Structure of light hypernuclei in the framework of Fermionic Molecular Dynamics

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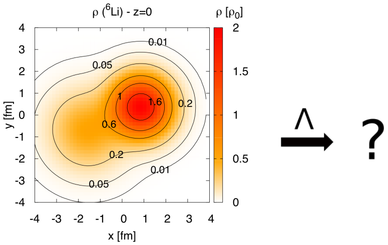
Introduction

Main goal

Study of light hypernuclei

- information about the ΛN (BB) interaction
- modification of the nuclear core
- cluster vs. shell nuclear structure
- Charge Symmetry Breaking (CSB) effects
- $\Lambda N - \Sigma N$ mixing
- 3-body YNN forces (neutron star structure)



The present work

- study of light hypernuclei within Fermionic Molecular Dynamics
- calculations of the ground and excited states of ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$, ${}^{5}_{\Lambda}\mathrm{He}$, and ${}^{7}_{\Lambda}\mathrm{Li}$
- $\mathrm{V}_{\Lambda\mathrm{N}}$ and V_{NN} potential model dependence
- cluster structure

Fermionic Molecular Dynamics

(H. Feldmeier, Nucl. Phys. **A 515** (1990) 147)
(T. Neff, H. Feldmeier, Nucl. Phys. **A 738** (2004) 367)

system of interacting fermions described by an antisymmetrized many-body state $|Q\rangle$

Antisymmetrization

- many-body wave function approximated by a **Slater determinant**

spatial part of a single-particle state represented by a **Gaussian wave packet**

$$\langle\vec{x}|\, q_k\rangle = \exp\left(-\frac{(\vec{x}-\vec{b}_k)^2}{2a_k}\right) \otimes \left|\chi_k^{\uparrow}, \chi_k^{\downarrow}\right\rangle \otimes |t\rangle$$

- complex width a_k, complex $\vec{b}$, complex $\chi^{\uparrow}$ and $\chi^{\downarrow}$ spin parameters (12 real parameters for each particle)

Minimization

Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{T}_\Lambda + \hat{V}_{NN} + \hat{V}_{\Lambda N} - \hat{T}_{\rm cm}$$

Binding energy

$$E_{\rm B} = \min_{q_1,\dots,q_n} \frac{\langle Q|\,\hat{H}\,|Q\rangle}{\langle Q|\,Q\rangle}$$

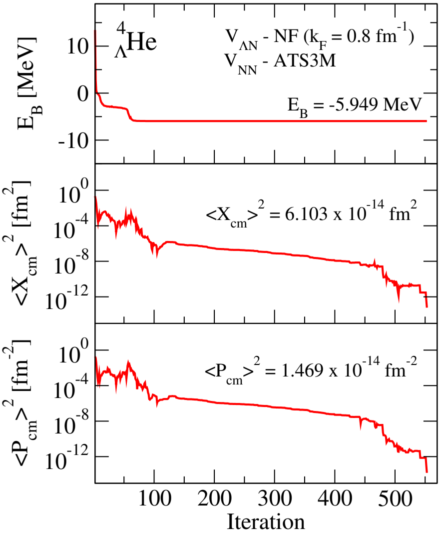
under conditions

$$< \hat{\mathbf{X}}_{\rm cm} >^2 = 0, \quad < \hat{\mathbf{P}}_{\rm cm} >^2 = 0, \quad Re(a_k) > 0$$

- single-particle state parameters $q_k = \{a_k, \vec{b}_k, \chi_k^\uparrow, \chi_k^\downarrow\}$

Result

- minimization yields an **intrinsic state** which is not parity and total angular momentum eigenstate J^π
- **broken symmetries** have to be **restored**



Projection techniques (T. Neff, H. Feldmeier, Eur. Phys. **J 156** (2008) 69)

Projections

Parity projection

$$\hat{P}^{\pi} = \frac{1}{2}(\hat{1} + \pi\hat{\Pi})$$

Total angular momentum projection

$$\hat{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int d\Omega D^{J\,\star}_{MK}(\Omega)\hat{R}(\Omega)$$

