Mapping the densities of exotic nuclei

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Introduction

Much information on the neutron density of neutron-rich nuclei is known:

- ➡ Halos
- ➡ Skins
- ➡ Nucleus-nucleus collisions
- ➡ Proton scattering in inverse kinematics

Not much is known of the proton density; reactions tend to concentrate on probing the neutron density only.

Mapping the density

Usually, we rely on more than one reaction to obtain information on the overall structures of nuclei:

- ➡ Proton scattering pn part of interaction dominates, probes neutron density
- ➡ Electron scattering Purely electromagnetic interaction. Longitudinal form factors probe directly the charge (proton) density.

Utilising both, self-consistently, provides a complete mapping of the matter density of the nucleus.

This requires, *a priori*, a self-consistent approach to both electron and proton (nucleon) scattering.

Requirements of both scatterings necessitates use of microscopic theories. The Shell Model is utilised to produce the underlying **one-body density matrix elements** (OBDME) which are used in both analyses of electron and nucleon scattering.

Electron scattering: de Forest and Walecka approach. (Adv. Phys. **15**, 1 (1966)); SK, *et al.* (PRC **51**, 2494 (1995)).

Nucleon scattering: Melbourne *g*-folding model. (K. Amos, *et al.*, Adv. Nucl. Phys. **25**, 275 (2000)).

Other predictions…

- Antonov *et al.* [PRC **72**, 044307 (2005)]: Form factors of He isotopes as obtained from densities calculated using large space shell model. Did not consider directly the effect of the halo in ^{6}He;
- Bertulani [JPG **34**, 315 (2007)]: Form factors of ^{6}He and ^{11}Li using densities obtained from a potential model. Did not find any effect of the halo on the elastic scattering charge form factors. Did find a significant effect of the proton halo on the charge form factor for ^{8}B.

Current work:

- Elastic scattering form factors of He and Li isotopes to investigate whether charge density changes with addition of neutrons and introduction of the halo;
- Inelastic scattering form factors of ^{6}He to see whether the halo has an effect therein.

Electron scattering form factors

Form factors:

$$\left|F_J^\eta\right|^2 = \frac{1}{2J+1}\frac{4\pi}{Z^2}\left|\left\langle J_f \left\| T_J^\eta(q) \right\| J_i \right\rangle\right|^2$$

η is the type. Assuming one-body operators:

$$\left\langle J_f \left\| T_J^\eta(q) \right\| J_i \right\rangle = \sqrt{\frac{1}{2J+1}}\mathrm{Tr}(SM).$$

- *S* is the matrix of one-body transitions densities;
- *M* contains the matrix elements of one-body longitudinal or transverse electromagnetic operators.

MEC included via Siegert's theorem in long-wavelength limit;
Darwin term included in the longitudinal operator.

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Formal theory of the optical potential

Optical potential: term associated with *elastic scattering only*. Split the Hilbert Space: P projects onto the elastic scattering channel; Q projects onto everything else. Thus:

$$P + Q = 1,\ PQ = QP = 0,\ Q\left|\Psi_{\mathrm{gs}}\right\rangle = 0$$

Schrödinger equation for the scattering state becomes:

$$\begin{aligned}(E - H_{PP})\,P\left|\Psi^+\right\rangle &= H_{PQ}Q\left|\Psi^+\right\rangle \\ (E - H_{QQ})\,Q\left|\Psi^+\right\rangle &= H_{QP}P\left|\Psi^+\right\rangle\end{aligned}$$

Recoupling, the S.E. for the projectile wave function is

$$\left\{E - H_0 - \left\langle\Phi_{\mathrm{gs}}\left|V\right|\Phi_{\mathrm{gs}}\right\rangle - \left\langle\Phi_{\mathrm{gs}}\left|VG^{(+)}_{QQ}V\right|\Phi_{\mathrm{gs}}\right\rangle\right\}\left|\chi^+\right\rangle = 0$$

where

$$G^{(+)} = \left[E - H_{QQ} + i\varepsilon\right]^{-1}.$$

Melbourne *g*-folding model

A refresher: for intermediate energy nucleon-nucleus scattering.

Effective NN potential - Melbourne model [K. Amos et al., Adv. Nucl. Phys. 25, 275 (2000)].

- Effective NN interaction obtained from g matrices.
- Bonn-B interaction used for the current examples.
- Momentum-space effective interaction mapped to coordinate space.
- Densities obtained from credible models of structure.

The g matrix is a solution of the Bethe-Goldstone equation:

$$g\left(\mathbf{q},\mathbf{q}';\mathbf{K}\right)=V\left(\mathbf{q}',\mathbf{q}\right)+\int V\left(\mathbf{q}',\mathbf{k}'\right)\frac{Q\left(\mathbf{k}',\mathbf{K};k_f\right)}{E\left(\mathbf{k},\mathbf{K}\right)-E\left(\mathbf{k}',\mathbf{K}\right)}g\left(\mathbf{k}',\mathbf{q};\mathbf{K}\right)\,d\mathbf{k}'$$

where Q is a Pauli operator and the energy denominator is dependent on auxiliary potentials (eg. effective mass operators).

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Construction of the optical potential

In coordinate space, the OMP for elastic scattering is

$$\begin{aligned} U\left(\mathbf{r},\mathbf{r}';E\right) &= \delta(\mathbf{r}-\mathbf{r}')\sum_i n_i \int \varphi_i^*\left(\mathbf{s}\right) g_D\left(\mathbf{r},\mathbf{s};E\right)\varphi_i\left(\mathbf{s}\right) d\mathbf{s} \\ &+ \sum_i n_i \varphi_i^*\left(\mathbf{r}'\right) g_E\left(\mathbf{r},\mathbf{r}';E\right)\varphi_i\left(\mathbf{r}\right) \\ &= U_D\left(\mathbf{r},E\right)\delta(\mathbf{r}-\mathbf{r}') - U_E\left(\mathbf{r},\mathbf{r}';E\right) \end{aligned}$$

- First term is the "$g\rho$" direct form of the optical potential.
- The nonlocality arises primarily and explicitly out of the exchange terms.
- Structure enters through the s.p. wave functions and occupations numbers.
- For nonzero spin targets, terms with nonzero spin coupling may be included via the DWA.

Structure of the target is critical.

Inelastic scattering

… is calculated within a distorted wave approximation.

$$T_{J_f J_i}^{M_f M_i \nu' \nu}(\theta) = \left\langle \chi_{\nu'}^{(-)} \right| \left\langle \Psi_{J_f M_f} \right| A g_{\text{eff}}(0,1) \mathcal{A}_{0,1} \left\{ \left| \chi_{\nu}^{(+)} \right\rangle \left| \Psi_{J_i M_i} \right\rangle \right\}$$

Nuclear structure

One-body density matrix elements (OBDME), obtained using the Shell Model.

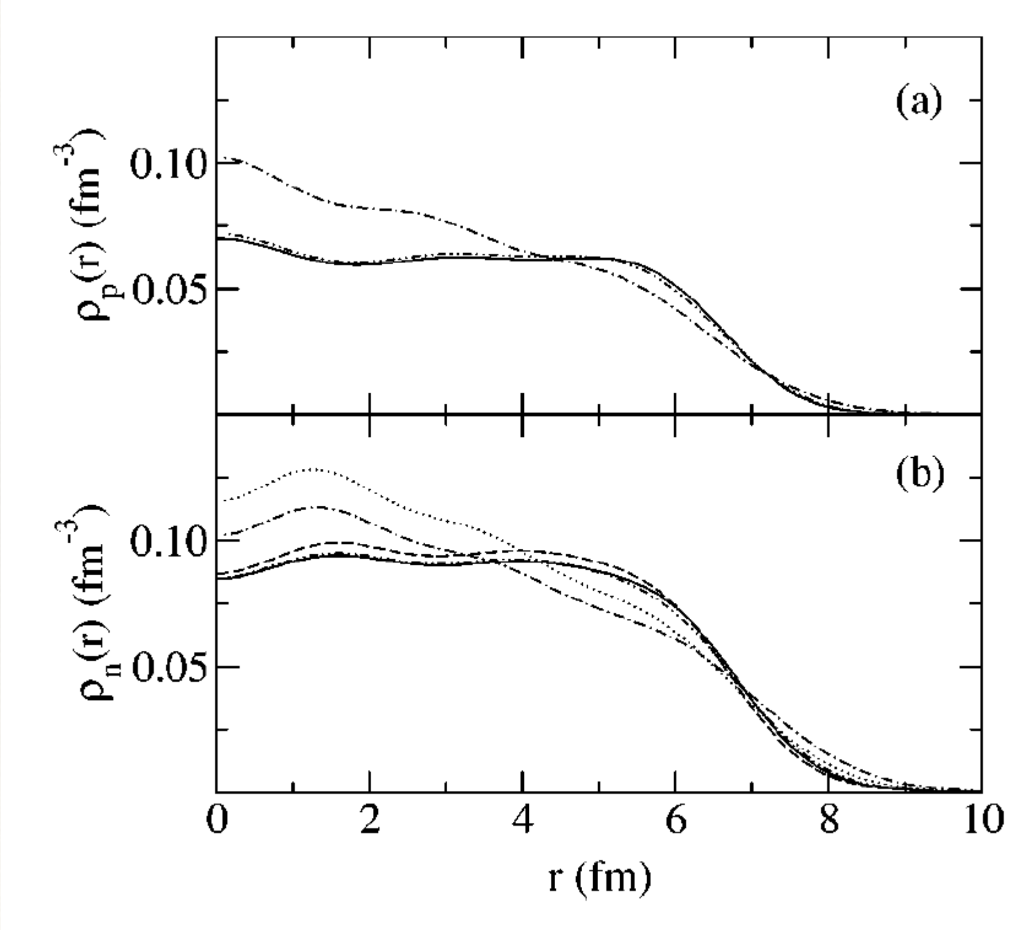
$$S_{j_1 j_2 J} = \left\langle J_f \left\| \left[a^\dagger_{j_2} \times \tilde{a}_{j_1} \right]^J \right\| J_i \right\rangle$$

Single-particle wave functions. Either:

- HO: (naive shell model) automatically gives the skin attributes;
- WS: with binding energy set to the single-nucleon separation energy gives the appropriate extension of the density consistent with a halo.

