The EOS of neutron matter, and the effect of Λ hyperons to neutron star structure

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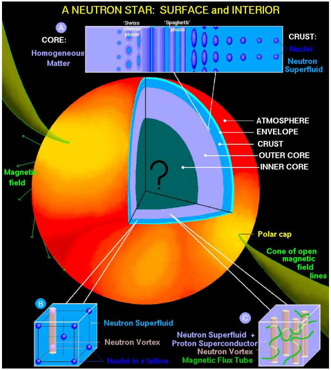
Neutron stars

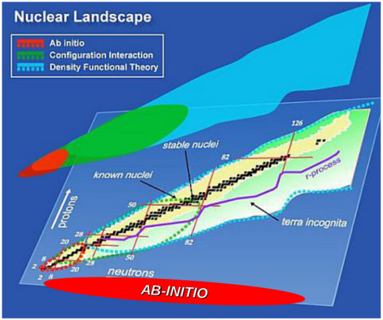
Neutron star is a wonderful natural laboratory

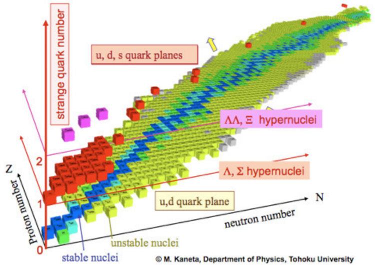


D. Page

- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? π or K condensates?

Nuclei and hypernuclei

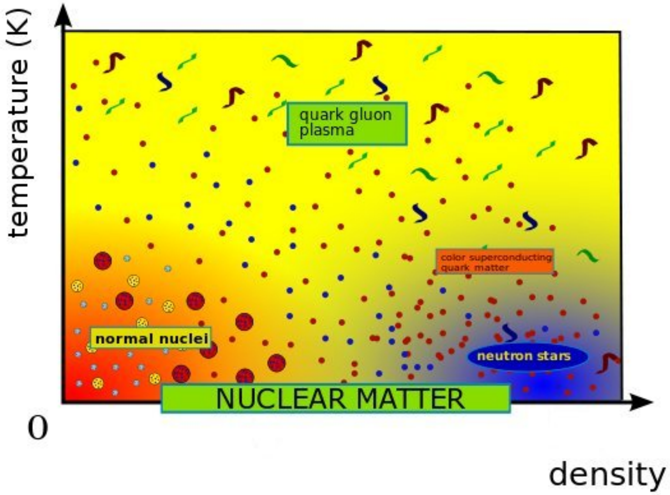




Few thousands of binding energies for normal nuclei are known.

Only few tens for hypernuclei.

Homogeneous neutron matter



Outline

- The model and the method
- Equation of state of neutron matter
- Neutron star structure (I) - radius
- Λ-hypernuclei
- Λ-neutron matter
- Neutron star structure (II) - maximum mass
- Conclusions

Nuclear Hamiltonian

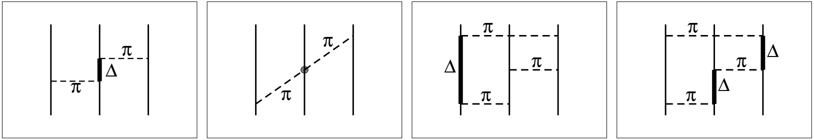
Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} NN (Argonne AV8') fitted on scattering data. Sum of operators:

$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij}), \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

Urbana–Illinois V_{ijk} models processes like



$+$ short-range correlations (spin/isospin independent).

Quantum Monte Carlo

$$H\,\psi(\vec{r}_1\ldots\vec{r}_N)=E\,\psi(\vec{r}_1\ldots\vec{r}_N)\qquad \psi(t)=e^{-(H-E_T)t}\psi(0)$$

Ground-state extracted in the limit of $t\to\infty$.

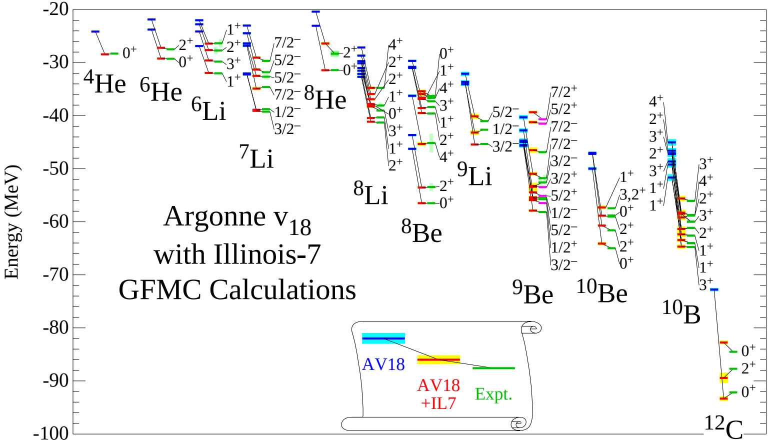
Propagation performed by

$$\psi(R,t)=\langle R|\psi(t)\rangle=\int dR'\,G(R,R',t)\psi(R',0)$$

- Importance sampling: $G(R,R',t)\to G(R,R',t)\,\Psi_I(R')/\Psi_I(R)$
- Constrained-path approximation to control the sign problem. Unconstrained calculation possible in several cases (exact).