Eigenstates

- total angular momentum and parity eigenstates are projected out of the minimized intrinsic state

$$|Q; J^{\pi} MK\rangle = \hat{P}^{J}_{MK}\hat{P}^{\pi}\,|Q\rangle$$

K-mixing

Orthogonal eigenstates

$$|Q; J^{\pi} M\kappa\rangle = \sum_{K} |Q; J^{\pi} MK\rangle\, C_K^{J^{\pi}\kappa}$$

Generalized eigenvalue problem

$$\hat{H}\,|Q; J^{\pi} M\kappa\rangle = E^{J^{\pi}\kappa}\,|Q; J^{\pi} M\kappa\rangle$$

- diagonalization of the $\hat{H}$ in a subspace spanned by the projected states $|Q; J^{\pi} M\kappa\rangle$

$$\sum_{K'} H^{J^{\pi}}_{K,K'}\, C_K^{J^{\pi}\kappa} = E^{J^{\pi}\kappa} \sum_{K''} N^{J^{\pi}}_{K,K''}\, C_K^{J^{\pi}\kappa}$$

$$H^{J^{\pi}}_{K,K'} = \langle Q|\,\hat{H}\hat{P}^{J}_{KK'}\hat{P}^{\pi}\,|Q\rangle$$
$$N^{J^{\pi}}_{K,K'} = \langle Q|\,\hat{P}^{J}_{KK'}\hat{P}^{\pi}\,|Q\rangle$$

V_{NN} and $V_{\Lambda N}$ potential input

NN two-body potentials

- V2-M0.0, V2-M0.6 (A. Volkov, Nucl. Phys. **74** (1965) 33)
- MTV (UCOM modified*)
  (R. Malfliet, J. Tjon, Nucl. Phys. **A 127** (1969) 161)
- ATS3M (UCOM modified*)
  (I. Afnan, Y. Tang, Phys. Rev. **175** (1968) 1337)

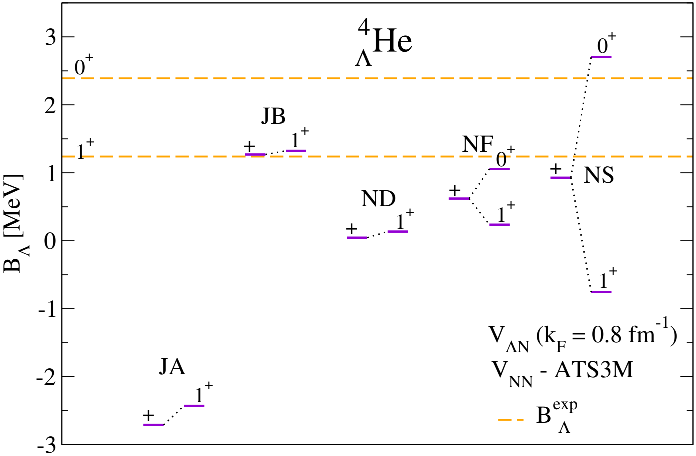
  * UCOM (H. Feldmeier, T. Neff, R. Roth, J.Schnack, Nucl. Phys. **A 632** (1998) 61)

ΛN two-body potential

- G-matrix transformed YNG (Jülich - JA, JB, Nijmegen - ND, NF, NS)
- k_F dependence (Y.Yamamoto et. al, PTP Suppl. **117** (1994) 361)

$$V_{\Lambda N}(r) = \sum_i^3 (a_i + b_i k_F + c_i k_F^2) \exp\left\{-\frac{r^2}{\beta_i^2}\right\}$$

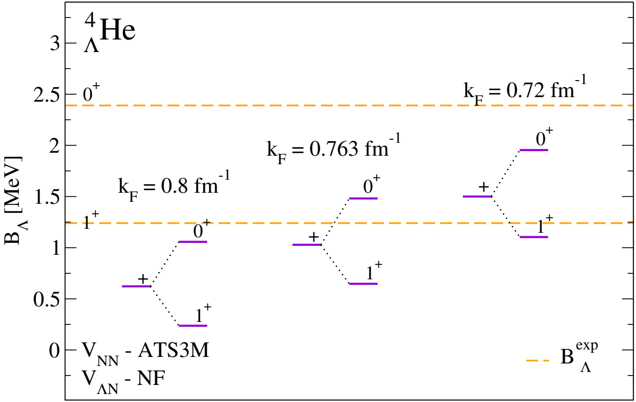
$V_{\Lambda N}$ potential model dependence



