Nuclear equation of state for core-collapse simulations with realistic nuclear forces

H. Togashi (RIKEN)

Outline

1 : Introduction

2 : Equation of state for uniform nuclear matter

3 : Equation of state for non-uniform nuclear matter

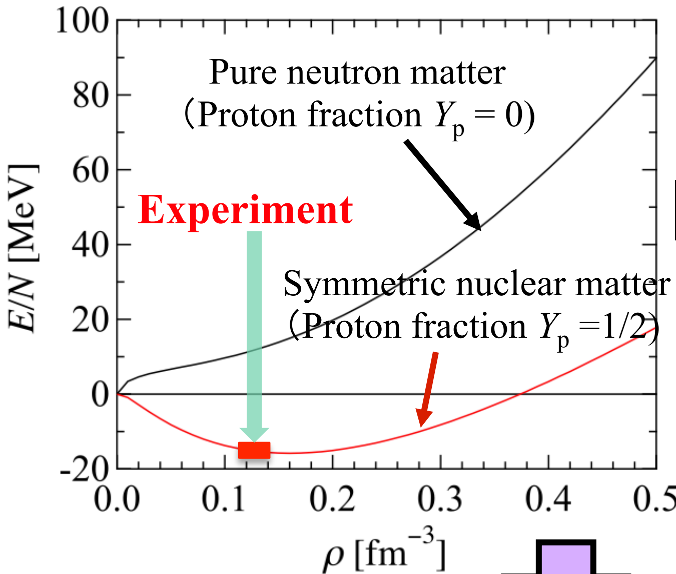
4 : Application to astrophysics

5 : Variational method for Hyperon matter

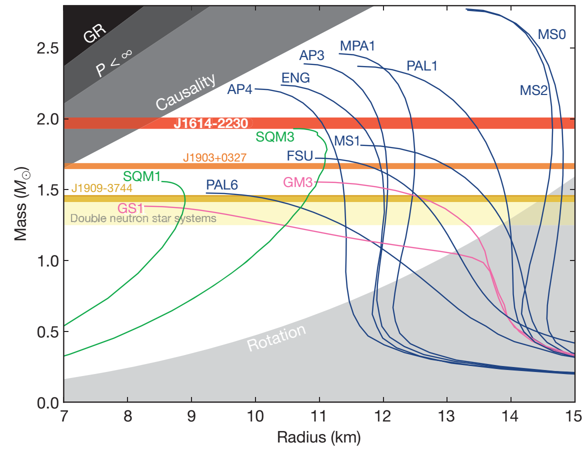
The 208th RIBF Nuclear Physics Seminar @ RIKEN, Oct. 27, 2015

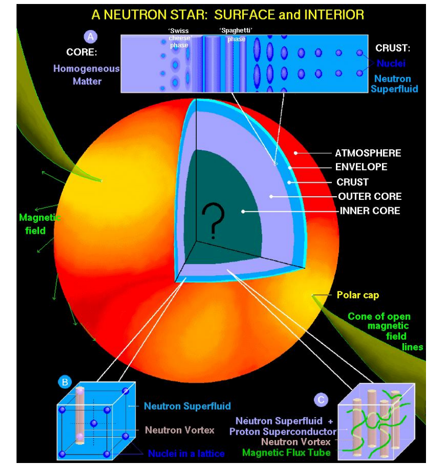
1: Introduction

The nuclear equation of state (EOS) plays important roles for astrophysical studies.



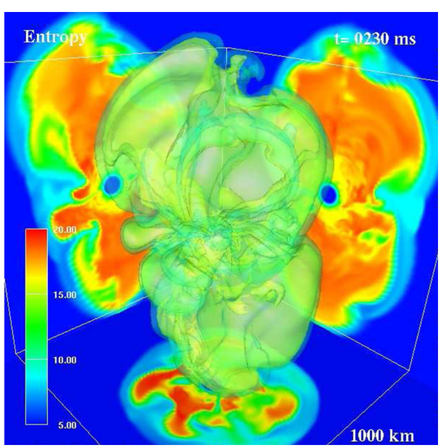
Cold Neutron Star Structure





(NATURE 467 (2010)) http://www.astro.umd.edu/~miller/nstar.html

Core-Collapse Supernovae



(APJ 786 (2014) 83)

Black Hole Formation

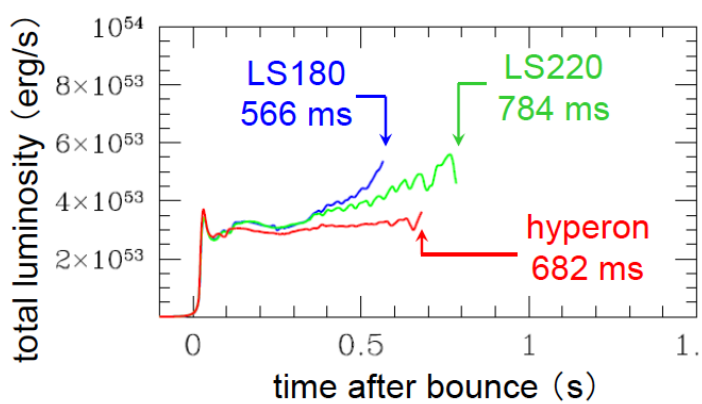
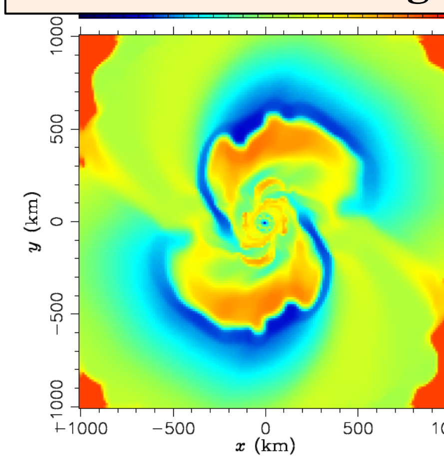


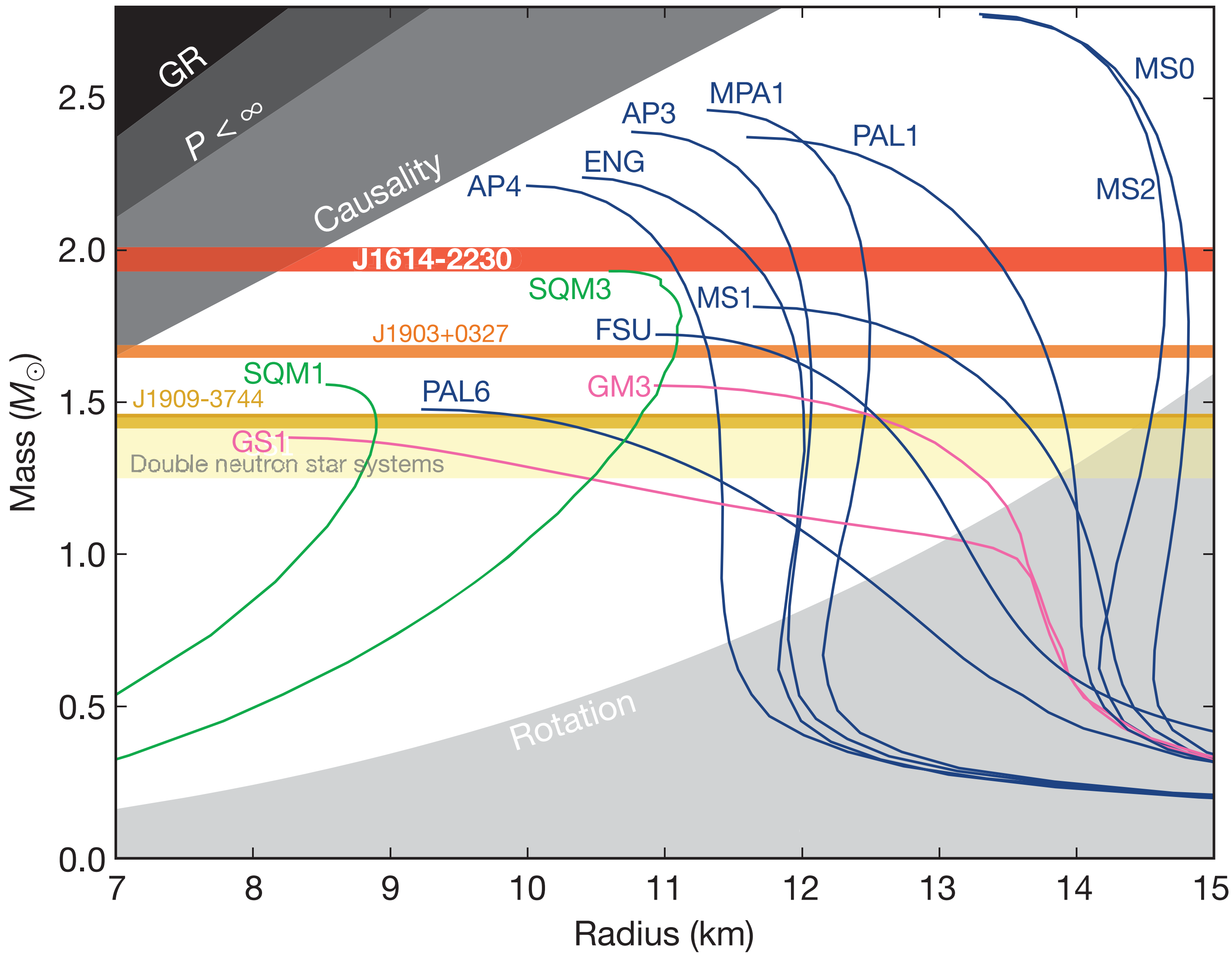
Figure by K. Nakazato

Neutron Star Merger



(APJL 789 (2014) L39)

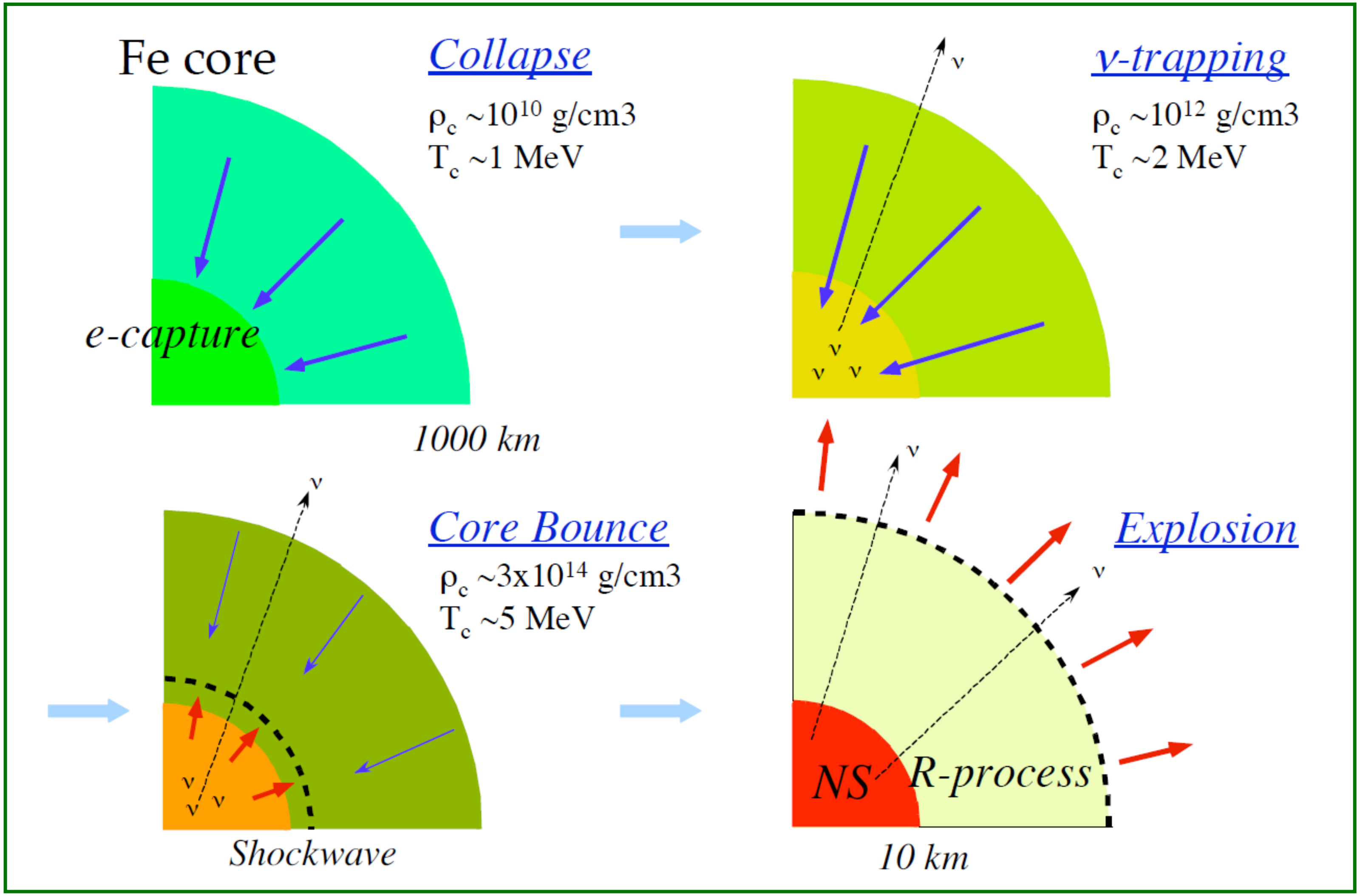
Nuclear EOS and Neutron star structures



P. B. Demorest et al., NATURE 467 (2010)

Core-Collapse Supernovae

The nuclear EOS at finite temperature is necessary for studies of core-collapse supernovae (SNe).



