Variational approach to neutron star matter with hyperons

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Outline

1: Introduction
2: Variational method for hyperonic nuclear matter
3: Application to neutron stars
4: Summary

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HYPERON PUZZLE

- Nuclear equation of state (EOS) becomes softer due to the hyperon mixing.
- Maximum mass of neutron star tends to be lower than the observational data.

The hyperon mixing in neutron stars has been studied with various nuclear theories.

- Brueckner-Hartree-Fock theory (H. Schulze, T. Rijken, PRC 84 (2011) 035801)
- Variational many-body theory (D. Lonardoni et al., PRL 114 (2015) 092301)
Our Variational Many-Body Theory

We have constructed the nuclear EOS for core-collapse supernovae with the variational method.

Collaboration with M. Takano (Waseda University), K. Sumiyoshi (Numazu College of Tech.), Y. Takehara, S. Yamamuro, K. Nakazato, H. Suzuki (Tokyo Univ. of Science)

**Method**: Cluster variational method  
**Potential**: AV18 + U1X

Cold and Hot Asymmetric Nuclear Matter for arbitrary particle fractions  

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**Energies of uniform matter**

**Application to Neutron Star**

**Application to Supernova simulation**

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*It is relatively easy to extend our simple variational method to calculate the EOS of hyperonic nuclear matter.*
Our Variational Many-Body Theory

We construct the EOS of nuclear matter containing $\Lambda$ and $\Sigma^{-}$ hyperons starting from bare baryon interactions by the cluster variational method.

It is relatively easy to extend our simple variational method to calculate the EOS of hyperonic nuclear matter.
Two-body Hamiltonian

$$H_2 = -\sum_i \left[ m_i c^2 + \frac{\hbar^2}{2m_i} \nabla_i^2 \right] + \sum_{i<j} V_{ij}$$

Two-body potential

$$V_{ij} = \sum_{N,Y} [V_{ij}^{NN} + V_{ij}^{YN} + V_{ij}^{YY}]$$

Three-body Hamiltonian

$$H_3 = \sum_{i<j<k} V_{ijk}$$

Three-nucleon potential: UIX

- **NN** potential: **AV18 two-body potential** (PRC 51 (1995) 38)
- **YN** and **YY** potentials: **Central three-range Gaussian potentials**
  - **ΛN interaction** (E. Hiyama et al., PRC 74 (2006) 054312)
  - **ΛΛ interaction** (E. Hiyama et al., PRC 66 (2002) 024007)
  - The *ab initio* variational calculations for Λ hypernuclei reproduce their experimental eigenvalues.

**Σ^-N interaction**: Based on the latest version of the Nijmegen model ESC08.
**ΛΛ interaction**

- ΛΛ interaction is constructed so as to reproduce the experimental ΛΛ binding energy given by the NAGARA event.

(E. Hiyama et al., PRC 66 (2002) 024007)

- Two Λs in the experimentally known double Λ hypernuclei are in the relative s orbit.

→ We determine only the even-state part of the ΛΛ interaction!

\[ \Lambda \Lambda \text{He} \]

\[ l = s \text{ wave} \]

### The experimental data on hypernuclei give no information on the odd-state part of the ΛΛ interactions.
The odd-state part of the $\Lambda\Lambda$ interaction

We prepare **four different models** for the odd-state part of the $\Lambda\Lambda$ interaction.

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td><em>The most attractive</em></td>
</tr>
<tr>
<td>Type 2</td>
<td>Less attractive</td>
</tr>
<tr>
<td>Type 3</td>
<td><em>Slightly repulsive</em></td>
</tr>
<tr>
<td>Type 4</td>
<td><em>The most repulsive</em></td>
</tr>
</tbody>
</table>

The repulsion strength of Type 4 is comparable to that of the odd-state repulsion of $\Lambda N$ interaction.

The repulsive effect increases monotonically from Type 1 to Type 4.