Ground–state obtained in a **non-perturbative way.** Systematic uncertainties within 1-2 %.

Light nuclei spectrum computed with GFMC

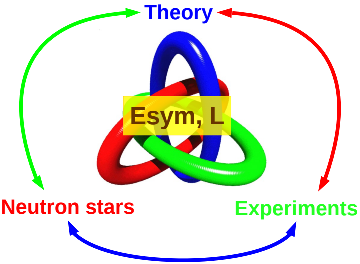


Carlson, *et al.*, arXiv:1412.3081, to appear in Rev. Mod. Phys.

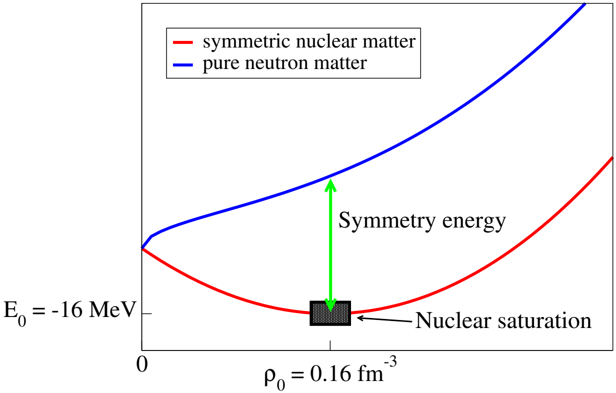
Neutron matter equation of state

Neutron matter is an "exotic" system. Why do we care?

- EOS of neutron matter gives the symmetry energy and its slope.
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments available.
- Determines radii of neutron stars.



What is the Symmetry energy?



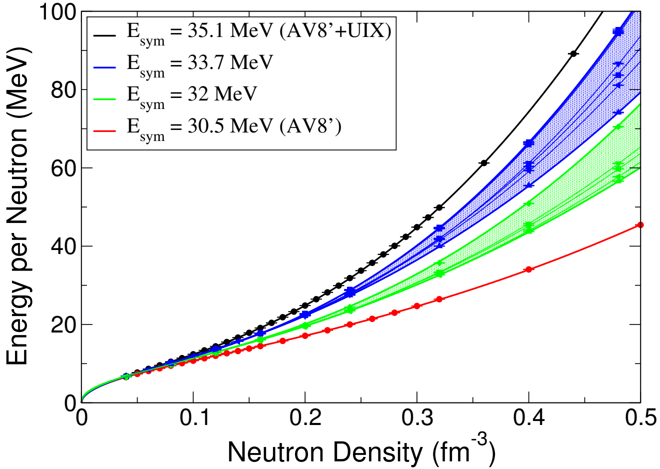
Assumption from experiments:

$$E_{SNM}(\rho_0) = -16 MeV\,, \quad \rho_0 = 0.16 fm^{-3}\,, \quad E_{sym} = E_{PNM}(\rho_0) + 16$$

At ρ_0 we access E_{sym} by studying PNM.

Neutron matter

Equation of state of neutron matter using Argonne forces:

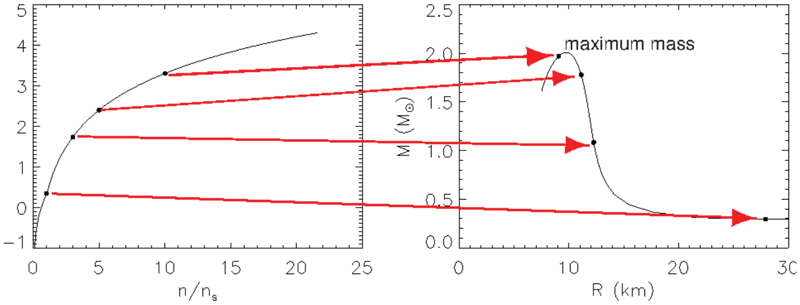


Gandolfi, Carlson, Reddy, PRC (2012)

TOV equations:

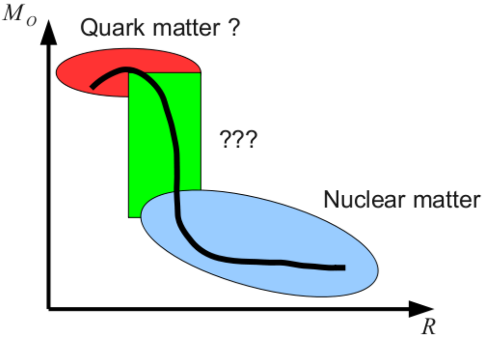
$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]} \,,$$

