Recent results from intermediate energy knockout reactions

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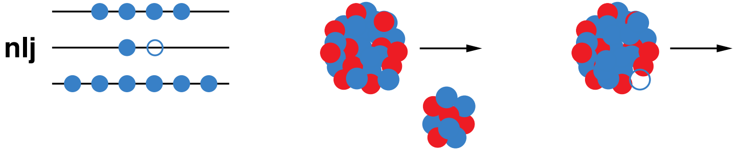
Direct reactions

- an excellent tool to study nuclear structure
- single-step and very fast, 10^{-22} s
  (time needed for the projectile to traverse the target)
- few nucleons participate, small momentum transfer
  $\rightarrow$ selectivity, use as a spectroscopic tool



- peripheral collisions, surface dominated
- for large impact parameters the core fragment remains largely unaffected

$$v_c \sim v_p$$

- experimentally: detect incident projectile and resulting fragment(s)
  $\rightarrow$ probe of peripheral character of the reaction

Factorization of the cross section

- removing a nucleon with quantum numbers $\alpha = (n, l, s, j, m, t_z)$: nucleon removal operator O_α acting on initial state $|\Psi_i^A\rangle$
- reaction amplitude:

$$A_\alpha^{if} = \langle \Psi_f^{A-1} | O_\alpha | \Psi_i^A \rangle$$

and cross section $\sigma_\alpha^{if} = |A_\alpha^{if}|^2$

- sudden approximation: reaction is fast compared to motion of nucleons: $O_\alpha \to (-1)^{j+m} a_{k,-m}$ proportional to annihilation operator a:

$$A_\alpha^{if} = C_\alpha^{if} \langle \Psi_f^{A-1} | a_\alpha | \Psi_i^A \rangle$$

- summing over final m, averaging over initial m projections:

$$\sigma_k^{if} = \frac{1}{2J_i + 1} \sum_{M_i, M_f} |C_k^{if}|^2 \left| \langle \Psi_f^{A-1} | a_{k,m} | \Psi_i^A \rangle \right|^2$$

- average over M_i, M_f, assuming spherical projectile, or $J_i = 0$:

$$\sigma_k^{if} = |C_k^{if}|^2 \frac{1}{2J_i + 1} \left| \langle \Psi_f^{A-1} || a_{k,m} || \Psi_i^A \rangle \right|^2 = \sigma_k^{sp} S_k^{if}$$

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- single-particle cross section $|C_k^{if}|^2 = \sigma_k^{\mathrm{sp}}$:

$$\sigma_k^{if} = \sigma_k^{\mathrm{sp}} \quad \text{if} \quad |\Psi_i^A\rangle = a_k^\dagger |\Psi_f^{A-1}\rangle$$

  described reaction dynamics only
- spectroscopic factor S_k^{if} only depends on the structure of initial and final states
- in proton-neutron formalism:
  $C^2 S_k^{if}(T) = C^2 S$ with isospin Clebsch-Gordan coefficient
- typically calculated in a harmonic oscillator basis $\rightarrow$ center of mass correction:

$$C^2 S \rightarrow \left(\frac{A}{A-1} \right)^N C^2 S$$

- spectroscopic factors are not observables, only the cross section is measured

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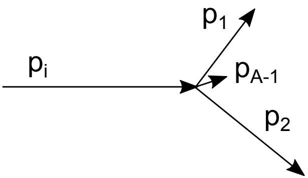
Reactions with stable beams

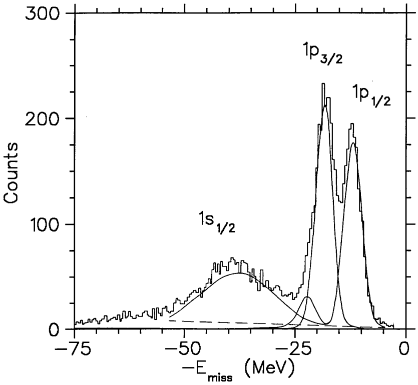
(p,2p) reactions on stable targets

- proton beam with several hundred MeV
- short wavelength, deep hole states
- NN cross section small
  → impulse approximation



- $p_{A-1} = p_i - p_1 - p_2$
  → excitation energy spectrum

$^{16}O(p,2p)^{15}N$ at 500 MeV (TRIUMF)



C. A. Miller et al., Phys. Rev. C **57** (1998) 1756

- proton-pair angular correlations
  → momentum distribution of protons in the nucleus
- determine orbital angular momentum *l*
- polarized protons → total angular momentum *j*

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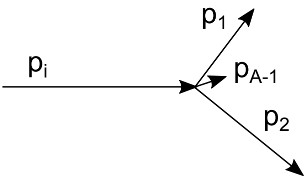

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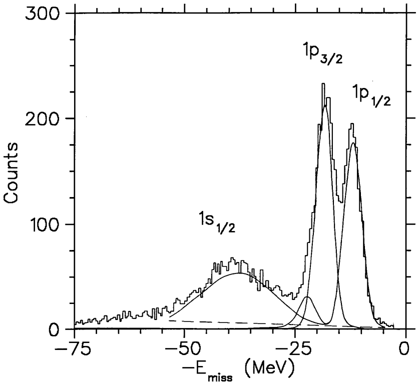

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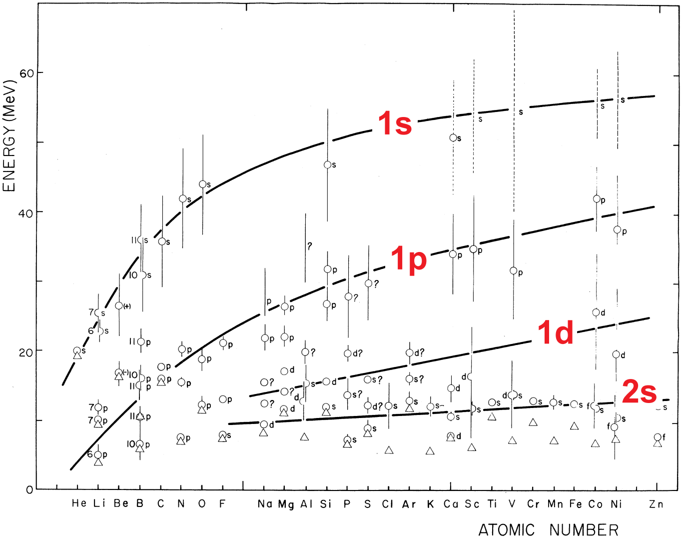
Quasi-free scattering on light nuclei



G. Jacob and Th. A. J. Maris, Rev. Mod. Phys. **45** (1973) 6

- challenge: limited resolution, heavier nuclei / higher level density not feasible

Electron induced nucleon removal (e,e′p)

- advantage: higher resolution
- nucleus is transparent to electrons $\longrightarrow$ study of inner shells
- less distortion of the associated momentum distributions
- disadvantage: small electro-magnetic cross section

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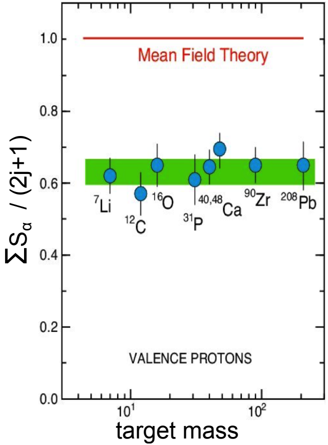
observation:

- summed spectroscopic strength $\sum S_\alpha$ compared to independent particle shell model $(2j+1)$
- reduction of the spectroscopic strength by 65 % $\longrightarrow$ correlations the are not included in the mean-field approximation
- depletion of states below the Fermi surface and population of states above it
- no in the (limited) model space of the theory



L. Lapikás, Nucl. Phys. A **553** (1993) 297

Short-range correlations

- repulsive core of the NN interaction at $r < 0.5$ fm
- uncertainty principle $\Delta p \Delta r < \hbar$
  $\rightarrow$ components in the wave function with $p \approx 400$ MeV/c
- extremely difficult to measure
- beyond the mean field theory (MFT)
  but for light nuclei: microscopic variational Monte-Carlo (VMC) calculations based on realistic NN interactions

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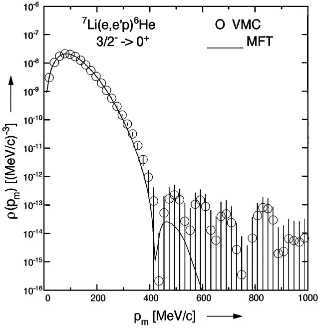
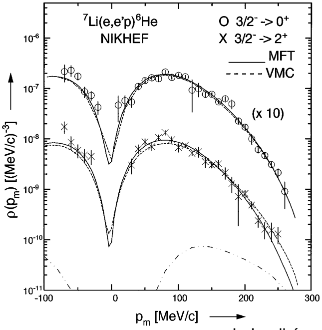
calculated momentum distributions



measurement



spectroscopic factor:

Model	$S(0^+ + 2^+)$
MFT	1
VMC	0.60
exp	0.58(5)

L. Lapikás et al., Phys. Rev. Lett. **82** (1999) 4404

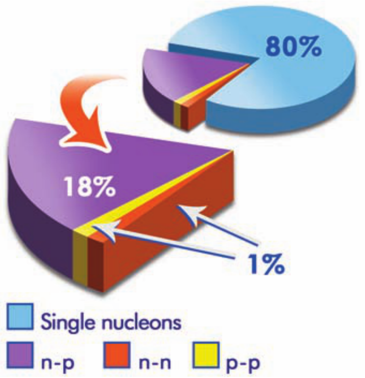
Correlated pairs

- only $\sim 65\ \%$ of the nucleons participate in the independent particle motion
- short-range correlations lead to pairs with large relative momentum and small center of mass momentum
- local density for pairs ~ 5 times larger than nuclear density
  $\rightarrow$ probing dense nuclear matter (neutron stars)

^{12}C(e,e$'$pN) at JLab

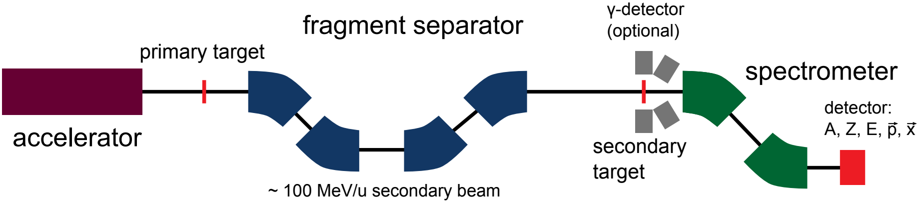
- if one partner of such a pair is struck: high relative momentum leads to recoil of the correlated nucleon as well
- measure (e,e$'$p) and (e,e$'$pN):
  $\sim 80\ \%$ of the nucleons act independently
  $\sim 20\ \%$ of the nucleons form correlated pairs
- measure (e,e$'$pp) and (e,e$'$pn):
  n-p pairs are 18 times more common
  $\rightarrow$ direct effect of the tensor force

