Few-body universality and halo nuclei

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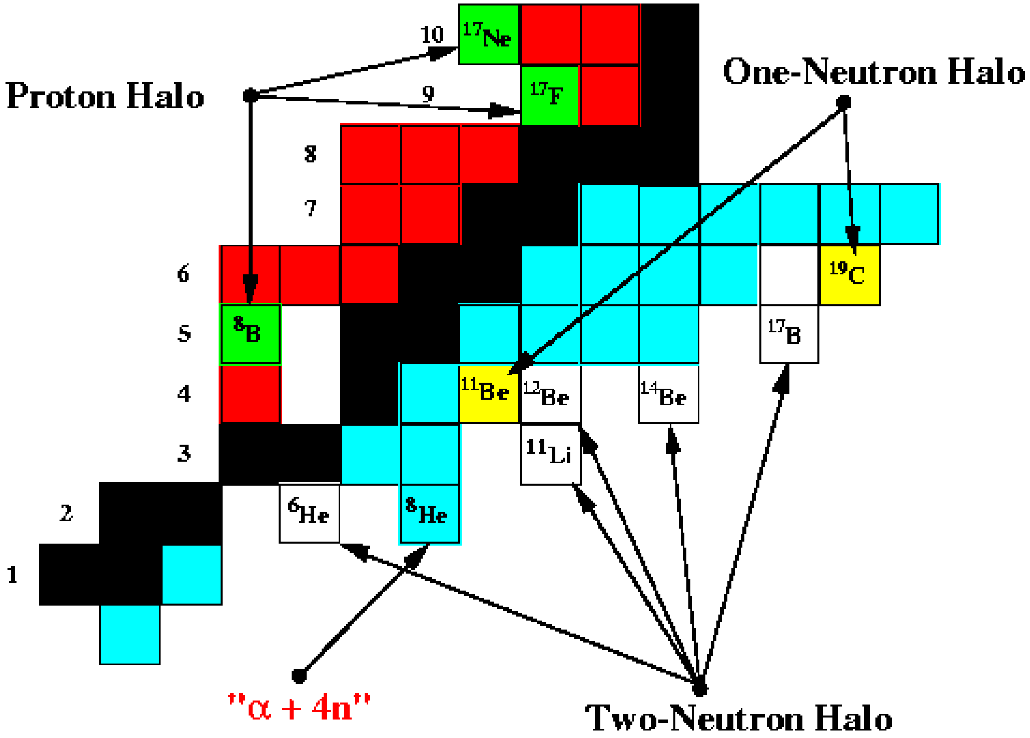
Outline

- Introduction
- Resonant Interactions and Universality
- Universality in Halo nuclei
  - Efimov Physics
  - Electromagnetic Properties
- Summary and Outlook

Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to “nucleon drip line” $\longrightarrow$ scale separation $\longrightarrow$ EFT



$$E \sim p^2/(2\mu) \sim 1/(2\mu R^2)$$

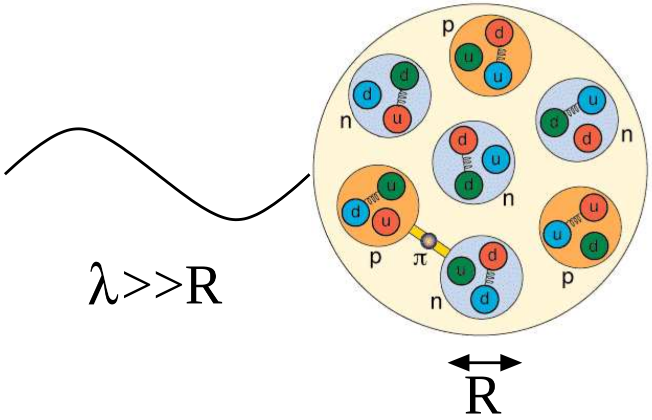
http://www.nupecc.org

- EFT for halo nuclei

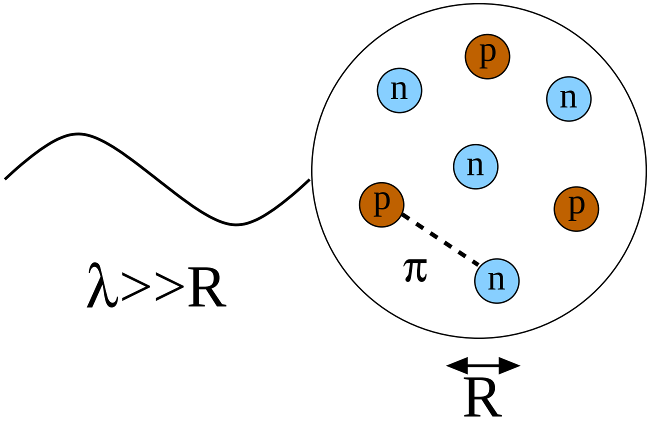
  (Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

Effective Field Theory

- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR



Effective Field Theory



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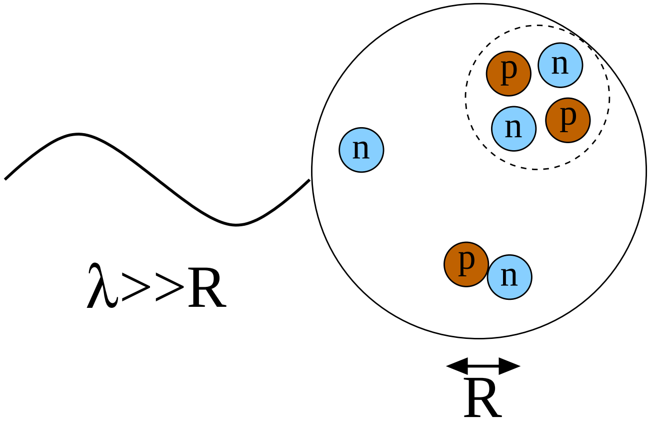
- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent

Effective Field Theory



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  $\longrightarrow$ expand in powers of kR