- We investigate the effects of **the odd-state part of bare $\Lambda\Lambda$ interactions** on the structure of neutron stars.
Expectation Value of $H_2$

Jastrow wave function

$$
\Psi = \text{Sym} \left( \prod_{i<j} f_{ij} \right) \Phi_F
$$

$\Phi_F$: The Fermi-gas wave function

Sym [ ]: Symmetrizer

$f_{ij}$: Two-body correlation function

$$
f_{ij} = \sum_{p=\pm} \sum_{\mu} \sum_{s=0}^1 \left[ f_{C\mu}^{\mu}(r_{ij}) + s f_{T\mu}^{\mu}(r_{ij}) S_{Tij} + s f_{SO\mu}^{\mu}(r_{ij})(L_{ij} \cdot s) \right] P_{\mu psi j}^{\mu}
$$

Central Tensor Spin-orbit

$E_2$ is the expectation value of $H_2$ with the Jastrow wave function in the two-body cluster approximation.

$$
E_2(n_p, n_n, n_\Lambda, n_{\Sigma^-}) = \frac{\langle H_2 \rangle_2}{A} \left[ f_{C\mu}^{\mu}, f_{T\mu}^{\mu}, f_{SO\mu}^{\mu} \right]
$$

$n_p$ : Proton number density
$n_n$ : Neutron number density
$n_\Lambda$ : $\Lambda$ number density
$n_{\Sigma^-}$ : $\Sigma^-$ number density

$E_2$ is minimized with respect to $f_{C\mu}^{\mu}(r), f_{T\mu}^{\mu}(r)$ and $f_{SO\mu}^{\mu}(r)$ by solving the Euler-Lagrange Equations with the appropriate constraints.
Energy per baryon

\[ E(n_p, n_n, n_\Lambda, n_{\Sigma^-}) = E_2(n_p, n_n, n_\Lambda, n_{\Sigma^-}) + E_3^N \]

Three-nucleon energy \( E_3^N \)

Based on the expectation value of \( H_3 \) with the Fermi-gas wave function

(NPA902 (2013) 53)

<table>
<thead>
<tr>
<th>( n_0 ) [fm(^{-3})]</th>
<th>( E_0 ) [MeV]</th>
<th>( K ) [MeV]</th>
<th>( E_{\text{sym}} ) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>-16.1</td>
<td>245</td>
<td>30.0</td>
</tr>
</tbody>
</table>
3: Application to neutron stars

Mass-radius relations of neutron stars

Maximum mass of neutron stars

<table>
<thead>
<tr>
<th>Type</th>
<th>Mass (M(_\odot))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>1.48 (M_\odot)</td>
</tr>
<tr>
<td>Type 2</td>
<td>1.53 (M_\odot)</td>
</tr>
<tr>
<td>Type 3</td>
<td>1.57 (M_\odot)</td>
</tr>
<tr>
<td>Type 4</td>
<td>1.62 (M_\odot)</td>
</tr>
<tr>
<td>free (Y)</td>
<td>1.31 (M_\odot)</td>
</tr>
<tr>
<td>without (Y)</td>
<td>2.22 (M_\odot)</td>
</tr>
</tbody>
</table>

The maximum mass increases. 
(1.48 \(M_\odot\) → 1.62 \(M_\odot\))

J0348+0432: Science 340 (2013) 1233232

Shaded region is the observationally suggested region by Steiner et al. (Astrophys. J. 722 (2010) 33)
Composition of Neutron Star Matter

Onset density of $\Lambda$: 0.42 fm$^{-3}$

Onset density of $\Sigma^{-}$: 0.76 fm$^{-3}$ → 0.72 fm$^{-3}$ → 0.70 fm$^{-3}$ → 0.68 fm$^{-3}$
Neutron Star Matter with Three-Baryon Force

We consider a phenomenological three-baryon repulsive force (TBF) as a density dependent two-body effective potential.

Y. Yamamoto et al., PRC 90 (2014) 045805

Mass-radius relations of neutron stars (Type 4)
Composition of neutron star matter (Type 4)
We construct the EOS of nuclear matter containing $\Lambda$ and $\Sigma^-$ hyperons by the cluster variational method.

We investigate the effects of the odd-state $\Lambda\Lambda$ interactions on the structure of neutron stars.

- The repulsion in the odd-state $\Lambda\Lambda$ interaction raises the maximum mass of neutron star. ($1.48 \, M_\odot \rightarrow 1.62 \, M_\odot$)
- The onset density of $\Sigma^-$ strongly depends on the odd-state $\Lambda\Lambda$ interaction.
- Maximum mass of neutron stars with TBF is consistent with the observational data.

Future Plans

- Taking into account mixing of other hyperons ($\Sigma^0, \Sigma^+, \Xi^0, \Xi^-$)
- Hyperon EOS at finite temperatures
- Employing more sophisticated baryon interactions (e.g. Nijmegen)