R. Subedi et al., Science **320** (2008) 1476



With radioactive beams

- production of radioactive ion beams by projectile fragmentation
- ideal beam energy range 50 – 1000 MeV/u

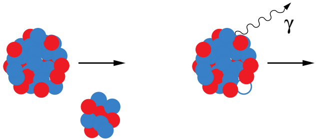


- cocktail beam requires fragment separator
- “bad” beam quality, momentum spread, contamination, emittance
- facilities:
  - NSCL A1900/S800: $\sim$ 100 MeV/u, $\Delta p = 0.1 - 5$ %, dispersion matching possible
  - GSI FRS: 500 – 1000 MeV/u, $\Delta p \leq 3$ %
  - GANIL SISSI/SPEG: $\sim$ 100 MeV/u, $\Delta p = 0.1$ %, energy loss mode
  - RIKEN BigRIPS/ZeroDegree: $\sim$ 200 MeV/u, $\Delta p \leq 6$ %
- intensities of a few particles per second required

$\rightarrow$ ideal conditions for nucleon removal reactions with radioactive beams

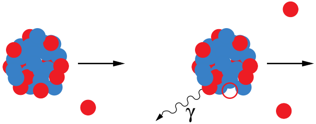
Types of reactions

nucleon knockout:
light nuclear target ^{9}Be or ^{12}C



- pioneering experiments using ^{11}Li breakup
  N. A. Orr et al., Phys. Rev. Lett. **69** (1992) 2050
- now extensively used at: NSCL, GANIL, GSI
- strong absorption: reaction happens at surface
- $\rightarrow$ probe the outer part of the wave-function

quasi-free scattering:
(p,2p) or (p,pn) using a hydrogen target



- in the past: (e,e′p) or (p,2p) on stable targets
- only way to determine absolute spectroscopic factors
  G. J. Kramer et al., Nucl. Phys. A **679** (2001) 267
- wide range from weakly bound (valence) to deeply bound (core) states
- $\rightarrow$ sample entire wave function

Knockout reactions:

experimental and theoretical methods

Nuclear targets

for this talk

- “knockout” refers to nucleon removal reactions with a light nuclear target such as ^{9}Be or ^{12}C
- “quasi-free scattering” to (p,2p) or (p,pn) reactions
- why do some people prefer knockout over quasi-free scattering for spectroscopy?

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- easy to make a thick, pure target
  (compared to CH_2 or liquid H)
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- absorptive disk, but core survives $\rightarrow$ peripheral collisions
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well-developed experimental and theoretical techniques allow to determine

- spectroscopic factors, occupation numbers
- spin and parity assignments through momentum distributions

Early experiments

- fast projectile mass A collides with nuclear target
- mass $(A-1)$ residues are detected
- light fragments are unobserved, final state tagging by γ-ray if needed
- sudden approximation:

$$\vec{k}_3 = \frac{A-1}{A}\vec{k}_A - \vec{k}_{A-1}$$

  momentum of the struck nucleon $\vec{k}_3$ is related to the residues $\vec{k}_{A-1}$
- first fragmentation experiment with radioactive beam at Bevalac/LBNL:

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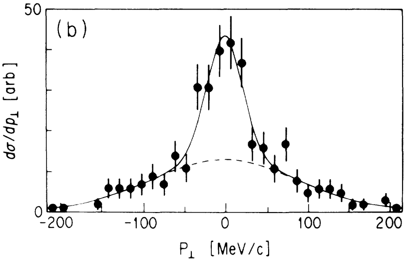
- first fragmentation experiment with radioactive beam at Bevalac/LBNL:

- two components in the transverse momentum distribution of ^{9}Li residues
- broad like for stable nuclei (^{12}C)
- very narrow
  $\rightarrow$ removal of weakly bound neutrons
  uncertainty relation $\rightarrow$ large spatial extent

  $\rightarrow$ signature of halo states

^{11}Li at 0.8 GeV/u on C target



T. Kobayashi et al., Phys. Rev. Lett. **60** (1988) 2599

Limitations

- Coulomb deflection and diffractive scattering affect the transverse distribution
  $\rightarrow$ measure parallel (longitudinal) momentum distributions
- however, much higher resolution is required:
  ex: $A = 50$ nucleus with energy of 100 MeV/u $p = 22$ GeV/c
  momentum width of nucleon 50 (halos) $-$ 300 MeV/c
  required resolution: $\Delta p/p \approx 0.5$ %
- momentum spread of incident beam: $\sim$ few %

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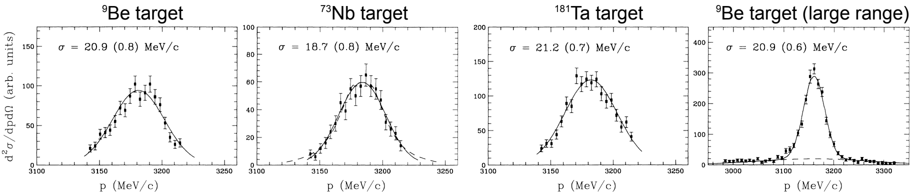
solution: dispersion matching

- target at dispersive image
- second magnet compensates, direct measure of k_{3z}
- ^{11}Li at 66 MeV/u on different targets

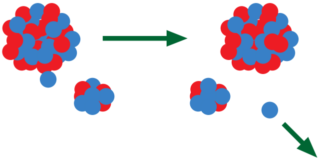


N. A. Orr et al., Phys. Rev. Lett. **69** (1992) 2050

The knockout reaction mechanism

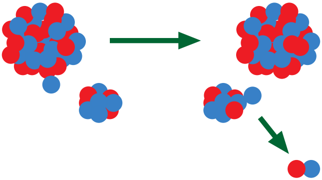
two processes contribute to the knockout reaction with nuclear targets

- diffractive or elastic breakup



- dissociation through two-body interaction with target (elastic)
- forward direction with beam velocity
- target remains in the ground state

- stripping or inelastic breakup

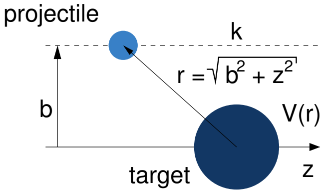


- removed nucleon reacts with target
- excites the target
- loses energy or picks up nucleons from the target

- for light targets Coulomb breakup negligible
- stripping typically dominant
- calculate both processes $\rightarrow$ incoherent sum compared to experiment

Eikonal theory

- scattering of a point projectile of a potential $V(r)$
- semi-classical approach: geometrical description in terms of the impact parameter b
- incident particle wave number k large wavelength small compared to changes in $V(r)$



- scattered wave: $\psi^{+}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\omega(\vec{r})$
  plane wave and modulating function ω (contains information on potential)
- Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right]\psi^{+}(\vec{r}) = E\psi^{+}(\vec{r})$$

$$\rightarrow \left[2i\nabla\omega(\vec{r})\cdot\vec{k} - \frac{2\mu}{\hbar^2}V(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

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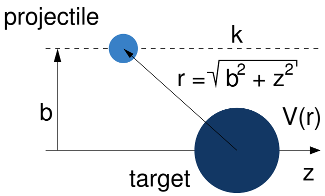

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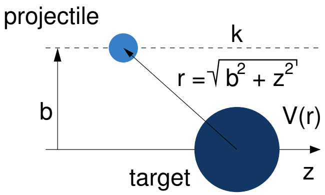
- align Z-axis along $\vec{k}$ $(b = \sqrt{x^2 + y^2})$

$$\frac{\partial \omega}{\partial z} = -\frac{i}{\hbar v} V(r)\omega(\vec{r}) \quad \rightarrow \quad \omega(\vec{r}) = \exp\left(-\frac{i}{\hbar v}\int_{-\infty}^{z} V(\sqrt{b^2 + z'^2})\mathrm{d}z'\right)$$

- neglecting $\nabla^2\omega(\vec{r})$ means: assuming a straight line trajectory
- $v = \hbar k/\mu$ classical incident velocity in the cm frame



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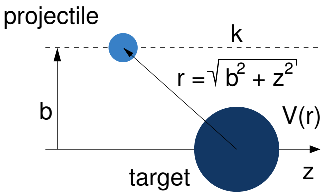
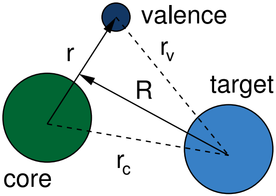


- the scattering wave function $(z \rightarrow \infty)$in eikonal approximation:

$$\psi^{\mathrm{eik}}(\vec{r}) \rightarrow \exp\left(-\frac{i}{\hbar v}\int_{-\infty}^{\infty} V(\sqrt{b^2 + z'^2})\mathrm{d}z'\right)\exp(i\vec{k}\cdot\vec{r}) = S(b)e^{i\vec{k}\cdot\vec{r}}$$

- $S(b)$: amplitude of the scattered wave, eikonal elastic S-matrix
- for a real potential $|S(b)|^2 = 1$
- rather simple: one dimensional integration through potential $V(r)$
- generalizes for few-body projectiles

Few-body models



- two-body projectile (bound): core c and valence particle v
- constituents interact with target through effective interactions V_{jt} ($j = $ v,c)
- V_{jt} can be obtained from: phenomenological optical models, or folding models
- at high energies ($>$ 50 MeV/u): double-folding of densities and effective NN interaction

$$V_{jt}(r_j) = \int d\vec{r}_1 \int d\vec{r}_2 \rho_j(r_1)\rho_t(r_2) t_{NN}(\vec{r}_j + \vec{r}_2 - \vec{r}_1)$$

- Schrödinger equation for incident projectile with $\vec{K}$ in cm frame

$$\left[T_R + U(\vec{r}, \vec{R}) + H_p - E\right] \psi^+(\vec{r}, \vec{R}) = 0$$

H_p projectile internal Hamiltonian, $U(\vec{r}, \vec{R})$ total projectile-target interaction