- Short-distance physics not resolved

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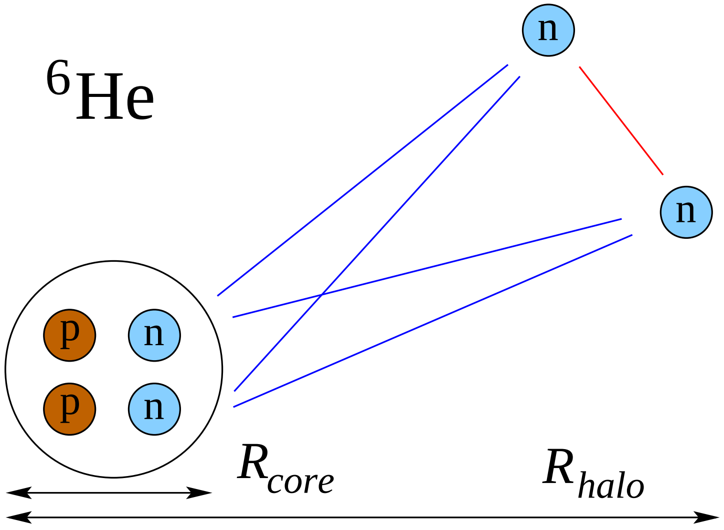
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent
- Very low energies: only short-range physics $\Longrightarrow$ pionless EFT
- Exploit cluster substructures $\Longrightarrow$ Halo EFT

Halo EFT

- Pionless EFT with explicit cluster degrees of freedom
- Scales: $R_{halo} \gg R_{core}$
- Antisymmetrization with respect to neutrons in core?
- Core neutrons not active dof in halo EFT



- Physics: exchange only contributes to observables if there is significant overlap between wave functions of core and halo nucleon

$\Longrightarrow$ small for $R_{core} \ll R_{halo}$

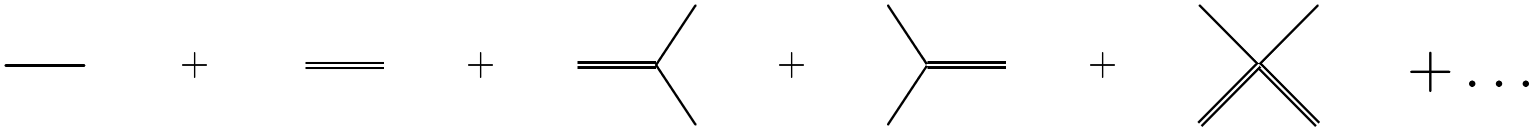
- Effects subsumed in low-energy constants, included perturbatively in expansion in R_{core}/R_{halo}

EFT Framework

- Exploit scale separation from shallow states: $B \sim 0 \;\Leftrightarrow\; 1/a \sim 0$
- Here: discuss S-wave case, higher L can be included

- Effective Lagrangian

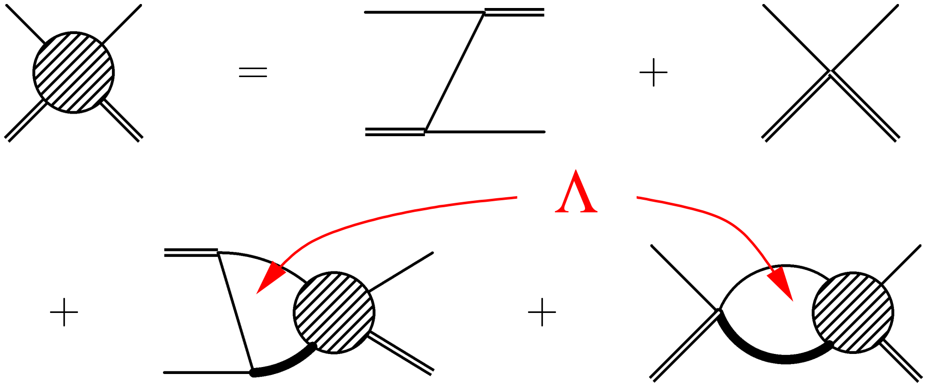
$\mathcal{L}_{eff} =$



- 2-body amplitude:

- 2-body coupling g_2 near fixed point: $1/a = 0 \quad \Longleftrightarrow$ unitary limit

- 3-body amplitude:

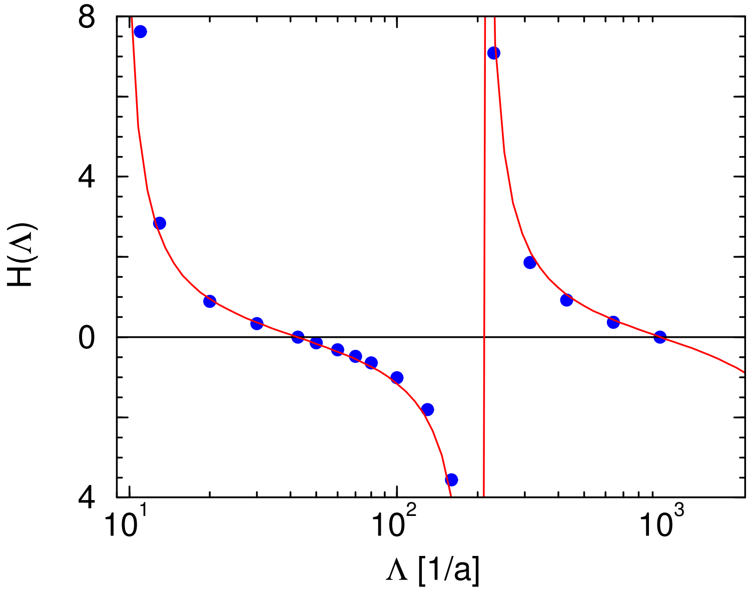


$g_3(\Lambda) \;\Rightarrow$ limit cycle

$\Rightarrow$ discrete scale inv.

Limit Cycle

- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3\Lambda^2/(9g_2^2)$



- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup

$$\boxed{H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624}$$

(Bedaque, HWH, van Kolck, 1999)

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance
- Observable Consequences? $\Longrightarrow$ Efimov effect, log-periodic behavior, universal correlations, ...

