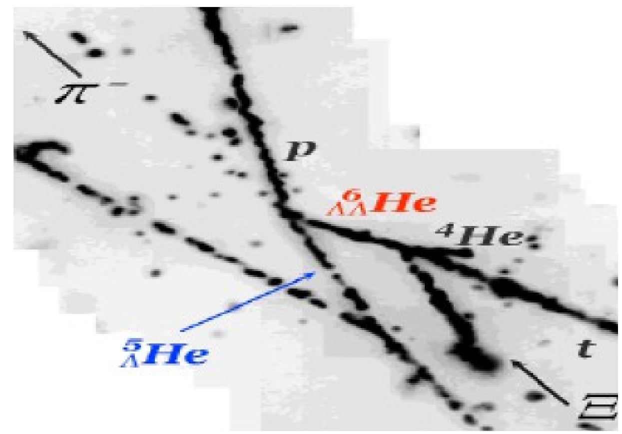
- Conclusions of the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 Collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV

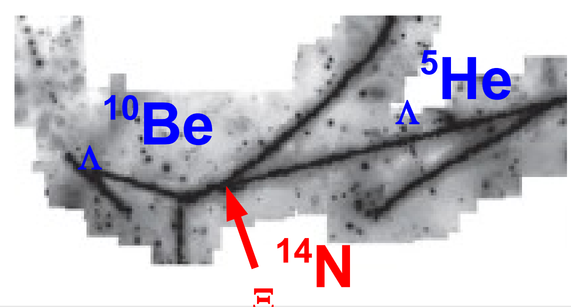


Ξ hypernucleus

- Conclusions of the "KISO Event"

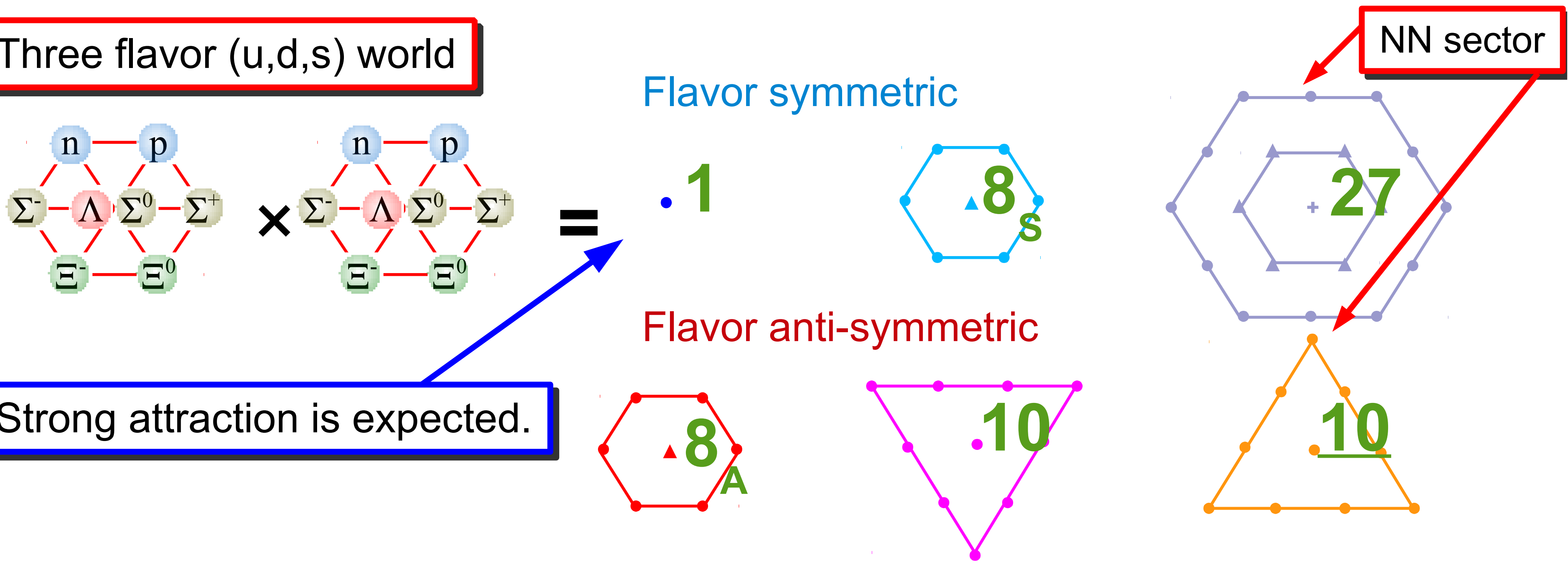
K.Nakazawa and KEK-E373 Collaborators

Ξ–N attraction



SU(3) feature of BB interaction

Three flavor (u,d,s) world



Strong attraction is expected.

In view of quark degrees of freedom

Oka, Shimizu and Yazaki NPA464 (1987)

- Short range repulsion in BB interaction could be a result of Pauli principle and color-magnetic interaction for the quarks.
- Strengths of repulsive core in YN and YY interaction are largely depend on their flavor structures.
- For the s-wave BB system, no repulsive core is predicted in flavor singlet state which is known as H-dibaryon channel.

Numerical results

Lists of channels

I=0 states

Strong attraction
(H-dibaryon)

Spin	BB channels			SU(3) representation		
$^1S_0$	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$	1	8s	27
$^3S_1$	--	$N\Xi$	--	8a	--	--

I=1 states

Attraction

Strong repulsion

Spin	BB channels			SU(3) representation		
$^1S_0$	$N\Xi$	--	$\Lambda\Sigma$	--	8s	27
$^3S_1$	$N\Xi$	$\Sigma\Sigma$	$\Lambda\Sigma$	8a	10	10*