- adiabatic (sudden) approximation $H_p \to -\varepsilon_0$ ground state energy

$$\left[T_R + U(\vec{r}, \vec{R}) - (E + \varepsilon_0)\right] \psi^{adj}(\vec{r}, \vec{R}) = 0$$

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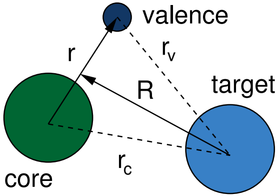

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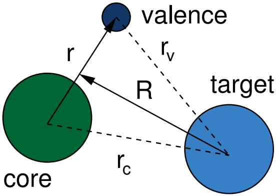

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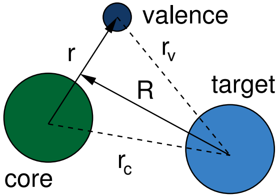

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Few-body eikonal model

- scattering wave product of incident wave and modulating function $\omega(\vec{r},\vec{R})$

$$\psi^{\text{adj}}(\vec{r},\vec{R}) = e^{i\vec{K}\cdot\vec{R}}\phi_0(\vec{r})\omega(\vec{r},\vec{R})$$

ϕ_0 projectile ground state wave function, $\hbar K = \sqrt{2\mu(E+\varepsilon_0)}$

- into Schrödinger equation and neglecting $\nabla^2\omega(\vec{r},\vec{R})$ gives

$$\rightarrow \quad \omega(\vec{r},\vec{R}) = \exp\left(-\frac{i}{\hbar v}\int_{-\infty}^{Z} U(\vec{r},\vec{R})\mathrm{d}Z'\right)$$

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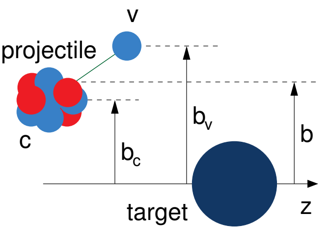
$$\psi^{\mathrm{eik}}(\vec{r},\vec{R}) \rightarrow S_{\mathrm{c}}(b_{\mathrm{c}})S_{\mathrm{v}}(b_{\mathrm{v}})e^{i\vec{K}\cdot\vec{R}}\phi_0(\vec{r})$$

$S_j(b_j)$ are the eikonal elastic S-matrices for independent scattering of v or c off the target

- adiabatic: $\vec{r}$ only parameter, S-matrices are calculated at fixed b_{c} and b_{v}



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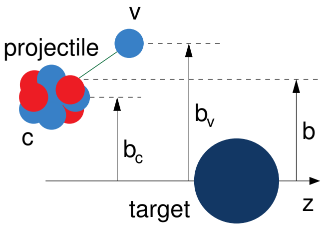

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Cross sections

- separation of dynamics (S_j) from structure (wave function to be probed)

$$S_\mathrm{p}(b) = \langle \phi_0 | S_\mathrm{c}(b_\mathrm{c}) S_\mathrm{v}(b_\mathrm{v}) | \phi_0 \rangle$$

- total cross section to populate state j ($\mathrm{d}\vec{b} = 2\pi b \mathrm{d}b$):

$$\sigma_j = \int \left| \langle \phi_j | S_\mathrm{c}(b_\mathrm{c}) S_\mathrm{v}(b_\mathrm{v}) | \phi_0 \rangle - \delta_{j0} \right|^2 2\pi b\, \mathrm{d}b$$

- elastic cross section:

$$\sigma_0 = \int \left| \langle \phi_0 | S_\mathrm{c}(b_\mathrm{c}) S_\mathrm{v}(b_\mathrm{v}) | \phi_0 \rangle - 1 \right|^2 2\pi b\, \mathrm{d}b$$

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$$\sigma_\mathrm{reac} = \int \left(1 - \left| \langle \phi_0 | S_\mathrm{c}(b_\mathrm{c}) S_\mathrm{v}(b_\mathrm{v}) | \phi_0 \rangle \right|^2 \right) 2\pi b\, \mathrm{d}b$$

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Elastic breakup cross section

- diffraction due to absorptive (imaginary) part and refraction in the real part of the potential together are called elastic breakup (diffraction)
- excite projectile to continuum with wave function $\phi_{\vec{k}}$
- integrate over continuum for projectile, target remains in ground state

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$$\sum_{\text{bound}} |\phi_b\rangle\langle\phi_b| + \int_0^\infty \mathrm{d}\vec{k} \, |\phi_{\vec{k}}\rangle\langle\phi_{\vec{k}}| = 1$$

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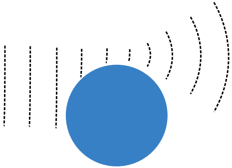

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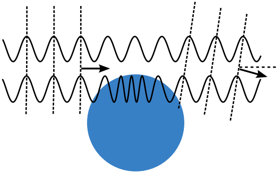

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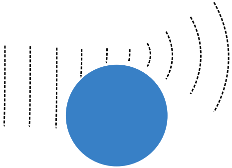

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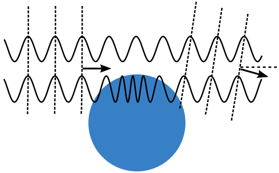

gives the total elastic (diffractive) cross section

$$\sigma_{\text{diff}} = \int \left(\langle\phi_0\big||S_c(b_c)S_v(b_v)|^2\big|\phi_0\rangle - |\langle\phi_0|S_c(b_c)S_v(b_v)|\phi_0\rangle|^2\right) 2\pi b\,\mathrm{d}b$$

under the assumption that there is only one bound state of the projectile

Stripping cross section

- total absorption cross section (target excitation):

$$\sigma_{\text{abs}} = \sigma_{\text{reac}} - \sigma_{\text{diff}} = \int \left(1 - \left\langle \phi_0 \left| |S_c(b_c)S_v(b_v)|^2 \right| \phi_0 \right\rangle\right) 2\pi b\, \mathrm{d}b$$

- $|S_j(b_j)|^2$ is the probability that $j =$ v,c survives the collision at impact parameter b_j and the target remains in the ground state
- $1 - |S_j(b_j)|^2$: probability that the target gets excited and j is absorbed from the elastic channel
- rewriting:

$$1 - |S_c S_v|^2 = |S_v|^2(1 - |S_c|^2) + |S_c|^2(1 - |S_v|^2) + (1 - |S_c|^2)(1 - |S_v|^2)$$

- cross section for stripping v from the projectile, exciting the target and c is only elastically scattered:

$$\sigma_{\text{str}} = \int \left\langle \phi_0 \left| |S_c(b_c)|^2 (1 - |S_v(b_v)|^2) \right| \phi_0 \right\rangle 2\pi b\, \mathrm{d}b$$

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Cross section for elastic break up

- two processes contribute, diffractive breakup and stripping
- they differ in their effect on the target, in stripping the target gets excited
  $\rightarrow$ measure the target excitation energy
- directly: experimentally not feasible (thick target, small energy)
  $\rightarrow$ determine the target excitation energy from missing mass spectroscopy
- need to measure the removed particle as well
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- peak in missing mass spectrum at $M_{\text{miss}} = M(^{9}\text{Be}) = 8.395$ GeV/c^2 reveals diffraction process

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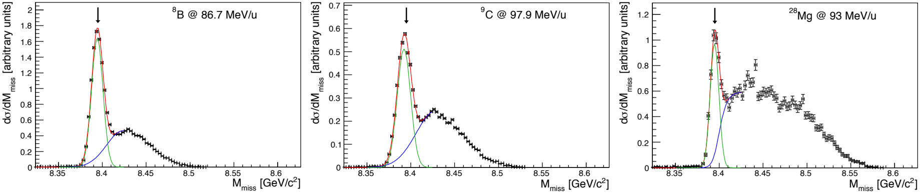

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Separation energy dependence

- earlier work suggested that the diffractive breakup cross section scales with separation energy as $1/\sqrt{S_p}$
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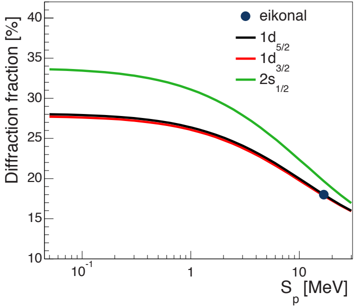
- $1/\sqrt{S_p}$ would suggest a factor of 6



- eikonal theory combined with USD shell model spectroscopic factors reproduces experiment
- for the range of separation energies studied here $S_p = 0.14$ (^{8}B) and 16.79 MeV (^{28}Mg)

$\rightarrow$ excellent agreement between the reaction theory and experiment

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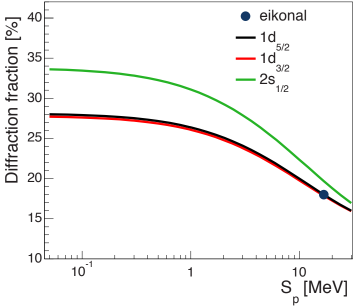

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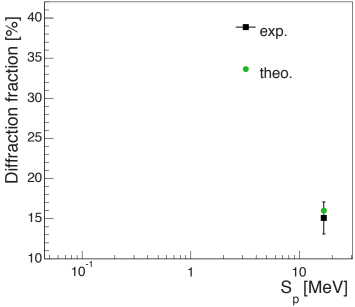

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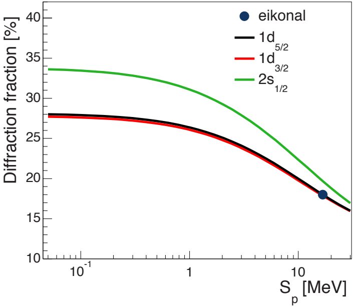

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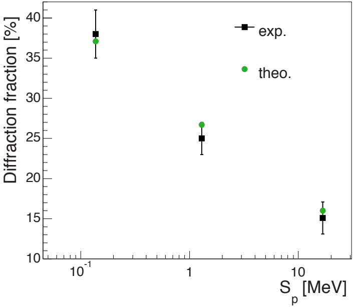
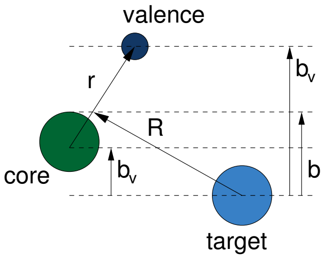


K. Wimmer et al., Phys. Rev. C **90** (2014) 064615

Momentum distributions



- momentum distribution of core (for stripping):

$$\frac{d\sigma}{d\vec{k}_c} = \int \left(1 - |S_v(b_v)|^2\right) \frac{dP(\vec{k}_c, b_v)}{d\vec{k}} \, 2\pi b_v \, db_v$$

