TRIUMF
Canada's national laboratory for particle and nuclear physics
Laboratoire national canadien pour la recherche en physique nucléaire
et en physique des particules

Ab initio calculations of nuclear reactions important for astrophysics

NIC-XIV School 2016
Niigata University (Ikarashi Campus)
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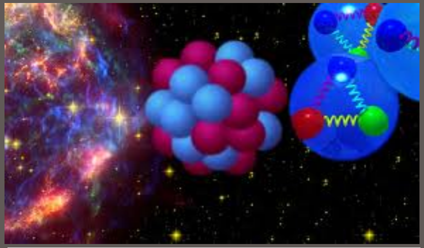
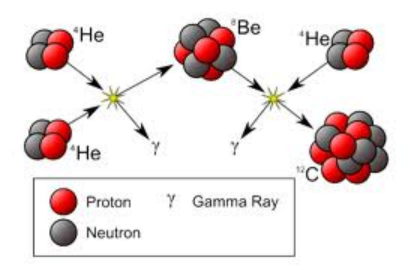
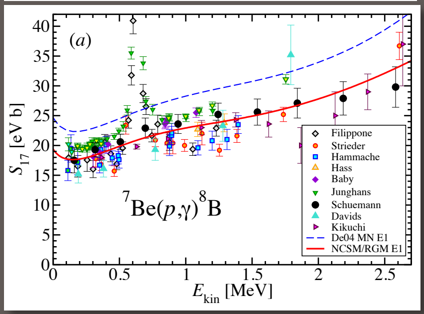
Petr Navratil | TRIUMF

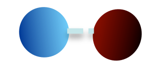




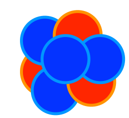


Outline

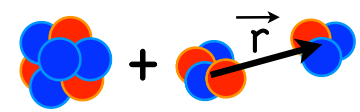
- Lecture 1
  - Nuclear forces
    - chiral EFT, two-nucleon, three-nucleon



  - Nuclear many-body calculations for bound states
    - No-core shell model (NCSM)



  - Unitary transformations
    - Similarity Renormalization Group
  - Nuclear many-body calculations including continuum

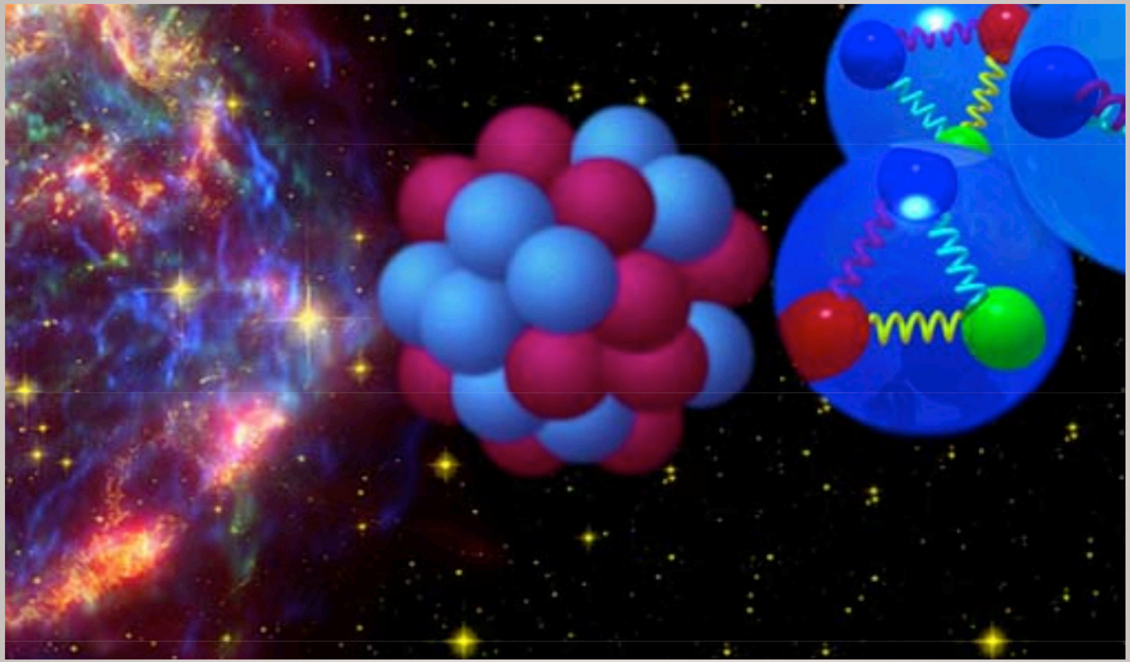


    - NCSM with the Resonating Group Method (NCSM/RGM)
    - NCSM with continuum (NCSMC)
- Lecture 2
  - *Ab initio* calculations of nuclear reactions important for astrophysics
    - ^{7}Be(p,γ)^{8}B, ^{3}He(α,γ)^{7}Be, ^{3}H(α,γ)^{7}Li, ^{3}He(d,p)^{4}He, ^{3}H(d,n)^{4}He
    - Progress towards ^{2}H(α,γ)^{6}Li, ^{4}He(nn,γ)^{6}He, ^{11}C(p,γ)^{12}N

Role of Nuclear Theory

Develop a unified theory of nuclei

Connecting to Astrophysics



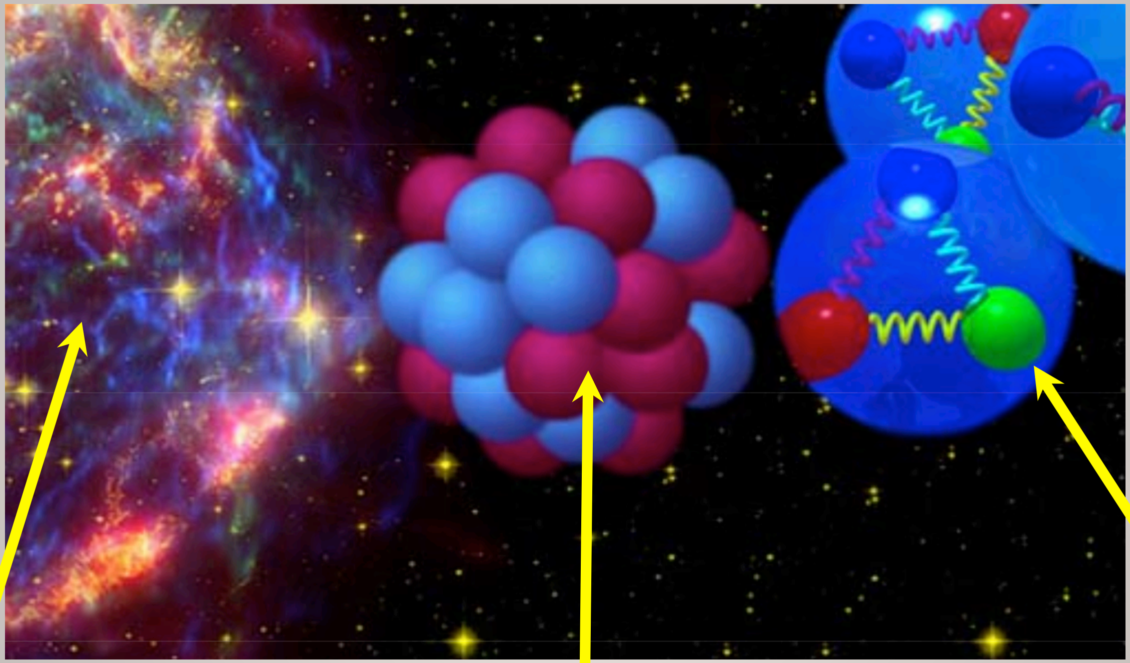
Connecting to QCD

Role of Nuclear Theory

Develop a unified theory of nuclei

more specific questions...

Connecting to Astrophysics



Connecting to QCD

- How does nuclear physics drive phenomena like the creation of elements and SN explosions?
- How do nuclear forces give rise to nuclear structure? How do we explain reactions?
- How do nuclear forces emerge from QCD?

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What is meant by *ab initio* in nuclear physics?

- **First principles for Nuclear Physics:**
  QCD
  - Non-perturbative at low energies
  - Lattice QCD in the future

- Degrees of freedom: NUCLEONS
  - Nuclei made of nucleons
  - Interacting by nucleon-nucleon and three-nucleon potentials

- *Ab initio*
  - ✧ All nucleons are active
  - ✧ Exact Pauli principle
  - ✧ Realistic inter-nucleon interactions
    - ✧ Accurate description of NN (and 3N) data
  - ✧ Controllable approximations

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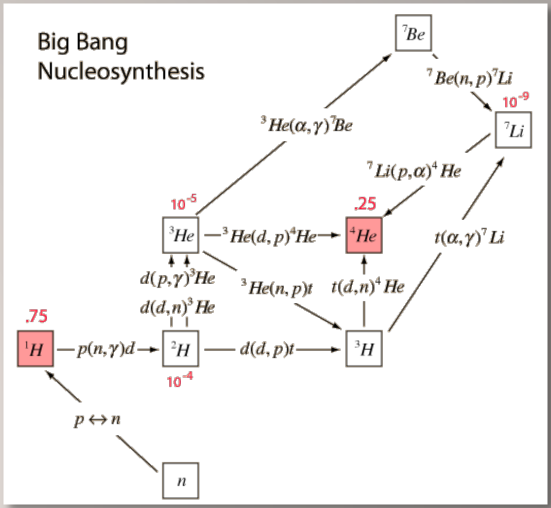
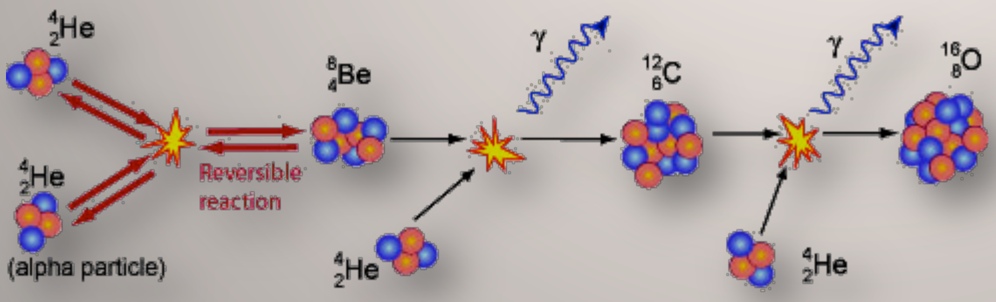
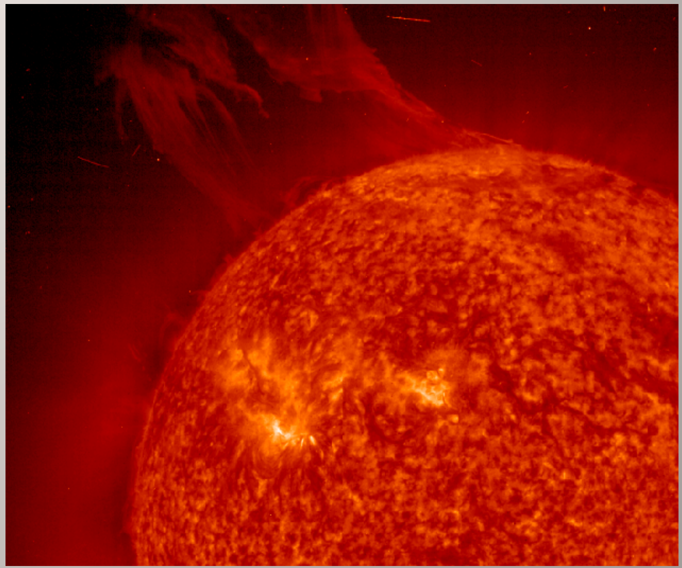
Why nuclei from first principles?

- **Goal**: Predictive theory of structure and reactions of nuclei
- Needed for
  - Physics of **exotic nuclei**, tests of fundamental symmetries
  - Understanding of nuclear reactions important for **astrophysics**
  - Understanding of reactions important for **energy generation**
  - **Double beta decay** nuclear matrix elements
  - **Neutrino-nucleus** cross sections
  - …

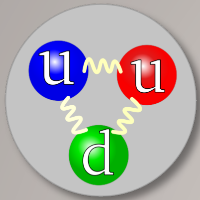


Understanding our Sun

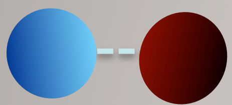




From QCD to nuclei

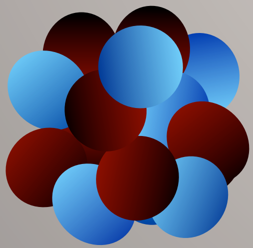


Low-energy QCD



NN+3N interactions from chiral EFT

…or accurate meson-exchange potentials



Nuclear structure and reactions

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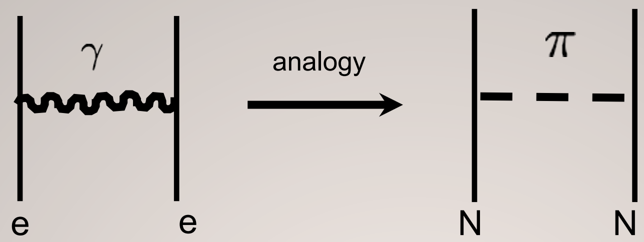
Nuclear forces

Nucleon-Nucleon force



$r \sim \frac{1}{m}$

one-pion exchange potential (OPE)

electromagnetic force:
infinite range → exchange of massless particle

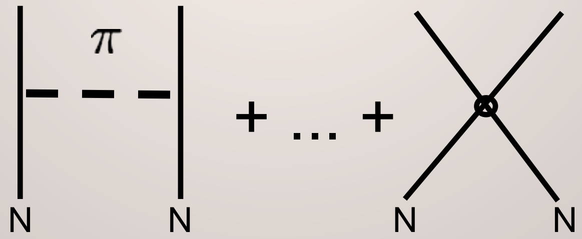
NN force:
finite range → exchange of massive particle

Yukawa
Nobel price in 1949

Nowadays:

New vision of Effective Field Theory   Links low energy physics to QCD in a systematic way

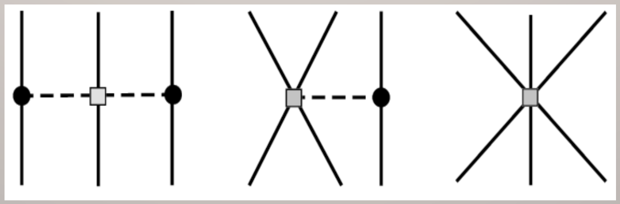
Nucleon-Nucleon force



Details of short distance physics not resolved, but captured in short range couplings → should come from QCD but are now fit to experiment

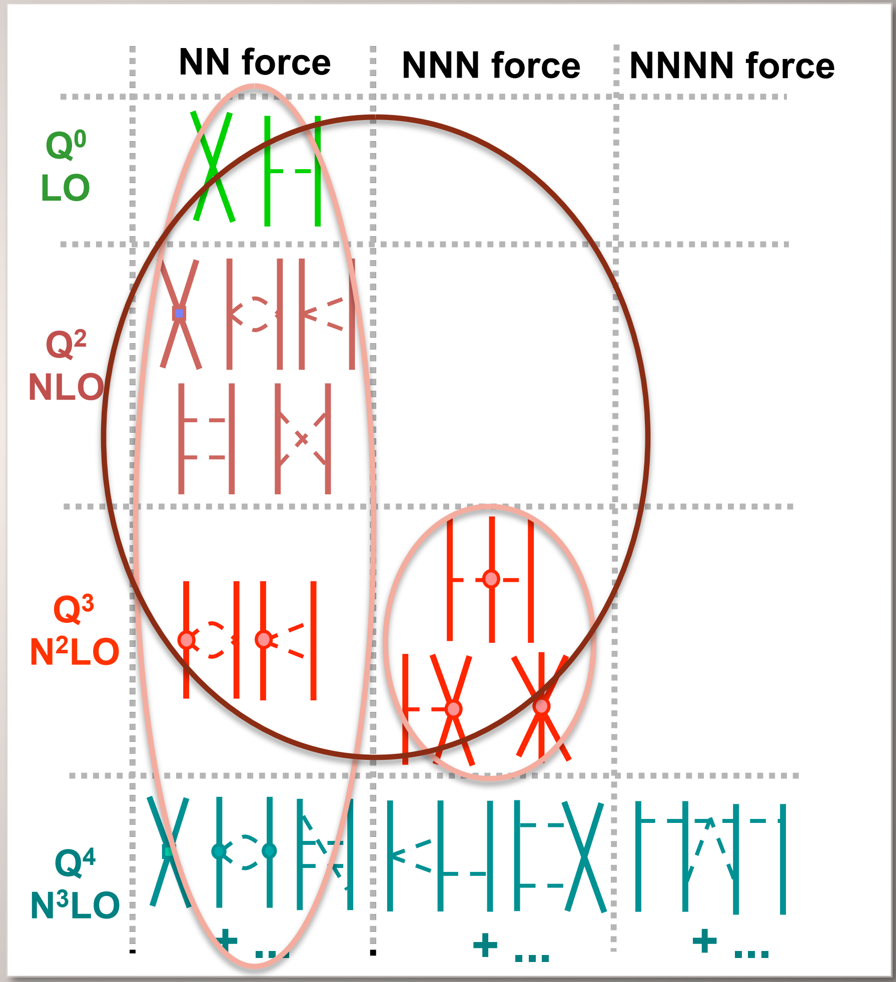
Many-Nucleon forces

Arise due to the effective nature of nuclear forces



Chiral Effective Field Theory

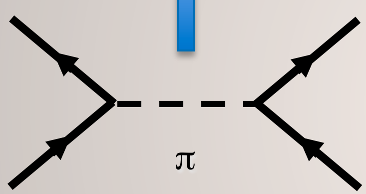
- Inter-nucleon forces from chiral effective field theory
  - Based on the symmetries of QCD
    - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order (Q/Λ_χ)
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD



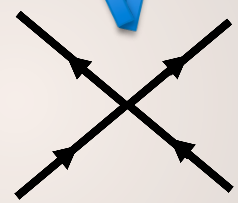
Λ_χ~1 GeV :
Chiral symmetry breaking scale

Chiral EFT NN interaction in the leading order (LO)

$$V^{\mathrm{LO}} = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M_\pi^2} + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$



one-pion exchange



contact

C_S , C_T :
Low-energy constants (LECs) fitted to NN data

$\vec{q} = \vec{k}' - \vec{k}$ …momentum transfer

g_A=1.29 …axial-vector coupling constant

F_π=92.4 MeV …pion decay constant

Regularized, e.g., by:

$$\exp\left(-(k'/\Lambda)^{2n} - (k/\Lambda)^{2n}\right)$$

Λ ~ 500 MeV << Λ_χ~ 1 GeV

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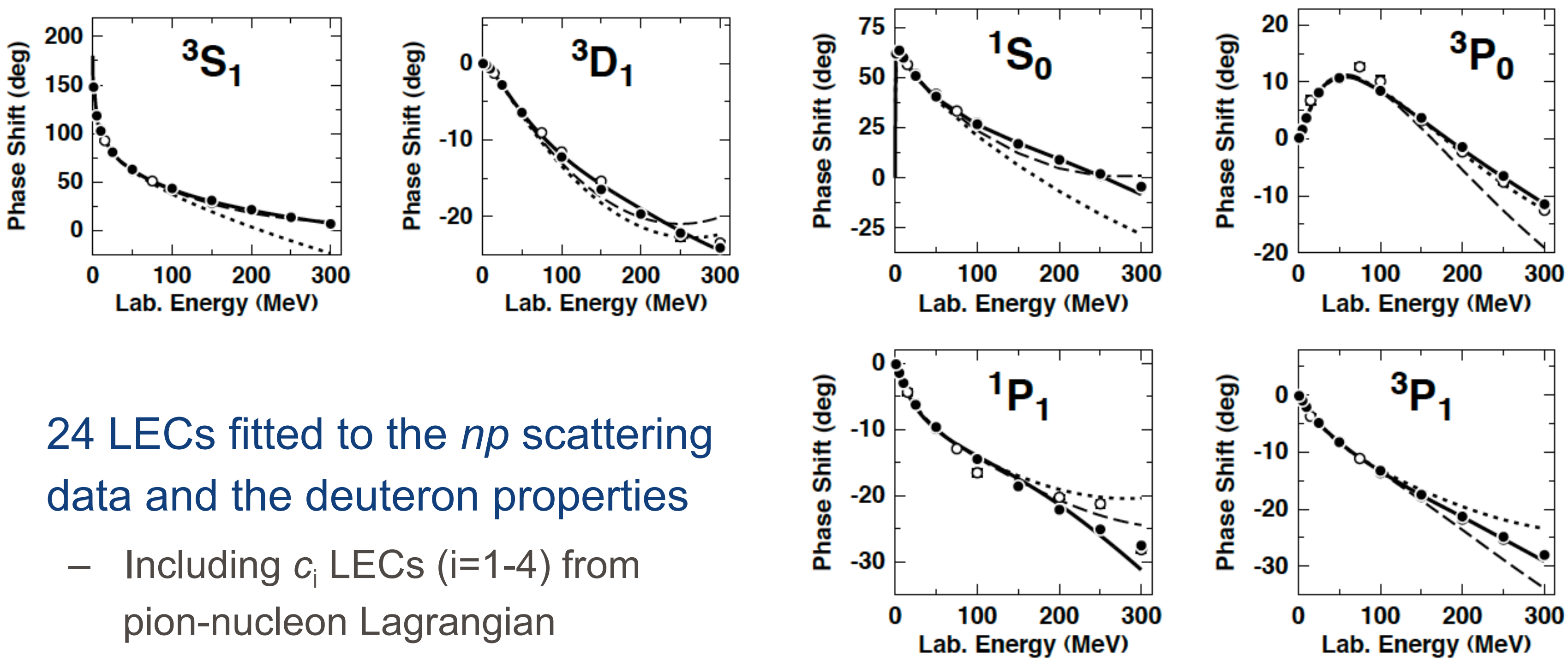
The NN interaction from chiral EFT

PHYSICAL REVIEW C **68**, 041001(R) (2003)

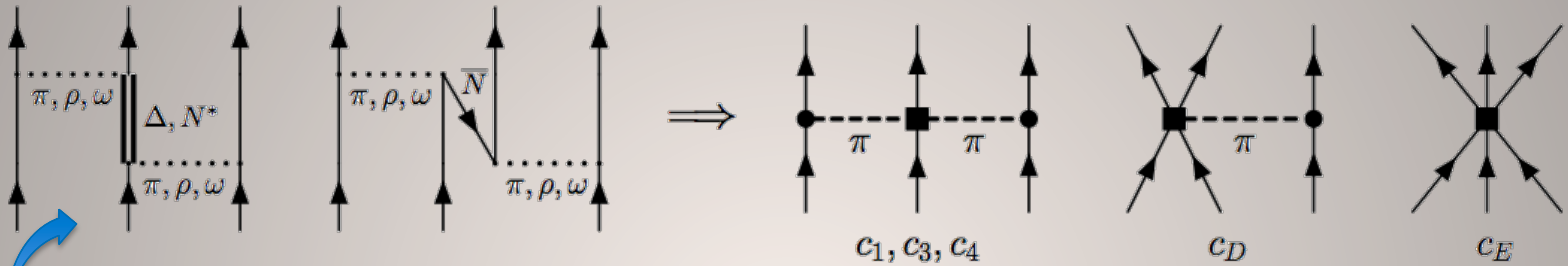
Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory

D. R. Entem[1,2,*] and R. Machleidt[1,†]



- 24 LECs fitted to the *np* scattering data and the deuteron properties
  - Including c_i LECs (i=1-4) from pion-nucleon Lagrangian

Three-nucleon forces why?



Eliminating degrees of freedom leads to three-body forces.

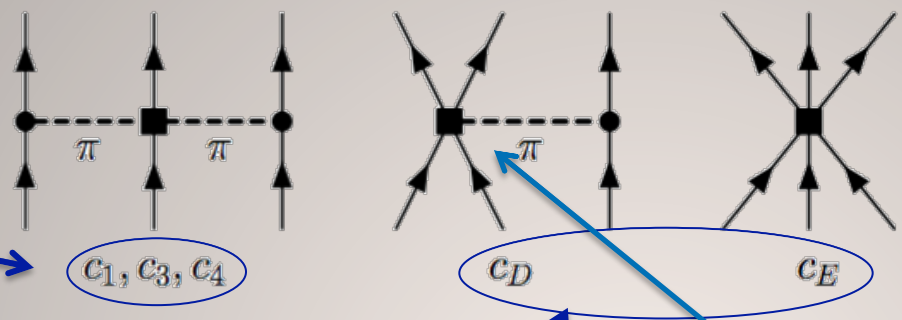
Two-pion exchange with **virtual Δ excitation** – Fujita & Miyazawa (1957)

- Leading three-nucleon force terms
  - Long-range two-pion exchange
  - Medium-range one-pion exchange + two-nucleon contact
  - Short range three-nucleon contact

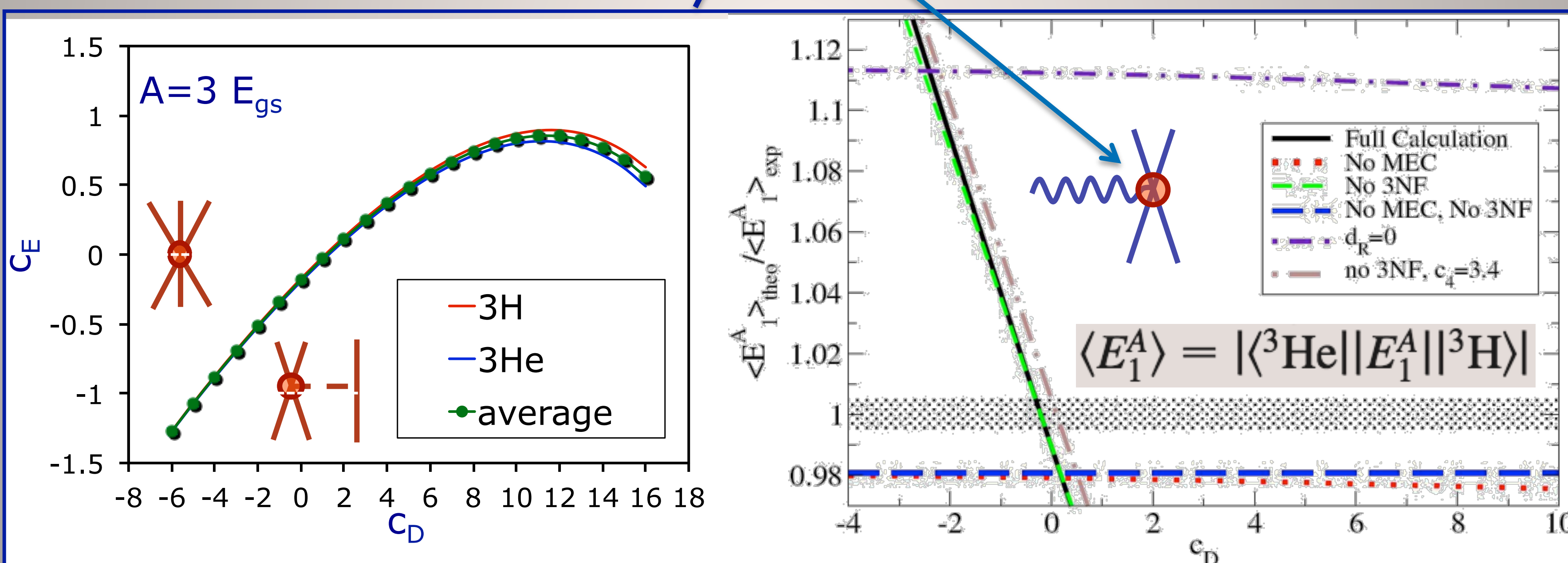
The question is not: Do three-body forces enter the description?
The only question is: How large are three-body forces?

Leading terms of the chiral NNN force

From NN & pion-nucleon scattering



Chiral EFT provides a link between the medium-range (c_D term) NNN force and the meson-exchange current appearing in nuclear beta decay



NNN parameters determined from the ^{3}H binding energy and half life

PRL 103, 102502 (2009) PHYSICAL REVIEW LETTERS week ending 4 SEPTEMBER 2009

Three-Nucleon Low-Energy Constants from the Consistency of Interactions and Currents in Chiral Effective Field Theory

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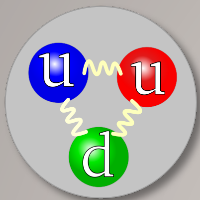
From QCD to nuclei

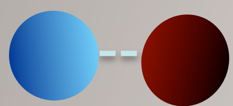


Low-energy QCD

↓

NN+3N interactions from chiral EFT

…or accurate meson-exchange potentials



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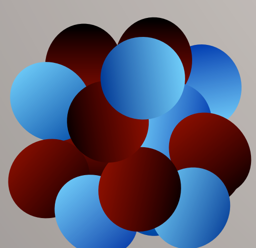
Many-Body methods

NCSM, NCSM/RGM, NCSMC, CCM, SCGF, GFMC, HH, Nuclear Lattice EFT…

$$H|\Psi\rangle = E|\Psi\rangle$$

↓

Nuclear structure and reactions



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The nuclear many-body problem

- Start with the microscopic *A*-nucleon Hamiltonian

$$H^{(A)} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} V^{2b}(\vec{r}_i - \vec{r}_j) + \left(\sum_{i<j<k=1}^{A} V_{ijk}^{3b} \right)$$

  – Nucleons interact with two- and three-nucleon forces: this yields complicated quantum correlations

- Solve the many-body Schrödinger equation

$$H^{(A)} \Psi^{(A)}(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A) = E \Psi^{(A)}(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A)$$

  – Negative energies (relative to a breakup threshold)– bound-state boundary conditions
    - Find eigenfunctions and eigenenergies
  – Continuum of positive energies – scattering boundary conditions
    - Find elements of the Scattering matrix

The nuclear many-body wave function

- *A* active nucleons – spatial, spin, and isospin degrees of freedom

$$\vec{r}_i \equiv \{\vec{r}_i,\ \vec{\sigma}_i,\ \vec{\tau}_i\},\ i = 1,2,\cdots,A$$

- Nucleons are fermions – wave function antisymmetric

$$\Psi^{(A)}(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_k, \ldots \vec{r}_j, \ldots \vec{r}_A) = -\Psi^{(A)}(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_j, \ldots \vec{r}_k, \ldots \vec{r}_A)$$

- Conserved total angular momentum *J* and parity π
  - approximately conserved total isospin *T*

- We are not interested in the motion of the center of mass, but only in the intrinsic motion
  - Look for translationally invariant wave function. Two options:
    - Work with *A* - 1 translational invariant coordinates known as Jacobi coordinates
    - Work with *A* single particle coordinates and aim at exact separation between intrinsic and center of mass motion

$$\Psi^{(A)}(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A) = \psi^{(A)}(\vec{\xi}_1, \vec{\xi}_2, \ldots \vec{\xi}_{A-1})\Psi_{CM}(\vec{R}_{CM})$$

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How to solve the many-body Schrödinger equation?

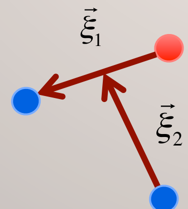
- The nuclear wave function must factorize, e.g., for free c.m. motion

$$\Psi^{(A)} = \psi^{(A)} \exp\left(-i \frac{\vec{P}_{CM}\vec{R}_{CM}}{\hbar}\right) \qquad E = \varepsilon + \frac{P_{CM}^2}{2Am}$$

- First option: solve eigenvalue problem for the intrinsic Hamiltonian
  - ☺ The c.m. motion is not present from the beginning
  - ☺ Work with 3(*A*-1) spatial degrees of freedom (Jacobi relative coordinates)
  - ☹ Jacobi coordinates do not treat the nucleons in a symmetric manner

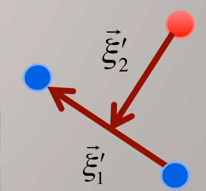
$$\hat{P}_{ij}\phi_{s,n}^{(A)}(\vec{\xi}_1,\ldots\vec{\xi}_{A-1}) = \phi_{s,n}^{(A)}(\hat{P}_{ij}\vec{\xi}_1,\ldots\ \hat{P}_{ij}\vec{\xi}_{A-1}) = \sum_{m=1}^{N} R_{nm}\phi_{s,m}^{(A)}(\vec{\xi}_1,\ldots\vec{\xi}_{A-1})$$

$$\begin{cases} \vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \\ \vec{\xi}_2 = \sqrt{\frac{2}{3}}\left[\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right] \end{cases}$$



$2 \leftrightarrow 3$

$$\begin{cases} \vec{\xi}'_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_3) \\ \vec{\xi}'_2 = \sqrt{\frac{2}{3}}\left[\frac{1}{2}(\vec{r}_1 + \vec{r}_3) - \vec{r}_2\right] \end{cases}$$

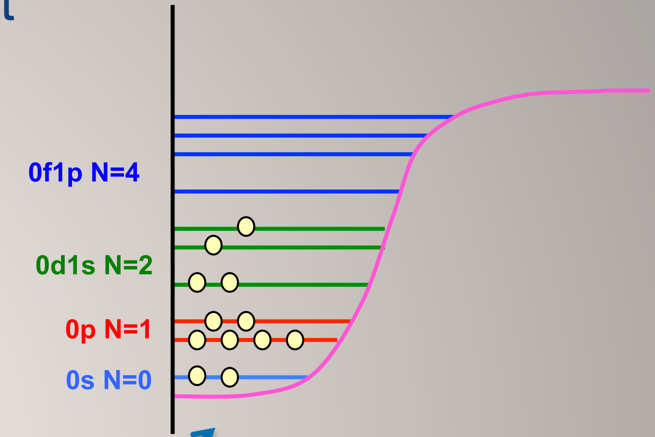


How to solve the many-body Schrödinger equation (for bound states)?

- Second option: tie the system to a fixed point

$$H_{SM}^{(A)} = \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + U_i(r_i)\right) + \sum_{i<j=1}^{A} V^{2b}(\vec{r}_i - \vec{r}_j) - \sum_{i=1}^{A} U_i(r_i)$$

mean field — residual interaction



– Sum of single particle Hamiltonians

$$\left(\frac{p^2}{2m} + U(r)\right)\varphi_k(\vec{r}) = \varepsilon_k \varphi_k(\vec{r})$$

The mean field determines the shell structure

– Antisymmetrized product of single-particle wfs: use these as *A*-body basis states

$$\phi_n^{(A)} = \frac{1}{\sqrt{A!}}\begin{vmatrix} \varphi_i(\vec{r}_1) & \varphi_i(\vec{r}_2) & \cdots & \varphi_i(\vec{r}_A) \\ \varphi_j(\vec{r}_1) & \varphi_j(\vec{r}_2) & & \varphi_j(\vec{r}_A) \\ & \vdots & \ddots & \vdots \\ \varphi_l(\vec{r}_1) & \varphi_l(\vec{r}_2) & \cdots & \varphi_l(\vec{r}_A) \end{vmatrix}$$

Slater Determinant (SD):

- Great to implement Pauli exclusion principle
- Very convenient, especially in second quantization formalism

How to solve the many-body Schrödinger equation for bound states?