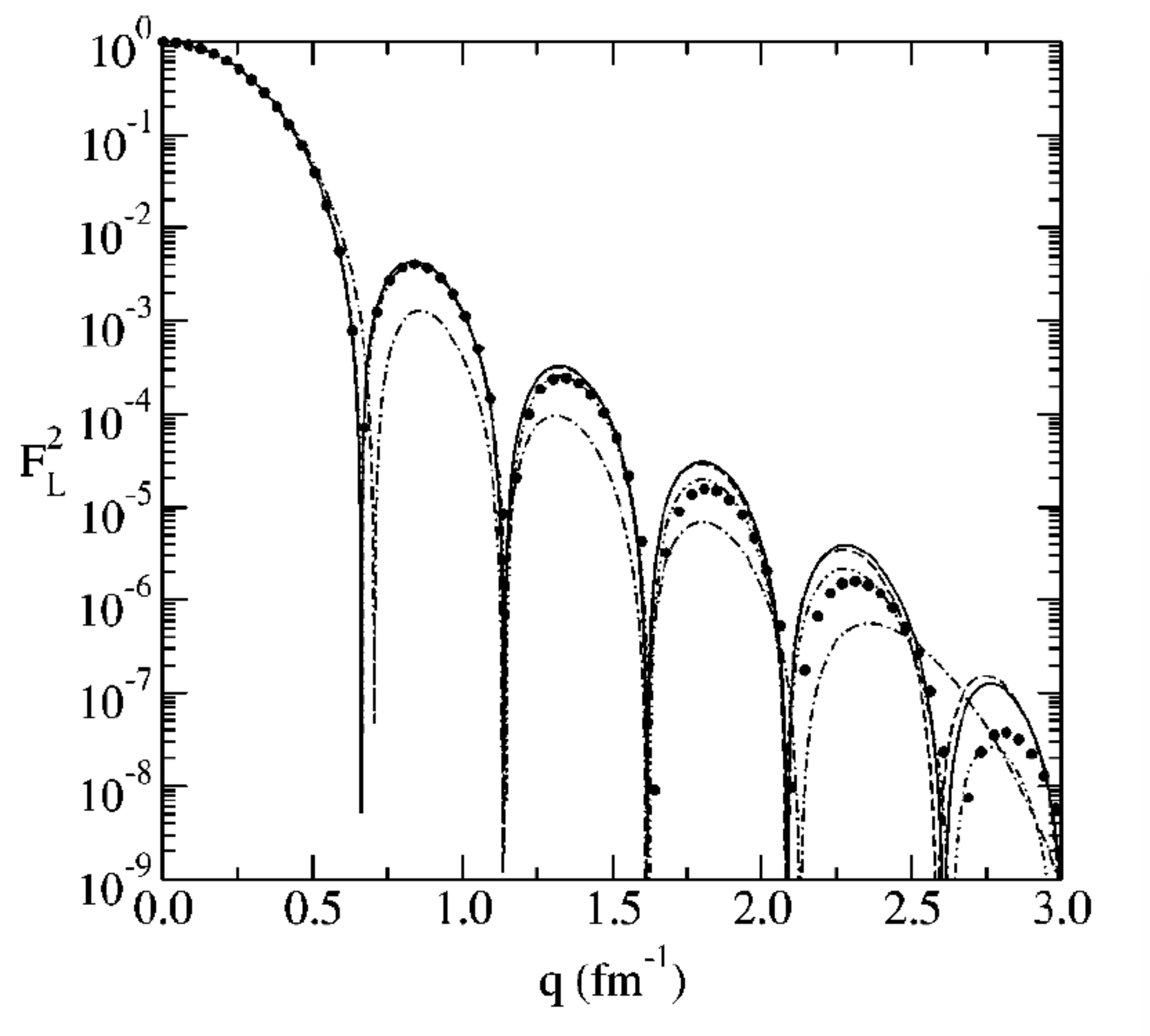
NB! No fitting…

Systematic error in cross-section magnitudes prevents accurate determination of densities by unfolding data. Instead, we use data to determine best model, and then use the model to calculate the densities. Example… ^{208}Pb skin thickness (SK, *et al.*, PRC **65**, 044306 (2002)).

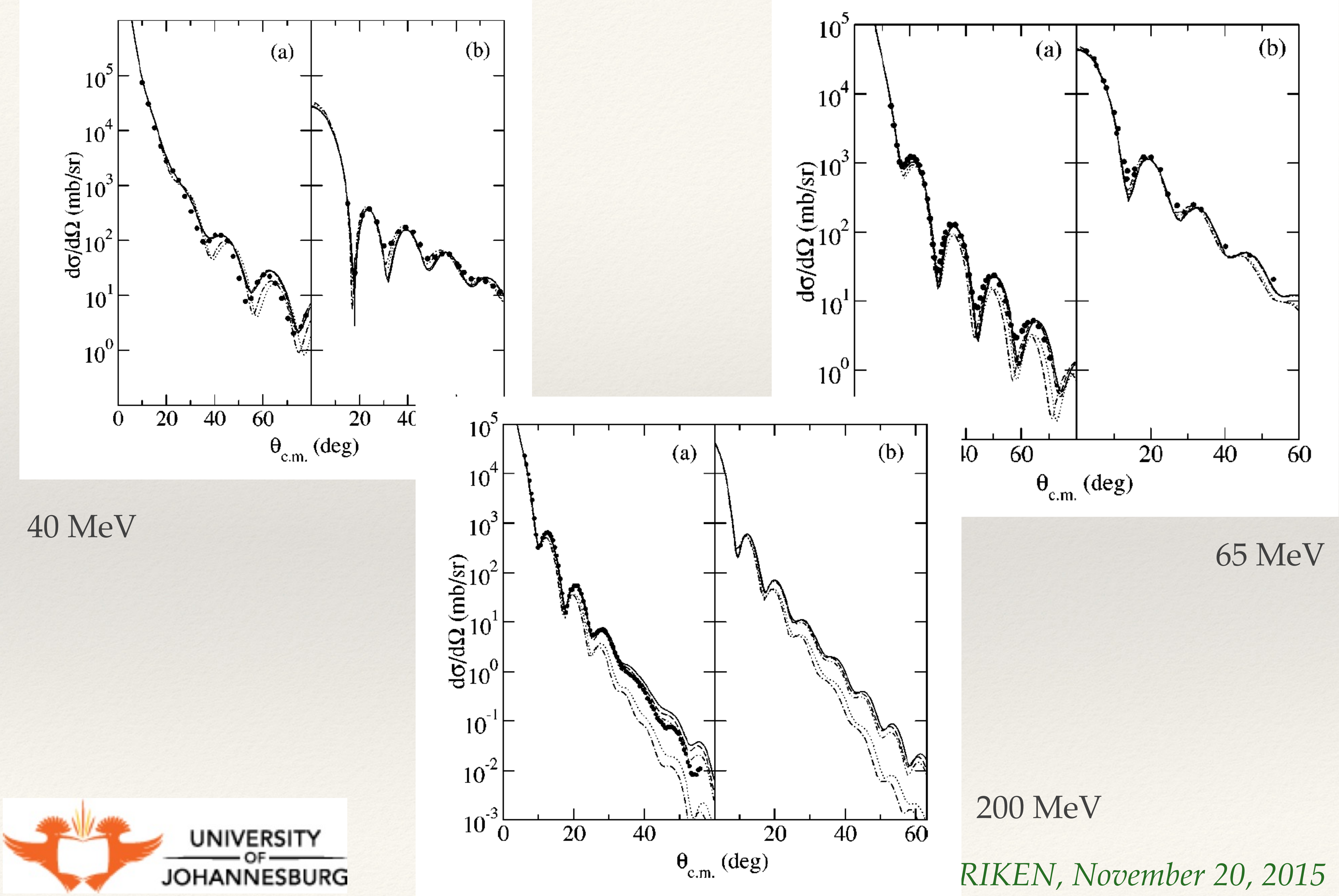


5 models: 2 oscillator, 3 Skyrme.

