No-Core Shell Model Approach to Hypernuclear Structure

Daniel Gazda

NPI Řež, ECT* Trento

12th International Conference on Hypernuclear and Strange Particle Physics
Sendai, Japan, September 7–12, 2015

Collaborators:
P. Navrátil (TRIUMF Vancouver)
R. Roth, R. Wirth (TU Darmstadt)
A. Gal (HU Jerusalem)
J. Mareš (NPI Řež)

Outline

- Motivation
- Hypernuclear No-Core Shell Model
- S-Shell Hypernuclei: $nn\Lambda$, CSB, ...
- P-Shell Hypernuclei
- Summary and Outlook

Strangeness in nuclear many-body systems

Interdisciplinary subject connecting particle physics, nuclear physics and astrophysics.

Related topical questions include:

- interaction of (anti)kaons with the nuclear medium
  - possible existence of deeply-bound K^-–nuclear states?
  - antikaons in dense matter?
- interaction of hyperons with the nuclear medium
  - S=-1 Λ hypernuclei, Σ-hypernuclei?
  - S=-2 $\Lambda\Lambda$-hypernuclei, Ξ hypernuclei
  - hyperons in dense nuclear matter and neutron stars?

Study of hypernuclei

- improve understanding of YN interaction
  - provide important constraints on YN interaction
  - precise experimental data on hypernuclear spectroscopy
  - supplement (very sparse) hyperon–nucleon scattering data base
- new precision experiments at J-PARC, J-Lab, FAIR, . . .
- modern developments of YN interaction
  - based on SU(3) chiral EFT
  - require advanced many-body computational methods to confront with hypernuclear structure measurements

Ab initio calculations of light hypernuclei

- given microscopic NN (+NNN) and YN interactions, calculate the energy spectra of A-body hypernuclear system with controllable approximations
- calculations so far limited to A=3,4 hypernuclear systems (Faddeev, Faddeev–Yakubovsky equations)
- recent developments in computational many-body methods

Our aim:

- develop a method applicable to heavier A≥5 hypernuclei
- study available boson-exchange and chiral YN interaction models

No-core shell model for hypernuclei

Ab initio

- all particles are active (no rigid core)
- exact Pauli principle
- realistic 2- and 3-body interactions (accurate description of NN and YN data)
- controllable approximations

- Hamiltonian is diagonalized in a *finite* A-particle harmonic oscillator basis
- NCSM results converge to exact results

No-core shell model for hypernuclei

- two independent NCSM formulations developed:

Slater-determinant HO basis

\+ starting with atisymmetrized basis

\+ second quantization methods

− c.m. degree of freedom present ⇒ huge basis

relative Jacobi-coordinate HO basis

\+ c.m. d.o.f. removed

⇒ smaller basis

⇒ larger model space possible

− the basis has to be antisymmetrized

Input V_{NN}, V_{NNN}, and V_{NY} potentials

NN+NNN interaction

- chiral N3LO NN potential
  (Entem, Machleidt, PRC 68 (2003) 041001)
- chiral N2LO NNN potential
  (Navrátil, FBS 41 (2007) 14)

YN interaction

- phenomenological meson-exchange Jülich04 potential
  (Haidenbauer, Meißner, PRC 72 (2006) 044005)
- chiral LO potential
  NLO version recently developed (Haidenbauer *et al.*, NPA 915 (2013) 24)

$\Lambda N - \Sigma N$ mixing explicitly taken into account:

$$V_{NY} = \begin{pmatrix} V_{\Lambda N-\Lambda N} & V_{\Lambda N-\Sigma N} \\ V_{\Sigma N-\Lambda N} & V_{\Sigma N-\Sigma N} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & m_\Sigma - m_\Lambda \end{pmatrix}$$

Similarity renormalization group effective interaction

- “bare” interactions require large model spaces for reasonable convergence
- series of unitary transformations of the original Hamiltonian H:

  $$H_\lambda = U_\lambda H U_\lambda^\dagger,\ H = H_\infty$$

  implemented as a flow equation in λ:

  $$\frac{\mathrm{d}H_\lambda}{\mathrm{d}\lambda} = -4/\lambda^5[[T, H_\lambda], H_\lambda]$$

- low- and high-momentum parts of H_λ being decoupled:

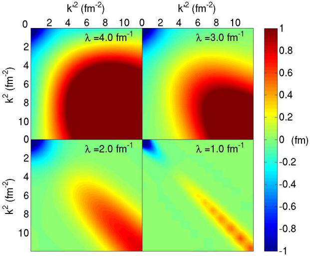


Figure: SRG evolved ${}^1\mathrm{S}_0$ partial wave of chiral N3LO V_{NN}. (Bogner, Furnstahl, Perry, PRC 75, 061001 (2007))