- Single-particle shell-model states are very convenient basis states for expanding the many-body wave function
- However, the introduction of the mean-field potential U destroys the invariance of the system with respect to translations
- The c.m. motion is no longer separable and remains mixed to intrinsic motion, giving rise in general to spurious effects

$$\Psi_{SM}^{(A)} = \sum_n \psi_n^{(A)}\left(\left\{\vec{\xi}_i\right\}\right) g_n(\vec{R}_{CM})$$

  - Factorization for H_{int} only when **complete convergence** reached (exact solution)

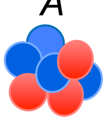
- Exception: harmonic oscillator (HO) potential is exactly separable

$$\sum_{i=1}^{A} \frac{1}{2} m\Omega^2 r_i^2 = \sum_{i<j=1}^{A} \frac{m\Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 + \frac{1}{2} A m\Omega^2 R_{CM}^2$$

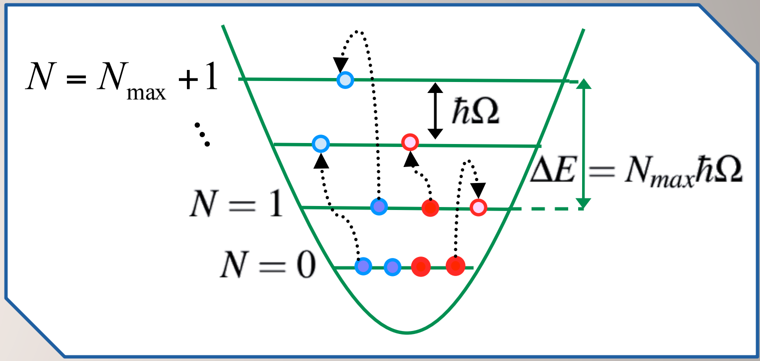
$$= \sum_{i=1}^{A-1} \frac{1}{2} m\Omega^2 \xi_i^2 + \frac{1}{2} A m\Omega^2 R_{CM}^2$$

Ab initio no-core shell model (NCSM)

- An *ab initio* approach to solve the many-body Schrödinger equation for bound states (narrow resonances) starting from
  - High-precision NN+NNN interactions (coordinate/momentum space)
- Uses large (but finite!) expansions in HO many-body basis states



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi^{HO}_{Ni}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)$$

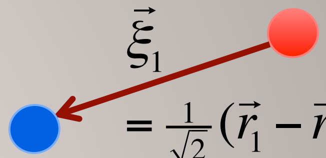


- Choice of either Jacobi relative or Cartesian single-particle coordinates according to the efficiency for the problem at hand
  - Translational invariance of the internal wave function is preserved also when single-particle Slater Determinant (SD) basis is used with $N_{\max}$ truncation
- Convergence to exact result using effective interactions (obtained from unitary transformations of the bare interaction)

$N_{\max}$ … maximal allowed HO excitation above the lowest possible *A*-nucleon configuration
Full $N_{\max}$ space: All basis states with $N \leq N_{\max}$ kept

HO multi-particle states in Jacobi coordinates

- Build many-body basis by adding one particle at the time
- Antisymmetrized two-particle states
  - Start with two-body basis states (LS coupled)



$$\vec{\xi}_1 = \tfrac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

$$\left\langle \vec{\xi}_1 \vec{\sigma}_1 \vec{\sigma}_2 \vec{\tau}_1 \vec{\tau}_2 \middle| n_2 \ell_2 s_2 j_2 t_2 \right\rangle$$

$$= R_{n_2\ell_2}(\xi_1)\left[Y_{\ell_2}(\hat{\xi}_1) \otimes \left[\chi^S_{\frac{1}{2}}(\vec{\sigma}_1) \otimes \chi^S_{\frac{1}{2}}(\vec{\sigma}_2) \right]^{s_2} \right]^{j_2} \left[\chi^T_{\frac{1}{2}}(\vec{\tau}_1) \otimes \chi^T_{\frac{1}{2}}(\vec{\tau}_2) \right]^{t_2}$$

  - Now keep only antisymmetric ones, that is only those for which

$$\hat{P}_{12} \left| n_2 \ell_2 s_2 j_2 t_2 \right\rangle = - \left| n_2 \ell_2 s_2 j_2 t_2 \right\rangle \;\Rightarrow\; (-1)^{\ell_2 + s_2 + t_2} = -1$$

  - Total energy

$$\varepsilon_N = (N + \tfrac{3}{2})\hbar\Omega \qquad\qquad N = 2n_2 + \ell_2$$

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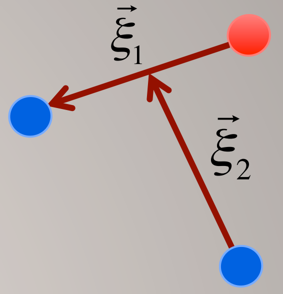
HO three-particle states in Jacobi coordinates

– Add one more body



$$\left\langle \vec{\xi}_2 \vec{\sigma}_3 \vec{\tau}_3 \middle| N_3 L_3 J_3 \right\rangle = R_{N_3 L_3}(\xi_2) \left[Y_{L_3}(\hat{\xi}_2) \otimes \chi^S_{\frac{1}{2}}(\vec{\sigma}_3) \right]^{J_3} \chi^T_{\frac{1}{2}}(\vec{\tau}_3)$$

– Three-body basis (JJ coupled)

$$\left\langle \vec{\xi}_1 \vec{\xi}_2 \vec{\sigma}_1 \vec{\sigma}_2 \vec{\sigma}_3 \vec{\tau}_1 \vec{\tau}_2 \vec{\tau}_3 \middle| \left[n_2 \ell_2 s_2 j_2 t_2 ; N_3 L_3 J_3 \right] JT \right\rangle$$

Note: antisymmetric for exchange of particle 1 and 2, but not for exchange of partcles 1 and 3 or 2 and 3

$$\left| \left[n_2 \ell_2 s_2 j_2 t_2 ; N_3 L_3 J_3 \right] JT \right\rangle$$

$$= \sum_{m_2, M_3} C^{JM}_{j_2 m_2, J_3 M_3} \sum_{m_2^t, M_3^t} C^{TM_T}_{t_2 m_2^t, T_3 M_3^t} \left| n_2 \ell_2 s_2 j_2 t_2 \right\rangle \left| N_3 L_3 J_3 \right\rangle$$

– Total energy: $\varepsilon_N = (N+3)\hbar\Omega$ with $N = 2n_2 + \ell_2 + 2N_3 + L_3$

– To find totally antisymmetric states, diagonalize: $\hat{A} = \frac{1}{3}(1 - \hat{P}_{13} - \hat{P}_{23})$

- Keep only antisymmetric eigenstates, that is those with eigenvalue 1

HO single-particle wave functions

- Start with single-particle HO spatial wave function, defined by radial quantum number n, orbital angular momentum l, and z-projection μ

$$\varphi_{nl\mu}(\vec{r}) = R_{nl}(r)Y_{l\mu}(\hat{r}) \qquad \varepsilon_{nl} = \left(2n + l + \tfrac{3}{2}\right)\hbar\Omega$$

- Now include the spin and isospin wave functions: $\chi^{S}_{\frac{1}{2}m_s}(\vec{\sigma}),\ \chi^{T}_{\frac{1}{2}m_t}(\vec{\tau})$
  - Uncoupled scheme

$$\varphi_{nl\mu\frac{1}{2}m_s\frac{1}{2}m_t}(\vec{r},\vec{\sigma},\vec{\tau}) = R_{nl}(r)Y_{l\mu}(\hat{r})\chi^{S}_{\frac{1}{2}m_s}(\vec{\sigma})\chi^{T}_{\frac{1}{2}m_t}(\vec{\tau})$$

  - j-coupled scheme

$$\varphi_{nljm_j\frac{1}{2}m_t}(\vec{r},\vec{\sigma},\vec{\tau}) = R_{nl}(r)\left[Y_l(\hat{r})\otimes\chi^{S}_{\frac{1}{2}}(\vec{\sigma})\right]^{j}_{m_j}\chi^{T}_{\frac{1}{2}m_t}(\vec{\tau})$$

$$\left[Y_l(\hat{r})\otimes\chi^{S}_{\frac{1}{2}}(\vec{\sigma})\right]^{j}_{m_j} = \sum_{\mu m_s} C^{j\,m_j}_{l\mu,\frac{1}{2}m_s} Y_{l\mu}(\hat{r})\ \chi^{S}_{\frac{1}{2}m_s}(\vec{\sigma})$$

Multi-particle states in the Slater Determinant basis

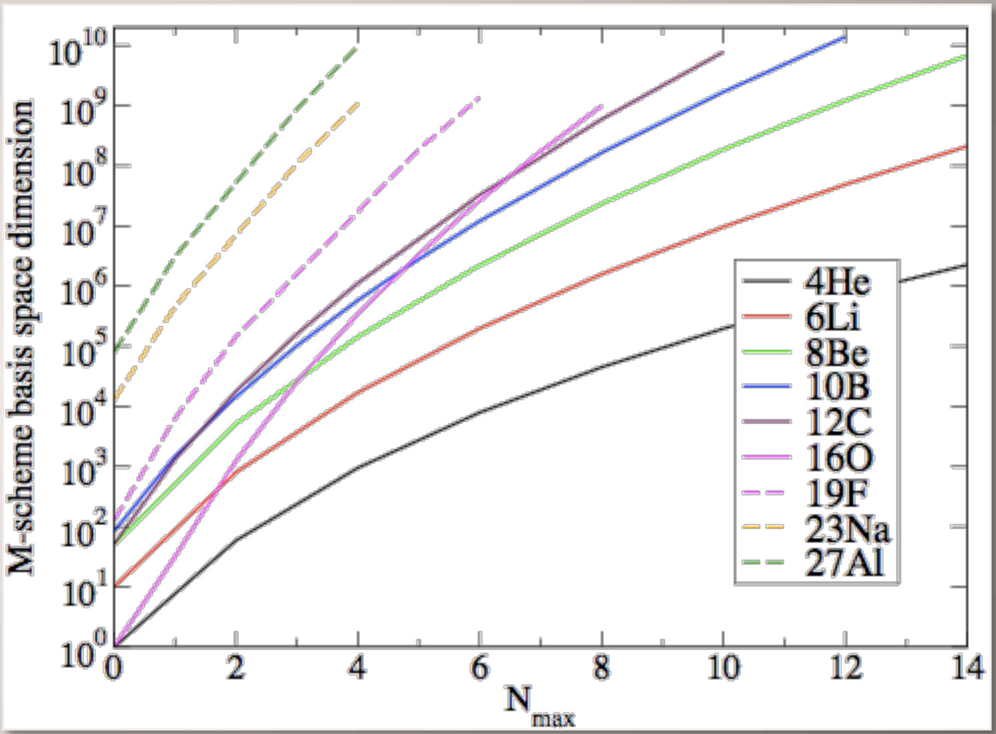
- Many-body HO Slater determinants

$$\left\langle \vec{r}_1\vec{\sigma}_1\vec{\tau}_1, \vec{r}_2\vec{\sigma}_2\vec{\tau}_2, \cdots, \vec{r}_A\vec{\sigma}_A\vec{\tau}_A \middle| a_l^+ \cdots a_j^+ a_i^+ \middle| 0 \right\rangle$$

$$= \frac{1}{\sqrt{A!}} \begin{vmatrix} \varphi_i(\vec{r}_1) & \varphi_i(\vec{r}_2) & \cdots & \varphi_i(\vec{r}_A) \\ \varphi_j(\vec{r}_1) & \varphi_j(\vec{r}_2) & & \varphi_j(\vec{r}_A) \\ & \vdots & \ddots & \vdots \\ \varphi_l(\vec{r}_1) & \varphi_l(\vec{r}_2) & \cdots & \varphi_l(\vec{r}_A) \end{vmatrix}$$

$$\varphi_{nljm_j\frac{1}{2}m_t}(\vec{r},\vec{\sigma},\vec{\tau})$$

$$= R_{nl}(r)\left[Y_l(\hat{r}) \otimes \chi^S_{\frac{1}{2}}(\vec{\sigma})\right]^j_{m_j} \chi^T_{\frac{1}{2}m_t}(\vec{\tau})$$



  - Antisymmetrization is trivial
  - Good M, M_T and parity quantum numbers, but not J and T
    - Huge number of basis states

Second Quantization

- One of the most useful representations in many-body theory
  - $|0\rangle$: the state with no particles (the vacuum)
  - a_i^+ : creation operator, creates a fermion in the state i $\qquad a_i^+|0\rangle = |i\rangle, \quad a_i^+|i\rangle = 0$
  - a_i : annihilation operator, annihilates a fermion in the state i $\qquad a_i|i\rangle = |0\rangle, \quad a_i|0\rangle = 0$
  - Anticommutation relations:

$$\{a_i^+,\ a_j^+\} = \{a_i\,,\ a_j\} = 0, \qquad \{a_i^+,\ a_j\} = \{a_i\,,\ a_j^+\} = \delta_{ij}$$

Pauli principle in second quantization $\quad a_i^+ a_j^+ = -a_j^+ a_i^+$

  - So that the Slater determinant can be written as:

$$\phi_n^{(A)} = \frac{1}{\sqrt{A!}}\begin{vmatrix} \varphi_i(\vec{r}_1) & \varphi_i(\vec{r}_2) & \dots & \varphi_i(\vec{r}_A) \\ \varphi_j(\vec{r}_1) & \varphi_j(\vec{r}_2) & & \varphi_j(\vec{r}_A) \\ & \vdots & \ddots & \vdots \\ \varphi_l(\vec{r}_1) & \varphi_l(\vec{r}_2) & \dots & \varphi_l(\vec{r}_A) \end{vmatrix} = a_l^+ \dots a_j^+ a_i^+ |0\rangle, \qquad l > \dots > j > i$$

implicitly **assumes** we have **already** chosen the form of the **single-particle states**, (i = 1,2,3, ... A) as dictated by some mean-field potential

Basis states: occupation representation

- How are Slater determinants actually represented in a computer program?
  - We are dealing with fermions, so a single-particle state is either occupied or empty, which in computer language translates to either 1's or 0's
  - A very useful approach is a bit representation known as M-scheme
    - If the mean-field is spherically symmetric, the single-particle states will have good j, m_j

$$a^{+}_{1,\frac{3}{2},-\frac{1}{2}}a^{+}_{1,\frac{3}{2},\frac{3}{2}}a^{+}_{1,\frac{1}{2},\frac{1}{2}}a^{+}_{0,\frac{1}{2},-\frac{1}{2}}\left|0\right\rangle =$$

(ℓ, j, j_z)

0p3/2				0p1/2		0s1/2		
0	1	0	1	0	1	1	0	$= 2^1 + 2^3 + 2^5 + 2^6 = 106$
-3	-1	1	3	-1	1	-1	1	$2m_j$

- A single integer represents a complicated slater determinant

  - While the many-body states will have good M, they do not have good J. States of good J must be projected and will be a combination of Slater determinants. Same for T and M_T.

Getting the eigenvalues and wave functions

- Setup Hamiltonian matrix $\langle \Phi_i | H | \Phi_j \rangle$ and diagonalize
- Lanczos algorithm
  - Bring matrix to tri-diagonal form ($\mathbf{v}_1$, $\mathbf{v}_2$ … orthonormal, H Hermitian)

$$
\begin{aligned}
H\mathbf{v}_1 &= \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2 \\
H\mathbf{v}_2 &= \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3 \\
H\mathbf{v}_3 &= \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4 \\
H\mathbf{v}_4 &= \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5
\end{aligned}
$$

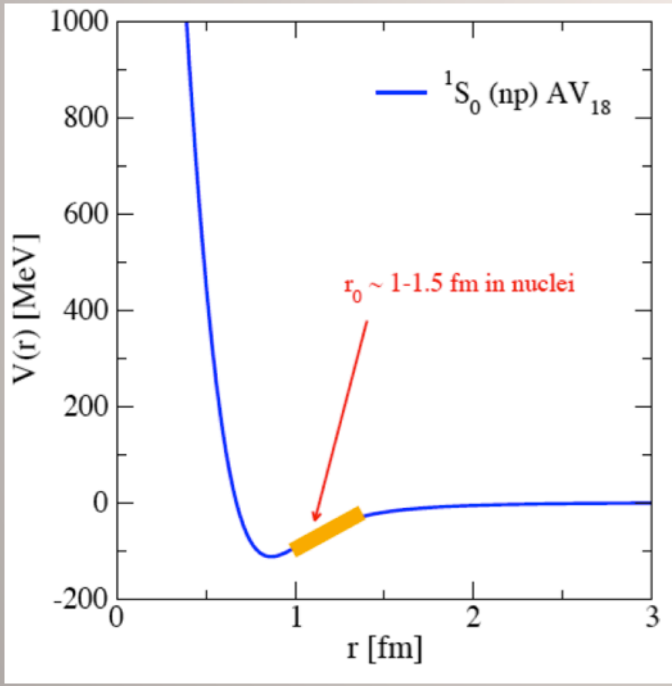
  - n^{th} iteration computes $2n^{th}$ moment
  - Eigenvalues converge to extreme (largest and smallest) values
  - ~ 100-200 iterations needed for 10 eigenvalues (even for 10^9 states)

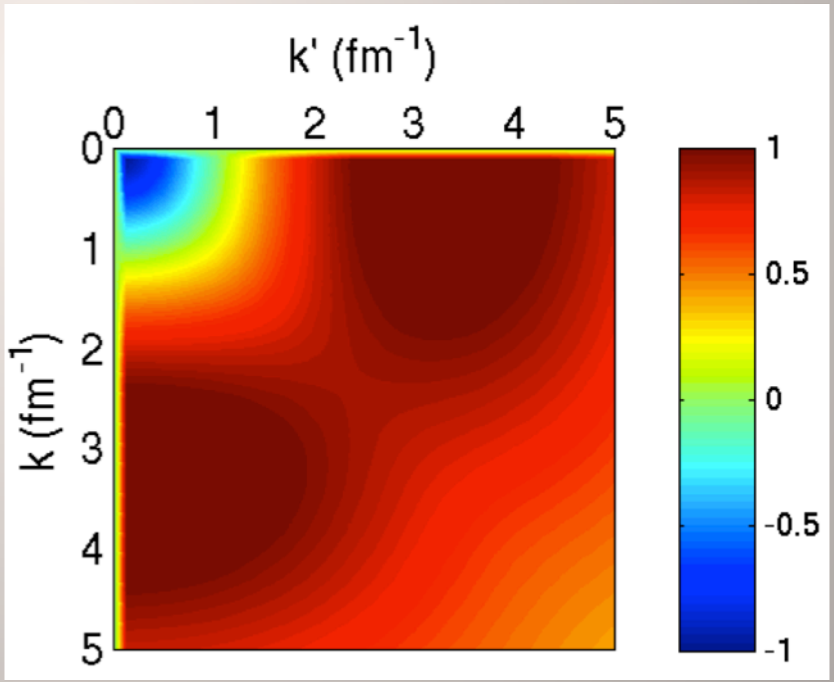
- Typically we use M-scheme:
  - Total M_J, M_T=$(Z$-$N)$/2 and parity conserved

TRIUMF

Accurate NN potentials are hard to use

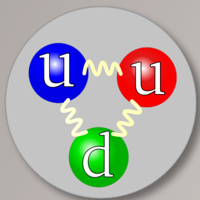
- Repulsive core of nuclear force introduces coupling to high momenta
  - Very large model spaces are required to reach convergent solution of the nuclear many-body problem





TRIUMF

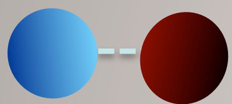
From QCD to nuclei



Low-energy QCD

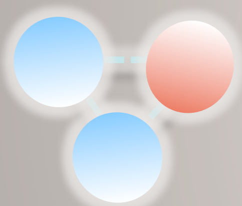
↓

NN+3N interactions from chiral EFT — …or accurate meson-exchange potentials



↓

Unitary/similarity transformations — Identity or SRG or OLS or UCOM … Softens NN, induces 3N



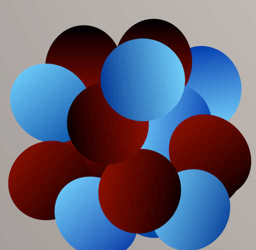
↓

$$H|\Psi\rangle = E|\Psi\rangle$$

Many-Body methods — NCSM, NCSM/RGM, CCM, GFMC, HH, Nuclear Lattice EFT…

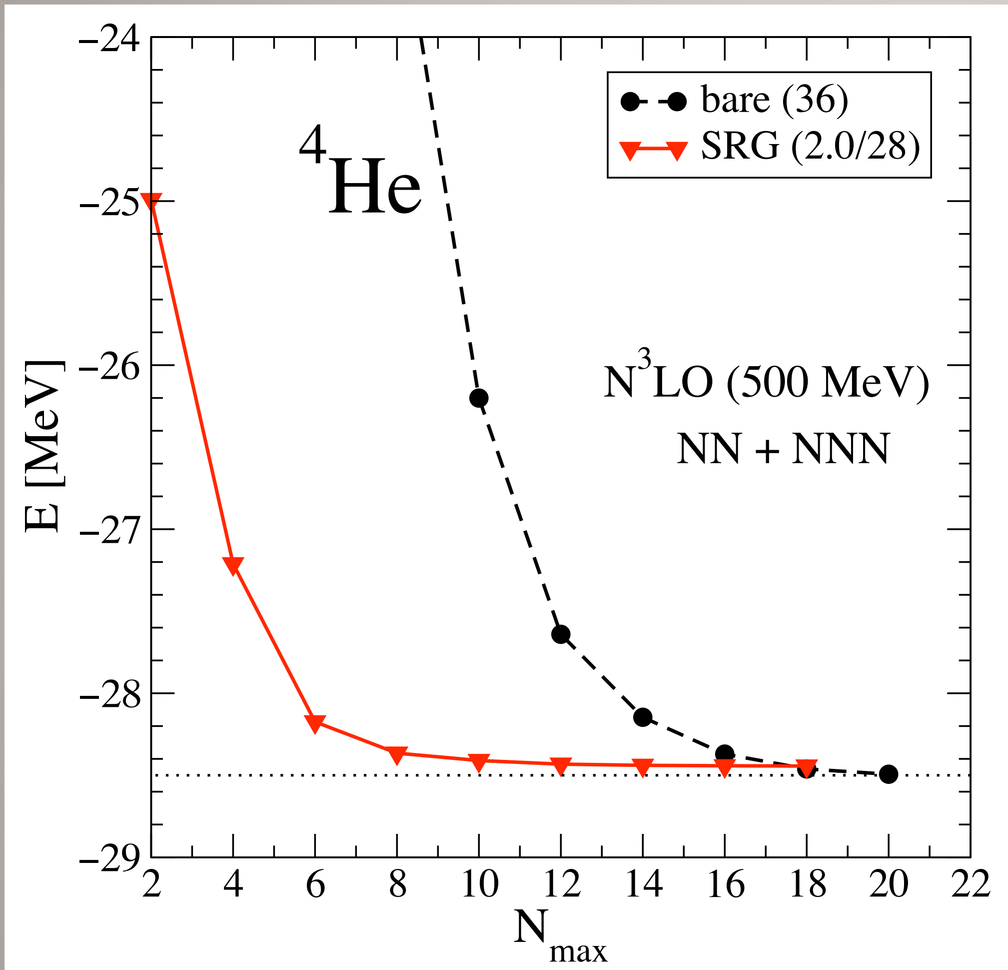
↓

Nuclear structure and reactions



TRIUMF

^{4}He from chiral EFT interactions: g.s. energy convergence



PRL 103, 082501 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Evolution of Nuclear Many-Body Forces with the Similarity Renormalization Group

E. D. Jurgenson,[1] P. Navrátil,[2] and R. J. Furnstahl[1]

A=3 binding energy and half life constraint
c_D=-0.2, c_E=-0.205, Λ=500 MeV

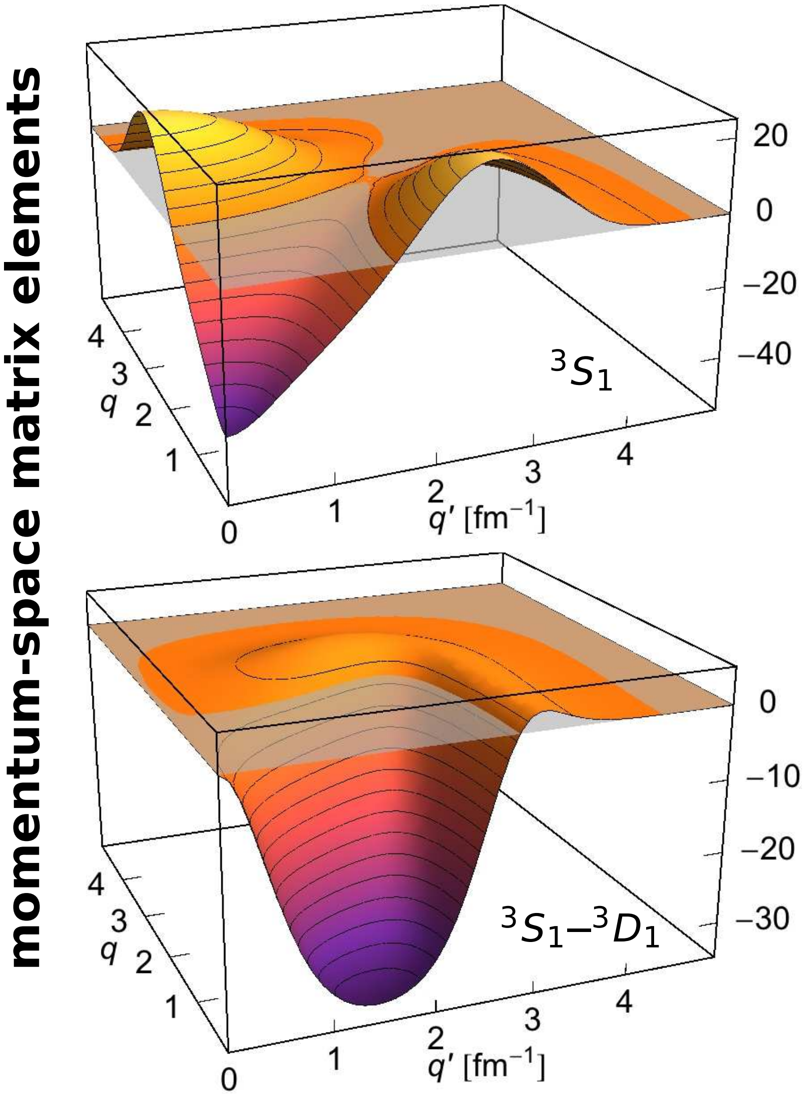
Chiral N^{3}LO NN plus N^{2}LO NNN potential

- Bare interaction (black line)
  - Strong short-range correlations
    - Large basis needed
- SRG evolved effective interaction (red line)
  - Unitary transformation

$$H_\alpha = U_\alpha H U_\alpha^+ \Rightarrow \frac{dH_\alpha}{d\alpha} = [[T, H_\alpha], H_\alpha] \quad \left(\alpha = 1/\lambda^4\right)$$

  - Two- plus *three*-body components, *four*-body omitted
  - Softens the interaction
    - Smaller basis sufficient

Why similarity renormalization?

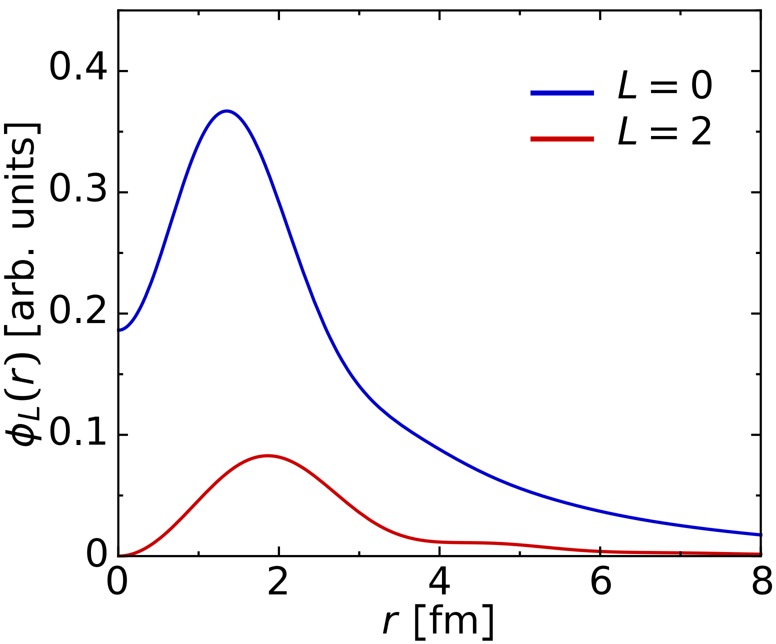


chiral N^{3}LO

Entem & Machleidt, 500 MeV

$J^\pi = 1^+, T = 0$

deuteron wave-function



Similarity Renormalization Group (SRG) evolution

- Continuous transformation driving Hamiltonian to band-diagonal form with respect to a chosen basis
- Unitary transformation $H_\alpha = U_\alpha H U_\alpha^+ \qquad U_\alpha U_\alpha^+ = U_\alpha^+ U_\alpha = 1$

$$\frac{dH_\alpha}{d\alpha} = \frac{dU_\alpha}{d\alpha} H U_\alpha^+ + U_\alpha H \frac{dU_\alpha^+}{d\alpha} = \frac{dU_\alpha}{d\alpha} U_\alpha^+ U_\alpha H U_\alpha^+ + U_\alpha H U_\alpha^+ U_\alpha \frac{dU_\alpha^+}{d\alpha}$$

$$= \frac{dU_\alpha}{d\alpha} U_\alpha^+ H_\alpha + H_\alpha U_\alpha \frac{dU_\alpha^+}{d\alpha} = \left[\eta_\alpha, H_\alpha\right]$$

$$\eta_\alpha \equiv \frac{dU_\alpha}{d\alpha} U_\alpha^+ = -\eta_\alpha^+$$

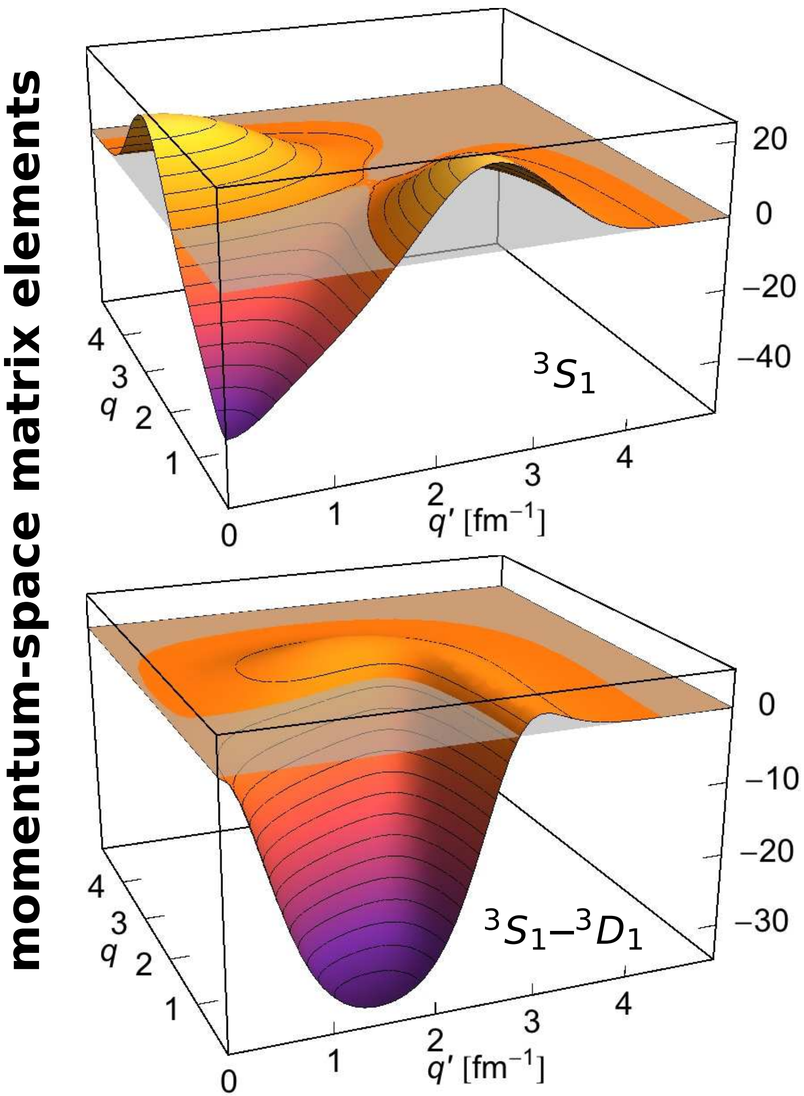
anti-Hermitian generator

- Setting $\eta_\alpha = \left[G_\alpha, H_\alpha\right]$ with Hermitian G_α

$$\frac{dH_\alpha}{d\alpha} = \left[\left[G_\alpha, H_\alpha\right], H_\alpha\right]$$

- Customary choice in nuclear physics $G_\alpha = T$ …kinetic energy operator
  - band-diagonal in momentum space plane-wave basis
- Initial condition $H_{\alpha=0} = H_{\lambda=\infty} = H \qquad \lambda^2 = 1/\sqrt{\alpha}$

SRG evolution in two-nucleon space

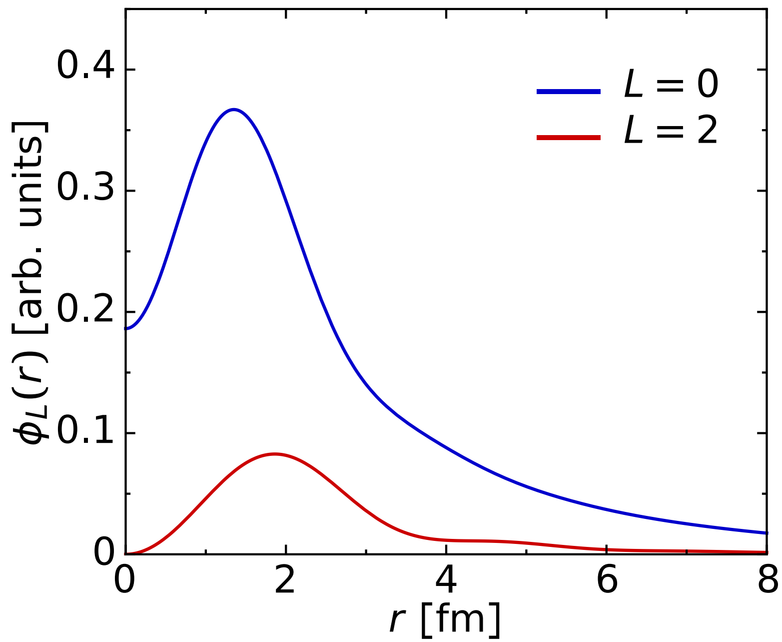


$$\alpha = 0.000\,\mathrm{fm}^4$$

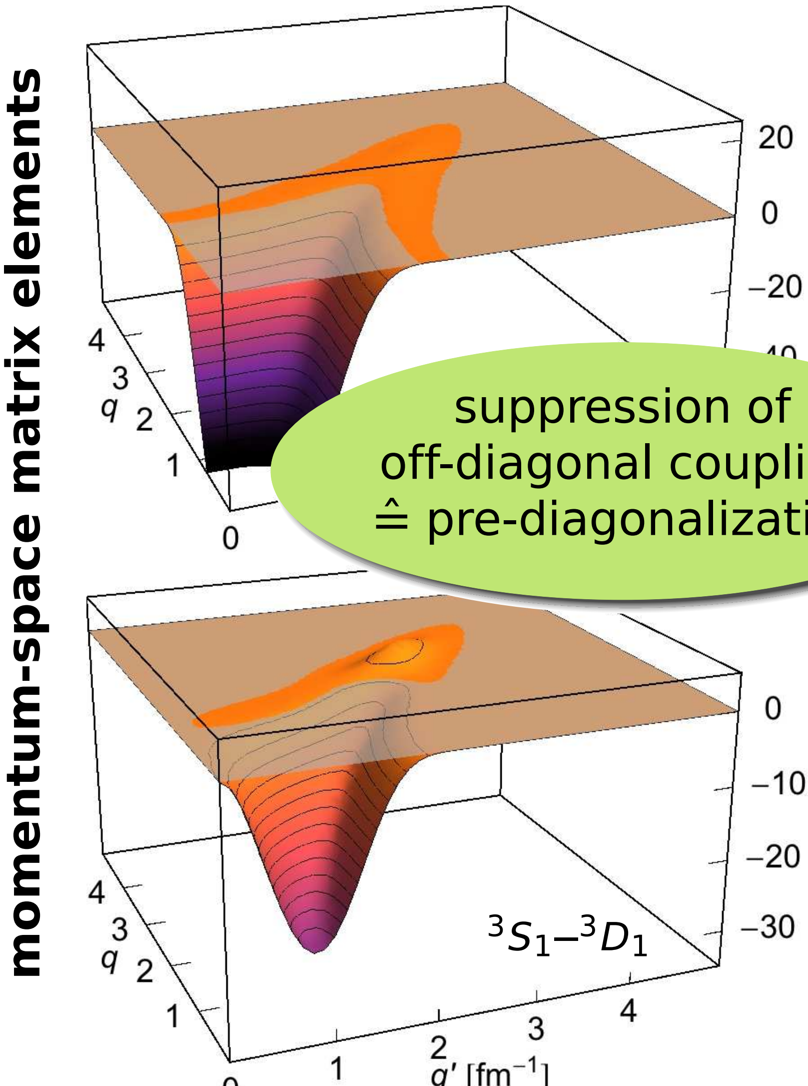
$\Lambda = \infty\,\mathrm{fm}^{-1}$

$J^\pi = 1^+, T = 0$

deuteron wave-function



SRG evolution in two-nucleon space



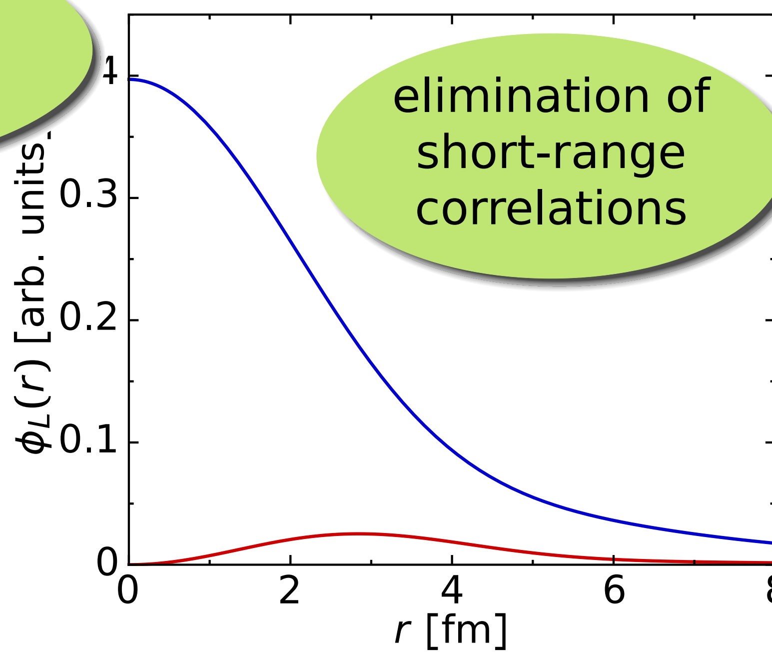
suppression of off-diagonal coupling ≙ pre-diagonalization