Electron scattering:



Nucleon scattering:



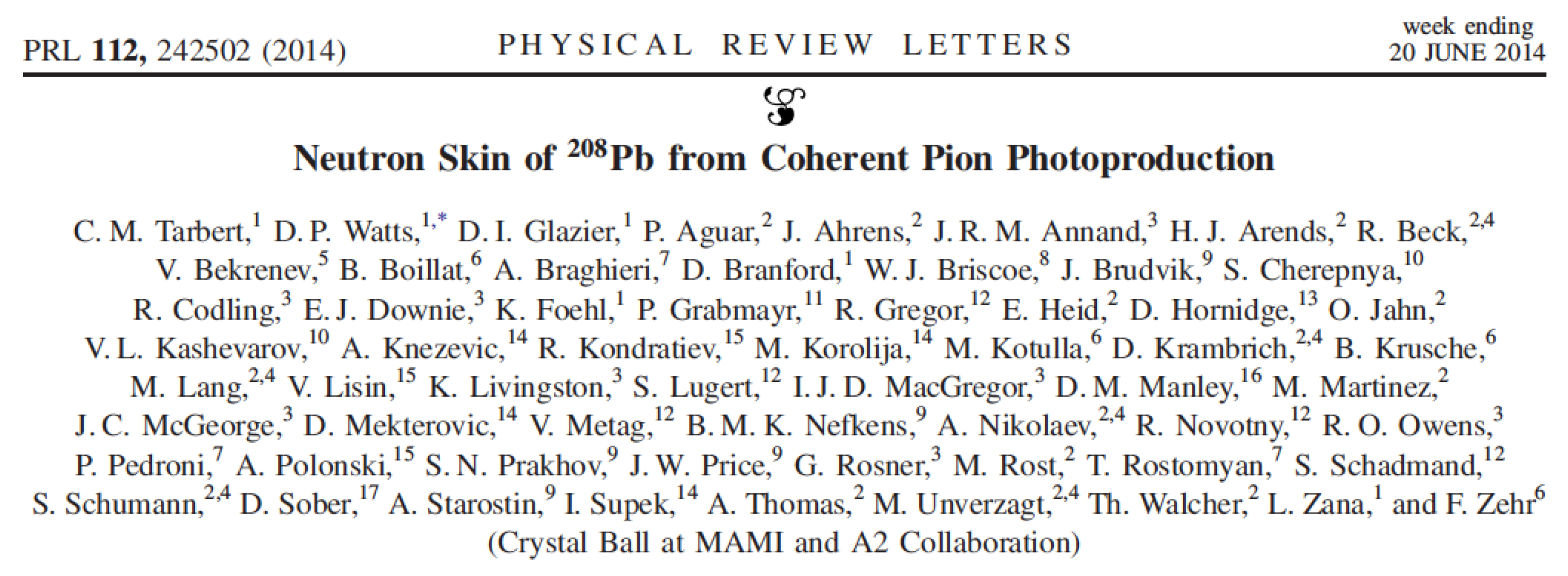
40 MeV

65 MeV

200 MeV

Skin thickness, ^{208}Pb

Best result, SKM* model: 0.17 fm



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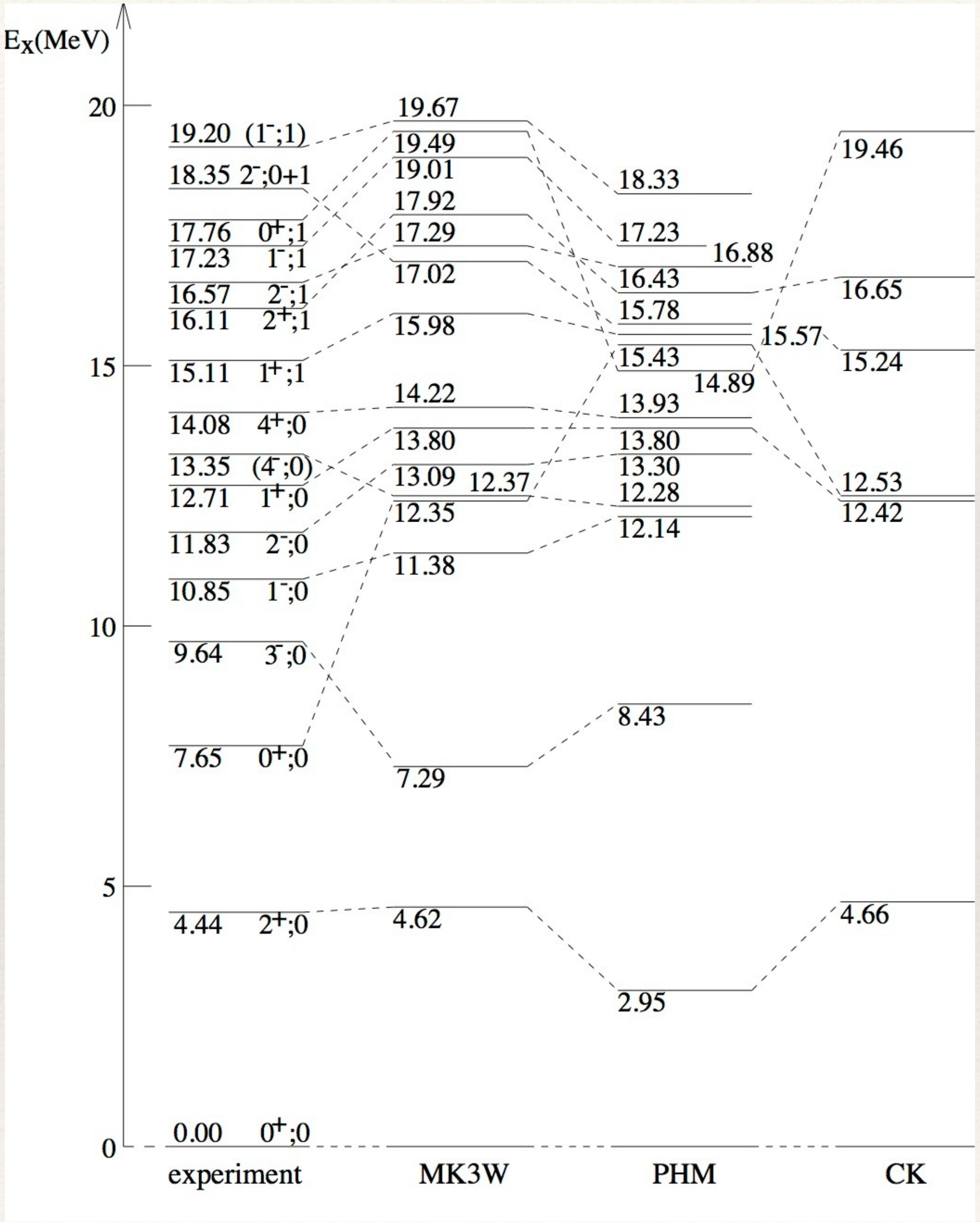
Neutron Skin of ^{208}Pb from Coherent Pion Photoproduction

C. M. Tarbert,[1] D. P. Watts,[1,*] D. I. Glazier,[1] P. Aguar,[2] J. Ahrens,[2] J. R. M. Annand,[3] H. J. Arends,[2] R. Beck,[2,4] V. Bekrenev,[5] B. Boillat,[6] A. Braghieri,[7] D. Branford,[1] W. J. Briscoe,[8] J. Brudvik,[9] S. Cherepnya,[10] R. Codling,[3] E. J. Downie,[3] K. Foehl,[1] P. Grabmayr,[11] R. Gregor,[12] E. Heid,[2] D. Hornidge,[13] O. Jahn,[2] V. L. Kashevarov,[10] A. Knezevic,[14] R. Kondratiev,[15] M. Korolija,[14] M. Kotulla,[6] D. Krambrich,[2,4] B. Krusche,[6] M. Lang,[2,4] V. Lisin,[15] K. Livingston,[3] S. Lugert,[12] I. J. D. MacGregor,[3] D. M. Manley,[16] M. Martinez,[2] J. C. McGeorge,[3] D. Mekterovic,[14] V. Metag,[12] B. M. K. Nefkens,[9] A. Nikolaev,[2,4] R. Novotny,[12] R. O. Owens,[3] P. Pedroni,[7] A. Polonski,[15] S. N. Prakhov,[9] J. W. Price,[9] G. Rosner,[3] M. Rost,[2] T. Rostomyan,[7] S. Schadmand,[12] S. Schumann,[2,4] D. Sober,[17] A. Starostin,[9] I. Supek,[14] A. Thomas,[2] M. Unverzagt,[2,4] Th. Walcher,[2] L. Zana,[1] and F. Zehr[6]
(Crystal Ball at MAMI and A2 Collaboration)