Similar to
The NN potential

Repulsion

I=2 states

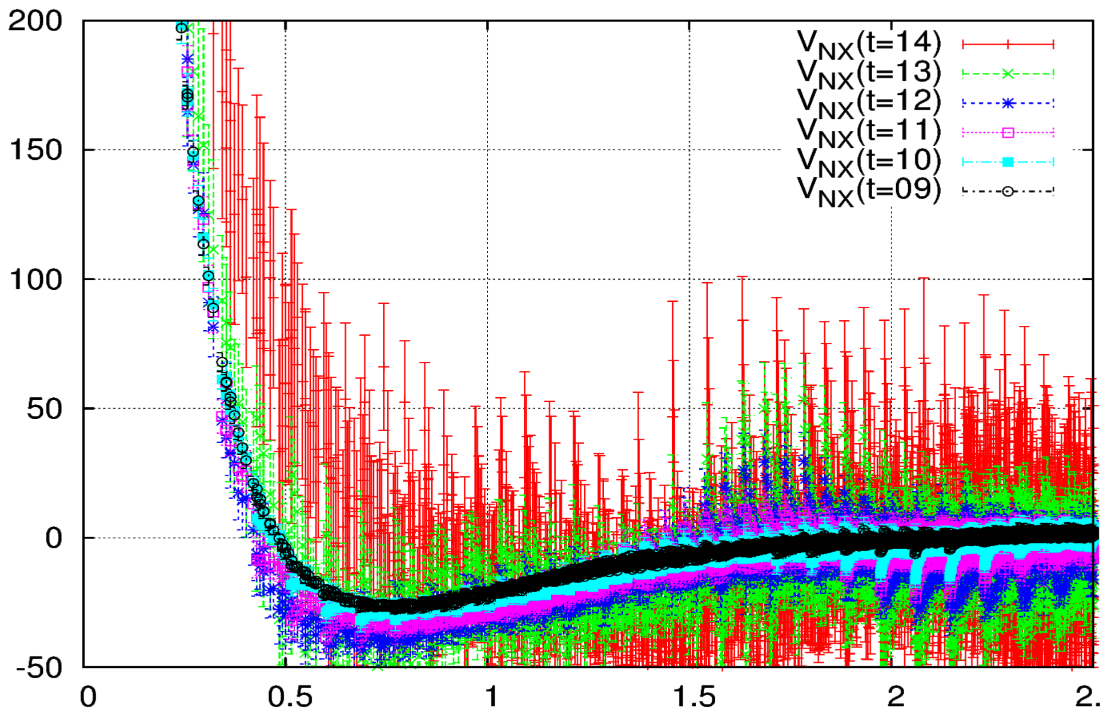
Spin	BB channels			SU(3) representation		
$^1S_0$	$\Sigma\Sigma$			--	--	27
~~$^3S_1$~~						

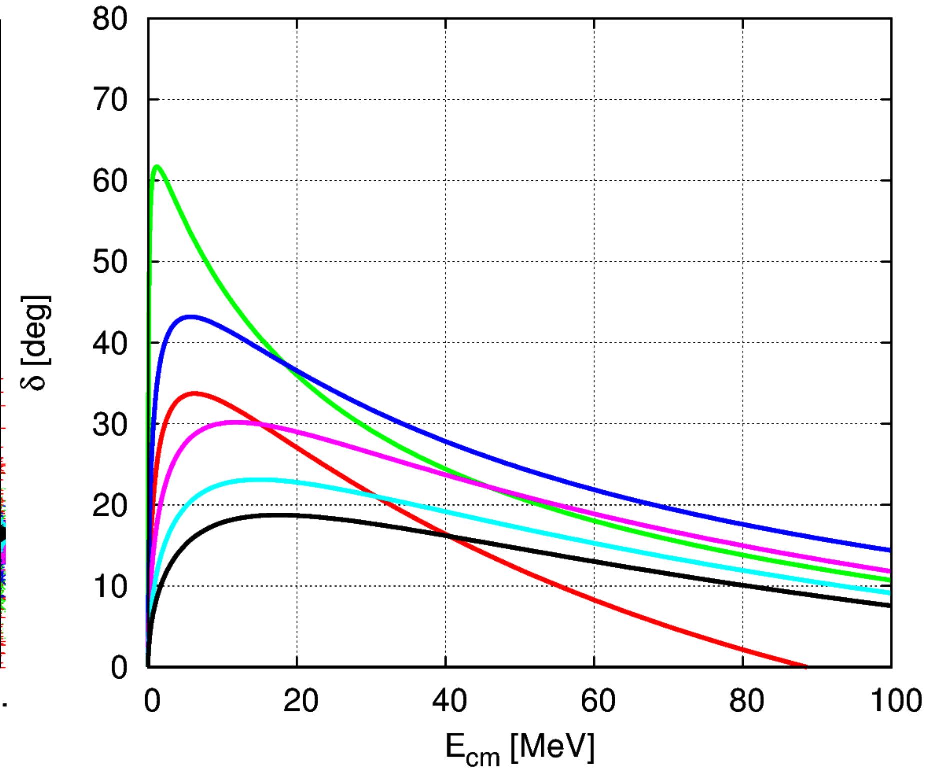
$N\Xi$ (I=0) $^3S_1$ potential

▶ N_f = 2+1 full QCD with L = 8fm, $m\pi$ = 145 MeV

Preliminary!

Belongs to 8a plet





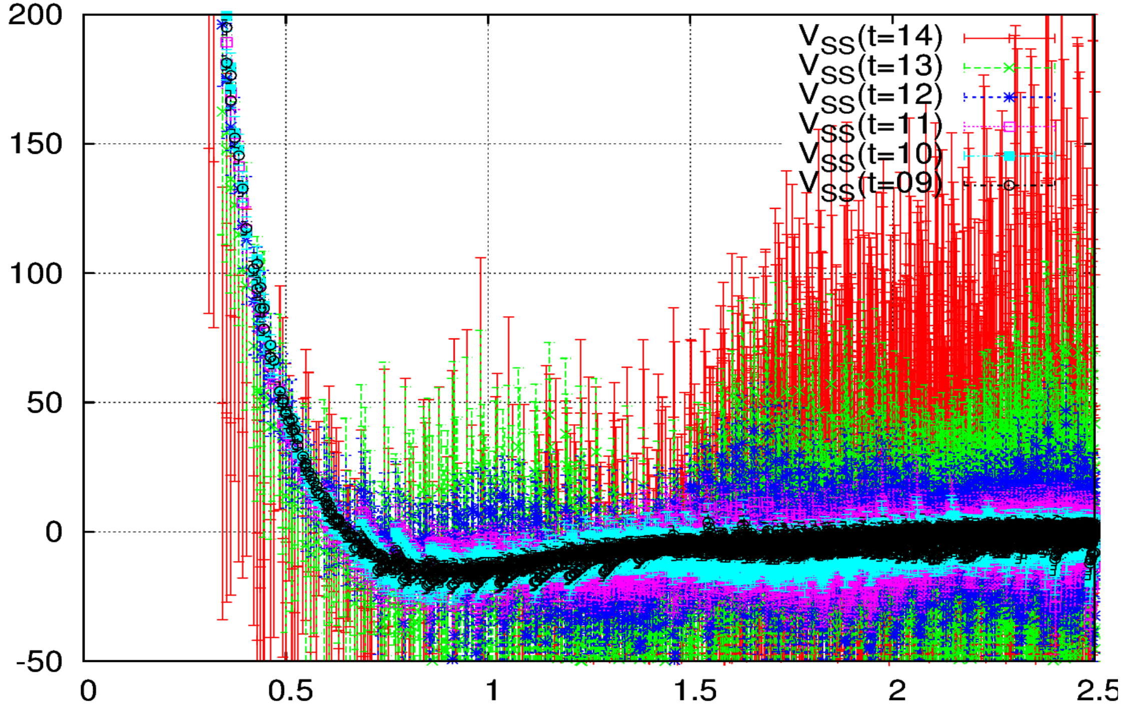
- **$N\Xi$ potential is strongly attractive.**
- We can see an attractive pocket at around r=0.7fm.
- We have to check that the potential is saturated well in this time range.