K. Sumiyoshi, CNS Summer School 2008

Equation of State for Supernova Simulations

1. **Supernova matter** contains uniform and non-uniform matter.

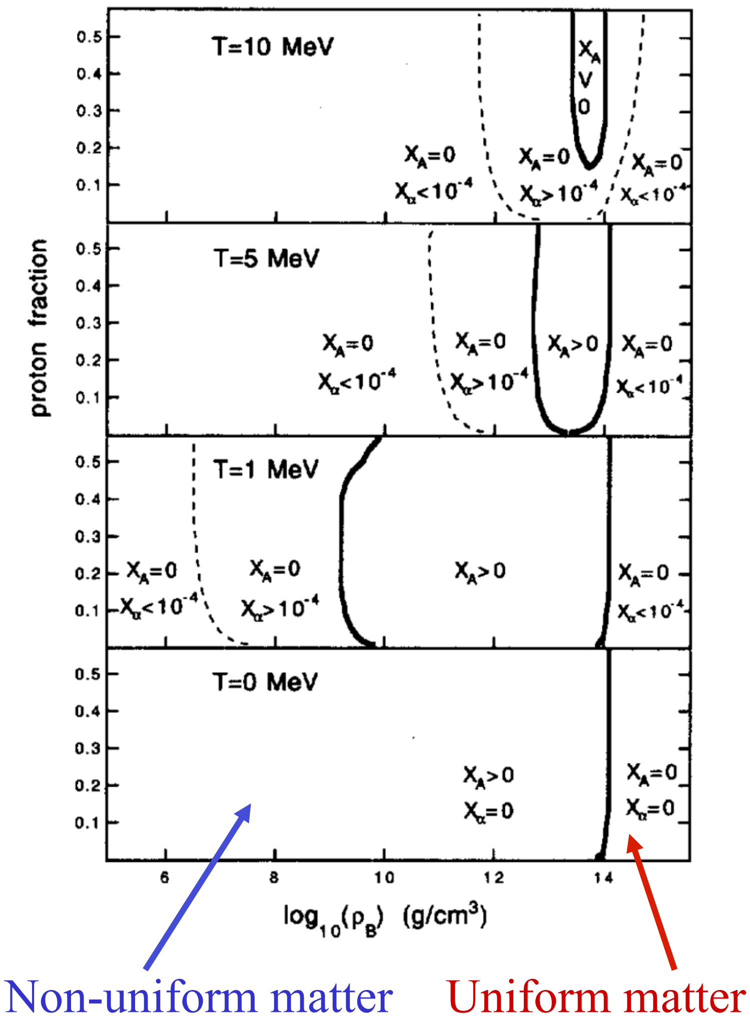
2. We have to calculate thermodynamic quantities in the following **wide ranges**.

Density ρ : $10^{5.1} \leq \rho_{\rm m} \leq 10^{16.0}$g/cm^3

Temperature T : $0 \leq T \leq 400$ MeV

Proton fraction $Y_{\rm p}$: $0 \leq Y_{\rm p} \leq 0.65$

(PTP 100 (1998) 1013)



List of SN-EOS

1. Lattimer-Swesty EOS : *The Skyrme-type interaction* (NPA 535 (1991) 331)

2. Shen EOS : *The Relativistic Mean Field Theory* (NPA 637 (1998) 435)

Nuclear Interaction	n_{sat} (fm^{-3})	BE/A (MeV)	K (MeV)	Q ($\frac{\mathrm{MeV}}{\mathrm{fm}^3}$)	J (MeV)	L (MeV)	type of int.	used in
SKa	0.155	16.0	263	-300	32.9	74.6	Skyrme	H&W
LS180	0.155	16.0	180	-451	28.6	73.8	Skyrme	LS180
LS220	0.155	16.0	220	-411	28.6	73.8	Skyrme	LS220, LS220Λ, LS220π
LS375	0.155	16.0	375	176	28.6	73.8	Skyrme	LS375
TMA	0.147	16.0	318	-572	30.7	90.1	RMF	HS(TMA)
NL3	0.148	16.2	272	203	37.3	118.2	RMF	SHT, HS(NL3)
FSUgold	0.148	16.3	230	-524	32.6	60.5	RMF	SHO(FSU1.7), HS(FSUgold)
FSUgold2.1	0.148	16.3	230	-524	32.6	60.5	RMF	SHO(FSU2.1)
IUFSU	0.155	16.4	231	-290	31.3	47.2	RMF	HS(IUFSU)
DD2	0.149	16.0	243	169	31.7	55.0	RMF	HS(DD2), BHBΛ, BHB$\Lambda\phi$
SFHo	0.158	16.2	245	-468	31.6	47.1	RMF	SFHo
SFHx	0.160	16.2	239	-457	28.7	23.2	RMF	SFHx
TM1	0.145	16.3	281	-285	36.9	110.8	RMF	STOS, FYSS, HS(TM1), STOSΛ, STOSY, STOSYπ, STOSπ, STOSπQ, STOSQ, STOSB139, STOSB145, STOSB155, STOSB162, STOSB165

SN-EOS list by M. Hempel

These EOSs are based on phenomenological models for uniform matter.

We aim to construct an EOS with the variational many-body theory starting from bare nuclear forces.

EOS with Variational Method

Fermi Hypernetted Chain (FHNC) method

Zero temperature : APR — A. Akmal et al., PRC58(1998)1804

Finite temperature : AM — A. Mukherjee, PRC 79(2009) 045811

Potential: AV18+UIX — Wave function: Jastrow wave function

For Symmetric Nuclear Matter and Pure Neutron Matter

SN-EOS table must cover in the following wide ranges.

Density ρ : $10^{5.1} \leq \rho_{\mathrm{m}} \leq 10^{16.0}\mathrm{g/cm^3}$	110 point
Temperature T : $0 \leq T \leq 400$ MeV	92 point
Proton fraction Y_{p} : $0 \leq Y_{\mathrm{p}} \leq 0.65$	66 point

We have to treat Asymmetric Nuclear Matter directly.

SN-EOS for Uniform Nuclear Matter is constructed with *the simplified cluster variational method.*

Our Plan to Construct the EOS for SN Simulations

Uniform Nuclear Matter

EOS is constructed with ***the cluster variational method***

Atomic Nuclei

We tune parameters in the uniform EOS with **the Thomas-Fermi calculation** for **atomic nuclei**.

Non-uniform Nuclear Matter

EOS is constructed with ***the Thomas-Fermi calculation***

★We are here.★

Completion of a Nuclear EOS table for SN simulations

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1 : Introduction

2 : Equation of state for uniform nuclear matter

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4 : Application to astrophysics

5 : Variational method for Hyperon matter

Collaborators : M. Takano (Waseda University)

NPA902 (2013) 53

2. Equation of state for uniform nuclear matter

Nuclear Hamiltonian $\boxed{H = H_2 + H_3}$

Two-body Hamiltonian

$$H_2 = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j}^{N} V_{ij}$$

The AV18 two-body nuclear potential

Three-body Hamiltonian

$$H_3 = \sum_{i<j<k}^{N} V_{ijk}$$

The UIX three-body nuclear potential

AV18 potential: (PRC 51 (1995) 38)

$$\begin{aligned} V_{ij} \quad = \quad & \sum_{t=0}^{1} \sum_{s=0}^{1} [V_{\mathrm{C}ts}(r_{ij}) + sV_{\mathrm{T}t}(r_{ij})S_{\mathrm{T}ij} + sV_{\mathrm{SO}t}(r_{ij})(\boldsymbol{L}_{ij} \cdot \boldsymbol{s}) \\ & + V_{\mathrm{qL}ts}(r_{ij}) \left|\boldsymbol{L}_{ij}\right|^2 + sV_{\mathrm{qSO}t}(r_{ij})(\boldsymbol{L}_{ij} \cdot \boldsymbol{s})^2] P_{tsij} \end{aligned}$$

UIX potential: (PRL 74 (1995) 4396) $\boxed{V_{ijk} = V_{ijk}^{\mathrm{R}} + V_{ijk}^{2\pi}}$

2. Equation of state for uniform nuclear matter

Nuclear Hamiltonian $\boxed{H = H_2 + H_3}$

Two-body Hamiltonian

$$\boxed{H_2 = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j}^{N} V_{ij}}$$

The AV18 two-body nuclear potential

Three-body Hamiltonian

$$\boxed{H_3 = \sum_{i<j<k}^{N} V_{ijk}}$$

The UIX three-body nuclear potential

Expectation values of these Hamiltonians are calculated separately.

Two-body Hamiltonian

Many-body calculations of H_2 have been investigated for a long time and reliable results have been obtained.

Three-body Hamiltonian

Three-body interaction has some uncertainty.

Expectation Value of H_2

Jastrow wave function

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}$$

Φ_{F}: The Fermi-gas wave function

Two-body correlation function f_{ij}

$$f_{ij} = \sum_{t=0}^{1}\sum_{\mu}\sum_{s=0}^{1}\Big[\underbrace{f^{\mu}_{Cts}(r_{ij})}_{\text{Central}} + s\,\underbrace{f^{\mu}_{Tt}(r_{ij})}_{\text{Tensor}}S_{Tij} + s\,\underbrace{f^{\mu}_{SOt}(r_{ij})}_{\text{Spin-orbit}}(\boldsymbol{L}_{ij}\cdot\boldsymbol{s})\Big]P^{\mu}_{tsij}$$

$P_{ts}{}^{\mu}$: Spin-isospin projection operators

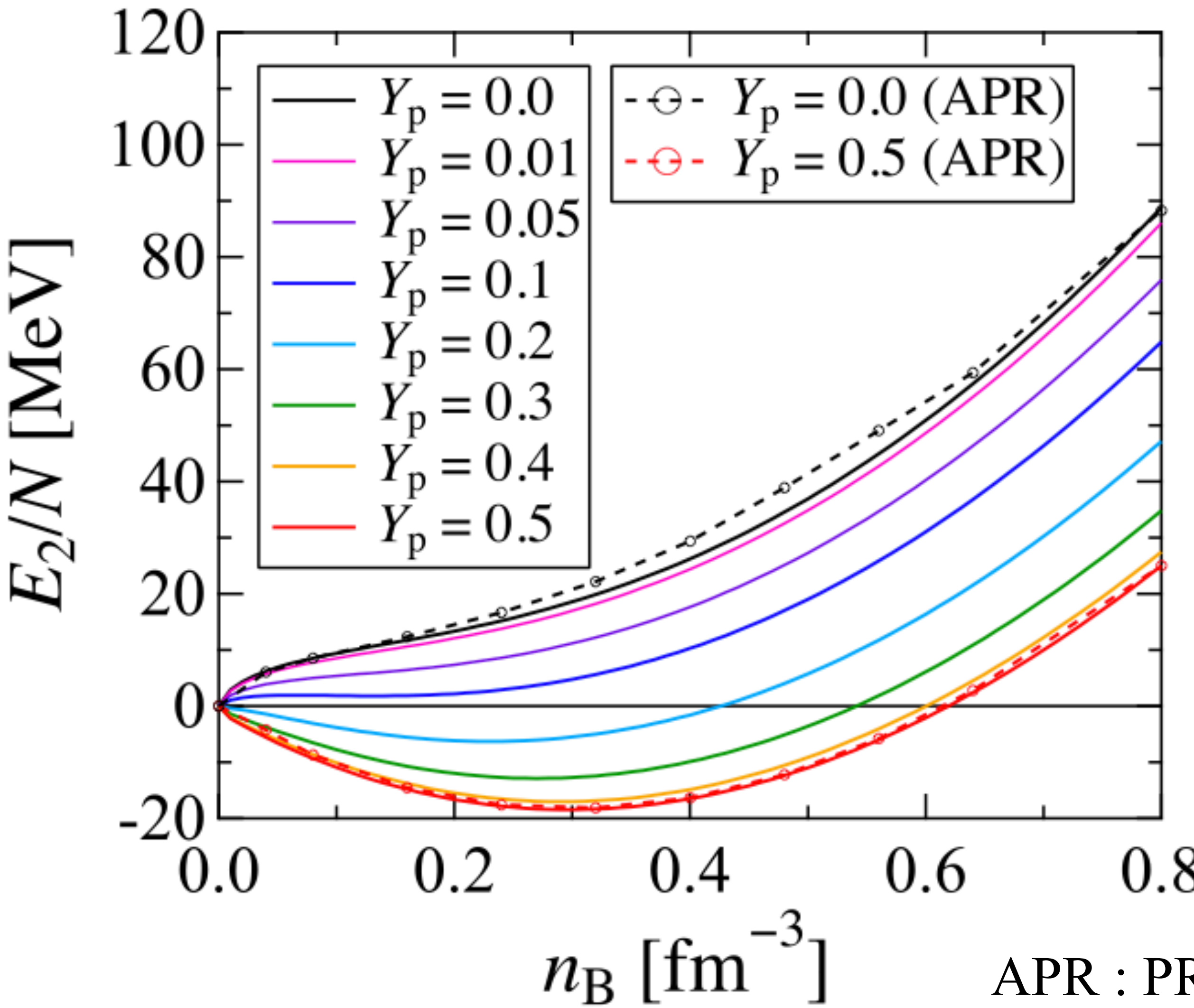
Expectation value of H_2 with *the Cluster Expansion*

$$\frac{\langle H_2\rangle}{N} = \frac{1}{N}\frac{\langle \Psi \mid H_2 \mid \Psi\rangle}{\langle \Psi \mid \Psi\rangle} = \frac{\langle H_2\rangle_2}{N} + \frac{\langle H_2\rangle_3}{N} + \cdots$$

E_2/N is the expectation value of H_2 in *the two-body cluster approximation*.