$$\frac{dm(r)}{dr} = 4\pi\epsilon r^2 \,,$$



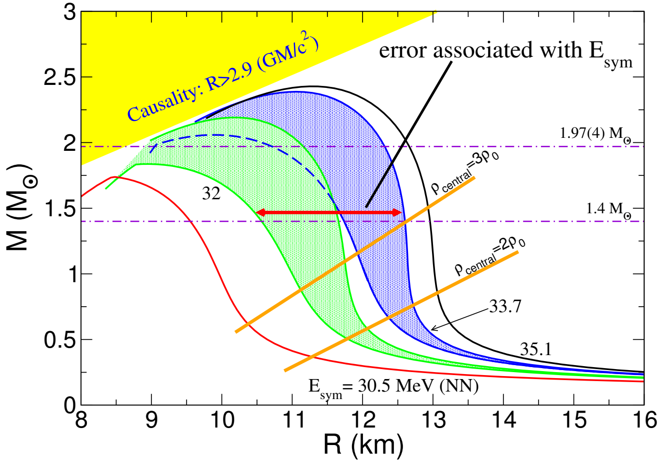
J. Lattimer

Neutron star matter



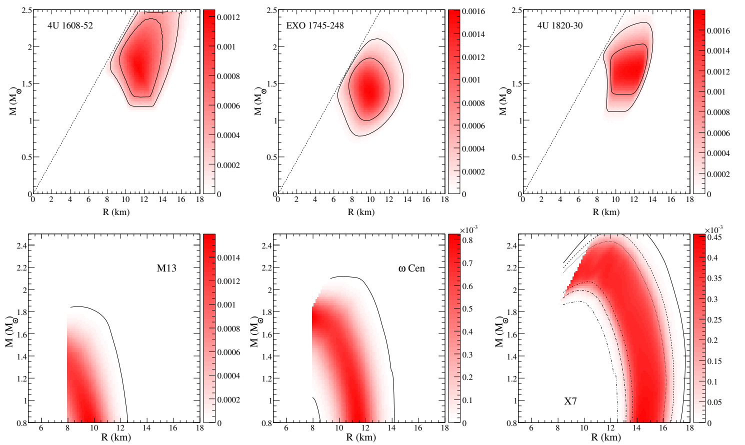
- Neutron star radius sensitive to EOS around ρ $(1-2)\rho_0$
- Maximum mass depends to higher densities

Neutron star structure



Gandolfi, Carlson, Reddy, PRC (2012).

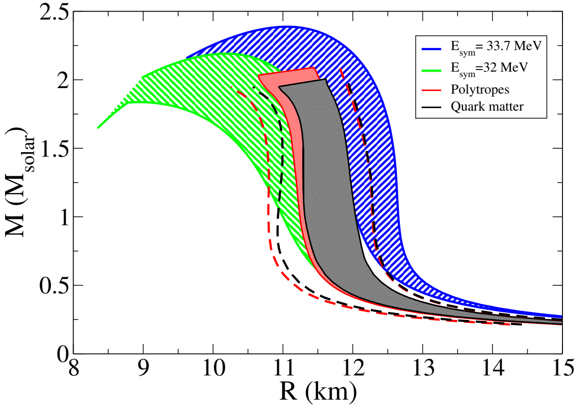
Observations of the mass-radius relation are becoming available:



Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to 'measure' the EOS and constrain E_{sym} and L. (Systematic uncertainties still under debate...)

Neutron star matter really matters!



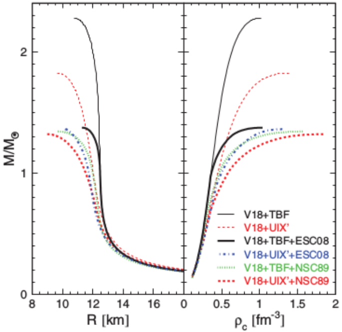
Steiner, Gandolfi, PRL (2012)

Gandolfi, Carlson, Reddy, Steiner, Wiringa, EPJA (2014)

High density neutron matter

If chemical potential large enough ($\rho \sim 2 - 3\rho_0$), nucleons produce Λ, Σ, ...

Non-relativistic BHF calculations suggest that available hyperon-nucleon Hamiltonians support an EOS with $M > 2M_\odot$:



Schulze and Rijken PRC (2011).

Vidana, Logoteta, Providencia, Polls, Bombaci EPL (2011).

Note: (Some) other relativistic model support $2M_\odot$ neutron stars.

$\longrightarrow$ *Hyperon puzzle* (Bombaci, topical session)

Λ-systems, outline

Λ-hypernuclei and hypermatter

$$H = H_N + \frac{\hbar^2}{2m_\Lambda}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij}^{\Lambda N} + \sum_{i<j<k} V_{ijk}^{\Lambda NN}$$

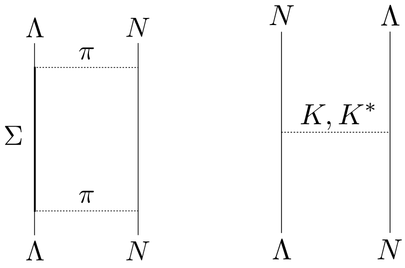
Λ-binding energy calculated as the difference between the system with and without Λ.

Λ-nucleon interaction

The Λ-nucleon interaction is constructed similarly to the Argonne potentials (Usmani).

Argonne NN: $v_{ij} = \sum_p v_p(r_{ij}) O^p_{ij}$, $O_{ij} = (1, \sigma_i \cdot \sigma_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \tau_i \cdot \tau_j)$

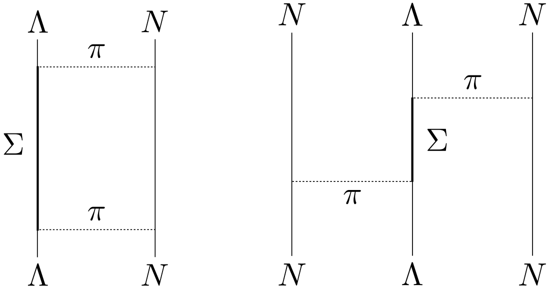
Usmani ΛN: $v_{ij} = \sum_p v_p(r_{ij}) O^p_{ij}$, $O_{\lambda j} = (1, \sigma_\lambda \cdot \sigma_j) \times (1, \tau^z_j)$