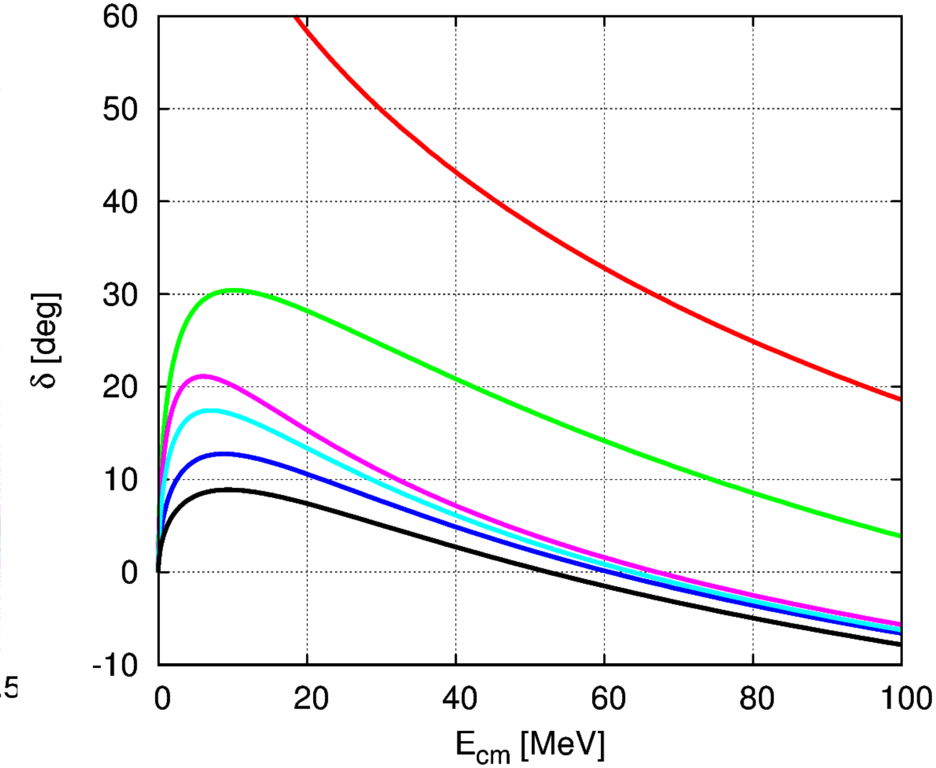
$\Sigma\Sigma$ (I=2) $^1S_0$ potential

▶ N_f = 2+1 full QCD with L = 8fm, $m\pi$ = 145 MeV

Preliminary!

Belongs to 27plet



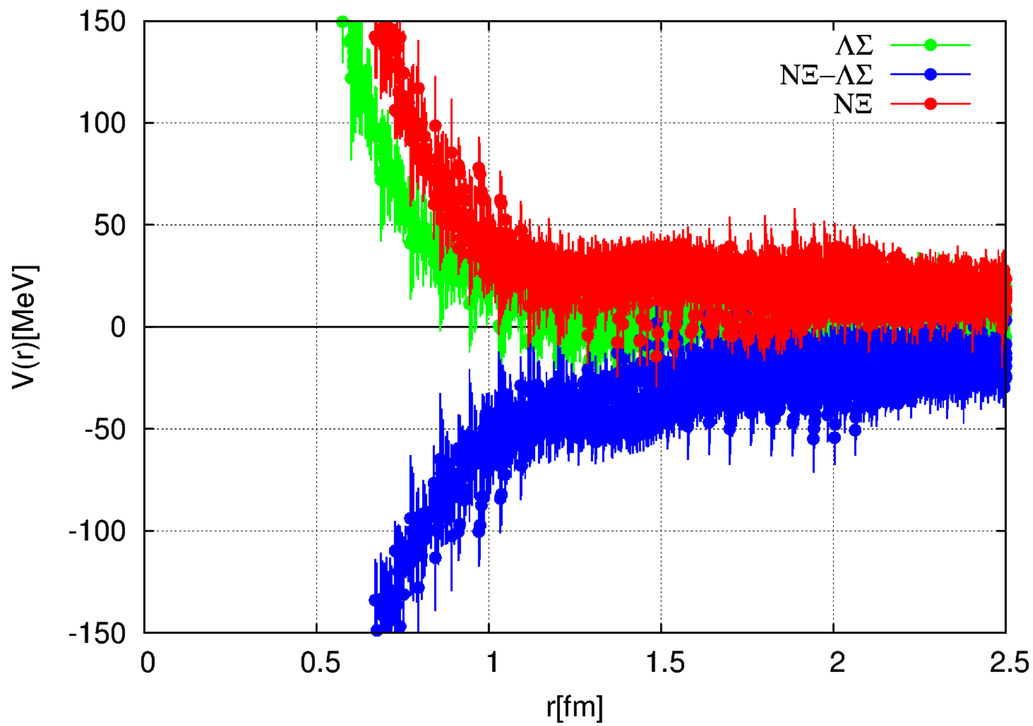


- We can see an attractive pocket at around r=0.7fm.
- We have to check that the potential is saturated well in this time range.

$N\Xi$, $\Lambda\Sigma$ (I=1) $^1S_0$ channel

▶ N_f = 2+1 full QCD with L = 8fm, ***$m\pi$ = 145 MeV***

Preliminary!