E_2/N is minimized with respect to $f_{Cts}{}^{\mu}(r)$, $f_{Tt}{}^{\mu}(r)$ and $f_{SOt}{}^{\mu}(r)$ with the appropriate constraints.

Two-Body Energy E_2/N



APR : PRC58(1998)1804

Our results are in good agreement with the results by APR (FHNC method).

Three-Body Energy

UIX potential

$$\boxed{V_{ijk} = V^{\mathrm{R}}_{ijk} + V^{2\pi}_{ijk}}$$

$V^{2\pi}_{ijk}$: 2π exchange part

V^{R}_{ijk}: Repulsive part

Three-Body Energy

$$\frac{E_3(n_{\mathrm{B}}, Y_{\mathrm{p}})}{N} = \frac{1}{N}\langle \sum_{i<j<k} \left[\alpha V^{\mathrm{R}}_{ijk} + \beta V^{2\pi}_{ijk}\right]\rangle_{\mathrm{F}} + \gamma n_{\mathrm{B}}^2 e^{-\delta n_{\mathrm{B}}} \left[1 - (1 - 2Y_{\mathrm{p}})^2\right]$$

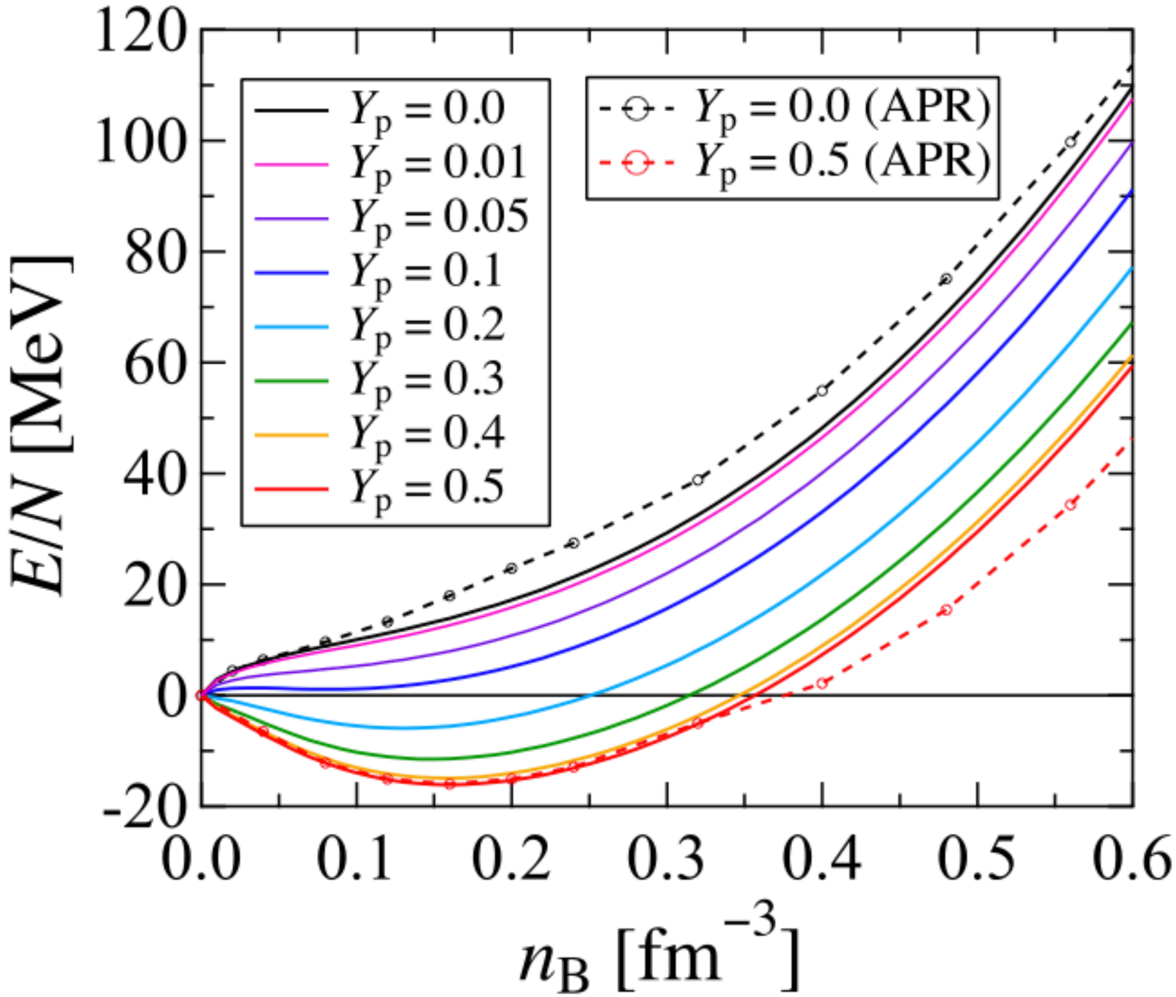
Correction term

Expectation value with the Fermi-gas wave function

α, β, γ, δ : Parameters in E_3/N

- **Total energy per nucleon** $\boldsymbol{E/N = E_2/N + E_3/N}$ reproduce the empirical saturation properties.
- Thomas-Fermi calculation of isolated atomic nuclei with $\boldsymbol{E/N}$ reproduces the gross feature of the experimental data.

Total Energy per Nucleon E/N



Our EOS : NPA902 (2013) 53

APR : PRC58(1998)1804

n_0[fm^{-3}]	E_0[MeV]	K [MeV]	E_{sym}[MeV]
0.16	−16.1	245	30.0

EOS of Uniform Nuclear Matter at Finite Temperature

We follow the prescription proposed by *Schmidt and Pandharipande.*

(Phys. Lett. 87B(1979) 11) (A. Mukherjee et al., PRC 75(2007) 035802)

Free energy F/N is expressed by the average occupation probabilities.

The average occupation probability

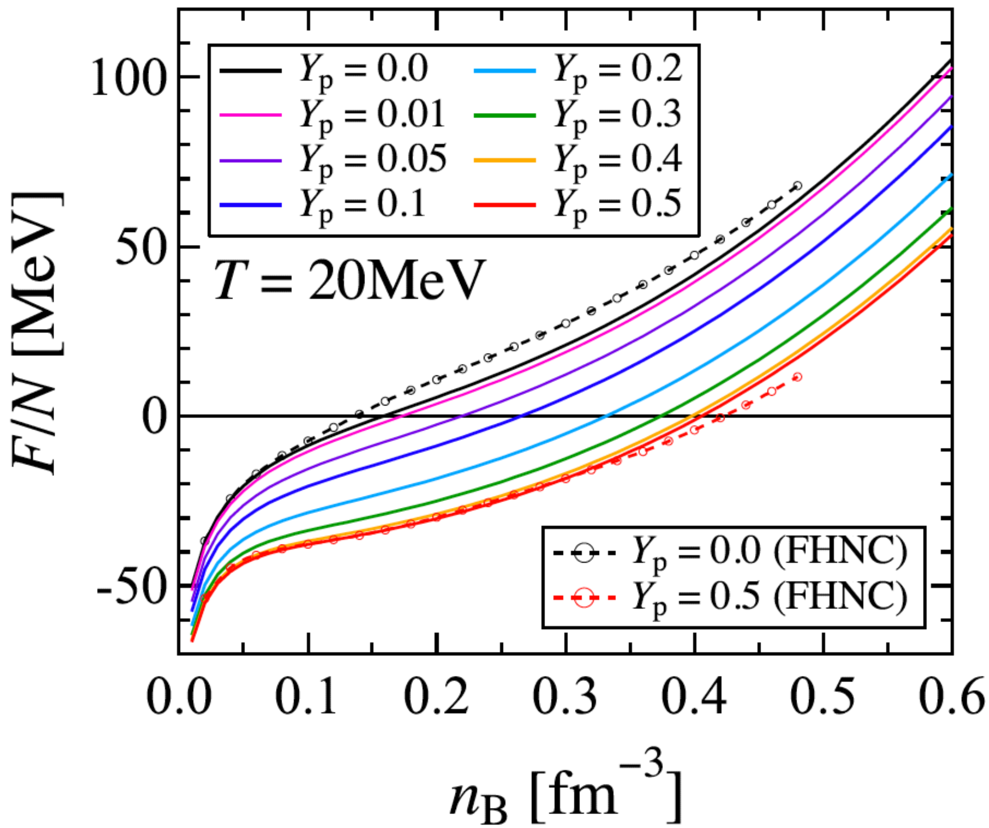
$$f_i(k) = \left\{1 + \exp\left[\frac{\varepsilon_i(k) - \mu_{0i}}{k_B T}\right]\right\}^{-1}$$

$\varepsilon_i(k)$: Single particle energy

$$\varepsilon_i(k) = \frac{\hbar^2 k^2}{2m_i^*} \quad (i = \mathrm{p, n})$$

m_i*: Effective mass of nucleons

Free energies are minimized with respect to m_p* and m_n*



Our EOS : NPA902 (2013) 53 FHNC : A. Mukherjee, PRC 79(2009) 045811

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Collaborators : M. Takano (Waseda University),

K. Nakazato, H. Suzuki, Y. Takehara, S. Yamamuro (Tokyo Univ. Sci.)

Thomas-Fermi method for isolated atomic nuclei

Binding energy (proton number Z, neutron number N)

$$-B(N,Z) = \int d\boldsymbol{r}\underbrace{\varepsilon(n_{\mathrm{p}}(r), n_{\mathrm{n}}(r))}_{\text{Bulk energy}} + \underbrace{F_0 \int d\boldsymbol{r}|\nabla n(r)|^2}_{\text{Gradient energy}} + \underbrace{\frac{e^2}{2}\int d\boldsymbol{r}\int d\boldsymbol{r}' \frac{n_{\mathrm{p}}(r) n_{\mathrm{p}}(r')}{|\boldsymbol{r}-\boldsymbol{r}'|}}_{\text{Coulomb energy}}$$

Energy density ε is derived from uniform EOS.

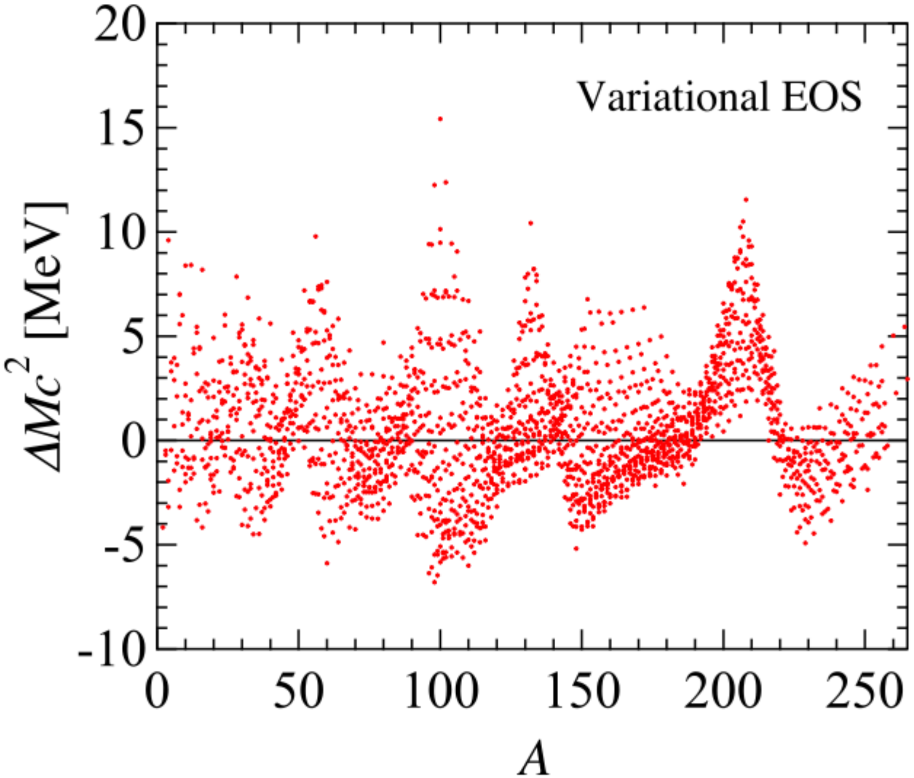
Proton/neutron density distribution

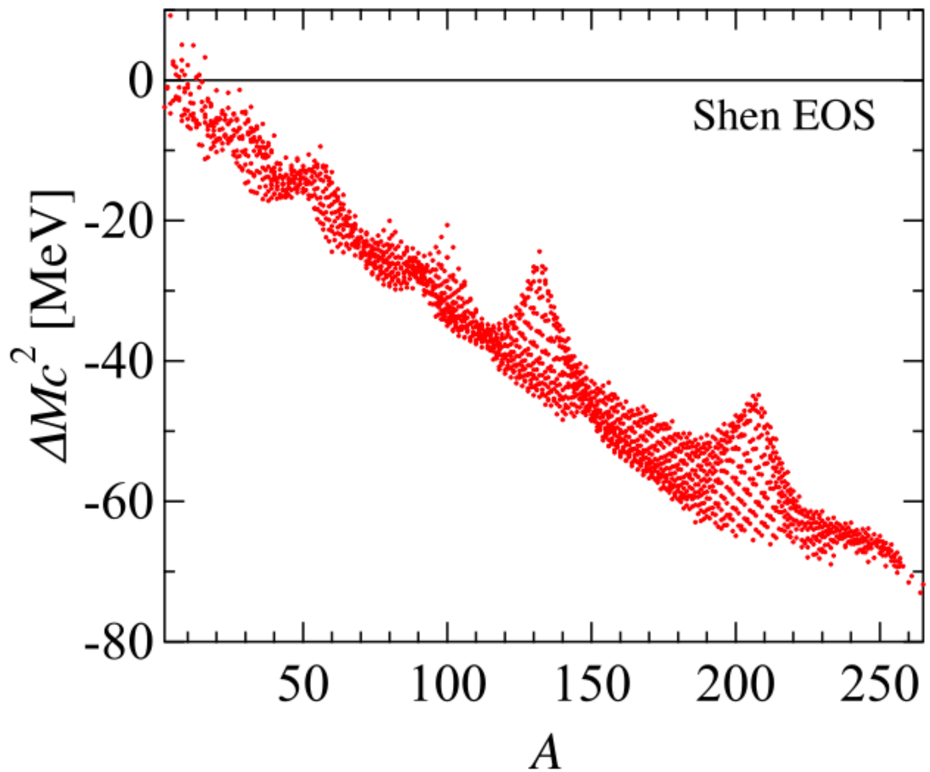
$$n_i(r) = \begin{cases} n_i^{\mathrm{in}}[1-(r/R_i)^{t_i}]^3 & (0 \le r \le R_i) \\ 0 & (R_i \le r) \end{cases} \qquad i = \mathrm{p, n}$$

Normalization condition : $Z = \int d\boldsymbol{r} n_{\mathrm{p}}(r)$ $\quad N = \int d\boldsymbol{r} n_{\mathrm{n}}(r)$

$-B(N, Z)$ is minimized with respect to $t_{\mathrm{n}}, R_{\mathrm{n}}, t_{\mathrm{p}}, R_{\mathrm{p}}$ for each nucleus.

