エクスクルーシヴ Drell-Yan 過程 $\pi^- p \to \ell^+ \ell^- n$ のQCDメカニズム

田中和廣　（順天堂大/KEK）

High momentum beam line at J-PARC

- Primary beam (proton)

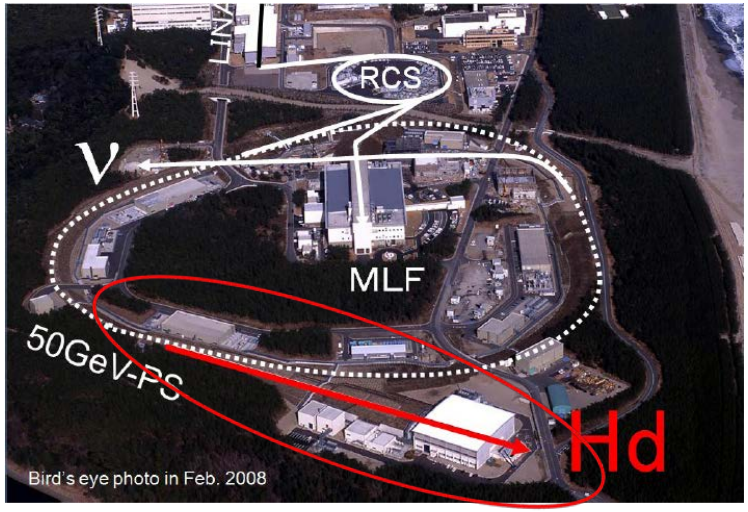
$E = 30\text{GeV}\ (\rightarrow 50\text{GeV?})$

$L = 10^{35}\text{cm}^{-2}\text{s}^{-1}$

$\leftrightarrow$ PANDA (anti-proton)

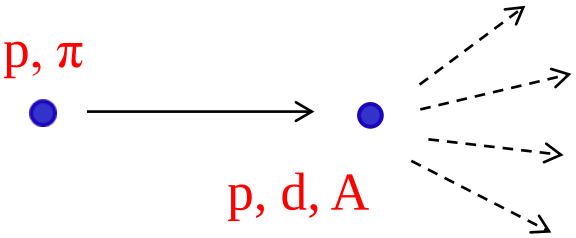
$E \leq 15\text{GeV}\ ,\ L = 10^{32}\text{cm}^{-2}\text{s}^{-1}$

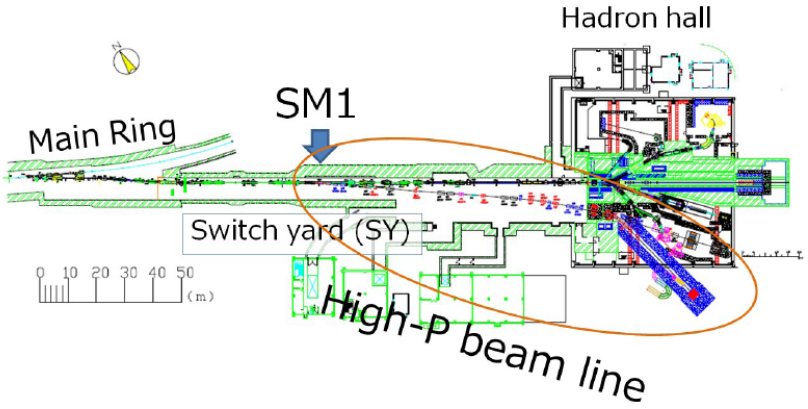
Hadron Facility at J-PARC



- Secondary beam (pion)

E = 15-20GeV



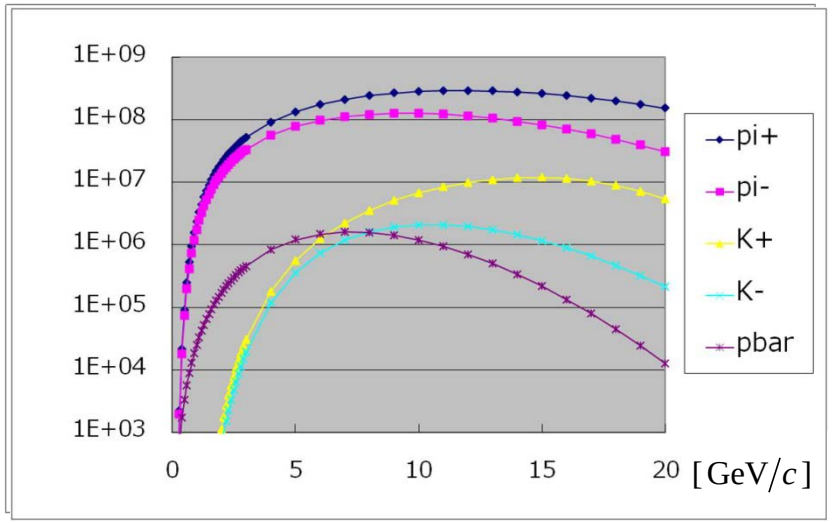


High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

Unseparated Secondary Beam Intensity

beam loss limit @ SM1:15kW
(limited by the thickness of the tunnel wall)



0° extraction angle

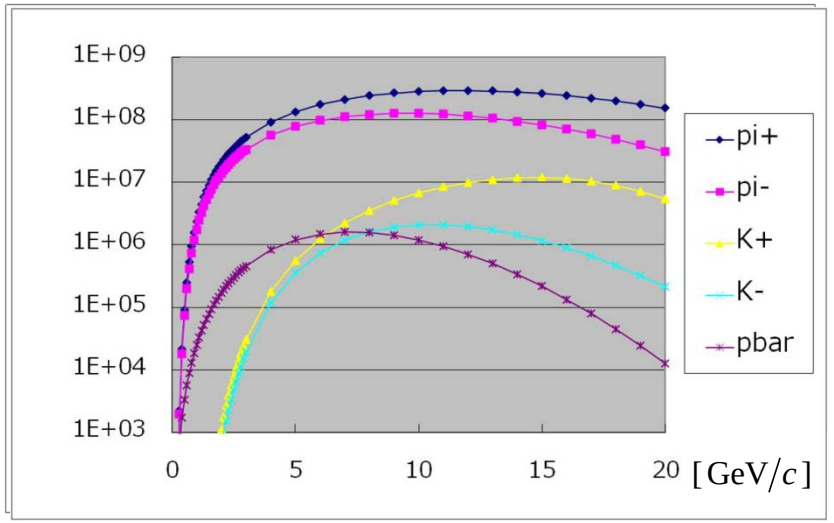
high intensity

High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

Unseparated Secondary Beam Intensity

beam loss limit @ SM1:15kW
(limited by the thickness of the tunnel wall)



0° extraction angle

high intensity

not too high energy $d\sigma \sim 1/s^a$

best suited to study meson-induced hard exclusive processes

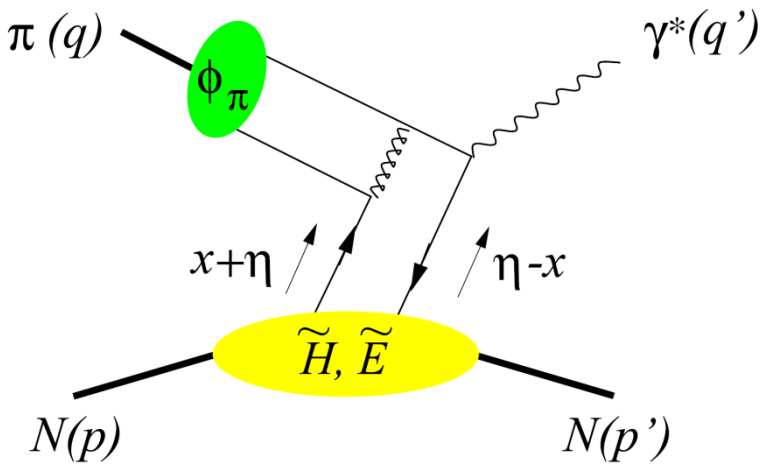
Exclusive lepton pair production in πN scattering

$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$

Berger, Diehl, Pire, PLB523(2001)265

"exclusive limit of DY"

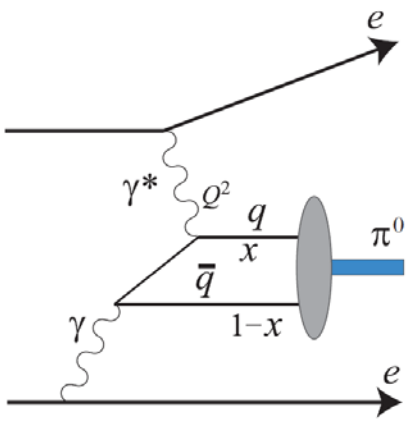
small $t = (q - q')^2$



Exclusive lepton pair production in πN scattering

$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$

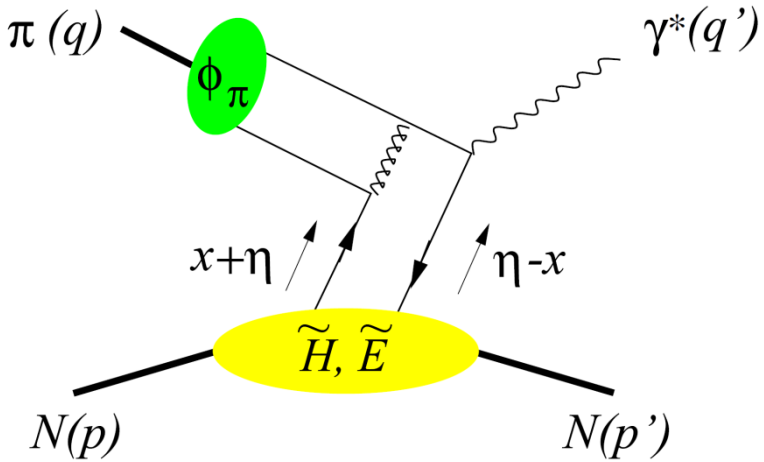
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

"exclusive limit of DY"

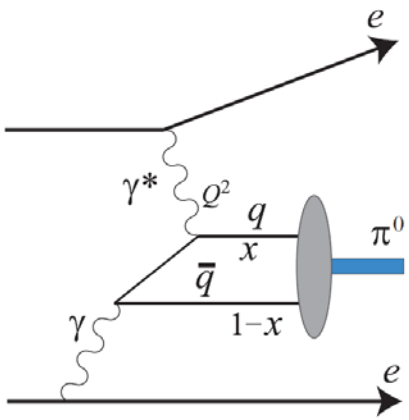
small $t = (q - q')^2$



Exclusive lepton pair production in πN scattering

$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$

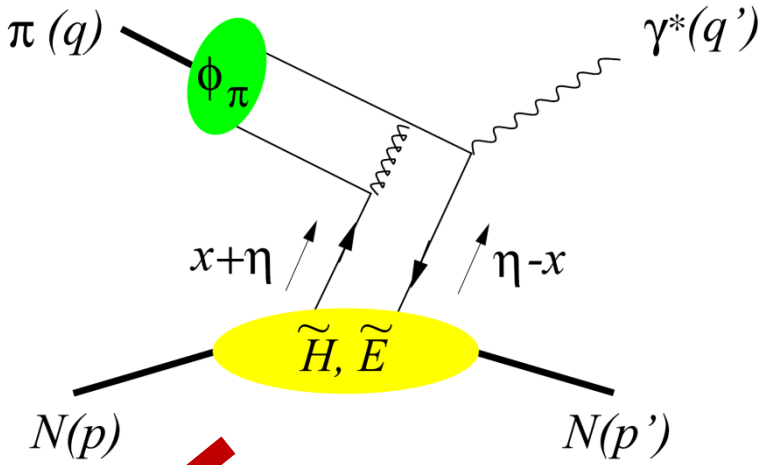
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

"exclusive limit of DY"

small $t = (q - q')^2$



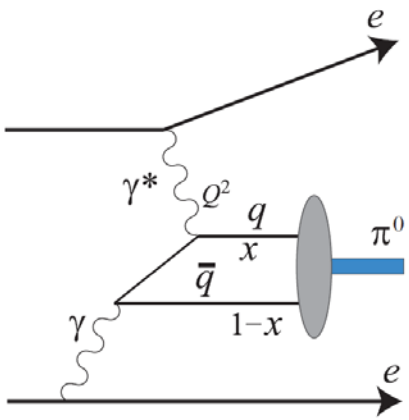
$t \to 0$

$\Delta q(x)$

Exclusive lepton pair production in πN scattering

$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$

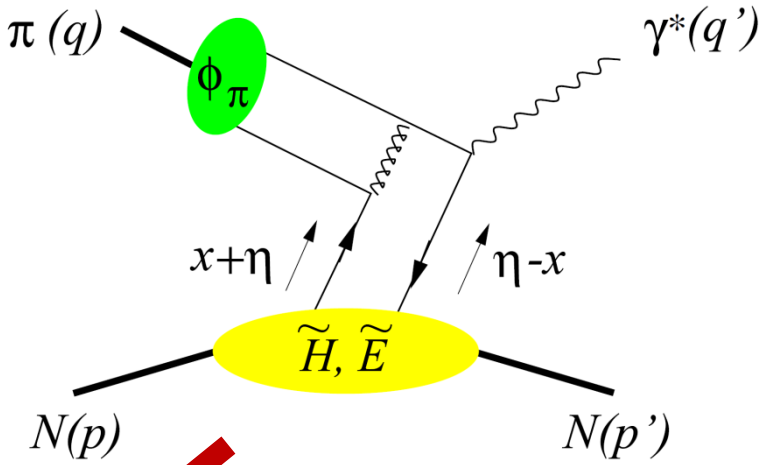
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