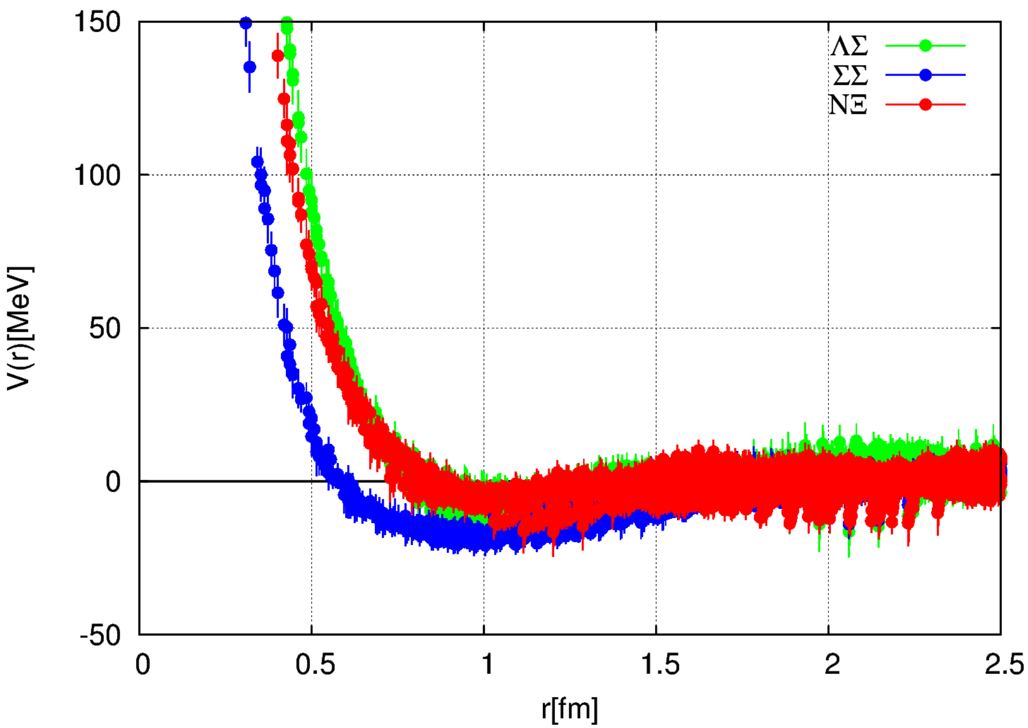
- All diagonal element are totally repulsive in whole range.
- **Diagonal $N\Xi$ potential is strongly repulsive unlike the I=0 $^3S_1$ case.**

  It means that the $N\Xi$ potential is strongly depend on the channel.

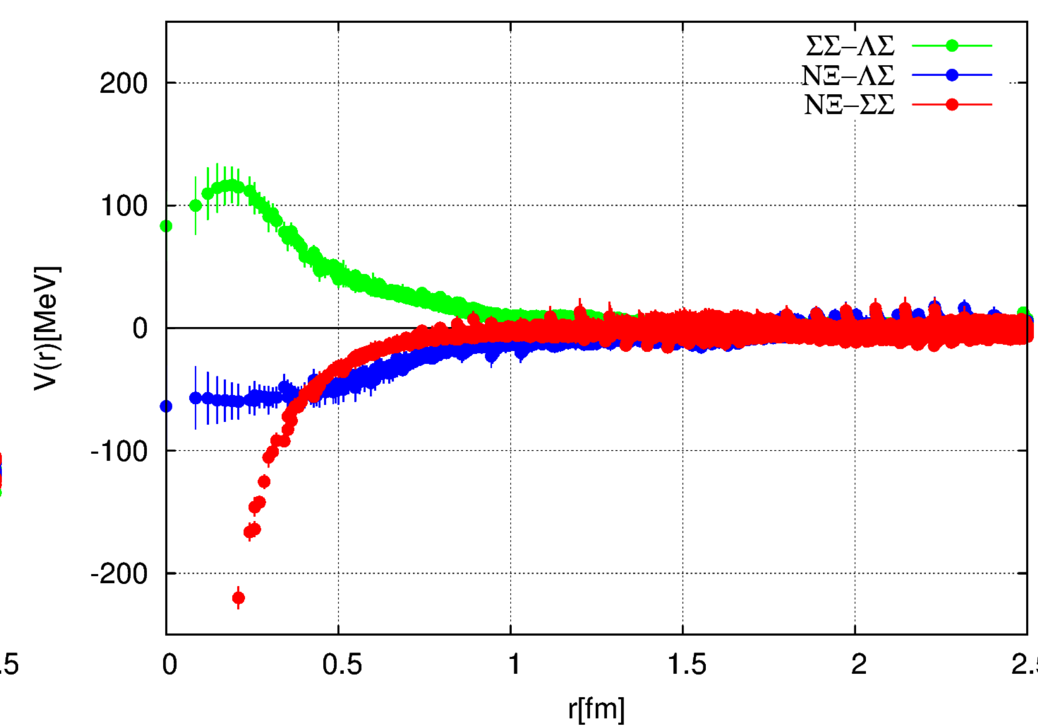
$N\Xi$, $\Lambda\Sigma$, $\Sigma\Sigma$ (I=1) $^3S_1$ channel