Deviation of the masses of the mass-measured nuclei





$\Delta M = M_{\mathrm{TF}} - M_{\mathrm{exp}}$

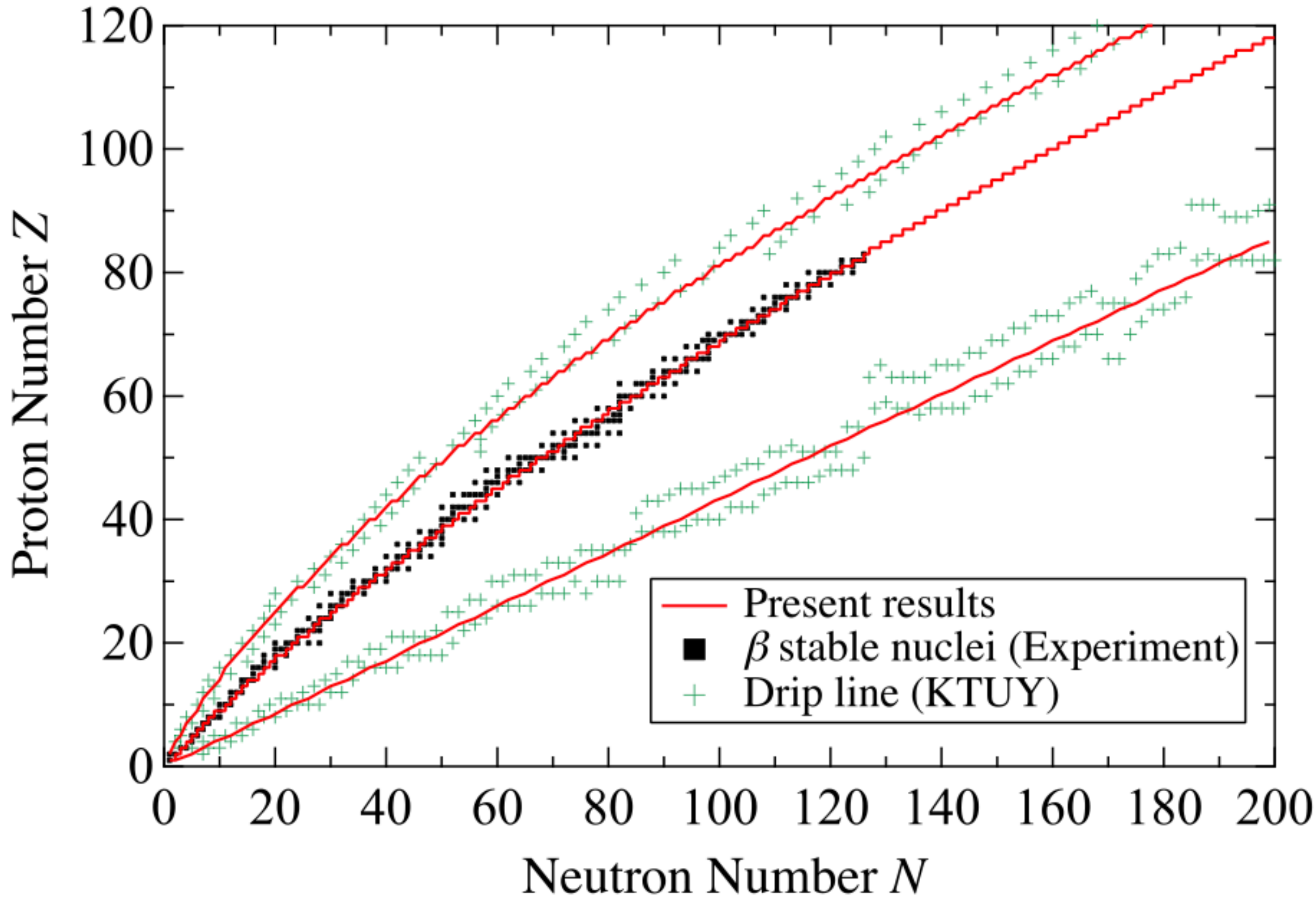
M_{TF} : Mass by the Thomas-Fermi calculation

M_{exp} : Experimental data
(G. Audi et al., Nucl. Phys. A **729** (2003) 337)

RMS deviation (for 2226 nuclei)

2.99 MeV (Variational EOS) **42.9 MeV (Shen EOS)**

β-stability line and drip lines



Our results are in good agreement with
the experimental data and the sophisticated atomic mass formula.

Thomas-Fermi method for non-uniform matter

We follow ***the Thomas-Fermi (TF) method*** *by Shen et. al.* (PTP100(1998) 1013) (APJS 197(2011) 20)

Free energy in the Wigner-Seitz (WS) cell

Bulk energy — Gradient energy

$$F = \int d\boldsymbol{r}\, \underline{f(n_{\mathrm{p}}(r), n_{\mathrm{n}}(r), n_\alpha(r))} + F_0 \int d\boldsymbol{r} |\nabla(n_{\mathrm{p}}(r) + n_{\mathrm{n}}(r))|^2$$
$$+ \frac{e^2}{2} \int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{[n_{\mathrm{p}}(r) + 2n_\alpha(r) - n_{\mathrm{e}}][n_{\mathrm{p}}(r') + 2n_\alpha(r') - n_{\mathrm{e}}]}{|\boldsymbol{r} - \boldsymbol{r}'|} + c_{\mathrm{bcc}} \frac{(Ze)^2}{a}$$

Coulomb energy

$F_0 = 68.00$ MeV fm^5 a : Lattice constant

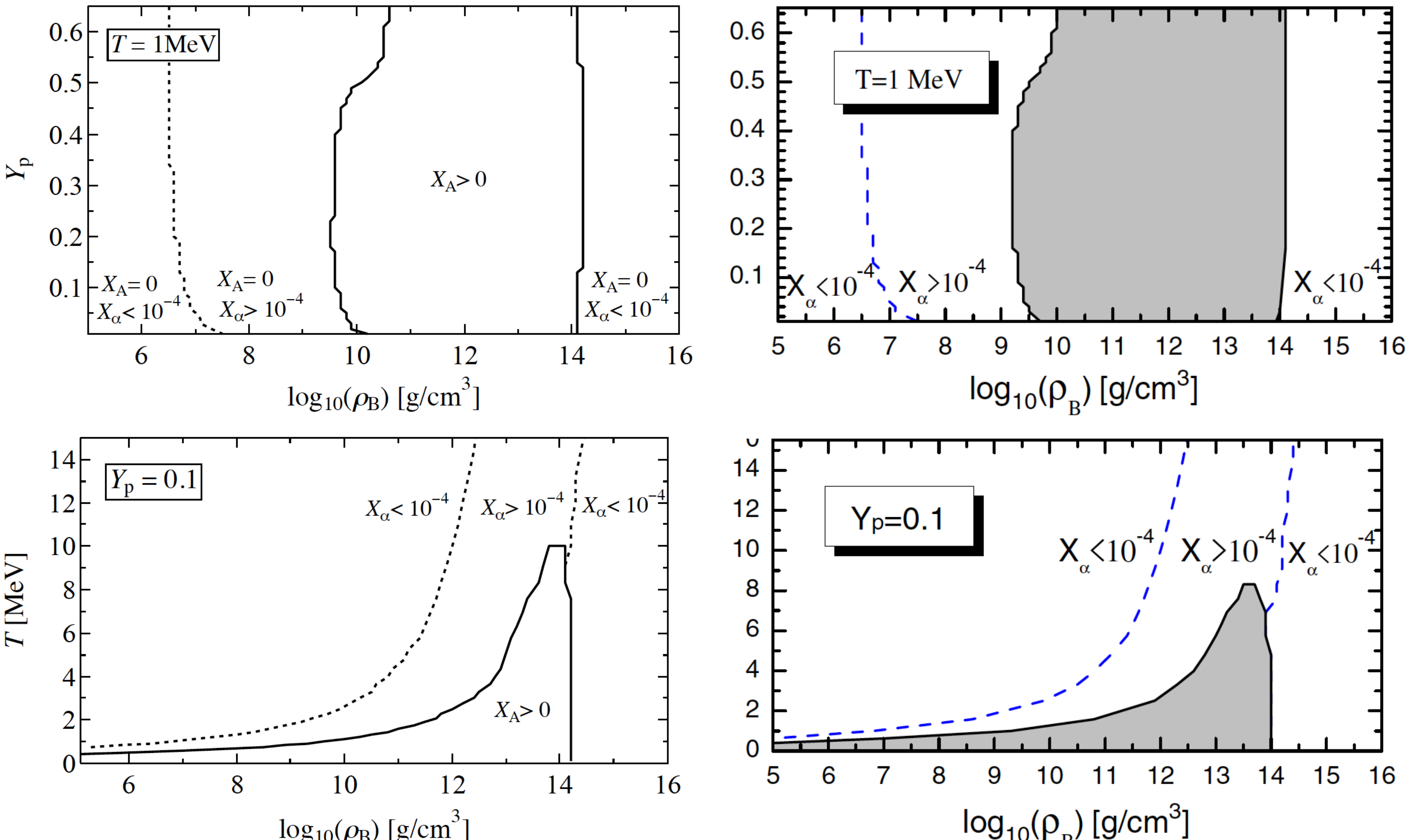
f **: Free energy density of uniform nuclear matter**

Parameter	Minimum	Maximum	Number
$\log_{10}(T)$ [MeV]	-1.08	1.52	66 + 1
Y_{p}	0.0	0.5	213
n_{B} [fm^{-3}]	0.000001	0.18	1980

67×213×1980
= 28256580 points

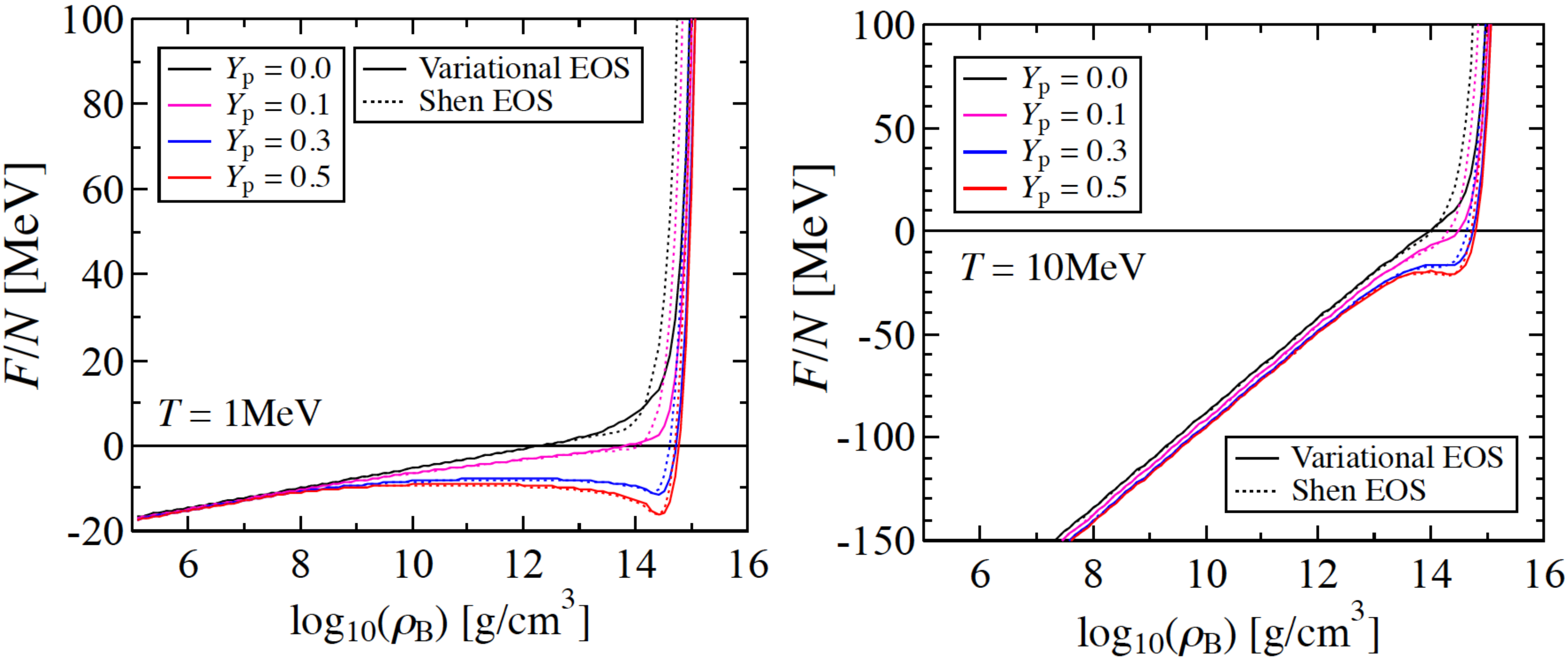
F/V_{cell} is minimized with respect to *parameters in* $n_{\mathrm{p}}(r)$, $n_{\mathrm{n}}(r)$, $n_\alpha(r)$ for various densities, temperatures and proton fractions.