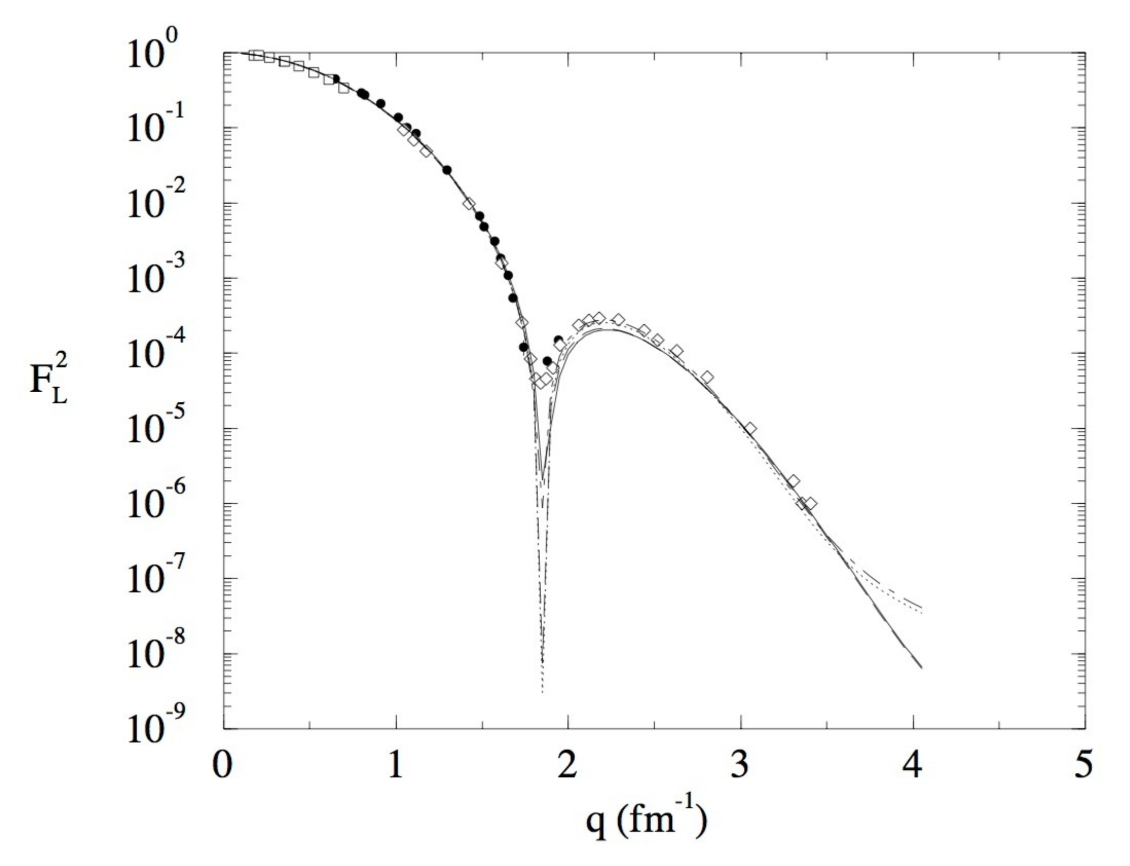
Measured value: 0.15±0.03 fm.

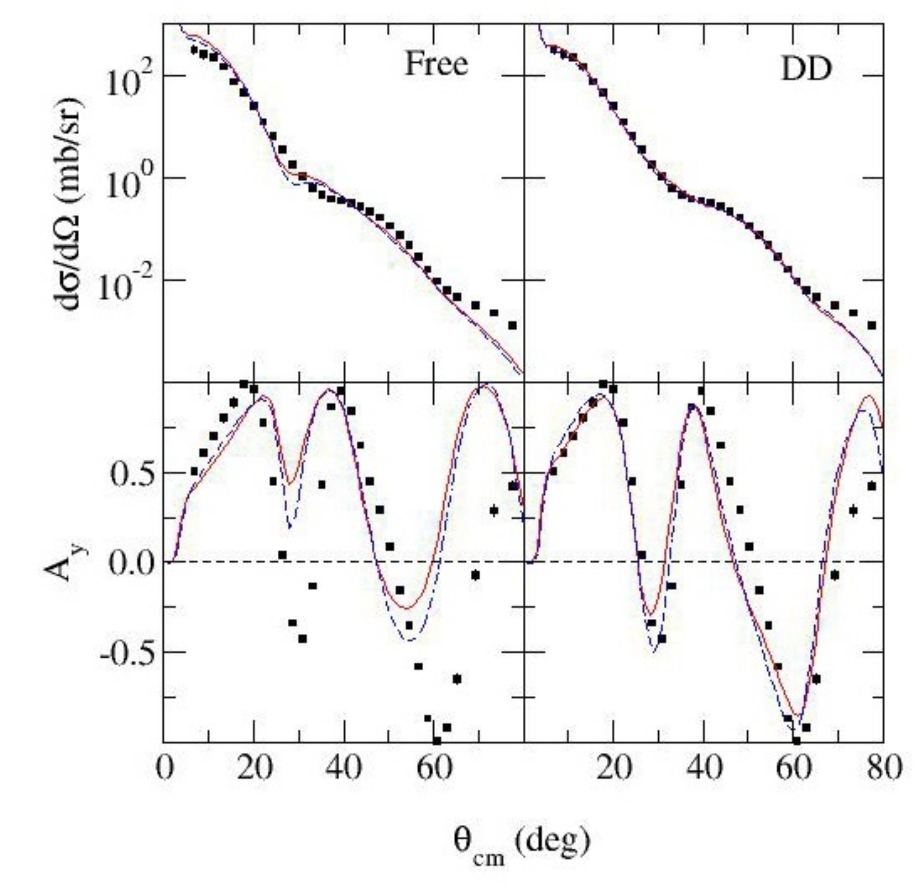
Example: ^{12}C

Spectrum
(SK, *et al.*, PRC **52**, 861 (1995)):



Elastic scattering (SK, *et al.*, PRC **52**, 861 (1995)):