▶ N_f = 2+1 full QCD with L = 8fm, **m_π = 145 MeV**

Diagonal elements



Off-diagonal elements



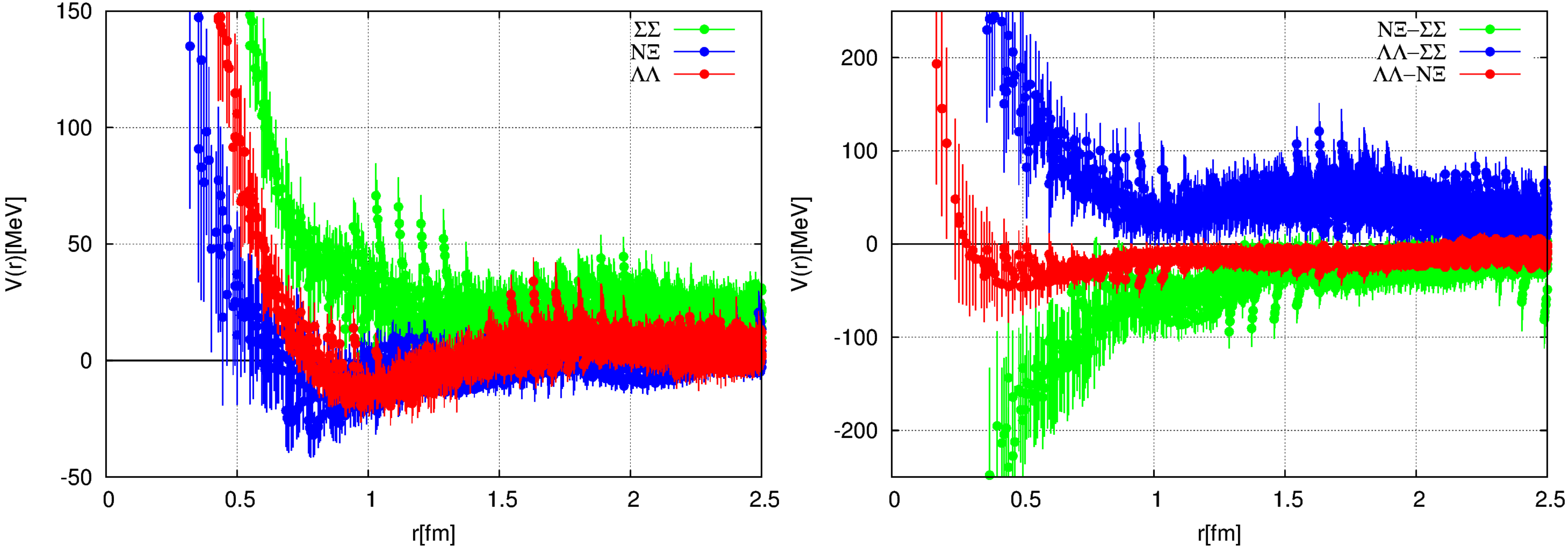
- All diagonal element have a repulsive core and shallow attractive pocket.
- **Diagonal NΞ potential has a shallow attractive pocket.**
- We find that NΞ−ΣΣ transition potential is relatively strong comparing to the other transition potentials

$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel near the physical point

- N_f = 2+1 full QCD with L = 8fm, m_π = 145 MeV

Preliminary!

Diagonal elements **Off-diagonal elements**



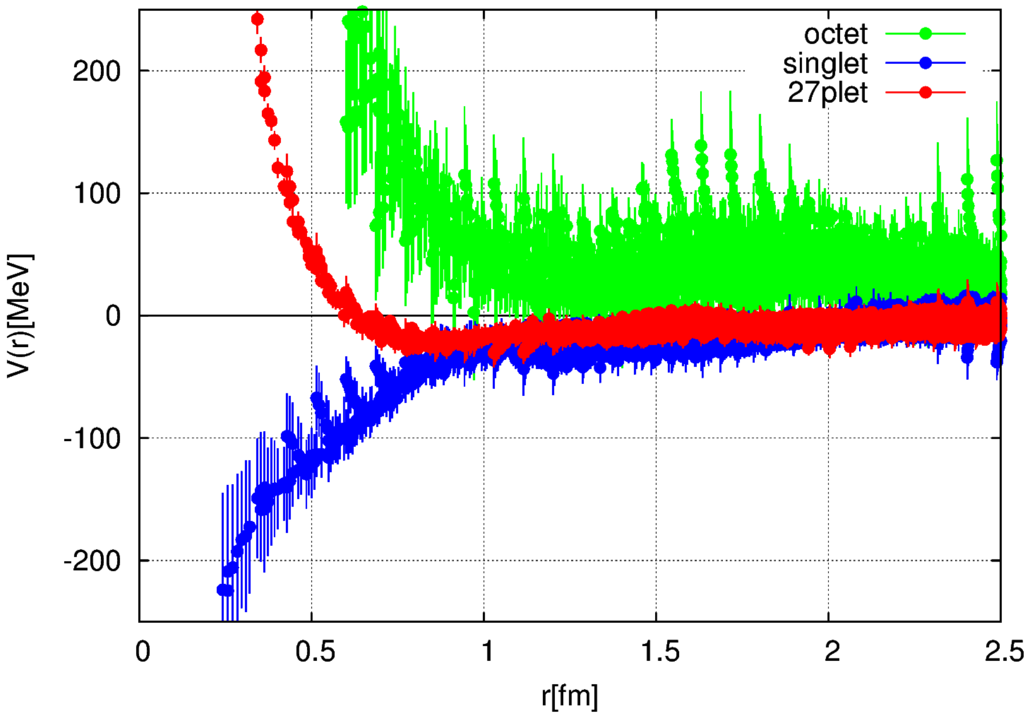
- All diagonal element have a repulsive core $\Sigma\Sigma$–$\Sigma\Sigma$ potential is strongly repulsive.
- **Diagonal $N\Xi$ potential is more attractive than the $\Lambda\Lambda$ potential.**
- We need more statistics to discuss physical observables through this potential.

Potentials in $^1S_0$ channel with SU(3) basis

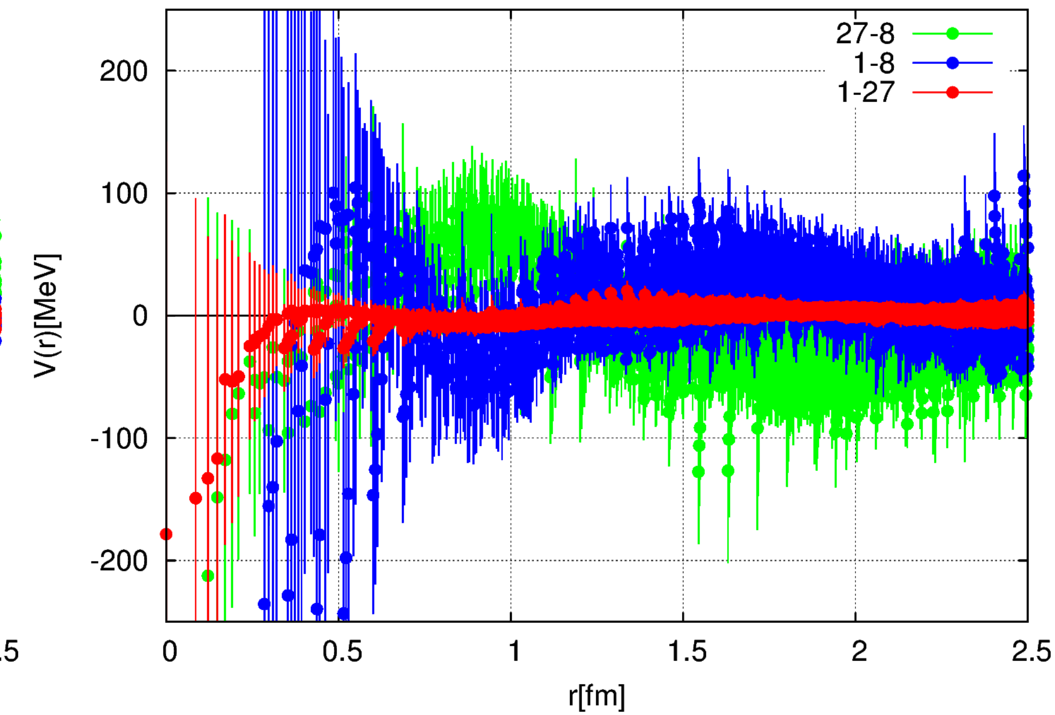
- **N_f = 2+1 full QCD with L = 8fm, *m_π = 145 MeV***