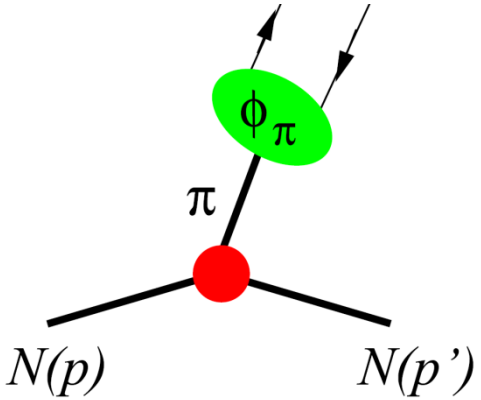
"exclusive limit of DY"

small $t = (q - q')^2$



$t \to 0$

$\Delta q(x)$

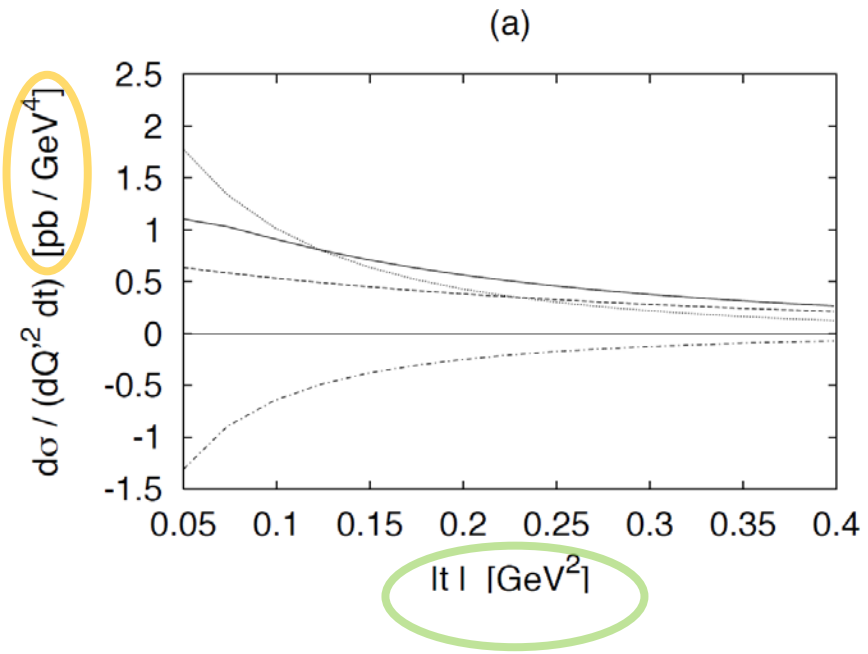


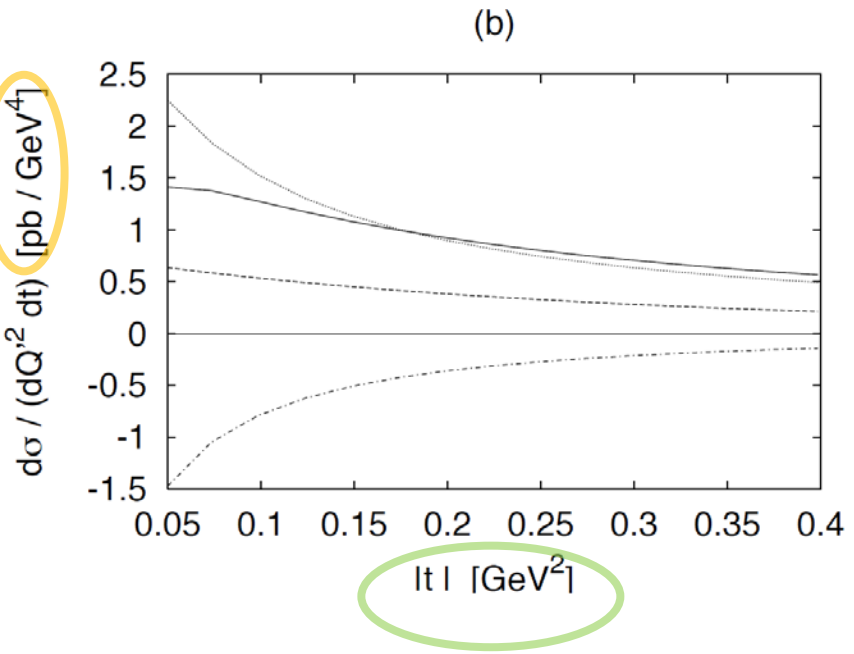
LO Estimates

Bjorken variable $\tau = \frac{Q'^2}{s - M^2}$

Berger, Diehl, Pire, PLB523(2001)265

$Q'^2 = 5 GeV^2 \qquad \tau = 0.2$





(dashed) $= |\tilde{\mathcal{H}}|^2$; **(dash-dotted)** $= \mathrm{Re}(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; **(dotted)** $= |\tilde{\mathcal{E}}|^2$

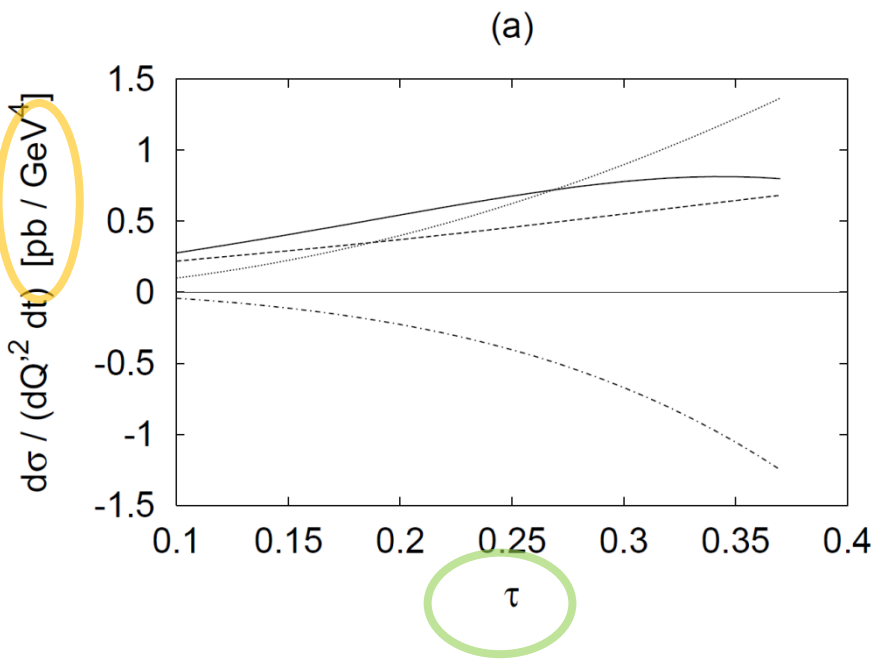
$$\frac{d\sigma}{dQ'^2\, dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{\mathrm{em}}^2}{27}\frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2) |\widetilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \mathrm{Re}\left(\widetilde{\mathcal{H}}^{du*} \widetilde{\mathcal{E}}^{du}\right) - \eta^2 \frac{t}{4M^2} |\widetilde{\mathcal{E}}^{du}|^2 \right]$$

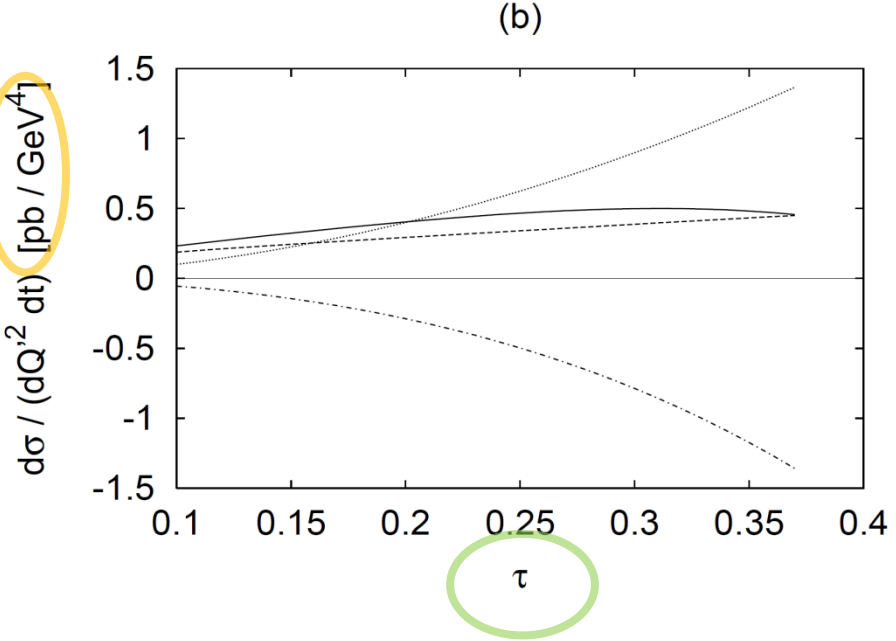
LO Estimates

Bjorken variable $\tau = \frac{Q'^2}{s - M^2}$

Berger, Diehl, Pire, PLB523(2001)265

$Q'^2 = 5 GeV^2 \qquad |t| = 0.2$

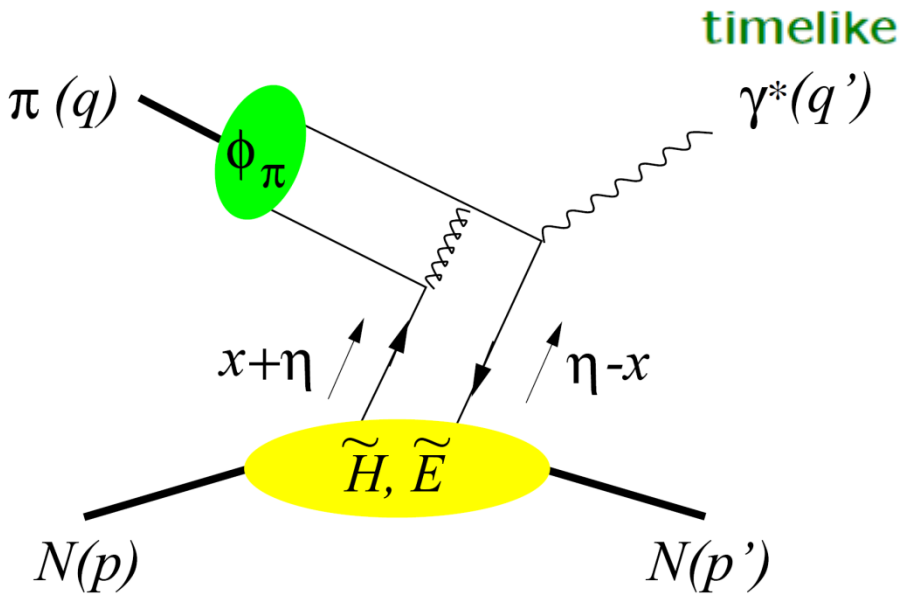


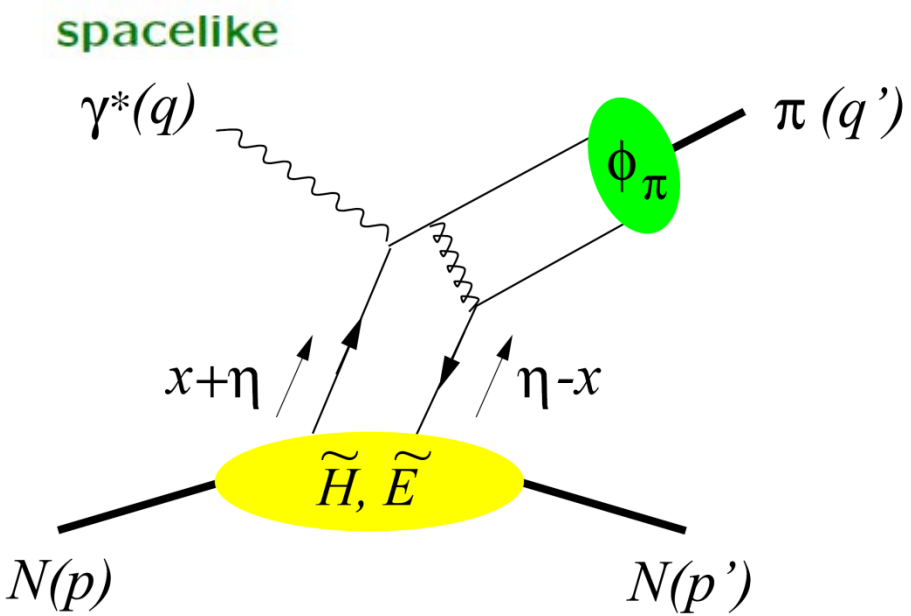


(dashed) $= |\tilde{\mathcal{H}}|^2$; **(dash-dotted)** $= \mathrm{Re}(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; **(dotted)** $= |\tilde{\mathcal{E}}|^2$

$$\frac{d\sigma}{dQ'^2\, dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{em}^2}{27}\frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2) |\widetilde{H}^{du}|^2 - 2\eta^2 \mathrm{Re}\left(\widetilde{H}^{du*} \widetilde{E}^{du}\right) - \eta^2 \frac{t}{4M^2} |\widetilde{E}^{du}|^2 \right]$$

Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions



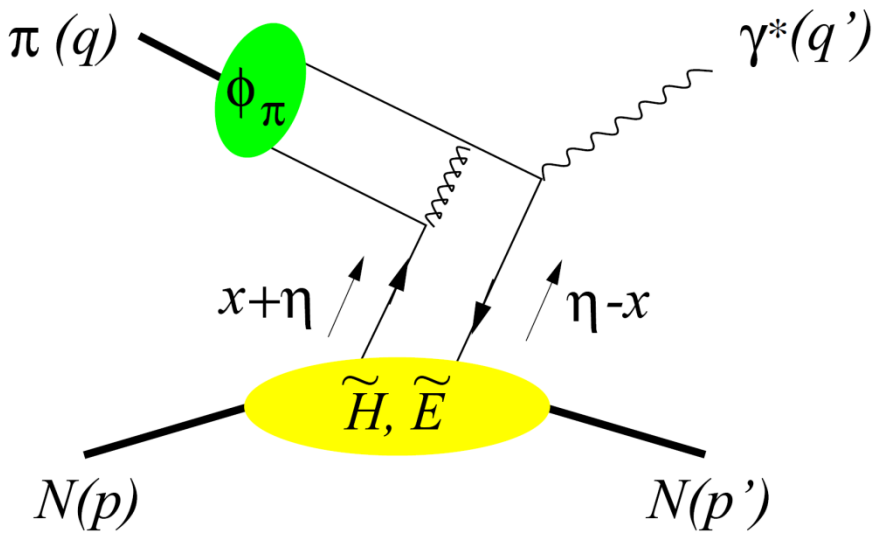


exDY@J-PARC

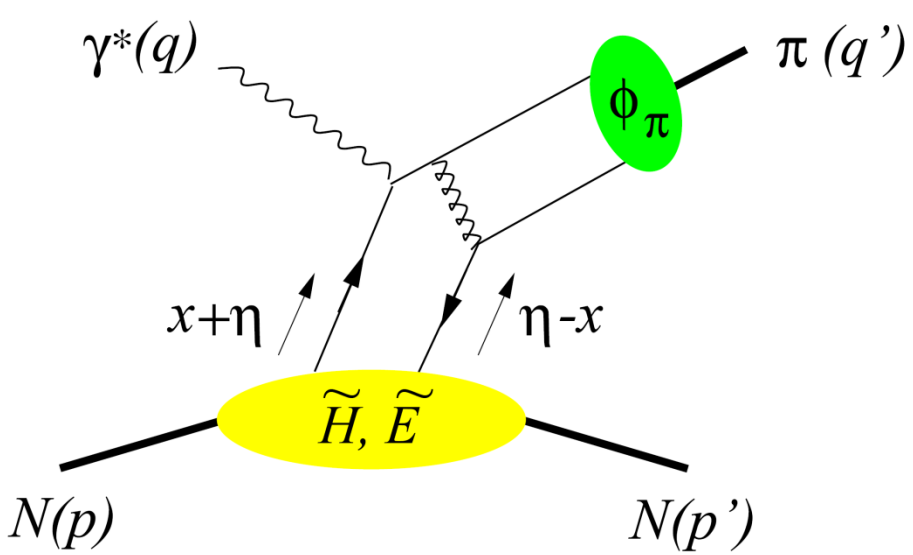
DVMP@JLab

Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions

timelike

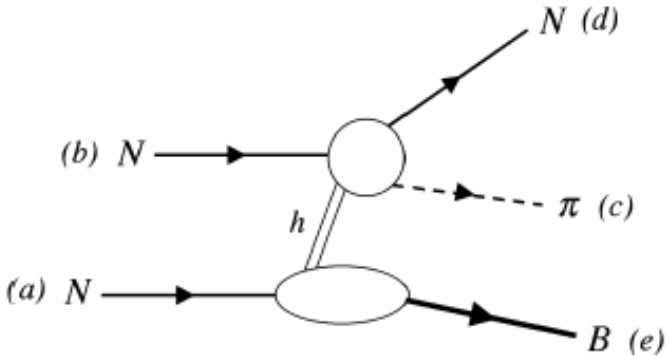


spacelike

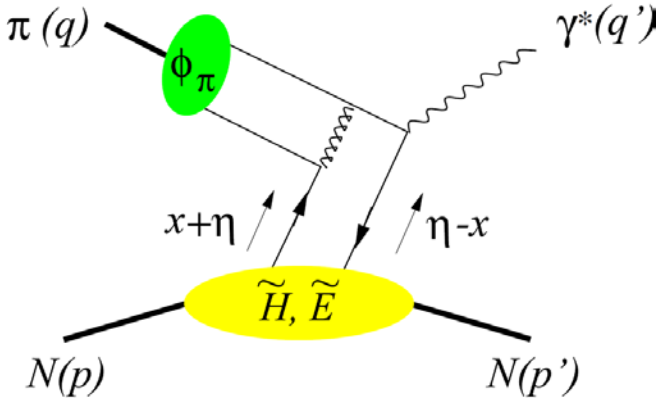


exDY@J-PARC

DVMP@JLab



Kumano,Sudoh,Strikman, PRD80(2000)074003.



Bjorken variable: $\tau = \dfrac{Q'^2}{2\,p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

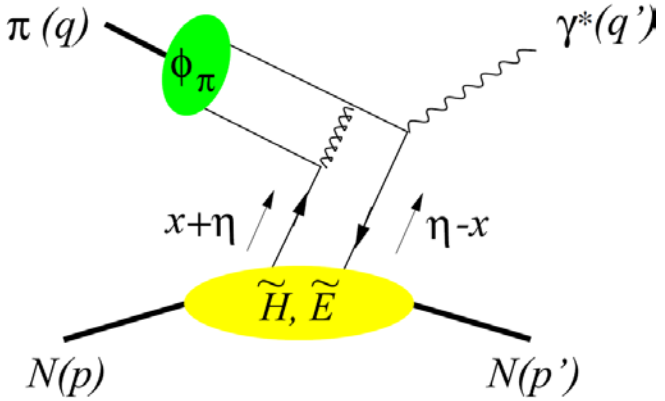
Berger, Diehl, Pire, PLB523(2001)

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$M^{0\lambda',\lambda}(\pi^- p\to\gamma^* n) = -ie\,\tfrac{4\pi}{3}\,\tfrac{f_\pi}{Q'}\,\tfrac{1}{(p+p')^+}\,\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\tfrac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1}dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde H^d(x,\eta,t) - \tilde H^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar P^+z^-}\langle p'|\bar\psi(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar P^+}\left[\tilde H^q(x,\eta,t)\bar u(p')\gamma^+\gamma_5 u(p) + \tilde E^q(x,\eta,t)\bar u(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$



Bjorken variable: $\tau = \dfrac{Q'^2}{2p\cdot q}$

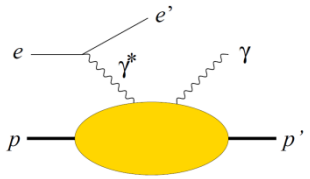
Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

$$\frac{d\sigma}{dQ'^2\, dt\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2 \sin^2\theta$$

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

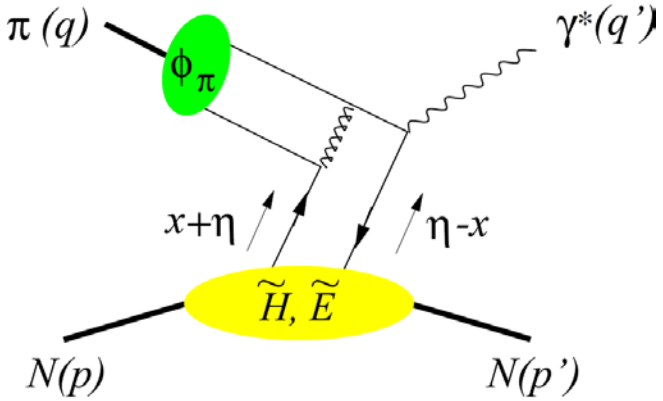
$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1} dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$



$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

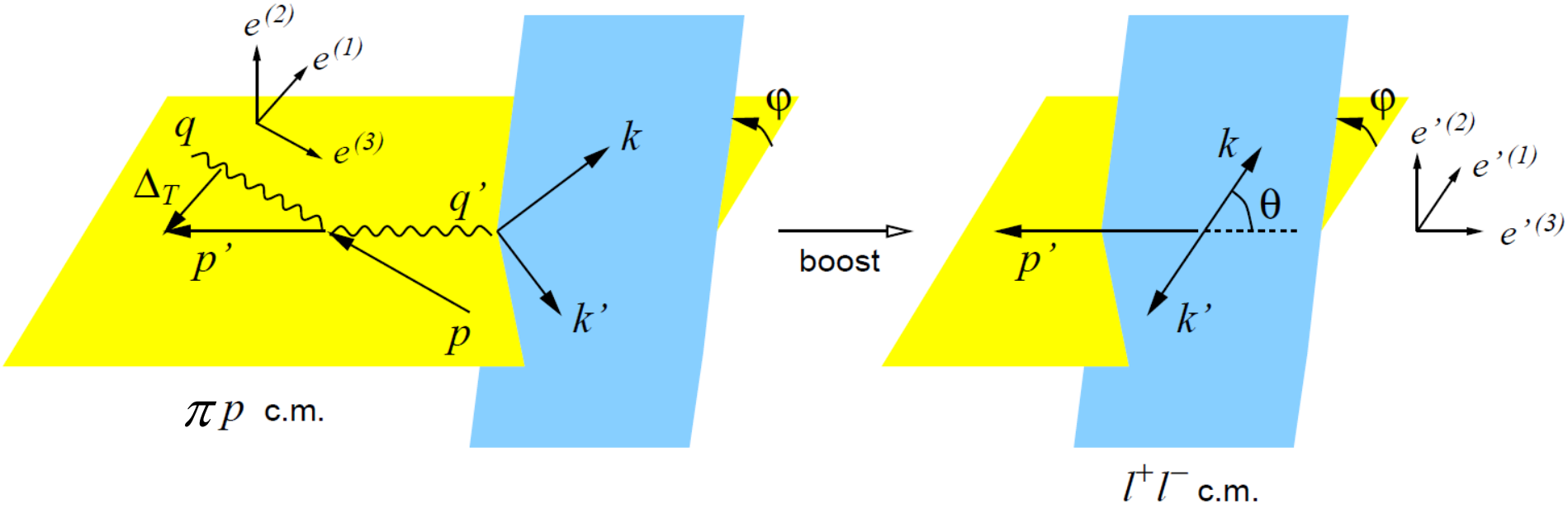
$$J_q = \frac{1}{2}\int_{-1}^{1} dx x\left(H^q(x,\eta,0) + E^q(x,\eta,0)\right)$$

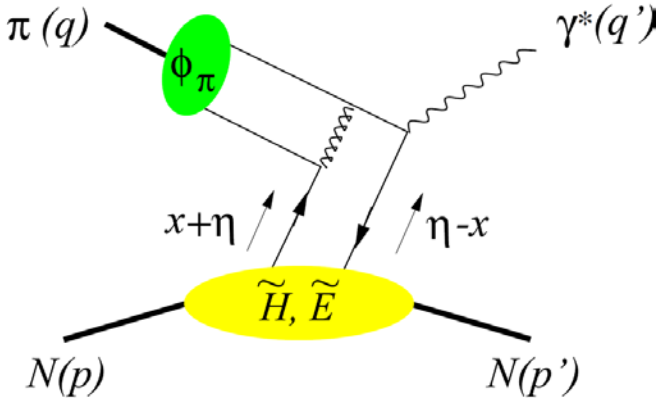


Bjorken variable: $\tau = \dfrac{Q'^2}{2\,p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\,\frac{\tau^2}{Q'^6}\,\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$





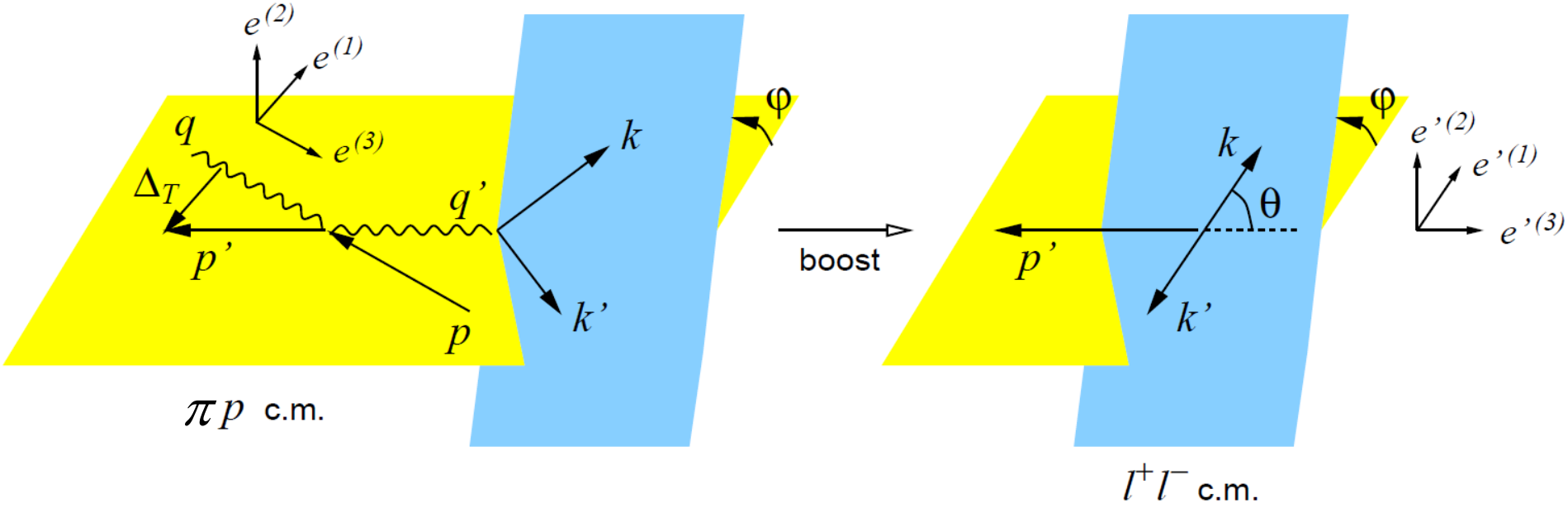
Bjorken variable: $\tau = \dfrac{Q'^2}{2\,p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

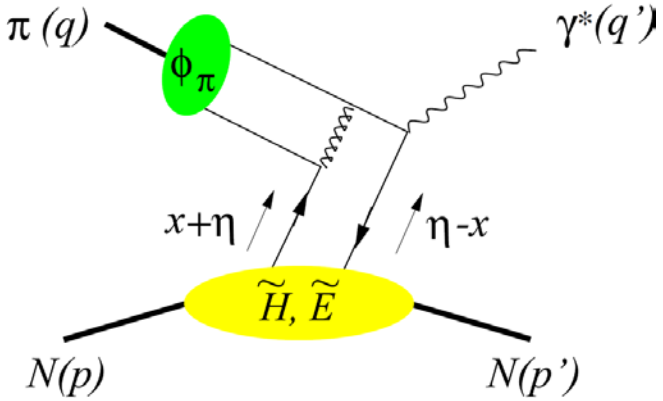
$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\,\frac{\tau^2}{Q'^6}\,\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\,\sin^2\theta$$

$$\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$$



πp c.m.

$l^+ l^-$ c.m.