Substantial difference between Λ separation energies as well as $|B_\Lambda(0^+) - B_\Lambda(1^+)|$ for various $V_{\Lambda N}$

Fermi momentum k_F dependence in $V_{\Lambda N}$

- value of k_F reflects the nuclear medium surrounding the Λ hyperon



$k_F = 0.8\ \mathrm{fm}^{-1}$ (Y.Yamamoto et al, PTP Suppl. **117** (1994) 361), $k_F = 0.763\ \mathrm{fm}^{-1}$ (^{3}He rms radius approximation), and $k_F = 0.72\ \mathrm{fm}^{-1}$(test value)

Strong Fermi momentum dependence in the $V_{\Lambda N}$ part (k_F acts as a scaling factor)

B_Λ differences of the ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H mirror hypernuclei

- Coulomb interaction included

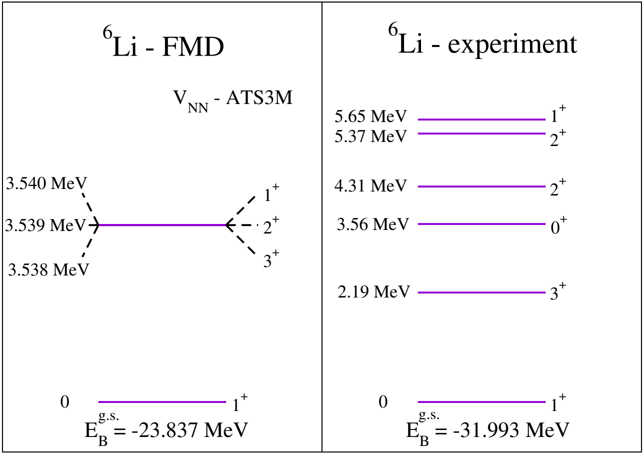
$B_\Lambda({}^{4}_{\Lambda}He) - B_\Lambda({}^{4}_{\Lambda}H)$	V_{NN}-MTV	V_{NN}-ATS3M	exp
0^+ [MeV]	-0.012	-0.032	0.35
1^+ [MeV]	-0.007	0.013	0.24

$V_{\Lambda N}$ - NF ($k_F = 0.763$ fm^{-1})

- opposite shift between the B_Λ spectra of the mirror hypernuclei ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He to that observed

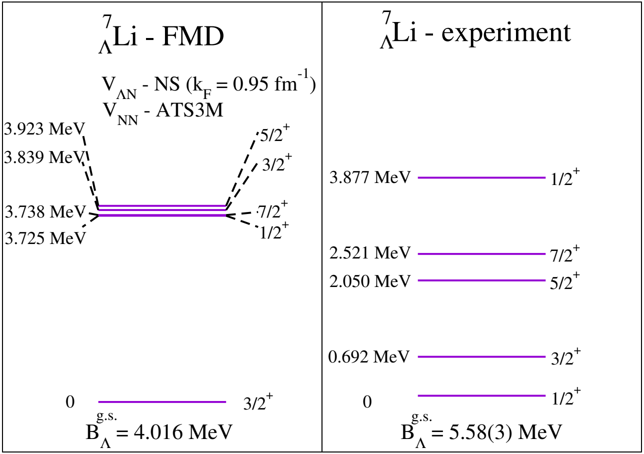
  → **missing ΛN − ΣN mixing** (R. H. Dalitz, F. Von Hippel, Phys. Rev. Lett. **10** (1964) 153)