- with $\vec{r} = \vec{r}_v - \vec{r}_c$ and $b_c = |\vec{b}_v - \vec{r}_\perp|$
- and no spin-orbit term

- core momentum at fixed b_v:

$$\frac{dP(\vec{k}_c, b_v)}{d\vec{k}_c} = \frac{1}{2\pi^3} \frac{1}{2l+1} \sum_m \left| \int e^{-i\vec{k}_c \cdot \vec{r}} S_c(b_c) \psi_{lm}(\vec{r}) d\vec{r} \right|^2$$

H. Esbensen et al., Phys. Rev. C **53** (1996) 2007

- integrating over the transversal components yields:

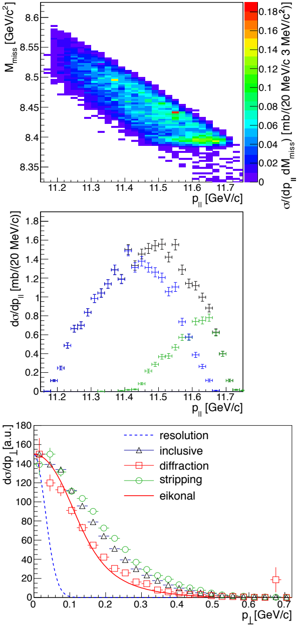
$$\frac{d\sigma}{dk_z} = \frac{1}{2\pi^2} \frac{1}{2l+1} \sum_m \int_0^\infty \left(1 - |S_v(b_v)|^2\right) \int_0^\infty |S_c(b_c)|^2 \left| \int_{-\infty}^{\infty} e^{-ik_z z} \psi_{lm}(\vec{r}) dz \right|^2 d^2(b_v - b_c) \, 2\pi b_v \, db_v$$

東京大学 THE UNIVERSITY OF TOKYO

Momentum distributions

- it is generally assumed that the momentum distribution for elastic breakup (diffraction) is the same as for stripping
- NSCL measurement suffers from acceptance issues
- clear difference in the transversal momentum distribution

K. Wimmer et al., in prep.



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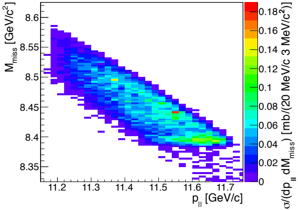
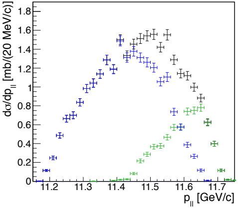
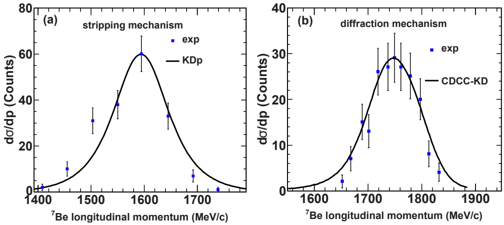
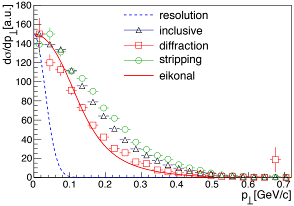




- new measurements of ^{8}B with proton - ^{7}Be coincidences



S. L. Jin et al., Phys. Rev. C **91** (2015) 054617



- limited resolution: → new experiments required

Beyond the eikonal approximation

the eikonal approximation

- does not conserve the energy
- does not include energy transfer between cm and relative motion degrees of freedom of residue and valence nucleon
- assumes a straight line path

$\rightarrow$ in the eikonal approximation the momentum distributions are symmetric

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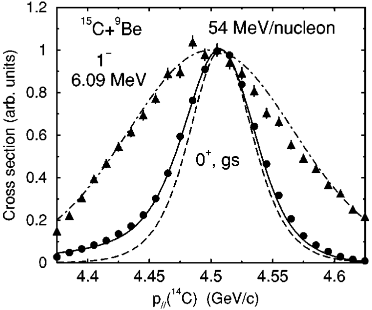

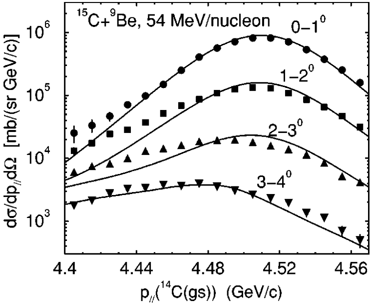

- asymmetry observed in the knockout from halo nuclei can be described by continuum discretized coupled channels (CDCC) calculations

J. A. Tostevin et al., Phys. Rev. C **66** (2002) 024607

Ingredients needed for calculation

- potentials for core-target and valence nucleon-target interactions

$$V_{jt}(r_j) = \int \mathrm{d}\vec{r}_1 \int \mathrm{d}\vec{r}_2 \rho_j(r_1)\rho_t(r_2)t_{NN}(\vec{r}_j + \vec{r}_2 - \vec{r}_1)$$

$\longrightarrow$ densities from Hartree-Fock calculations

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- wave function ϕ_0: many-body overlap function
  in practice not available
  $\longrightarrow$ calculate single-particle wave function in a Woods-Saxon potential

$$V(r) - V_0 f(r) + (\vec{l}\cdot\vec{s}\,)V_{SO}\frac{\mathrm{d}}{\mathrm{d}r}f(r) \quad \text{with} \quad f(f) = \frac{1}{1 + e^{(r-r_0)}/a_0}$$

radius r_0 to reproduce the HF rms radius for the orbit,
set V to reproduce the experimental binding energy

Hartree-Fock calculations

Hartree-Fock calculations are performed to obtain the

- density distribution of the core
- rms radii of the valence nucleon orbits

using the Skyrme X interaction

B. A. Brown et al., Phys. Rev. C **58** (1998) 220

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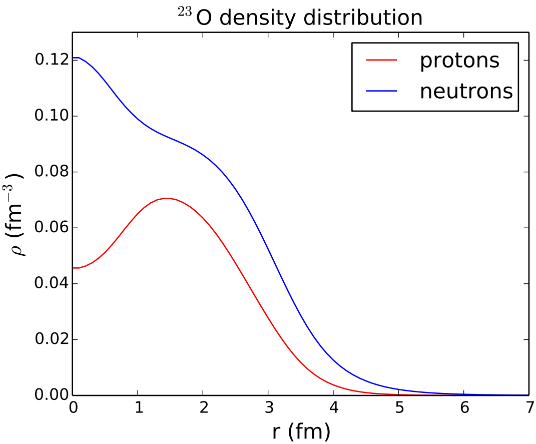
B. A. Brown et al., Phys. Rev. C **58** (1998) 220

Example: neutron knockout from ^{24}O



orbital	E (MeV)	r_{rms} (fm)
$1s_{1/2}$	-27.046	2.257
$1p_{3/2}$	-17.056	2.857
$1p_{1/2}$	-12.528	2.952
$1d_{5/2}$	-6.301	3.430
$2s_{1/2}$	-3.708	4.072
$1d_{3/2}$	-0.209	4.539

S-matrix

S-matrices for core and valence particle on target need:

- potentials for core-target and valence nucleon-target interactions
- double-folding integral of densities and effective NN-interaction
- for the core: use Hartree-Fock result
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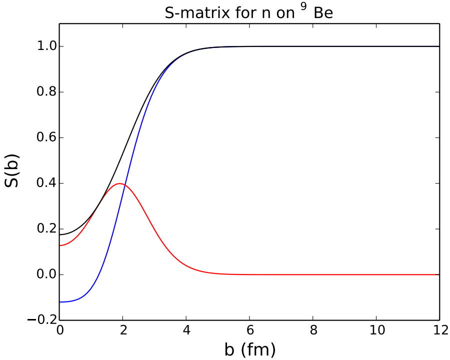

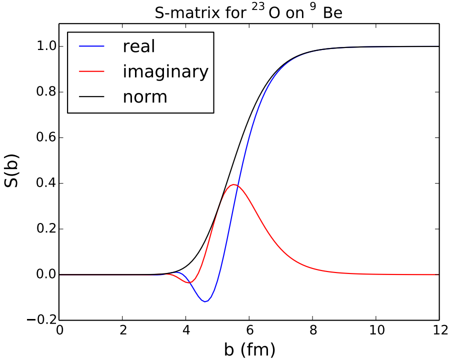

Wave functions

- initial bound state wave function or radial overlap function
- calculated in a Woods-Saxon potential with V_0 adjusted to reproduce the experimental binding energy $(S_n + E(j^\pi))$
- fixed diffuseness $a_0 = 0.7$ fm
- spin-orbit strength $V_{SO} = 6$ MeV, same r_0,a_0
- radius is constrained by the Hartree-Fock calculations: choose r_0 such that the wave function has a rms radius of

$$r_{\text{sp}} = \sqrt{\frac{A}{A-1}}\, r_{\text{HF}}$$

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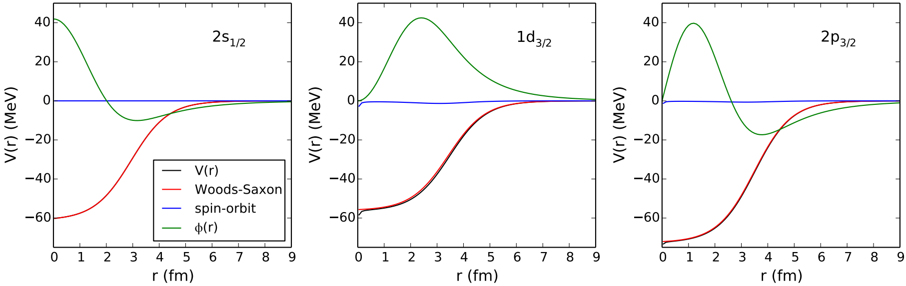
example: neutron single-particle wave functions with a core of ^{23}O:



Cross sections

- in the experiment only the residue is detected, not removed nucleon or the target
- calculate the single-particle cross section (neglecting Coulomb breakup)

$$\sigma_{\mathrm{sp}} = \sigma_{\mathrm{diff}} + \sigma_{\mathrm{str}}$$

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  from the *S*-matrices and the overlap functions calculated previously:

orbit	σ_{str} (mb)	σ_{diff} (mb)	σ_{sp} (mb)
$1d_{5/2}$	18.5	6.0	24.5
$2s_{1/2}$	17.1	5.4	22.6
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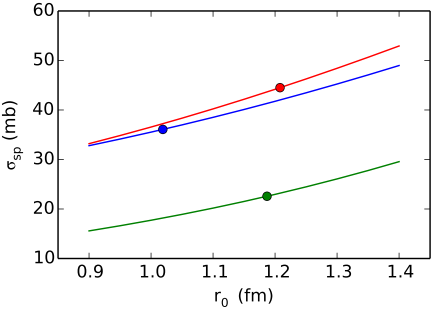
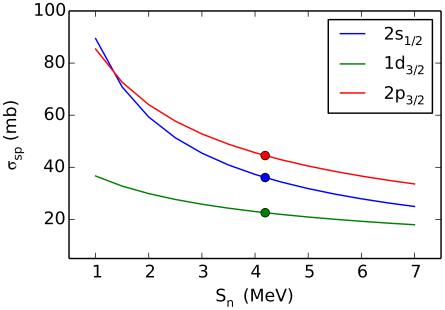
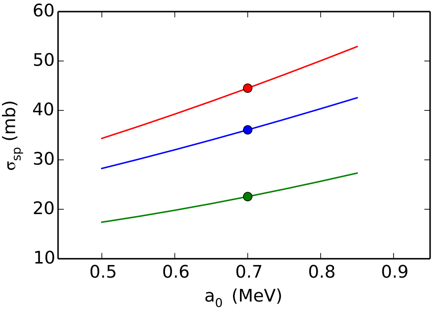
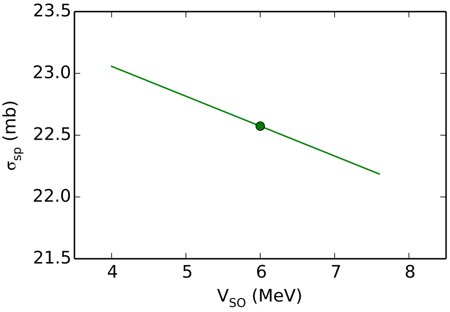
- σ_{sp} depends strongly on the chosen r_0
  $\rightarrow$ constrain r_{sp} by Hartree-Fock rms
- dependence on a_0 rather weak $\rightarrow$ constant $a_0 = 0.7$ fm for consistency
- V_{SO} has little influence

Cross sections



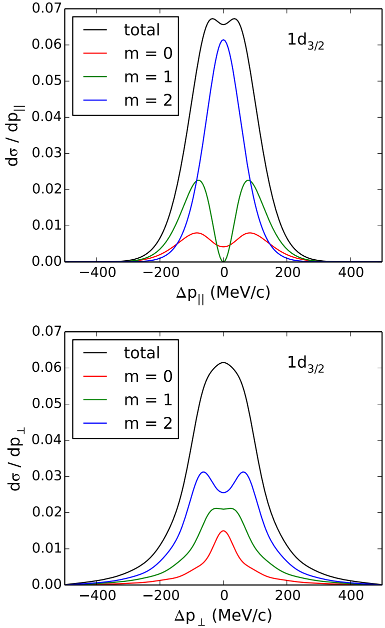






Momentum distributions

- calculation of parallel $\frac{d\sigma}{dp_{\|}}$ and transversal $\frac{d\sigma}{dp_{\perp}}$ momentum distributions
- using same input *S*-matrices and wave functions
- eikonal approximation
  $\rightarrow$ symmetric distributions



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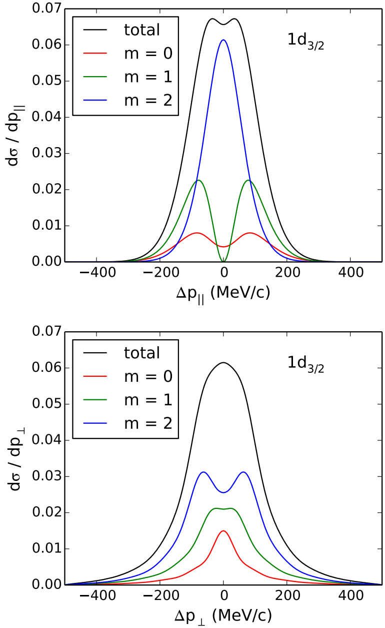
- calculations for different m states:
  $m = l$ dominant
- $m = l$ nucleon orbit will be perpendicular to the *z*-axis (beam direction):
  $\rightarrow$ high probability to hit the target, with the core further away surviving the collision
- $m = 0$ nucleon orbit aligned with beam direction:
  $\rightarrow$ if nucleon hits target, core will be absorbed as well



Momentum distributions

- Coulomb deflection and diffractive scattering affect the transverse distribution
  $\longrightarrow$ measure parallel (longitudinal) momentum distributions
- width (and shape) of the parallel momentum distribution allows to make spin and parity assignments
- common use of knockout reactions in combination with γ-ray spectroscopy for nuclear structure studies

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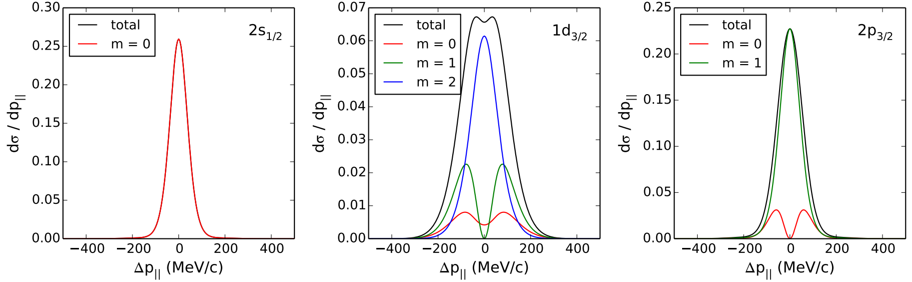

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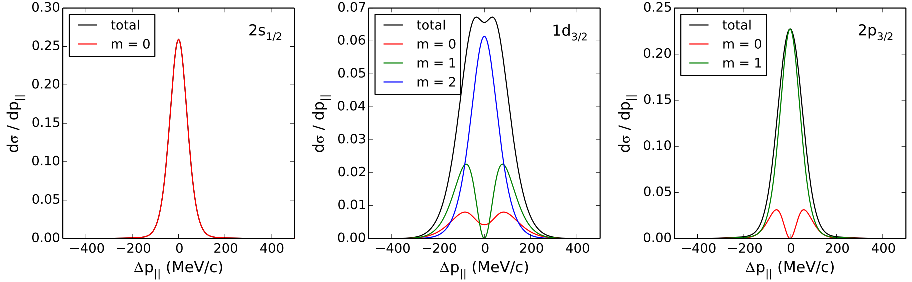

- for comparison with experiment:
  → transformation into laboratory system and convolution with resolution

Sensitivity to single-particle structure

- one-nucleon knockout probability

$$P(\vec{b}\,) = |S_c(\vec{b}\,)|^2 \int |\phi_{nlj}(\vec{r}\,)|^2 \left(1 - |S_v(\vec{b}_v)|^2\right) \, \mathrm{d}\vec{r}$$

- core survival probability $|S_c|^2$
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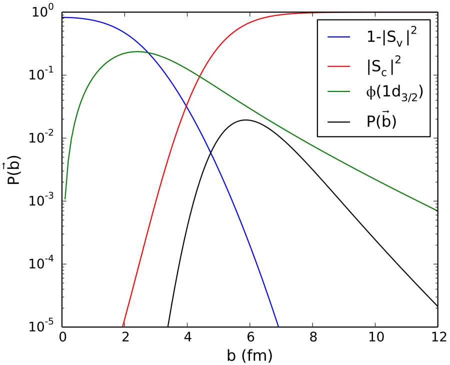

example: $1d_{3/2}$ neutron knockout from ^{24}O on ^{9}Be at 100 MeV/u

- sensitivity to the surface
- probing the valence space
- asymptotic normalization coefficients:

$$R(r_a) = C_l \frac{W_{-\eta,l+1/2}(2kr_a)}{r_a}$$

$R(r_a)$ radial wave function at asymptotic distance r_a, W Whittaker function

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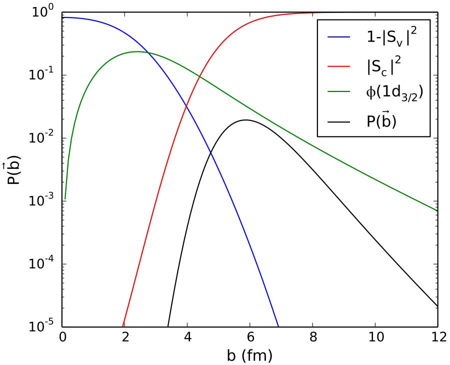

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Some results and open questions

The spectroscopic factor

Theoretical partial cross section for the removal of a nucleon from a single-particle state j^π

- populating state f in the residue nucleus (excitation energy E_f^*, effective separation energy $S_f^* = S + E_f^*$)

$$\sigma_{\text{th}}(f) = \left(\frac{A}{A-1}\right)^N C^2S(f,j^\pi)\sigma_{\text{sp}}(j,S_f^*)$$

- N harmonic oscillator shell number for center of mass correction
- $C^2S(f,j^\pi)$ shell model spectroscopic factor
- inclusive cross section: sum over all bound states:

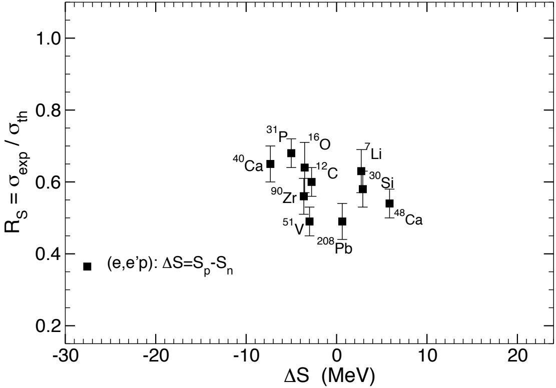
$$\sigma_{\text{th}} = \sum_{\text{bound}} \sigma_{\text{th}}(f)$$

- many input parameters into σ for the reaction geometry
- comparison to theory by cross section ratio

$$R_S = \frac{\sigma_{\text{exp}}}{\sigma_{\text{th}}}$$

Asymmetry in binding energy

- in (e,e′p) experiments on stable target a reduction of the spectroscopic strength of $R_S \approx 0.65$ was found.
- stable nuclei have a limited range of proton to neutron asymmetry $\Delta S = S_p - S_n$
- radioactive nuclei at the drip-lines like ^{32}Ar or ^{20}C have $|\Delta S| \approx 20$