Bjorken variable: $\tau = \dfrac{Q'^2}{2p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

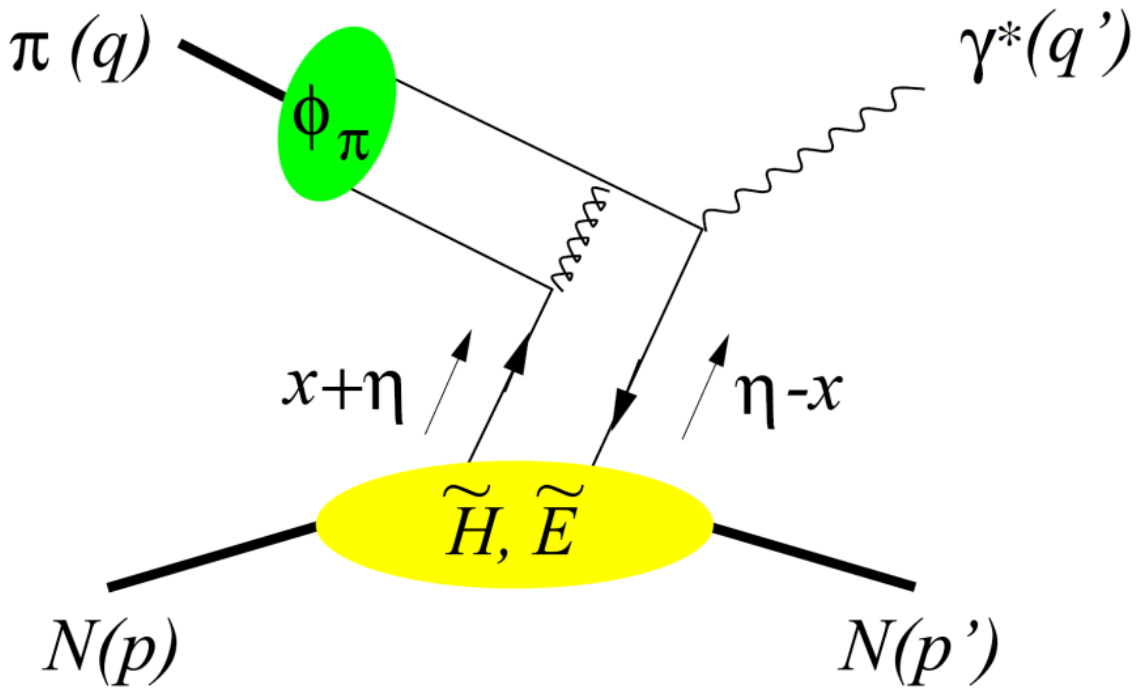
$$\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$$

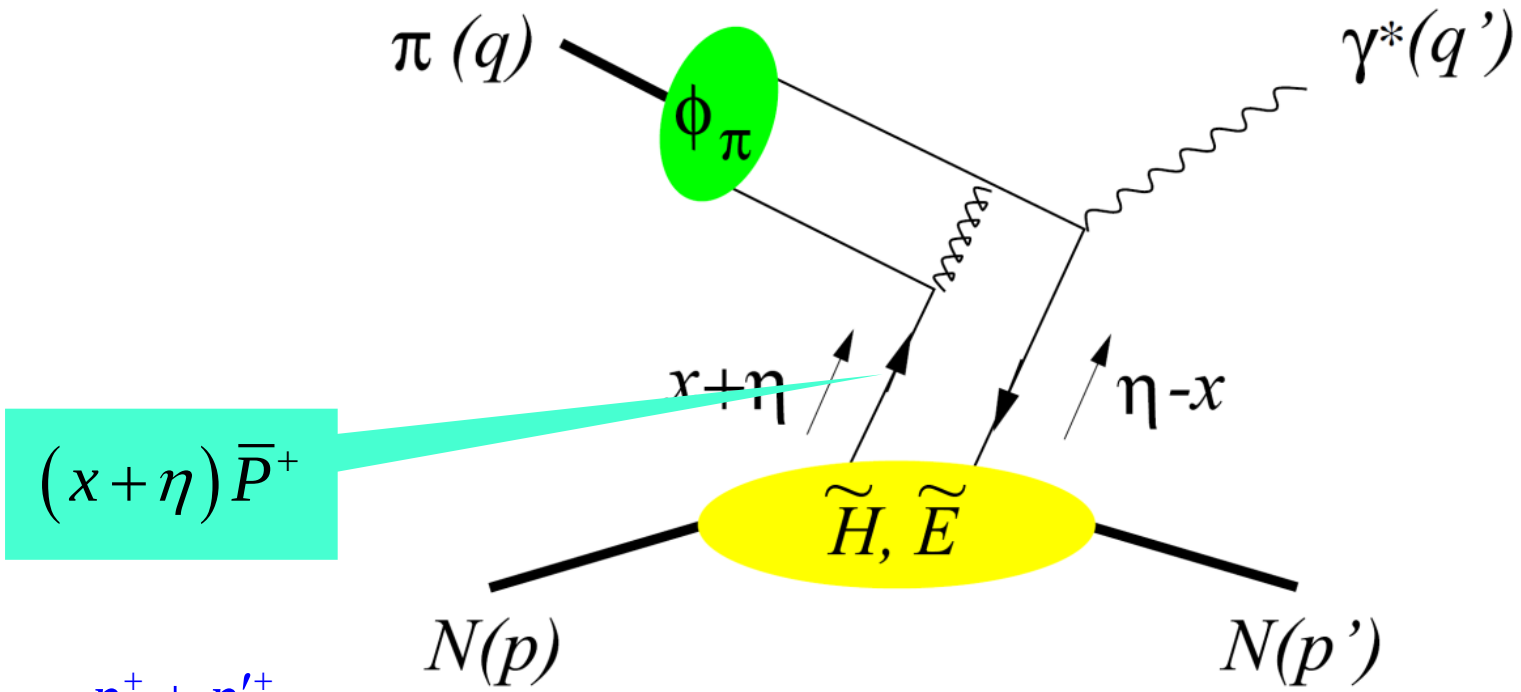
$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$M^{0\lambda',\lambda}(\pi^- p\to\gamma^* n) = -ie\,\tfrac{4\pi}{3}\,\tfrac{f_\pi}{Q'}\,\tfrac{1}{(p+p')^+}\,\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\tfrac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

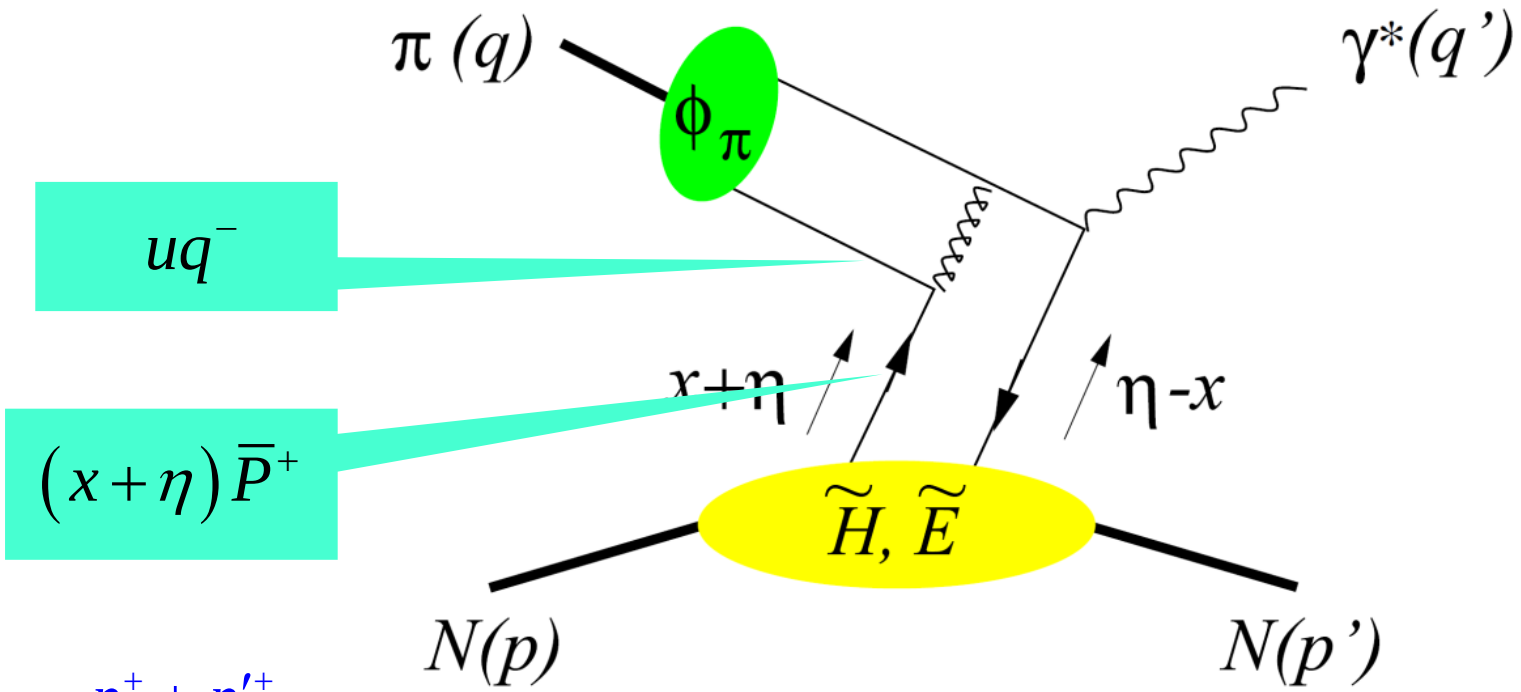
$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1}dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde H^d(x,\eta,t) - \tilde H^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar P^+z^-}\langle p'|\bar\psi(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar P^+}\left[\tilde H^q(x,\eta,t)\bar u(p')\gamma^+\gamma_5 u(p) + \tilde E^q(x,\eta,t)\bar u(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

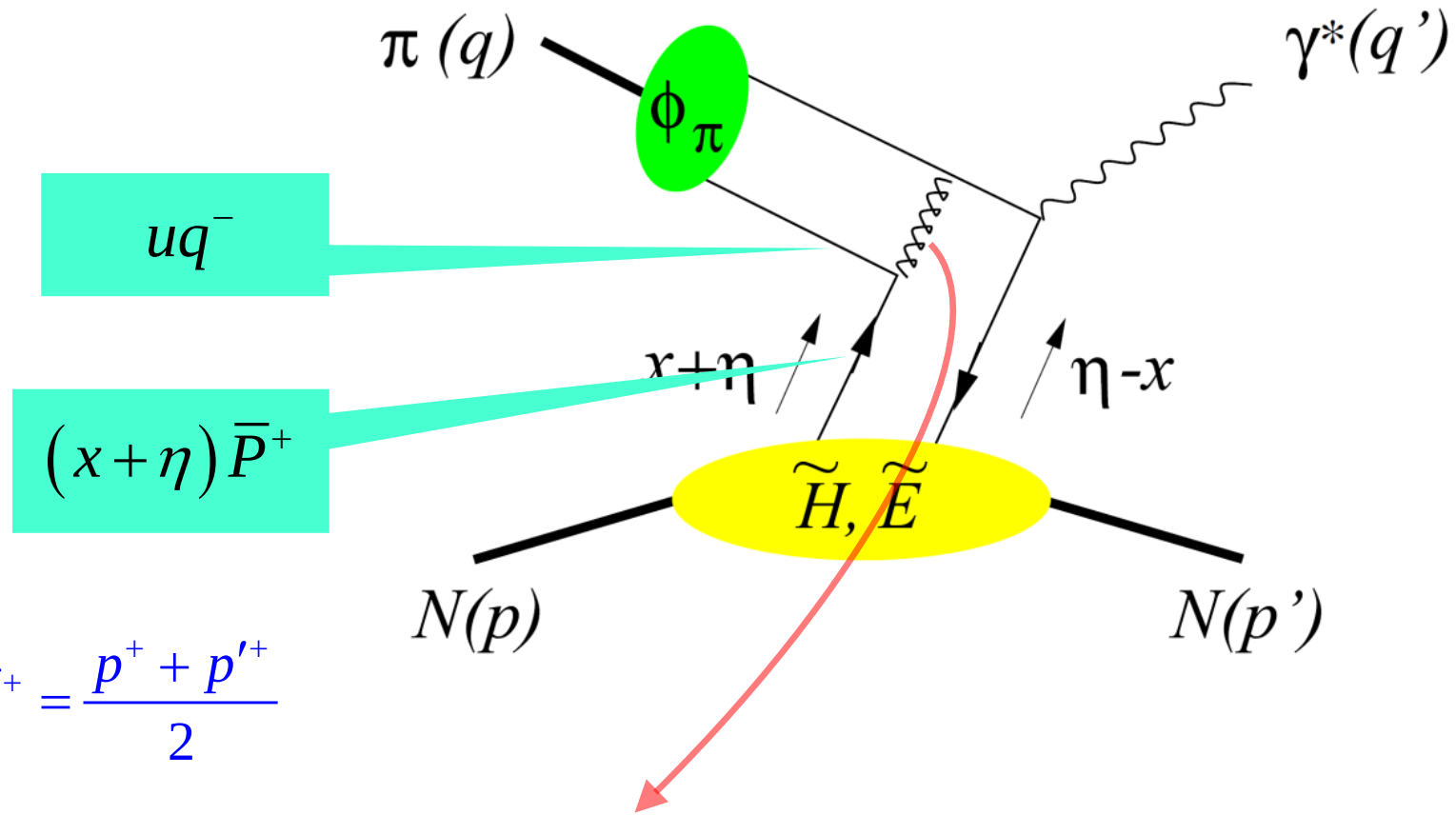

π (q)
γ*(q')
ϕπ
x+η
η-x
H̃, Ẽ
N(p)
N(p')



$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

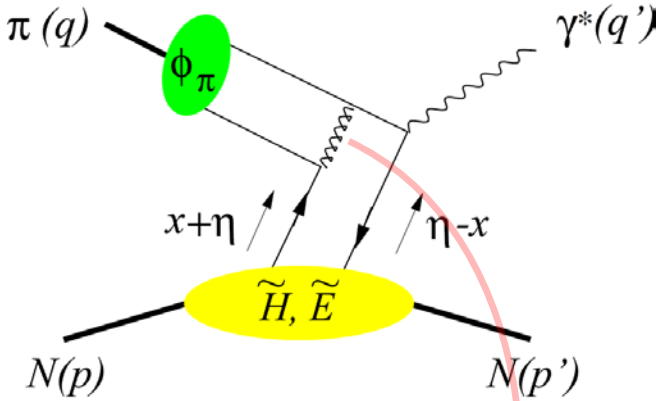


$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$



$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^2} \simeq \frac{1}{u(x+\eta)s}$$



Bjorken variable: $\tau = \dfrac{Q'^2}{2\,p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

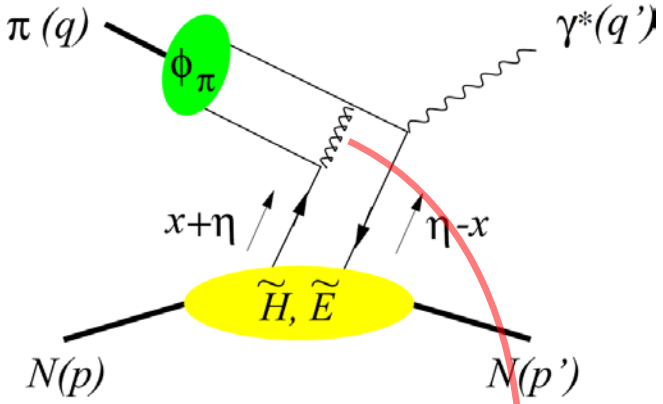
long. photon $\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$M^{0\lambda',\lambda}(\pi^- p\to\gamma^* n) = -ie\,\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\,\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde H^d(x,\eta,t) - \tilde H^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar P^+z^-}\langle p'|\bar\psi(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar P^+}\left[\tilde H^q(x,\eta,t)\bar u(p')\gamma^+\gamma_5 u(p) + \tilde E^q(x,\eta,t)\bar u(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$



Bjorken variable: $\tau = \dfrac{Q'^2}{2p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2 \sin^2\theta$$