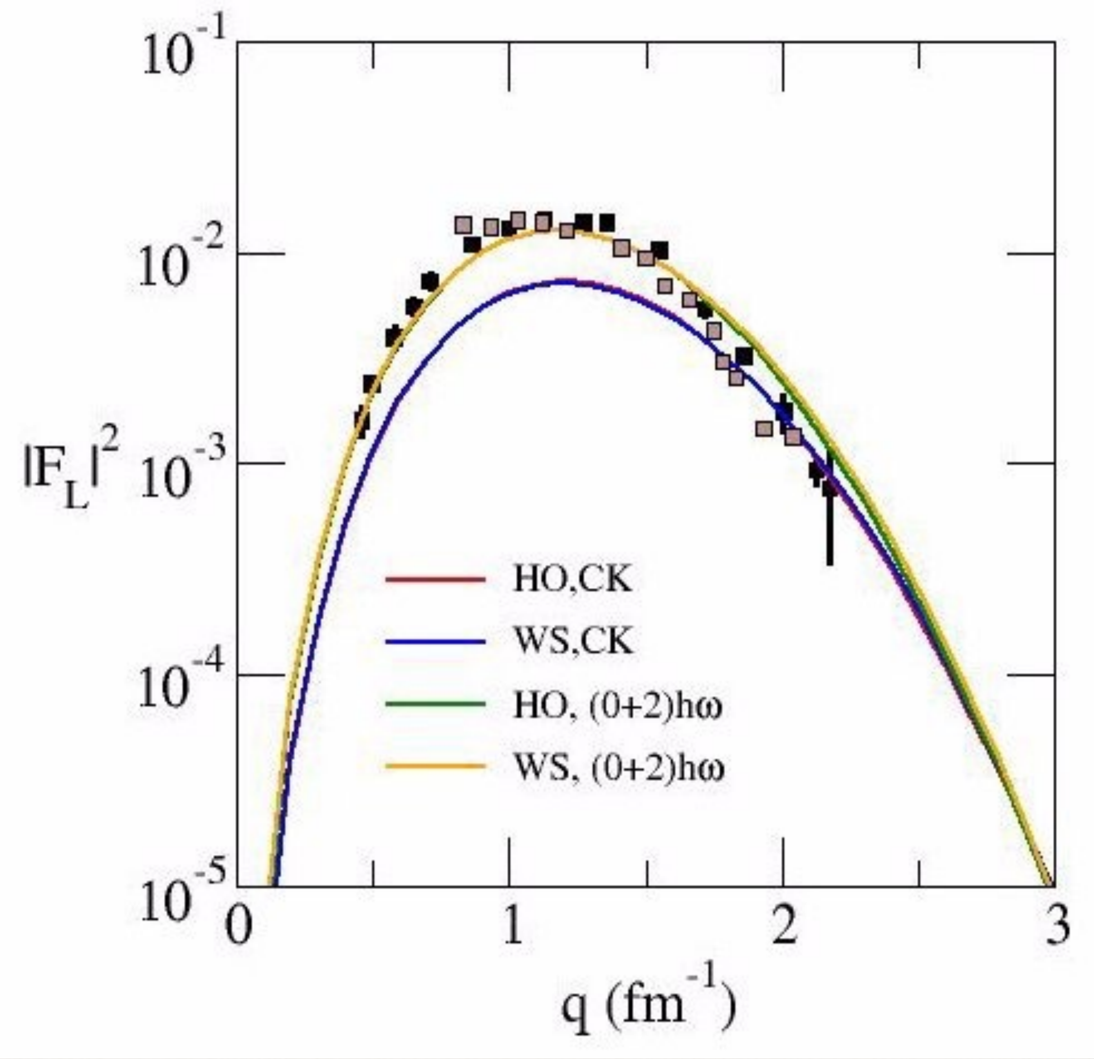




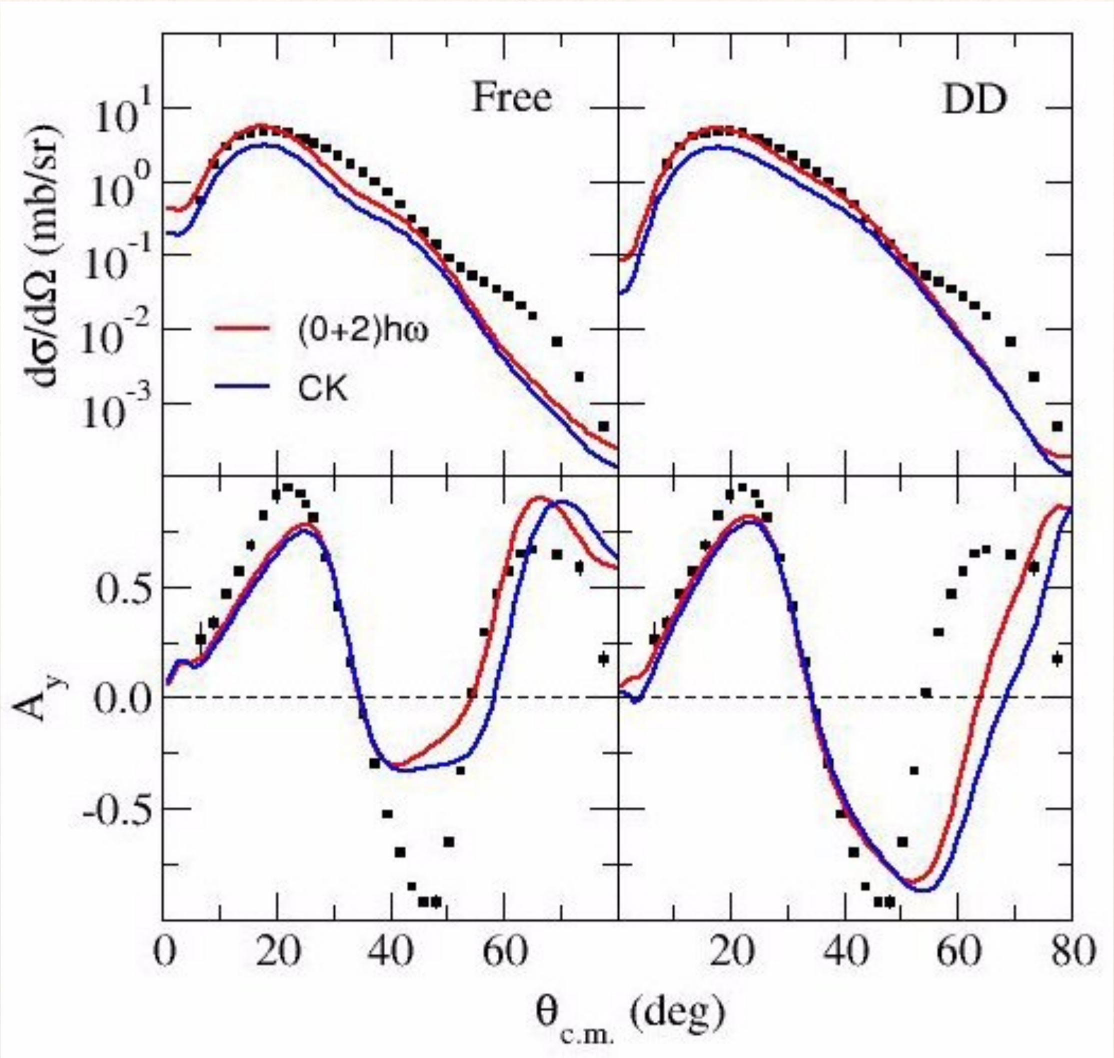
Elastic electron scattering

Elastic 200 MeV proton scattering

Inelastic scattering (2^+ state, 4.44 MeV)



Longitudinal form factor



200 MeV proton scattering

Densities, exotic nuclei

4$\hbar\omega$ shell model, Zheng interaction

(SK., *et al.*, PRC **61**, 024319 (2000)).

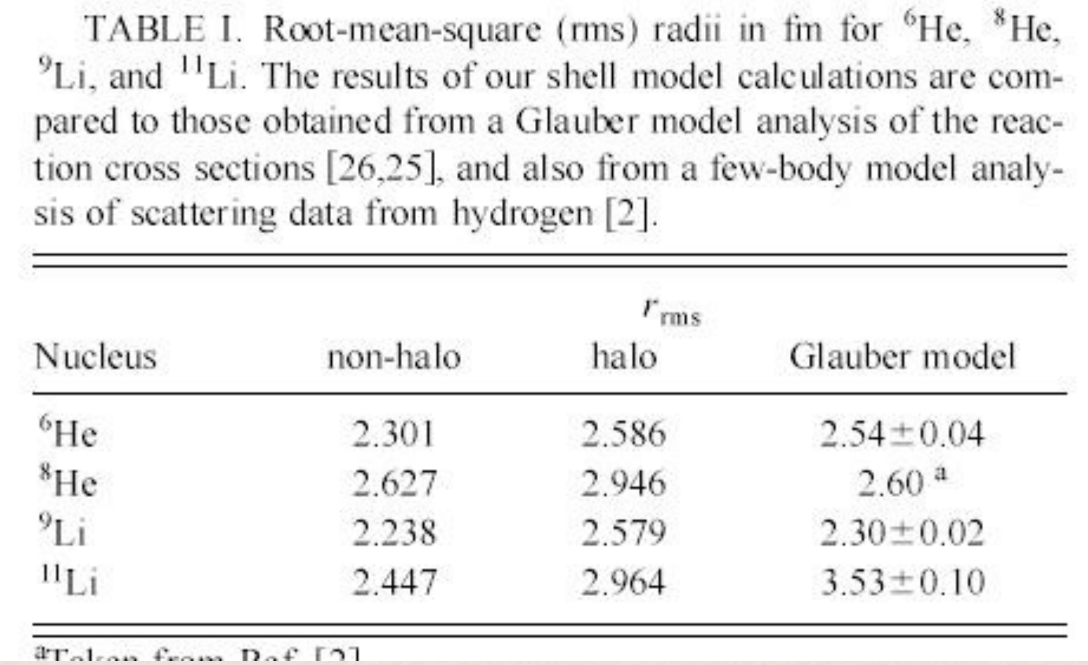
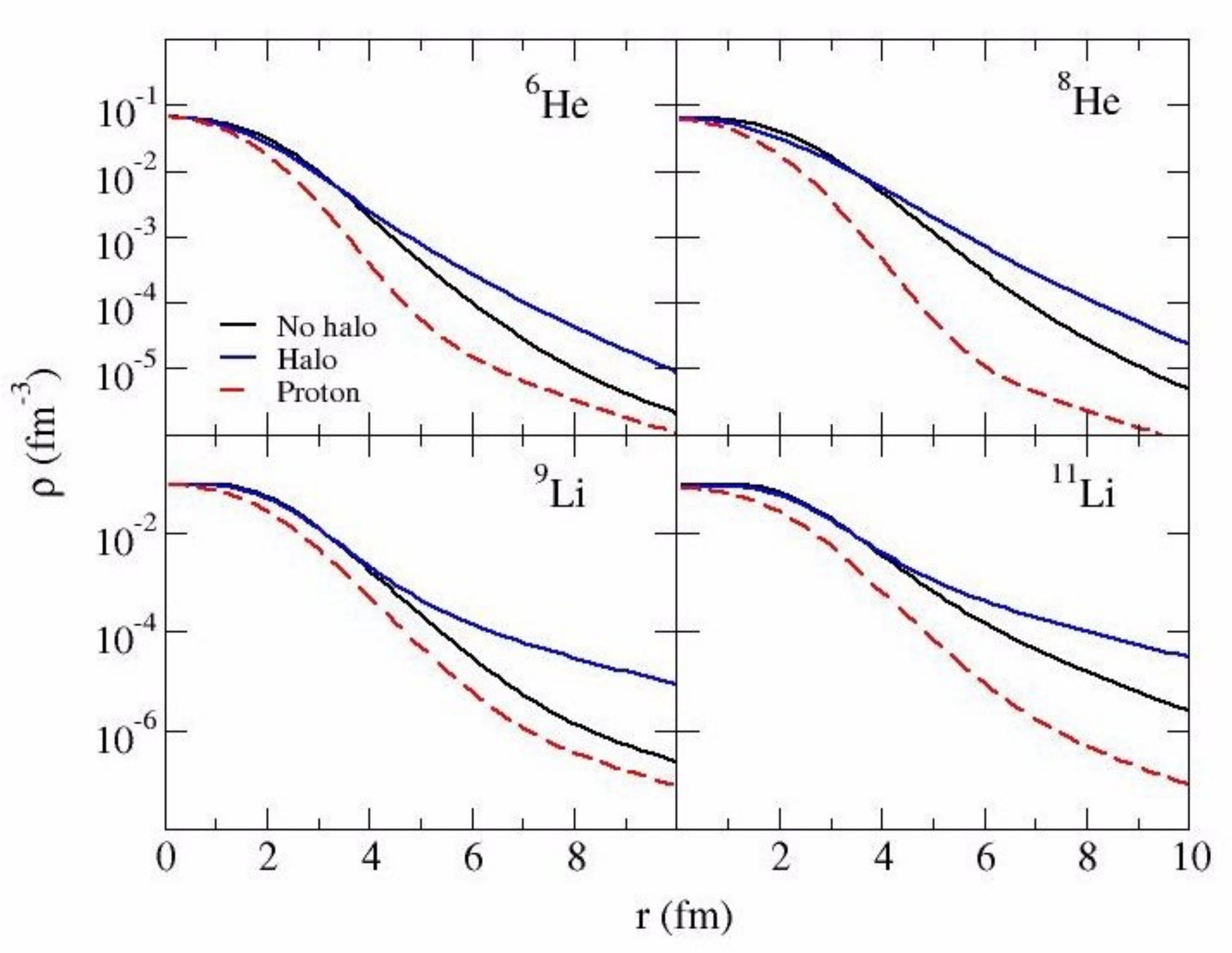