$\alpha = 0.320\,\mathrm{fm}^4$

$\Lambda = 1.33\,\mathrm{fm}^{-1}$

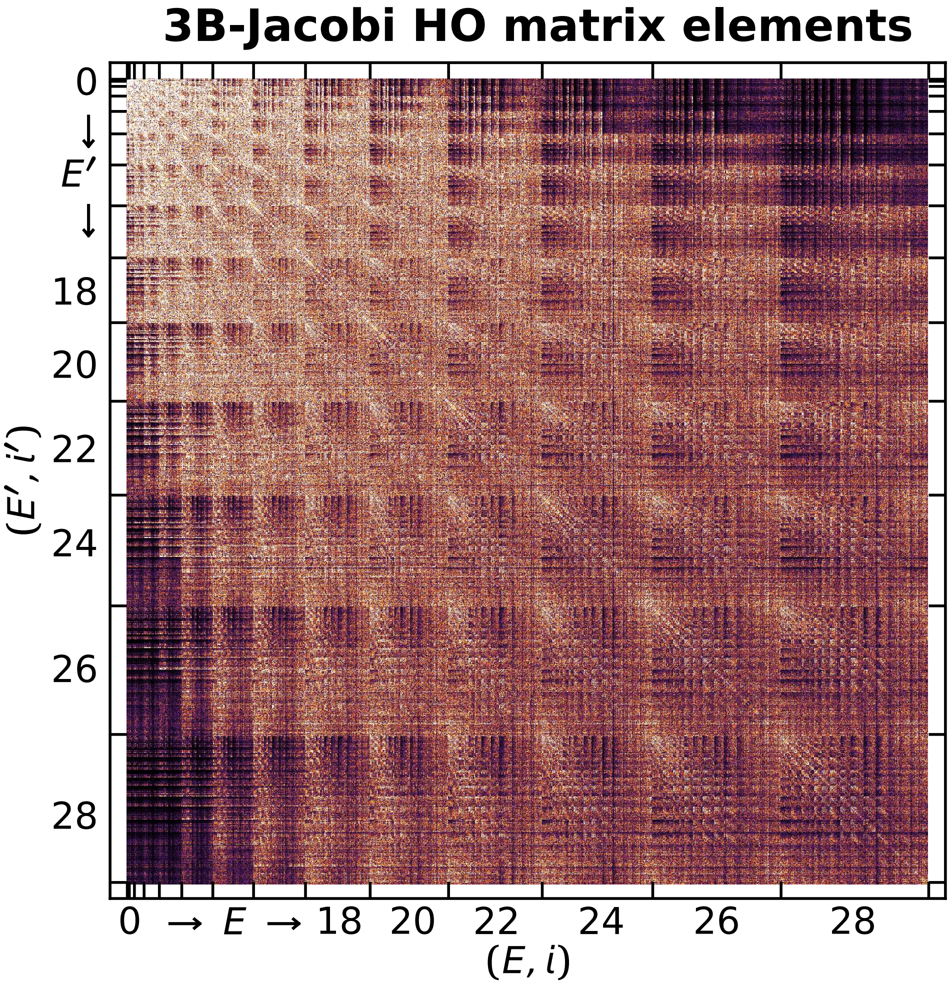
$J^\pi = 1^+, T = 0$

deuteron wave-function



elimination of short-range correlations

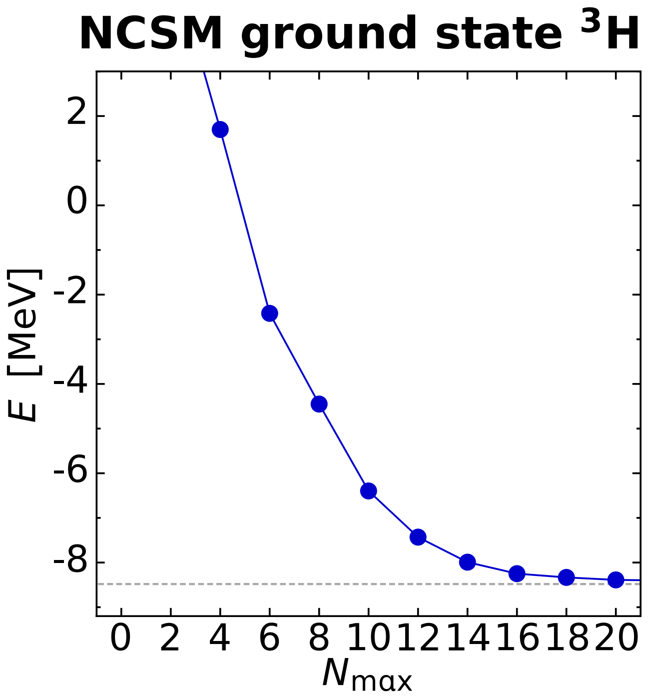
SRG evolution in three-nucleon space



$\alpha = 0.000\,\mathrm{fm}^4$

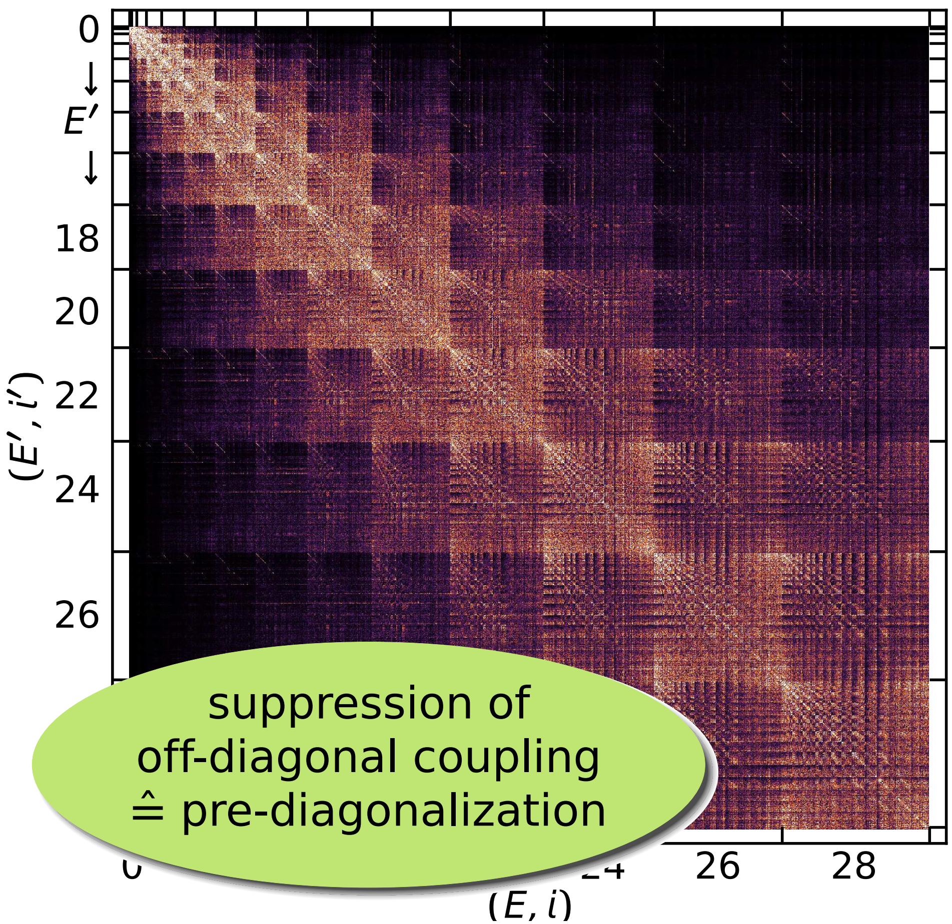
$\Lambda = \infty\,\mathrm{fm}^{-1}$

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 28\,\mathrm{MeV}$



SRG evolution in three-nucleon space

3B-Jacobi HO matrix elements



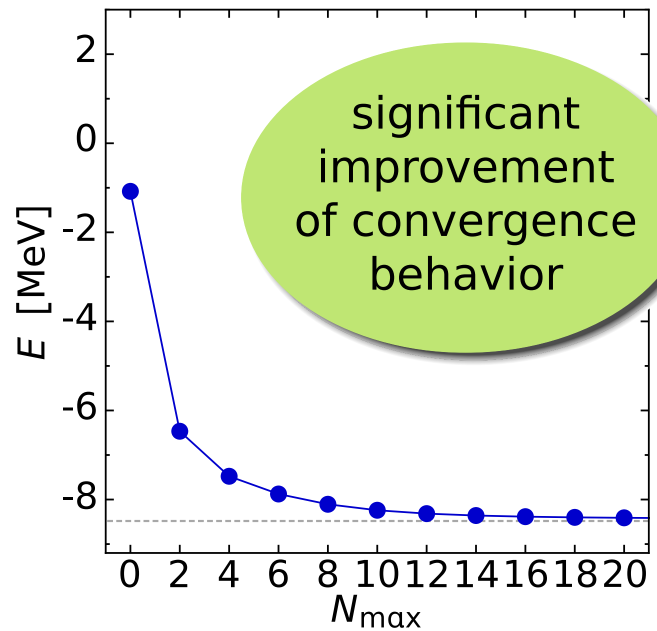
suppression of
off-diagonal coupling
≙ pre-diagonalization

$$\alpha = 0.320\,\mathrm{fm}^4$$

$\Lambda = 1.33\,\mathrm{fm}^{-1}$

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 28\,\mathrm{MeV}$

NCSM ground state ^{3}H



significant
improvement
of convergence
behavior

TRIUMF

SRG evolution for *A*-nucleon system

- Evolution induces many-nucleon terms (up to *A*)

$$\tilde{H}_\alpha = \tilde{H}_\alpha^{[1]} + \tilde{H}_\alpha^{[2]} + \tilde{H}_\alpha^{[3]} + \tilde{H}_\alpha^{[4]} + \ldots + \tilde{H}_\alpha^{[A]}$$

- In actual calculations so far only terms up to $\tilde{H}_\alpha^{[3]}$ kept

- Three types of SRG-evolved Hamiltonians used
  - **NN only**: Start with initial $T+V_{NN}$ and keep $\tilde{H}_\alpha^{[1]} + \tilde{H}_\alpha^{[2]}$
  - **NN+3N-induced**: Start with initial $T+V_{NN}$ and keep $\tilde{H}_\alpha^{[1]} + \tilde{H}_\alpha^{[2]} + \tilde{H}_\alpha^{[3]}$
  - **NN+3N-full**: Start with initial $T+V_{NN}+V_{NNN}$ and keep $\tilde{H}_\alpha^{[1]} + \tilde{H}_\alpha^{[2]} + \tilde{H}_\alpha^{[3]}$

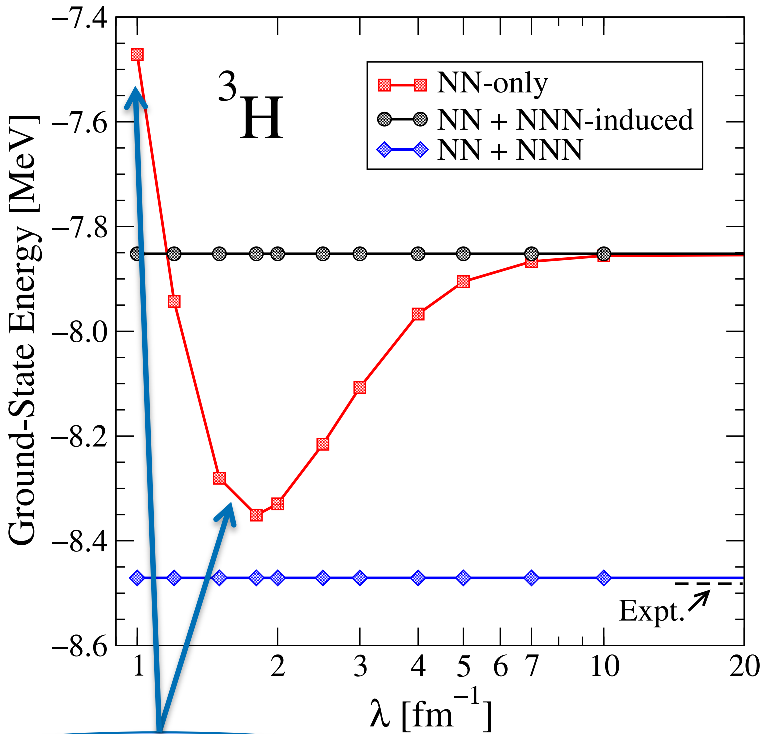
α variation (Λ variation) provides a diagnostic tool to asses the contribution of omitted many-body terms, tests the unitarity of the SRG transformation

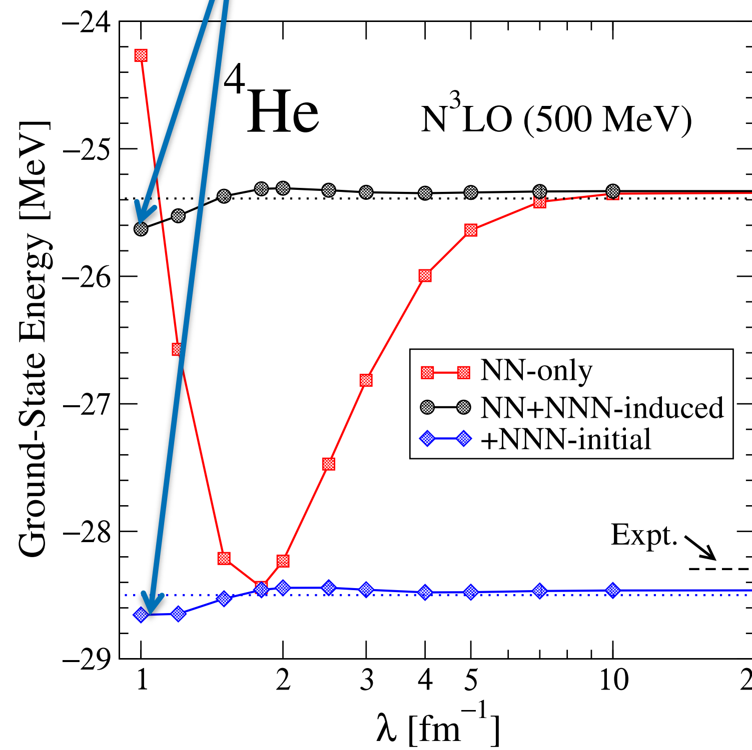
SRG evolution: ^{3}H and ^{4}He

PRL 103, 082501 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Evolution of Nuclear Many-Body Forces with the Similarity Renormalization Group

E. D. Jurgenson,[1] P. Navrátil,[2] and R. J. Furnstahl[1]



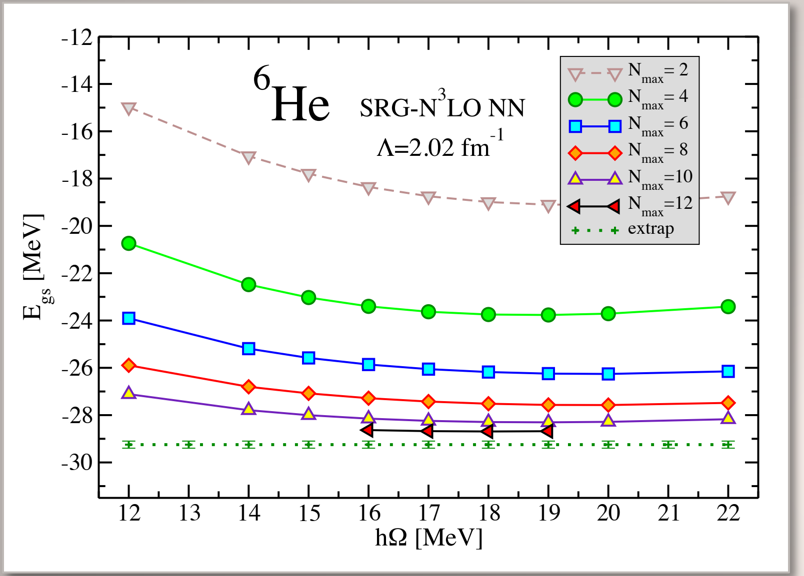


omitted induced 4N

omitted induced 3N

Ab initio calculations (NCSM, in this case)
used also for SRG evolution of NNN force (in HO basis)

NCSM calculations of ^{6}He g.s. energy



Dependence on:

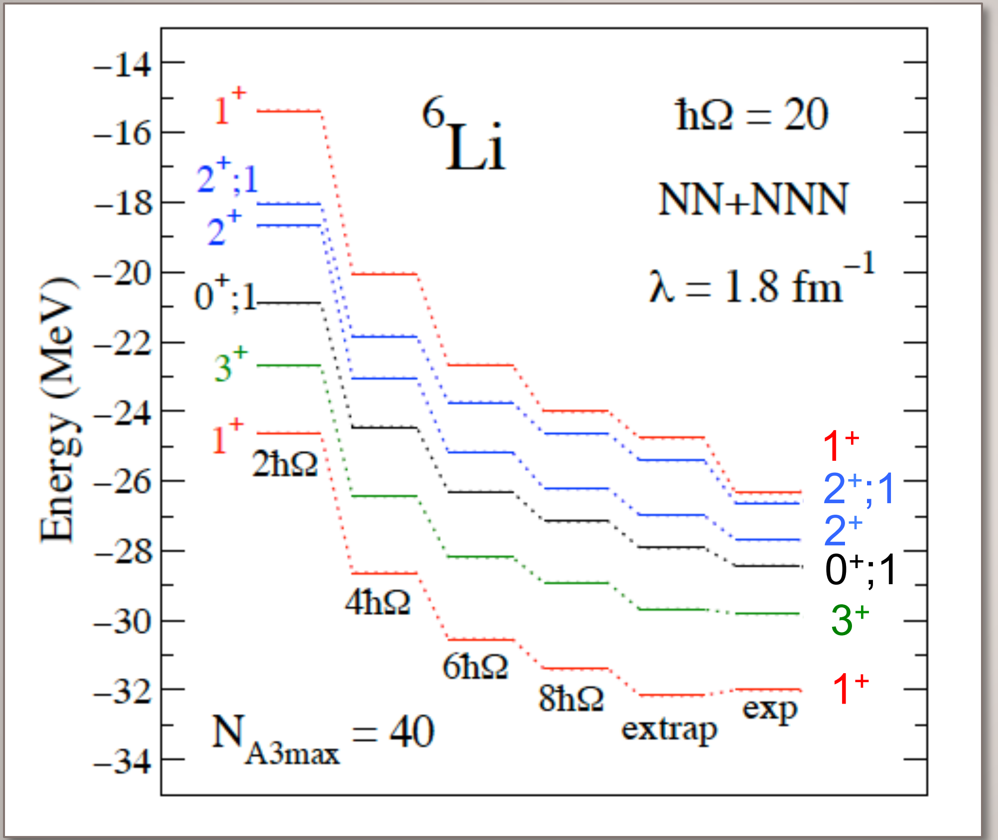
Basis size – N_{max}

HO frequency – hΩ

- Soft SRG evolved NN potential
- ✓ N_{max} convergence OK
- ✓ Extrapolation feasible

$E_{g.s.}$ [MeV]	^{4}He	^{6}He
NCSM N_{max}=12	-28.05	-28.63
NCSM extrap.	-28.22(1)	-29.25(15)
Expt.	-28.30	-29.27

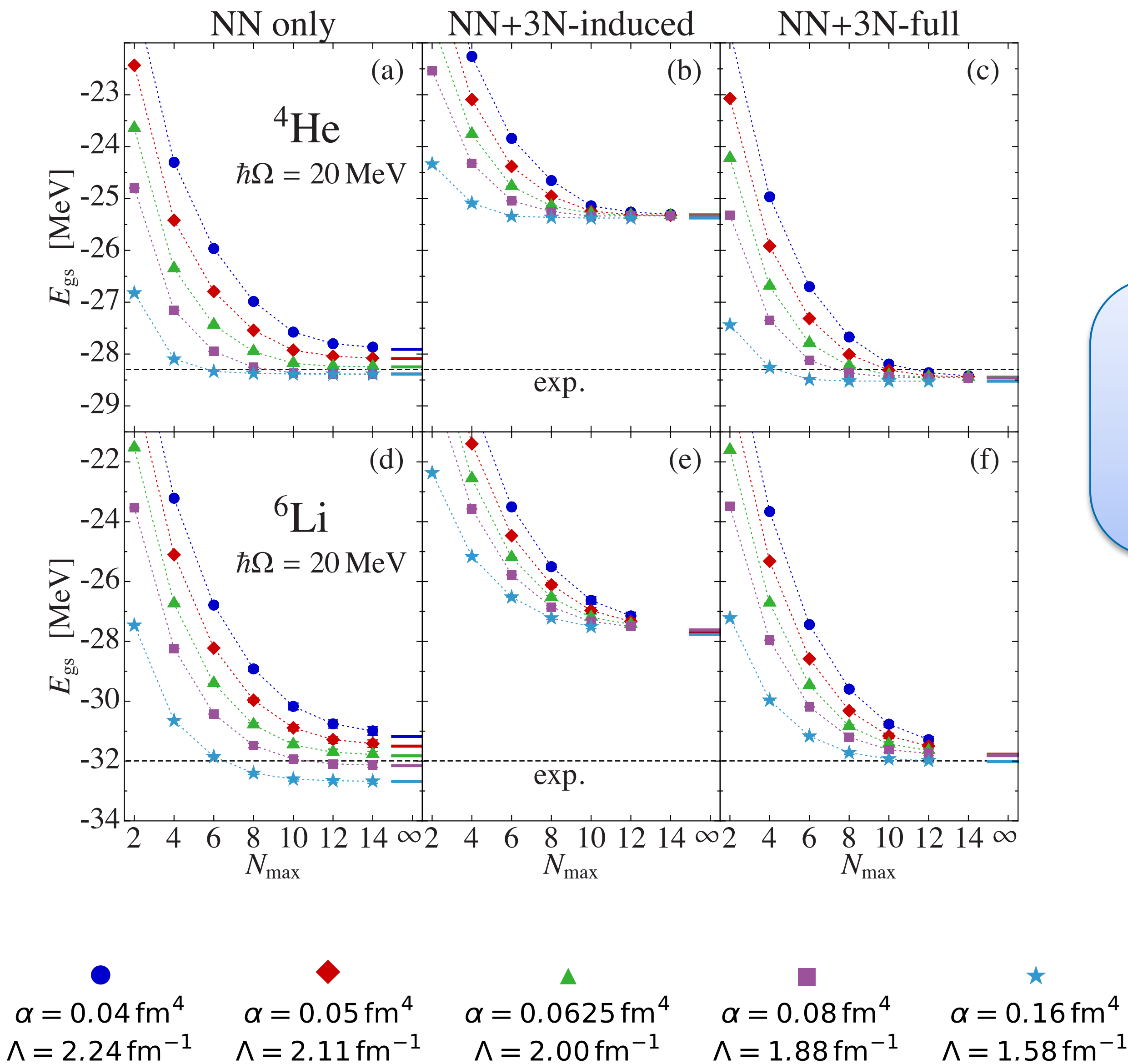
^{6}Li from chiral EFT interactions: Ground-state & excitation energies



A=3 binding energy & half life constraint
c_D=-0.2, c_E=-0.205, Λ=500 MeV

SRG with 2- plus 3-body: Good convergence, extrapolation to infinite basis space possible

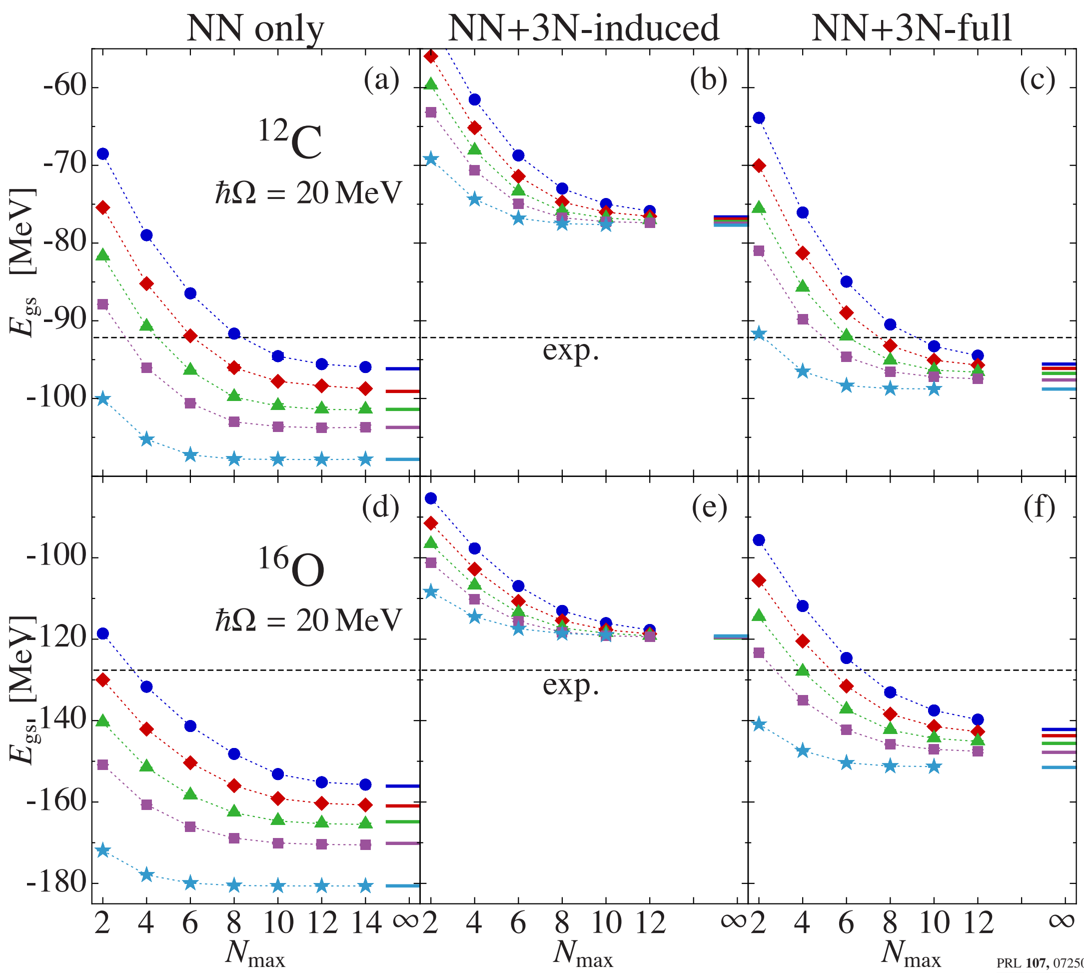
Light nuclei with SRG evolved interactions



- Fast convergence
- Significant 3N induced interaction
- No 4N induced interaction

TRIUMF

Heavier p-shell nuclei with SRG evolved interactions



- ● $\alpha = 0.04\,\text{fm}^4$, $\Lambda = 2.24\,\text{fm}^{-1}$
- ◆ $\alpha = 0.05\,\text{fm}^4$, $\Lambda = 2.11\,\text{fm}^{-1}$
- ▲ $\alpha = 0.0625\,\text{fm}^4$, $\Lambda = 2.00\,\text{fm}^{-1}$
- ■ $\alpha = 0.08\,\text{fm}^4$, $\Lambda = 1.88\,\text{fm}^{-1}$
- ★ $\alpha = 0.16\,\text{fm}^4$, $\Lambda = 1.58\,\text{fm}^{-1}$

- Fast convergence
- Significant 3N induced interaction
- 4N induced interaction when chiral 3N included

4N induced suppressed by lowering the chiral 3N cutoff to 400 MeV

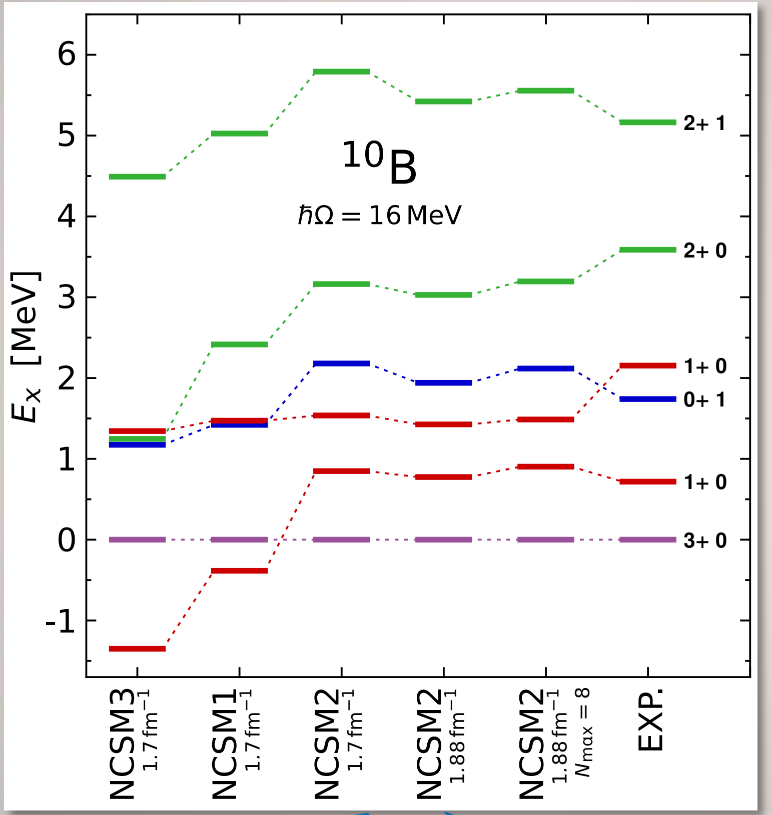
PRL 107, 072501 (2011) PHYSICAL REVIEW LETTERS week ending 12 AUGUST 2011

Similarity-Transformed Chiral *NN* + 3*N* Interactions for the *Ab Initio* Description of ^{12}C and ^{16}O

Robert Roth,[1,*] Joachim Langhammer,[1] Angelo Calci,[1] Sven Binder,[1] and Petr Navrátil[2,3]

TRIUMF

^{10}B states very sensitive to 3N interaction



chiral NN

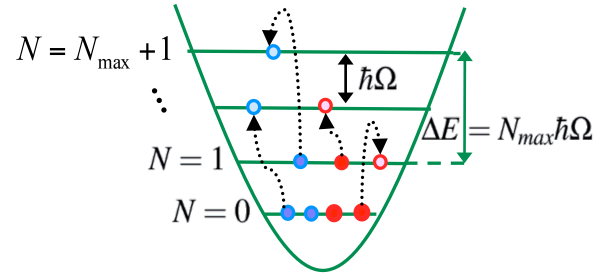
chiral NN+3N(400)

chiral NN+3N(500)

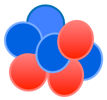
TRIUMF

No-core shell model

- No-core shell model (NCSM)
  - *A*-nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances



A



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

$$\Psi^{(A)} = \sum_\lambda c_\lambda \left| {}^{(A)} \right. \text{[nucleus]}, \lambda \rangle$$

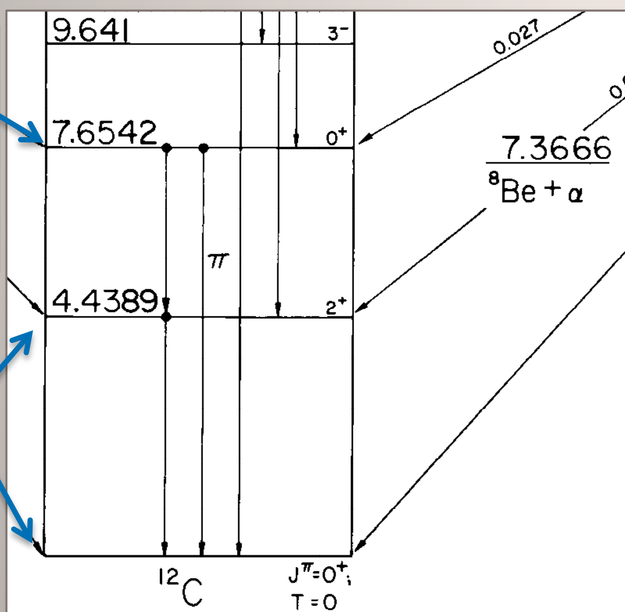
Unknowns

Light & medium mass nuclei from first principles

- Nuclear **structure** and **reaction** theory for light nuclei cannot be uncoupled
  - Well-bound nuclei, e.g. ^{12}C, have low-lying **cluster-dominated resonances**
  - Bound states of exotic nuclei, e.g. ^{11}Be, manifest **many-nucleon correlations**

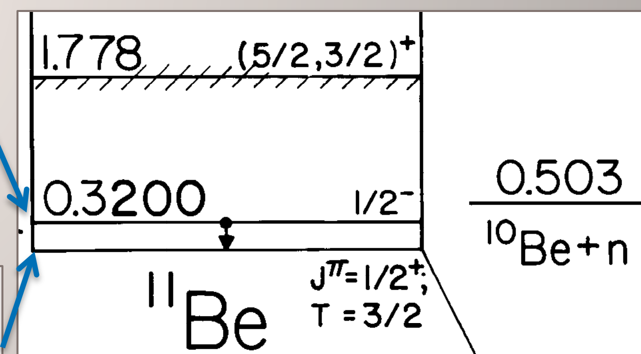
Hoyle state: α-cluster resonance with *p*-shell components

p-shell states



p-shell state with extended tail

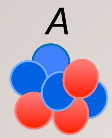
^{10}Be$_{gs}$ + (*l=0*) *n* with significant ^{10}Be* components



Extending no-core shell model beyond bound states

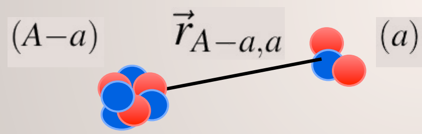
Include more many nucleon correlations…

NCSM →



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

+



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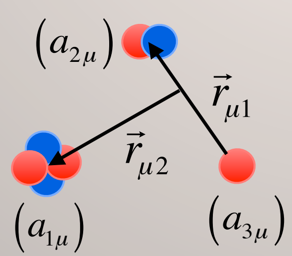


$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

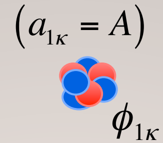
+

…

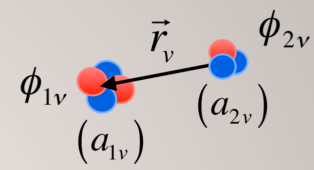
…using the Resonating Group Method (RGM) ideas

Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_\kappa \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$



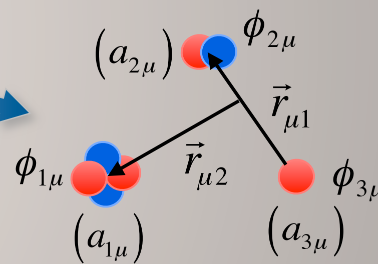
$$+ \sum_{\nu} \hat{A}_\nu \phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right) g_\nu(\vec{r}_\nu)$$



$$a_{1\nu} + a_{2\nu} = A$$

$$+ \sum_{\mu} \hat{A}_\mu \phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right) G_\mu(\vec{r}_{\mu 1}, \vec{r}_{\mu 2})$$

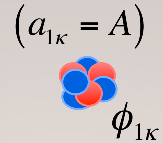
$$+ \cdots$$



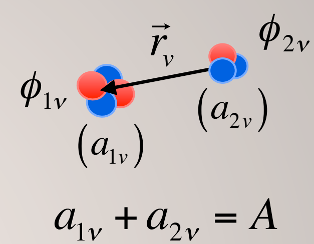
$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_{\kappa} \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$

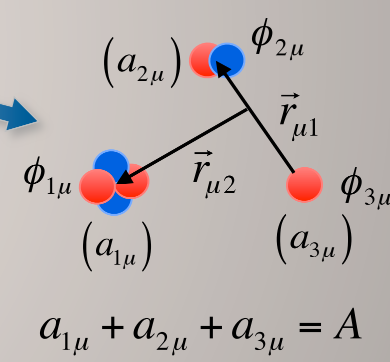


$$+ \sum_{\nu} \hat{A}_{\nu} \phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right) \phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right) g_{\nu}(\vec{r}_{\nu})$$



$$+ \sum_{\mu} \hat{A}_{\mu} \phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right) \phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right) \phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right) G_{\mu}(\vec{r}_{\mu 1}, \vec{r}_{\mu 2})$$

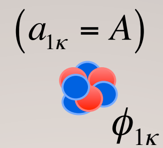
$$+ \cdots$$



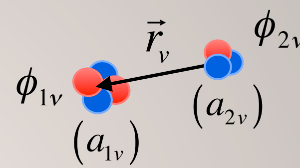
- ϕ : antisymmetric cluster wave functions
  - $\{\xi\}$: Translationally invariant internal coordinates (Jacobi relative coordinates)
  - These are known, they are an input

Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_{\kappa} \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$



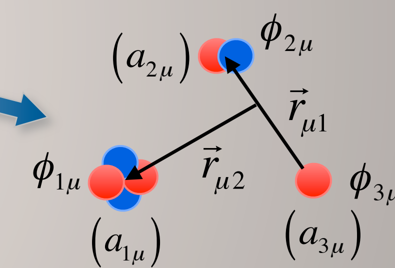
$$+ \sum_{\nu} \hat{A}_{\nu} \phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right) \phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right) g_{\nu}(\vec{r}_{\nu})$$



$$a_{1\nu} + a_{2\nu} = A$$

$$+ \sum_{\mu} \hat{A}_{\mu} \phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right) \phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right) \phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right) G_{\mu}(\vec{R}_{\mu 1}, \vec{R}_{\mu 2})$$

$$+ \cdots$$



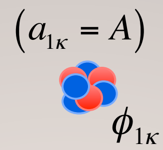
$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

- A_{ν}, A_{μ} : intercluster antisymmetrizers
  - Antisymmetrize the wave function for exchanges of nucleons between clusters
  - Example:

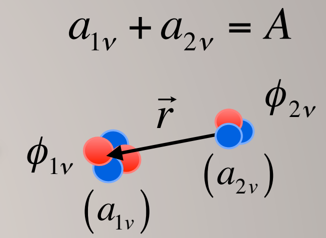
$$a_{1\nu} = A - 1,\ a_{2\nu} = 1 \quad \Rightarrow \quad \hat{A}_{\nu} = \frac{1}{\sqrt{A}}\left[1 - \sum_{i=1}^{A-1} \hat{P}_{iA}\right]$$

Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} \boxed{c_{\kappa}} \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$

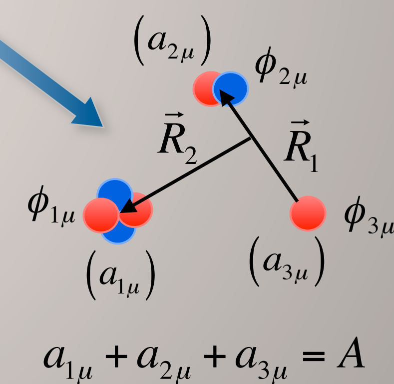


$$+ \sum_{\nu} \int \boxed{g_{\nu}(\vec{r}\,)}\, \hat{A}_{\nu}\left[\phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right)\delta(\vec{r} - \vec{r}_{\nu})\right] d\vec{r}$$



$$+ \sum_{\mu} \iint \boxed{G_{\mu}(\vec{R}_1, \vec{R}_2)}\, \hat{A}_{\mu}\left[\phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right)\delta(\vec{R}_1 - \vec{R}_{\mu 1})\delta(\vec{R}_2 - \vec{R}_{\mu 2})\right] d\vec{R}_1 d\vec{R}_2$$

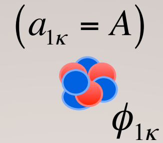
$$+ \cdots$$



- ***c***, ***g*** and ***G***: discrete and continuous linear variational amplitudes
  - Unknowns to be determined

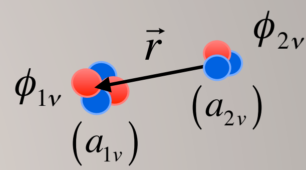
Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_{\kappa} \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$



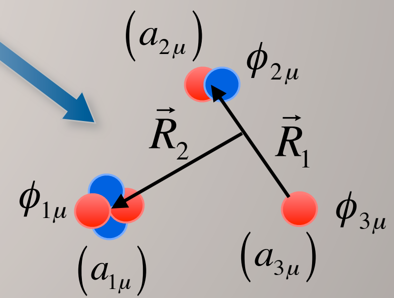
$$+ \sum_{\nu} \int g_{\nu}(\vec{r}\,)\, \hat{A}_{\nu}\left[\phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right)\delta(\vec{r} - \vec{r}_{\nu})\right] d\vec{r}$$

$$a_{1\nu} + a_{2\nu} = A$$



$$+ \sum_{\mu} \iint G_{\mu}(\vec{R}_1, \vec{R}_2)\, \hat{A}_{\mu}\left[\phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right)\delta(\vec{R}_1 - \vec{R}_{\mu 1})\delta(\vec{R}_2 - \vec{R}_{\mu 2})\right] d\vec{R}_1 d\vec{R}_2$$

$$+ \cdots$$



$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

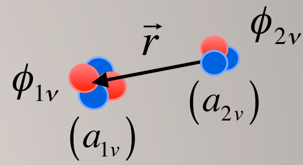
- Discrete and continuous set of basis functions
  - Non-orthogonal
  - Over-complete

Binary cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_{\kappa} \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$

$$+ \sum_{\nu} \int g_{\nu}(\vec{r}\,)\ \hat{A}_{\nu}\left[\phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right)\delta(\vec{r} - \vec{r}_{\nu})\right]d\vec{r}$$

$$a_{1\nu} + a_{2\nu} = A$$



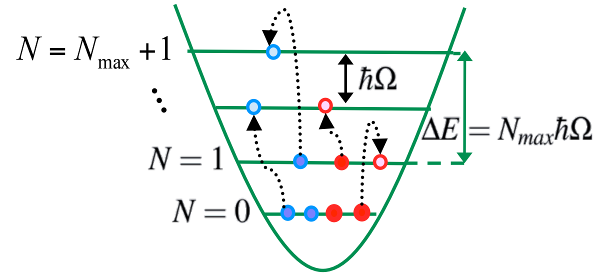
$$+ \sum_{\mu} \iint G_{\mu}(\vec{R}_1, \vec{R}_2)\ \hat{A}_{\mu}\left[\phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right)\delta(\vec{R}_1 - \vec{R}_{\mu 1})\delta(\vec{R}_2 - \vec{R}_{\mu 2})\right]d\vec{R}_1 d\vec{R}_2$$

$$+ \cdots$$

- In practice: function space limited by using relatively simple forms of Ψ chosen according to physical intuition and energetical arguments
  - Most common: expansion over binary-cluster basis

No-core shell model with RGM

- No-core shell model (NCSM)
  - *A*-nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances



- NCSM with Resonating Group Method (NCSM/RGM)
  - cluster expansion
  - proper asymptotic behavior
  - long-range correlations

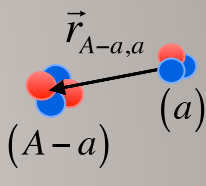
$$\Psi^{(A)} = \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left| \underset{(A-a)}{} \xrightarrow{\vec{r}} \underset{(a)}{}, \nu \right\rangle$$

Unknowns

Binary cluster Resonating Group Method

- Working in partial waves ($\nu \equiv \left\{ A-a\ \alpha_1 I_1^{\pi_1} T_1 ; a\ \alpha_2 I_2^{\pi_2} T_2 ; s\ell \right\}$)

$$\left|\psi^{J^\pi T}\right\rangle = \sum_\nu \int \frac{g_\nu^{J^\pi T}(r)}{r} \hat{A}_\nu \left[\left(\underbrace{\left|A-a\ \alpha_1 I_1^{\pi_1} T_1\right\rangle}_{\text{Target}} \underbrace{\left|a\ \alpha_2 I_2^{\pi_2} T_2\right\rangle}_{\text{Projectile}} \right)^{(sT)} Y_\ell(\hat{r}) \right]^{(J^\pi T)} \delta(\vec{r} - \vec{r}_{A-a,a})\ r^2 dr\, d\hat{r}$$



- Now introduce partial wave expansion of delta function

$$\delta(\vec{r} - \vec{r}_{A-a,a}) = \sum_{\lambda\mu} \frac{\delta(r - r_{A-a,a})}{r r_{A-a,a}} Y_{\lambda\mu}^*(\hat{r}) Y_{\lambda\mu}(\hat{r}_{A-a,a})$$

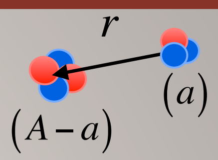
- After integration in the solid angle one obtains:

$$\left|\psi^{J^\pi T}\right\rangle = \sum_\nu \int \frac{g_\nu^{J^\pi T}(r)}{r} \underbrace{\hat{A}_\nu \left[\left(\left|A-a\ \alpha_1 I_1^{\pi_1} T_1\right\rangle \left|a\ \alpha_2 I_2^{\pi_2} T_2\right\rangle \right)^{(sT)} Y_\ell(\hat{r}_{A-a,a}) \right]^{(J^\pi T)} \frac{\delta(r - r_{A-a,a})}{r r_{A-a,a}}}_{\left|\Phi_{\nu r}^{J^\pi T}\right\rangle} r^2 dr$$

$\left|\Phi_{\nu r}^{J^\pi T}\right\rangle$ (Jacobi) channel basis

Binary cluster RGM equations

- Trial wave function: $\left|\psi^{J^\pi T}\right\rangle = \sum_\nu \int \frac{g_\nu^{J^\pi T}(r)}{r} \hat{A}_\nu \left|\Phi_{\nu r}^{J^\pi T}\right\rangle r^2 dr$



- Projecting the Schrödinger equation on the channel basis yields:

$$\sum_\nu \int \left[H_{\nu'\nu}^{J^\pi T}(r',r) - E\, N_{\nu'\nu}^{J^\pi T}(r',r) \right] \frac{g_\nu^{J^\pi T}(r)}{r} r^2\, dr = 0$$

$$\left\langle \Phi_{\nu' r'}^{J^\pi T} \right| \hat{A}_{\nu'} H \hat{A}_\nu \left| \Phi_{\nu r}^{J^\pi T} \right\rangle$$

Hamiltonian kernel

$$\left\langle \Phi_{\nu' r'}^{J^\pi T} \right| \hat{A}_{\nu'} \hat{A}_\nu \left| \Phi_{\nu r}^{J^\pi T} \right\rangle$$

Overlap (or norm) kernel

- Breakdown of approach:
  1. Build channel basis states from input target and projectile wave functions
  2. Calculate Hamiltonian and norm kernels
  3. Solve RGM equations: find unknown relative motion wave functions
     - Bound-state / scattering boundary conditions

How to calculate the RGM kernels?

- Since we are using NCSM wave functions, it is convenient to introduce Jacobi channel states in the HO space

$$\left|\Phi^{J^\pi T}_{\nu n}\right\rangle = \left[\left(\left|A-a\;\alpha_1 I_1^{\pi_1} T_1\right\rangle \left|a\;\alpha_2 I_2^{\pi_2} T_2\right\rangle\right)^{(sT)} Y_\ell(\hat{r}_{A-a,a})\right]^{(J^\pi T)} R_{n\ell}(r_{A-a,a})$$

- Note :
  - The coordinate space channel states are given by

$$\left|\Phi^{J^\pi T}_{\nu r}\right\rangle = \sum_n R_{n\ell}(r) \left|\Phi^{J^\pi T}_{\nu n}\right\rangle$$

    - We used the closure properties of HO radial wave functions

Trick #1

$$\frac{\delta(r - r_{A-a,a})}{r\, r_{A-a,a}} = \sum_n R_{n\ell}(r) R_{n\ell}(r_{A-a,a})$$

  - We call them Jacobi channel states because they describe only the internal motion
    - Target and projectile wave functions are both translational invariant NCSM eigenstates calculated in the Jacobi coordinate basis

Norm kernel (Pauli principle)

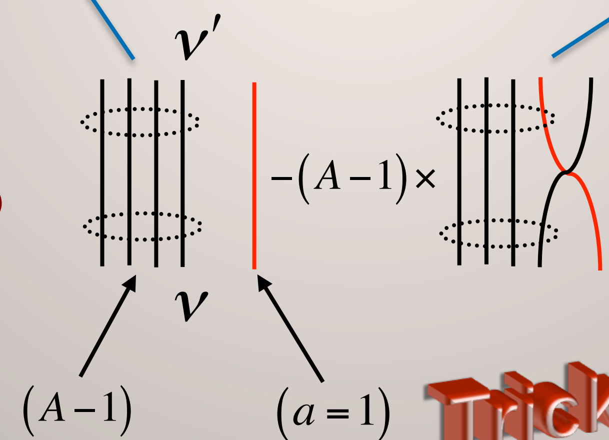
Single-nucleon projectile

$$\left\langle \Phi^{J^\pi T}_{\nu' r'} \middle| \hat{A}_{\nu'} \hat{A}_{\nu} \middle| \Phi^{J^\pi T}_{\nu r} \right\rangle = \left\langle \; (A-1) \;\; r' \;\; (a'=1) \; \middle| \; 1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \; \middle| \; (a=1) \;\; r \;\; (A-1) \; \right\rangle$$

$$N^{J^\pi T}_{\nu'\nu}(r',r) = \delta_{\nu'\nu} \frac{\delta(r'-r)}{r'r} - (A-1) \sum_{n'n} R_{n'\ell'}(r') R_{n\ell}(r) \left\langle \Phi^{J^\pi T}_{\nu' n'} \middle| \hat{P}_{A-1,A} \middle| \Phi^{J^\pi T}_{\nu n} \right\rangle$$

$${}_{SD}\left\langle \psi^{(A-1)}_{\mu_1} \middle| a^+ a \middle| \psi^{(A-1)}_{\nu_1} \right\rangle_{SD}$$

Direct term:
Treated exactly!
(in the full space)



Exchange term:
Obtained in the model space!
(Many-body correction due to the exchange part of the inter-cluster antisymmetrizer)

Trick #1
$$\frac{\delta(r - r_{A-a,a})}{r\, r_{A-a,a}} = \sum_n R_{n\ell}(r) R_{n\ell}(r_{A-a,a})$$

Trick #2 Target wave functions expanded in the SD basis, the CM motion exactly removed

Solving the NCSM/RGM equations

- Other technical details
  - Because of the norm kernel, the radial wave functions solutions of the RGM equation are not Schrödinger wave functions
  - However, the RGM equations can be orthogonalized

$$\sum_{\nu'} \int dr' r'^2 \left[N^{-\frac{1}{2}} H N^{-\frac{1}{2}} \right]_{\nu\nu'} (r, r') \frac{u_{\nu'}(r')}{r'} = E \frac{u_\nu(r)}{r}$$

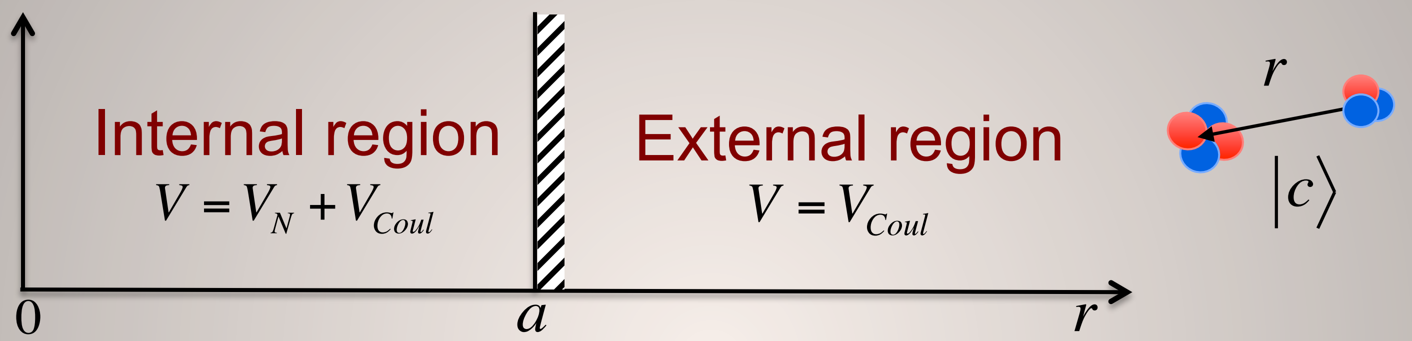
    - Explained, e.g., in Phys. Rev. C 79, 044606 (2009)

- In the end, a set of integral-differential coupled channel equations with a non-local potential of the type:

$$\left[T_{rel}(r) + \bar{V}_{Coul}(r) - (E - \varepsilon_{\alpha_1} - \varepsilon_{\alpha_2}) \right] u_\nu(r) + \sum_{\nu'} \int dr' r' \; W_{\nu\nu'}(r, r') \; u_{\nu'}(r') = 0$$

Microscopic *R*-matrix theory

- Separation into "internal" and "external" regions at the channel radius a



- W.f. matching through the Bloch operator: $L_c = \frac{\hbar^2}{2\mu_c}\delta(r-a)\left(\frac{d}{dr} - \frac{B_c}{r}\right)$
- System of Bloch-Schrödinger equations:

$$\left[\hat{T}_{rel}(r) + L_c + \overline{V}_{Coul}(r) - (E - E_c)\right]u_c(r) + \sum_{c'} \int dr' \, r' W_{cc'}(r,r') u_{c'}(r') = L_c u_c(r)$$

Microscopic *R*-matrix theory

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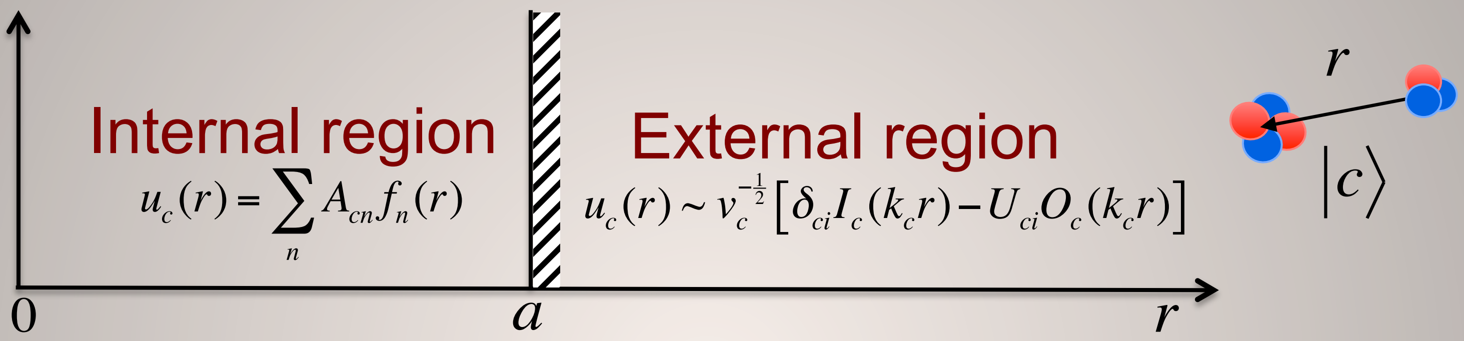


  - W.f. matching through the Bloch operator: $L_c = \dfrac{\hbar^2}{2\mu_c} \delta(r-a) \left(\dfrac{d}{dr} - \dfrac{B_c}{r} \right)$
  - System of Bloch-Schrödinger equations:

$$\left[\hat{T}_{rel}(r) + L_c + \overline{V}_{Coul}(r) - (E - E_c) \right] u_c(r) + \sum_{c'} \int dr' \, r' W_{cc'}(r,r') u_{c'}(r') = L_c u_c(r)$$

  - Internal region: expansion on square-integrable basis $u_c(r) = \sum_n A_{cn} f_n(r)$
  - External region: asymptotic form for large r

$$u_c(r) \sim C_c W(k_c r) \quad \text{or} \quad u_c(r) \sim v_c^{-\frac{1}{2}} \left[\delta_{ci} I_c(k_c r) - U_{ci} O_c(k_c r) \right]$$

Bound state | Scattering state | Scattering matrix

TRIUMF

To find the Scattering matrix

- After projection on the basis $f_n(r)$:

$$\sum_{c'n'}\left[C_{cn,c'n'} - (E - E_c)\delta_{cn,c'n'}\right]A_{c'n'} = \frac{\hbar^2 k_c}{2\mu_c v_c^{1/2}}\left\langle f_n \middle| L_c \middle| I_c\delta_{ci} - U_{ci}O_c\right\rangle$$

$$\left\langle f_n \middle| \hat{T}_{rel}(r) + L_c + \overline{V}_{Coul}(r) \middle| f_{n'}\right\rangle \delta_{cc'} + \left\langle f_n \middle| W_{cc'}(r,r') \middle| f_{n'}\right\rangle$$

Lagrange basis associated with Lagrange mesh:

$$\{ax_n \in [0,a]\}$$

$$\int_0^1 g(x)dx \approx \sum_{n=1}^{N} \lambda_n g(x_n)$$

$$\int_0^a f_n(r) f_{n'}(r)dr \approx \delta_{nn'}$$

1. Solve for A_{cn}
2. Match internal and external solutions at channel radius, a

$$\sum_{c'} R_{cc'} \frac{k_{c'}a}{\sqrt{\mu_{c'}v_{c'}}}\left[I'_{c'}(k_{c'}a)\delta_{ci} - U_{c'i}O'_{c'}(k_{c'}a)\right] = \frac{1}{\sqrt{\mu_c v_c}}\left[I_c(k_c a)\delta_{ci} - U_{ci}O_c(k_c a)\right]$$

- In the process introduce R-matrix, projection of the Green's function operator on the channel-surface functions

$$R_{cc'} = \sum_{nn'} \frac{\hbar}{\sqrt{2\mu_c a}} f_n(a)\left[C - EI\right]^{-1}_{cn,c'n'} \frac{\hbar}{\sqrt{2\mu_{c'} a}} f_{n'}(a)$$

To find the Scattering matrix

3. Solve equation with respect to the scattering matrix U

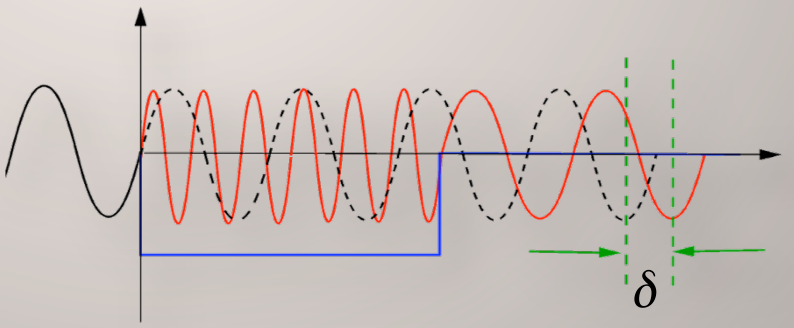
$$\sum_{c'} R_{cc'} \frac{k_{c'}a}{\sqrt{\mu_{c'}v_{c'}}}\left[I'_{c'}(k_{c'}a)\delta_{ci} - U_{c'i}O'_{c'}(k_{c'}a)\right] = \frac{1}{\sqrt{\mu_c v_c}}\left[I_c(k_c a)\delta_{ci} - U_{ci}O_c(k_c a)\right]$$

4. You can demonstrate that the solution is given by:

$$U = Z^{-1}Z^{*}, \qquad Z_{cc'} = \left(k_{c'}a\right)^{-1}\left[O_c(k_c a)\delta_{cc'} - k_{c'}a\, R_{cc'}\; O'_{c'}(k_{c'}a)\right]$$

- Scattering phase shifts are extracted from the scattering matrix elements

$$U = \exp(2i\delta)$$



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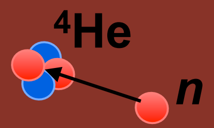
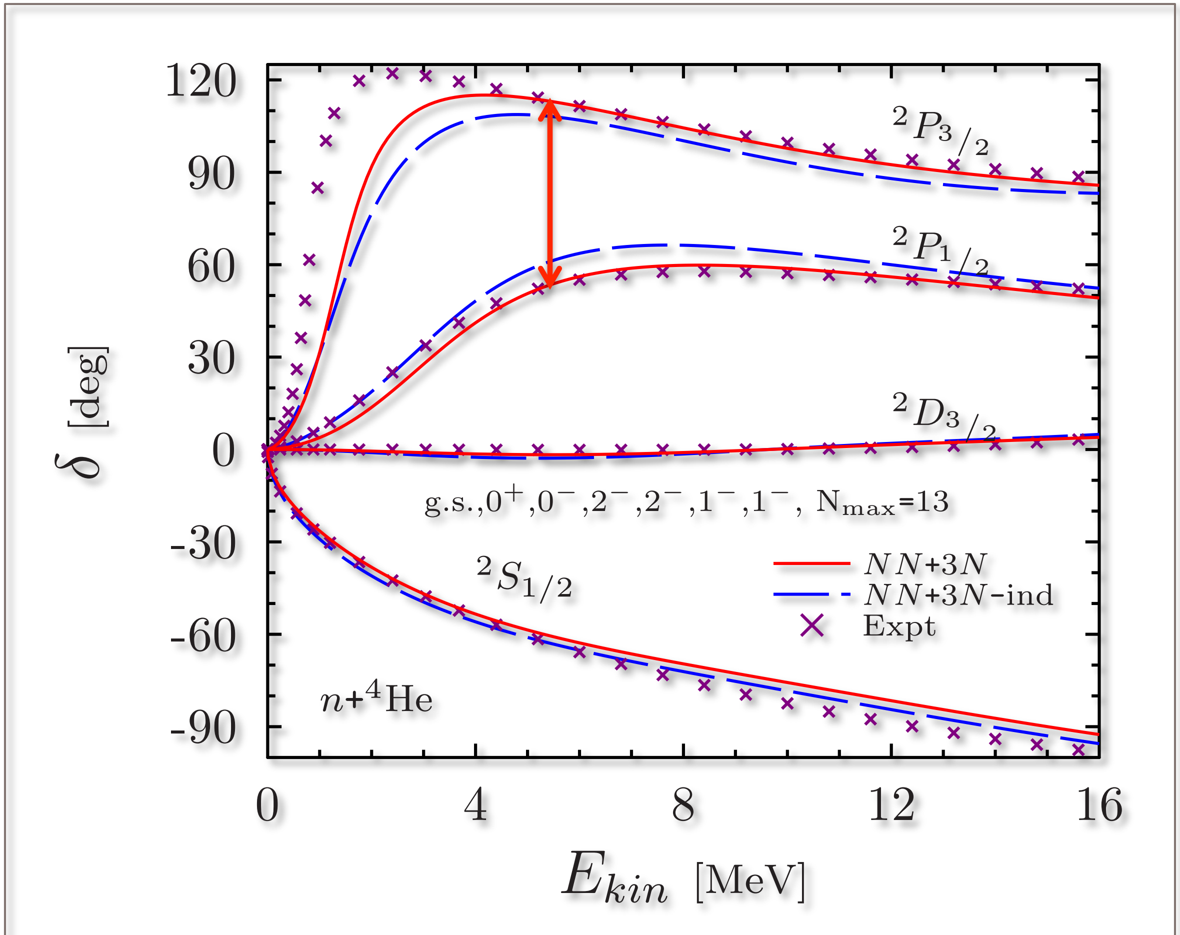


n-^{4}He scattering within the NCSM/RGM



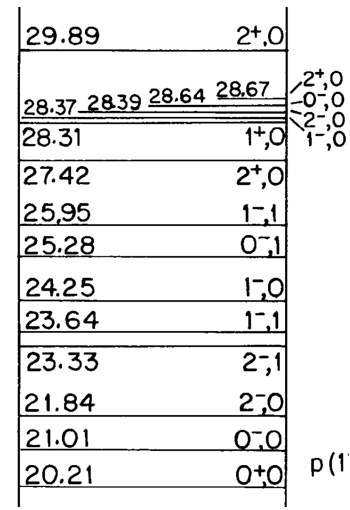
PHYSICAL REVIEW C **88**, 054622 (2013)

Ab initio **many-body calculations of nucleon-^{4}He scattering with three-nucleon forces**

Guillaume Hupin,[1,*] Joachim Langhammer,[2,†] Petr Navrátil,[3,‡] Sofia Quaglioni,[1,§] Angelo Calci,[2,‖] and Robert Roth[2,¶]

chiral NN+NNN(500)
chiral NN+NNN-induced
SRG λ=2 fm^{-1}
HO N_{max}=13, hΩ=20 MeV

^{4}He g.s. and 6 excited states



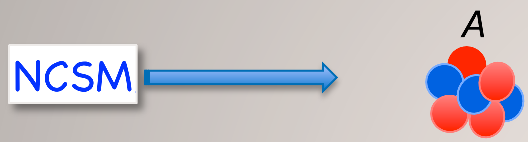
A larger splitting between the *P*-waves obtained with the chiral NN+NNN interaction

The 3/2$^-$ resonance still off:
Interaction or **CONVERGENCE?**

TRIUMF

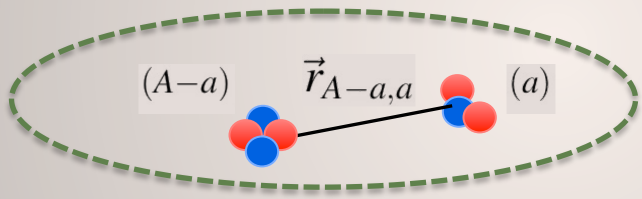
Extending no-core shell model beyond bound states

Include more many nucleon correlations…

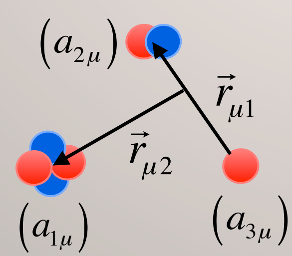


$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

+



+



$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

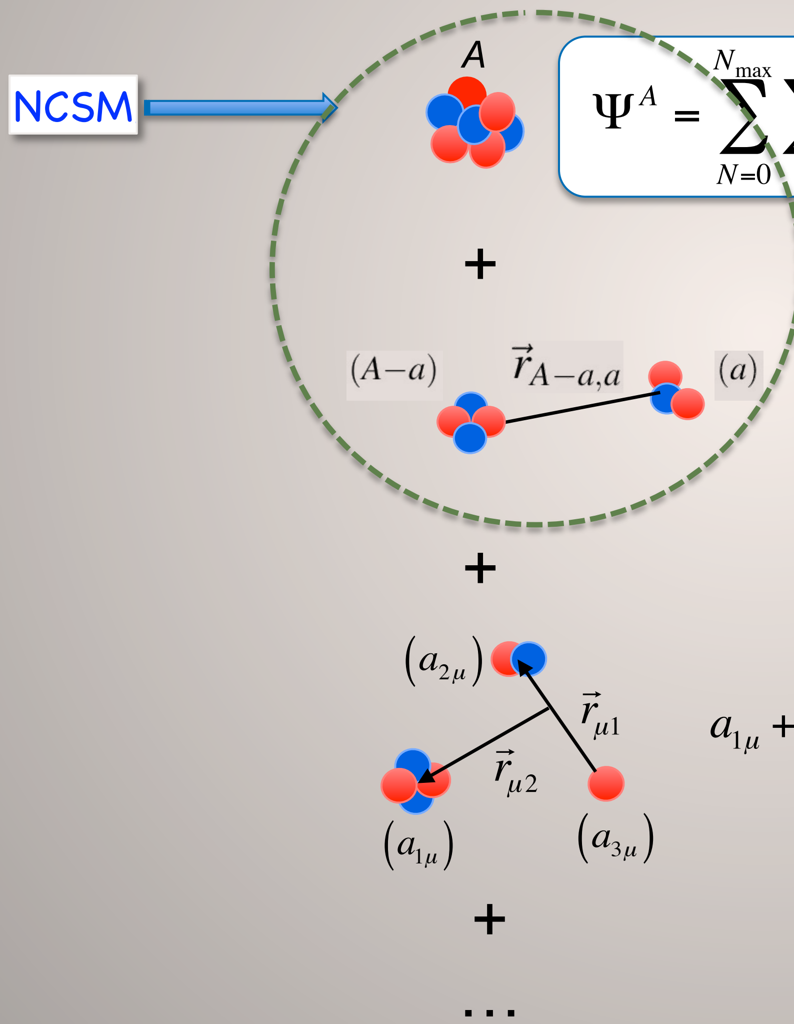
+

…

…using the Resonating Group Method (RGM) ideas

Extending no-core shell model beyond bound states

Include more many nucleon correlations…



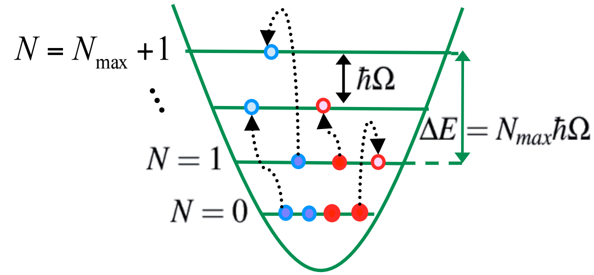
$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

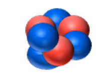
…using the Resonating Group Method (RGM) ideas

Unified approach to bound & continuum states; to nuclear structure & reactions

- *Ab initio* no-core shell model
  - Short- and medium range correlations
  - Bound-states, narrow resonances

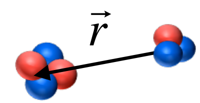


Harmonic oscillator basis



NCSM

- …with resonating group method
  - Bound & scattering states, reactions
  - Cluster dynamics, long-range correlations



NCSM/RGM

S. Baroni, P. Navratil, and S. Quaglioni, PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

- Most efficient: *ab initio* no-core shell model with continuum

NCSMC

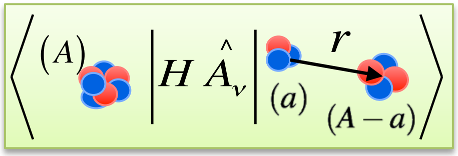
NCSM eigenstates **NCSM/RGM channel states**

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| {}^{(A)} \, , \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \; \gamma_{\nu}(\vec{r}) \; \hat{A}_{\nu} \left| {}_{(A-a)} \; \vec{r} \; {}_{(a)} \, , \nu \right\rangle$$

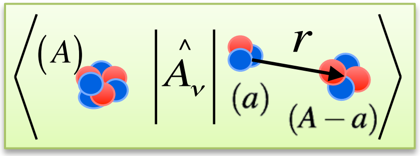
Unknowns

Coupled NCSMC equations

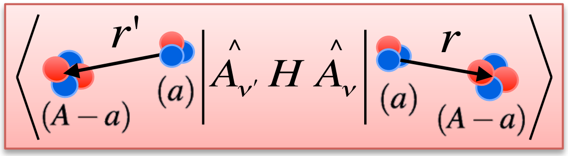
$$E_\lambda^{NCSM}\,\delta_{\lambda\lambda'}$$

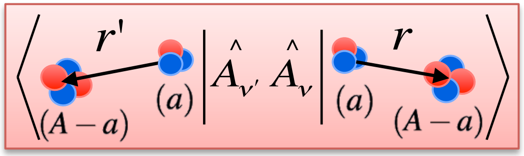


$$\delta_{\lambda\lambda'}$$



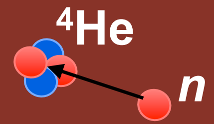
$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix}$$





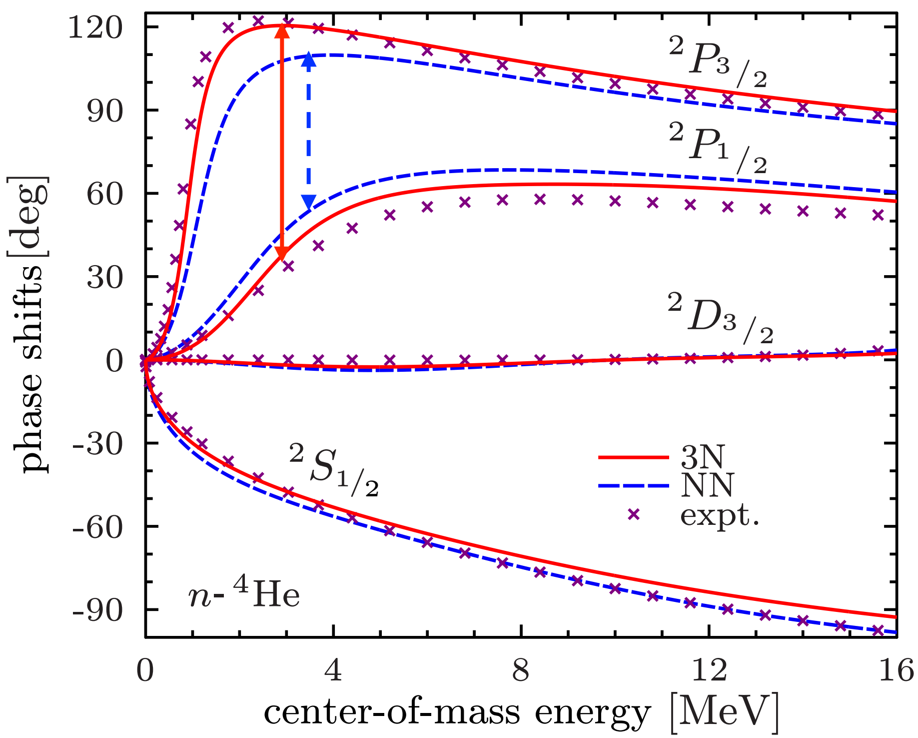
Scattering matrix (and observables) from matching solutions to known asymptotic with microscopic *R*-matrix on Lagrange mesh

TRIUMF

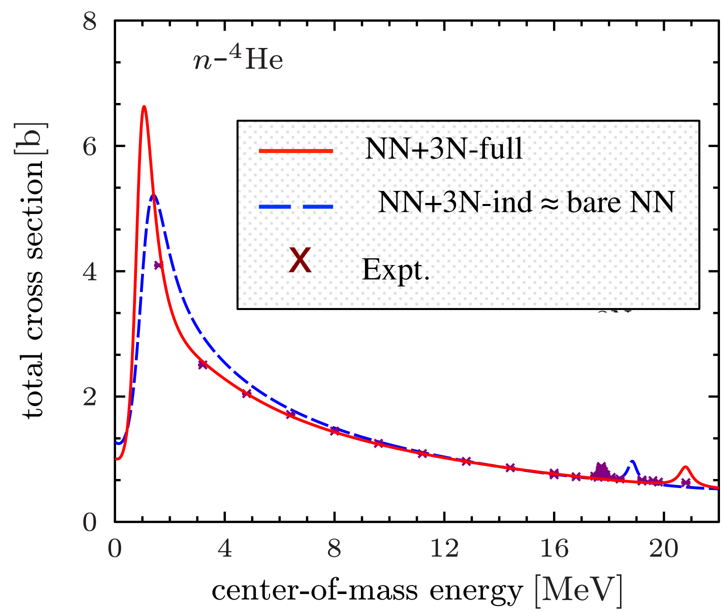


n-^{4}He scattering within NCSMC

n-^{4}He scattering phase-shifts for chiral NN and NN+3N potential



Total n-^{4}He cross section with NN and NN+3N potentials



3N force enhances 1/2$^-$ ←→ 3/2$^-$ splitting: Essential at low energies!