Lee–Suzuki effective interaction

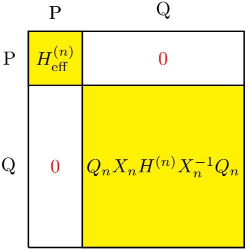
- preserves exactly part of the Hamiltonian spectrum

$$\sigma(H) = \{E_1, E_2, E_3, \ldots, E_P, \ldots, E_\infty\}$$

$$\sigma(H_{\text{eff}}) = \{E_1, E_2, E_3, \ldots, E_P\}$$

- decoupling condition

$$QXHX^{-1}P = 0 \rightarrow H_{\text{eff}} = PXHX^{-1}P$$



$$\begin{array}{c|c|c} & \text{P} & \text{Q} \\ \hline \text{P} & H_{\text{eff}}^{(n)} & 0 \\ \hline \text{Q} & 0 & Q_n X_n H^{(n)} X_n^{-1} Q_n \end{array}$$

- n-body cluster approximation ($2 \leq n \leq A$): $H_{\text{eff}}^{(n)}$ n-body operator
  $n = 2$: takes into account 2-body correlations from outside of the model space (← repulsive core)

S-shell hypernuclei: ${}^{3}_{\Lambda}\mathrm{H}$

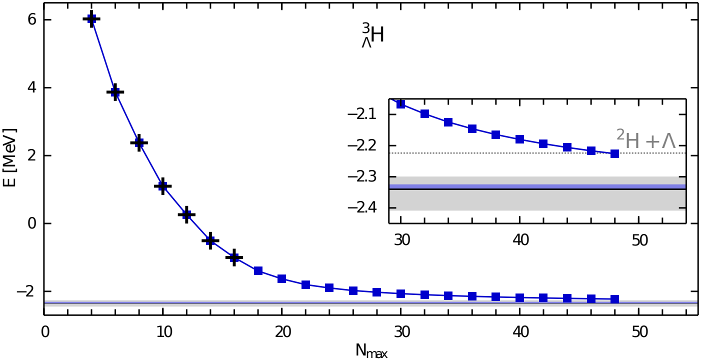


Figure: Ground state energy of ${}^{3}_{\Lambda}\mathrm{H}$ as a function of the size of the model space, with bare chiral LO @ 600 MeV interactions.

Particle-stable $nn\Lambda$ system?

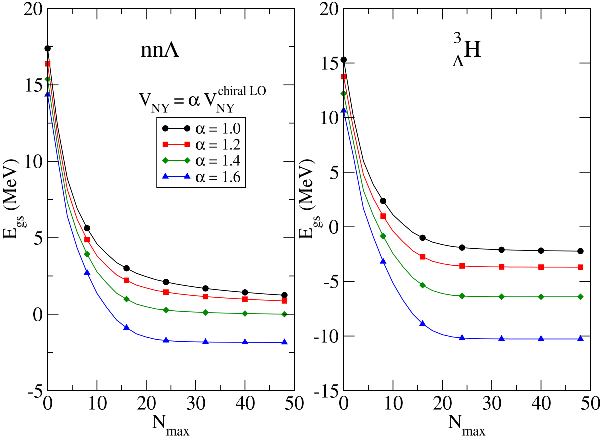


Figure: Ground state energies of $nn\Lambda$ and $^3_\Lambda$H with scaled V_{NY} potential as a function of the size of the model space.

S-shell hypernuclei: ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He

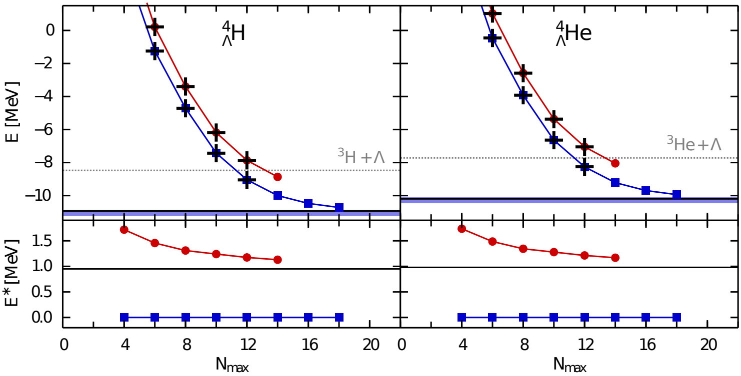


Figure: Ground state (blue) and excited state (red) energy of ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He as a function of the size of the model space, with chiral LO @ 600 MeV interactions.