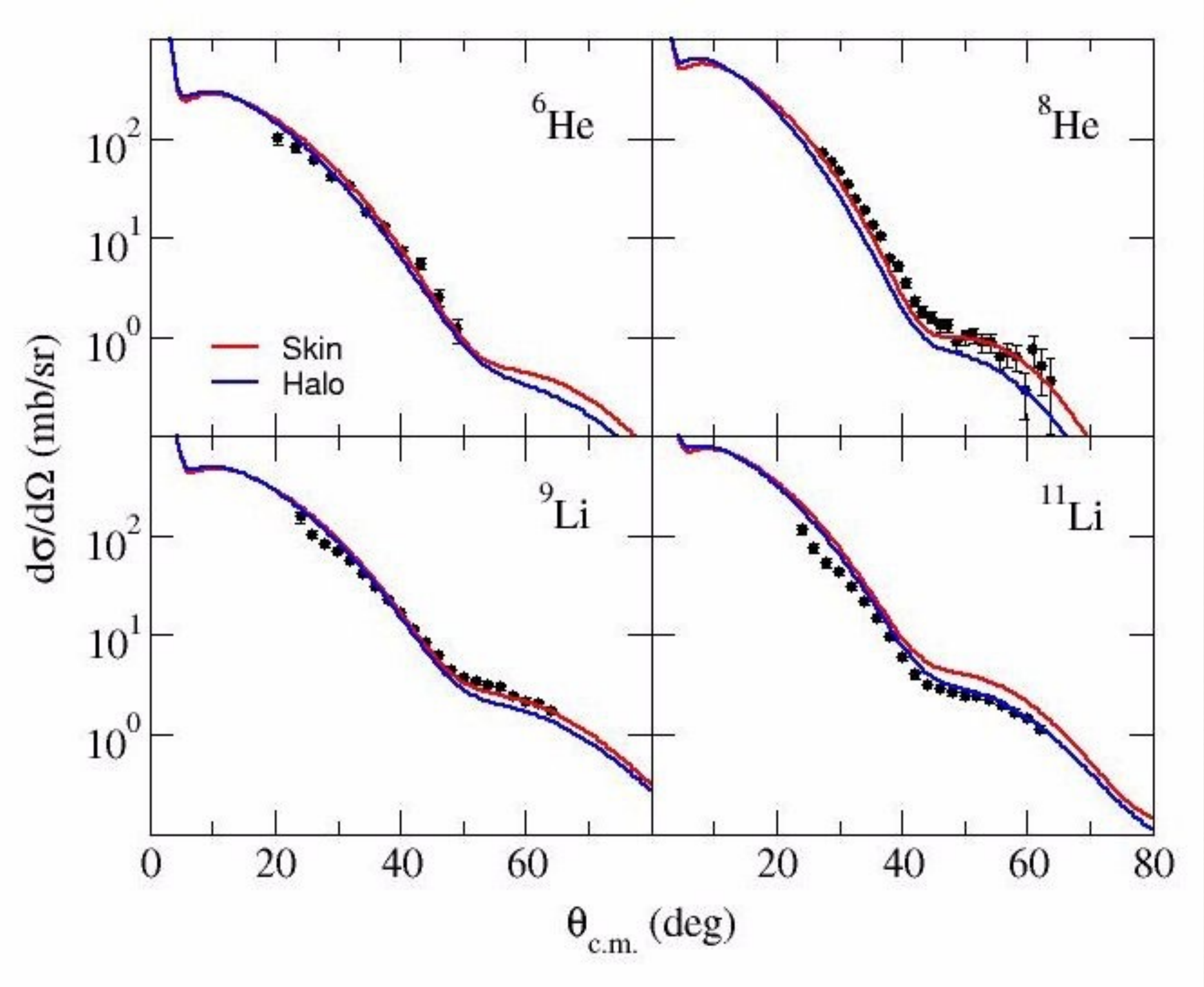
TABLE I. Root-mean-square (rms) radii in fm for ^{6}He, ^{8}He, ^{9}Li, and ^{11}Li. The results of our shell model calculations are compared to those obtained from a Glauber model analysis of the reaction cross sections [26,25], and also from a few-body model analysis of scattering data from hydrogen [2].

	r_{rms}		
Nucleus	non-halo	halo	Glauber model
^{6}He	2.301	2.586	2.54±0.04
^{8}He	2.627	2.946	2.60 [a]
^{9}Li	2.238	2.579	2.30±0.02
^{11}Li	2.447	2.964	3.53±0.10

[a] Taken from Ref. [2].