IOP Publishing | Royal Swedish Academy of Sciences — Physica Scripta

Phys. Scr. 91 (2016) 053002 (38pp) — doi:10.1088/0031-8949/91/5/053002

Invited Comment

Unified *ab initio* approaches to nuclear structure and reactions

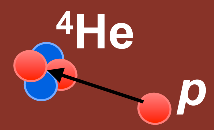
Petr Navrátil[1], Sofia Quaglioni[2], Guillaume Hupin[3,4], Carolina Romero-Redondo[2] and Angelo Calci[1]

PHYSICAL REVIEW C **88**, 054622 (2013)

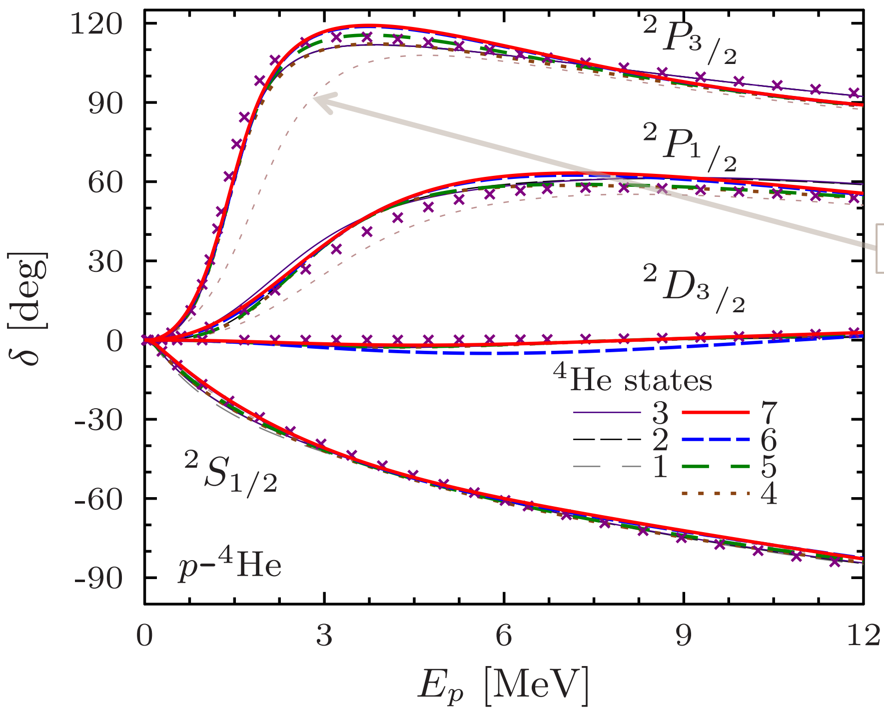
***Ab initio* many-body calculations of nucleon-^{4}He scattering with three-nucleon forces**

Guillaume Hupin,[1,*] Joachim Langhammer,[2,†] Petr Navrátil,[3,‡] Sofia Quaglioni,[1,§] Angelo Calci,[2,‖] and Robert Roth[2,¶]

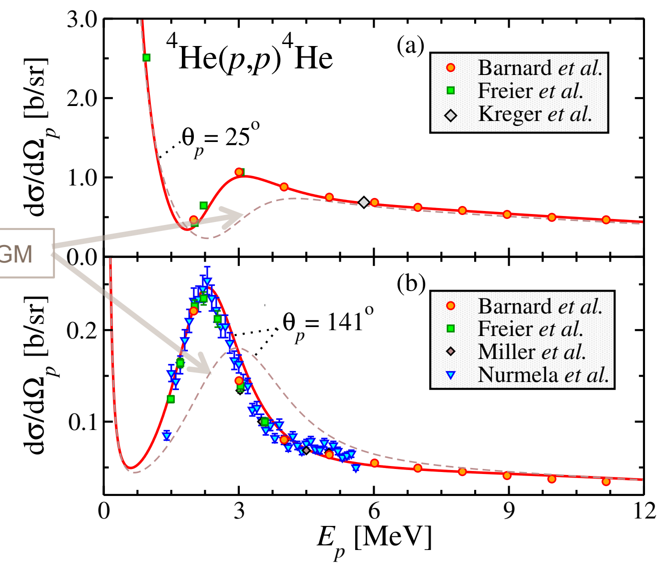
p-^{4}He scattering within NCSMC

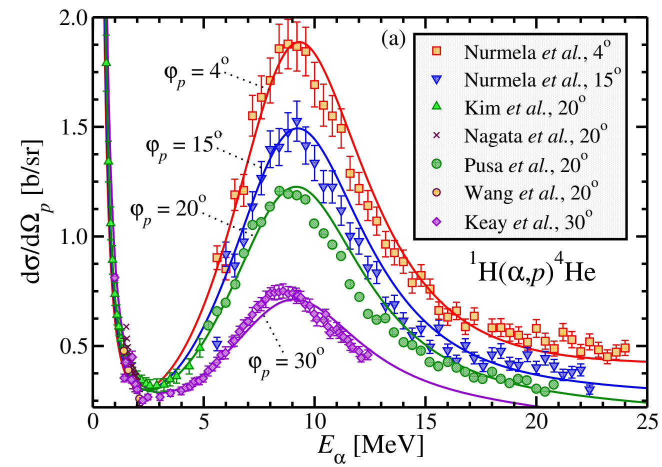


p-^{4}He scattering phase-shifts for NN+3N potential: Convergence



Differential p-^{4}He cross section with NN+3N potentials





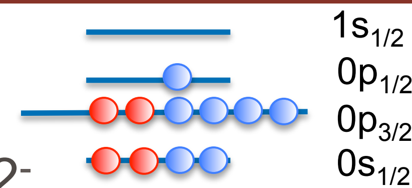
Predictive power in the 3/2$^-$ resonance region: Applications to material science

PHYSICAL REVIEW C **90**, 061601(R) (2014)

Predictive theory for elastic scattering and recoil of protons from ^{4}He

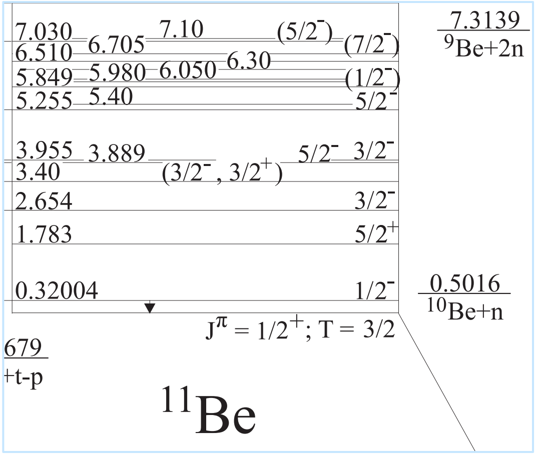
Guillaume Hupin,[1,*] Sofia Quaglioni,[1,†] and Petr Navrátil[2,‡]

Neutron-rich halo nucleus ^{11}Be



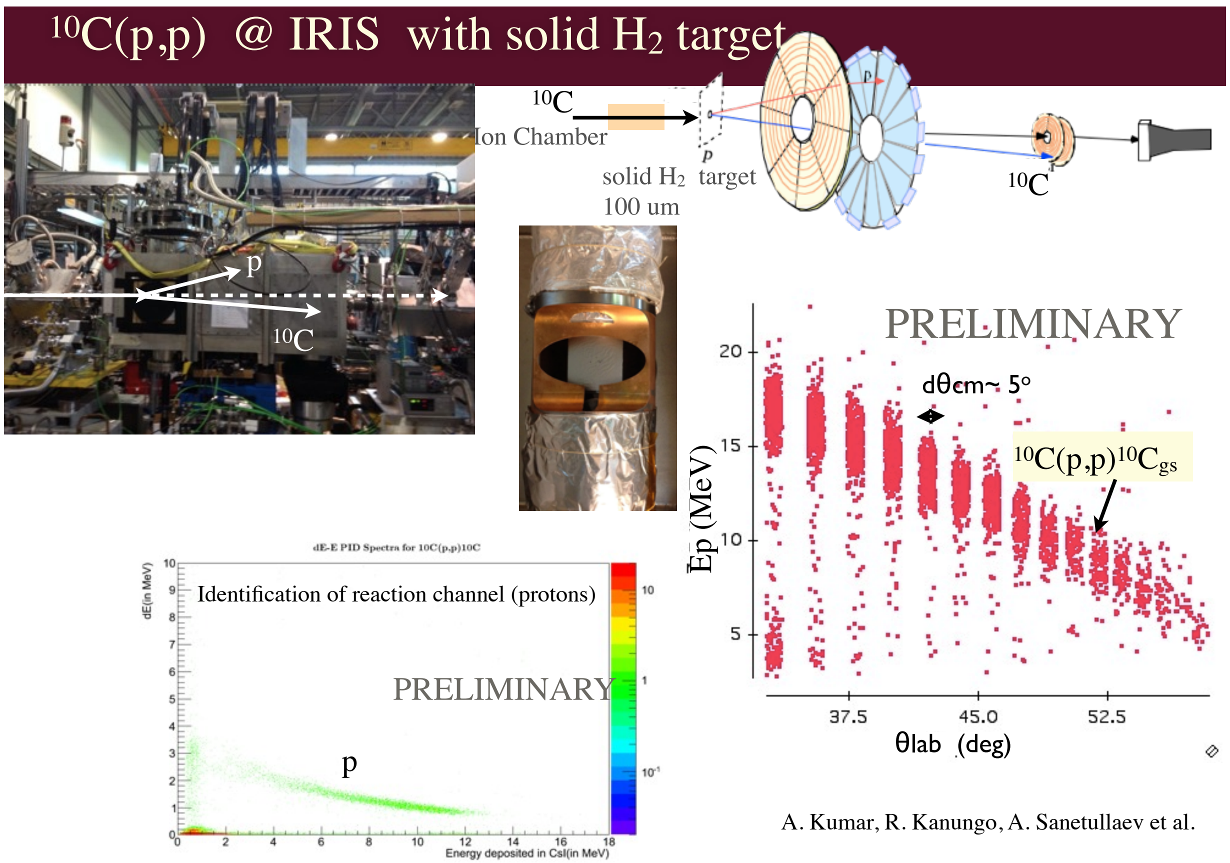
- Z=4, N=7
  - In the shell model picture g.s. expected to be $J^\pi=1/2^-$
    - Z=6, N=7 ^{13}C and Z=8, N=7 ^{15}O have $J^\pi=1/2^-$ g.s.
  - In reality, ^{11}Be g.s. is **$J^\pi=1/2^+$** - parity inversion
  - Very weakly bound: E_{th}=-0.5 MeV
    - Halo state – dominated by ^{10}Be-n in the *S*-wave
  - The $1/2^-$ state also bound – only by 180 keV

- Can we describe ^{11}Be in *ab initio* calculations?
  - Continuum must be included
  - Does the 3N interaction play a role in the parity inversion?



$^{10}C(p,p)$ @ IRIS with solid H_2 target

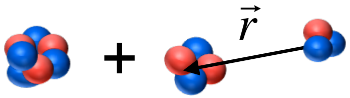
- New experiment at TRIUMF with the novel IRIS solid H_2 target
  - First re-accelerated ^{10}C beam at TRIUMF
  - $^{10}C(p,p)$ angular distributions measured at E_{CM} ~ 4.16 MeV and 4.4 MeV



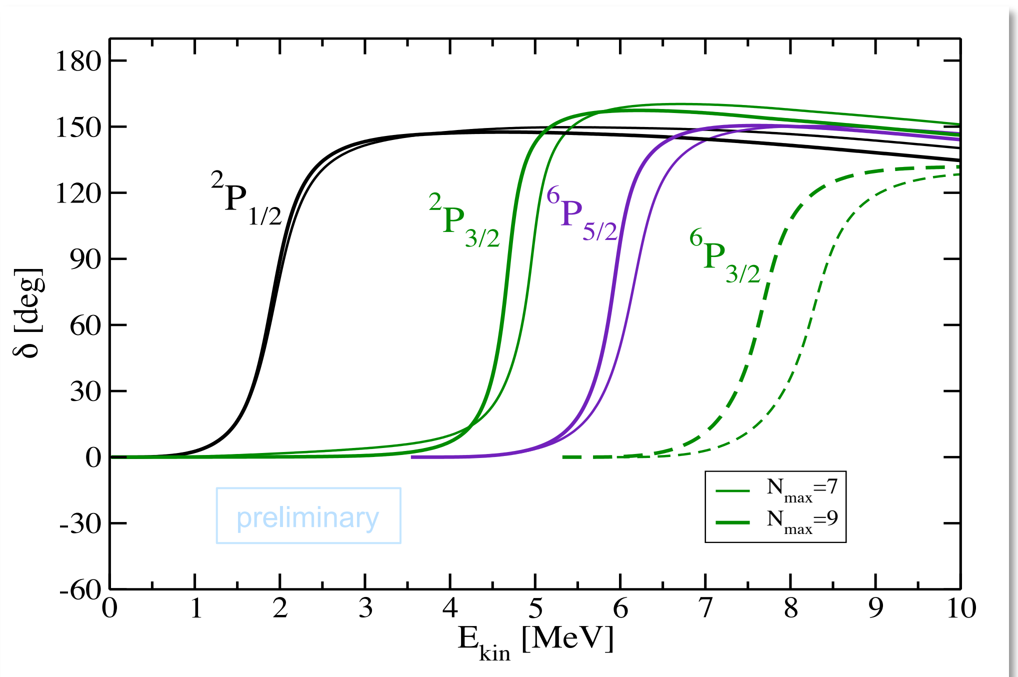
IRIS collaboration:
A. Kumar, R. Kanungo,
A. Sanetullaev *et al.*

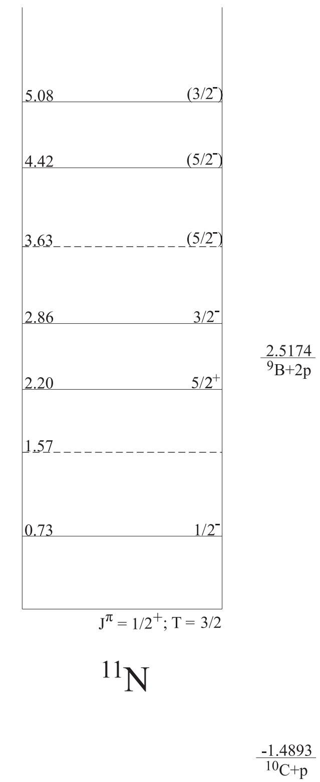
$p + {}^{10}C$ scattering: structure of ${}^{11}N$ resonances

- NCSMC calculations with **chiral NN+3N** (N^3LO NN+N^2LO 3NF400, NNLOsat)
  - p-${}^{10}C$ + ${}^{11}N$
    - ${}^{10}C$: 0^+, 2^+, 2^+ NCSM eigenstates
    - ${}^{11}N$: ≥4 $\pi = -1$ and ≥3 $\pi = +1$ NCSM eigenstates



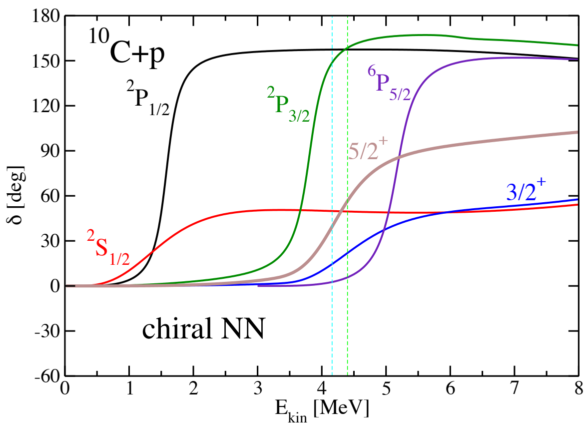
chiral NN+3NF400

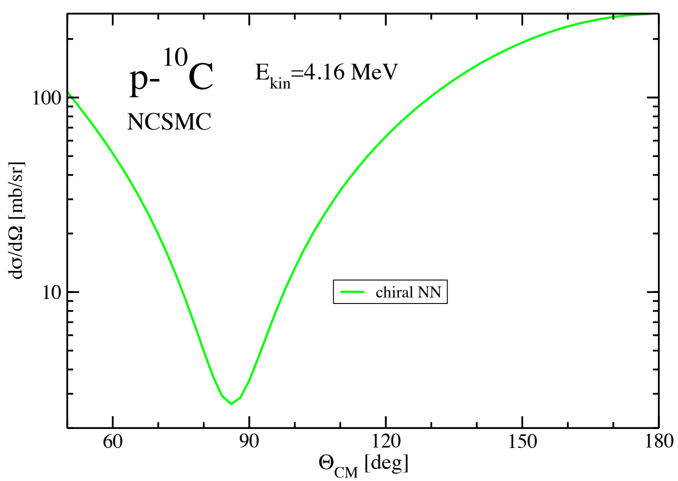


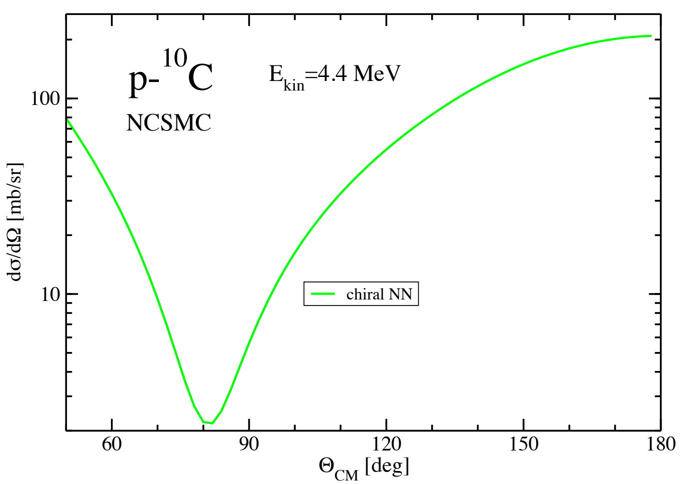


A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al* with IRIS collaboration, in preparation

p+^{10}C scattering: structure of ^{11}N resonances







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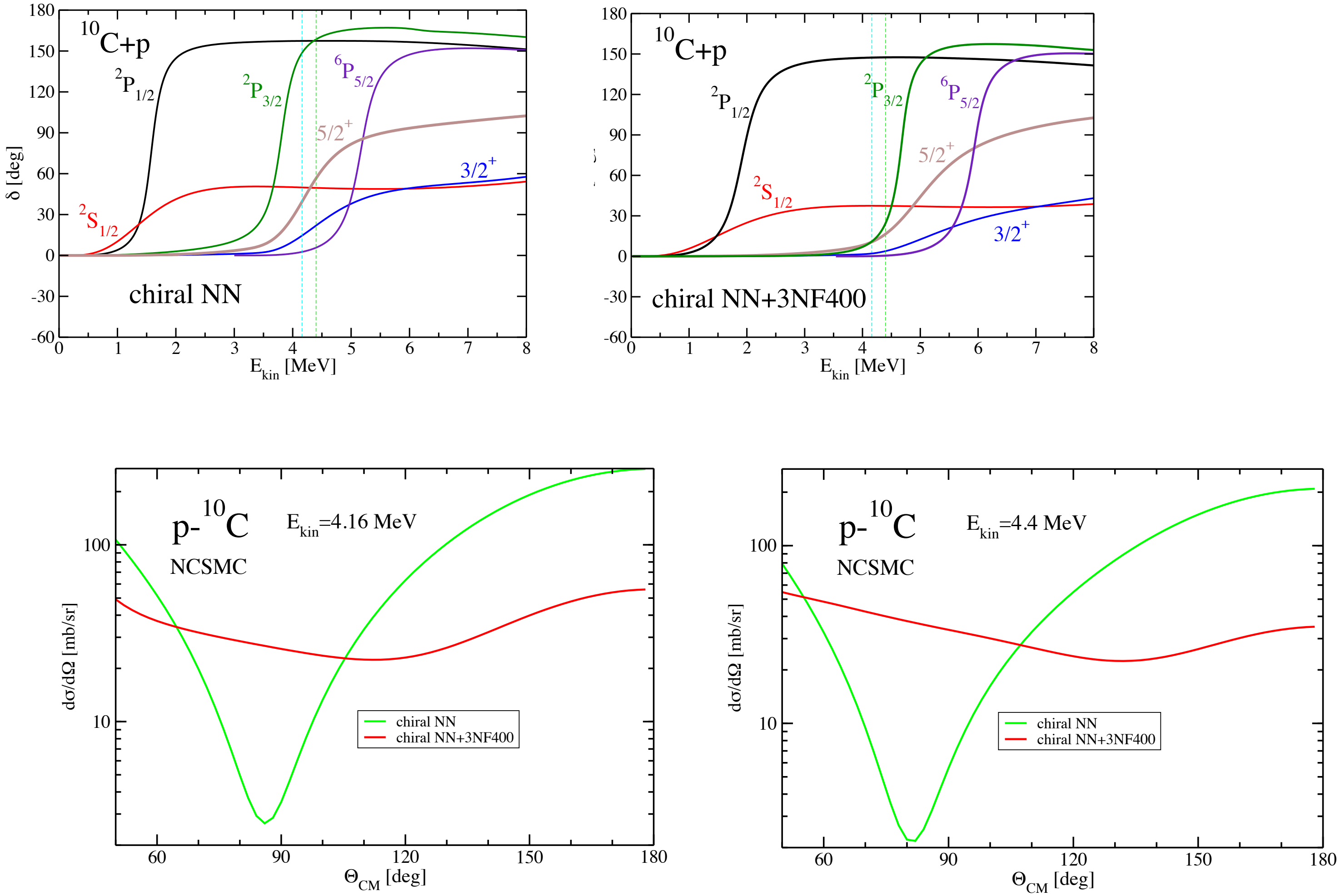

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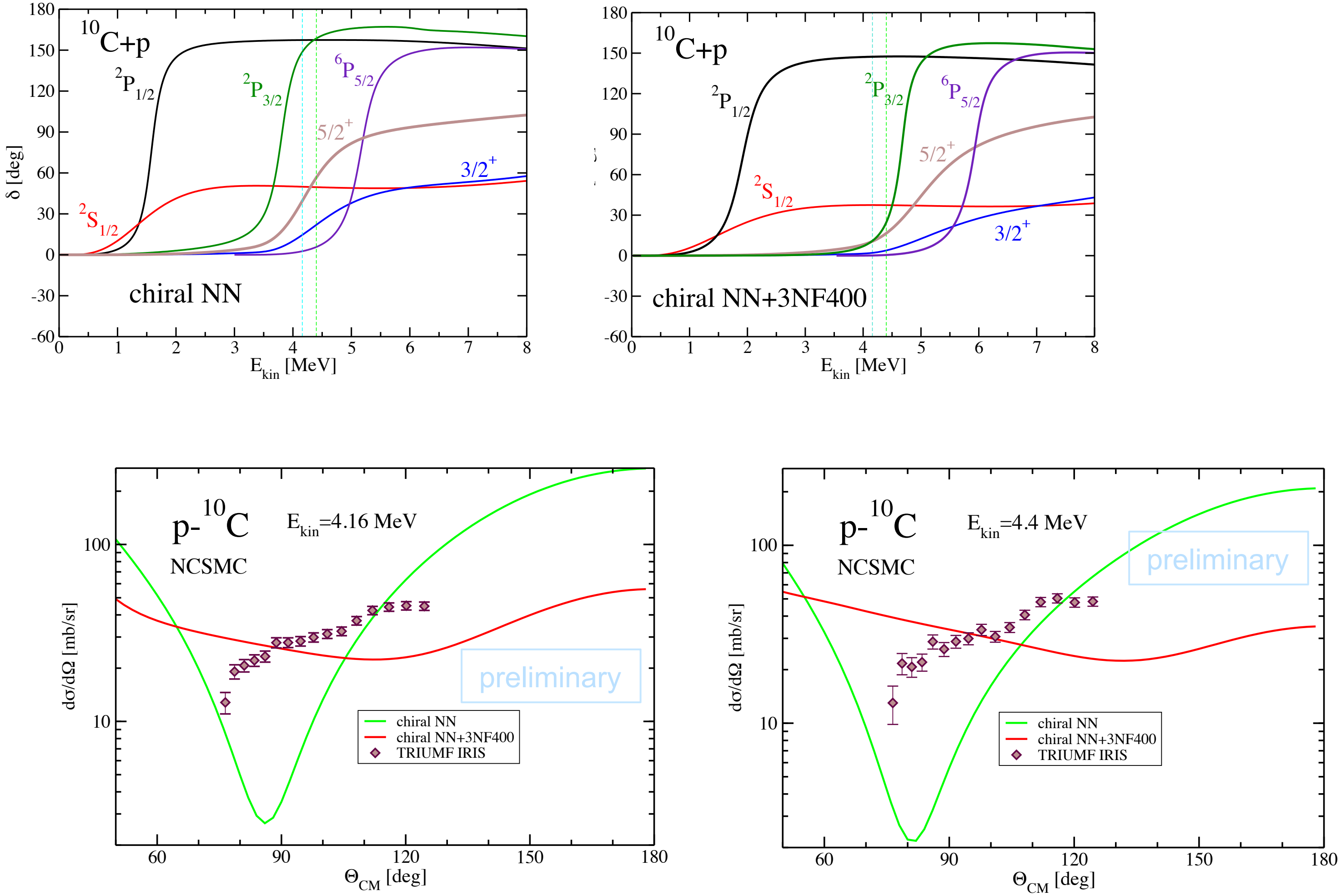

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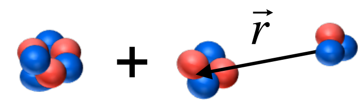
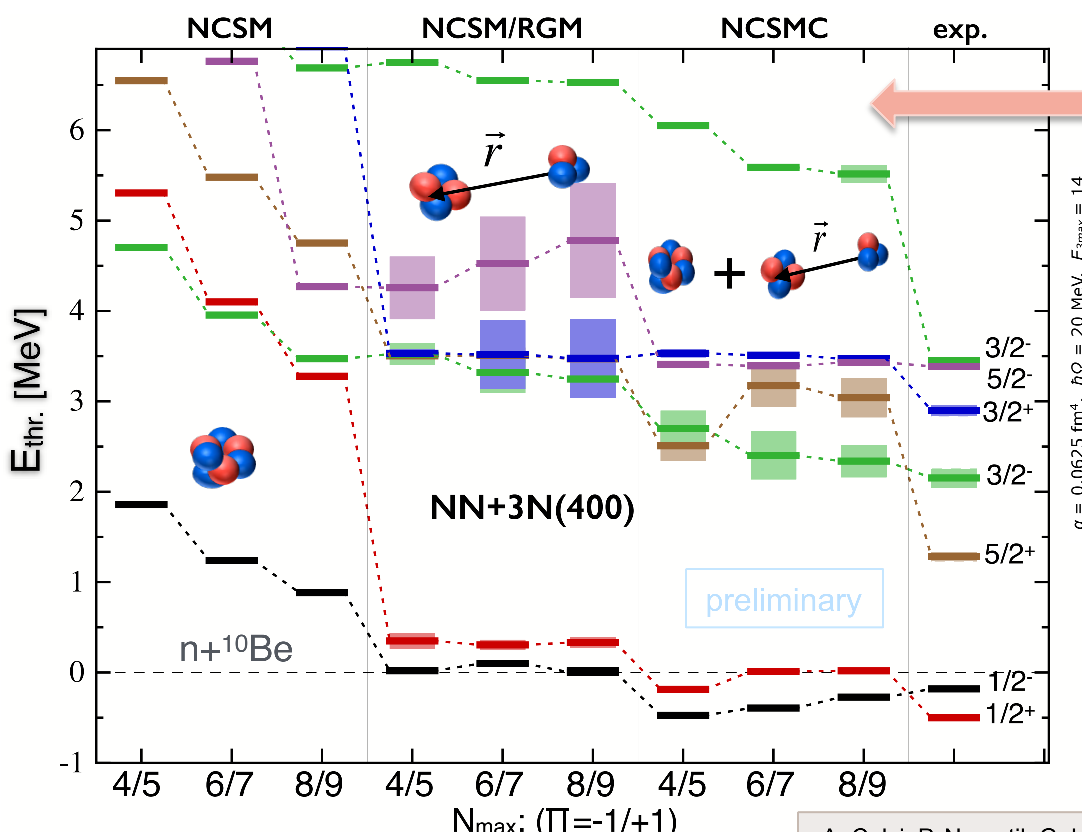
A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al* with IRIS collaboration, in preparation

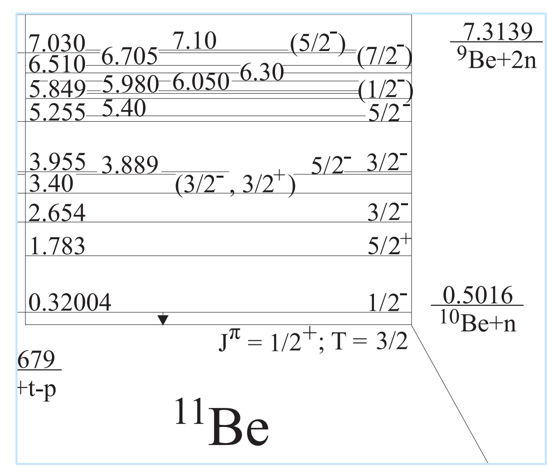
Structure of ^{11}Be from chiral NN+3N forces

- NCSMC calculations **including chiral 3N** (N^3LO NN+N^2LO 3NF400)
  - n-^{10}Be + ^{11}Be
    - ^{10}Be: 0^+, 2^+, 2^+ NCSM eigenstates
    - ^{11}Be: ≥6 π = -1 and ≥3 π = +1 NCSM eigenstates