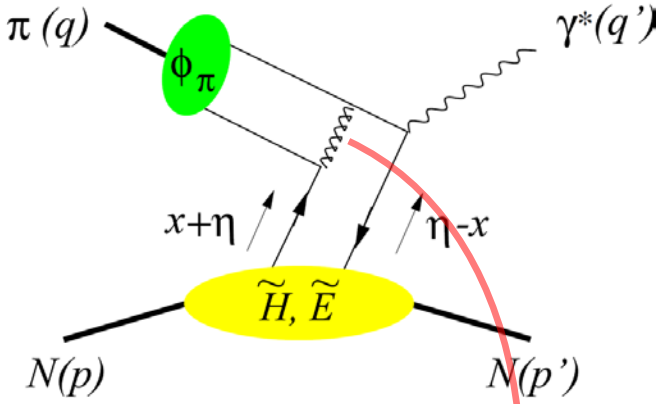
$$\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$$

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

$$\phi_\pi(u) \sim u(1-u)$$



Bjorken variable: $\tau = \dfrac{Q'^2}{2p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2 \sin^2\theta$$

$$\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$$

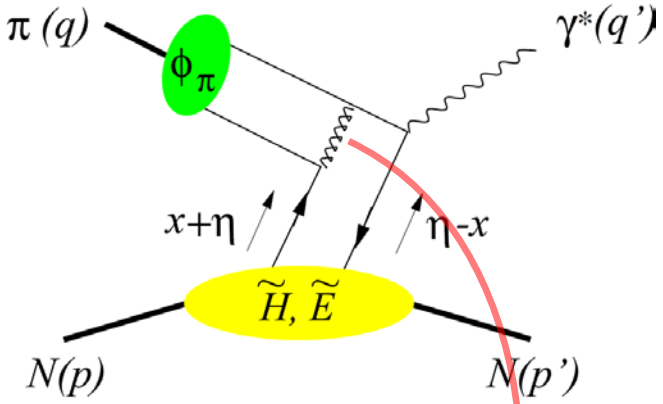
$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

$$\phi_\pi(u) \sim u(1-u)$$

$$M^{\pm1,\lambda;\lambda'}(\pi^- p \to \gamma^* n) \sim \frac{\alpha_s}{Q'^2}\int_0^1 du\,\frac{\phi_p(u)}{u(1-u)} \otimes \frac{1}{(\eta \pm x + i\varepsilon)^2} \otimes \left\{H_T^q(x,\eta,t), \tilde{H}_T^q(x,\eta,t), E_T^q(x,\eta,t), \tilde{E}_T^q(x,\eta,t)\right\}$$



Bjorken variable: $\tau = \dfrac{Q'^2}{2p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

long. photon ↘

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2 \sin^2\theta$$

$$\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$$

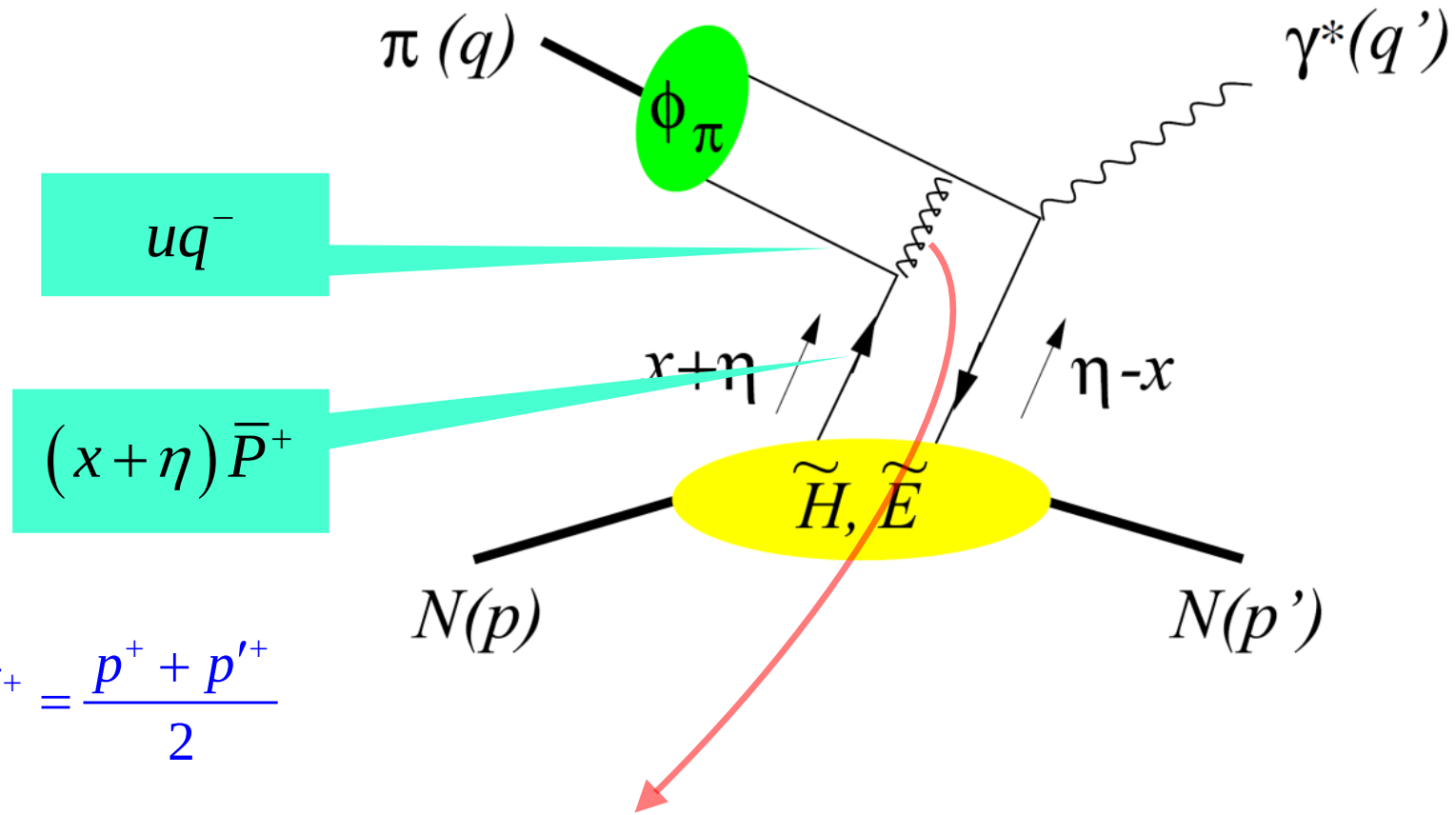
$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

$$\phi_\pi(u) \sim u(1-u) \qquad \phi_p(u) \sim 1$$

$$M^{\pm1,\lambda;\lambda'}(\pi^- p \to \gamma^* n) \sim \frac{\alpha_s}{Q'^2}\int_0^1 du\,\frac{\phi_p(u)}{u(1-u)} \otimes \frac{1}{(\eta\pm x+i\varepsilon)^2} \otimes \left\{H_T^q(x,\eta,t), \tilde{H}_T^q(x,\eta,t), E_T^q(x,\eta,t), \tilde{E}_T^q(x,\eta,t)\right\}$$



$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

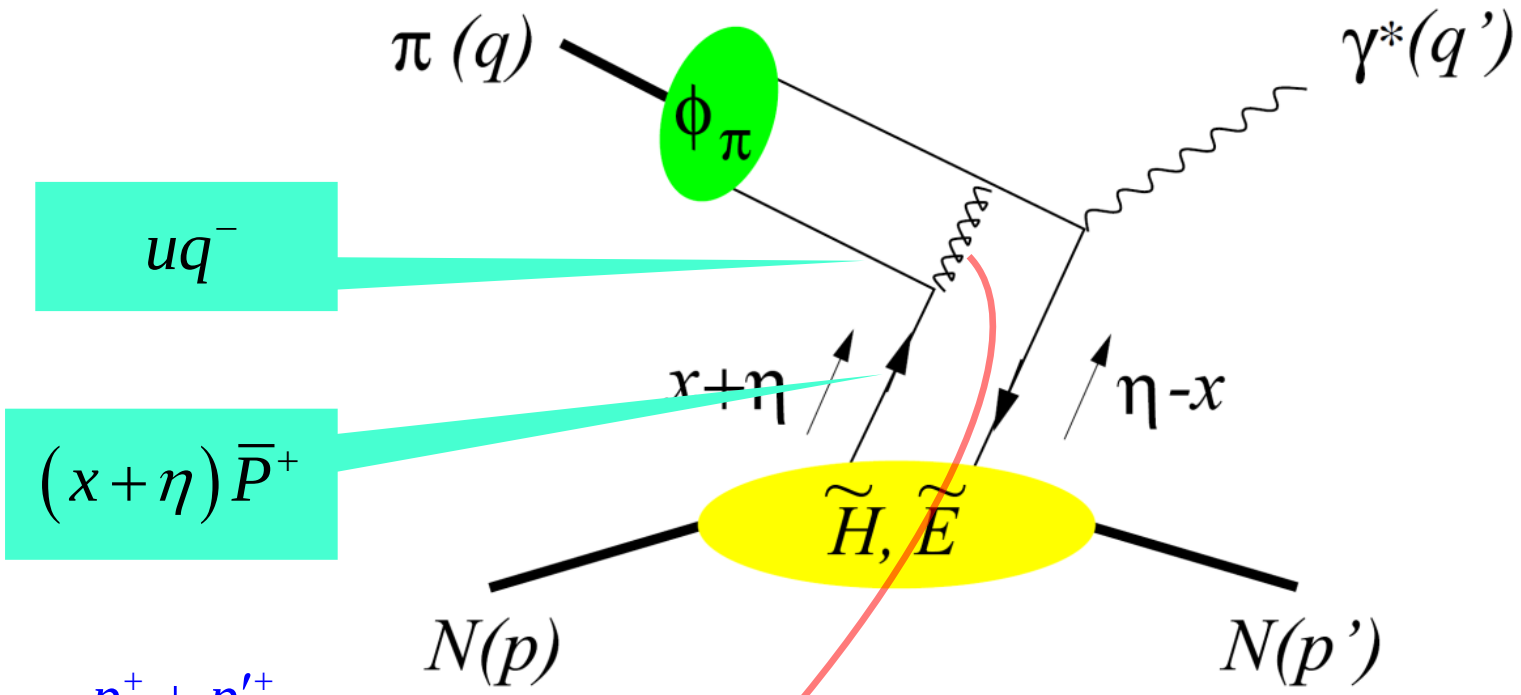
$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^2} \simeq \frac{1}{u(x+\eta)s}$$

Collinear factorization does not work at twist-3:

- quark $k_\perp$ ("k_T-factorization") Goloskokov, Kroll

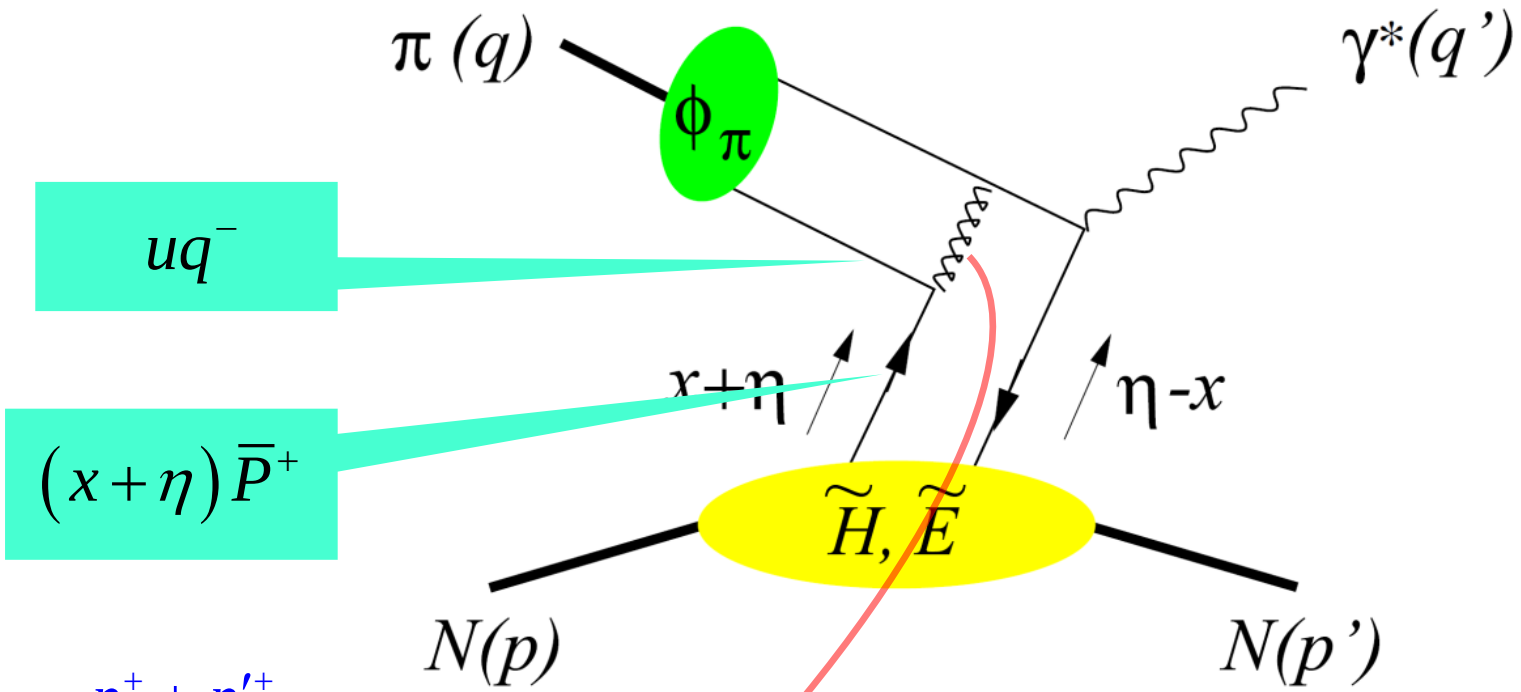
  with Sudakov resummation

  Li, Sterman



$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^2} \simeq \frac{1}{u(x+\eta)s}$$



$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^2 - \boldsymbol{k}_\perp^2} \simeq \frac{1}{u(x+\eta)s - \boldsymbol{k}_\perp^2}$$

Collinear factorization does not work at twist-3:

- quark $k_\perp$ ("k_T-factorization") Goloskokov, Kroll

  with Sudakov resummation

  Li, Sterman

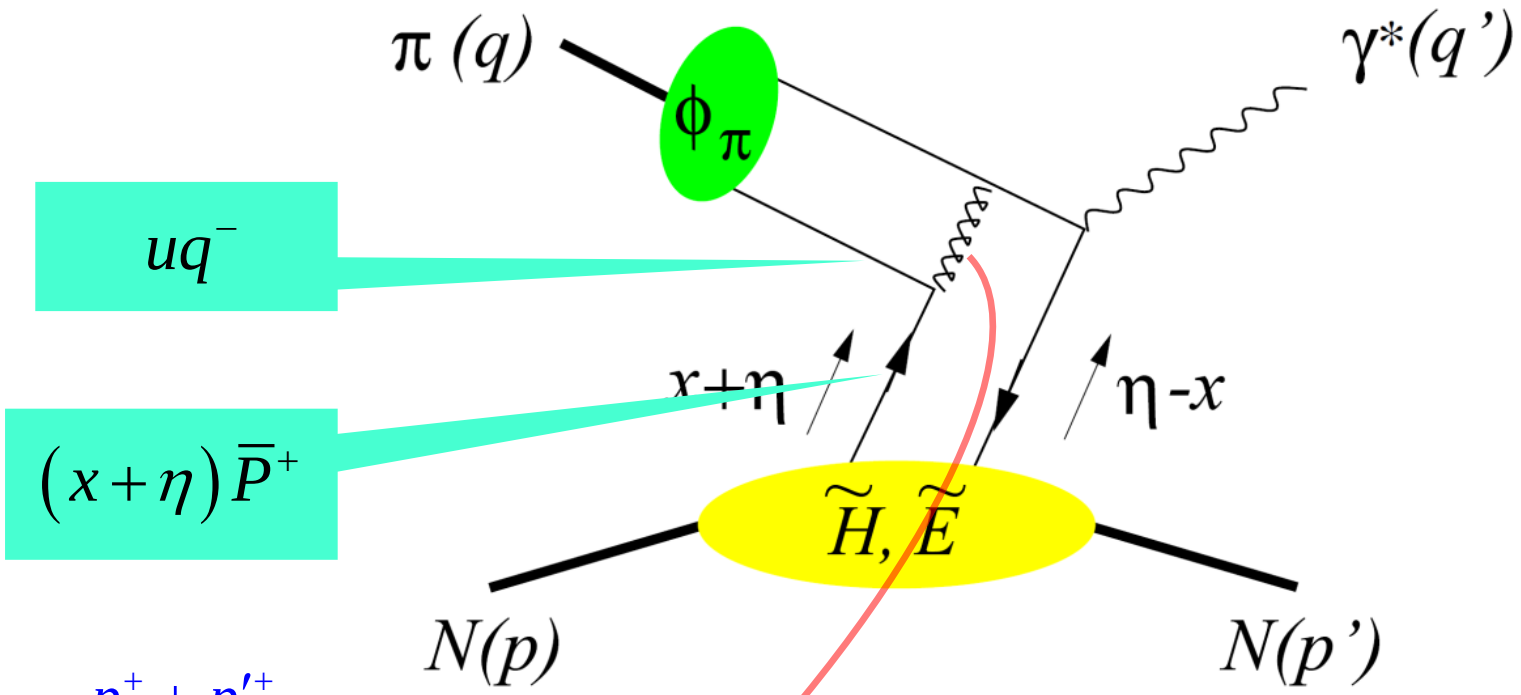
Collinear factorization does not work at twist-3:

- quark $k_\perp$ ("k_T-factorization") Goloskokov, Kroll

  with Sudakov resummation

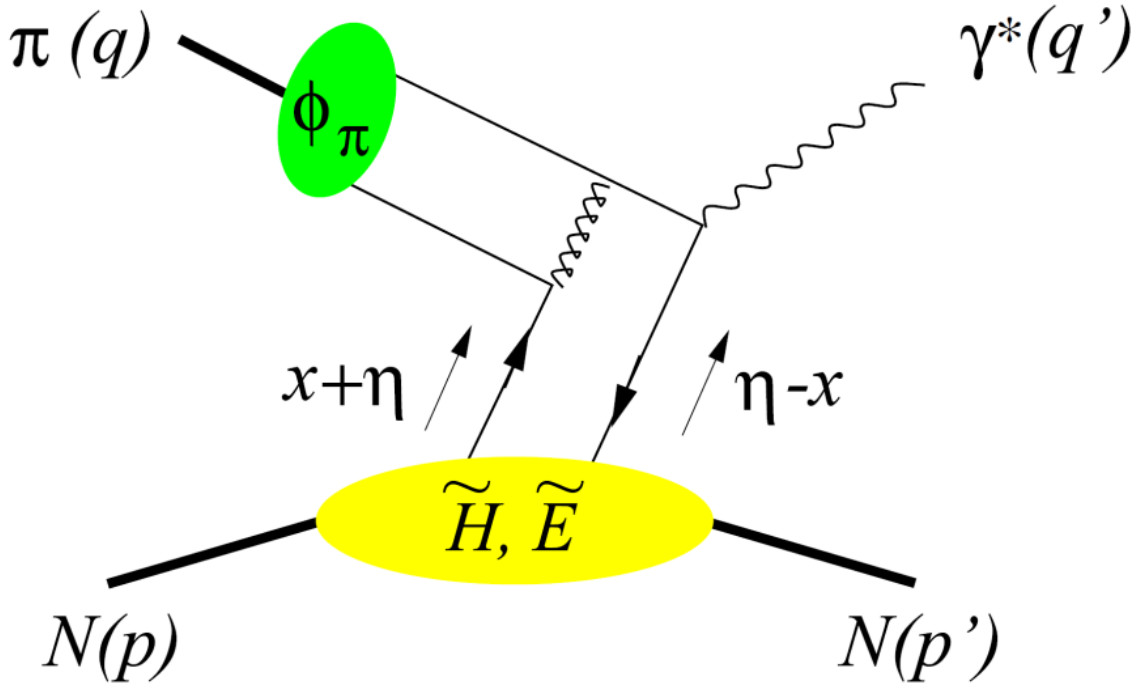
  Li, Sterman

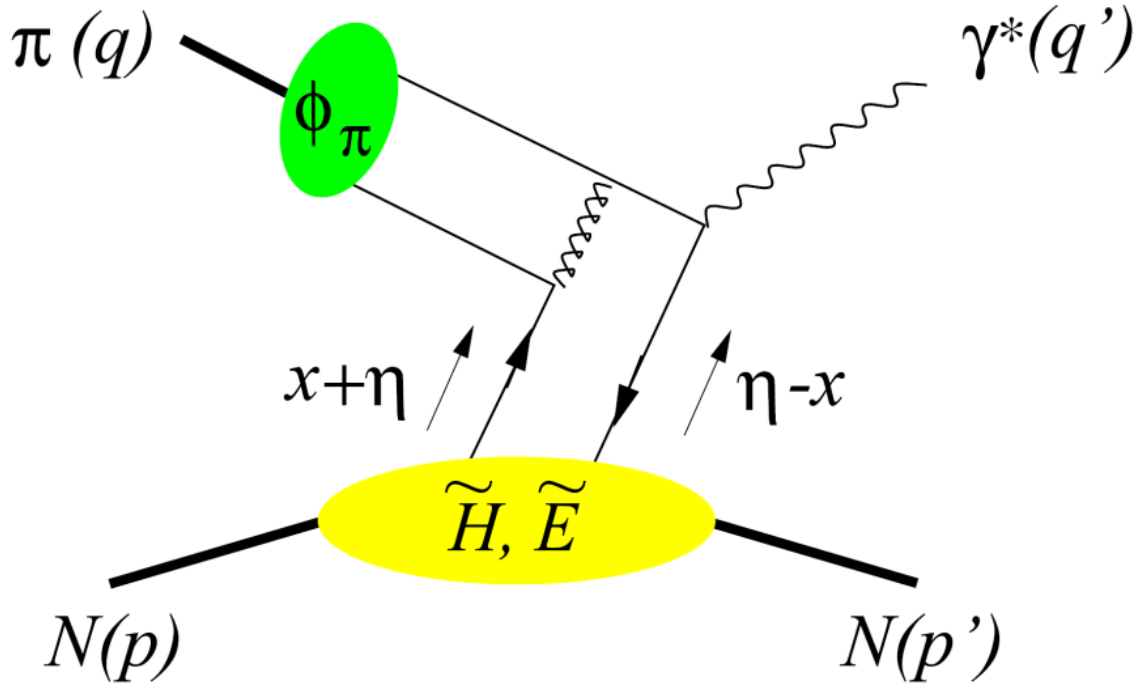
- include "soft" propagator in long-distance part

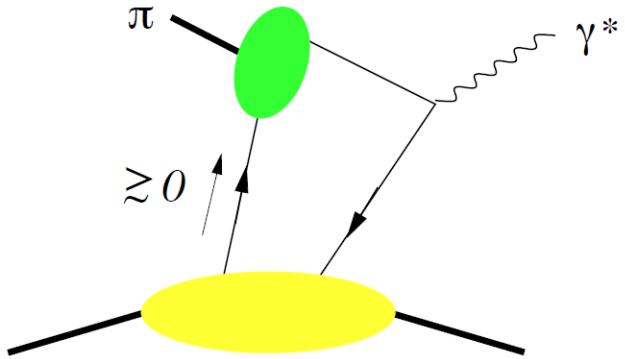


$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

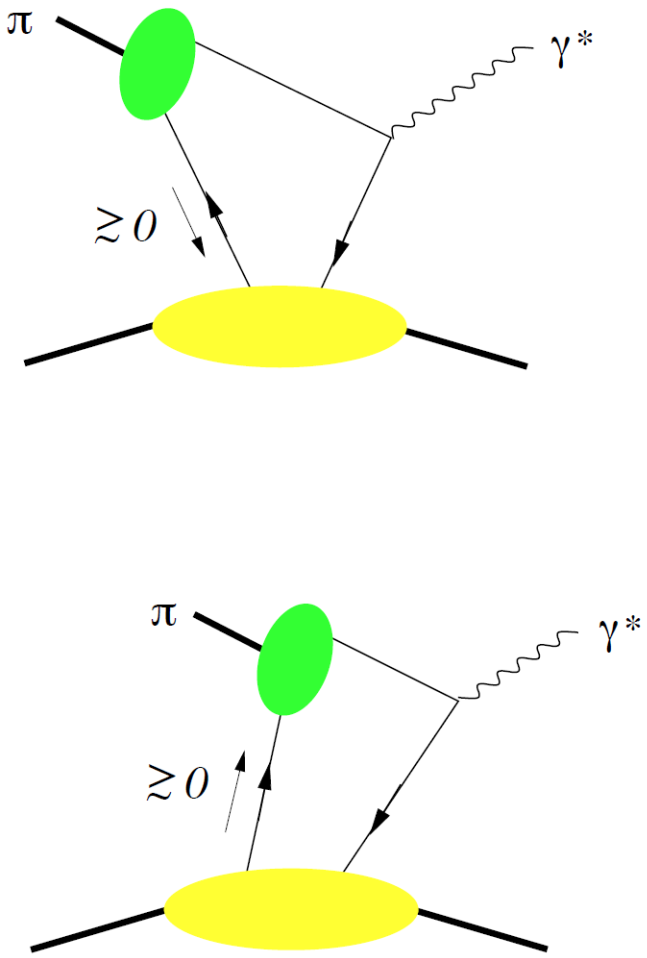
$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^2} \simeq \frac{1}{u(x+\eta)s}$$


π (q)
γ*(q')
ϕ_π
x+η
η-x
$\widetilde{H}, \widetilde{E}$
N(p)
N(p')

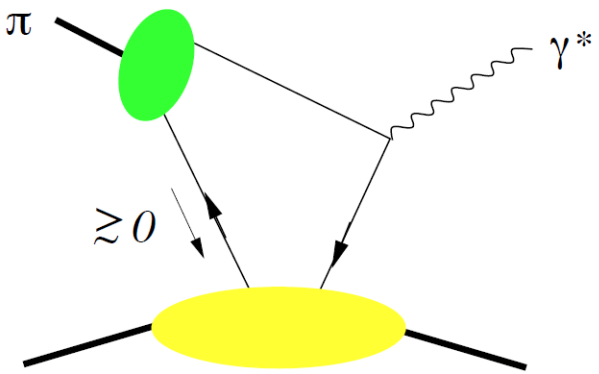

π (q)
γ*(q')
ϕ_π
x+η
η-x
$\widetilde{H}, \widetilde{E}$
N(p)
N(p')