Limit Cycle: Efimov Effect

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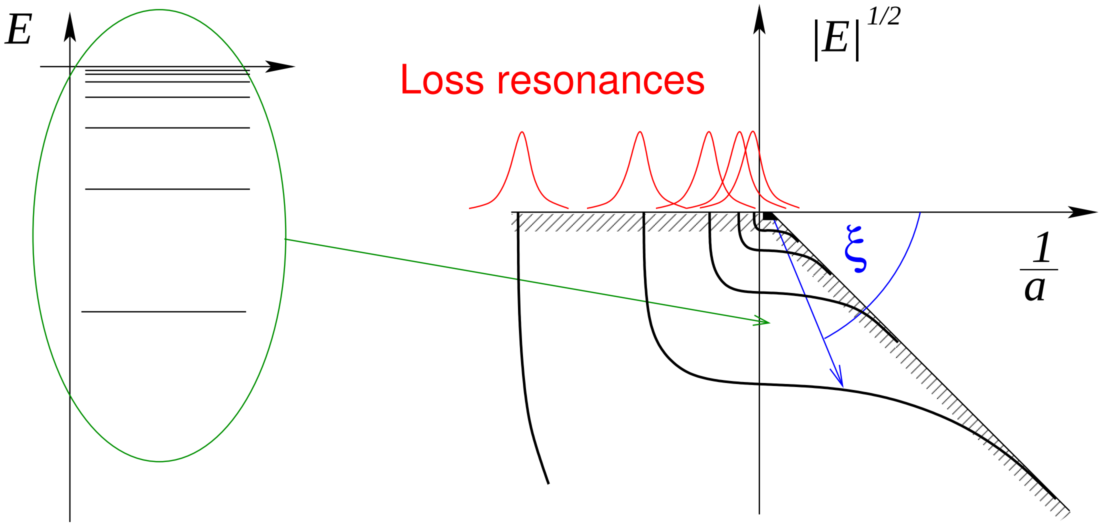
- Universal spectrum of three-body states (Efimov, 1970)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$\boxed{B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a\to 0} \quad \left(e^{\pi/s_0}\right)^2 = 515.035\ldots}$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances
  $\Longrightarrow$ drive RF transitions

Efimov States in Ultracold Atoms

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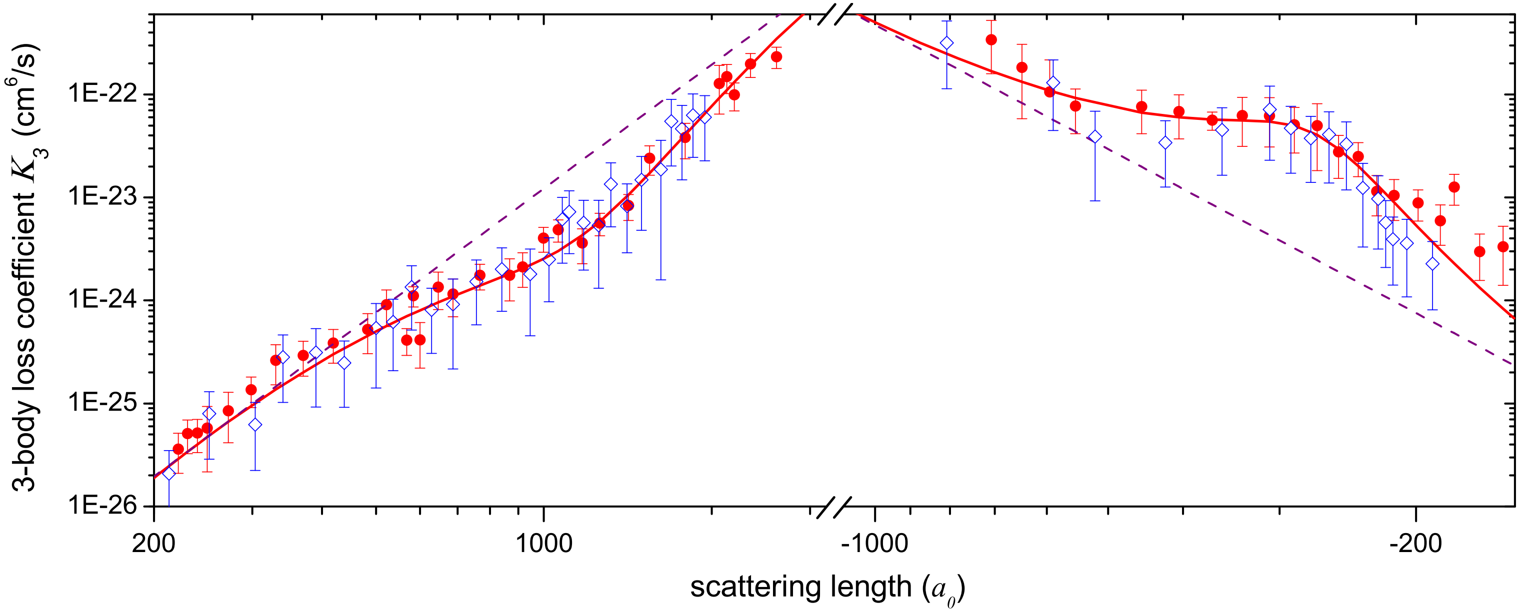
- First experimental evidence in ^{133}Cs (Krämer et al. (Innsbruck), 2006)
  now also ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb, ^{6}Li/^{133}Cs
- Example: Efimov spectrum in ^{7}Li ($|m_F = 0\rangle$, $|m_F = 1\rangle$)
  (Gross et al. (Bar-Ilan Univ.), Phys. Rev. Lett. **105** (2010) 103203)



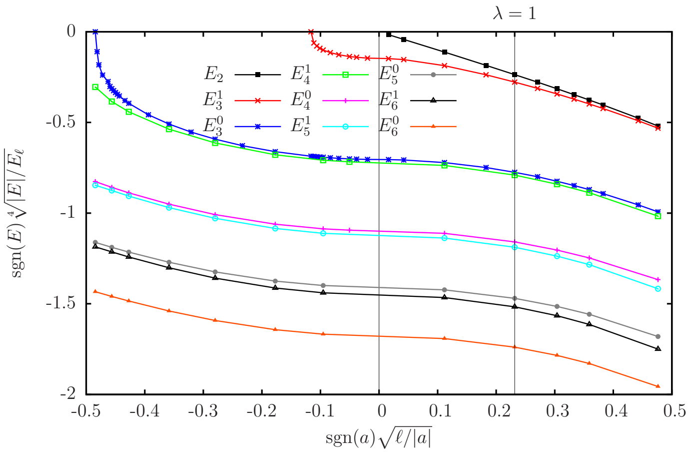
- Van der Waals tail determines $\Lambda_* \Rightarrow a_-/l_{vdW} \approx -10\ (\pm 15\%)$
  (Wang et al., 2012; Naidon et al., 2012, 2014; ...)
  . . . but not width parameter η_* . . .