S-shell hypernuclei: $\Lambda N - \Sigma N$ mixing in ${}^{4}_{\Lambda}$He

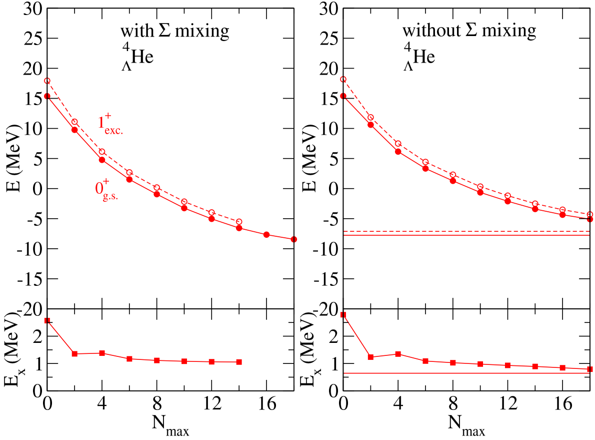


Figure: The ground state (0^{+}) and excited state (1^{+}) energies of ${}^{4}_{\Lambda}$He with and without $\Lambda N - \Sigma N$ mixing as a function of the size of the model space.

S-shell hypernuclei: charge symmetry breaking in ${}^4_\Lambda$He-${}^4_\Lambda$H

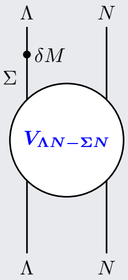
Electromagnetic $\Lambda - \Sigma^0$ mixing

Physical Λ and Σ^0 hyperons have mixed isospin composition in terms of the SU(3) pure isospin Λ (I=0) and Σ (I=1) hyperons, with mixing angle proportional to electromagnetic mass matrix element related to isospin-breaking mass differences

$$\langle\Sigma|\delta M|\Lambda\rangle = \frac{1}{\sqrt{3}}[(M_{\Sigma^0} - M_{\Sigma^+}) - (M_n - M_p)] = 1.14(05)\ \text{MeV}$$

(Dalitz, von Hippel, Phys. Lett. 10, 153 (1964).)

Charge symmetry breaking ΛN interaction



- Λ–Σ^0 mixing results in *effective* $\pi^0\Lambda\Lambda$ ($\rho^0\Lambda\Lambda$) coupling

$$g_{\pi\Lambda\Lambda} = -2\frac{\langle\Sigma^0|\delta M|\Lambda\rangle}{M_{\Sigma^0} - M_\Lambda} g_{\pi\Lambda\Sigma} = -0.0297 g_{\pi\Lambda\Sigma}$$

- For models with explicit $\Lambda N - \Sigma N$ conversion

$$\langle N\Lambda|V^{CSB}_{\Lambda N}|N\Lambda >= -0.0297\tau_{N3}\frac{1}{\sqrt{3}}\langle N\Sigma|V_{\Lambda N-\Sigma N}|N\Lambda\rangle$$

(A. Gal, PLB 744, 352 (2015))

S-shell hypernuclei: charge symmetry breaking in ${}^4_\Lambda$He-${}^4_\Lambda$H

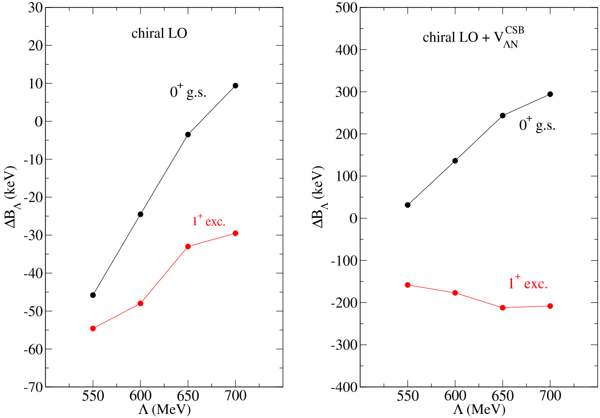


Figure: Chiral YN cutoff Λ dependence of ${}^4_\Lambda$He$-{}^4_\Lambda$H ground state ($0^+_{\text{g.s.}}$) an excited state ($1^+_{\text{exc.}}$) Λ hyperon binding energy differences calulated without (left) and with V_{CSB} generated by $\Lambda N-\Sigma N$ conversion.