Phase Diagram of Hot Nuclear Matter



Shen EOS (APJ Suppl.197 (2011) 20)

Free Energy for Hot Nuclear Matter

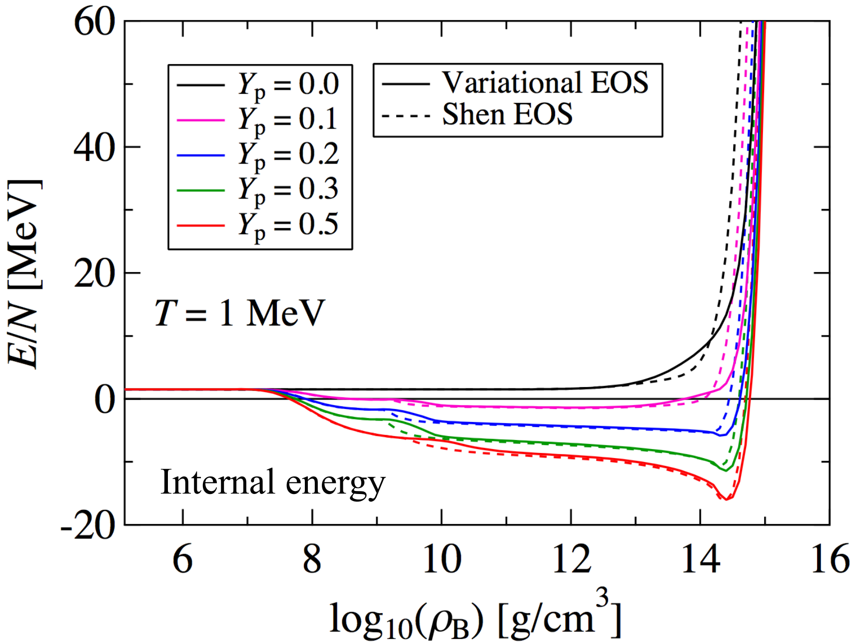


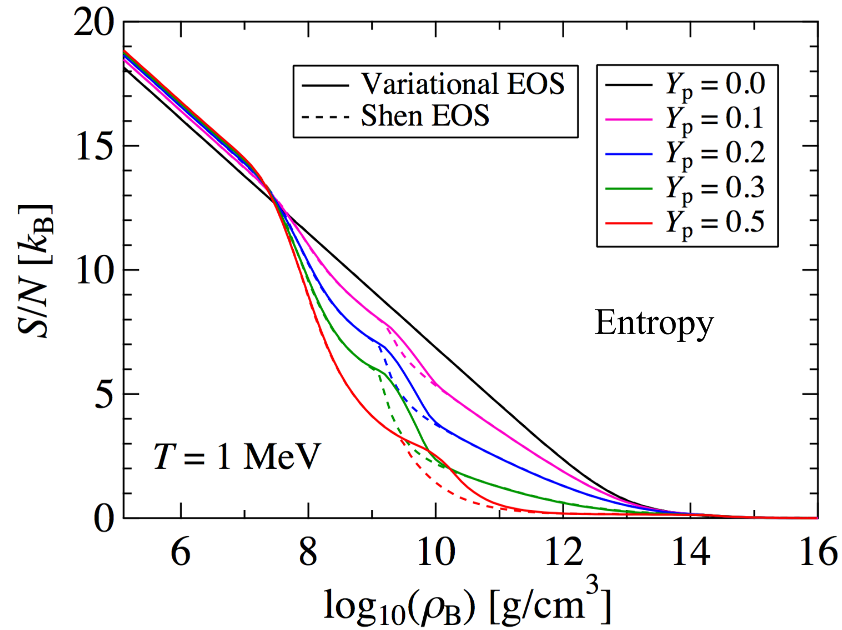
Free energy per nucleon at $T = 1$ MeV

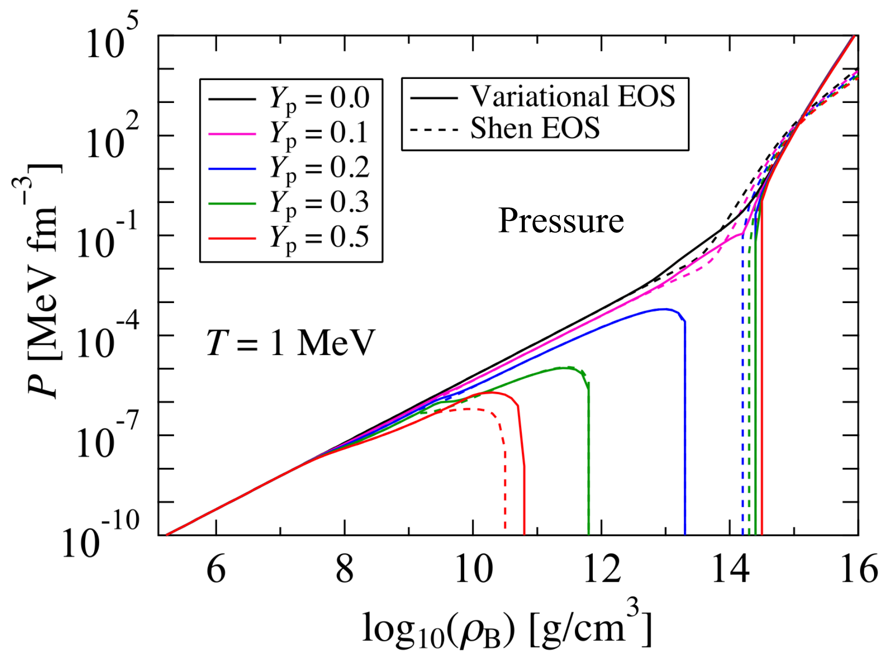
Free energy per nucleon at $T = 10$ MeV

Shen EOS (APJ Suppl.197 (2011) 20)

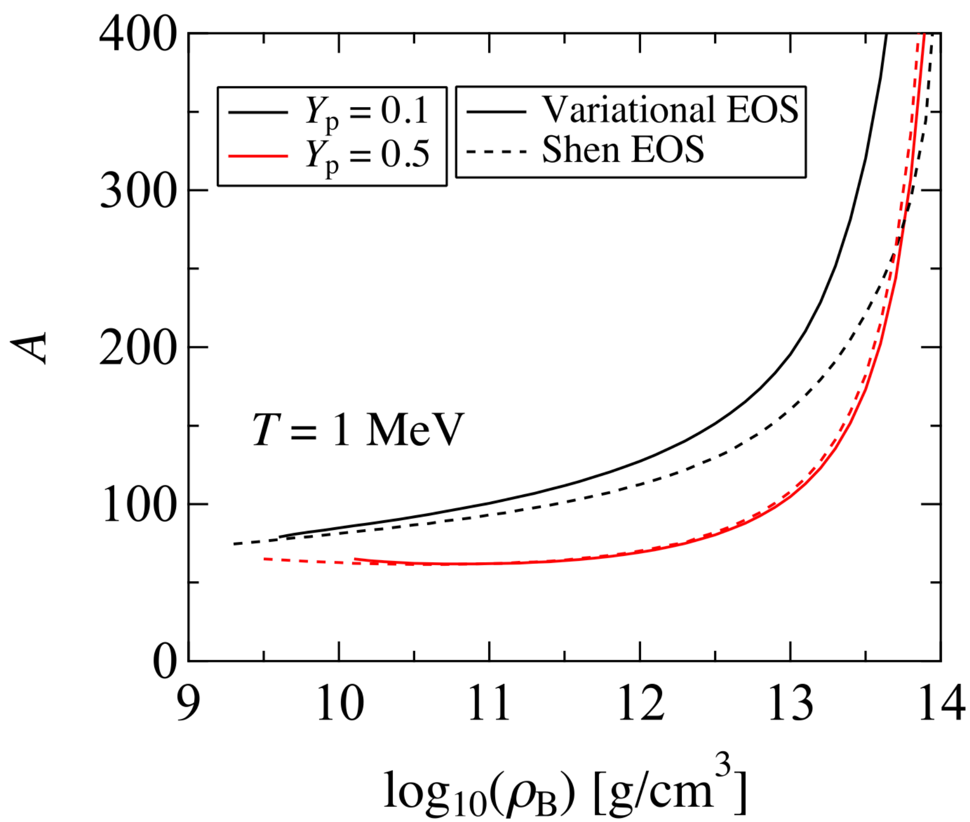
Other thermodynamic quantities



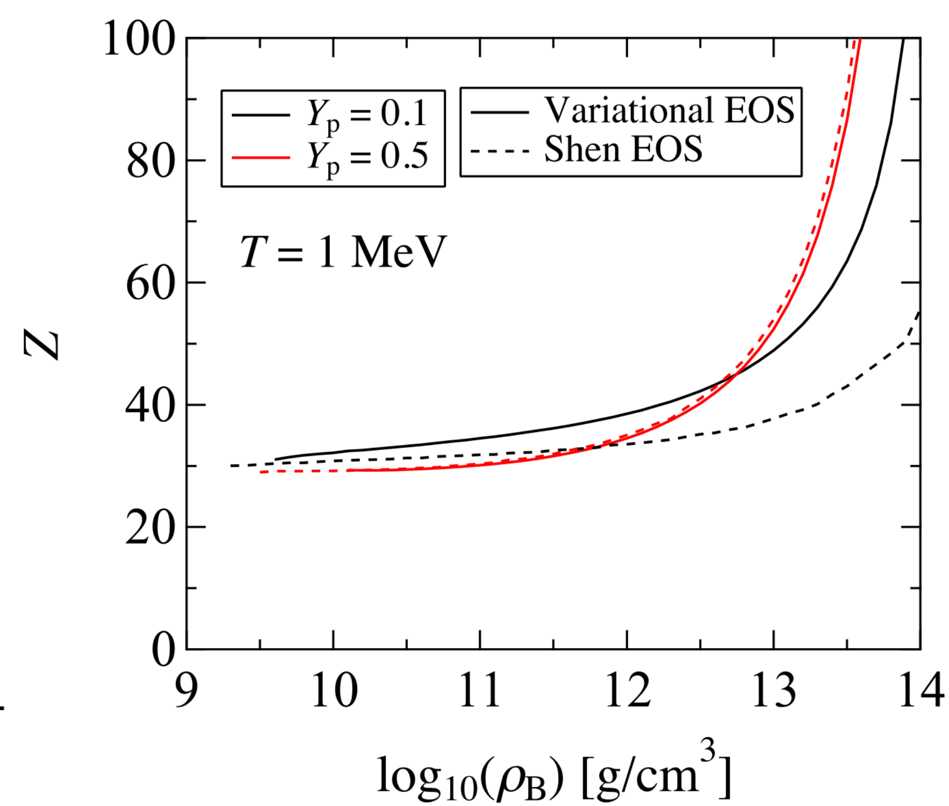




Mass Number and Proton Number



Mass number A



Proton number Z

Shen EOS (APJ Suppl.197 (2011) 20)

A and *Z* of heavy nuclei appearing in neutron-rich non-uniform phase are larger than those in the Shen EOS.