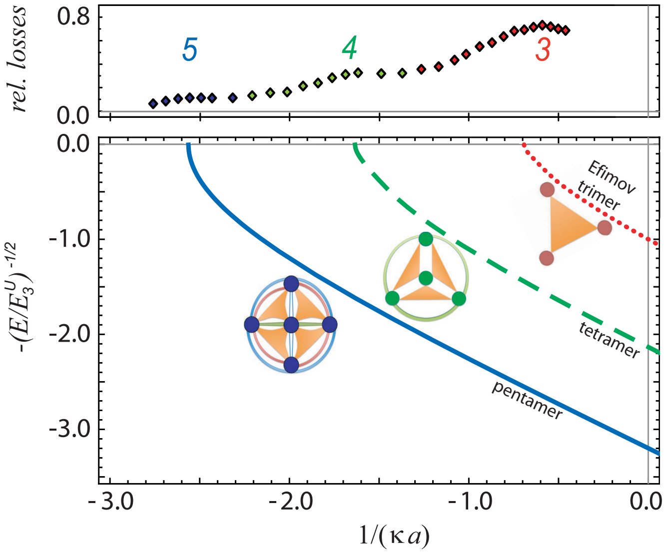
Universal Tetramers and Beyond

- No four-body force required at LO (Platter, HWH, Meißner, 2004)
- Universal tetramers: $B_4^{(0)} = 4.610(1)\ B_3\,,\quad B_4^{(1)} = 1.00227(1)\ B_3$

  (Platter, HWH, 2007; von Stecher et al., 2009; Deltuva 2010-2013)
- Universal states up to $N = 16$ calculated

  (von Stecher, 2010, 2011; Gattobigio, Kievsky, Viviani, 2011-2014)
- Observation up to $N = 5$ in Cs losses (Grimm et al. (Innsbruck), 2009, 2013)

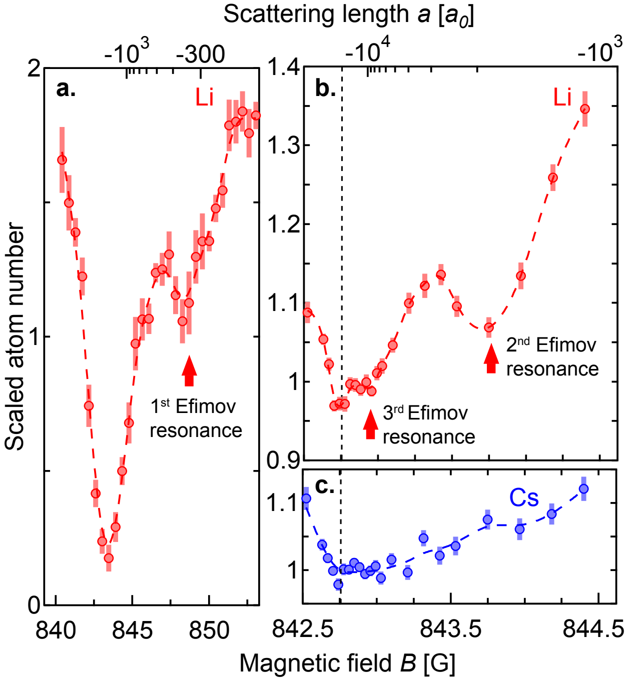


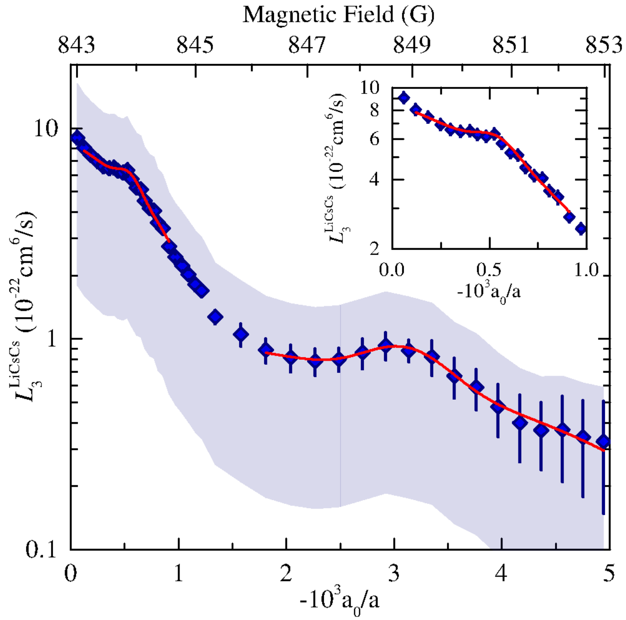


Efimov Physics in Mixtures

- Scaling factor can be significantly reduced in mixtures
- Example: Efimov spectrum in ^{6}Li/^{133}Cs mixture ($\lambda_0 \approx 4.9$)

Tung et al. (Chicago), PRL113, 240402 ('14)   Pires et al. (Heidelberg), PRL112, 250404 ('14)