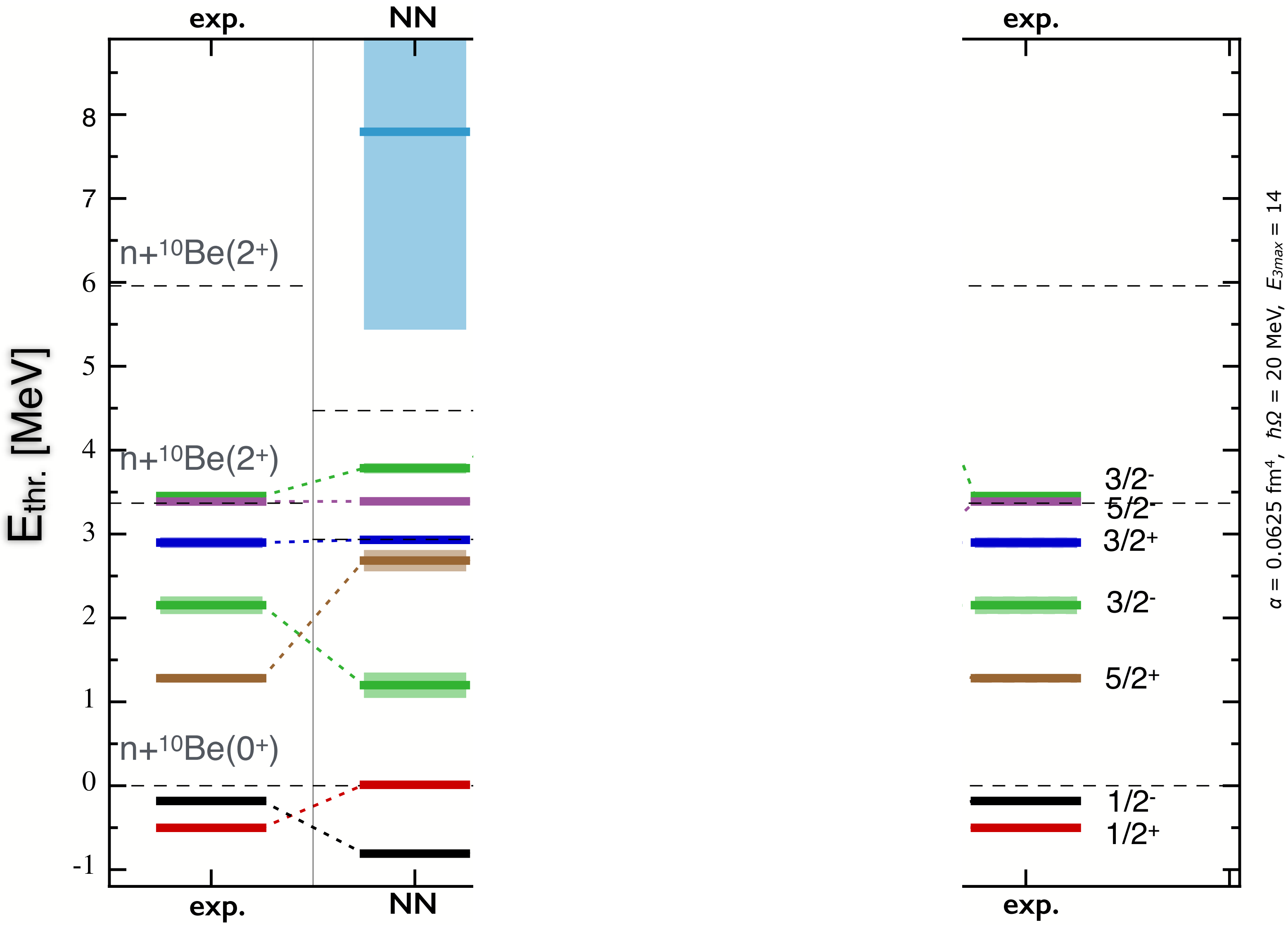


Continuum effects



A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al.*, in preparation

^{11}Be within NCSMC: Discrimination among chiral nuclear forces



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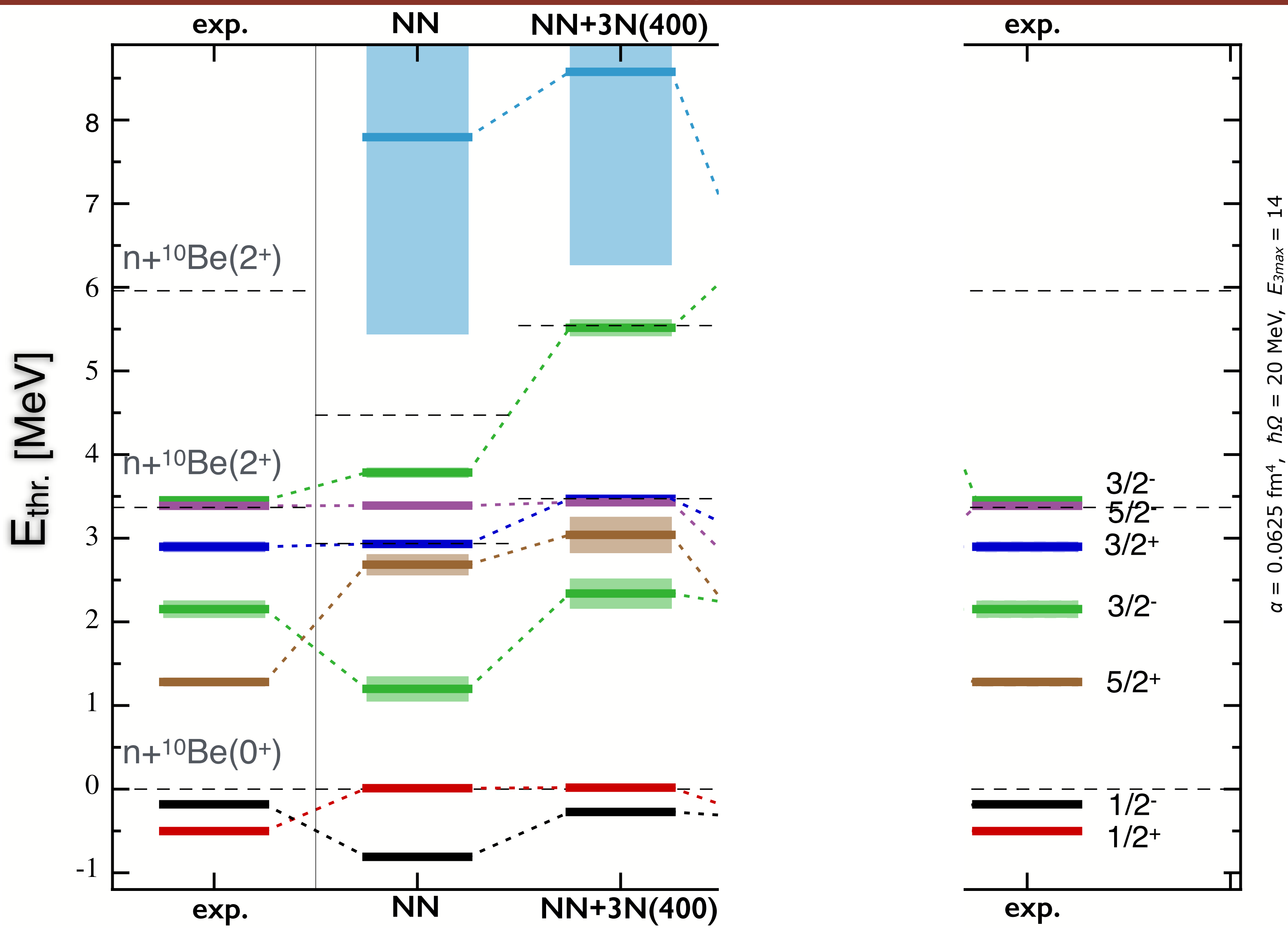

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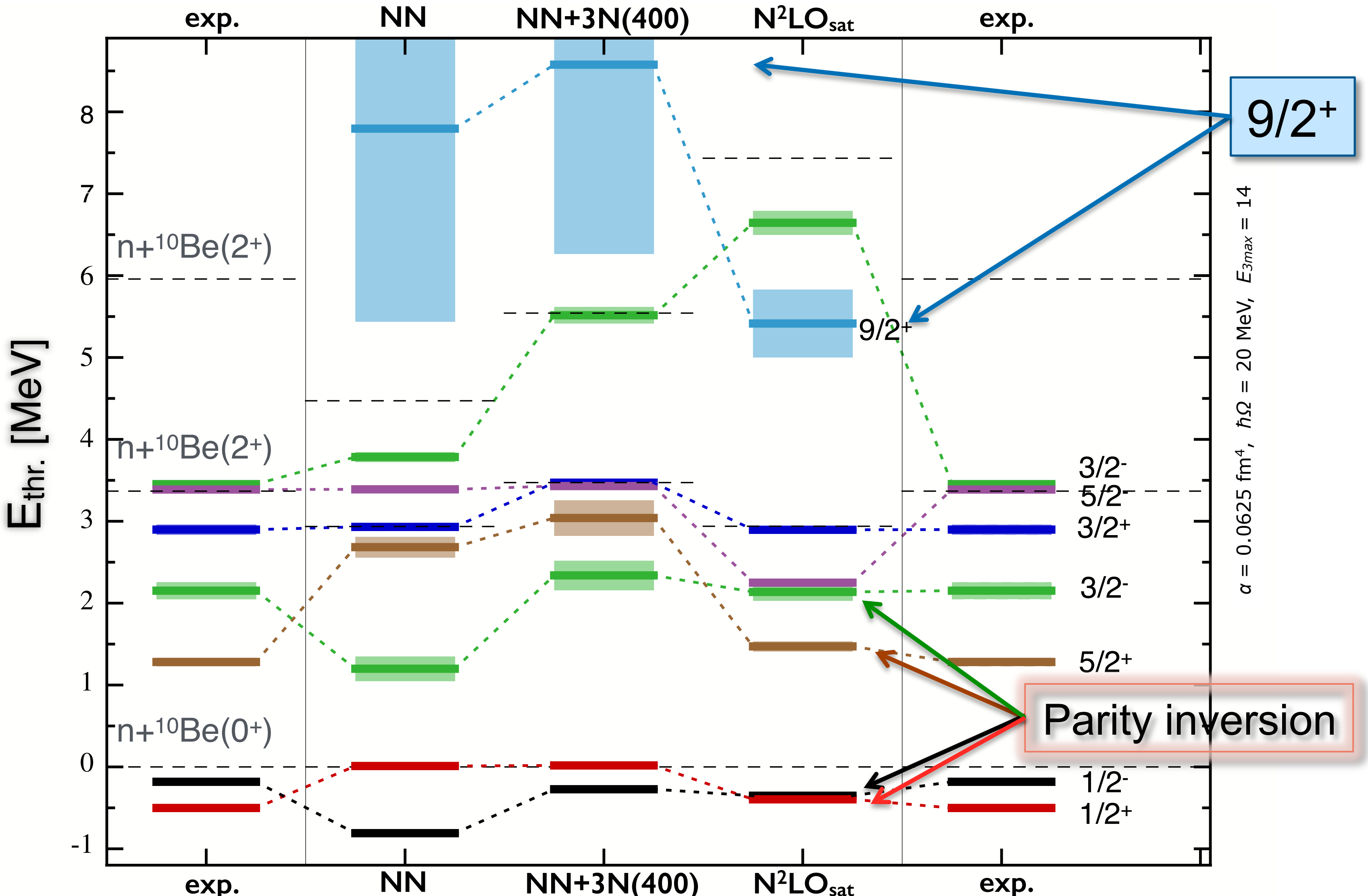

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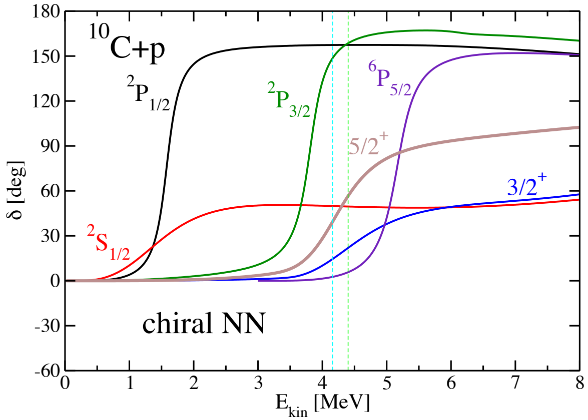

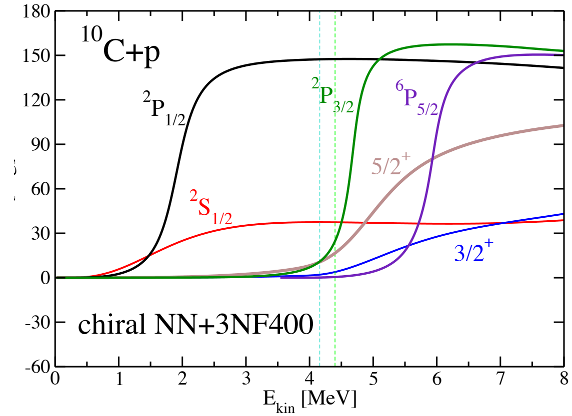

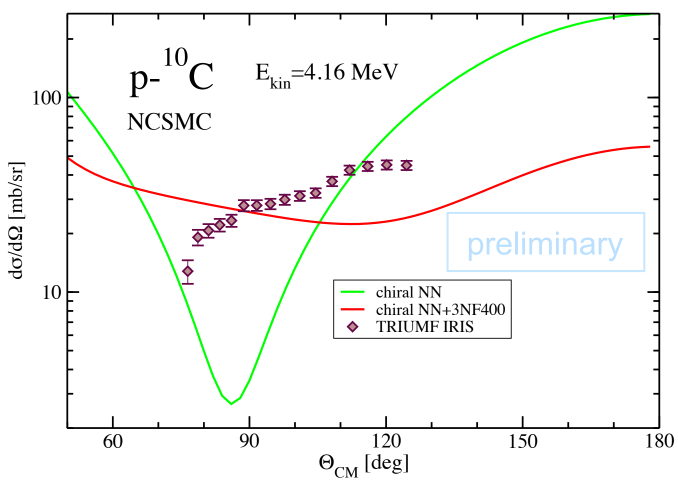

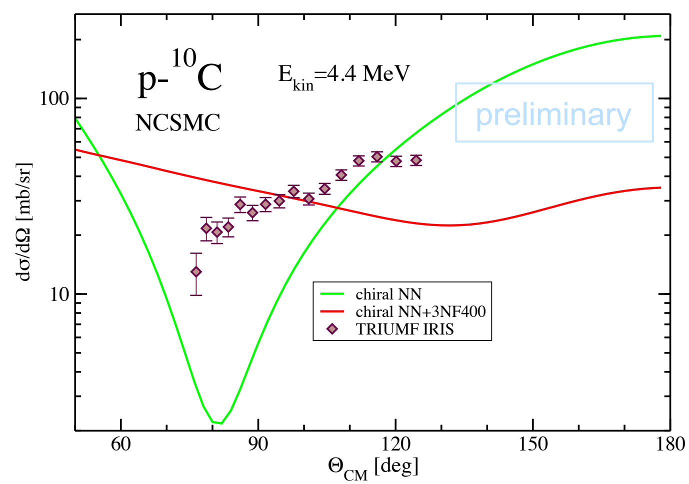

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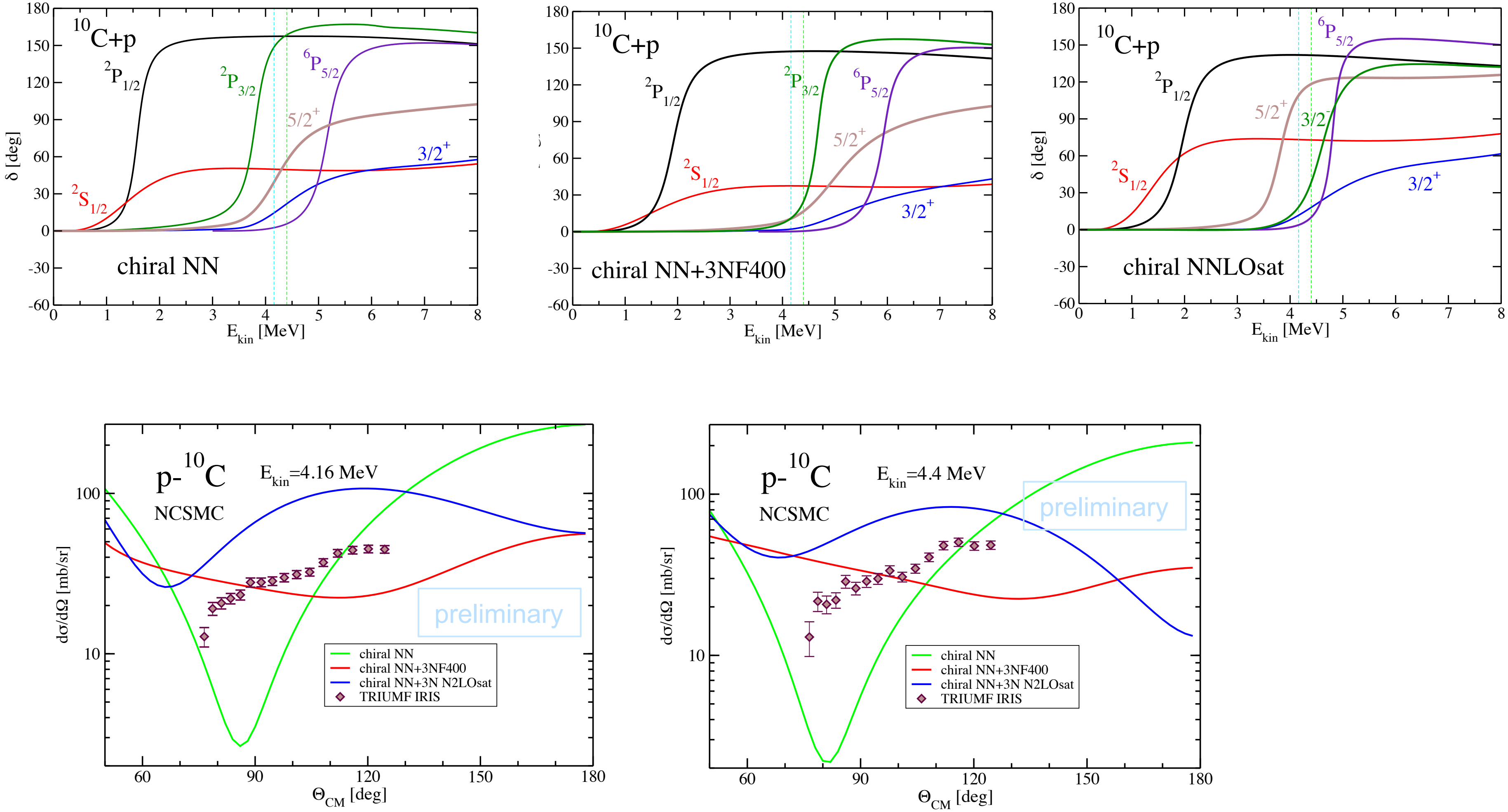

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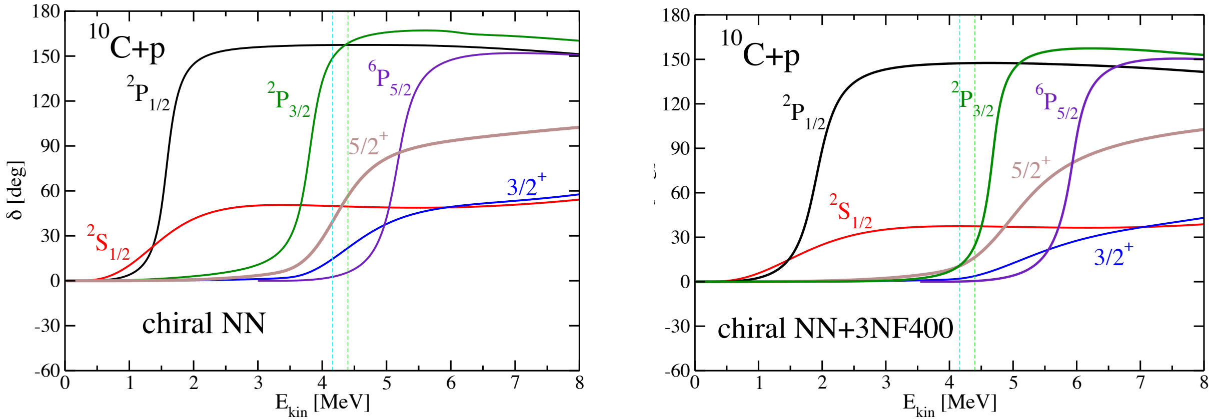
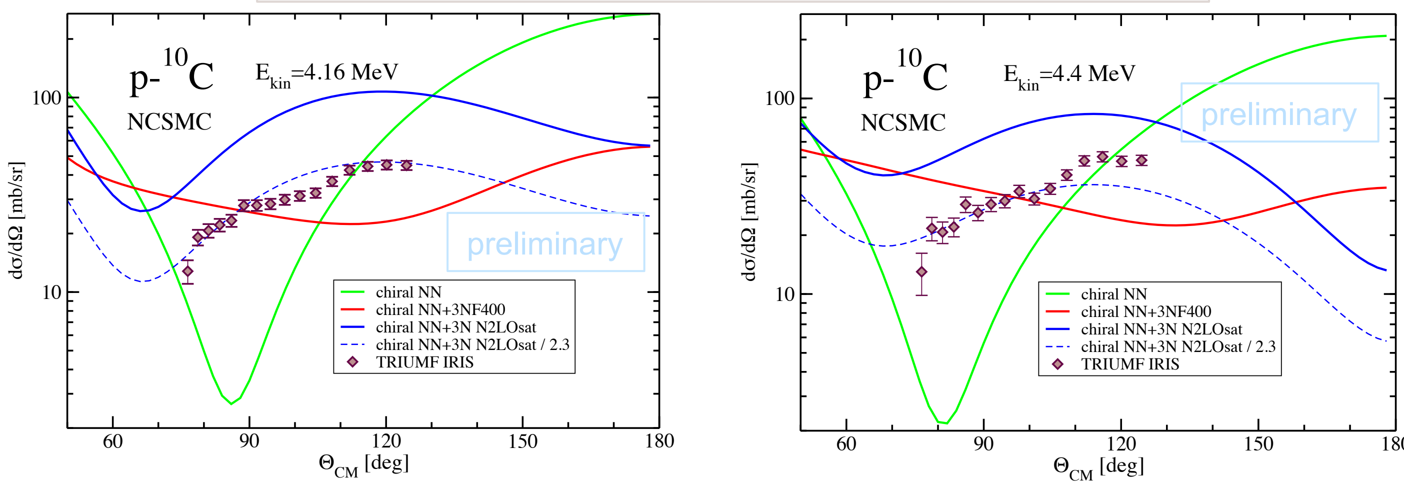
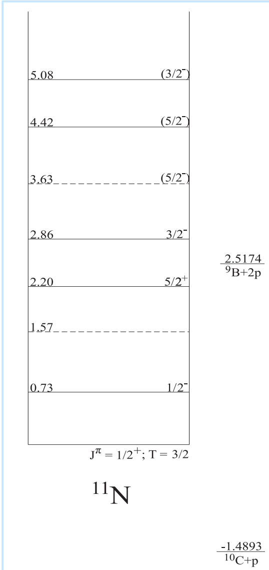


Discrimination among chiral nuclear forces





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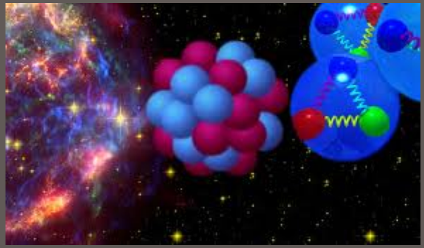
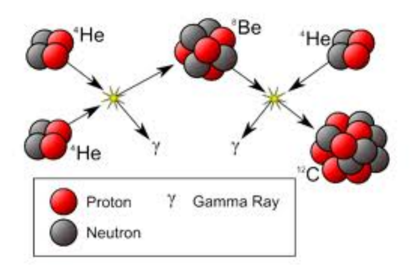
Canada's national laboratory for particle and nuclear physics
Laboratoire national canadien pour la recherche en physique nucléaire
et en physique des particules

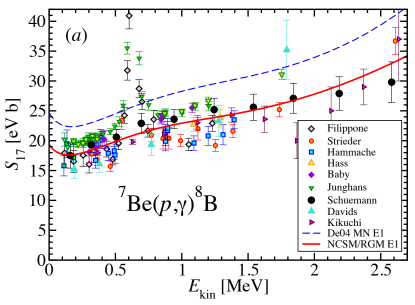
Ab initio calculations of nuclear reactions important for astrophysics

NIC-XIV School 2016
Niigata University (Ikarashi Campus)
Niigata, Japan
June 13-17, 2016

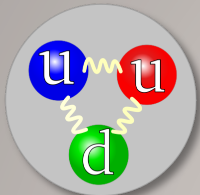
Petr Navratil | TRIUMF

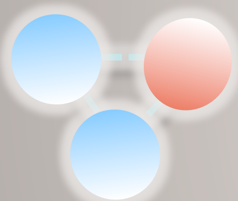




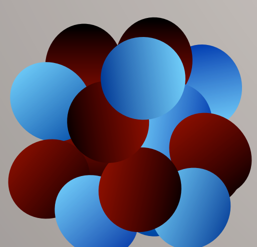


From QCD to nuclei





$$H|\Psi\rangle = E|\Psi\rangle$$



Low-energy QCD

↓

NN+3N interactions from chiral EFT — …or accurate meson-exchange potentials

↓

Unitary/similarity transformations — Identity or SRG or OLS or UCOM … Softens NN, induces 3N

↓

Many-Body methods — NCSM, NCSM/RGM, CCM, GFMC, HH, Nuclear Lattice EFT…

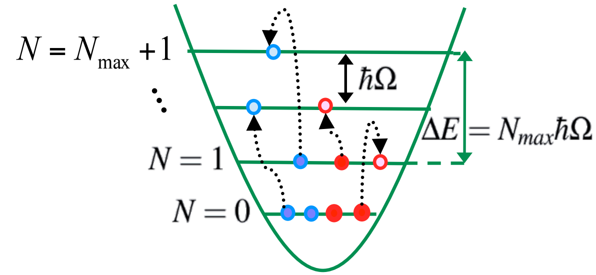
↓

Nuclear structure and reactions

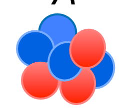
Applications to nuclear astrophysics

No-core shell model

- No-core shell model (NCSM)
  - *A*-nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances



A



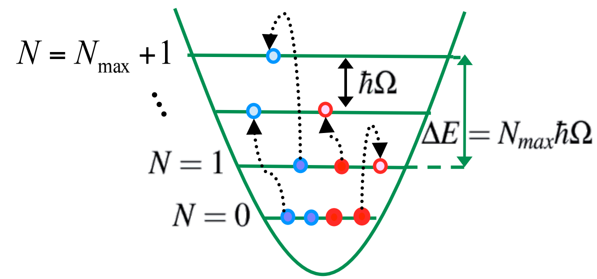
$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

$$\Psi^{(A)} = \sum_\lambda c_\lambda \left|^{(A)} \, , \lambda \right\rangle$$

Unknowns

No-core shell model with RGM

- No-core shell model (NCSM)
  - *A*-nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances



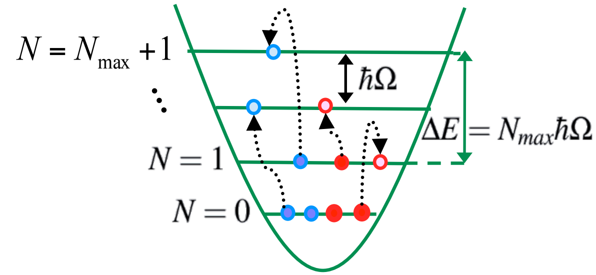
- NCSM with Resonating Group Method (NCSM/RGM)
  - cluster expansion
  - proper asymptotic behavior
  - long-range correlations

$$\Psi^{(A)} = \sum_{\nu} \int d\vec{r}\, \gamma_{\nu}(\vec{r}\,)\, \hat{A}_{\nu} \left| (A-a) \xrightarrow{\vec{r}} (a), \nu \right\rangle$$

Unknowns

No-core shell model with continuum

- No-core shell model (NCSM)
  - *A*-nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances



- NCSM with Resonating Group Method (NCSM/RGM)
  - cluster expansion
  - proper asymptotic behavior
  - long-range correlations

S. Baroni, P. N., and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

The most efficient:
No-Core Shell Model with Continuum (NCSMC)

NCSM eigenstates

NCSM/RGM channel states

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| {}^{(A)} , \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r})\; \hat{A}_{\nu} \left| {}_{(A-a)} \; \vec{r} \; {}_{(a)} , \nu \right\rangle$$

Unknowns

NCSMC wave function

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| (A) , \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left| \underset{(A-a)}{} \xrightarrow{\vec{r}} (a) , \nu \right\rangle$$

$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{\lambda} |A\lambda J^\pi T\rangle \left[\sum_{\lambda'} (N^{-\frac{1}{2}})^{\lambda\lambda'} \bar{c}_{\lambda'} + \sum_{\nu'} \int dr'\, r'^2 (N^{-\frac{1}{2}})^{\lambda}_{\nu' r'} \frac{\bar{\chi}_{\nu'}(r')}{r'} \right]$$

$$+ \sum_{\nu\nu'} \int dr\, r^2 \int dr'\, r'^2 \hat{\mathcal{A}}_{\nu} \left|\Phi^{J^\pi T}_{\nu r}\right\rangle \mathcal{N}^{-\frac{1}{2}}_{\nu\nu'}(r, r') \left[\sum_{\lambda'} (N^{-\frac{1}{2}})^{\lambda'}_{\nu' r'} \bar{c}_{\lambda'} + \sum_{\nu''} \int dr''\, r''^2 (N^{-\frac{1}{2}})_{\nu' r' \nu'' r''} \frac{\bar{\chi}_{\nu''}(r'')}{r''} \right].$$

Asymptotic behavior $r \longrightarrow \infty$:

$$\bar{\chi}_{\nu}(r) \sim C_{\nu} W(k_{\nu} r) \qquad \bar{\chi}_{\nu}(r) \sim \mathrm{v}_{\nu}^{-\frac{1}{2}} \left[\delta_{\nu i} I_{\nu}(k_{\nu} r) - U_{\nu i} O_{\nu}(k_{\nu} r) \right]$$

Bound state — Scattering state — Scattering matrix

E1 transitions in NCSMC

$$\Psi^{(A)} = \sum_\lambda c_\lambda \left| (A), \lambda \right\rangle + \sum_\nu \int d\vec{r}\, \gamma_\nu(\vec{r}\,)\, \hat{A}_\nu \left| \vec{r}, (A-a), (a), \nu \right\rangle$$

$$\vec{E}1 = e\sum_{i=1}^{A-a} \frac{1+\tau_i^{(3)}}{2}\left(\vec{r}_i - \vec{R}^{(A-a)}_{\rm c.m.}\right) + e\sum_{j=A-a+1}^{A} \frac{1+\tau_j^{(3)}}{2}\left(\vec{r}_i - \vec{R}^{(a)}_{\rm c.m.}\right) + e\frac{Z_{(A-a)}a - Z_{(a)}(A-a)}{A}\vec{r}_{A-a,a}\,.$$

$$\mathcal{M}^E_{1\mu} = e\sum_{j=1}^{A} \frac{1+\tau_j^{(3)}}{2}\left|\vec{r}_j - \vec{R}^{(A)}_{\rm c.m.}\right| Y_{1\mu}(\widehat{r_j - R^{(A)}_{\rm c.m.}})$$

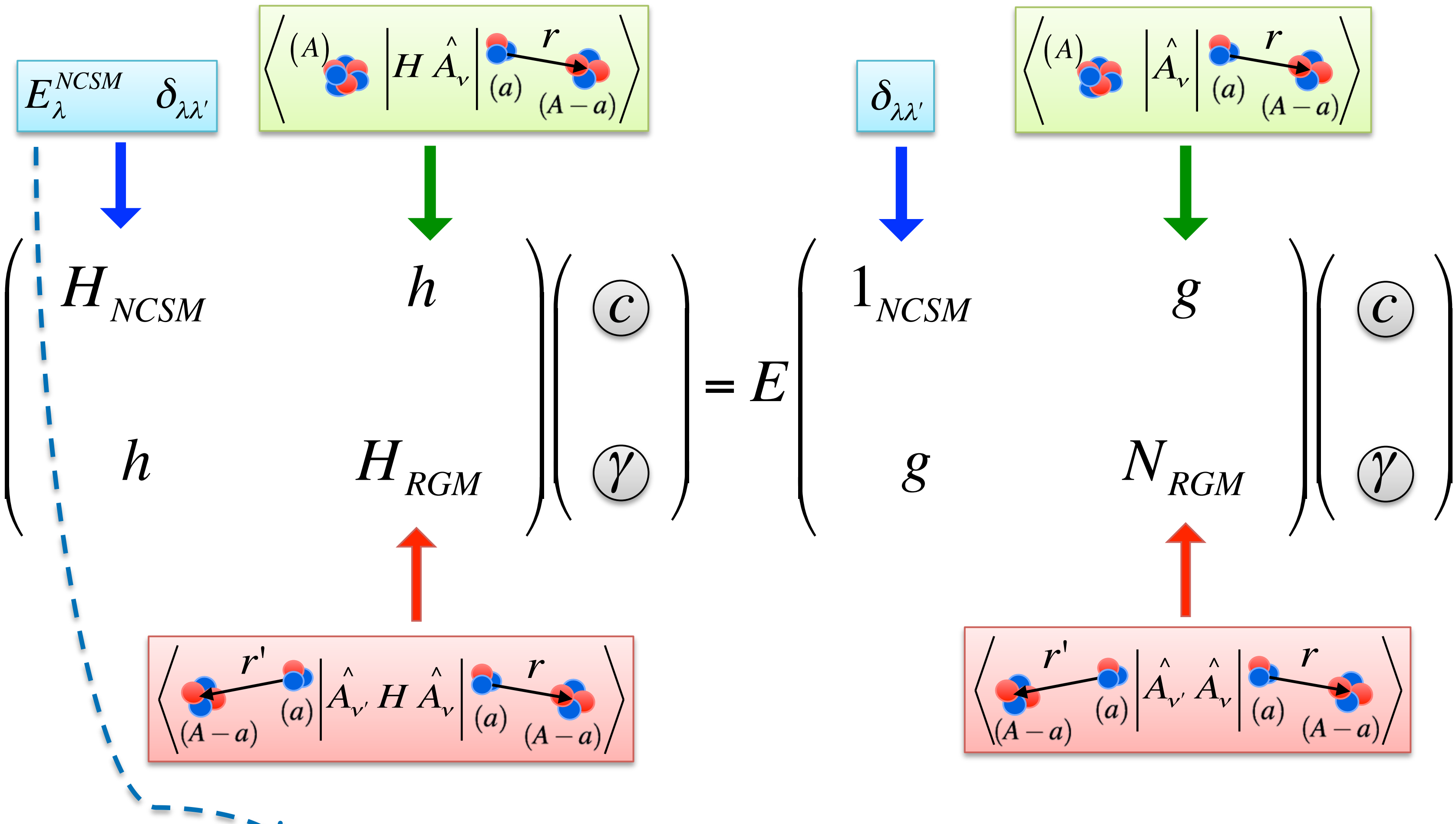
$$\begin{aligned}\mathcal{B}^{E1}_{fi} = & \sum_{\lambda\lambda'} c^{*f}_{\lambda'} \langle A\lambda' J^{\pi_f}_f T_f || \mathcal{M}^E_1 || A\lambda J^{\pi_i}_i T_i\rangle\, c^i_\lambda \\ & + \sum_{\lambda'\nu} \int dr r^2\, c^{*f}_{\lambda'} \langle A\lambda' J^{\pi_f}_f T_f || \mathcal{M}^E_1 \hat{\mathcal{A}}_\nu || \Phi^i_{\nu r}\rangle \frac{\gamma^i_\nu(r)}{r} \\ & + \sum_{\lambda\nu'} \int dr' r'^2 \frac{\gamma^{*f}_{\nu'}(r')}{r'} \langle \Phi^f_{\nu' r'} || \hat{\mathcal{A}}_{\nu'} \mathcal{M}^E_1 || A\lambda J^{\pi_i}_i T_i\rangle\, c^i_\lambda \\ & + \sum_{\nu\nu'} \int dr' r'^2 \int dr r^2 \frac{\gamma^{*f}_{\nu'}(r')}{r'} \langle \Phi^f_{\nu' r'} || \hat{\mathcal{A}}_{\nu'} \mathcal{M}^E_1 \hat{\mathcal{A}}_\nu || \Phi^i_{\nu r}\rangle \frac{\gamma^i_\nu(r)}{r}\end{aligned}$$

TRIUMF

Photo-disassociation of ^{11}Be

Bound to bound	NCSM	NCSMC-phenom	Expt.
B(E1; $1/2^+ \rightarrow 1/2^-$) [e^2 fm^2]	5×10^{-6}	0.118	0.102(2)

NCSMC phenomenology



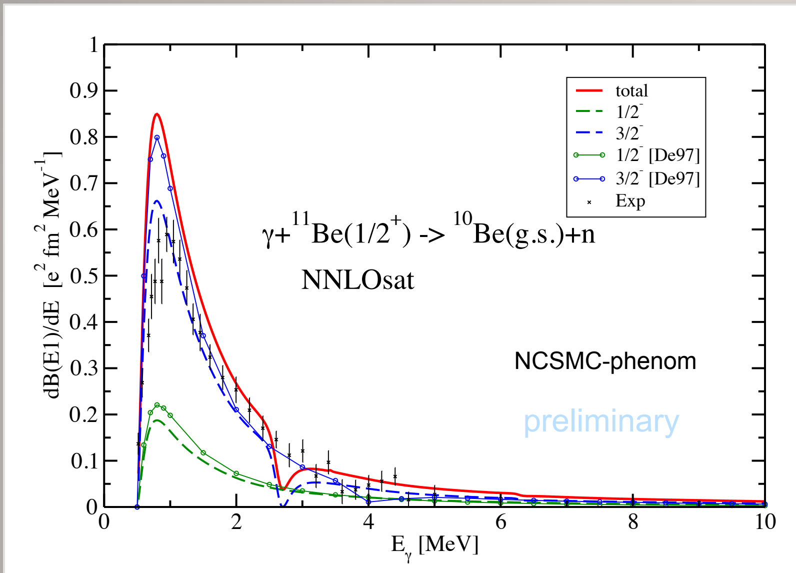
$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix}$$