Energy levels in ^{6}Li



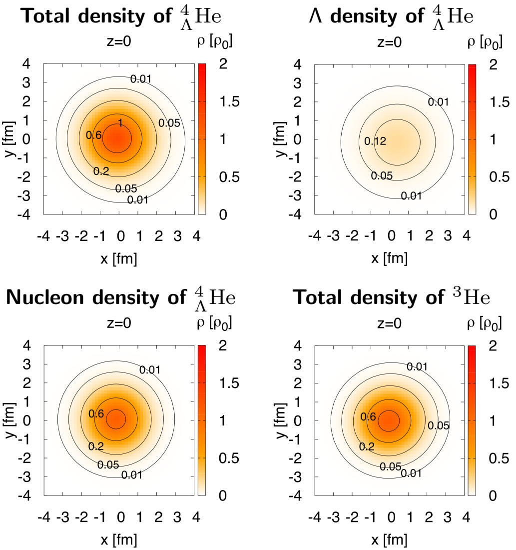
Considerable inconsistency between calculated and experimentally measured excitation spectra – attributed to the rather simple ATS3M potential (no **LS** interaction)

Energy levels in $^{7}_{\Lambda}$Li



Considerable inconsistency between calculated and experimentally measured excitation spectra – attributed to the rather simple V_{NN} potential

Cluster structure: s-shell hypernucleus ${}^{4}_{\Lambda}\mathrm{He}$

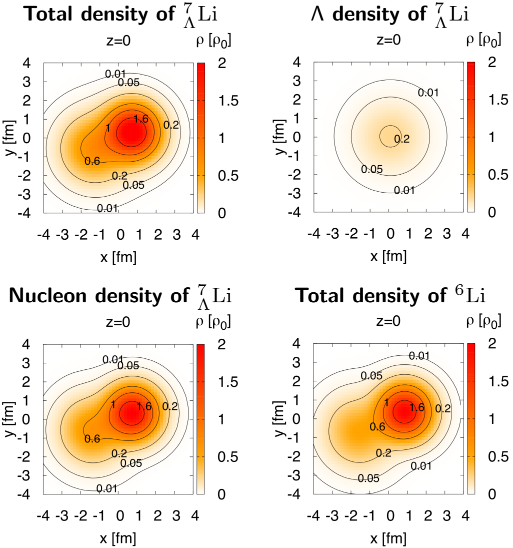


$\mathrm{V_{NN}}$ - ATS3M, and $\mathrm{V_{\Lambda N}}$ - NF $(\mathrm{k_F} = 0.8\ \mathrm{fm}^{-1})$

Findings

- after variation, the Λ hyperon is located very close to the center of nuclear core
- modifications of compact nuclear core (${}^{3}\mathrm{H}$, ${}^{3}\mathrm{He}$, and ${}^{4}\mathrm{He}$) in ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$, and ${}^{5}_{\Lambda}\mathrm{He}$ due to the presence of Λ are negligible

Cluster structure: p-shell hypernucleus $^{7}_{\Lambda}$Li



V_{NN} - ATS3M, and $V_{\Lambda N}$ - NS ($k_F = 0.95$ fm^{-1})

Findings

- clear evidence of the internal $\alpha + d$ cluster structure of the ^{6}Li nuclear core
- after variation, the Λ hyperon is located in the middle between the α and d clusters
- Λ hyperon pulls the α and d cluster closer together

$R_{\mathrm{T}}^{g.s.}(^{6}\mathrm{Li}) = 2.049$ fm

$R_{\mathrm{core}}^{g.s.}(^{7}_{\Lambda}\mathrm{Li}) = 1.929$ fm

$\Delta R_{\mathrm{core}}(^{7}_{\Lambda}\mathrm{Li}) = -0.120$ fm

- confirmation of the "glue-like" role of the Λ hyperon (H. Tamura et al, Nucl. Phys. **A 670** (2000) 249)

Conclusions

In this work :