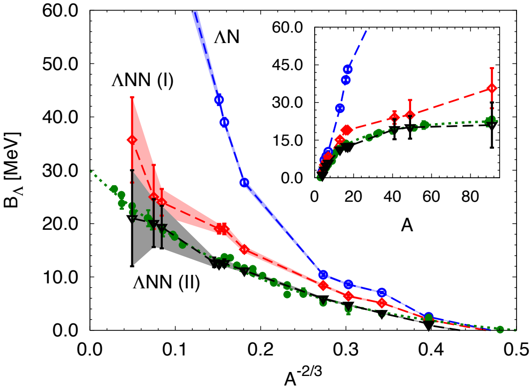


Unfortunately... ∼4500 NN data, ∼30 of ΛN data.

ΛN and ΛNN interactions

ΛNN has the same range of ΛN





Lonardoni, Gandolfi, Pederiva, PRC (2013) and PRC (2014).

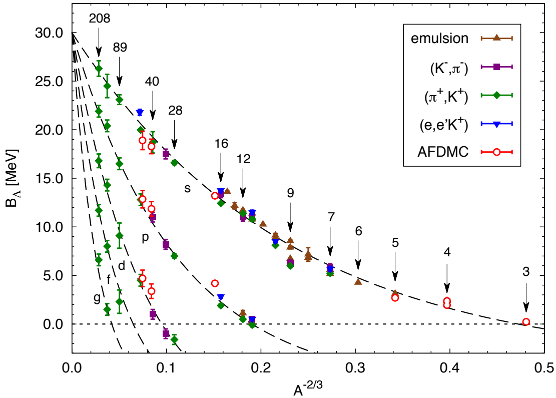
$V^{\Lambda NN}$ (II) is a new form where the parameters have been readjusted.

ΛNN crucial for saturation.

see Lonardoni this afternoon

Λ hypernuclei

Λ in different states:



Pederiva, Catalano, Lonardoni, Lovato, Gandolfi, arXiv:1506.04042

see Lonardoni this afternoon

Hyper-neutron matter

Neutrons and Λ particles:

$$\rho = \rho_n + \rho_\Lambda\,, \qquad\qquad x = \frac{\rho_\Lambda}{\rho}$$

$$E_{\rm HNM}(\rho,x) = \Big[E_{\rm PNM}((1-x)\rho)+m_n\Big](1-x)+\Big[E_{\rm P\Lambda M}(x\rho)+m_\Lambda\Big]x+f(\rho,x)$$

where $E_{P\Lambda M}$ is the non-interacting energy (no $v_{\Lambda\Lambda}$ interaction),

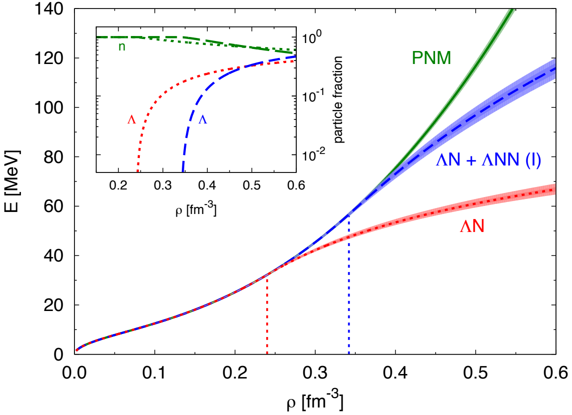
$$E_{PNM}(\rho) = a\left(\frac{\rho}{\rho_0}\right)^\alpha + b\left(\frac{\rho}{\rho_0}\right)^\beta$$

and

$$f(\rho,x) = c_1\frac{x(1-x)\rho}{\rho_0} + c_2\frac{x(1-x)^2\rho^2}{\rho_0^2}$$

All the parameters are fit to Quantum Monte Carlo results.

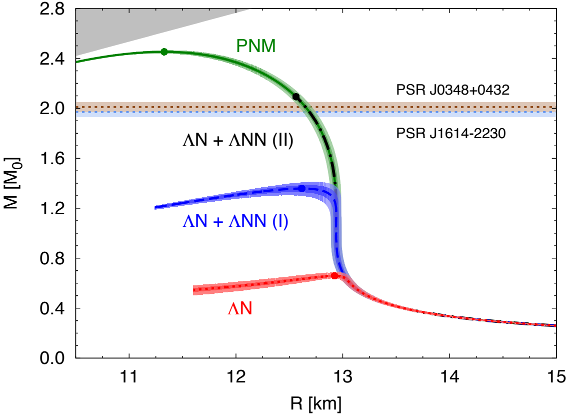
EOS obtained by solving for $\mu_\Lambda(\rho, x) = \mu_n(\rho, x)$



Lonardoni, Lovato, Gandolfi, Pederiva, PRL (2015)

No hyperons up to $\rho = 0.5$ fm^{-3} using ΛNN (II)!!!

Λ-neutron matter



Lonardoni, Lovato, Gandolfi, Pederiva, PRL (2015)

Drastic role played by ΛNN. Calculations can be compatible with neutron star observations.

Note: no $v_{\Lambda\Lambda}$, no protons, and no other hyperons included yet...