E_λ^{NCSM} energies treated as adjustable parameters

Cluster excitation energies set to experimental values

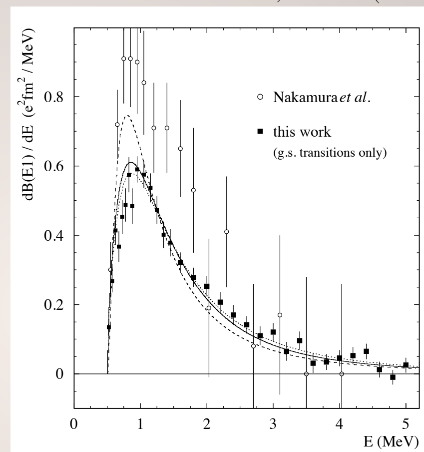
Photo-disassociation of ^{11}Be

Bound to continuum

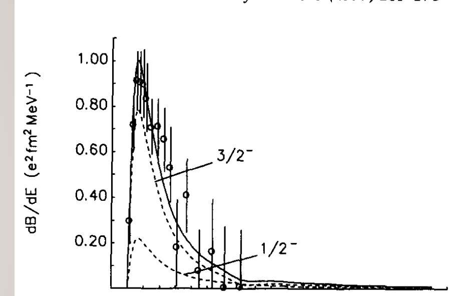


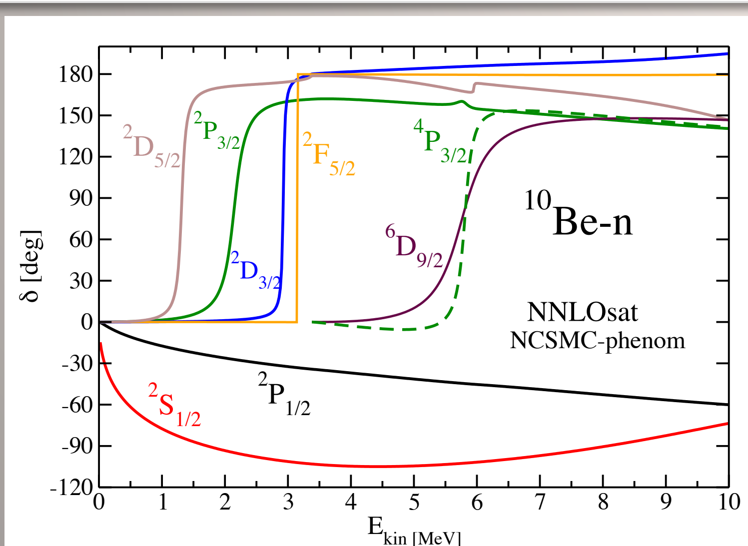
Bound to bound	NCSM	NCSMC-phenom	Expt.
B(E1; $1/2^+ \rightarrow 1/2^-$) [e^2 fm^2]	5×10^{-6}	0.118	0.102(2)

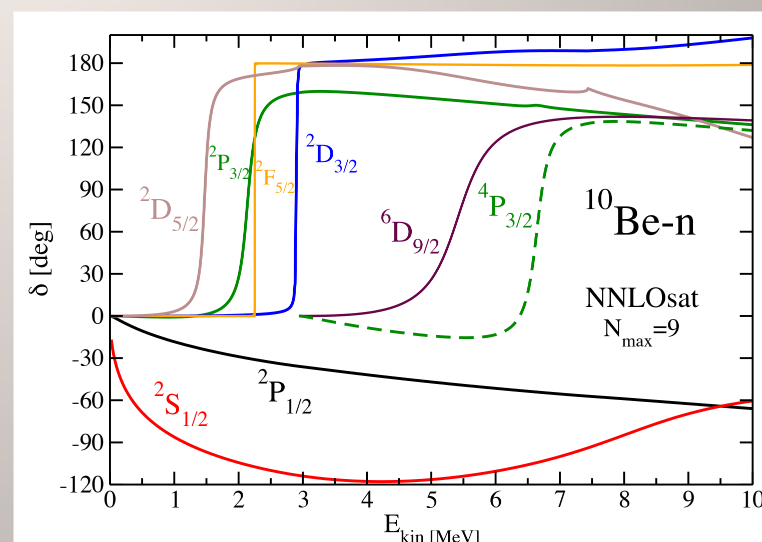
PHYSICAL REVIEW C **68**, 034318 (2003)



P. Descouvemont/Nuclear Physics A 615 (1997) 261–276







Next: p+^{11}C scattering and ^{11}C(p,γ)^{12}N capture

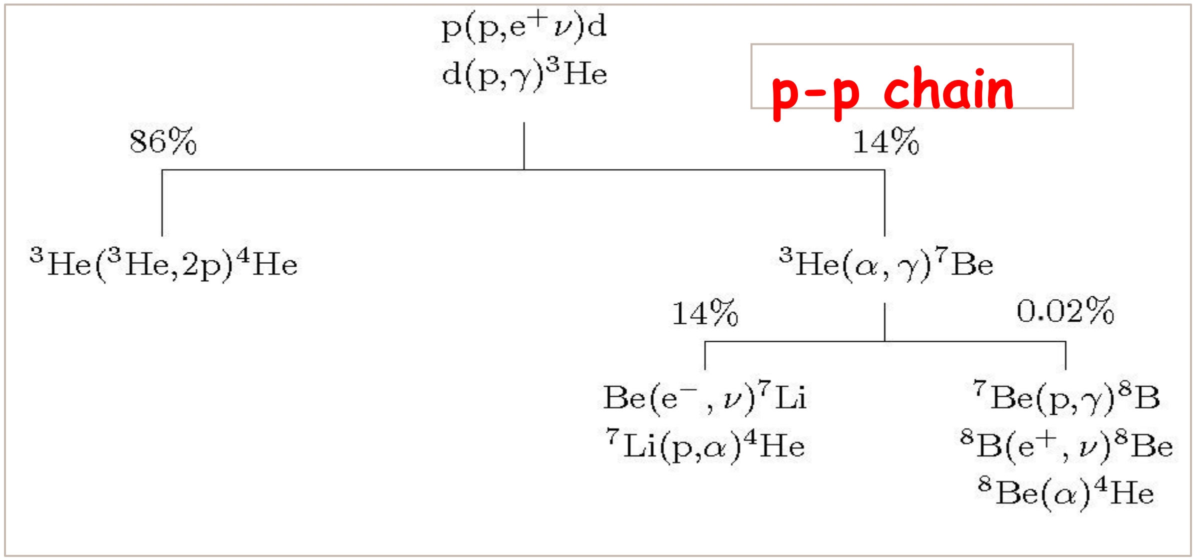
- Measurement of ^{11}C(p,p) resonance scattering planned at TRIUMF
  - TUDA facility
  - ^{11}C beam of sufficient intensity produced

- NCSMC calculations of ^{11}C(p,p) with chiral NN+3N under way

- Obtained wave functions will be used to calculate ^{11}C(p,γ)^{12}N capture relevant for astrophysics

Next: p+^{11}C scattering and ^{11}C(p,γ)^{12}N capture

- ^{11}C(p,γ)^{12}N capture relevant in hot *p-p* chain: Link between pp chain and the CNO cycle - bypass of slow triple alpha capture ^{4}He(αα,γ)^{12}C



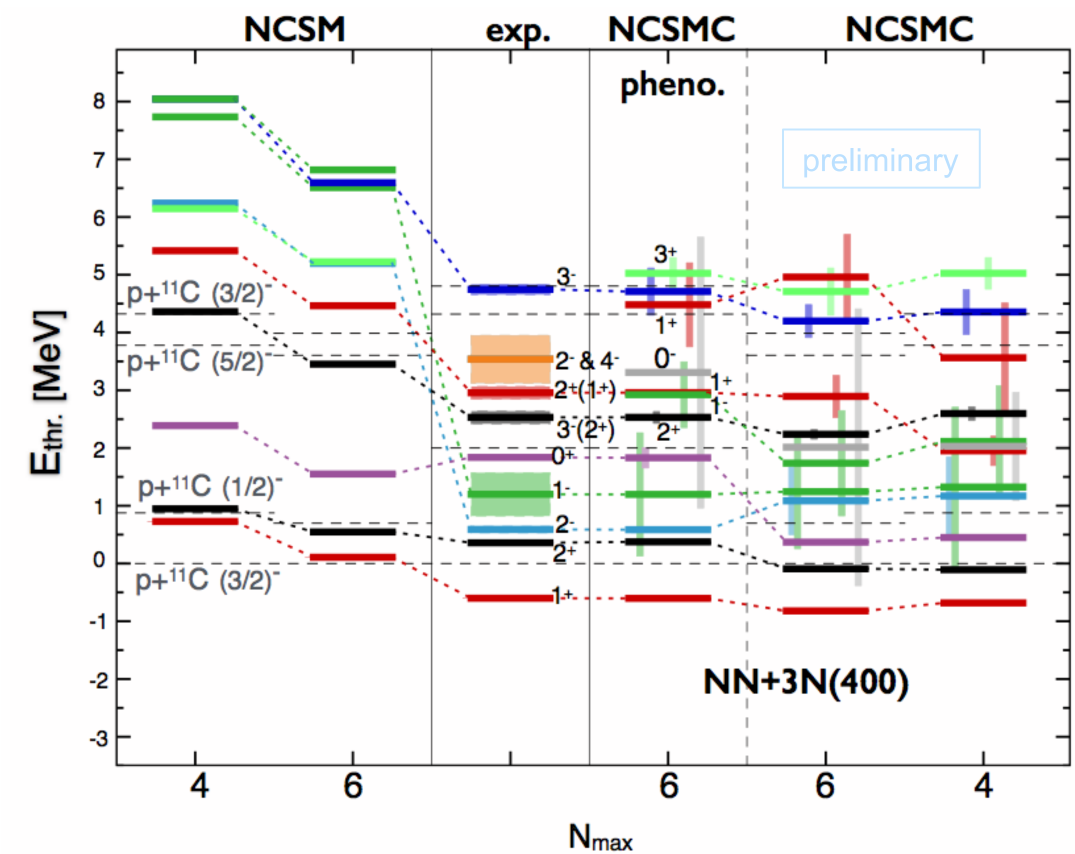
$$^{3}He(\alpha,\gamma)^{7}Be(\alpha,\gamma)^{11}C(p,\gamma)^{12}N(p,\gamma)^{13}O(\beta^{+},\nu)^{13}N(p,\gamma)^{14}O$$

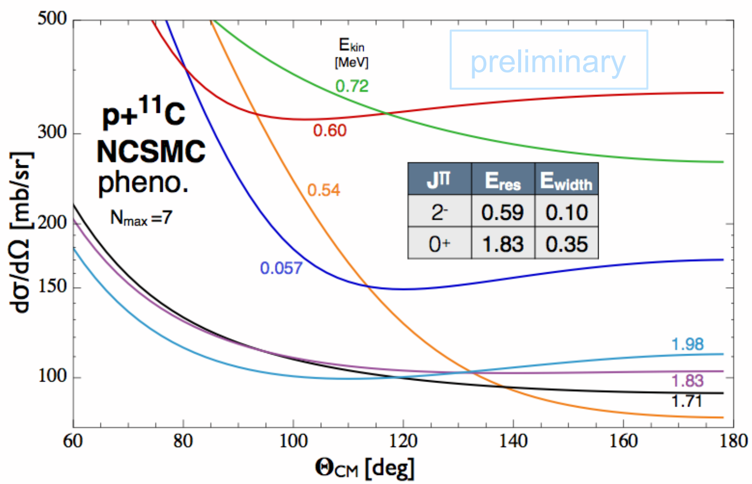
$$^{3}He(\alpha,\gamma)^{7}Be(\alpha,\gamma)^{11}C(p,\gamma)^{12}N(\beta^{+},\nu)^{12}C(p,\gamma)^{13}N(p,\gamma)^{14}O$$

$$^{11}\mathrm{C}(\beta^{+}\ \nu)^{11}\mathrm{B}(p,\alpha)^{8}\mathrm{Be}(^{4}\mathrm{He},^{4}\mathrm{He})$$

Next: p+^{11}C scattering and ^{11}C(p,γ)^{12}N capture

- NCSMC calculations of ^{11}C(p,p) with chiral NN+3N under way





Jπ	Eres	Ewidth
2-	0.59	0.10
0+	1.83	0.35

NCSMC calculations to be validated by measured cross sections and applied to calculate the ^{11}C(p,γ)^{12}N capture

Solar *p-p* chain

p-p chain

p(p,$e^+\nu$)d
d(p,γ)^{3}He

86% | 14%

^{3}He(^{3}He,2p)^{4}He

^{3}He(α,γ)^{7}Be

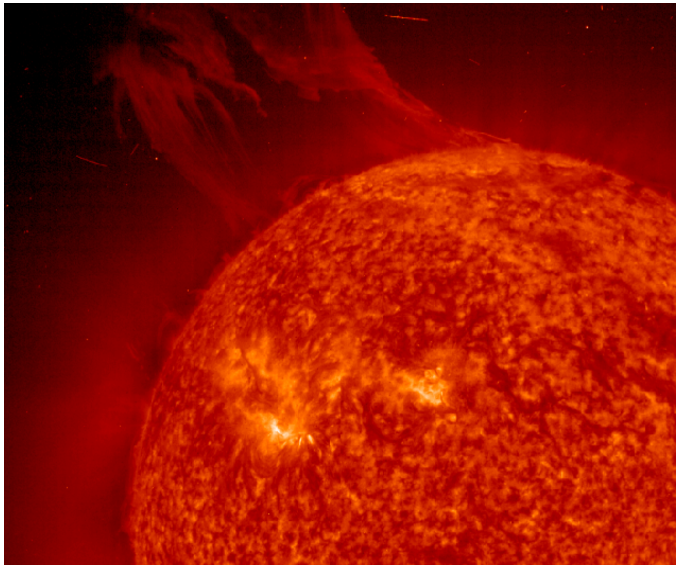
14% | 0.02%

^{7}Be(e^-,ν)^{7}Li
^{7}Li(p,α)^{4}He

^{7}Be(p,γ)^{8}B
^{8}B(e^+,ν)^{8}Be
^{8}Be(α)^{4}He

Solar neutrinos

E_ν < 15 MeV



TRIUMF

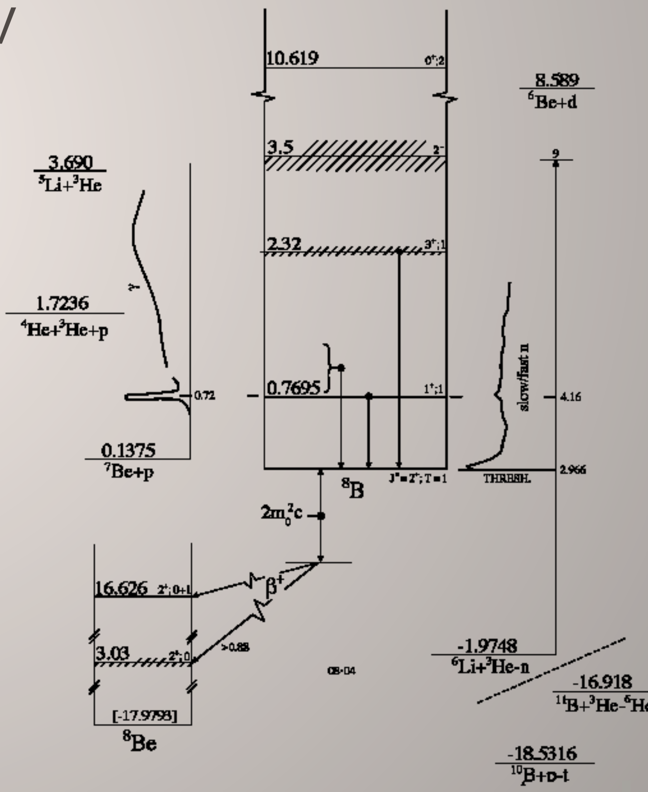
$^{7}Be(p,\gamma)^{8}B$ S-factor

- S_{17} one of the main inputs for understanding the solar neutrino flux
  - Needs to be known with high precision
- Current evaluation has uncertainty ~ 10%
  - Theory needed for extrapolation to ~ 10 keV

$$S(E) = E\sigma(E)\exp[2\pi\eta(E)]$$

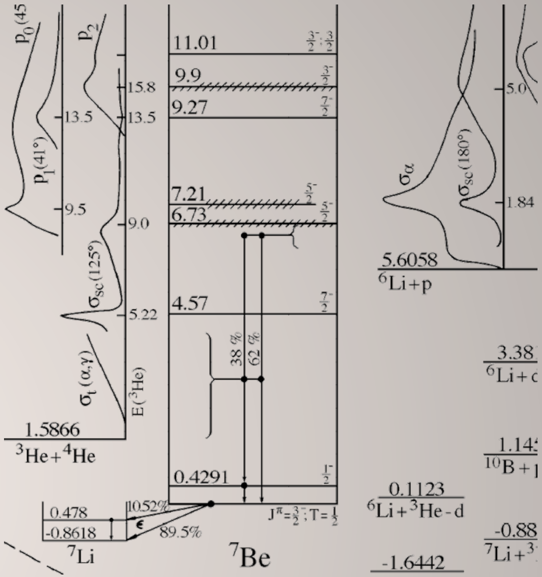
$$\eta(E) = Z_{A-a} Z_a e^2 / \hbar v_{A-a,a}$$

$$\left\langle {}^{8}\mathrm{B}_{\mathrm{g.s.}} \left| E1 \right| {}^{7}\mathrm{Be}_{\mathrm{g.s.}} + \mathrm{p} \right\rangle$$

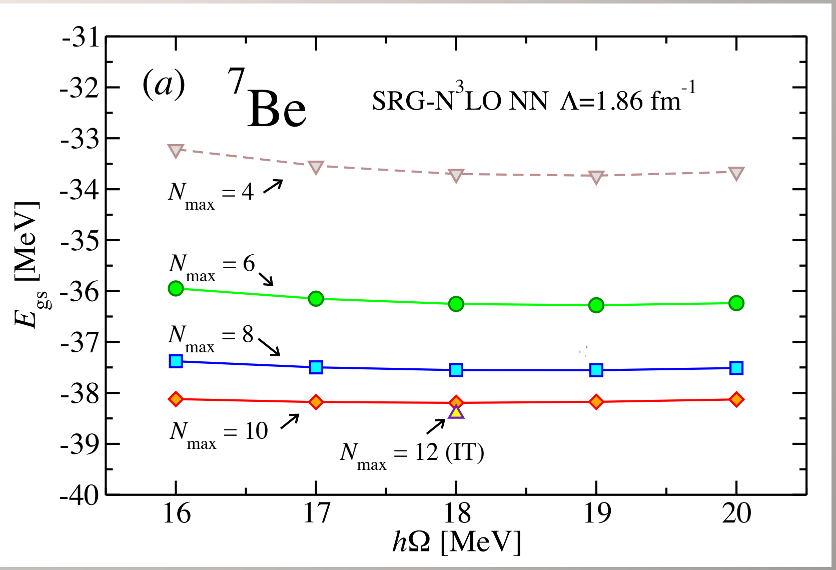


$^7Be(p,\gamma)^8B$ radiative capture: Input - *NN* interaction, ^7Be eigenstates

- Similarity-Renormalization-Group (SRG) evolved chiral N^3LO *NN* interaction
  - Accurate
  - Soft: Evolution parameter Λ
    - Study dependence on Λ

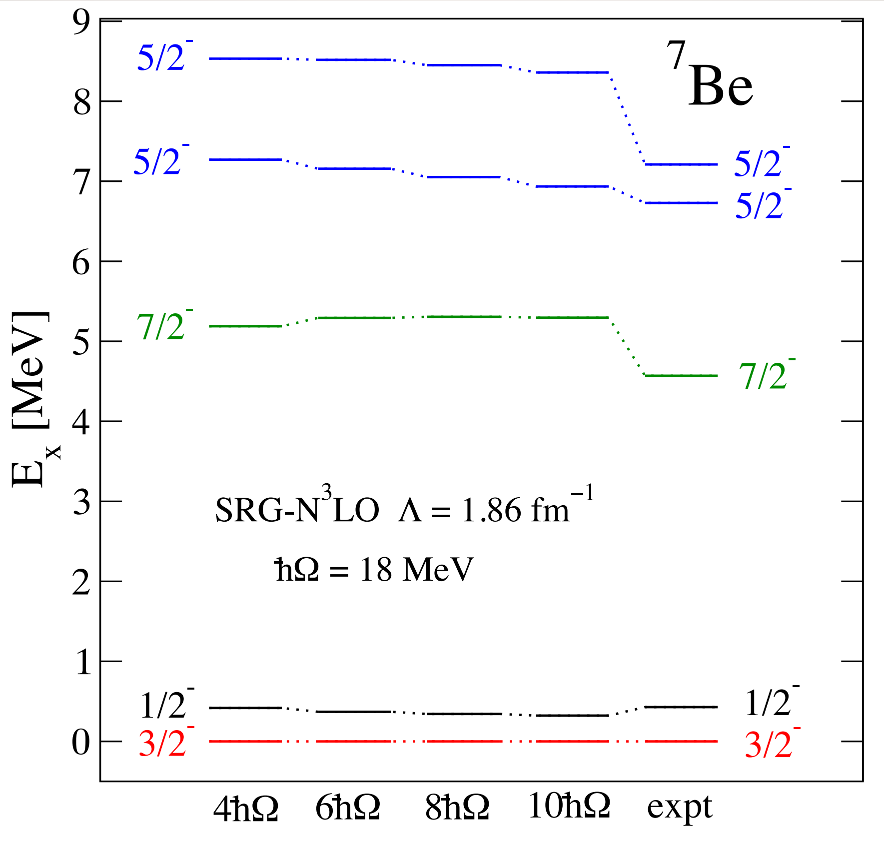


- ^7Be
  - NCSM up to N_{max}=10, Importance Truncated NCSM up to N_{max}=14
  - Variational calculation
    - optimal HO frequency from the ground-state minimum
    - For the selected NN potential with Λ=1.86 fm^{-1}: hΩ=18 MeV

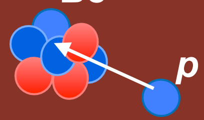


Input: ^{7}Be eigenstates

- Excited states at the optimal HO frequency, ħΩ=18 MeV

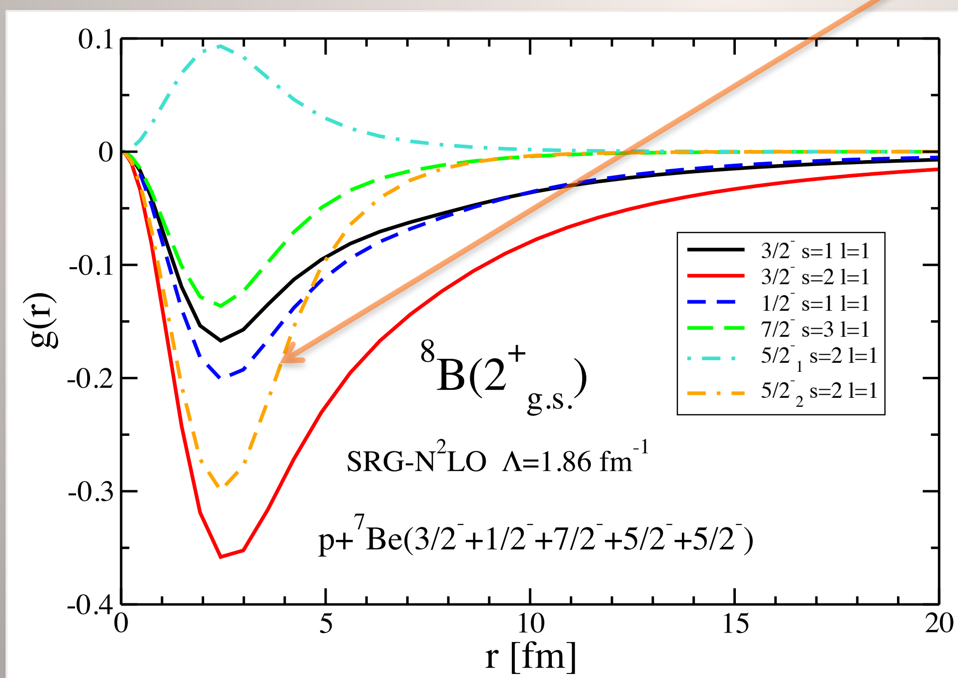


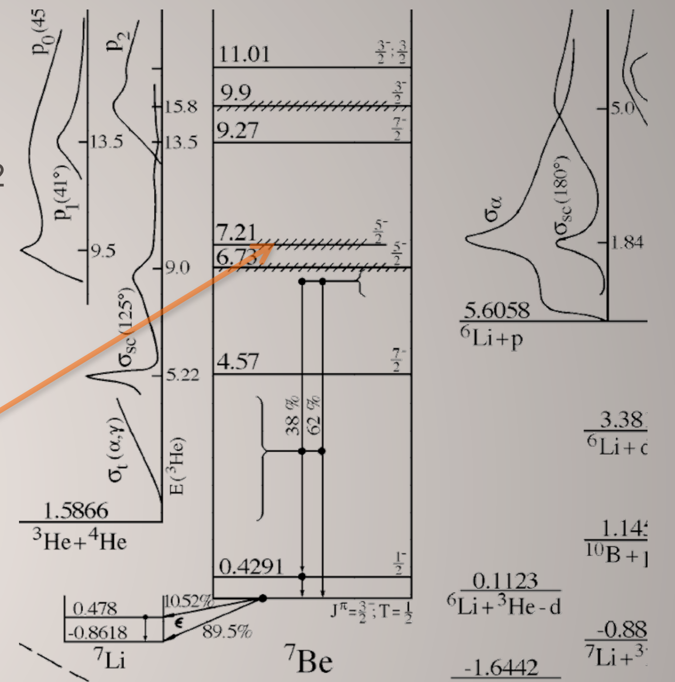
TRIUMF



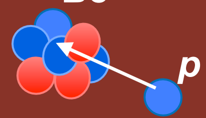
Structure of the ^{8}B ground state

- NCSM/RGM p-^{7}Be calculation
  - five lowest ^{7}Be states: $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN SRG-N^{3}LO with Λ = 1.86 fm^{-1}
- ^{8}B 2^+ g.s. bound by 136 keV (Expt 137 keV)
  - Large *P*-wave $5/2^-_2$ component





5/2$^-_2$ state of ^{7}Be should be included in ^{7}Be(p, γ)^{8}B calculations



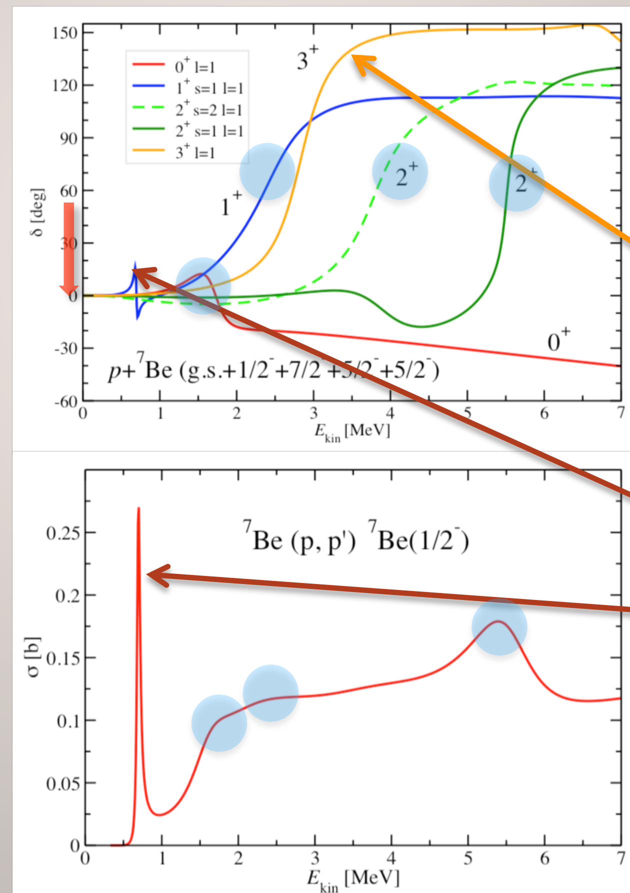
p-^{7}Be scattering

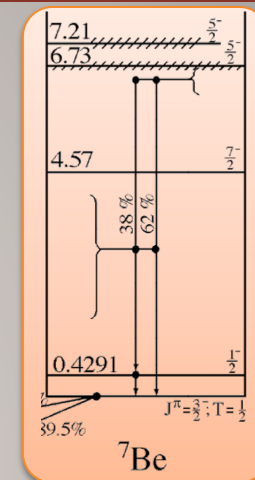
- NCSM/RGM calculation of p-^{7}Be scattering
  - ^{7}Be states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.86 fm^{-1})

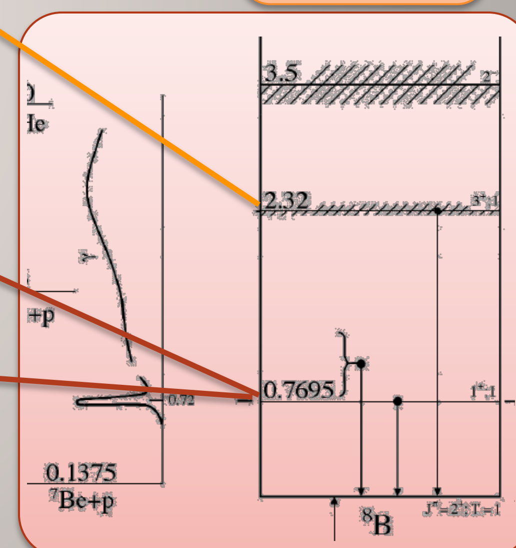
^{8}B 2^+ g.s. **bound** by 136 keV (expt. bound by 137 keV)

New 0^+, 1^+, and two 2^+ resonances **predicted**

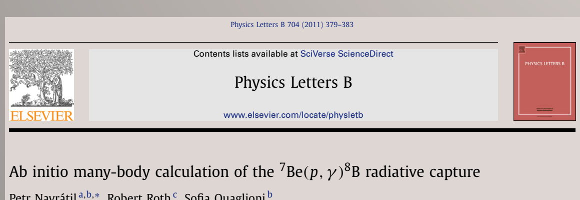
s =1 l =1 2^+ clearly visible in (p,p') cross sections







PRC **82**, 034609 (2010)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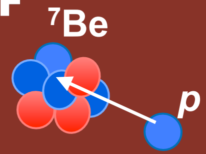
Ab initio many-body calculation of the ^{7}Be$(p,\gamma)^8$B radiative capture

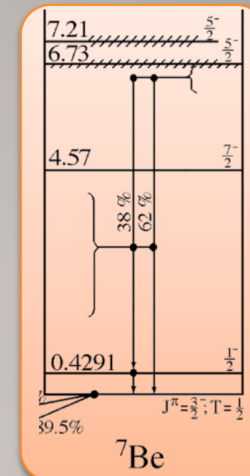
Petr Navrátil[a,b,*], Robert Roth[c], Sofia Quaglioni[b]

TRIUMF



^{7}Be(p,γ)^{8}B radiative capture

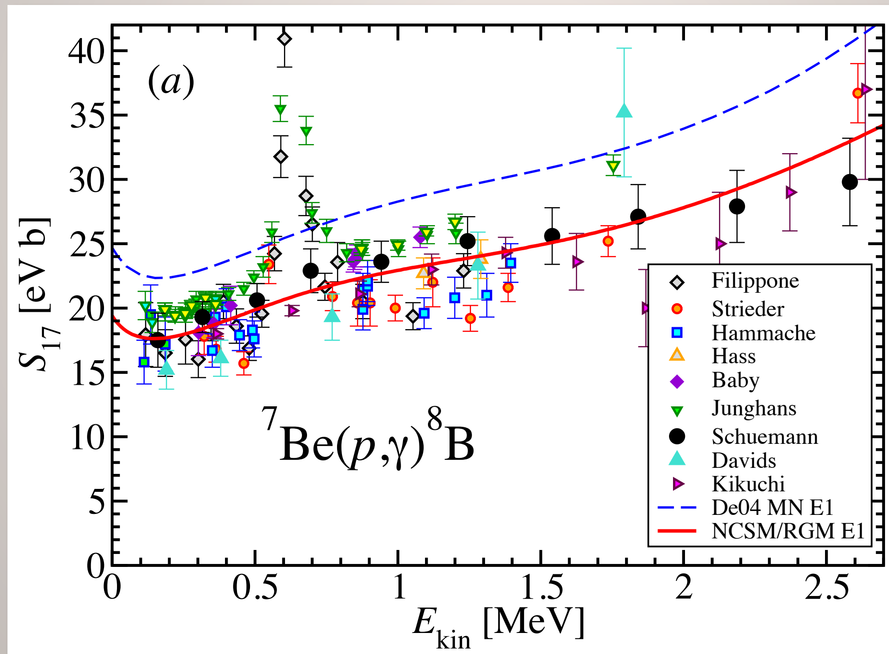
- NCSM/RGM calculation of ^{7}Be(p,γ)^{8}B radiative capture
  - ^{7}Be states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.86 fm^{-1})

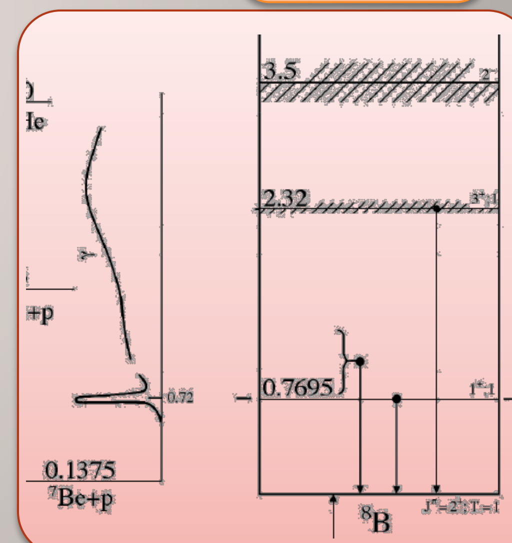


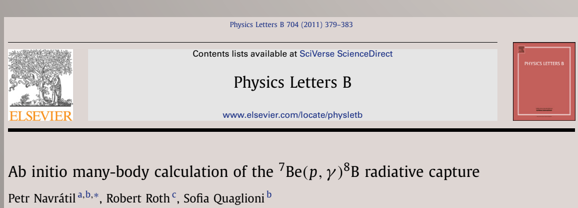
^{8}B 2^+ g.s. **bound** by 136 keV (expt. 137 keV)

S(0) ~ 19.4(0.7) eV b

Data evaluation: S(0)=20.8(2.1) eV b



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Ab initio many-body calculation of the ^{7}Be(p, γ)^{8}B radiative capture