S-shell hypernuclei: ${}^{5}_{\Lambda}$He with Lee–Suzuki eff. interaction

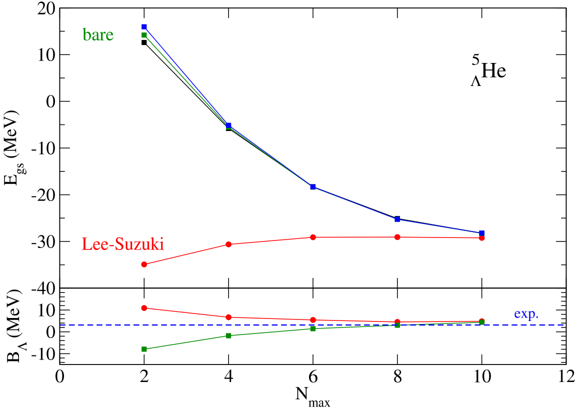


Figure: Ground state energy and Λ binding energy in ${}^{5}_{\Lambda}$He with bare and Lee–Suzuki effective interactions as a function of the size of the model space, with chiral LO @ 600 MeV interactions.

P-shell hypernuclei: ${}^{7}_{\Lambda}\mathrm{Li}$

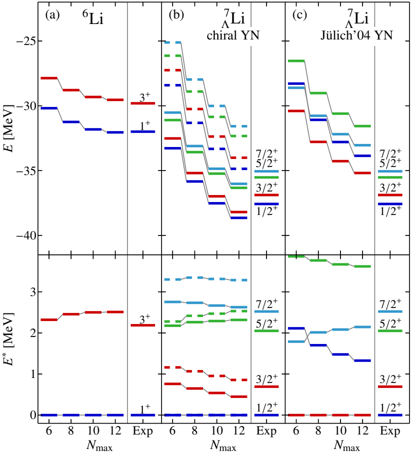


Figure: Calculations of ${}^{7}_{\Lambda}\mathrm{Li}$ with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

P-shell hypernuclei: ${}^{9}_{\Lambda}\mathrm{Be}$

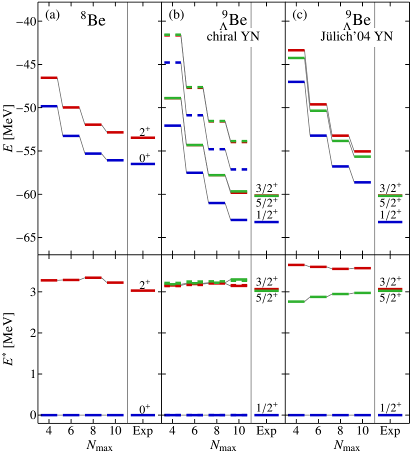


Figure: Calculations of ${}^{9}_{\Lambda}$Be with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

P-shell hypernuclei: ${}^{13}_{\Lambda}$C

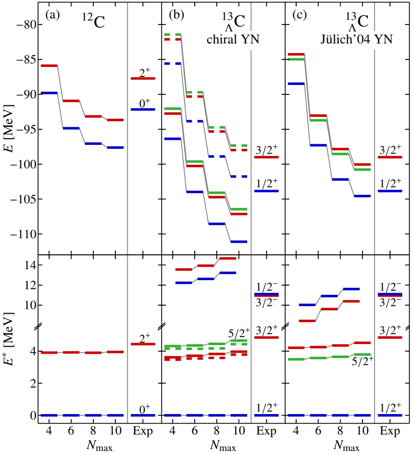


Figure: Calculations of ${}^{13}_{\Lambda}$C with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

Summary and outlook

Calculations of light hypernuclei within NCSM

- *no indication* of particle-stable $nn\Lambda$ system
- electromagnetic $\Lambda - \Sigma^0$ mixing generates *substantial contributions* to the CSB

- reliable *ab initio* calculations of p-shell hypernuclei with microscopic interactions are now possible
- systematic study of p-shell hypernuclei *improves* understanding of YN interactions
- LO chiral YN interactions are *consistent* with measured low-lying energy levels of light hypernuclei
- indication of *deficiencies* for higher relative partial waves of LO chiral YN interactions

Gazda, Mareš, Navrátil, Roth, Wirth, Few-Body Syst. 55, 857 (2014).
Wirth, Gazda, Navrátil, Calci, Langhammer, Phys. Rev. Lett. 113, 192502 (2014).

Outlook

- benchmark calculations
- study chiral NLO NY interaction
- lattice NY interactions, $S{=}-2$ systems, . . .