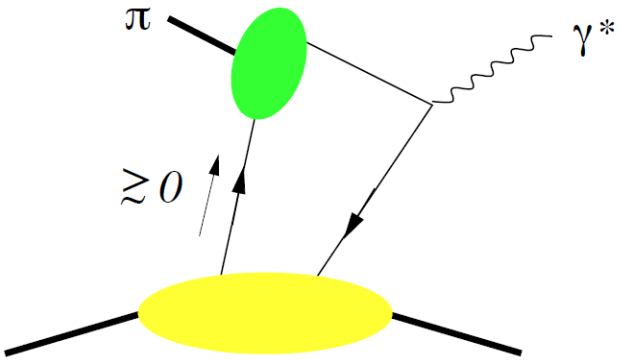

π
γ*
$\gtrsim 0$

"nonfactorizable" mechanism

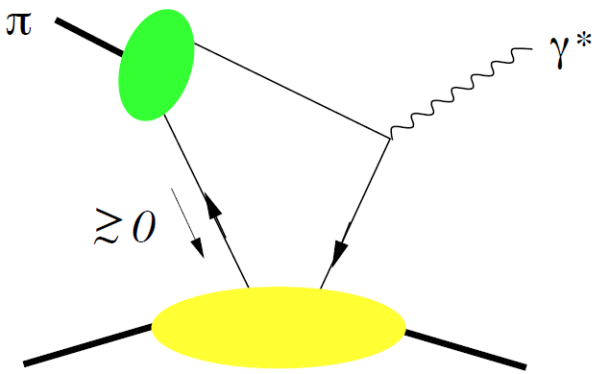


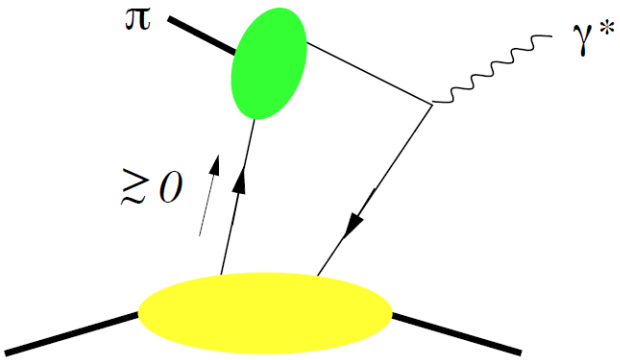
"nonfactorizable" mechanism





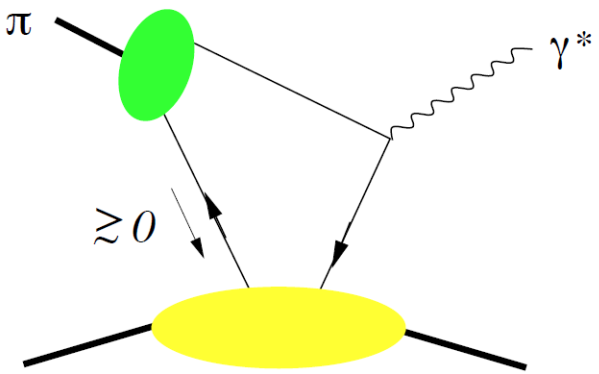
"nonfactorizable" mechanism



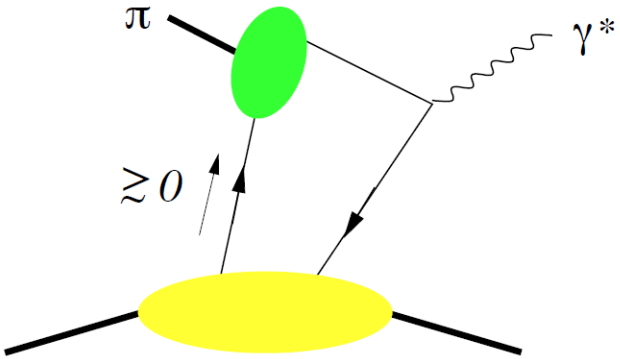


lower order in α_s

"nonfactorizable" mechanism

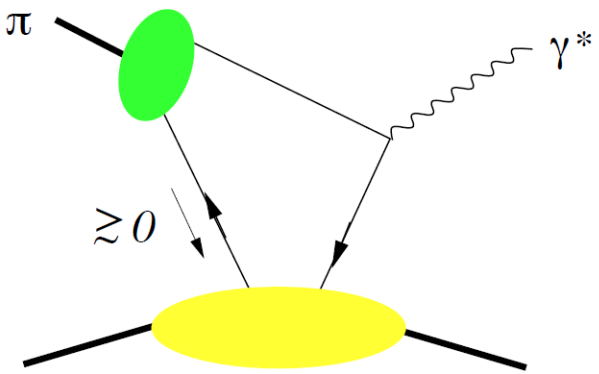


lower order in α_s



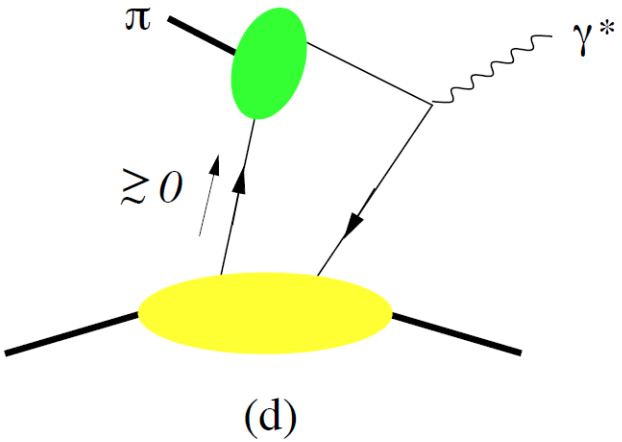
"Feynman mechanism"

"nonfactorizable" mechanism

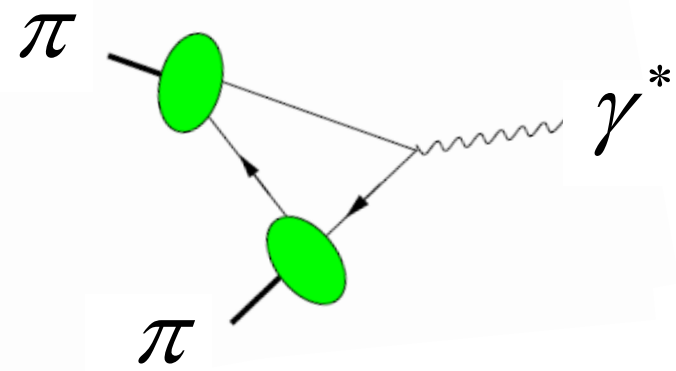


lower order in α_s

"Feynman mechanism"



(d)



Collinear factorization does not work at twist-3:

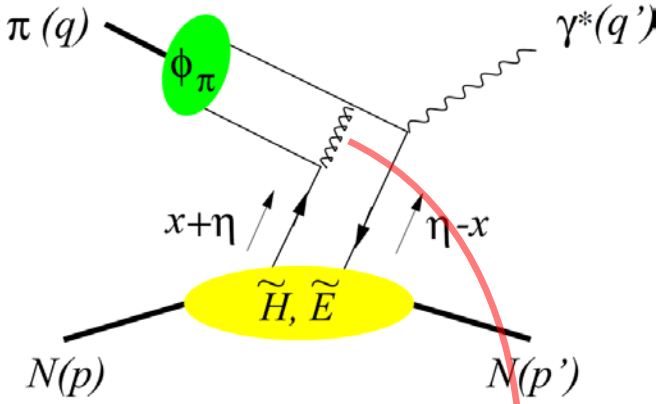
- quark $k_\perp$ ("k_T-factorization") Goloskokov, Kroll

  with Sudakov resummation

  Li, Sterman

- include "soft" propagator in long-distance part

  nonfactorizable "Feynman mechanism"
  at lower order in α_s



Bjorken variable: $\tau = \dfrac{Q'^2}{2\,p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2 \sin^2\theta$$

$$\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$$

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

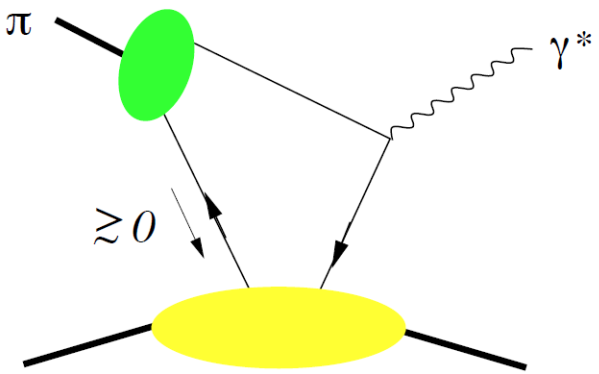
$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|\,p\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

$$\phi_\pi(u) \sim u(1-u) \qquad \phi_p(u) \sim 1$$

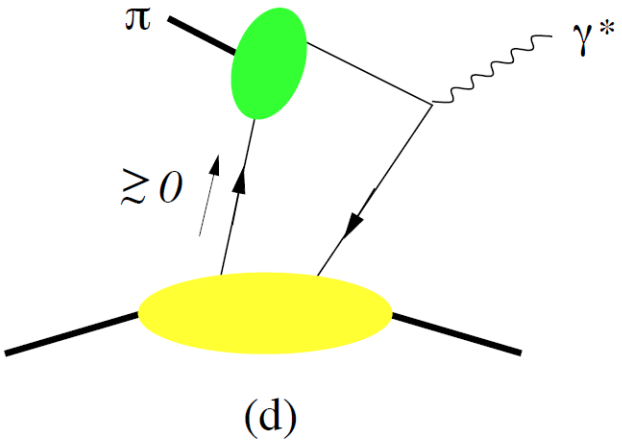
$$M^{\pm1,\lambda;\lambda'}(\pi^- p \to \gamma^* n) \sim \frac{1}{Q'^2}\int_0^1 du\,\frac{\phi_p(u)}{u(1-u)} \otimes \frac{1}{(\eta \pm x + i\varepsilon)^2} \otimes \left\{H_T^q(x,\eta,t), \tilde{H}_T^q(x,\eta,t), E_T^q(x,\eta,t), \tilde{E}_T^q(x,\eta,t)\right\}$$

"nonfactorizable" mechanism

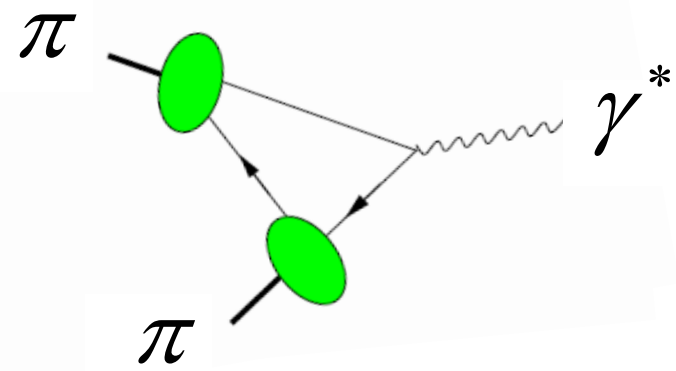


lower order in α_s

"Feynman mechanism"



(d)



Collinear factorization does not work at twist-3:

- quark $k_\perp$ ("k_T-factorization") Goloskokov, Kroll

  with Sudakov resummation

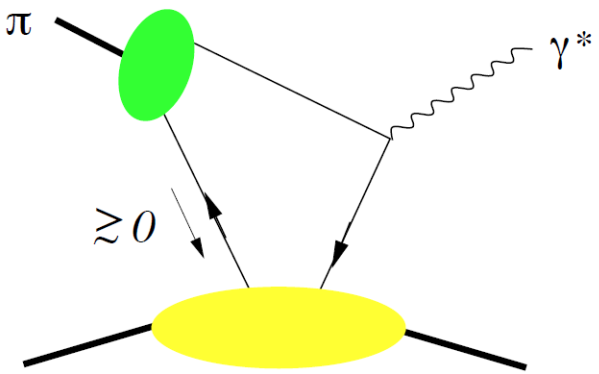
  Li, Sterman

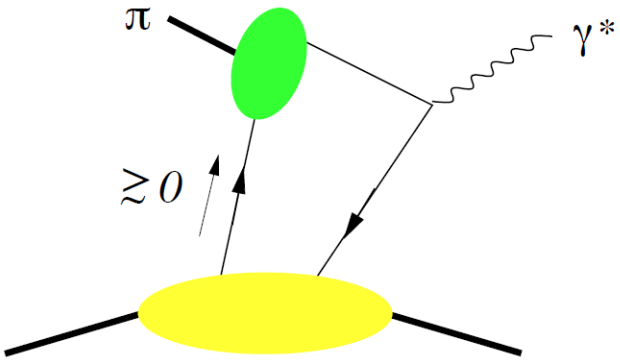
- include "soft" propagator in long-distance part

  nonfactorizable "Feynman mechanism" at lower order in α_s

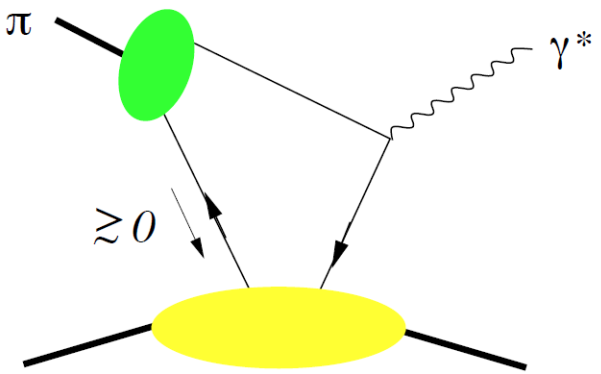
  relevent also for leading twist!