Preliminary!

Diagonal elements



Off-diagonal elements



- **Potential of flavor singlet channel does not have a repulsive core**
- Potential of flavor octet channel is strongly repulsive which reflects Pauli effect.
- Off-diagonal potentials are visible only in r<1fm region.

H-dibaryon channel (2-ch calculation)

Effective two channel potential

Original coupled channel equation

$$\begin{pmatrix} \left(E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda}\right)R^{\Lambda\Lambda}(\vec{r},t) \\ \left(E^{\Xi N}-H_0^{\Xi N}\right)R^{\Xi N}(\vec{r},t) \\ \left(E^{\Sigma\Sigma}-H_0^{\Sigma\Sigma}\right)R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} V^{\Lambda\Lambda}_{\Lambda\Lambda}(\vec{r}) & V^{\Lambda\Lambda}_{\Xi N}(\vec{r}) & V^{\Lambda\Lambda}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Xi N}_{\Lambda\Lambda}(\vec{r}) & V^{\Xi N}_{\Xi N}(\vec{r}) & V^{\Xi N}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Sigma\Sigma}_{\Lambda\Lambda}(\vec{r}) & V^{\Sigma\Sigma}_{\Xi N}(\vec{r}) & V^{\Sigma\Sigma}_{\Sigma\Sigma}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \\ R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix}$$

Truncation of ΣΣ channel

Reduced coupled channel equation

$$\begin{pmatrix} \left(E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda}\right)R^{\Lambda\Lambda}(\vec{r},t) \\ \left(E^{\Xi N}-H_0^{\Xi N}\right)R^{\Xi N}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \overline{V^{\Lambda\Lambda}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Lambda\Lambda}_{\Xi N}}(\vec{r}) \\ \overline{V^{\Xi N}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Xi N}_{\Xi N}}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \end{pmatrix}$$

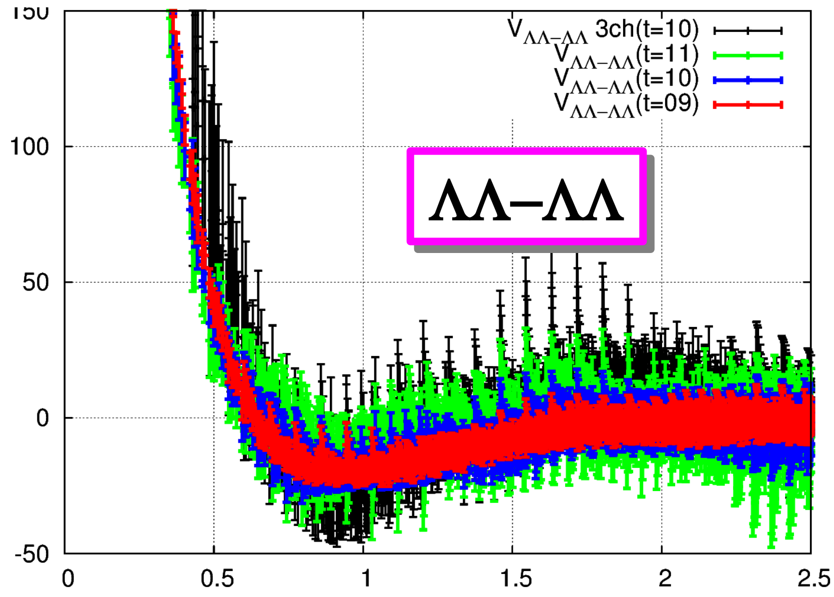
Effective ΛΛ–NΞ potential

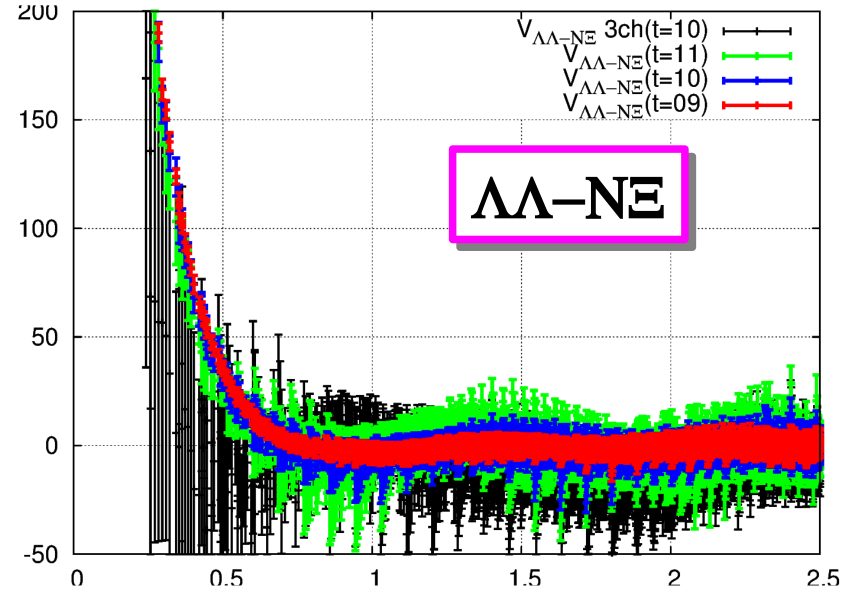
- The same scattering phase shift would be expected in a low energy region.
- Non-locality (energy dependence, higher derivative contribution) of potential matrix could be enhanced.