Summary

- EOS of pure neutron matter qualitatively well understood.
- Λ-nucleon data very limited, but ΛNN seems very important.
- Role of Λ in neutron stars far to be understood. Conclusion of the conclusions? We cannot conclude **anything** for neutron stars with present models...
  We cannot solve the puzzle, too many pieces are missing!

My wishes:

- More ΛN experimental data needed. Input from Lattice QCD?
- Light and medium Λ-nuclei measurements needed, especially $N \neq Z$.

Acknowledgments

- J. Carlson (LANL)
- D. Lonardoni, A. Lovato (ANL)
- F. Pederiva (Trento)

Extra slides

Nuclear Hamiltonian

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

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v_{ij} NN fitted on scattering data. Sum of operators:

$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij}), \qquad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

NN interaction - Argonne AV8' and AV6'.

Variational wave function

$$E_0 \le E = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{\int dr_1 \ldots dr_N \, \psi^*(r_1 \ldots r_N) H \psi^*(r_1 \ldots r_N)}{\int dr_1 \ldots dr_N \, \psi^*(r_1 \ldots r_N) \psi^*(r_1 \ldots r_N)}$$

$\rightarrow$ Monte Carlo integration. Variational wave function:

$$|\Psi_T\rangle = \left[\prod_{i<j} f_c(r_{ij})\right] \left[\prod_{i<j<k} f_c(r_{ijk})\right] \left[1 + \sum_{i<j,p} \prod_k u_{ijk} f_p(r_{ij}) O_{ij}^p\right] |\Phi\rangle$$

where O^p are spin/isospin operators, f_c, u_{ijk} and f_p are obtained by minimizing the energy. About 30 parameters to optimize.

$|\Phi\rangle$ is a mean-field component, usually HF. Sum of many Slater determinants needed for open-shell configurations.

BCS correlations can be included using a Pfaffian.

Quantum Monte Carlo

Projection in imaginary-time t:

$$H\,\psi(\vec{r}_1\ldots\vec{r}_N) = E\,\psi(\vec{r}_1\ldots\vec{r}_N) \qquad \psi(t) = e^{-(H-E_T)t}\psi(0)$$

Ground-state extracted in the limit of $t \to \infty$.

Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR'\,G(R,R',t)\psi(R',0)$$

- $dR' \to dR_1\,dR_2\ldots,\ G(R,R',t) \to G(R_1,R_2,\delta t)\,G(R_2,R_3,\delta t)\ldots$
- Importance sampling: $G(R,R',\delta t) \to G(R,R',\delta t)\,\Psi_I(R')/\Psi_I(R)$
- Constrained-path approximation to control the sign problem. Unconstrained calculation possible in several cases (exact).

Ground–state obtained in a **non-perturbative way.** Systematic uncertainties within 1-2 %.

Overview

Recall: propagation in imaginary-time

$$e^{-(T+V)\Delta\tau}\psi \approx e^{-T\Delta\tau}e^{-V\Delta\tau}\psi$$

Kinetic energy is sampled as a diffusion of particles:

$$e^{-\nabla^2\Delta\tau}\psi(R) = e^{-(R-R')^2/2\Delta\tau}\psi(R) = \psi(R')$$

The (scalar) potential gives the weight of the configuration:

$$e^{-V(R)\Delta\tau}\psi(R) = w\psi(R)$$

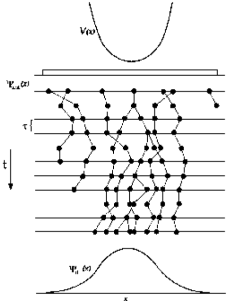
Algorithm for each time-step:

- do the diffusion: $R' = R + \xi$
- compute the weight w
- compute observables using the configuration R' weighted using w over a trial wave function ψ_T.

For spin-dependent potentials things are much worse!

Branching

The configuration weight w is efficiently sampled using the branching technique:



Configurations are replicated or destroyed with probability

$$int[w + \xi] \tag{1}$$

Note: the re-balancing is the bottleneck limiting the parallel efficiency.

Because the Hamiltonian is state dependent, all spin/isospin states of nucleons must be included in the wave-function.

Example: spin for 3 neutrons (radial parts also needed in real life):

GFMC wave-function:

$$\psi = \begin{pmatrix} a_{\uparrow\uparrow\uparrow} \\ a_{\uparrow\uparrow\downarrow} \\ a_{\uparrow\downarrow\uparrow} \\ a_{\uparrow\downarrow\downarrow} \\ a_{\downarrow\uparrow\uparrow} \\ a_{\downarrow\uparrow\downarrow} \\ a_{\downarrow\downarrow\uparrow} \\ a_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

A correlation like

$$1 + f(r)\sigma_1 \cdot \sigma_2$$

can be used, and the variational wave function can be very good. Any operator accurately computed.

AFDMC wave-function:

$$\psi = \mathcal{A}\left[\xi_{s_1}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}\xi_{s_2}\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}\xi_{s_3}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix}\right]$$

We must change the propagator by using the Hubbard-Stratonovich transformation:

$$e^{\frac{1}{2}\Delta t O^2} = \frac{1}{\sqrt{2\pi}}\int dx e^{-\frac{x^2}{2}+x\sqrt{\Delta t}O}$$

Auxiliary fields x must also be sampled. The wave-function is pretty bad, but we can simulate larger systems (up to $A \approx 100$). Operators (except the energy) are very hard to be computed, but in some case there is some trick!