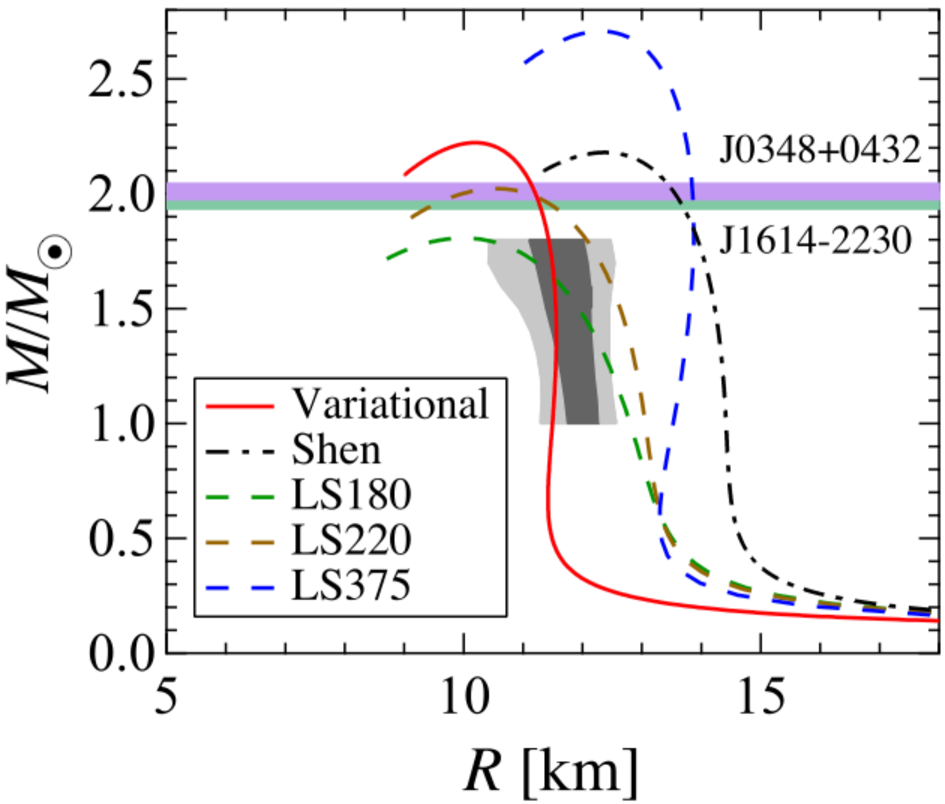
Outline

Collaborators : M. Takano (Waseda University),

K. Sumiyoshi (Numazu College), K. Nakazato (Tokyo Univ. Sci.)

NPA902 (2013) 53
PTEP 2014 (2014) 023D05

Application to neutron stars



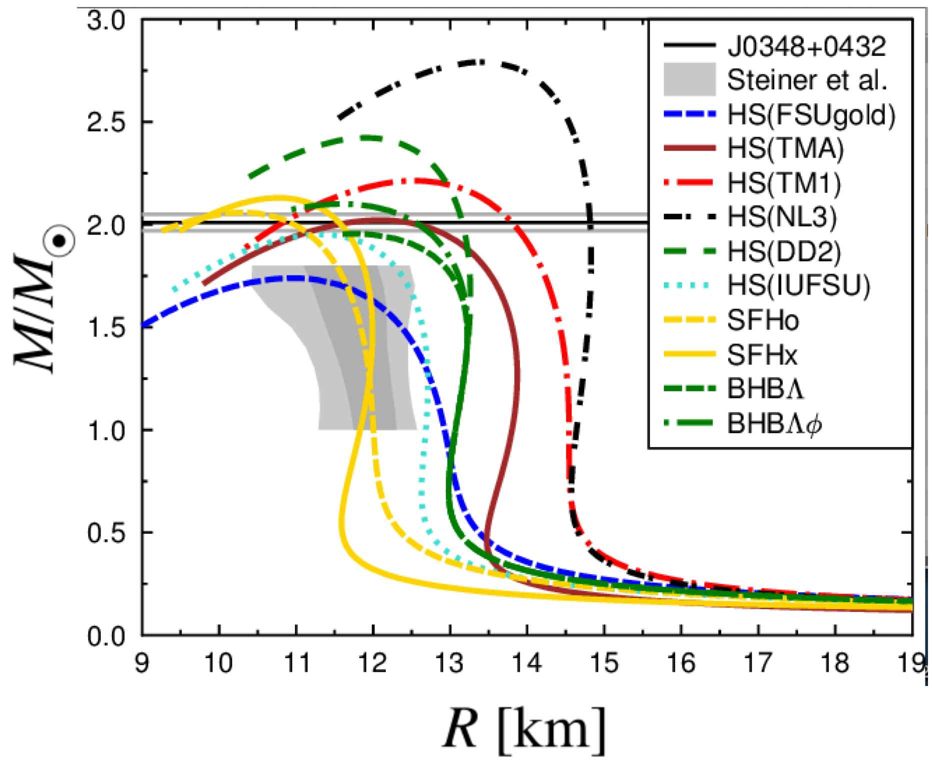
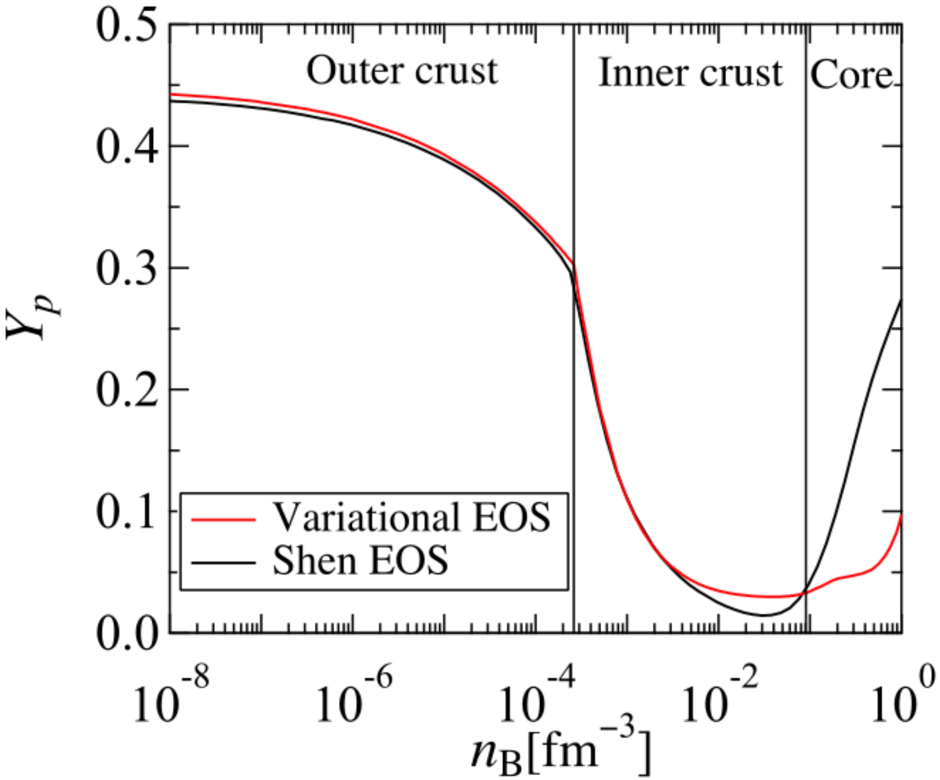


Figure by M. Hempel

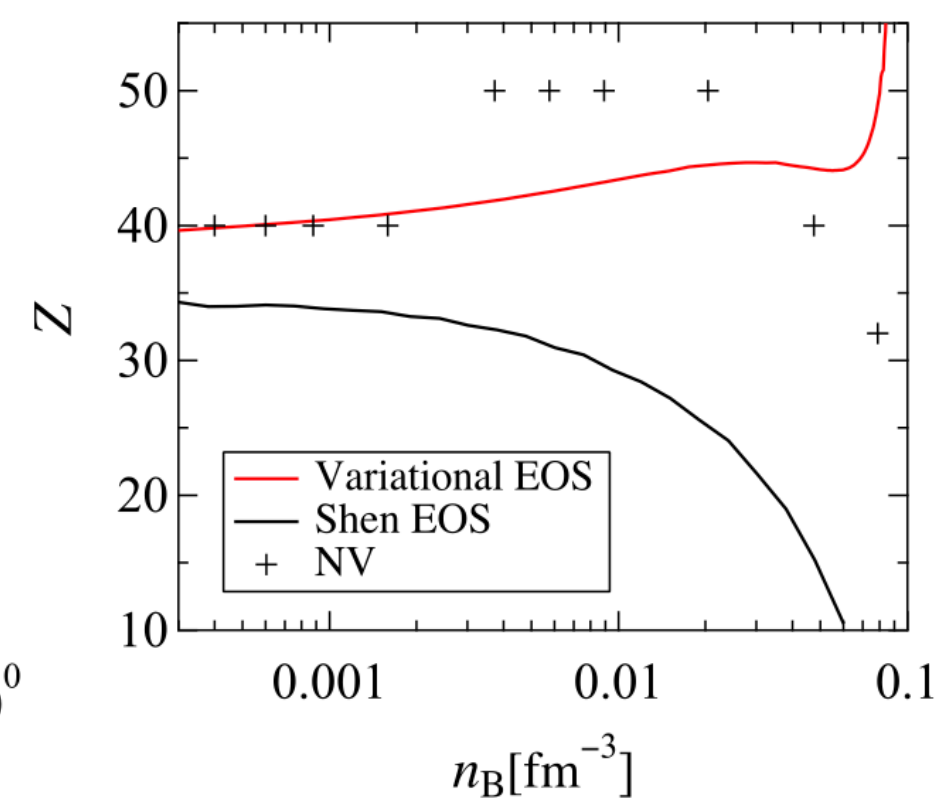
J0348+0432: Science 340 (2013) 1233232 J1614-2230: Nature 467 (2010) 1081

Shaded region is the observationally suggested region by Steiner et al. (Astrophys. J. 722 (2010) 33)

Composition of neutron star matter



Proton fraction in neutron star matter



Proton number in the inner crust

NV : Hartree-Fock calculation by J. W. Negele and D. Vautherin
(Nucl. Phys. A 207 (1973) 298)

Application to core-collapse supernovae

With use of our EOS for uniform matter, we construct the SN-EOS.

(PTEP 2014 (2014) 023D05)

Density ρ : $10^{14.0} \leq \rho_B \leq 10^{16.0}\text{g/cm}^3$	21 point
Temperature T : $0 \leq T \leq 400$ MeV	92 point
Proton fraction x : $0 \leq x \leq 0.65$	66 point

Shen-EOS is adopted in $\rho_B \leq 10^{14.0}\text{g/cm}^3$.

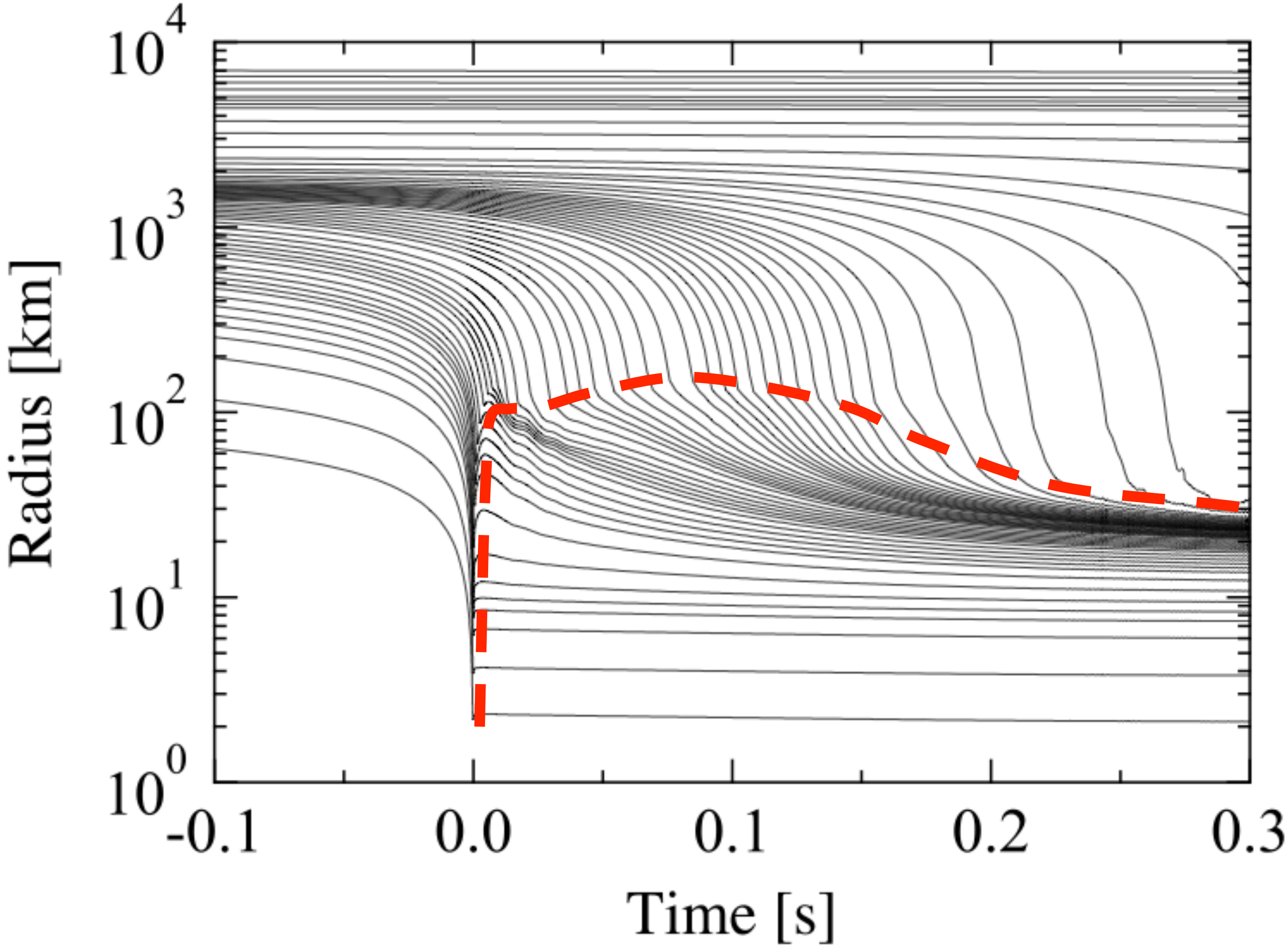
We perform the basic SN simulation as follows.