J. A. Tostevin and A. Gade, Phys. Rev. C **90** (2014) 057602

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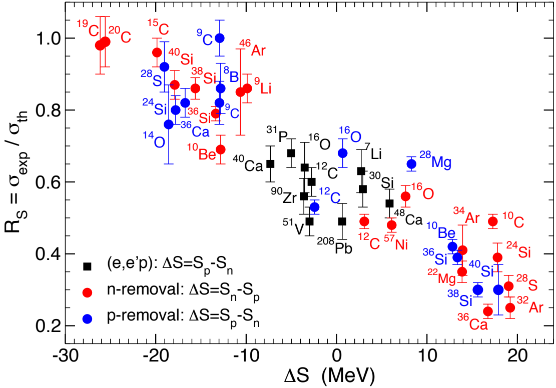

J. A. Tostevin and A. Gade, Phys. Rev. C **90** (2014) 057602

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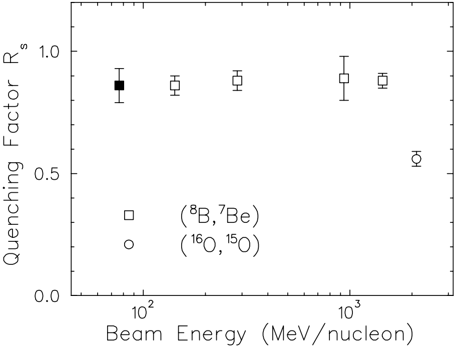
Dependence on the beam energy

- claim: 100 MeV/u is too low for the eikonal approximation
- measurements of proton knockout from ^{8}B from 76 to 1440 MeV/u

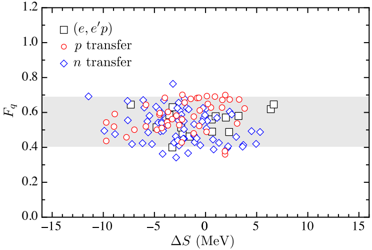


E (MeV/u)	σ_{exp} (mb)	R_S
76	130(11)	0.86(7)
142	109(1)	0.86(1)
285	89(2)	0.88(2)
936	94(9)	0.89(9)
1400	96(3)	0.88(3)

J. Enders et al., Phys. Rev. C **67** (2003) 064301, B. Blank et al., Nucl. Phys. A **624** (1997) 242,
D. Cortina-Gil et al., Phys. Lett. B **529** (2002) 36, D. Cortina-Gil et al., Eur. Phys. J. A **10** (2001)49,
B. A. Brown et al., Phys. Rev. C **65** (2002) 061601

- consistent results over a large range of energies
- $\rightarrow$ need to cover a larger range of ΔS as well

Quenching in transfer reactions



- reanalysis of transfer reactions with stable nuclei
- (d,p), (p,d), ^{3}He and α induced reactions
- all consistent with (e,e′p)

B. P. Kay et al., Phys. Rev. Lett. **111** (2013) 042502

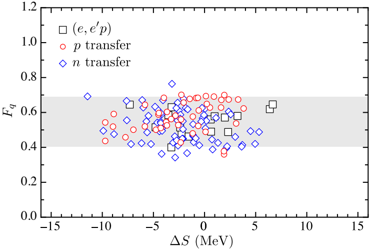
- (p,d) transfer reactions with Ar isotopes at 33 MeV/u
- no dependence on ΔS observed, but strong dependence on choice of optical model

J. Lee et al., Phys. Rev. Lett. **104** (2010) 112701

- similar observations for d(^{14}O,t,^{3}He) at 18 MeV/u

F. Flavigny et al., Phys. Rev. Lett. **110** (2013) 122503

Quenching in transfer reactions



- reanalysis of transfer reactions with stable nuclei
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B. P. Kay et al., Phys. Rev. Lett. **111** (2013) 042502

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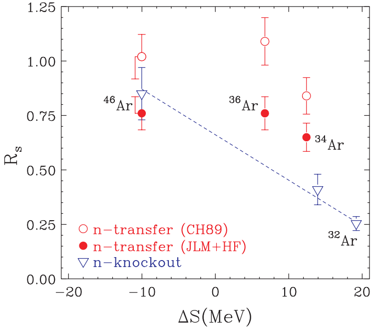

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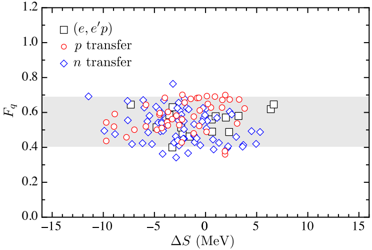

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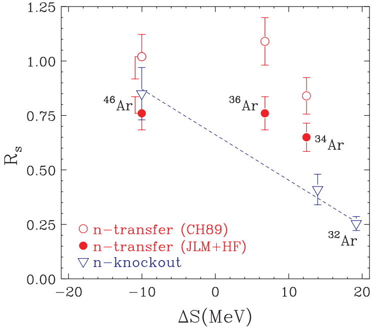

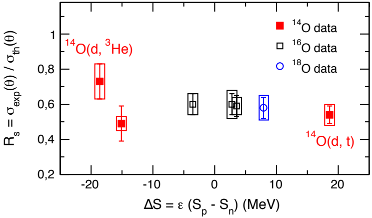

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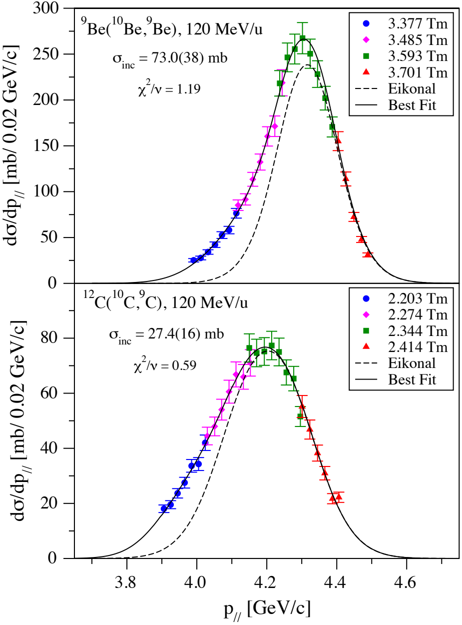
Light nuclei: test of ab-initio methods



- scanning the momentum distribution
- precise measurements of absolute cross sections of light *p*-shell nuclei
- deviations from the eikonal theory
- *p*-shell nuclei can be calculated in ab-initio methods
- overlap function derived from variational Monte-Carlo (VMC) and no-core shell model (NCSM)
- systematic difference at large radii
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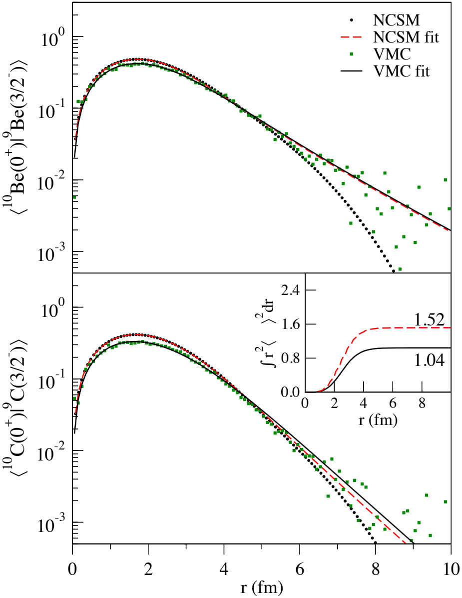

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Systematic studies of light nuclei

- $A = 10$ results for neutron knockout:

projectile	σ_{exp} (mb)	SM (mb)	NCSM (mb)	VMC (mb)
^{10}Be	73(4)	96.6	86.9(16)	72.8(13)
^{10}C	23.2(10)	48.0	43.4(9)	30.8(6)

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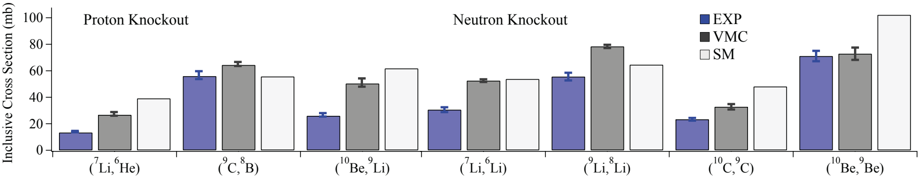
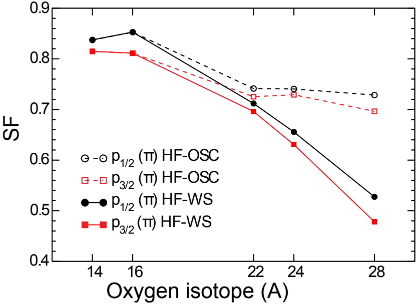


- systematic study for several nuclei
- VMC agrees for removal of deeply bound nucleons
- less good description for weakly bound

G. F. Grinyer et al., Phys. Rev. C **86** (2012) 024315

The role of the continuum

- experimentally: more bound → more reduction factor
- explore the role of the continuum and the effect on the removal strength for weakly bound nucleons
- ab-initio coupled cluster theory for oxygen isotopes $^{14-28}$O



- spectroscopic factors calculated with continuum states included (HF-WS) show a quenching towards the drip-line
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- data required for the neutron-rich oxygen isotopes

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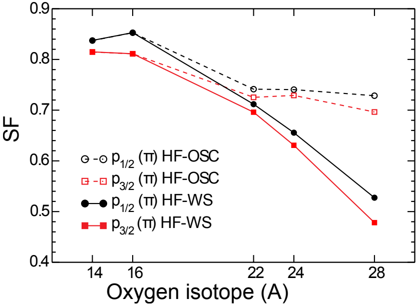

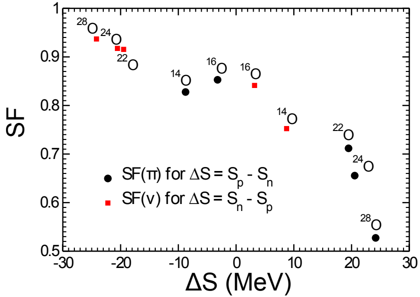

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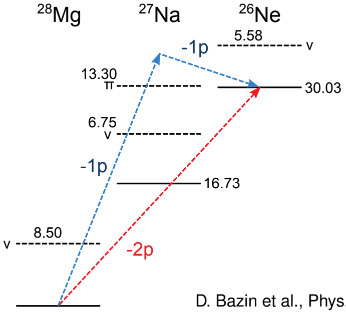
Two-nucleon knockout