Propagator

We first rewrite the potential as:

$$V = \sum_{i<j} [v_\sigma(r_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j + v_t(r_{ij})(3\vec{\sigma}_i \cdot \hat{r}_{ij}\vec{\sigma}_j \cdot \hat{r}_{ij} - \vec{\sigma}_i \cdot \vec{\sigma}_j)] =$$

$$= \sum_{i,j} \sigma_{i\alpha} A_{i\alpha;j\beta} \sigma_{j\beta} = \frac{1}{2}\sum_{n=1}^{3N} O_n^2 \lambda_n$$

where the new operators are

$$O_n = \sum_{j\beta} \sigma_{j\beta} \psi_{n,j\beta}$$

Now we can use the HS transformation to do the propagation:

$$e^{-\Delta\tau \frac{1}{2}\sum_n \lambda O_n^2}\psi = \prod_n \frac{1}{\sqrt{2\pi}} \int dx e^{-\frac{x^2}{2} + \sqrt{-\lambda\Delta\tau} x O_n}\psi$$

Computational cost $\approx (3N)^3$.

Three-body forces

Three-body forces, Urbana, Illinois, and local chiral N^{2}LO can be exactly included in the case of neutrons.

For example:

$$
\begin{aligned}
O_{2\pi} &= \sum_{cyc} \left[\{X_{ij}, X_{jk}\} \{\tau_i \cdot \tau_j, \tau_j \cdot \tau_k\} + \frac{1}{4} [X_{ij}, X_{jk}] [\tau_i \cdot \tau_j, \tau_j \cdot \tau_k] \right] \\
&= 2 \sum_{cyc} \{X_{ij}, X_{jk}\} = \sigma_i \sigma_k f(r_i, r_j, r_k)
\end{aligned}
$$

The above form can be included in the AFDMC propagator.

The Sign problem in one slide

Evolution in imaginary-time:

$$\psi_I(R')\Psi(R',t+dt)=\int dR\,G(R,R',dt)\frac{\psi_I(R')}{\psi_I(R)}\psi_I(R)\Psi(R,t)$$

note: $\Psi(R,t)$ must be positive to be "Monte Carlo" meaningful.

Fixed-node approximation: solve the problem in a restricted space where $\Psi>0$ (Bosonic problem) $\Rightarrow$ upperbound.

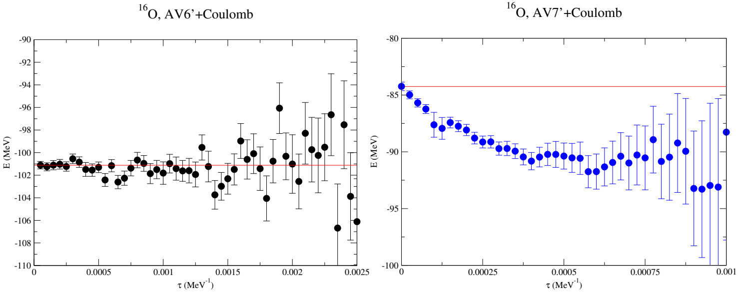
If Ψ is complex:

$$|\psi_I(R')||\Psi(R',t+dt)|=\int dR\,G(R,R',dt)\left|\frac{\psi_I(R')}{\psi_I(R)}\right||\psi_I(R)||\Psi(R,t)|$$

Constrained-path approximation: project the wave-function to the real axis. Multiply the weight term by $\cos\Delta\theta$ (phase of $\frac{\Psi(R')}{\Psi(R)}$), $Re\{\Psi\}>0$ $\Rightarrow$ not necessarily an upperbound.

Unconstrained-path

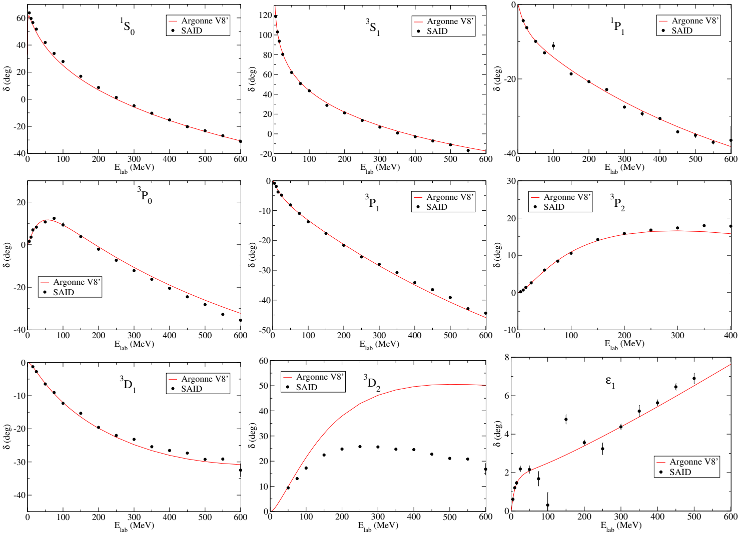
After some equilibration within constrained-path, release the constraint:



The difference between CP and UP results is mainly due to the presence of LS terms in the Hamiltonian. Same for heavier systems.

Work in progress to improve Ψ and to "fully" include three-body forces.