- FMD for hypernuclei developed
  - calculations of s-shell hypernuclei ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$, and ${}^{5}_{\Lambda}\mathrm{He}$ and the p-shell hypernucleus ${}^{7}_{\Lambda}\mathrm{Li}$
  - substantial difference between various $\mathrm{V}_{\Lambda\mathrm{N}}$ potential models
  - strong $\mathrm{k_F}$ dependence
    ($\mathrm{k_F}$ acts as a scaling parameter of YNG $\mathrm{V}_{\Lambda\mathrm{N}}$ potentials)
  - opposite shift between the B_Λ spectra of ${}^{4}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{He}$
    $\rightarrow$ missing $\Lambda N - \Sigma N$ mixing
  - the nuclear core modifications in s-shell hypernuclei are negligible
  - confirmation of the "glue-like" role of the Λ hyperon in ${}^{7}_{\Lambda}\mathrm{Li}$

Next steps :

- calculations of heavier p-shell hypernuclei
- more sophisticated interactions (Argonne V18, $\mathrm{V}_{\Lambda\mathrm{N}}$ potentials with $\Lambda - \Sigma$ mixing, chiral V_{NN} and $\mathrm{V}_{\Lambda\mathrm{N}}$ potentials)
- $\Lambda\Lambda$ hypernuclei
- 3-body *YNN* forces (neutron star structure)

Variational parameters

Single-particle wave function

$$\langle\vec{x}|\ q_k\rangle = \exp\left(-\frac{(\vec{x}-\vec{b}_k)^2}{2a_k}\right) \otimes \left|\chi_k^{\uparrow}, \chi_k^{\downarrow}\right\rangle \otimes |t\rangle$$

Spatial part

Complex width

- $a_k = \mathrm{Re}(a_k) + \mathrm{iIm}(a_k)$

Complex vector parameter $\vec{b}_k$

- position and velocity
- $\vec{b}_k = (b_{k1}, b_{k2}, b_{k3})$
- **8** real parameters

Spin part parameters

- the most general form ensures a rotation of an arbitrary angle

$$\left|\chi_k^{\uparrow}, \chi_k^{\downarrow}\right\rangle = \begin{pmatrix} \mathrm{Re}(\chi_\mathrm{k}^{\uparrow}) + \mathrm{iIm}(\chi_\mathrm{k}^{\uparrow}) \\ \mathrm{Re}(\chi_\mathrm{k}^{\downarrow}) + \mathrm{iIm}(\chi_\mathrm{k}^{\downarrow}) \end{pmatrix}$$

- **4** real parameters

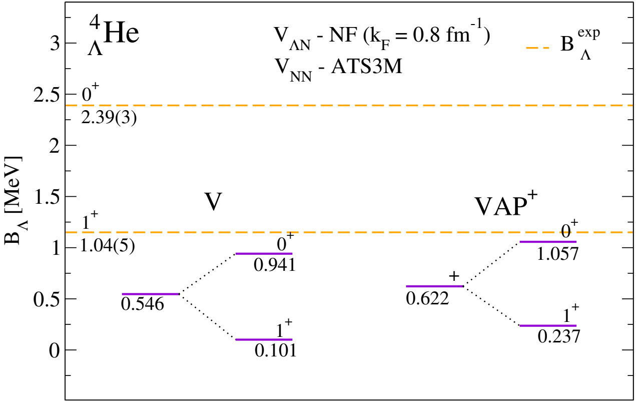
Position, momentum, and the spread

$$\vec{r} = \frac{\mathrm{Re}(a)\mathrm{Re}(\vec{b}) + \mathrm{Im}(a)\mathrm{Im}(\vec{b})}{\mathrm{Re}(a)} \qquad \vec{p} = \frac{\mathrm{Im}(\vec{b})}{\mathrm{Re}(a)} \qquad (\Delta r)^2 = 3\frac{\mathrm{Re}(a)^2 + \mathrm{Im}(a)^2}{2\mathrm{Re}(a)}$$