Two-proton knockout reactions from neutron-rich nuclei

- give access to even more exotic nuclei
- are direct reactions



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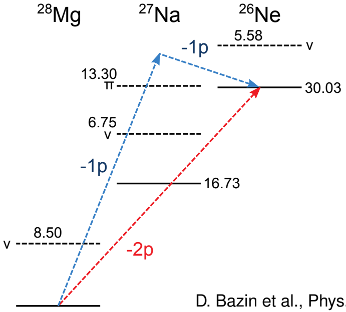

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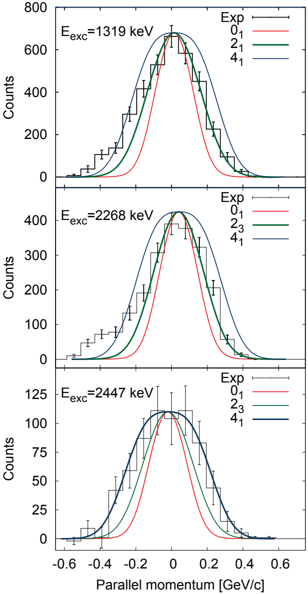

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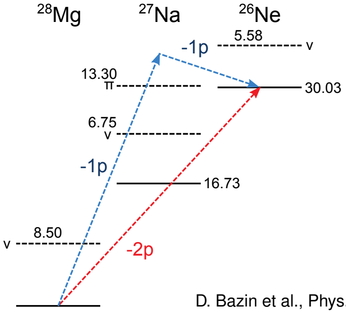

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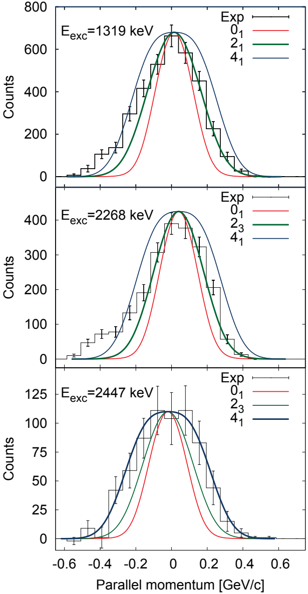

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Two-nucleon knockout cross section

- two-nucleon overlap functions:
  remove two nucleons from orbitals $(nlj)_{1,2}$ coupled to I, μ

$$\Psi^{(f)}_{J_i M_i} = \langle \Phi_{J_f M_f}(A) | \Psi_{J_i M_i}(A,1,2) \rangle = \sum_{I\mu\alpha} C_\alpha^{J_i J_f I} \langle I\mu J_f M_f | J_i M_i \rangle \left[\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)} \right]_{I\mu}$$

  with $\alpha = n_1 l_1 j_1 n_2 l_2 j_2$, ϕ_j single-particle wave functions
  $\rightarrow C^{J_i J_f I}$ signed two-nucleon amplitudes (equivalent of spectroscopic factors)

- stripping cross section to final state f:

$$\sigma^{(f)}_{\text{str-str}} = \int |S_c|^2 \frac{1}{2J_i+1} \sum_{M_i} \left\langle \Psi^{(f)}_{J_i M_i} \middle| (1-|S_1|^2)(1-|S_2|^2) \middle| \Psi^{(f)}_{J_i M_i} \right\rangle 2\pi b \, \mathrm{d}b$$

  under the assumption that the S-matrix is diagonal with respect to the different states $S_f \rightarrow S_c$

J. A. Tostevin and B. A. Brown, Phys. Rev. C **74** (2006) 064604

- reminder for one-nucleon knockout:

$$\sigma_{\text{str}} = \int \left\langle \phi_0 \middle| |S_c(b_c)|^2 (1-|S_v(b_v)|^2) \middle| \phi_0 \right\rangle 2\pi b \, \mathrm{d}b$$

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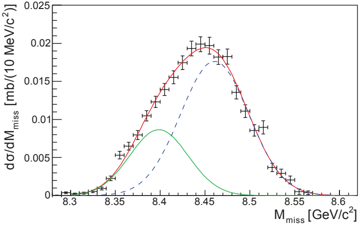
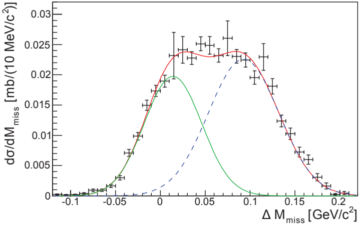
J. A. Tostevin and B. A. Brown, Phys. Rev. C **74** (2006) 064604

Elastic and inelastic breakup contributions

- test the reaction theory by measuring exclusive cross sections
- detection of the knocked out particles
  → missing mass indicated the state of the target nucleus





- disentangle different contributions

	diff-diff	diff-str	str-str	tot.
σ_{exp} (mb)	0.11(3)	0.44(23)	0.87(23)	1.43(5)
fraction (%)	8(2)	31(16)	61(16)	
$\sigma_{theo} \cdot R_S(2N)$ (mb)	0.09	0.55	0.83	1.475
fraction_{theo} (%)	6.3	37.4	56.3	

- good agreement for relative contributions of the reaction processes

K. Wimmer et al., Phys. Rev. C **85** (2012) 051603(R)

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- removal of a pair
  $\rightarrow$ states with J_f^π corresponding to the coefficients of fractional parentage
- $B_0([1d_{5/2}]^6) = 1/6$
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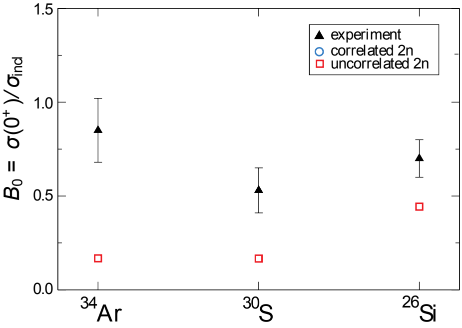

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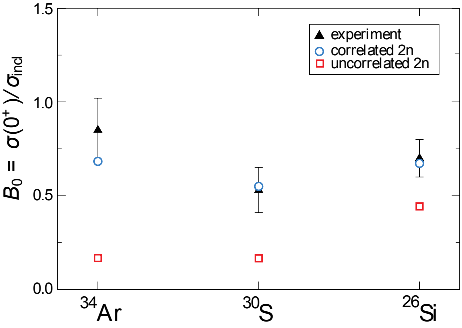

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- good agreement with USD shell model calculations for all cases
- these reactions can be used to constrain theoretical (structure) calculations

Two-nucleon knockout as a tool

- for several cases the inclusive cross section has been measured
- in comparison with shell model calculations a reduction is observed:

$$R_s(2N) = \frac{\sigma_{\text{exp}}}{\sigma_{\text{th}}}$$

- $R_s(2N) = 0.5$ for all cases measured
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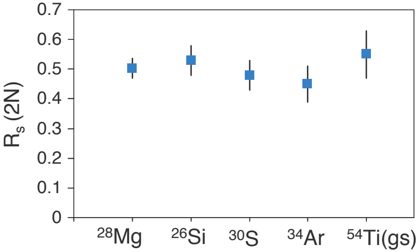

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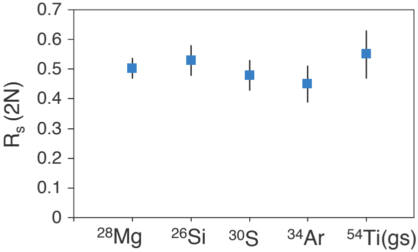

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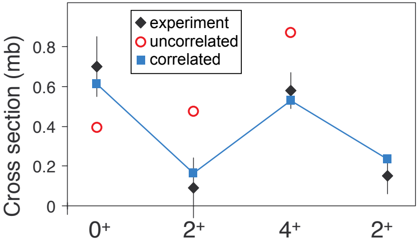

- with $R_s(2N)$ included the shell model reproduces also exclusive cross sections
- $\rightarrow$ test predictions for TNA throughout the nuclear chart

D. Bazin et al., Phys. Rev. Lett. **91** (2003) 012501

New developments and future directions

Deformed projectiles

- in the eikonal model the overlap is determined by the size of target and core
- orientation of projectile symmetry axis with respect to target matters

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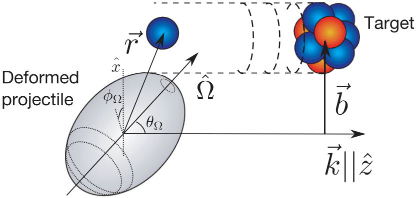
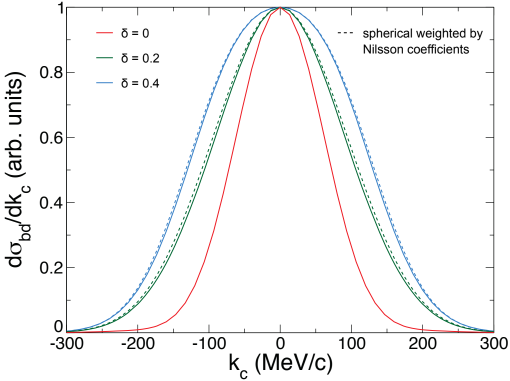
- orientation of projectile symmetry axis with respect to target matters

- first study in a simplified absorptive disk model





- large, prolate deformation: knockout from prolate-like Nilsson states reduced
- oblate-like Nilsson states: cross sections increased
- momentum distributions remain characteristic of the orbital angular momentum of the initial state

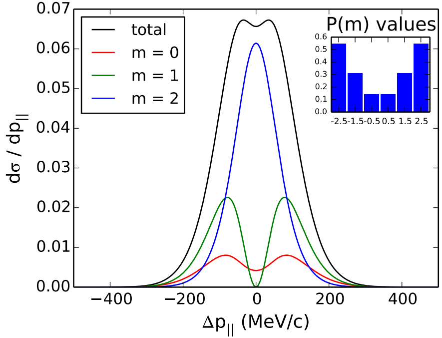
E. C. Simpson and J. A. Tostevin, Phys. Rev. C **86** (2012) 054603