- Spectrum of universal states for $N > 3$

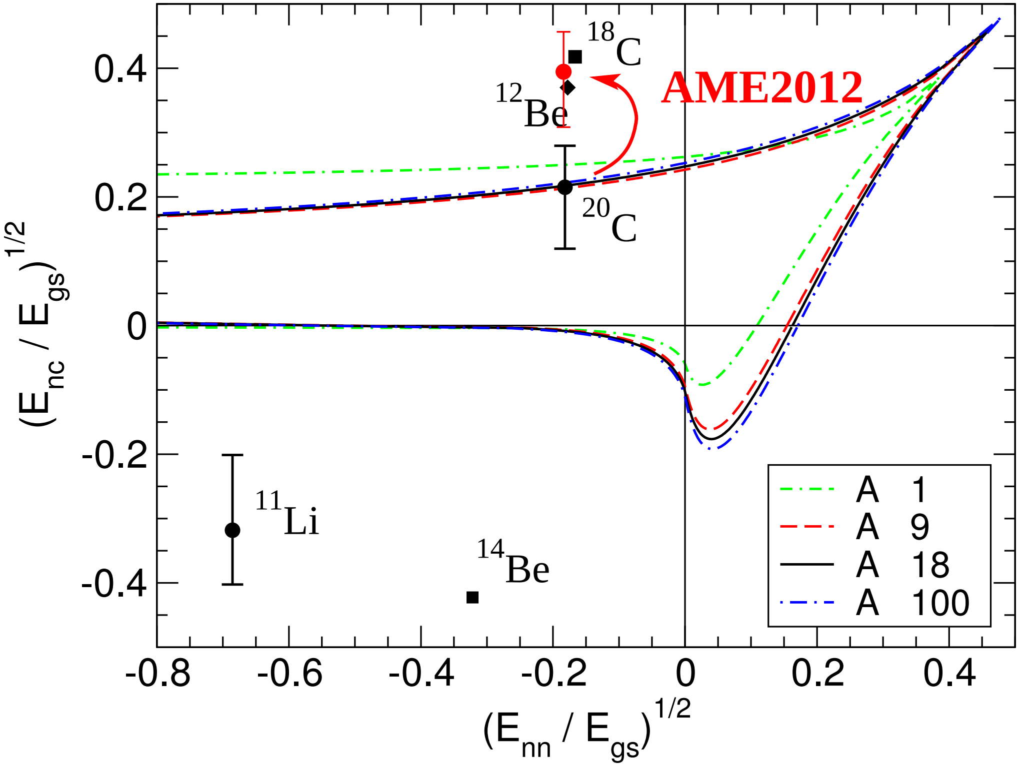
  (Blume, Yang, 2014; Schmickler, HWH, Hiyama, in progress)

Efimov Physics in Halo Nuclei

Efimov Physics in Halo Nuclei

- Efimov effect in halo nuclei? (Fedorov, Jensen, Riisager, 1994)

  $\Longrightarrow$ excited states obeying scaling relations
- Correlation plot: $E_{nn} \leftrightarrow E_{nc}$ (Amorin, Frederico, Tomio, 1997)



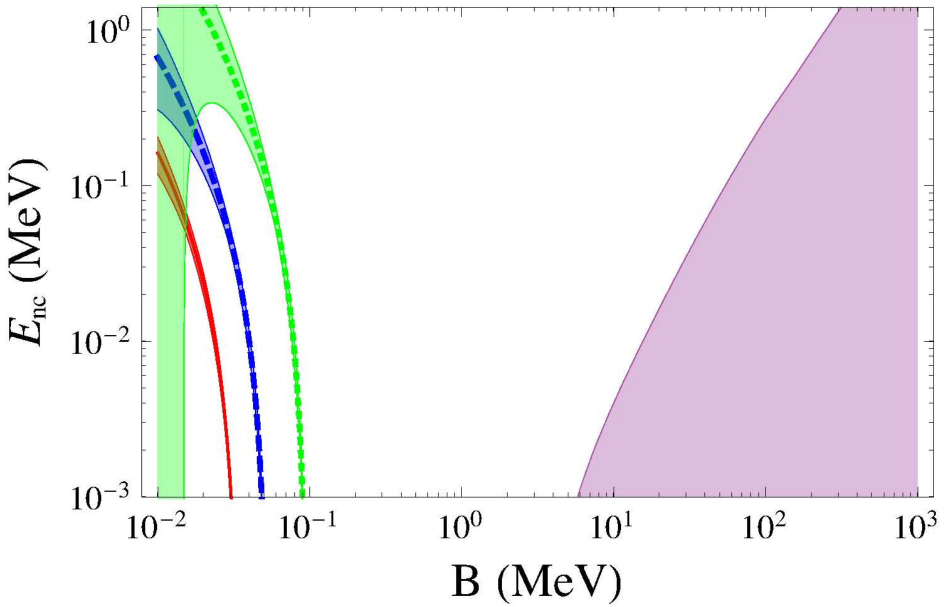
Canham, HWH, Eur. Phys. J. A **37** (2008) 367

Efimov Physics in ^{22}C

- Matter radius from ^{22}C + p & Glauber: $\langle r_0^2 \rangle^{1/2} = 5.4(9)$ fm

  (Tanaka et al., Phys. Rev. Lett. **104** (2010) 062701)
- Halo EFT analysis of impact on other observables in ^{22}C

  (Acharya, Ji, Phillips, Phys. Lett. B **723** (2013) 196)



Plots for $\langle r_0^2 \rangle^{1/2} = 4.5,\ 5.4,\ 6.3$ fm

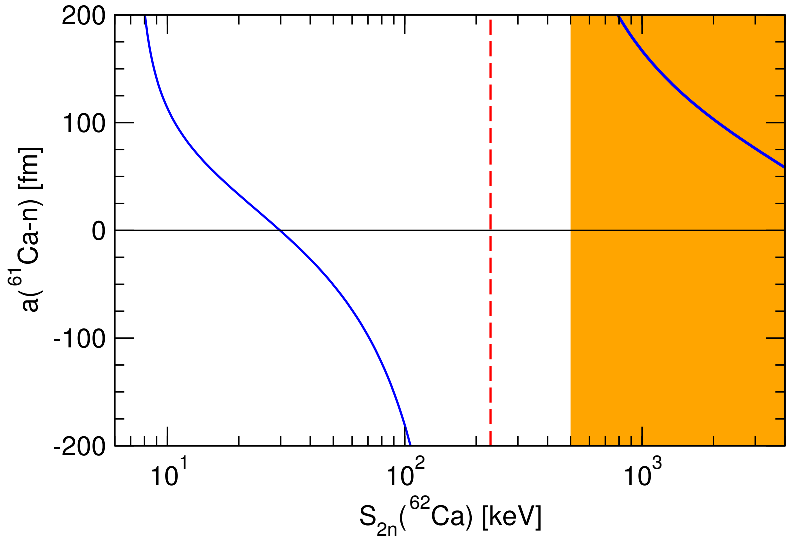
- Excited Efimov states in ^{22}C appear to be ruled out