Parity projection before variation

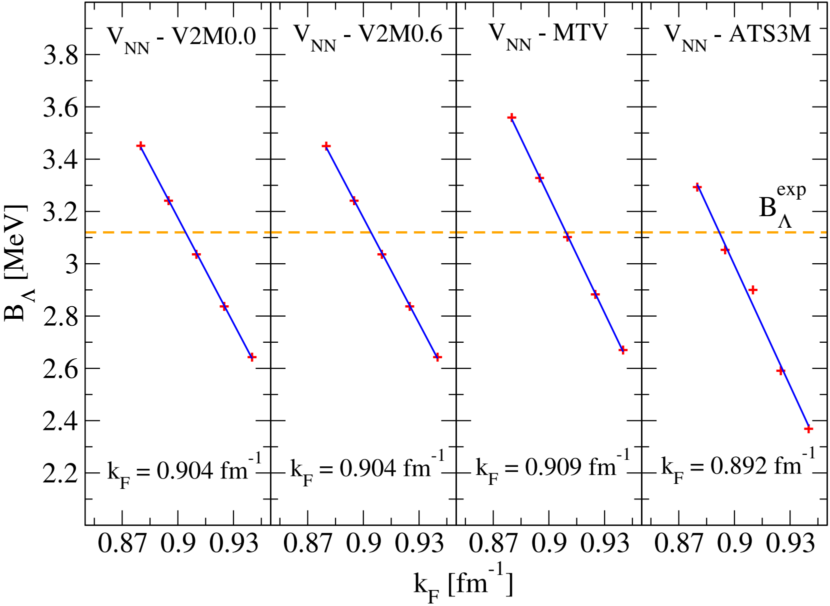
$\mathrm{V} \rightarrow$ variation using basic FMD trial state $|Q\rangle$

$\mathrm{VAP}^{+} \rightarrow$ variation using the even parity projected trial state $|Q;+\rangle = \frac{1}{2}\left(|Q\rangle + \hat{\Pi}\,|Q\rangle\right)$

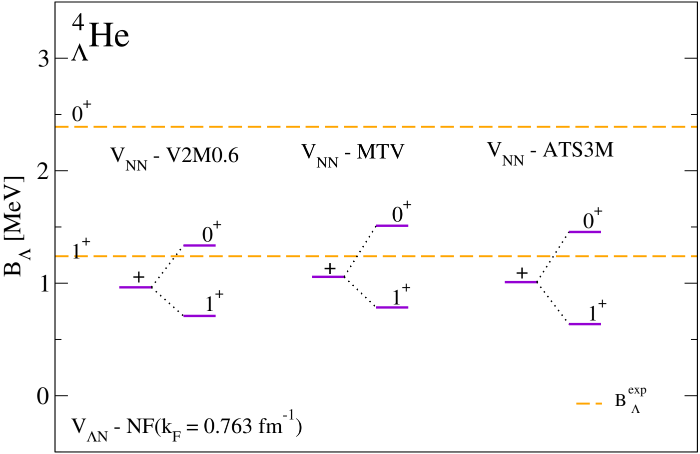


The VAP^{π} with the parity coinciding with the parity of the ground or excited states provides better description of the variated system

Fermi momentum k_F dependence in ${}^5_\Lambda\mathrm{He}$



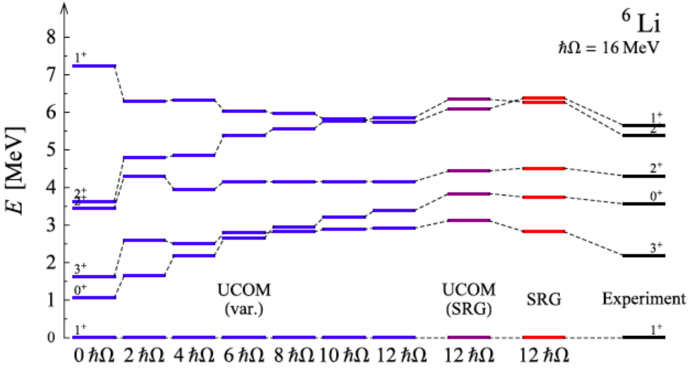
V_{NN} potential model dependence



Λ separation energy (B_Λ) slightly changes with various V_{NN} potential models

Energy levels in ^{6}Li

Energy levels for UCOM modified Argonne v18 V_{NN} potential



(R. Roth, T. Neff, H. Feldmeier, Prog. Nuc. Phys. **65** (2010) 50)

Consistency between calculated and experimentally measured excitation spectra in ^{6}Li for more sophisticated Argonne v18 V_{NN} potential – especially **LS** interaction included