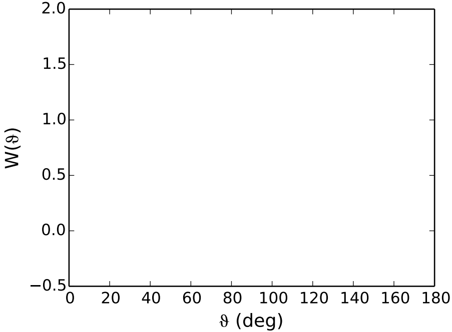
Alignment

- knockout reaction produce significant alignment of states

$$P^J(m) = \sigma_m^J / \sum_m \sigma_m^J = \sigma_m^J / \sigma^J$$

- example: $1d_{5/2}$ neutron knockout from ^{24}O on ^{9}Be at 100 MeV/u
- determine multipolarity by γ-ray angular distribution
  but: limited coverage and resolution
- gating on central part of momentum distribution ($|\Delta p_{||}| < 50$ MeV/c)
  enhances $P(m = 2)$ from 54 to 82 %





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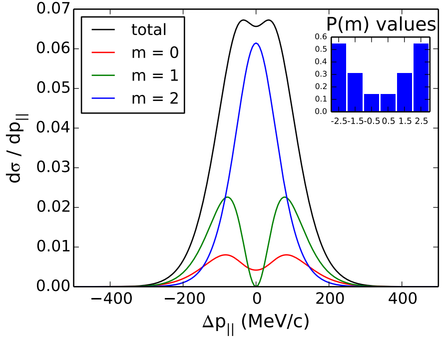

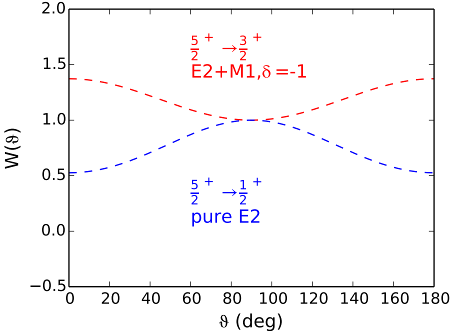

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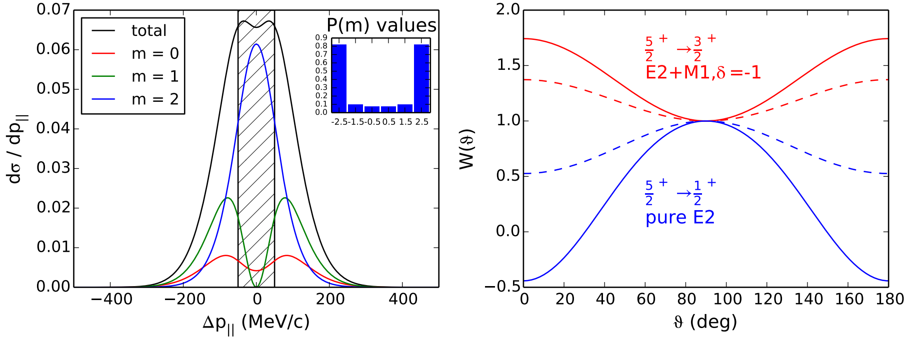
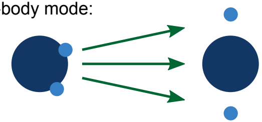


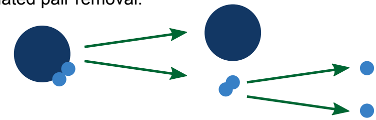
Correlations in two-nucleon knockout

there are several way how the two nucleons can be knocked out:

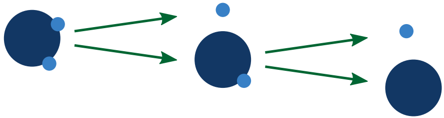
three-body mode:



correlated pair removal:



two-step process (excluded by separation energy):



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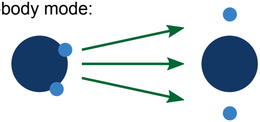

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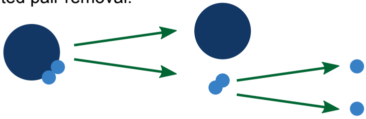

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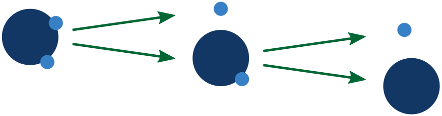
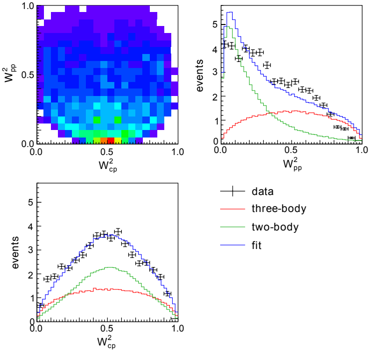


- Dalitz plots of pairs of invariant masses



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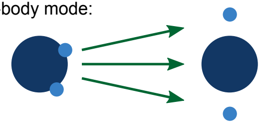

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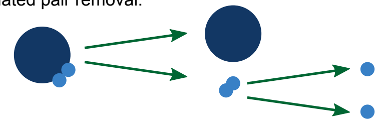

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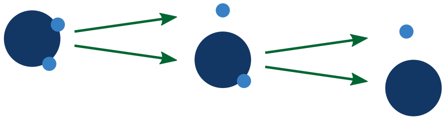

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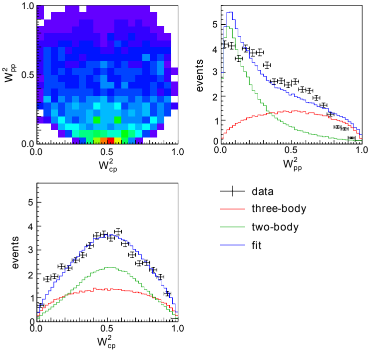

- significant correlation of the two protons
- small relative momentum
- $\rightarrow$ surface localization and spacial proximity

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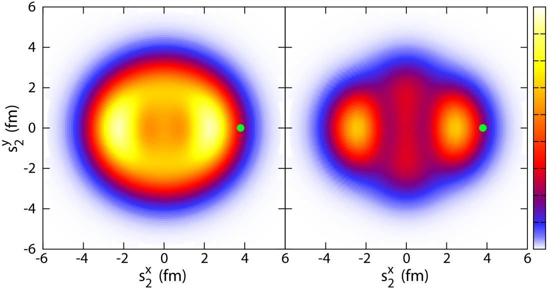
Correlations

- two-nucleon joint position probabilities in the impact parameter plane: $P(\mathbf{s}_1, \mathbf{s}_2)$ integrated over $z_{1,2}$ ($z =$ beam axis), proton 1 $\mathbf{s}_1$ at the surface
- $S = 0$ enhances spacial correlation

all   only $S = 0$



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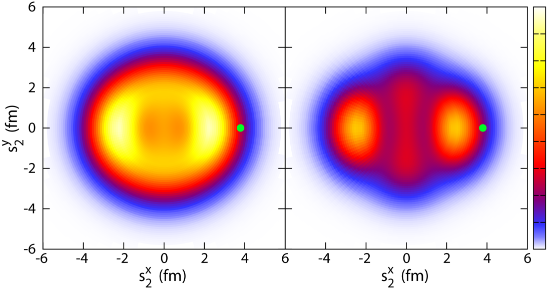

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- 56(12) % correlated proton pair fraction measured

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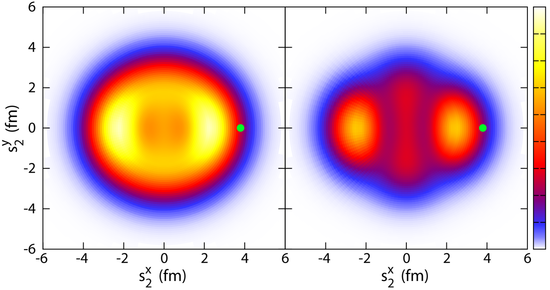

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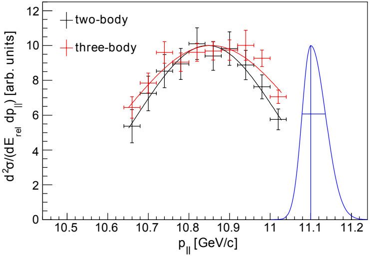
- $S = 0$ has a more narrow momentum distribution



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$\rightarrow$ a new probe of the spin correlations of valence nucleons

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Nuclear targets versus (p,pN)

quasi-free scattering experiments with radioactive beams

- probe valence and deeply bound states
- do not limit the sampling of the wave function to the surface
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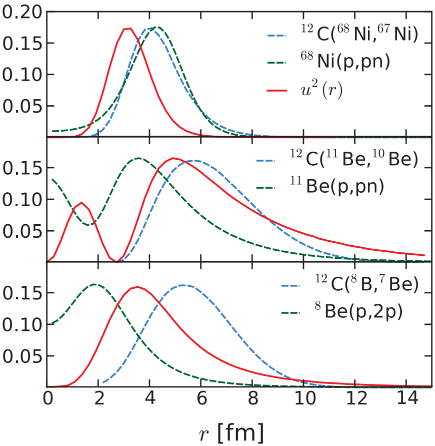

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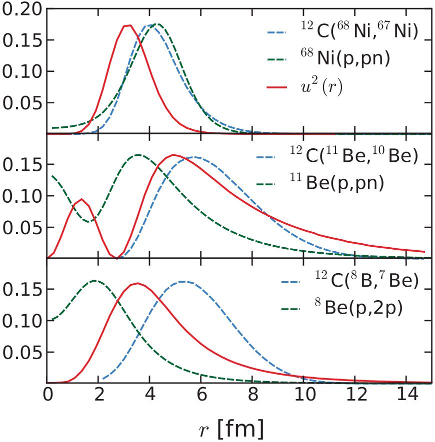
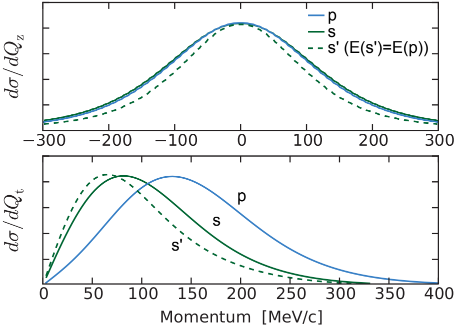


- momentum distributions are challenging
- → measure transverse momentum



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Summary

- nucleon removal reactions are an excellent tool to study the single-particle structure of nuclei
- with radioactive beams on light targets the give access to the most exotic nuclei, neutron and proton-rich
- at intermediate energies the eikonal and sudden approximations give an excellent description of many experiments
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- new approaches and techniques are developed at many places for both theory and experiment

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