Efimov Physics in ^{62}Ca

(G. Hagen, P. Hagen, HWH, Platter, Phys. Rev. Lett. **111** (2013) 132501)

- The Many and the Few: emergence of halo degrees of freedom
- Coupled cluster calculations of ^{60}Ca and ^{61}Ca using chiral N2LO two-body force and schematic three-body force:

$\Rightarrow$ ^{61}Ca is a weakly bound S-wave state (or virtual state)



- Prospects for excited Efimov states in ^{62}Ca:

$S_{\text{deep}} = 1/(\mu_{cn} r_{cn}^2) \approx 500$ keV

scaling factor $\lambda_0 \approx 16$

$\Longrightarrow$ possible if $S_{2n} \gtrsim 230$ keV

Production of Excited Efimov States

- How to study excited Efimov states experimentally?

  (A. Macchiavelli, Few-Body Syst. **56**, 773 (2015)

- Consider transfer reactions for candidate nucleus ${}^{A}Z_N$

  (a) One-neutron transfer: ${}^{(A-1)}Z_{(N-1)}(d,p)^{A}Z_N$

  (b) Two-neutron transfer: ${}^{(A-2)}Z_{(N-2)}(t,p)^{A}Z_N$

  (c) Inelastic scattering: ${}^{A}Z_N(X,X')^{A}Z_N$

- Back-of-the-envelope estimate:

$$\frac{d\sigma_{ex}}{d\sigma_{gs}} = \frac{S_{ex}}{S_{gs}}\frac{\sigma_{ex,0}}{\sigma_{gs,0}}$$

$\Longrightarrow$ suggests (a) is most promising

- Reaction calculation in Halo EFT would be useful

Electromagnetic Properties

Electromagnetic Structure of ^{11}Be

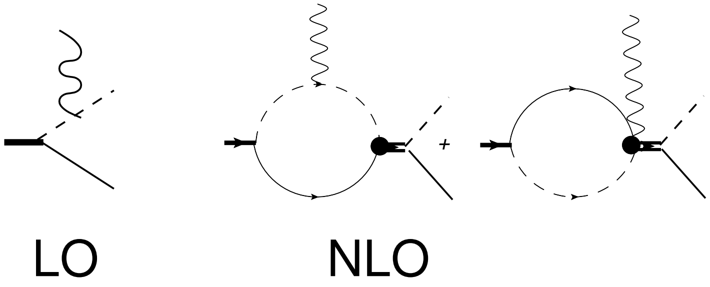
- Coupling to photons straightforward
  - Covariant derivative: $\partial_\mu \to \partial_\mu + ie\,\hat{Q}A_\mu$
  - Contact terms (separately gauge invariant)
- Properties of ^{11}Be
  - ^{11}Be ground state: $J^P = 1/2^+$, $S_n = 504$ keV
  - ^{11}Be excited state: $J^P = 1/2^-$, $S_n = 184$ keV
- Properties of ^{10}Be core
  - ^{10}Be ground state: $J^P = 0^+$
  - ^{10}Be first excitation: 3.4 MeV above g.s.
- Separation of scales: $E_{lo}/E_{hi} \approx \frac{0.5}{3.5} = \frac{1}{7} \quad \Rightarrow \quad R_{core}/R_{halo} \approx 0.4$

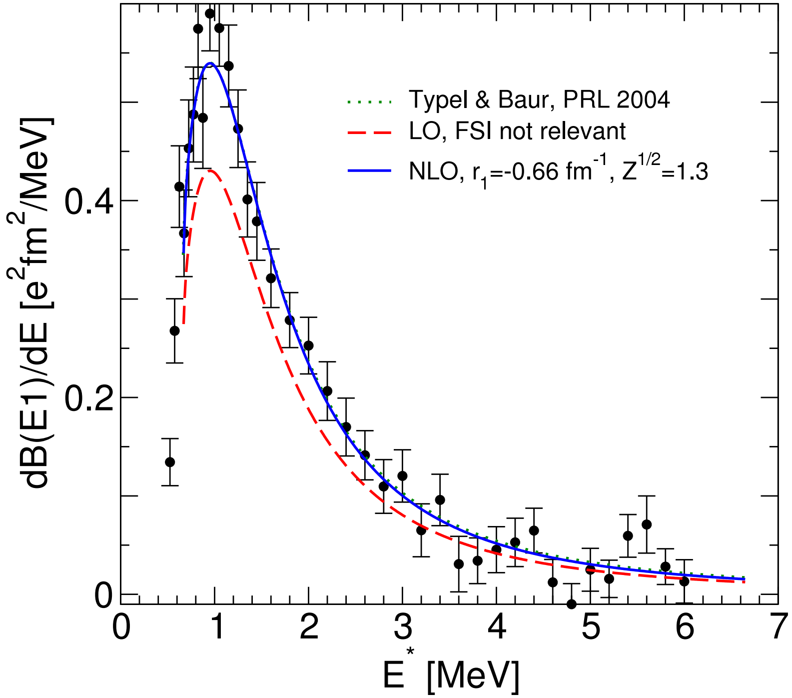
$\Rightarrow$ one neutron halo picture for ^{11}Be appropriate

- EM properties in halo EFT: form factors, radii,...