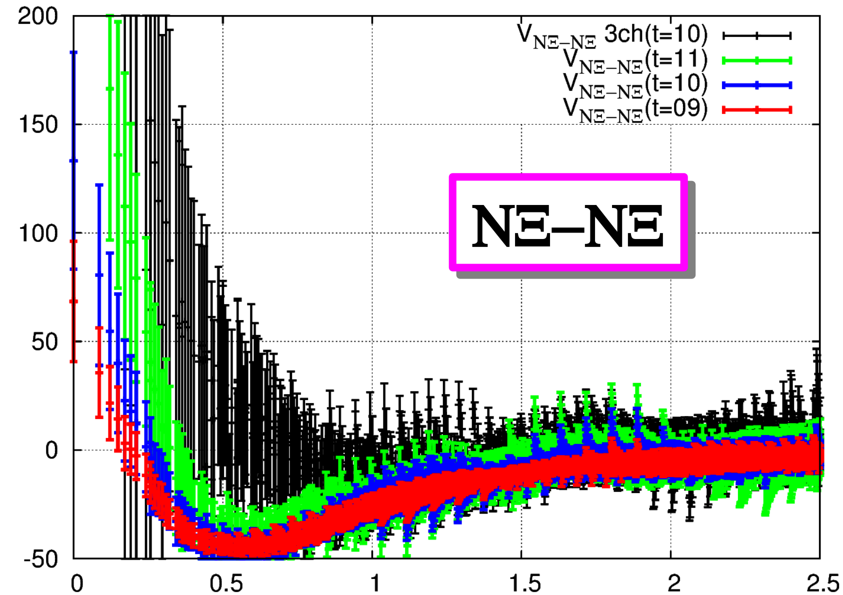
$\Lambda\Lambda$, $N\Xi$ (I=0) $^1S_0$ potential (2ch calc.)

- N_f = 2+1 full QCD with L = 8fm, m_π = 145 MeV

Preliminary!





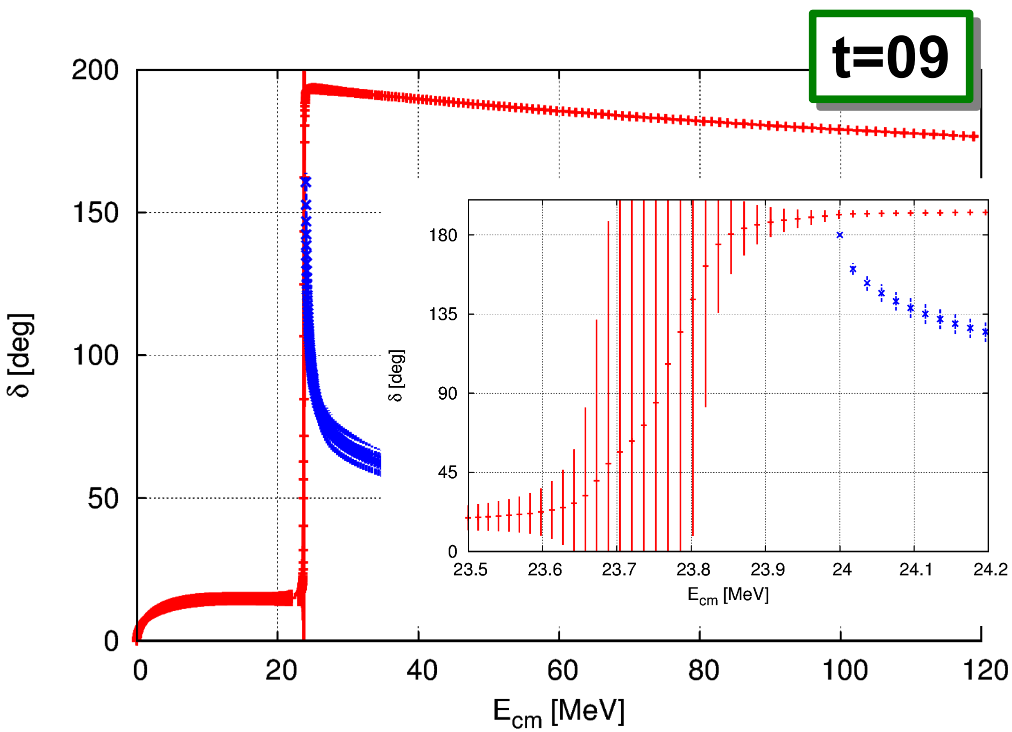


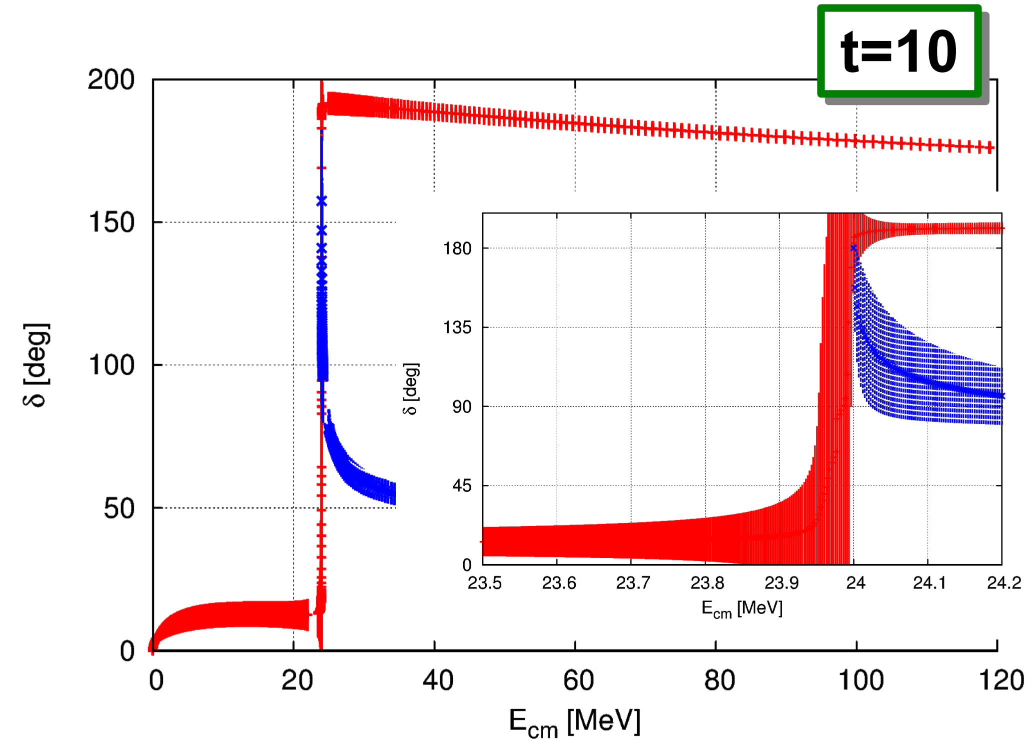
- Potential calculated by only using $\Lambda\Lambda$ and $N\Xi$ channels
- Long range part of potential is stable against the time slice
- Short range part of $N\Xi$ potential is largely changed.
- Deviation from potential in 3ch calc. can be seen mainly in r<1fm.