Phase shifts, AV8'



Difference AV8′-AV18 less than 0.2 MeV per nucleon up to A=12.

Scattering data and neutron matter

Two neutrons have

$$k \approx \sqrt{E_{lab}\, m/2}\,, \qquad \to k_F$$

that correspond to

$$k_F \to \rho \approx (E_{lab}\, m/2)^{3/2}/2\pi^2\,.$$

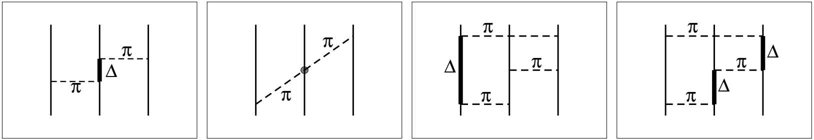
E_{lab}=150 MeV corresponds to about 0.12 fm^{-3}.

E_{lab}=350 MeV to 0.44 fm^{-3}.

Argonne potentials useful to study dense matter above ρ_0=0.16 fm^{-3}

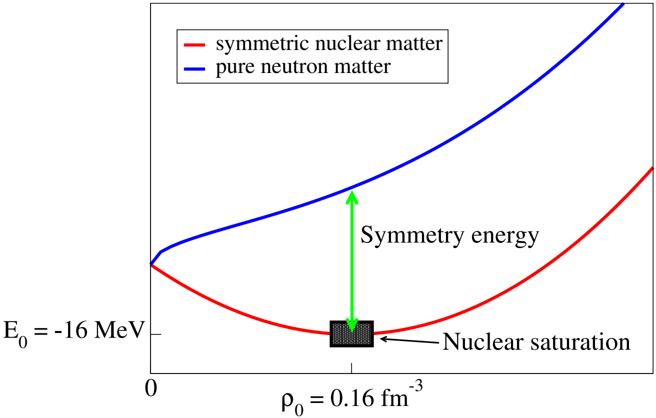
Three-body forces

Urbana–Illinois V_{ijk} models processes like



$+$ short-range correlations (spin/isospin independent).

What is the Symmetry energy?



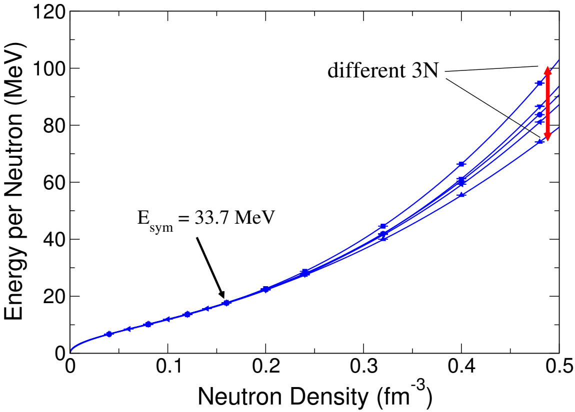
Assumption from experiments:

$$E_{SNM}(\rho_0) = -16 MeV \,, \quad \rho_0 = 0.16 fm^{-3} \,, \quad E_{sym} = E_{PNM}(\rho_0) + 16$$

At ρ_0 we access E_{sym} by studying PNM.

Neutron matter

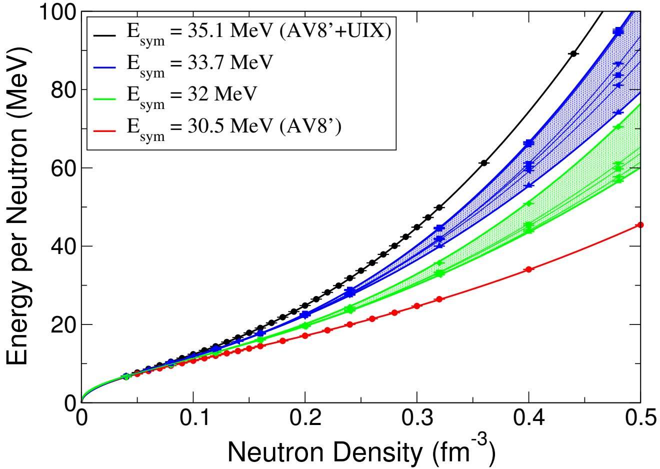
We consider different forms of three-neutron interaction by only requiring a particular value of E_{sym} at saturation.



different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several μ)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$

Equation of state of neutron matter using Argonne forces:

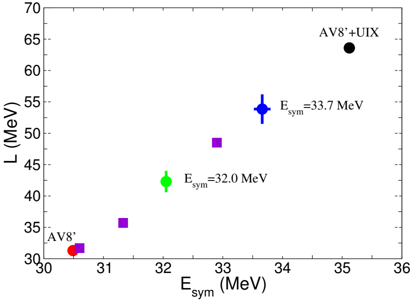


Gandolfi, Carlson, Reddy, PRC (2012)

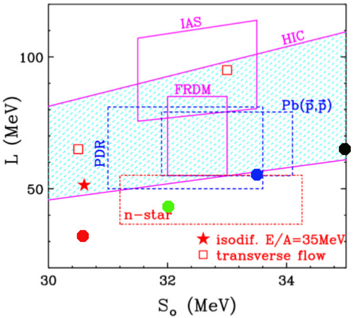
Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



Gandolfi *et al.*, EPJ (2014)