Coulomb Dissociation of ^{11}Be

- Transition to the continuum:





- Reasonable convergence
- At LO: impulse approximation
- At NLO: FSI from excited state

$$r_1 = -0.66\ \text{fm}^{-1} \qquad [B(E1)]$$

$$\sqrt{Z_\sigma} = 1.3 \quad \Rightarrow \quad r_0 = 2.7\ \text{fm}$$

- Detector resolution folded in

Data: Palit et al., PRC **68** (2003) 034318

Calculation: HWH, Phillips, NPA **865** (2011) 17

Universal Correlations

- EFT gives correlations between different observables
- Example: $B(E1)$ for S-to-P transition and radius of ^{11}Be*

$$B(E1) = \frac{2e^2Q_c^2}{15\pi}\left(\langle r_c^2\rangle_{^{11}\mathrm{Be}^*} - \langle r_c^2\rangle_{^{10}\mathrm{Be}}\right) x \left[\frac{1+2x}{(1+x)^2}\right]^2 + \ldots\,,$$

where $x = \sqrt{B_1/B_0}$

- Adapt strategy to experimental situation
- ^{11}Be* radius relative to ^{10}Be core from $B(E1)$

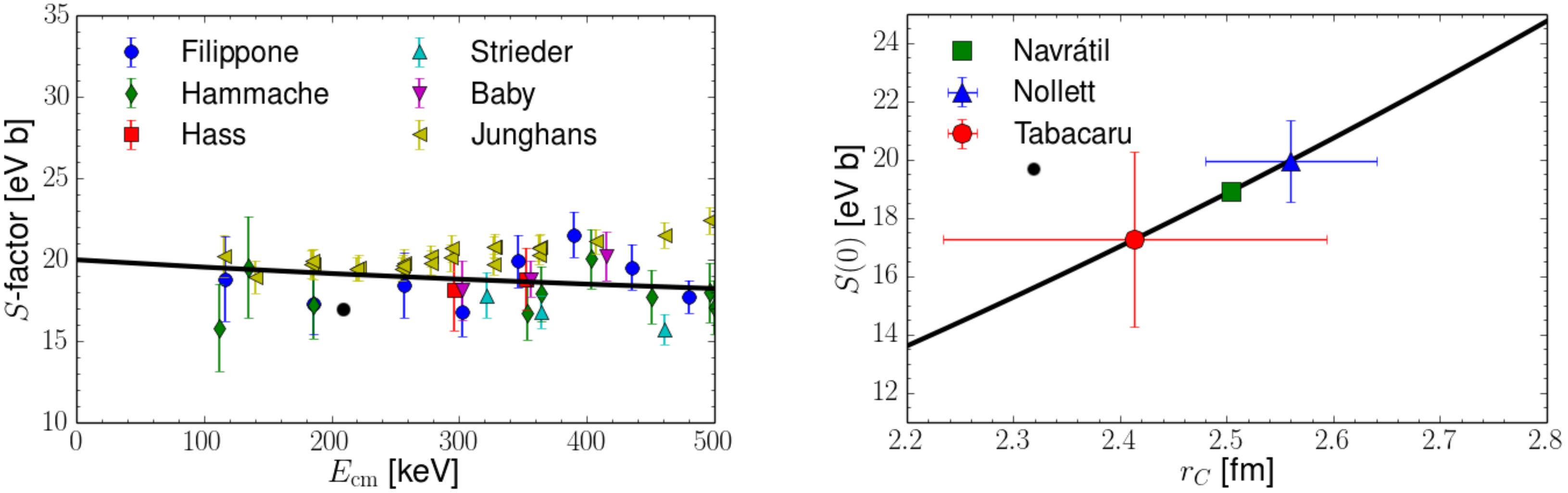
$$\langle r_c^2\rangle_{^{11}\mathrm{Be}^*} - \langle r_c^2\rangle_{^{10}\mathrm{Be}} = 0.35...0.39 \text{ fm}^2$$

Universality: can be applied to any one-neutron halo nucleus with shallow S- and/or P-Wave State

More Universal Correlations

- Radiative proton capture on ^{7}Be determines high-energy part of solar neutrino spectrum
- EFT calculation of S-factor using ab initio ANCs

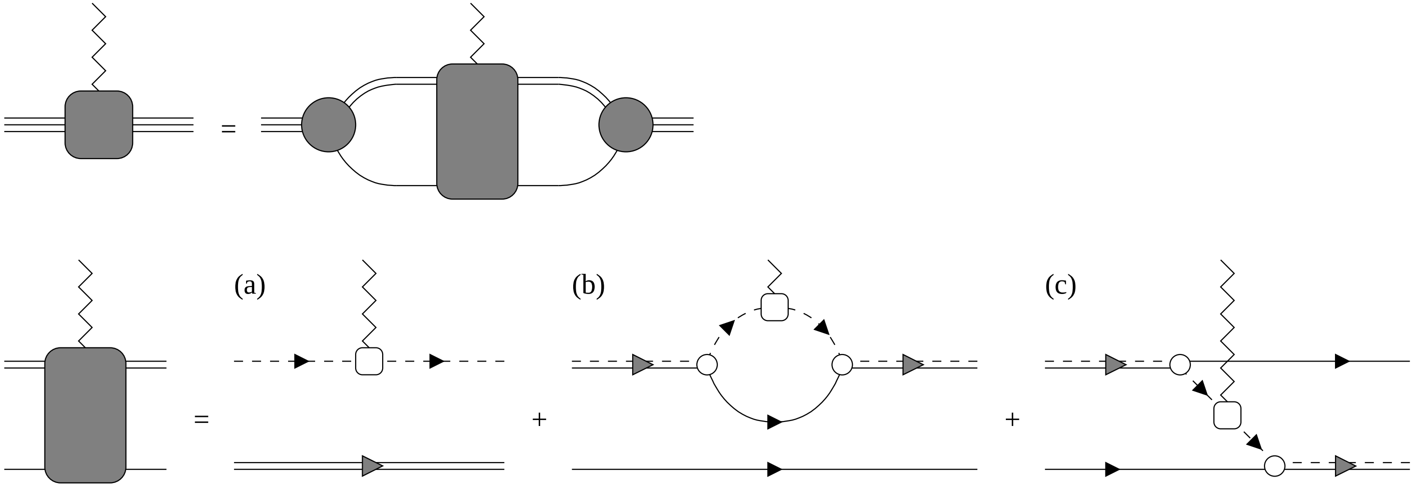
  (Zhang, Nollett, Phillips, 2014, 2015)
- S-factor for proton capture on ^{7}Be and charge radius of ^{8}B are correlated