"nonfactorizable" mechanism

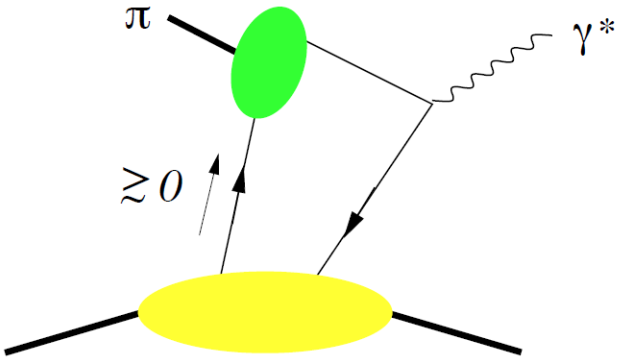




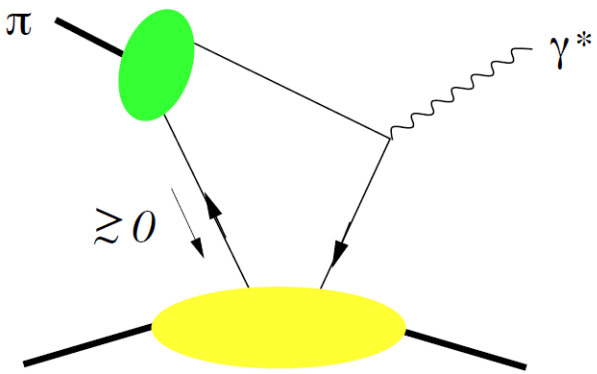
"nonfactorizable" mechanism

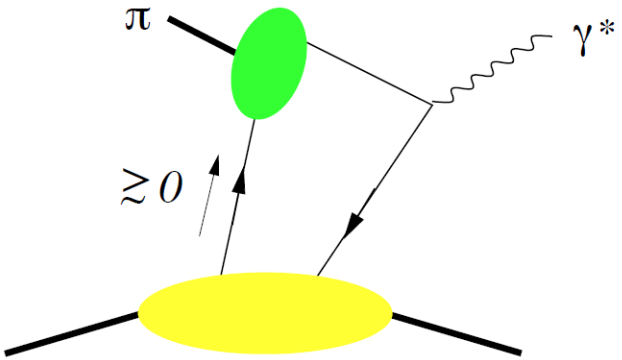


$q^2 \neq m_\pi^2$

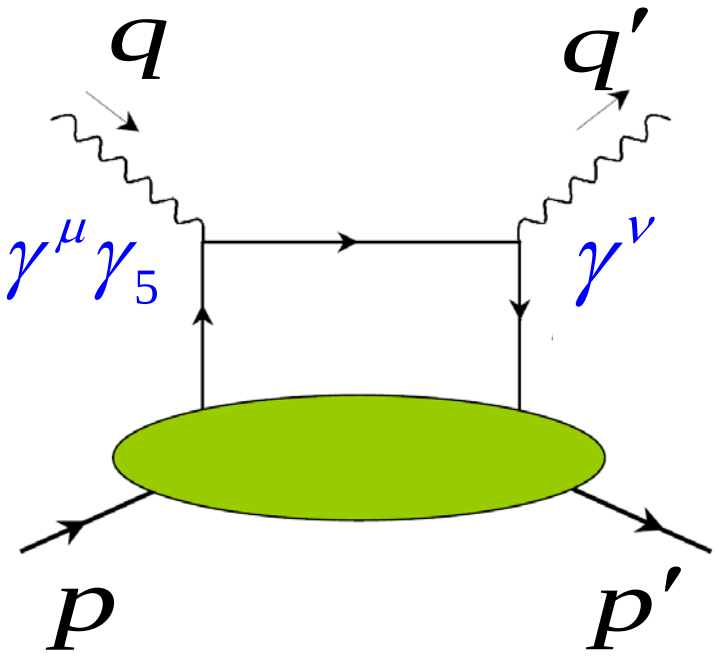


"nonfactorizable" mechanism

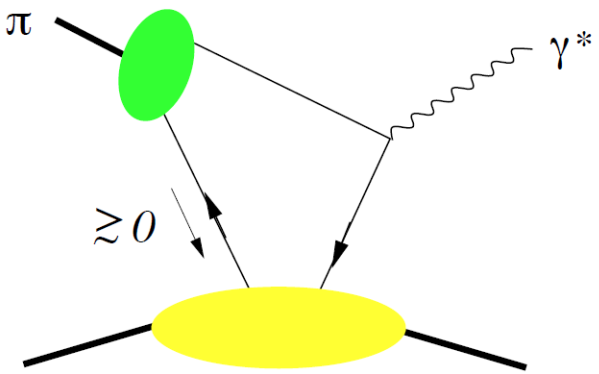


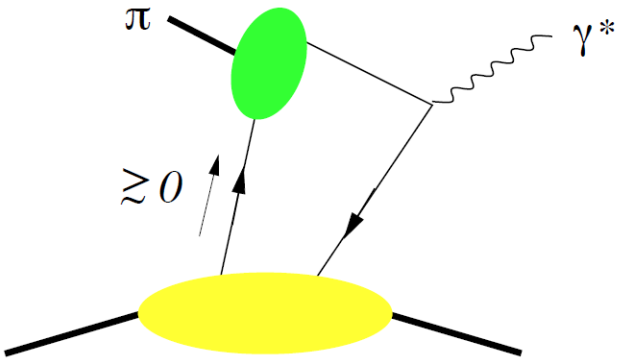


$q^2 \neq m_\pi^2$

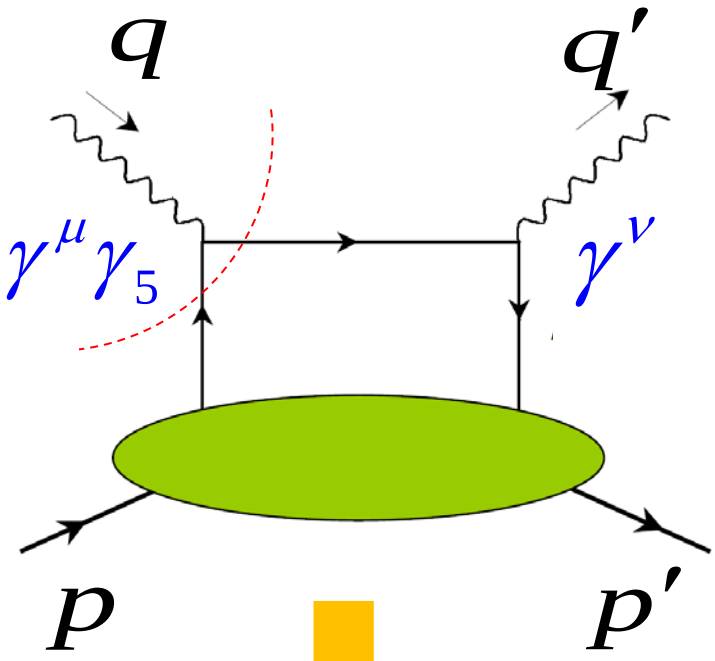


"nonfactorizable" mechanism



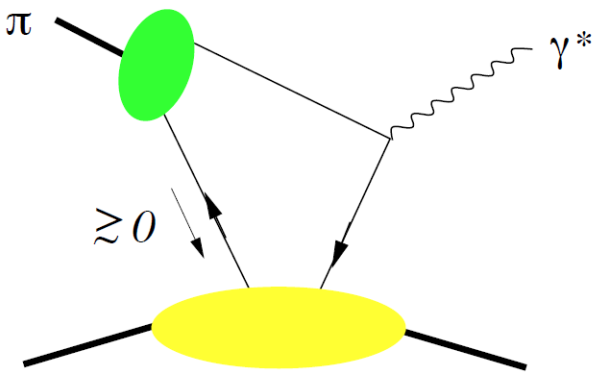


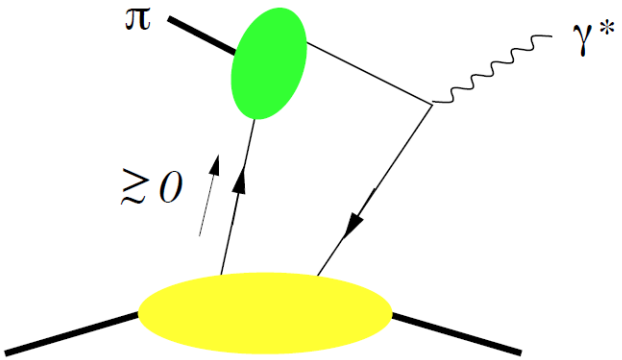
$q^2 \neq m_\pi^2$



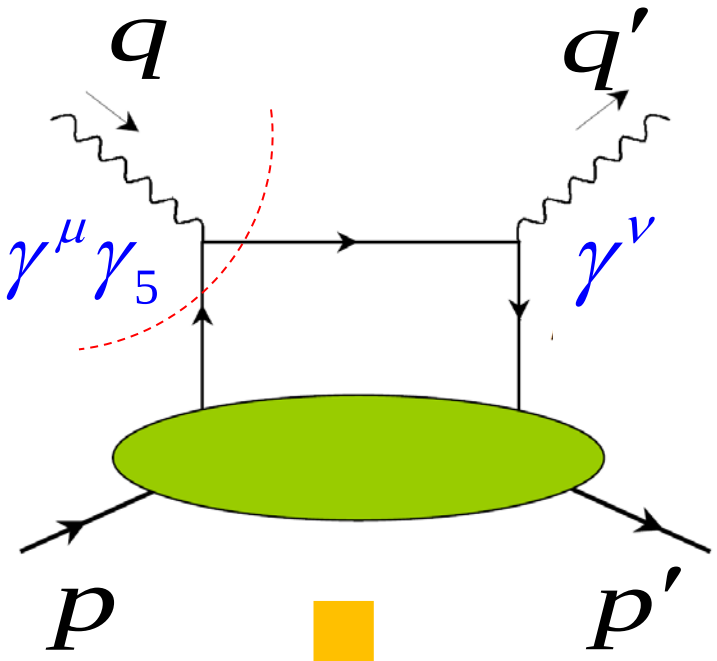
dispersion relation

"nonfactorizable" mechanism





$q^2 \neq m_\pi^2$

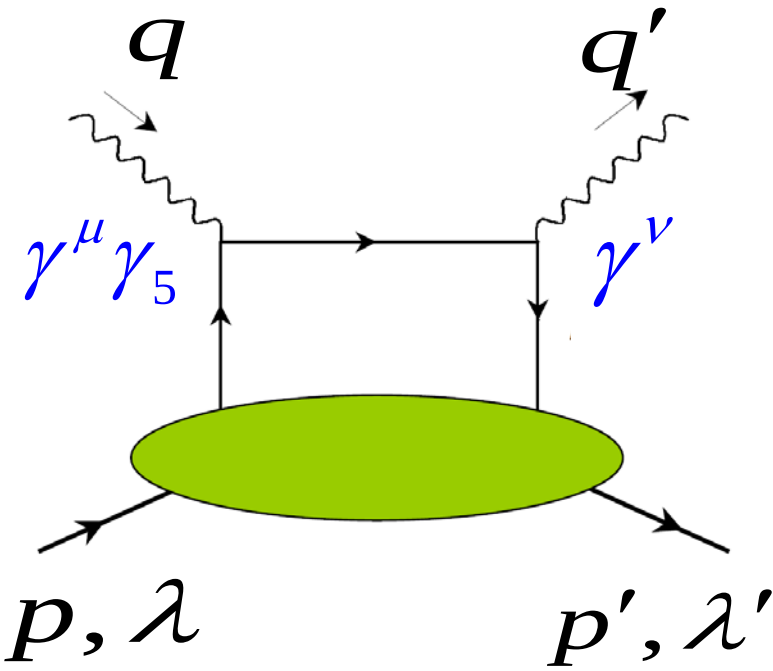


dispersion relation

quark-hadron duality

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j_\mu^5(0)\, j_\nu^{\mathrm{em}}(x) | p\lambda \rangle$$

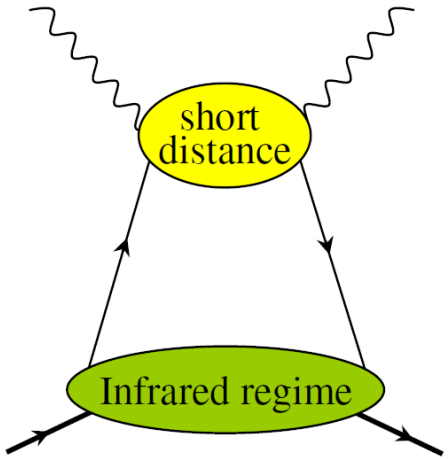
$$\equiv -iT_{\mu\nu}$$

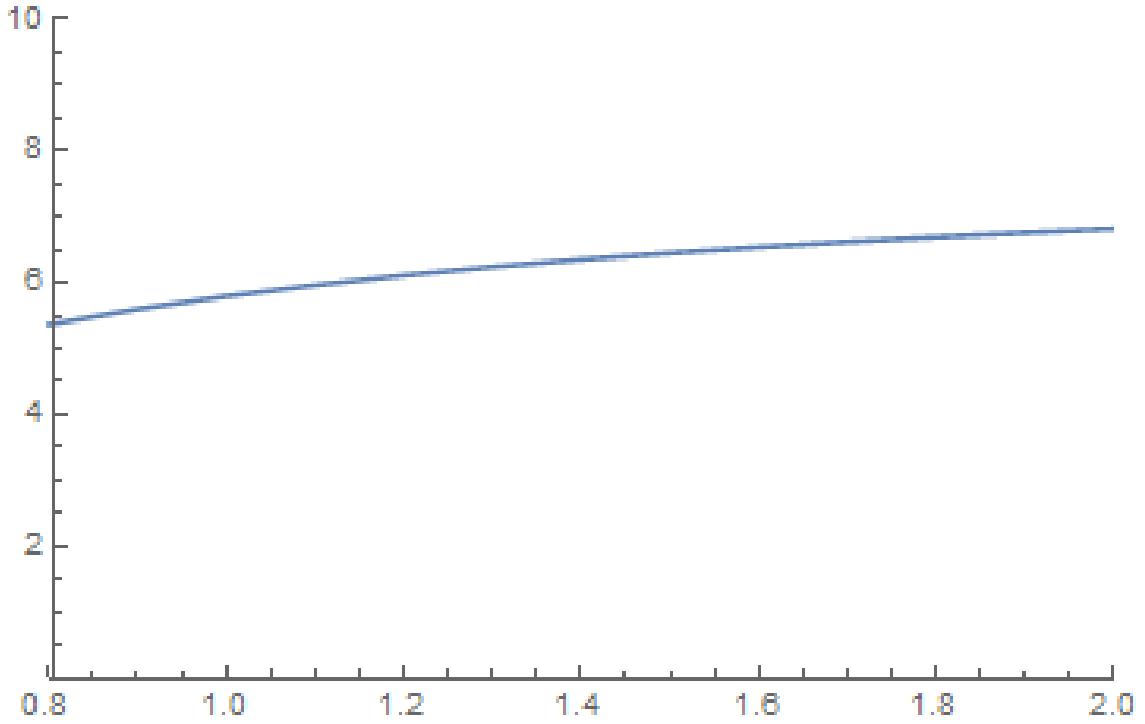


$$j_\mu^5 = \bar{d}\gamma_\mu\gamma_5 u$$

$$j_\nu^{\mathrm{em}} = e_u \bar{u}\gamma_\nu u + e_d \bar{d}\gamma_\nu d$$

$$|q^2|, |q'^2| \gg \Lambda_{\mathrm{QCD}}^2$$





Borel trnsf.: $\hat{L}_{M_B}\left(\frac{1}{m^2-q^2}\right)=\frac{1}{M_B^2}e^{-\frac{m^2}{M_B^2}}$

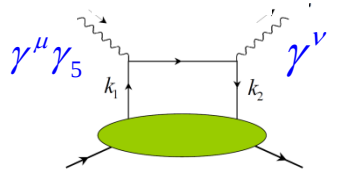
Summary

$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$ at J-PARC GPDs

LO $\left(O(\alpha_s^2)\right)$ factorization formula is known, but it misses soft nonfactorizable mechanism (SNM)

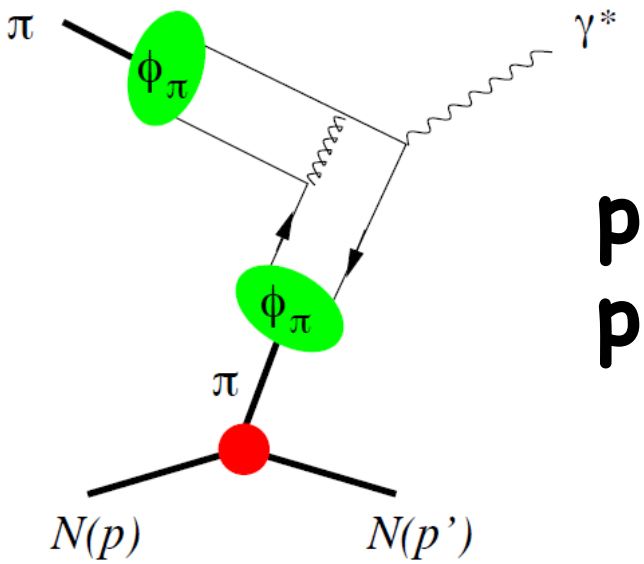
LCSR at LO $\left(O(\alpha_s^0)\right)$ is derived for largely model-independent estimate for SNM

$\tilde{H}$, $\tilde{E}$, $q^2_{\rm th}$ $(\sim 0.7\ {\rm GeV}^2)$



- numerical calculation at LO!
- NLO LCSR ⟷ quark $k_\perp$, pion pole contri.
- twist-3 LCSR → $M^{\pm 1\lambda',\lambda}_{LCSR}(\pi^- p \to \gamma^* n)$

interplay of soft/hard QCD mechanism



pion-pole contribution using pion form factor $F_\pi(Q'^2)$

Goloskokov, Kroll

$F_\pi(Q'^2)$: important soft nonfactorizable contr. was shown with LCSR

Braun, Khodjamirian, Maul