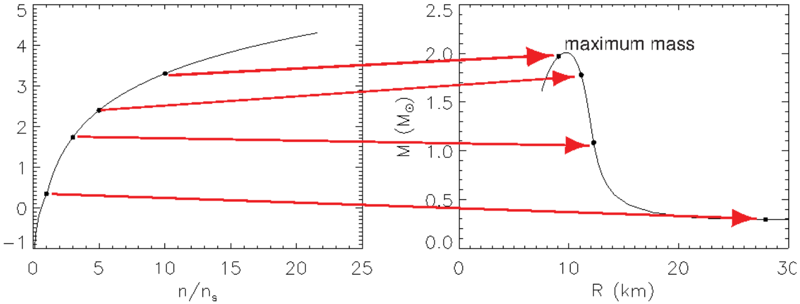
Tsang *et al.*, PRC (2012)

Very weak dependence to the model of 3N force for a given E_{sym}.
Chiral Hamiltonians give compatible results.

TOV equations:

$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]},$$

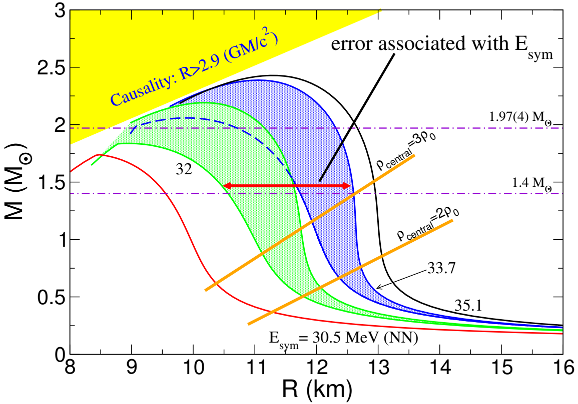
$$\frac{dm(r)}{dr} = 4\pi\epsilon r^2,$$



J. Lattimer

Neutron star structure

EOS used to solve the TOV equations.

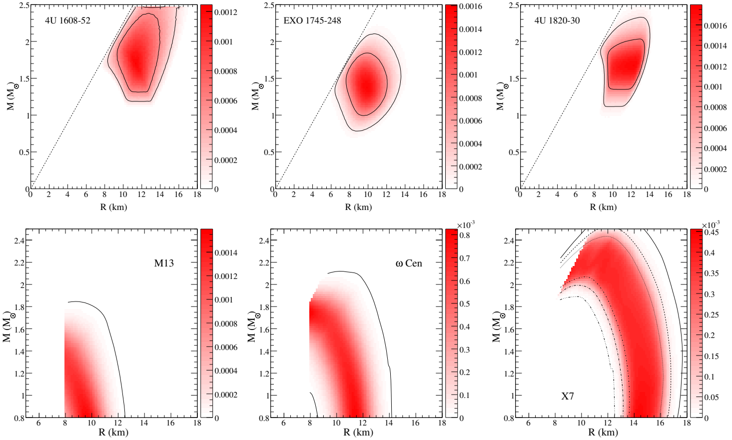


Gandolfi, Carlson, Reddy, PRC (2012).

Accurate measurement of E_{sym} put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain E_{sym}!

Neutron stars

Observations of the mass-radius relation are becoming available:



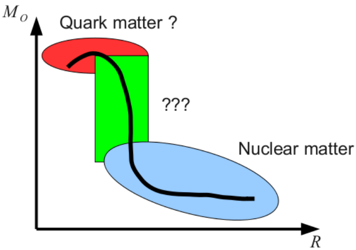
Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to 'measure' the EOS and constrain E_{sym} and L.

Neutron star matter

Neutron star matter model:

$$E_{NSM} = a \left(\frac{\rho}{\rho_0}\right)^{\alpha} + b \left(\frac{\rho}{\rho_0}\right)^{\beta} , \quad \rho < \rho_t$$



(form suggested by QMC simulations),

and a high density model for $\rho > \rho_t$

i) two polytropes

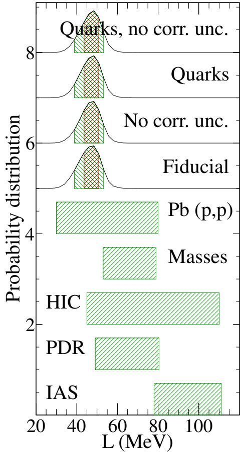
ii) polytrope+quark matter model

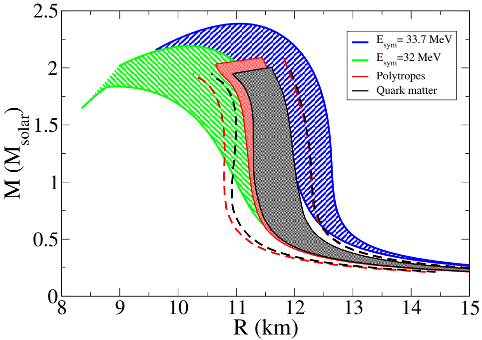
Neutron star radius sensitive to the EOS at nuclear densities!

Direct way to extract E_{sym} and L from neutron stars observations:

$$E_{sym} = a + b + 16 , \quad L = 3(a\alpha + b\beta)$$

Here an 'astrophysical measurement'





$32 < E_{sym} < 34\,MeV,\ 43 < L < 52\,MeV$

Steiner, Gandolfi, PRL (2012).