- Fully GR spherically symmetric simulation（1D）
- Progenitor: Woosley Weaver 1995, $15M_\odot$

Astrophys. J. Suppl. 101 (1995) 181

K. Sumiyoshi, et al., APJ 629 (2005) 922

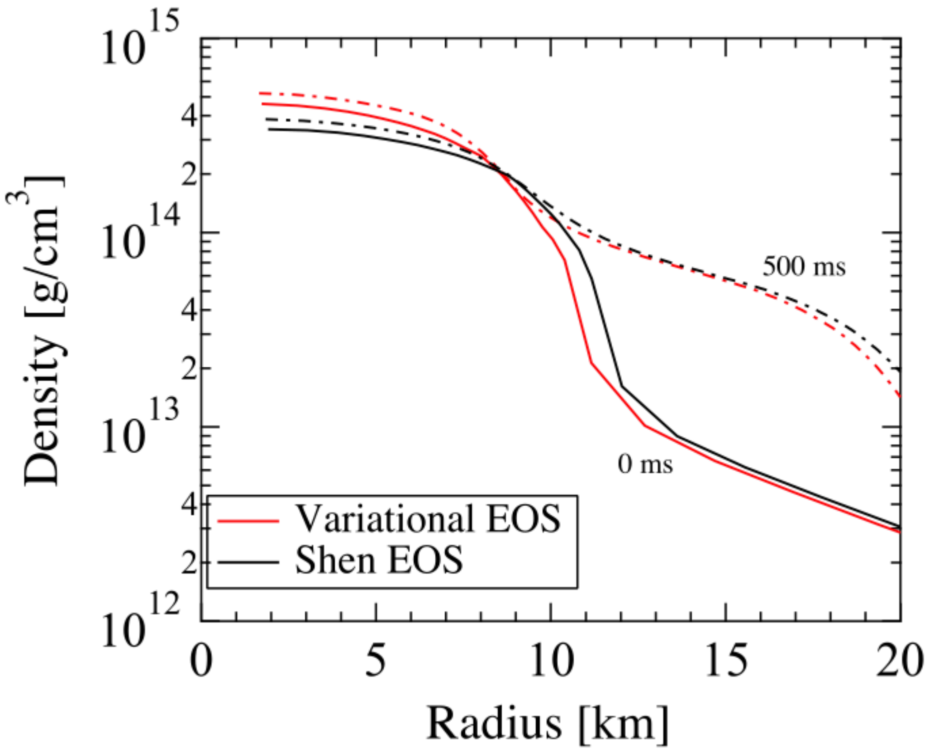
Radial Trajectories of Mass Elements



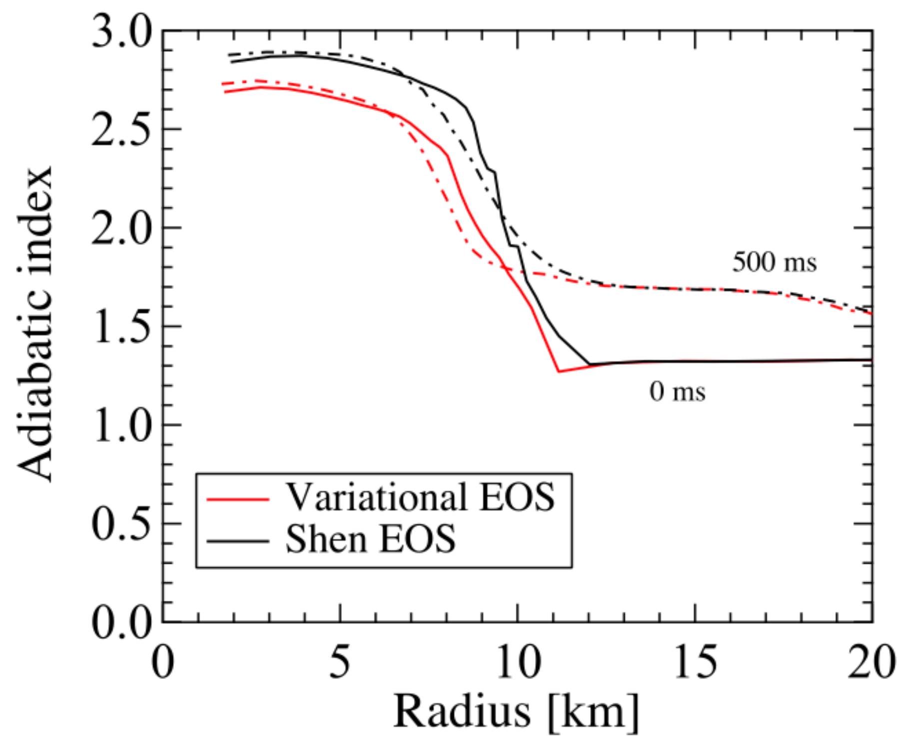
The shock wave stalls due to the energy loss by the neutrino emission.

→ This result is consistent with other modern 1D SN simulations.

Comparison with Shen EOS



The density profiles at t_{pb} = 0, 500 ms



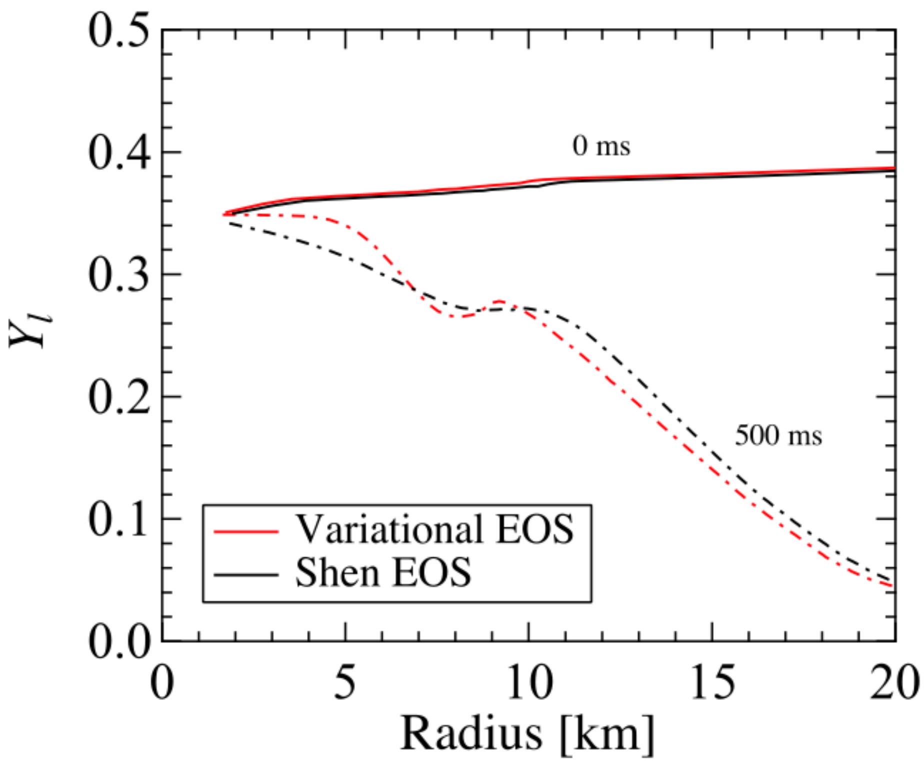
The adiabatic indices at t_{pb} = 0, 500 ms

Adiabatic index: $\Gamma = \left.\dfrac{d \log P}{d \log \rho}\right|_S$

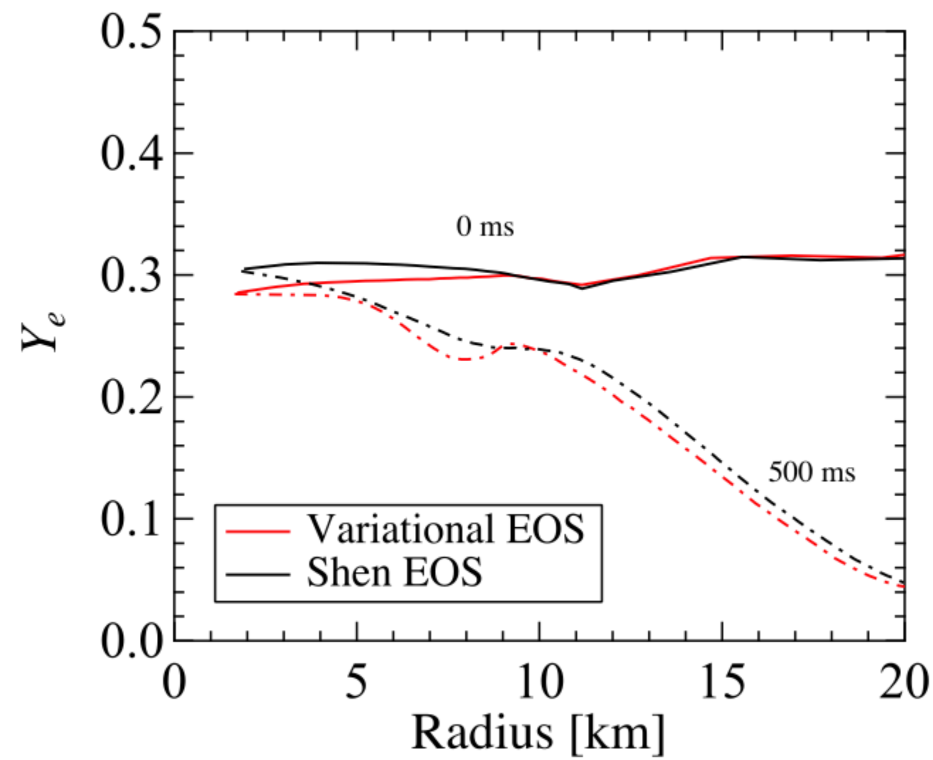
Variational EOS is softer than the Shen EOS.

→ Our EOS is advantageous for SN explosion!

Lepton Fraction and Electron Fraction



The lepton fractions Y_l



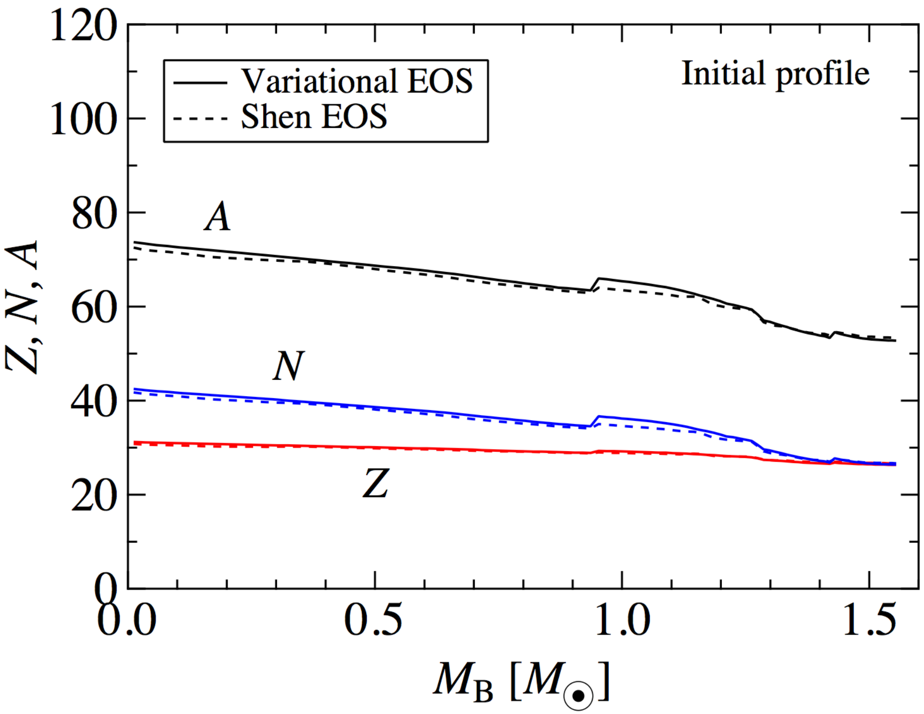
The electron fractions Y_e

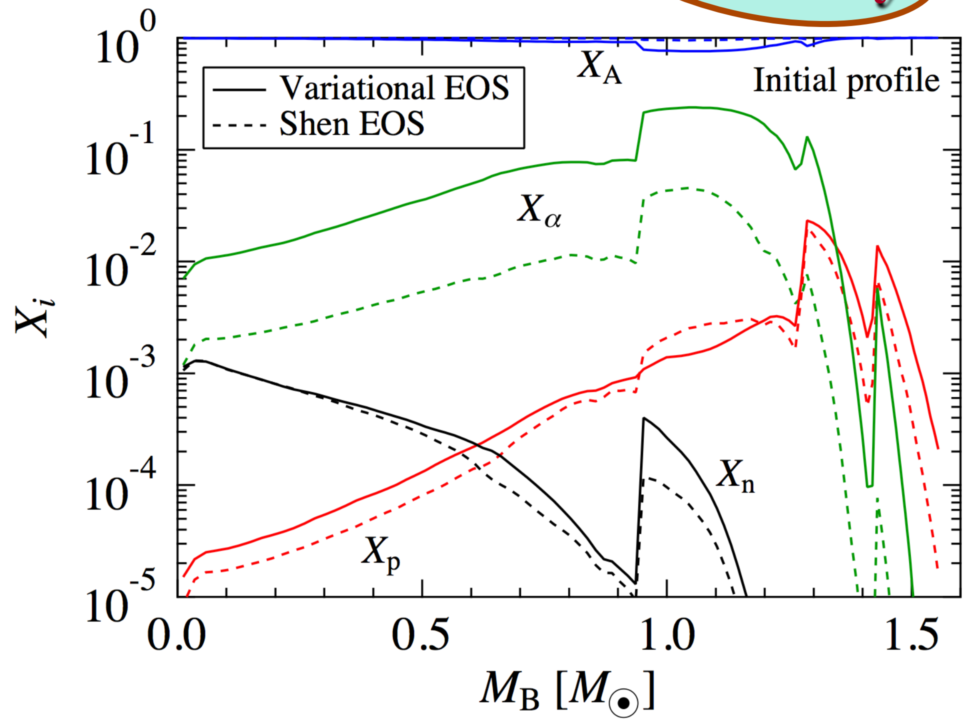
the core in the central region is more neutron-rich with the variational EOS than that with the Shen EOS.

→ The symmetry energy of the variational EOS（30.0 MeV）is smaller than that of the Shen EOS（36.9 MeV）.

Compositions of Central Core

Preliminary





Compositional differences lead to changes in electron capture rates, neutrino interaction rates, and dynamics of the SN explosion.