ΛΛ and NΞ (I=0) $^1S_0$ phase shift (2ch calculation)

▶ N_f = 2+1 full QCD with L = 8fm, m_π = 145 MeV

Preliminary!





- ΛΛ and NΞ phase shift is calculated by using 2ch effective potential.
- For both cases, we found the sharp resonance just below the NΞ threshold.
- Time slice saturation should be checked.
- 3ch calculation(need more statistics)

Summary and outlook

- We have investigated coupled channel S=-2 baryon-baryon interactions from lattice QCD.
  - Simulation is performed near the physical point
    - $m\pi$=145MeV, La=8fm.

- This talk focused on the S=-2 BB interactions
  - $\Lambda\Lambda$ potential is weakly attractive.
  - $N\Xi$ potential is largely depend on the channel.
  - H-dibaryon channel
    - There is strongly attractive potential in flavor singlet state.
    - It is not enough statistics to calculate several observables and to discuss the fate of H-dibayon.

- Further investigation will be performed with high statistical data.

Backup

Introduction

Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

Lattice QCD simulation

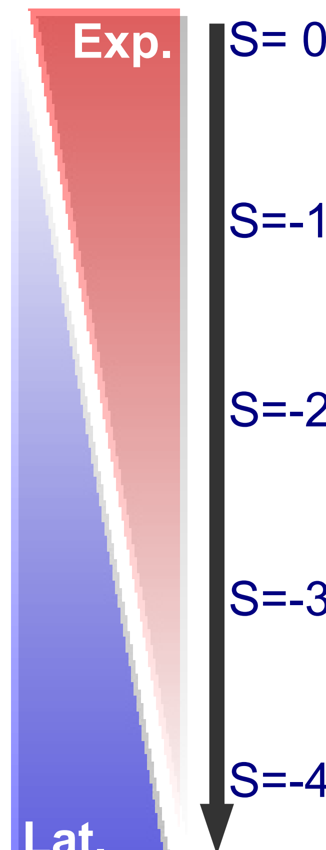
Difficult to calculate light quarks

Suffered from statistical noise

High performance for more data

Massively parallel super computer

Accessibility



Experiment

Difficult to perform collision experiment

Collision data are scarce

More intensity for more data

Huge experimental facility

They would be complement each other to complete knowledge of generalized BB interaction.

BB interaction from NBS wave function

$$\boxed{\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})=\int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r\,'}$$

Derivative (velocity) expansion of U is performed to deal with its nonlocality.

- For the case of oct-oct system,

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_T(r)\right]}_{\text{Leading order part}}+\left[\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)\right]+O(\nabla^2)$$

- For the case of dec-oct and dec-dec system,

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_{T_1}(r)+S_{ii}V_{T_2}(r)+O(Spin\,op^3)\right]}_{\text{Leading order part}}+O(\nabla^2)$$

$$\equiv \left[V_C^{eff}(r)\right]+O(\nabla^2)$$

$$((\vec{r}\cdot\vec{S}_1)^2-\frac{\vec{r}^{\,2}}{3}\vec{S}_1^{\,2}+(\vec{r}\cdot\vec{S}_2)^2-\frac{\vec{r}^{\,2}}{3}\vec{S}_2^{\,2})V_{T^2}(r)$$

We consider the effective central potential which contains not only the genuine central potential but also tensor parts.

Coupled channel Schrödinger equation

NBS wave function with ith energy eigen state

$$\Psi^{\alpha}(E_i,\vec{r}) = \langle 0|(B_1 B_2)^{\alpha}(\vec{r})|E_i\rangle$$

$$\Psi^{\beta}(E_i,\vec{r}) = \langle 0|(B_1 B_2)^{\beta}(\vec{r})|E_i\rangle$$

Two-channel coupling case

$$\int dr\, \tilde{\Psi}_{\beta}(E',\vec{r})\Psi^{\gamma}(E,\vec{r}) = \delta(E'-E)\delta_{\beta}{}^{\gamma}$$

We define potentials which satisfy a coupled channel Schrodinger equation

$$\begin{pmatrix} (p_{\alpha}^2+\nabla^2)\Psi^{\alpha}(E_i,\vec{r}) \\ (p_{\beta}^2+\nabla^2)\Psi^{\beta}(E_i,\vec{r}) \end{pmatrix} = \int dr' \begin{pmatrix} U^{\alpha}{}_{\alpha}(\vec{r},\vec{r}\,') & U^{\alpha}{}_{\beta}(\vec{r},\vec{r}\,') \\ U^{\beta}{}_{\alpha}(\vec{r},\vec{r}\,') & U^{\beta}{}_{\beta}(\vec{r},\vec{r}\,') \end{pmatrix} \begin{pmatrix} \Psi^{\alpha}(E_i,\vec{r}\,') \\ \Psi^{\beta}(E_i,\vec{r}\,') \end{pmatrix}$$

Leading order of velocity expansion and time-derivative method

$$\begin{pmatrix} \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_{\alpha}}\right)R^{\alpha}_{E_0}(t,\vec{r}) \\ \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_{\beta}}\right)R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}{}_{\alpha}(t) & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_0}(t,\vec{r}) \\ R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix}$$

$$\Delta_{\beta}^{\alpha} = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

Considering two different energy eigen states

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}_{\beta} \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}_{\alpha} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_{\alpha}}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_{\beta}}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E1}(t,\vec{r}) \\ \left(\frac{\nabla^2}{2\mu_{\alpha}}-\frac{\partial}{\partial t}\right)R^{\beta}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_{\beta}}-\frac{\partial}{\partial t}\right)R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E0}(t,\vec{r}) & R^{\alpha}_{E1}(t,\vec{r}) \\ R^{\beta}_{E0}(t,\vec{r}) & R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix}^{-1}$$