Ryberg, Forssen, HWH, Platter, Eur. Phys. J. A **50** (2014) 170

Charge Form Factors

- Charge form factor of $2n$ halo nuclei (Hagen, HWH, Platter, EPJA **49**, 118 ('13))



$$i\bar{\Gamma}(P,K) = (-i\mathcal{Z}e)\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\bar{G}(P,p)\bar{\Gamma}^{red}(P;K,p,k)\bar{G}(K,k)$$

- Charge form factor $\mathcal{F}_{\mathsf{E}}(\mathbf{Q}^2)$ from $\bar{\Gamma}(P,K)$ in Breit frame
- Implementation using trimer auxilliary field
- Renormalization and current conservation explicitly verified

Charge Radii

c cn cnn	E_c^* [MeV] $\frac{B_{cn}}{E_c^*}$ $\frac{B_{cnn}}{E_c^*}$	B_{c-n} [MeV] $\frac{B_{cn}}{B_{c-n}}$ $\frac{B_{cnn}}{B_{c-n}}$	$\delta \left\langle r_{\mathsf{E}}^2 \right\rangle$ [fm^2]	$\delta \left\langle r_{\mathsf{E}}^2 \right\rangle_{\mathsf{exp}}$ [fm^2]
^{9}Li	2.69	4.06		
^{10}Li	-0.10^2	-0.08^2	unbound	
^{11}Li	0.37^2	0.30^2	$1.68(62)$	$1.171(120)$ [2]
^{20}C	1.59 [1]	$2.9(3)$		
^{21}C	-0.09^2	-0.07^2	unbound	
^{22}C	0.26^2	0.20^2	$1.66^{+??}_{-0.49}$	—

[1] M. Stanoiu et al., Phys. Rev. C **78**, 034315 (2008),
[2] R. Sánchez et al., Phys. Rev. Lett. **96**, 033002 (2006).

all other data from NNDC

- LO Halo EFT (no P-wave resonance in ^{9}Li-n)

Coulomb Dissociation of $2n$ Halos

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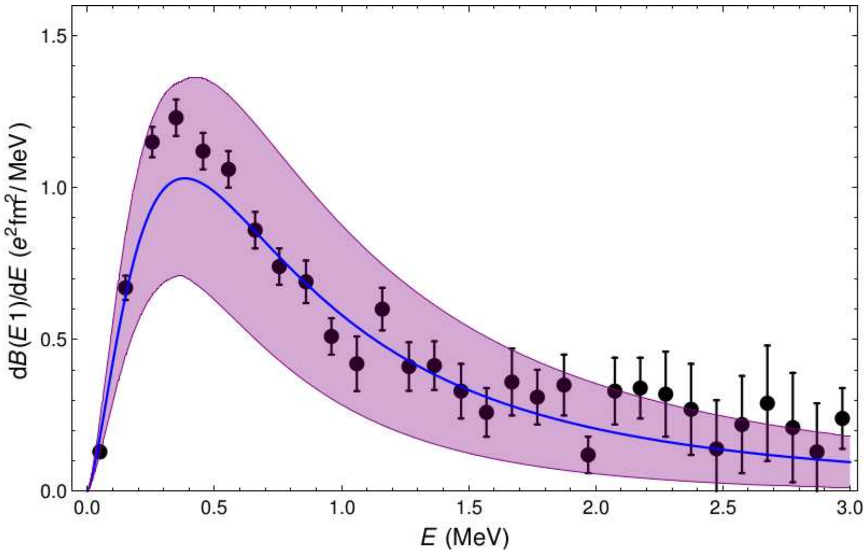
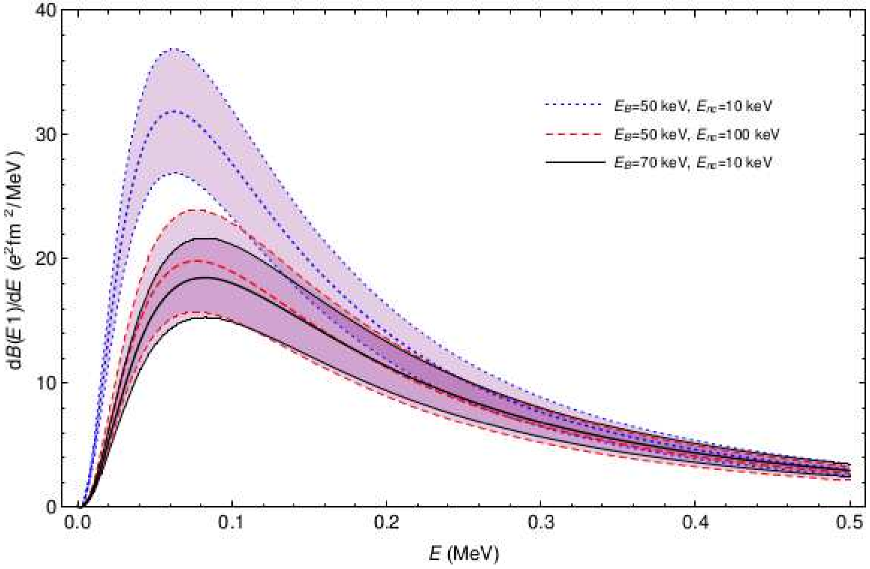
- Preliminary results for $B(E1)$ strength distribution
- LO Halo EFT with FSI included (no P-wave resonance in ^{9}Li-n)



^{11}Li



^{22}C

(Acharya, Hagen, HWH, Phillips, in preparation)

- Data for ^{11}Li: Nakamura et al., Phys. Rev. Lett. **96**, 252502 (2006)

Summary and Outlook

- Cluster EFT for halo nuclei

  $\Longrightarrow$ large scattering length/shallow states
  - Controlled, systematic approach
  - Straightforward inclusion of external currents
- Universal theory has many applications
  - ultracold atoms
  - light nuclei, halo nuclei $\Longrightarrow$ Halo EFT
  - hadronic molecules
- Universality predicts correlations between observables

  $\Longrightarrow$ input from theory or experiment

  $\Longrightarrow$ combine with ab initio approaches
- Reactions with photons
- Proton halos and Coulomb interaction
- Manifestation of Efimov physics in halo nuclei?