→ *These differences of compositions should be examined further in realistic SN simulations with neutrino transfer !!*

Outline

1 : Introduction

2 : Equation of state for uniform nuclear matter

3 : Equation of state for non-uniform nuclear matter

4 : Application to astrophysics

5 : Variational method for Hyperon matter

Collaborators : E. Hiyama (RIKEN) ,
M. Takano (Waseda University), Y. Yamamoto (RIKEN)

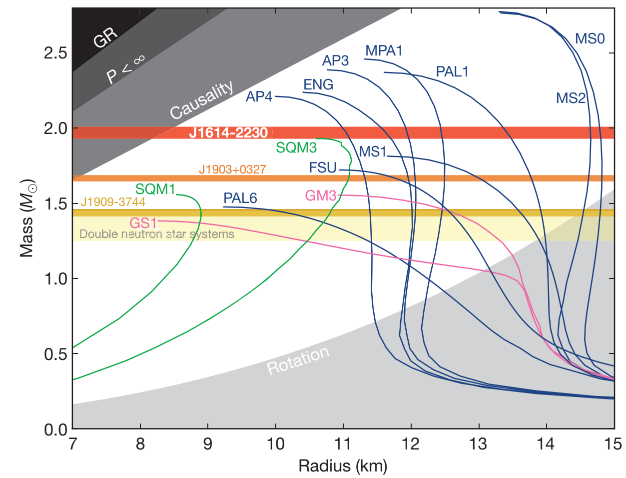
Equation of state for nuclear matter with hyperons

Hyperon interactions play important roles in the study of neutron stars.

Hyperons (Λ, Σ, Ξ) are expected to appear in the core of neutron stars.

HYPERON PUZZLE

- Nuclear equation of state (EOS) becomes softer due to the hyperon mixing.
- Maximum mass of neutron star tends to be lower than the observational data.



P. B. Demorest et al., NATURE 467 (2010)

The hyperon mixing in neutron stars has been studied with various nuclear theories.

- **Relativistic mean field theory** (C. Ishizuka et al., J. Phys. G 35 (2008) 085201)
- **Relativistic Hartree-Fock theory** (T. Miyatsu, et al., PRC 88 (2013) 01802)
- **Brueckner-Hatree-Fock theory** (H. Schulze, T. Rijken, PRC 84 (2011) 035801)
- **Variational many-body theory** (D. Lonardoni et al., PRL 114 (2015) 092301)

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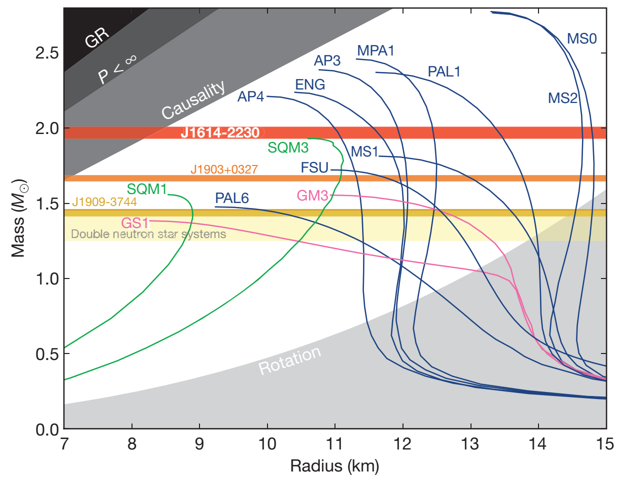

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Hamiltonian and hyperon interactions

Two-body Hamiltonian

$$H_2 = -\sum_i \left[m_i c^2 + \frac{\hbar^2}{2m_i} \nabla_i^2 \right] + \sum_{i<j} V_{ij}$$

Two-body potential

$$V_{ij} = \sum_{N,Y} [V_{ij}^{NN} + V_{ij}^{YN} + V_{ij}^{YY}]$$

Three-body Hamiltonian

$$H_3 = \sum_{i<j<k} V_{ijk}$$

Three-nucleon potential : UIX

- *NN* potential: **AV18 two-body potential** (PRC 51 (1995) 38)
- *YN* and *YY* potentials: **Central three-range Gaussian potentials**

Λ*N* interaction (E. Hiyama et al., PRC 74 (2006) 054312)

ΛΛ interaction (E. Hiyama et al., PRC 66 (2002) 024007)

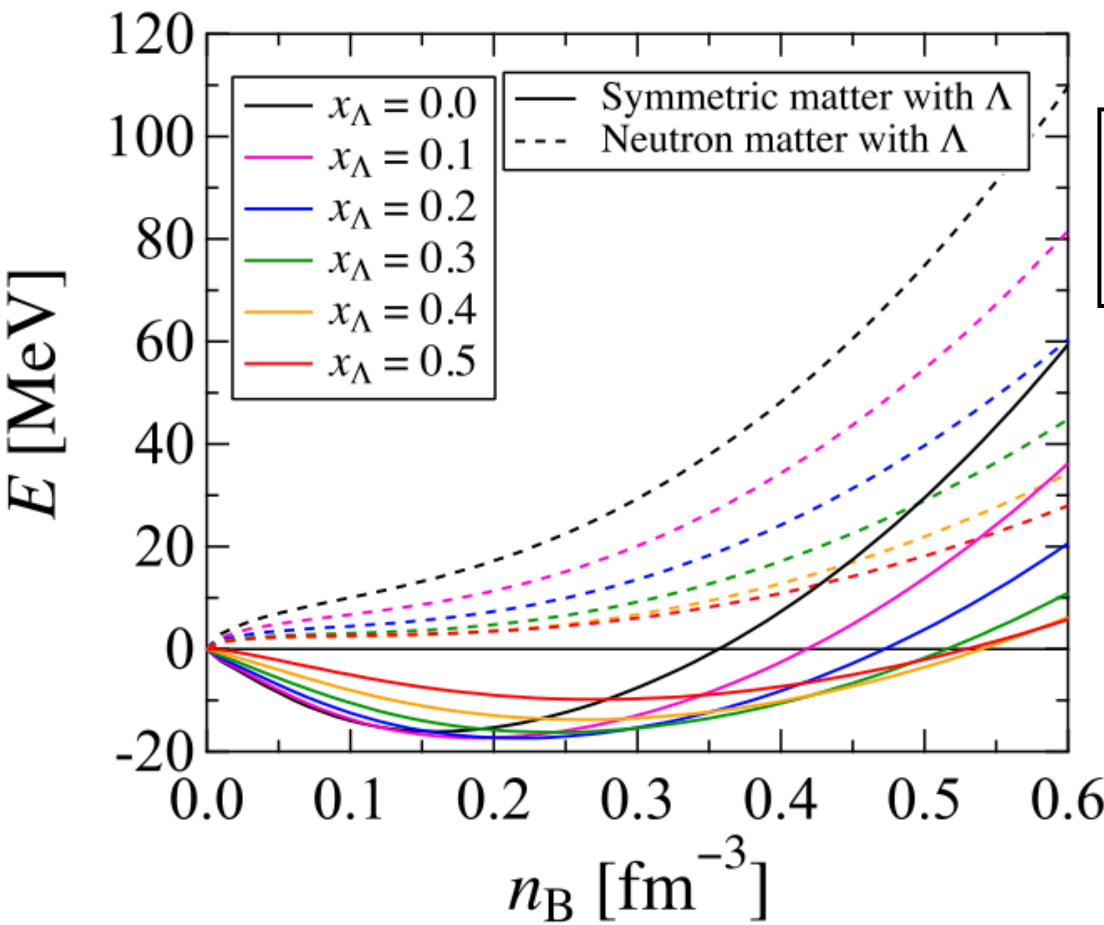
- **The *ab initio* variational calculations for Λ hypernuclei reproduce their experimental eigenvalues.**

Σ^-*N* interaction : Based on the latest version of the **Nijmegen model ESC08**.

Energy of Hyperonic Nuclear Matter

Energy per baryon $\boxed{E(n_{\mathrm{p}}, n_{\mathrm{n}}, n_{\Lambda}, n_{\Sigma^-}) = E_2(n_{\mathrm{p}}, n_{\mathrm{n}}, n_{\Lambda}, n_{\Sigma^-}) + E_3^{\mathrm{N}}}$

E_2 is the expectation value of H_2 with the Jastrow wave function **in the two-body cluster approximation**.

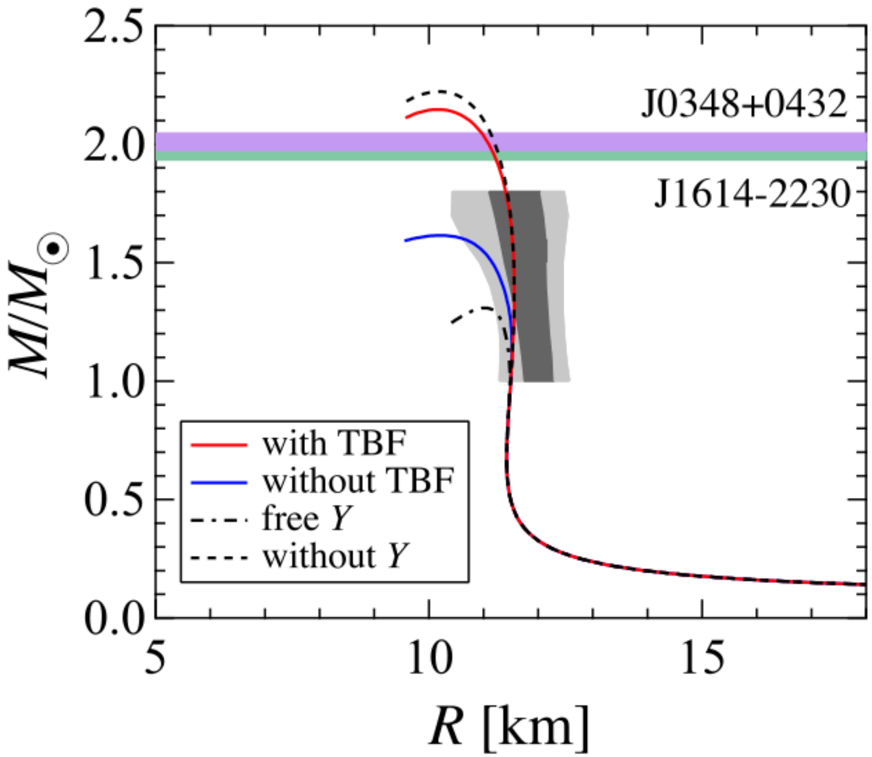


Two-body correlation function

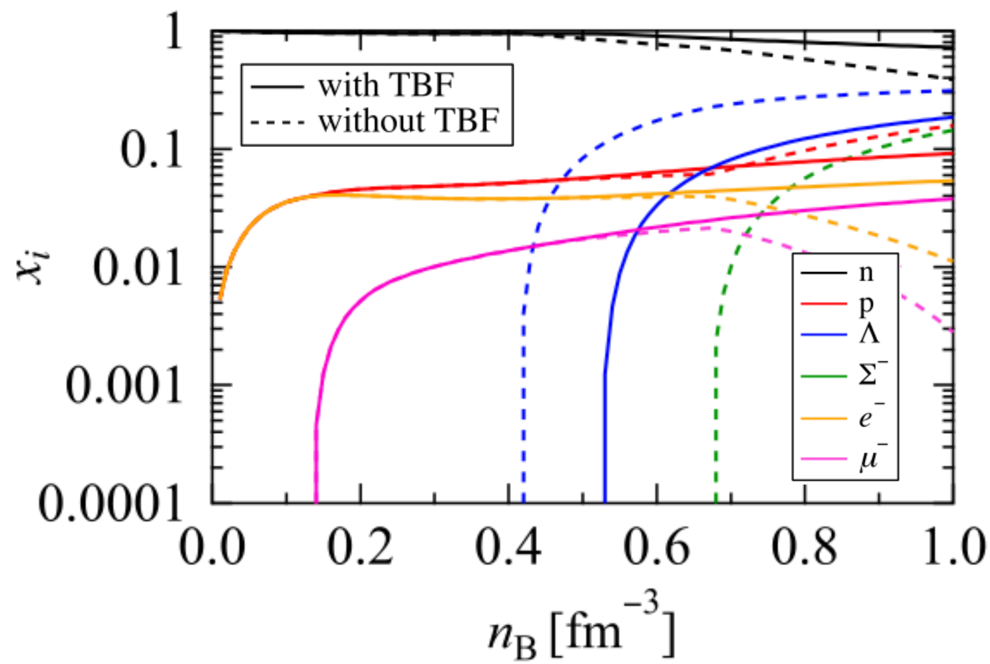
$$f_{ij} = \sum_{N,Y} [f_{ij}^{NN} + f_{ij}^{YN} + f_{ij}^{YY}]$$

E_3^{N} : Three-nucleon energy

Application to neutron stars



Mass-radius relations of neutron stars



Composition of neutron star matter

with TBF: considering **a phenomenological Three-Baryon repulsive Force (TBF)**

(Y. Yamamoto et al., PRC 90 (2014) 045805)

Summary

New nuclear EOS for SN simulations is constructed with realistic nuclear forces (AV18 + UIX).

- *EOS of **uniform nuclear matter** is constructed with the cluster variational method.*
- *EOS of **non-uniform nuclear matter** is constructed with the Thomas-Fermi calculation.*
- **NS mass-radius relation is consistent with observational data.**
- **Our EOS is softer than Shen EOS in 1D SN simulations.**
- *We extend the cluster variational method to calculate the energy of **hyperon matter**.*

Future Plans

- Application to another simulations of high-energy astrophysics
- Hyperon EOS at finite temperatures