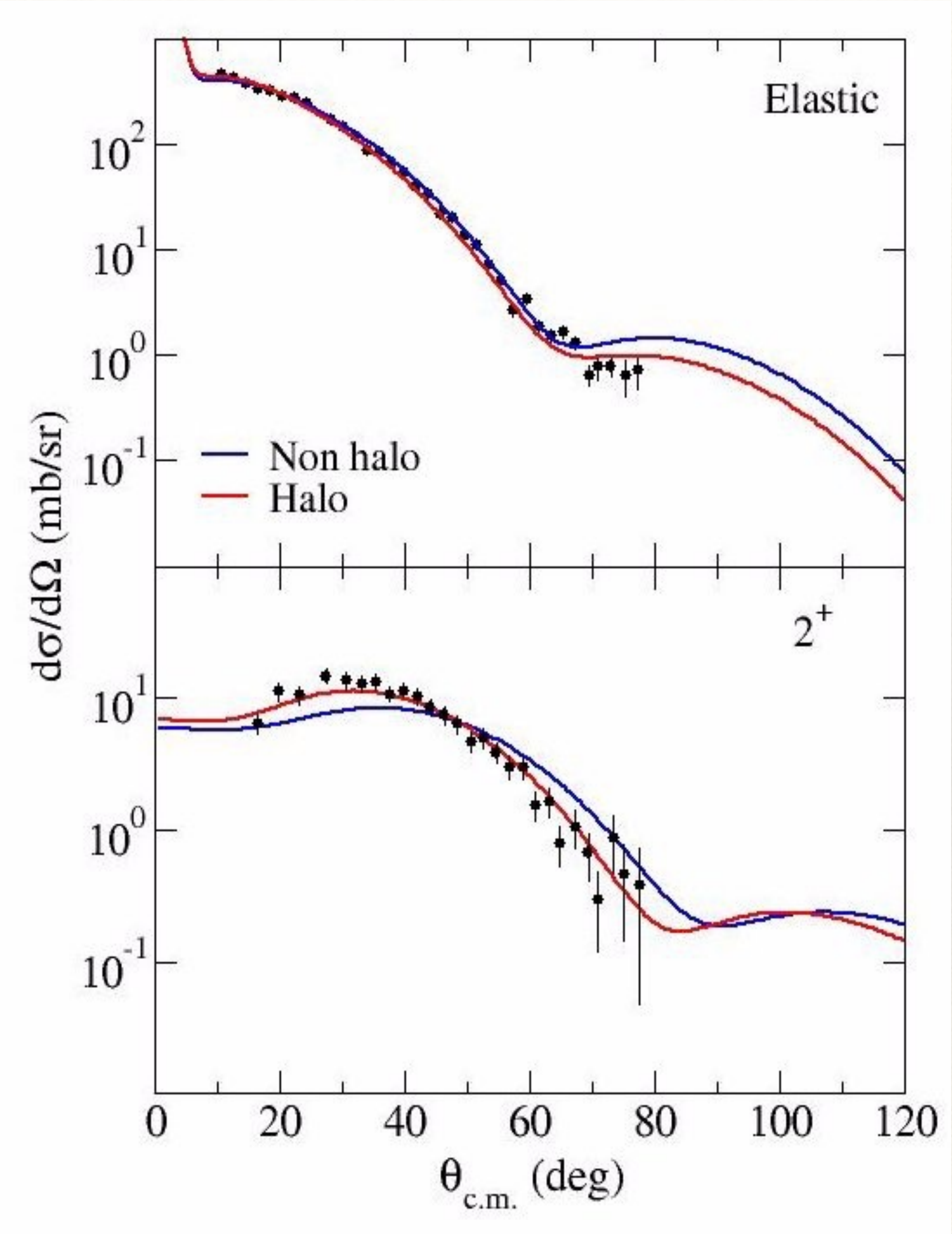
Elastic scattering



71*A* MeV

62*A* MeV

p-^{6}He scattering



Reaction cross section

Predicted

$$\sigma_R = 353 \text{ mb (nonhalo)}$$
$$= 406 \text{ mb (halo)}$$

Measured

$$\sigma_R = 409 \pm 22 \text{ mb}$$

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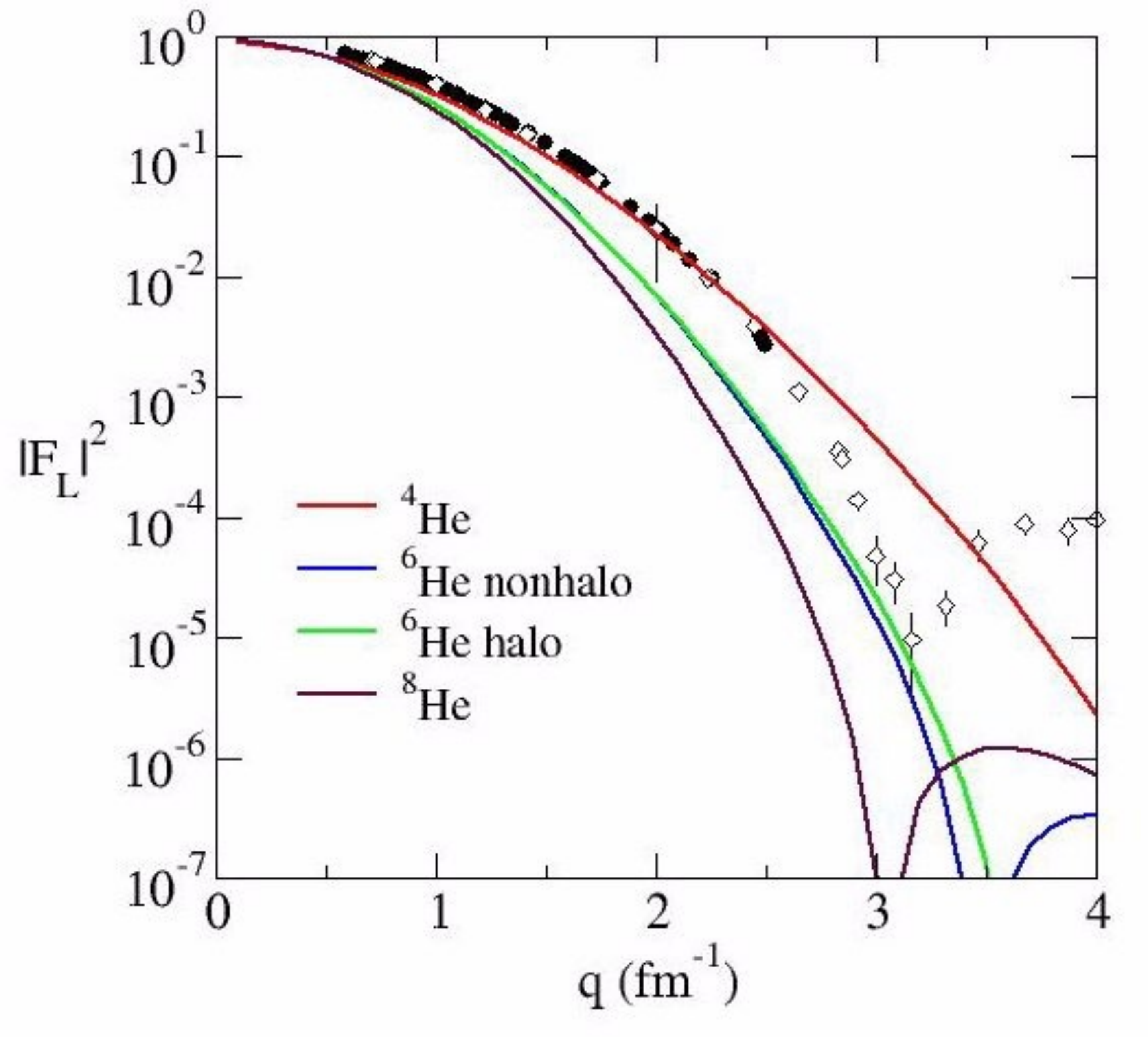
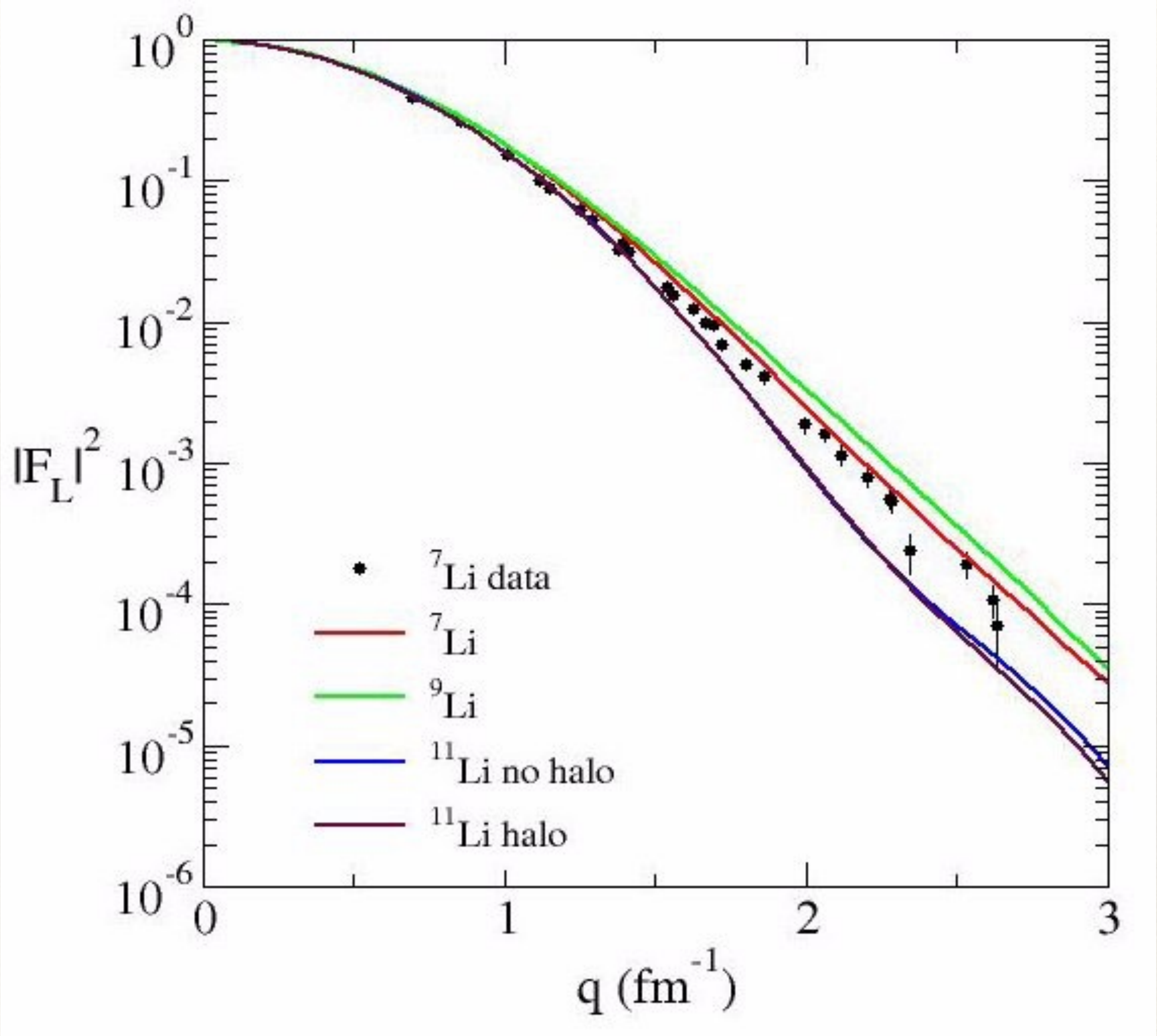
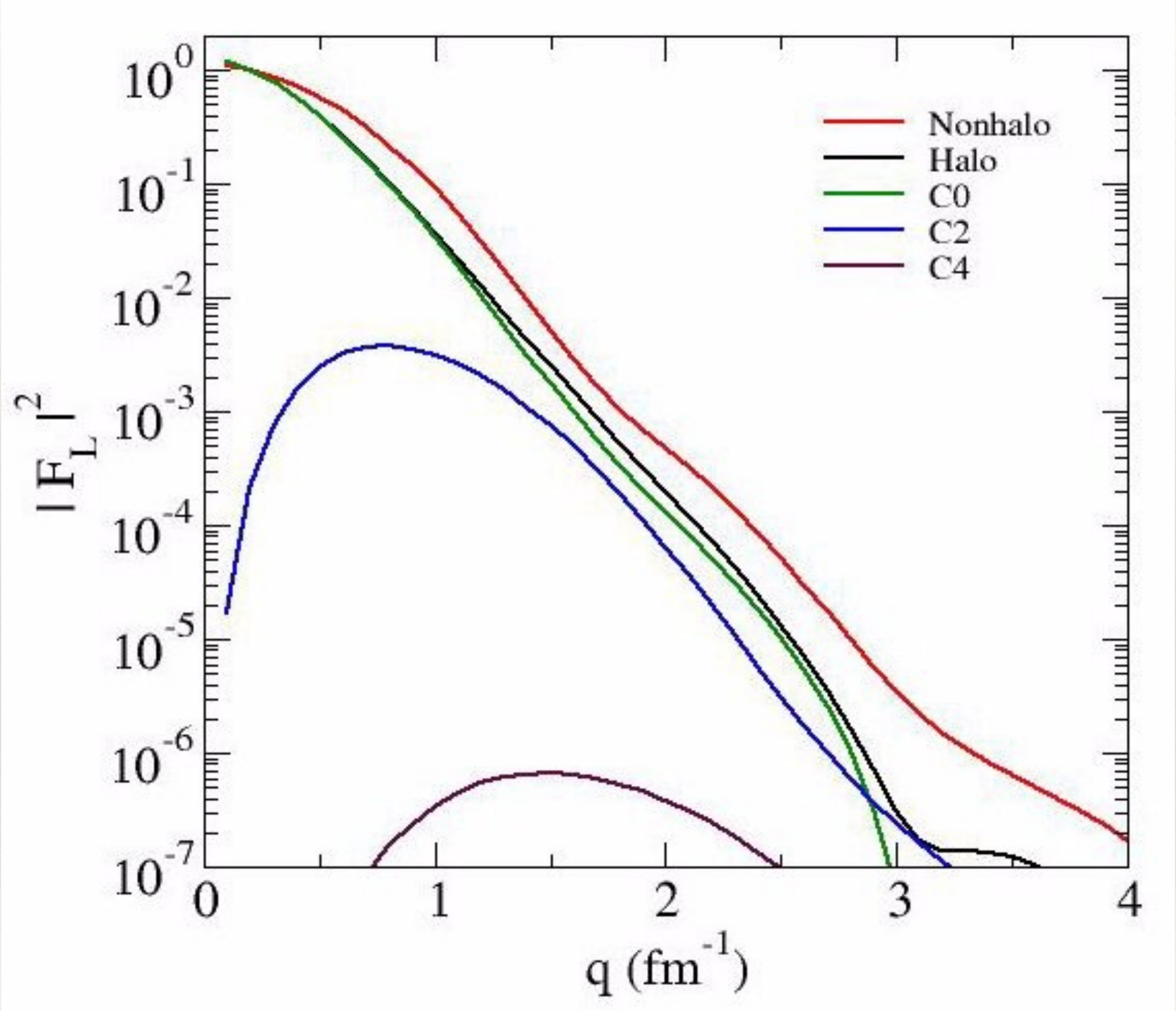
Electron scattering, He isotopes



Electron scattering, Li isotopes



Electron scattering, ^8B



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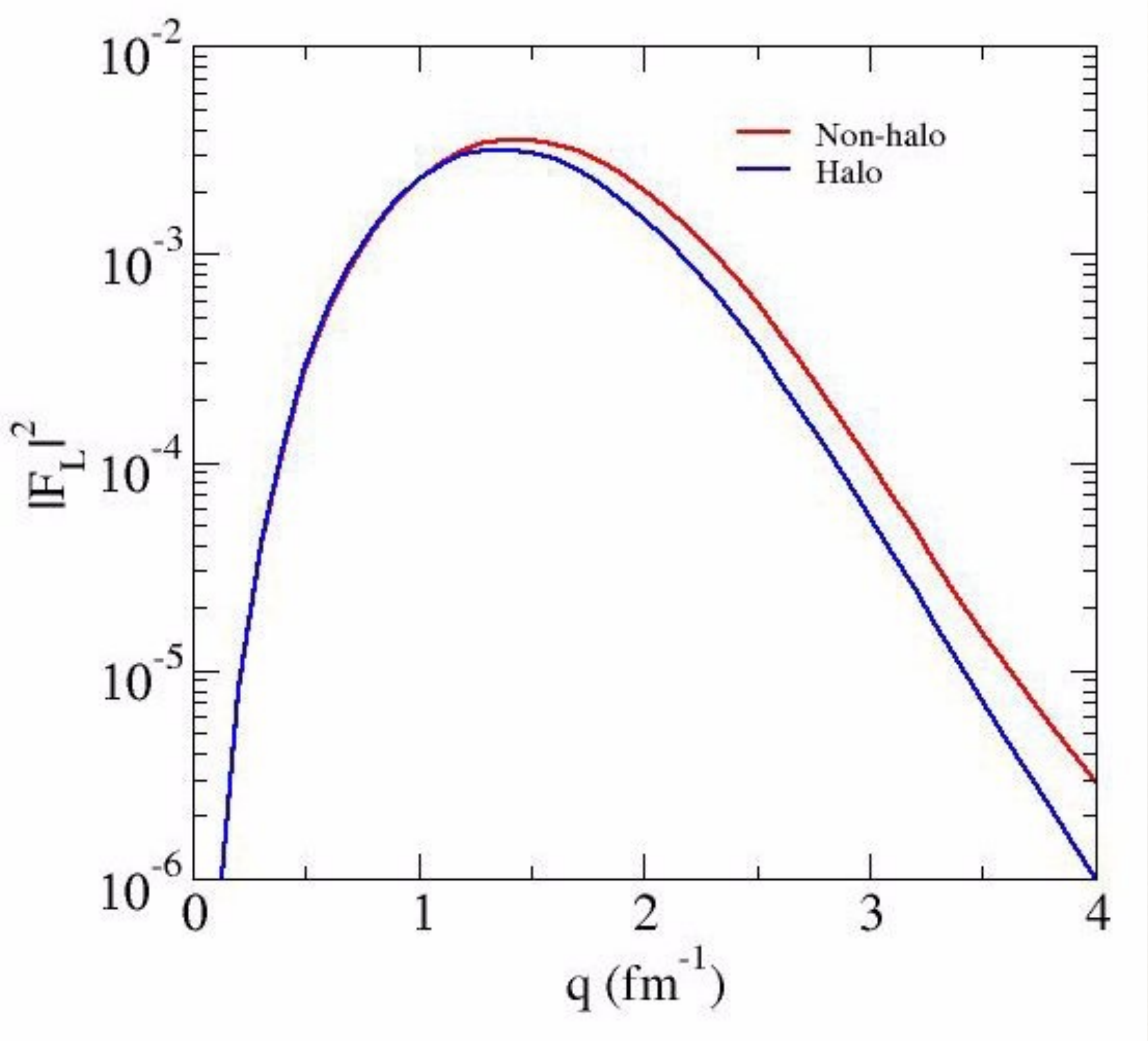
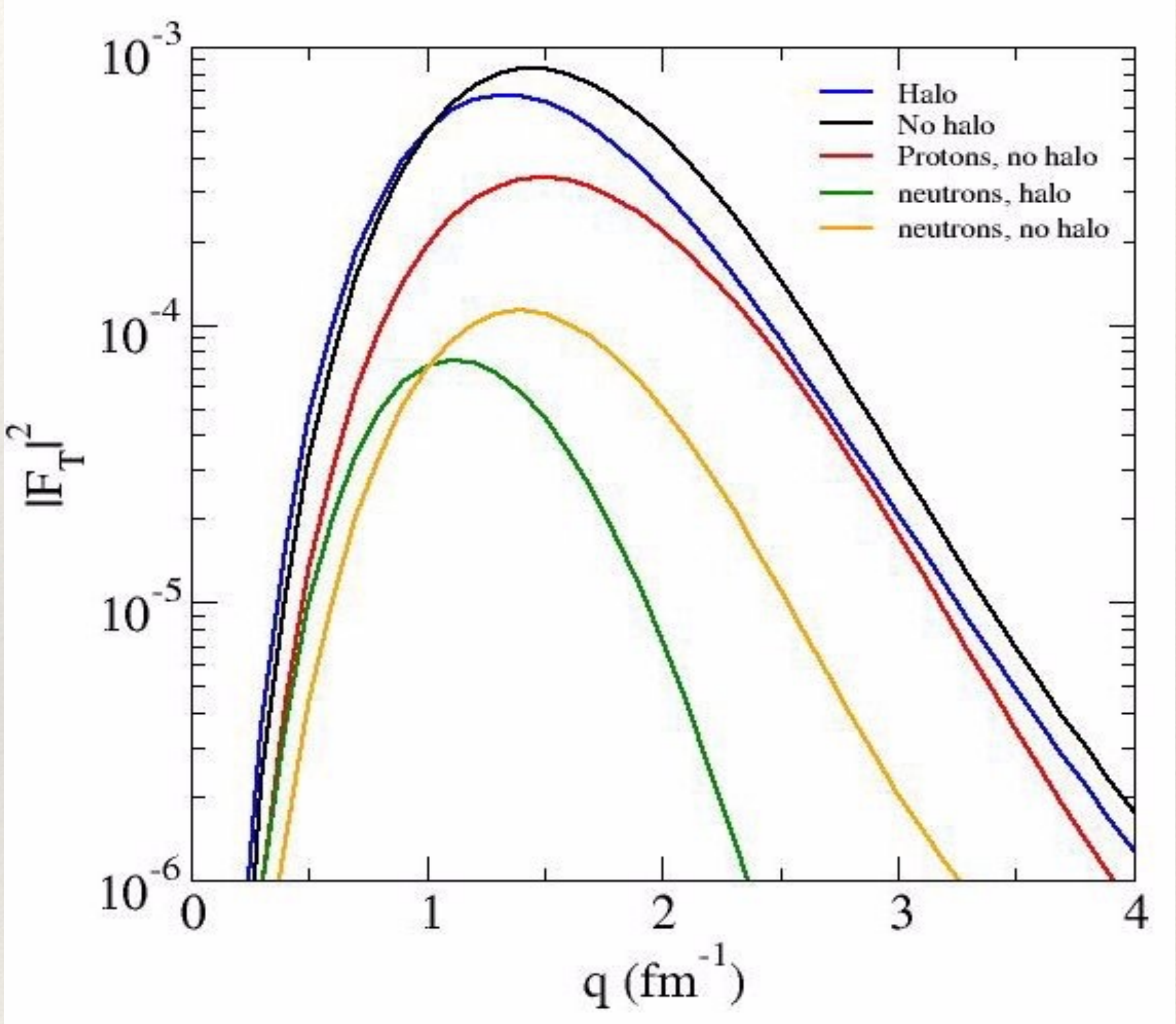
Inelastic electron scattering, ^{6}He





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Conclusions

- Presented results for calculations of electron and proton scattering from light stable and exotic nuclei.
- For the He isotopes, the results of the calculations for the elastic longitudinal form factors follow a natural mass dependence.
- For the Li isotopes, the results of the calculations also follow a natural mass dependence.
- For ^{8}B, the proton halo **does** significantly change the prediction of the form factor.
- The inelastic scattering form factor for ^{6}He does show some effect due to the neutron halo, as consistent with the results of proton scattering.