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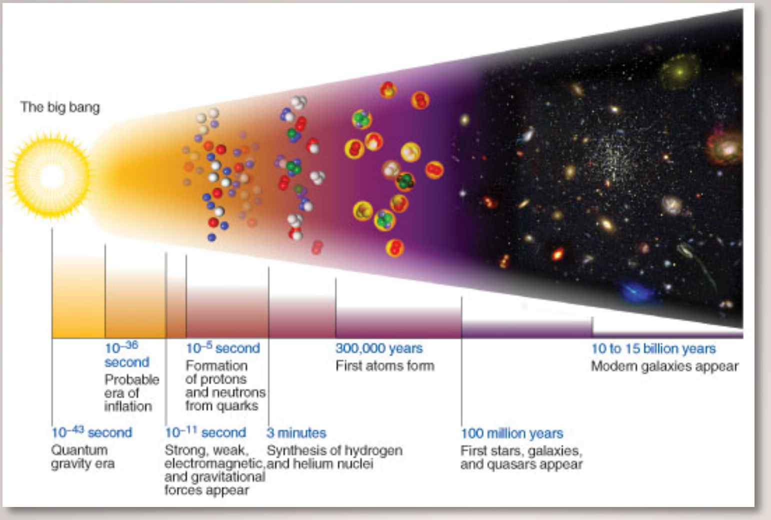
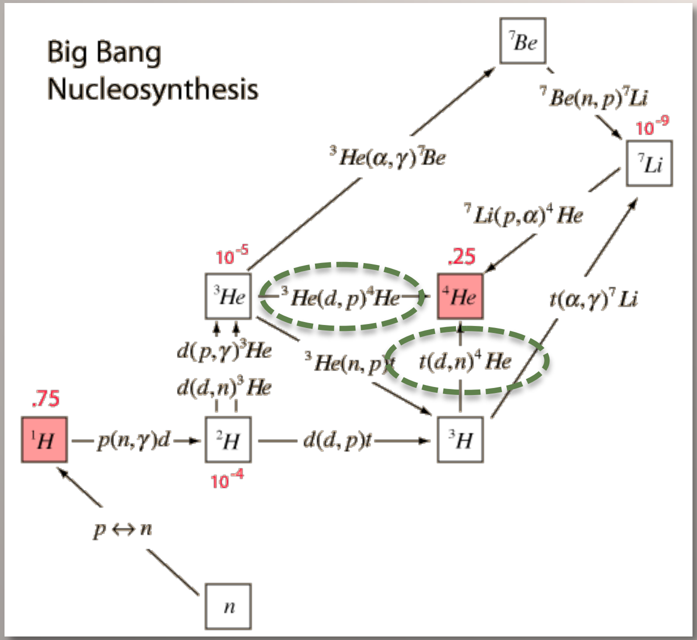
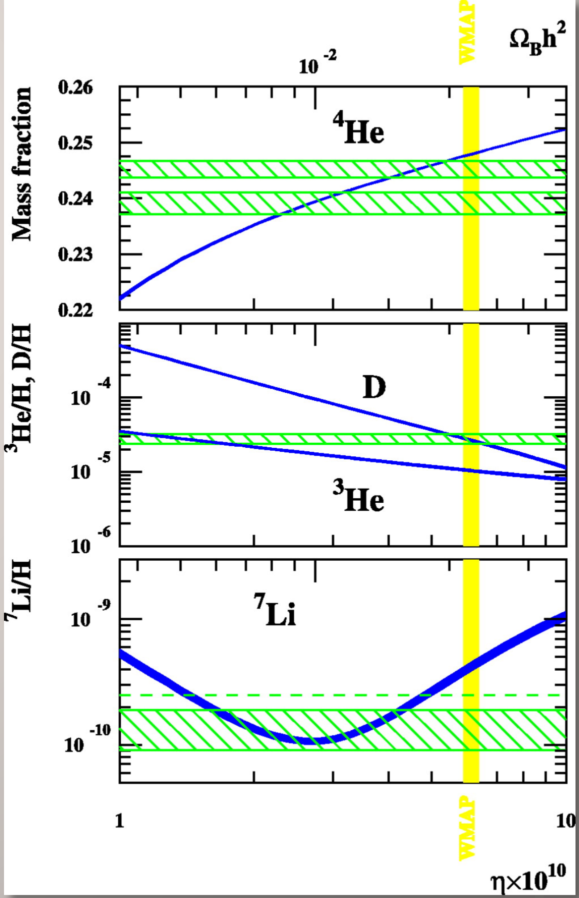
NCSMC calculations with chiral NN+3N forces in progress

Big Bang nucleosythesis





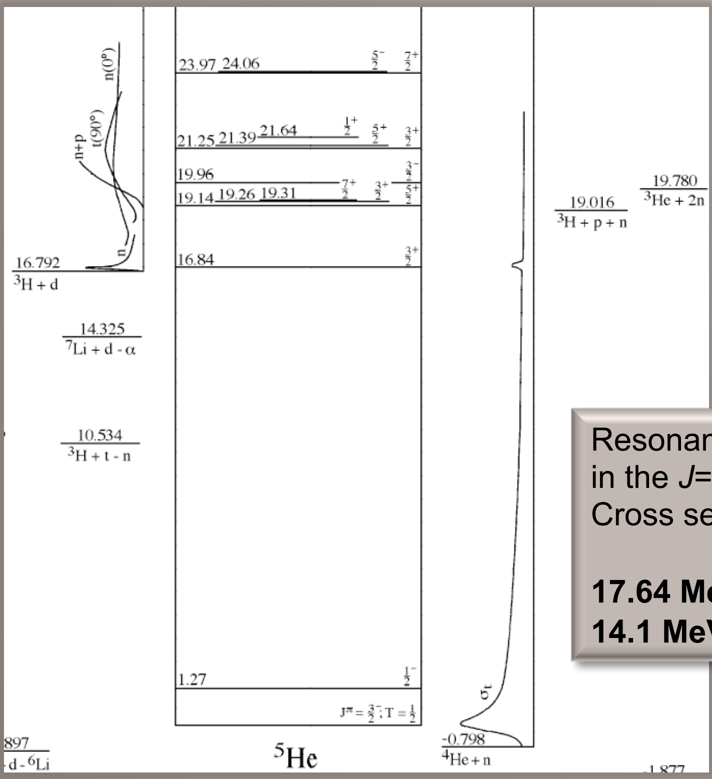
Key reactions

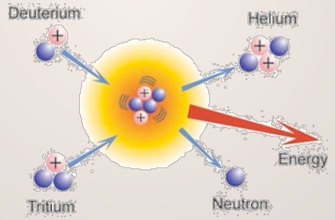


^{7}Li puzzle

Deuterium-Tritium fusion: a future energy source

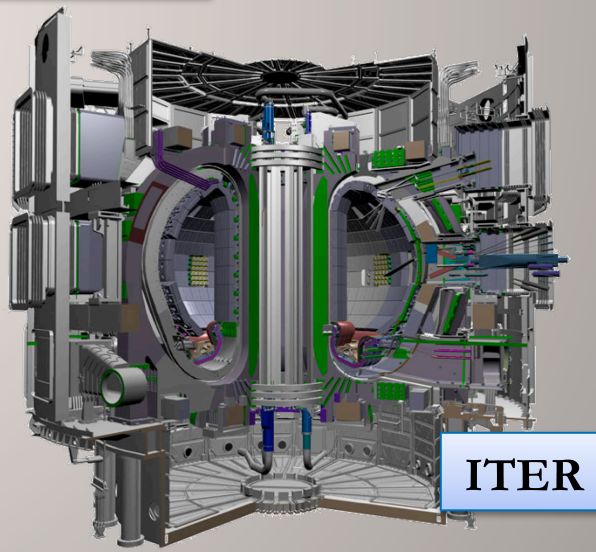
- The d+^{3}H→n+^{4}He reaction
  - The most promising for the production of fusion energy in the near future
  - Will be used to achieve inertial-confinement (laser-induced) fusion at NIF, and magnetic-confinement fusion at ITER
  - With its mirror reaction, ^{3}He(d,p)^{4}He, important for Big Bang nucleosynthesis



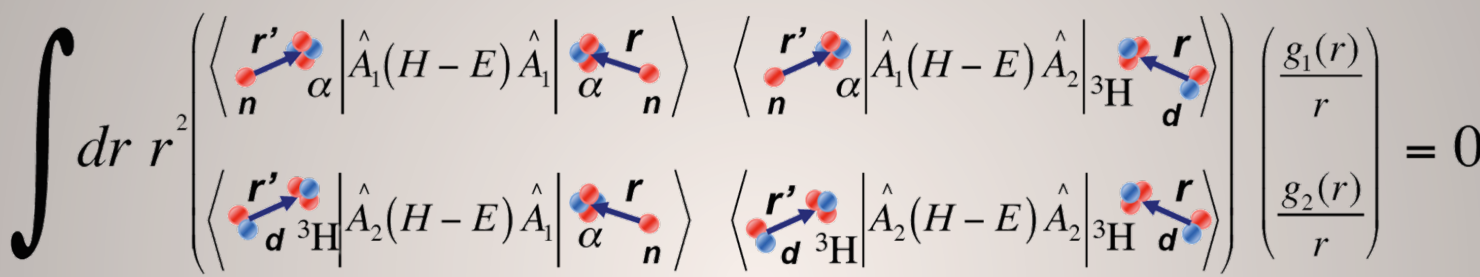


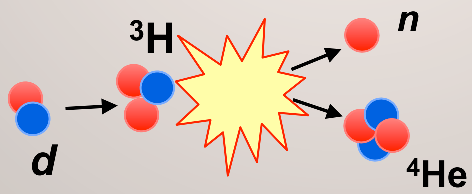
Resonance at E_{cm} =48 keV (E_d=105 keV) in the J=3/2$^+$ channel
Cross section at the peak: 4.88 b

17.64 MeV energy released:
14.1 MeV neutron and 3.5 MeV alpha

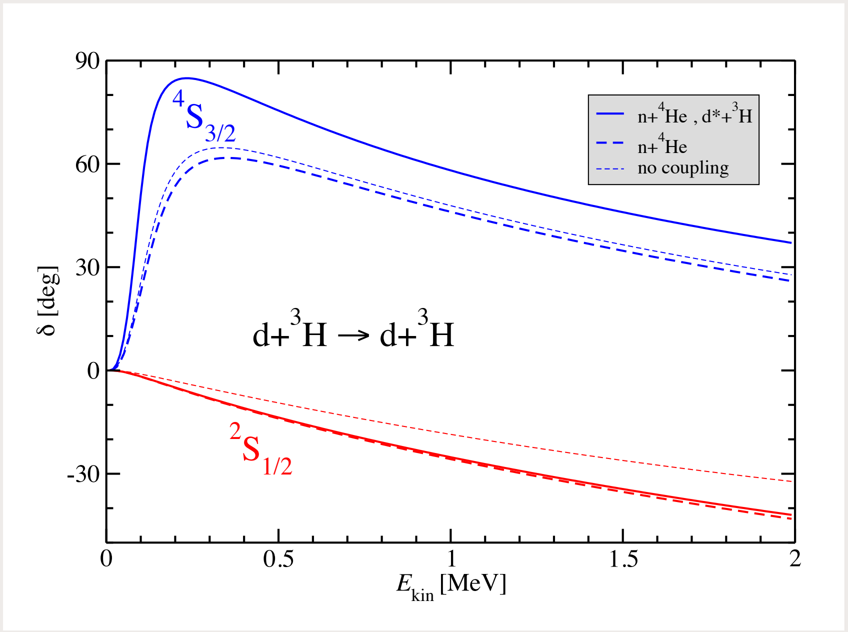


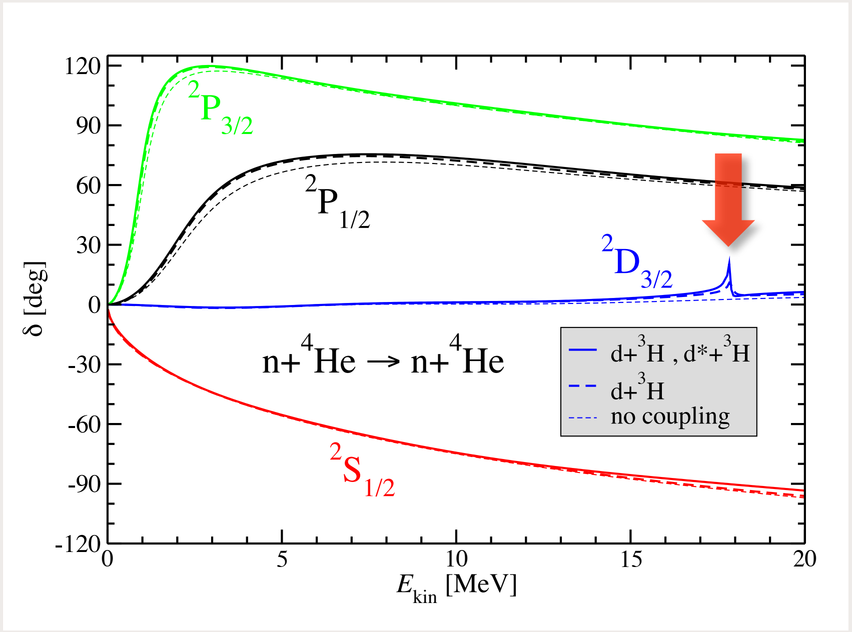
Ab initio calculation of the ^{3}H(d,n)^{4}He fusion





d+^{3}H and n+^{4}He elastic scattering: phase shifts





- *d*+^{3}H elastic phase shifts:
  - Resonance in the $^4S_{3/2}$ channel
  - Repulsive behavior in the $^2S_{1/2}$ channel ➔ Pauli principle

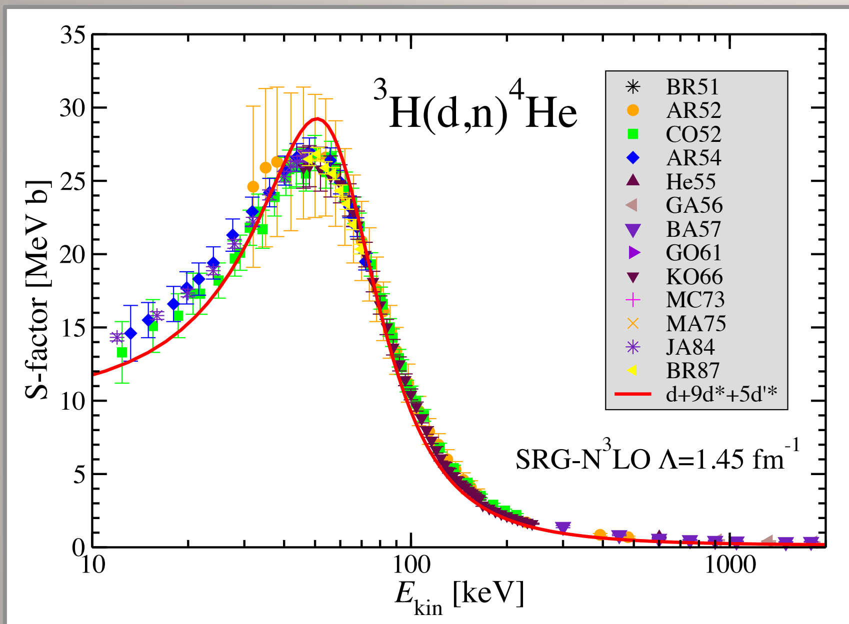
*d** deuteron pseudo state in $^3S_1$-$^3D_1$ channel: deuteron polarization, virtual breakup

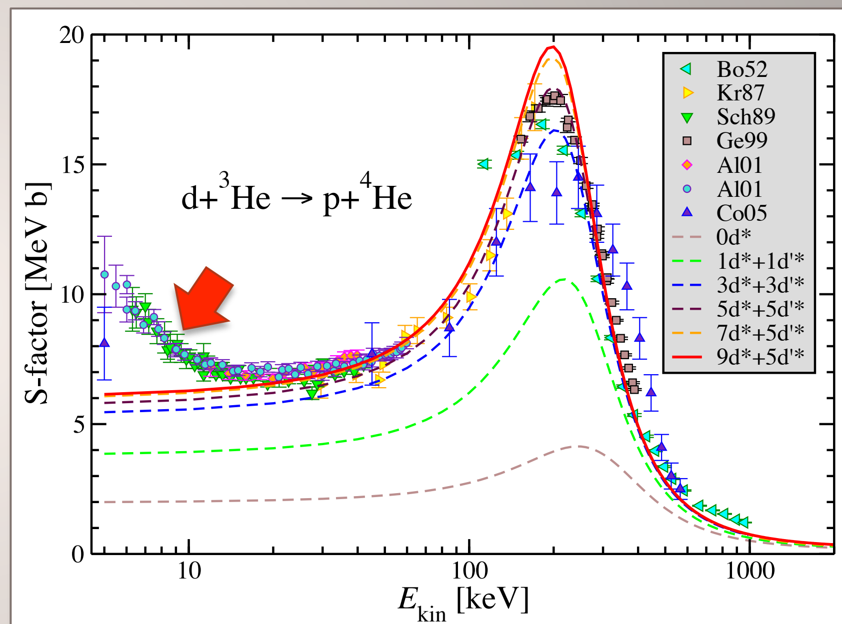
- *n*+^{4}He elastic phase shifts:
  - *d*+^{3}H channels produces slight increase of the *P* phase shifts
  - Appearance of resonance in the 3/2$^+$ *D*-wave, just above *d*-^{3}H threshold

The *d*-^{3}H fusion takes place through a transition of *d*+^{3}H is *S*-wave to *n*+^{4}He in *D*-wave: Importance of the **tensor force**

^{3}H(d,n)^{4}He & ^{3}He(d,p)^{4}He fusion

- NCSM/RGM with *SRG-N^{3}LO NN* potentials





Potential to address unresolved fusion research related questions:
^{3}H(d,n)^{4}He fusion with polarized deuterium and/or tritium,
^{3}H(d,$n\gamma$)^{4}He bremsstrahlung, electron screening at very low energies …

NCSMC calculations
with chiral NN+3N forces in progress…

PRL **108,** 042503 (2012) PHYSICAL REVIEW LETTERS week ending 27 JANUARY 2012

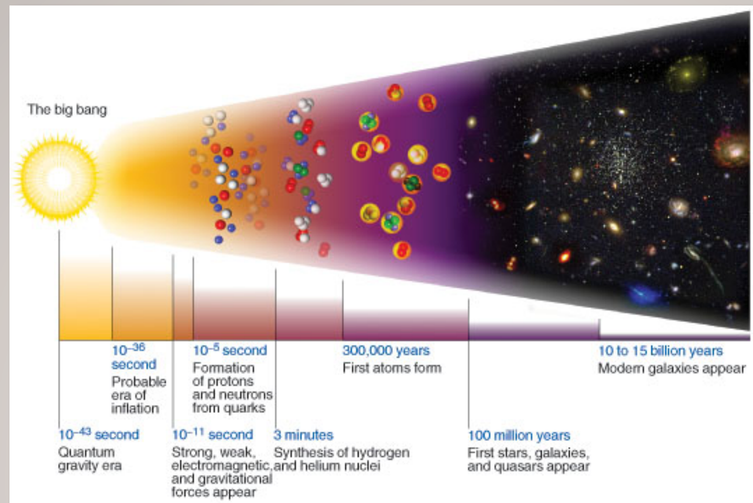
Ab Initio **Many-Body Calculations of the ^{3}H(d, n)^{4}He and ^{3}He(d, p)^{4}He Fusion Reactions**

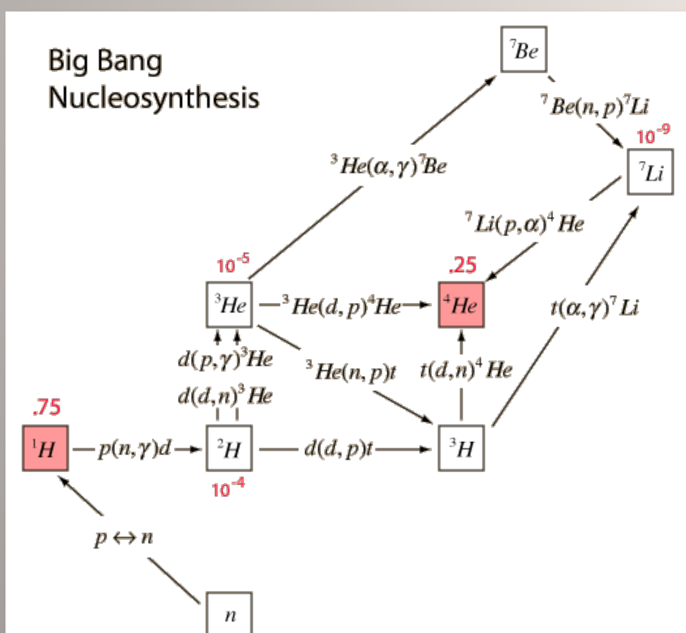
Petr Navrátil[1,2] and Sofia Quaglioni[2]

Big Bang nucleosythesis

^{6}Li puzzle

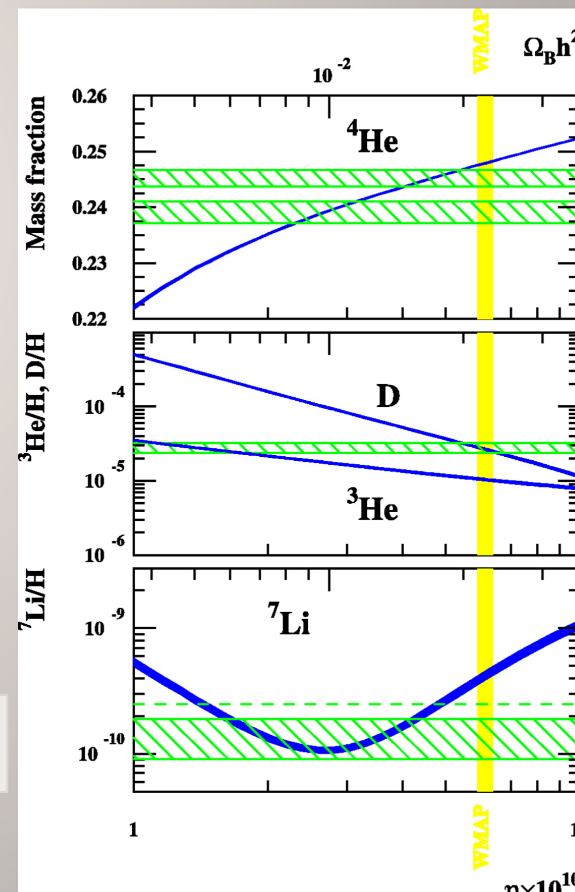
^{2}H(α,γ)^{6}Li





Key reactions

^{7}Li puzzle



Unified description of ^{6}Li structure and d+^{4}He dynamics

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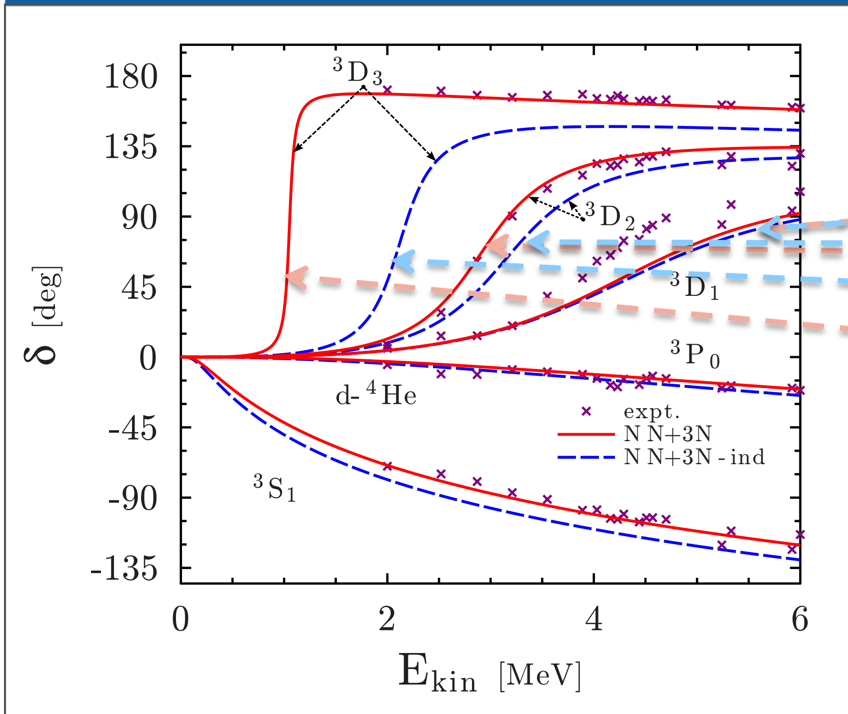
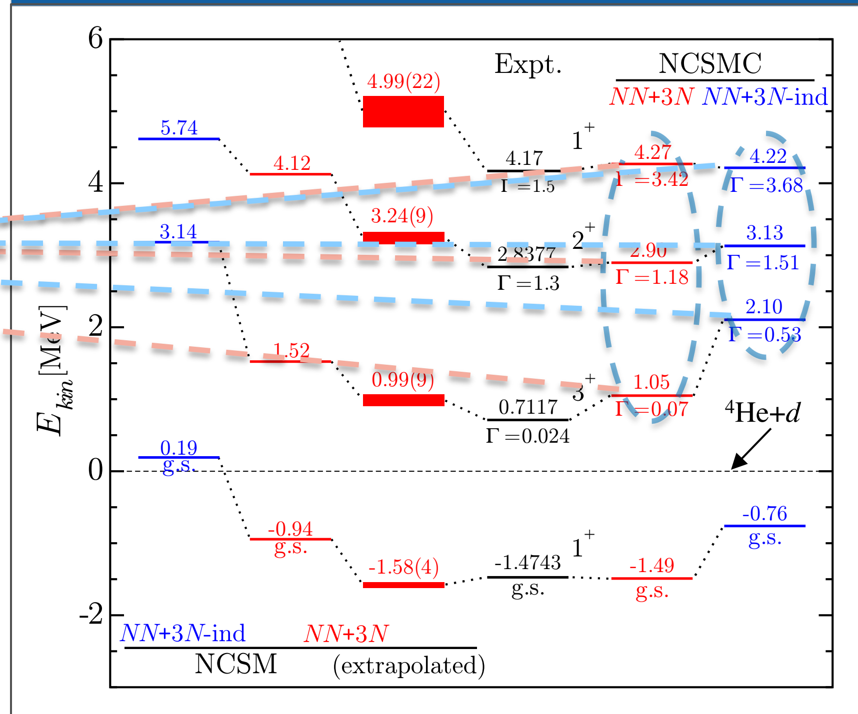
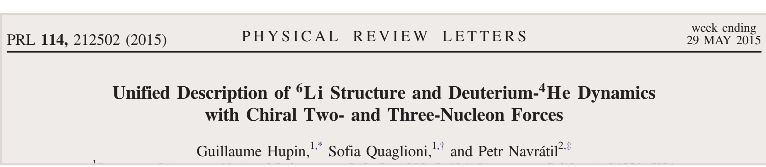
d+^{4}He Scattering Phase Shifts



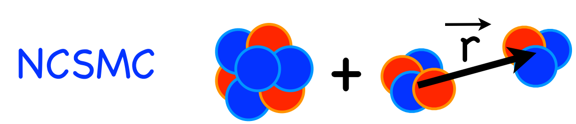
^{6}Li vs. (^{4}He+d)+^{6}Li calculation




PRL **114,** 212502 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

Unified Description of ^{6}Li Structure and Deuterium-^{4}He Dynamics with Chiral Two- and Three-Nucleon Forces

Guillaume Hupin,[1,*] Sofia Quaglioni,[1,†] and Petr Navrátil[2,‡]



TRIUMF

Unified description of ^{6}Li structure and d+^{4}He dynamics

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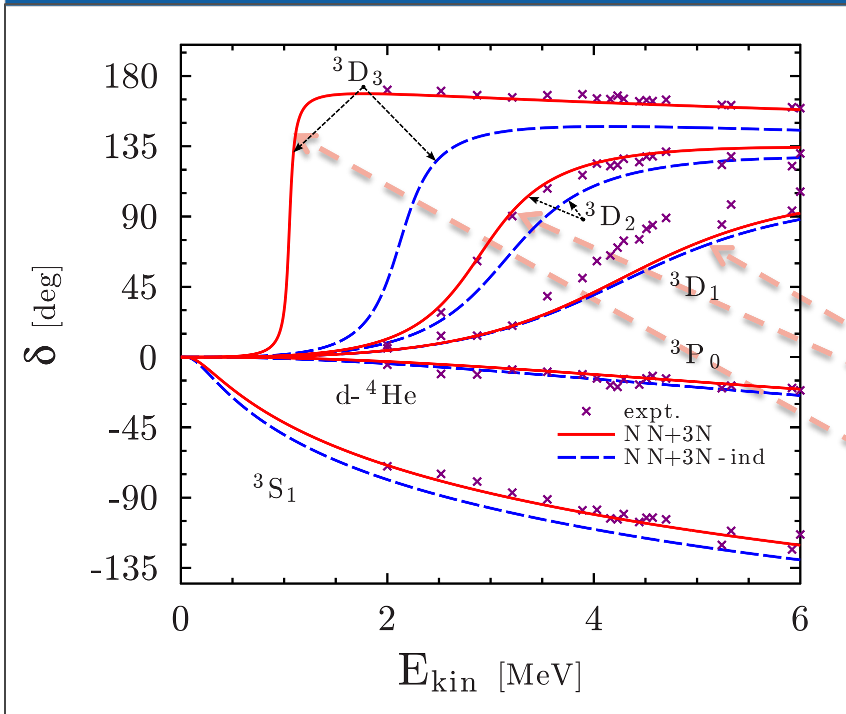

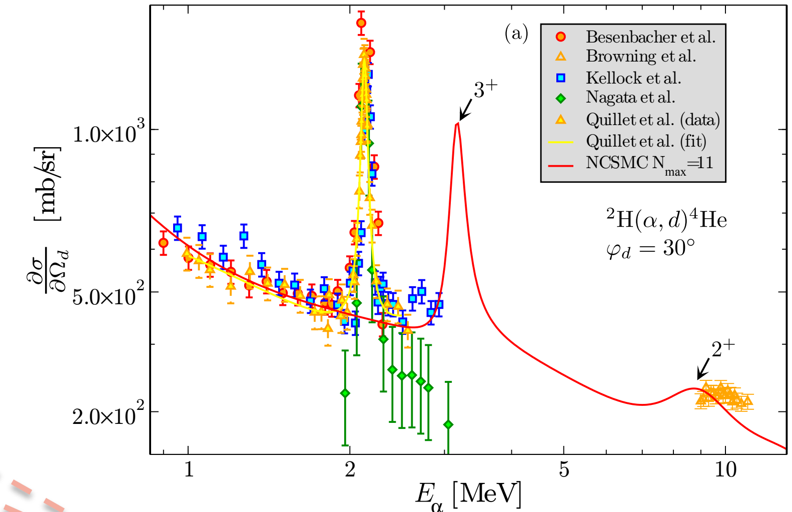

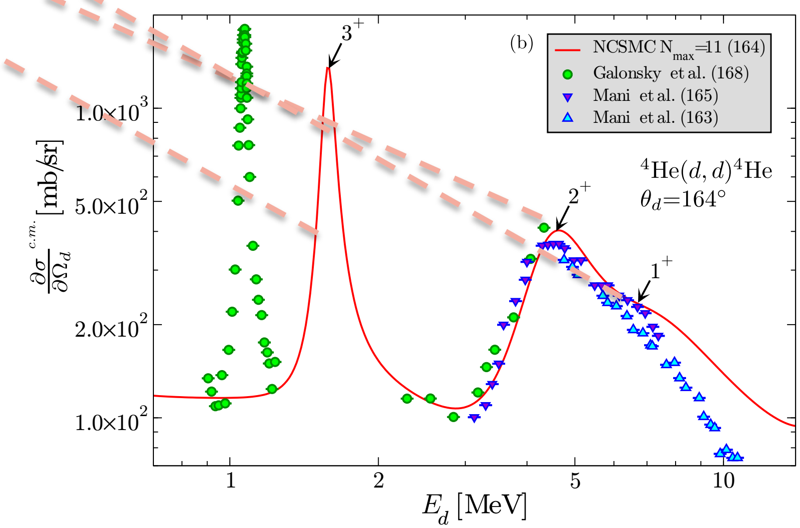

PRL 114, 212502 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

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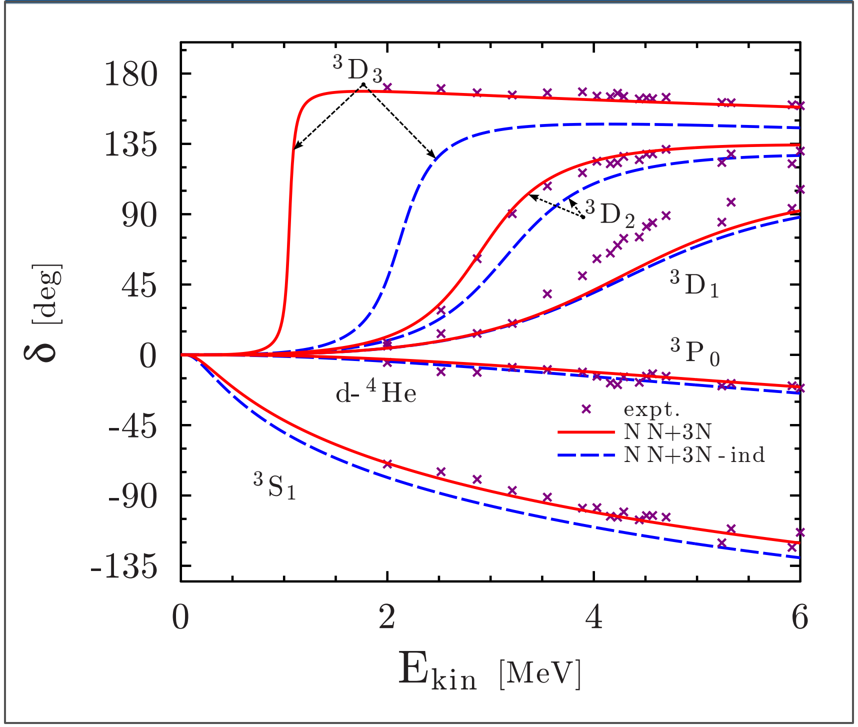
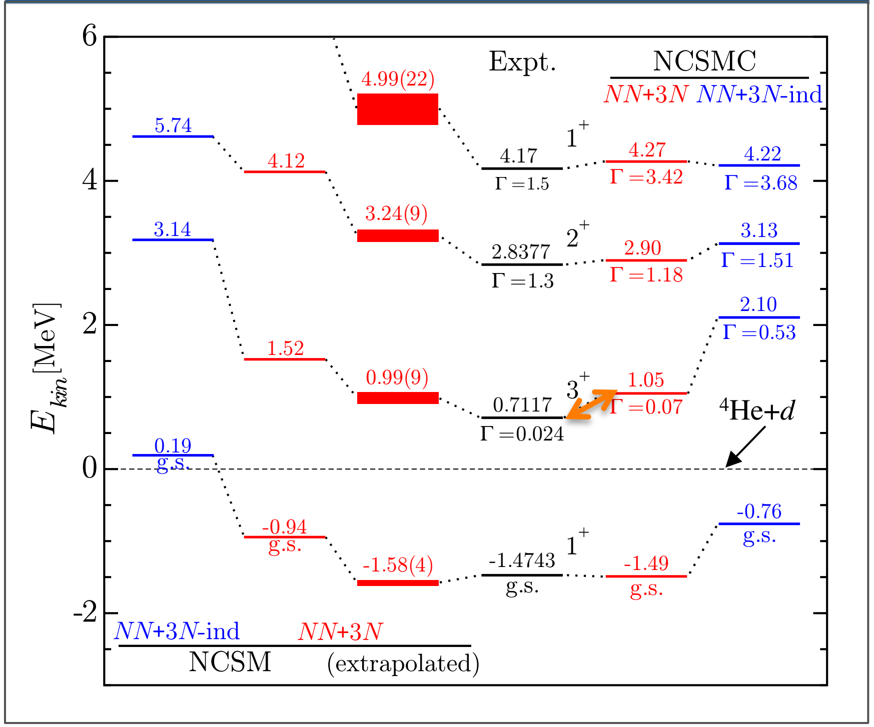
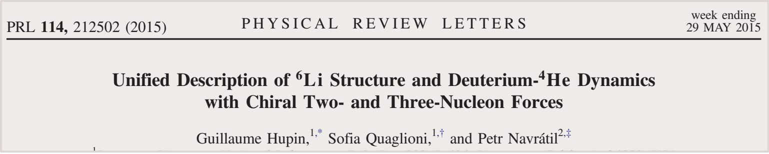


^{6}Li vs. (^{4}He+d)+^{6}Li calculation



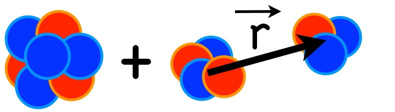


PRL 114, 212502 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

Unified Description of ^{6}Li Structure and Deuterium-^{4}He Dynamics with Chiral Two- and Three-Nucleon Forces

Guillaume Hupin,[1,*] Sofia Quaglioni,[1,†] and Petr Navrátil[2,‡]

NCSMC



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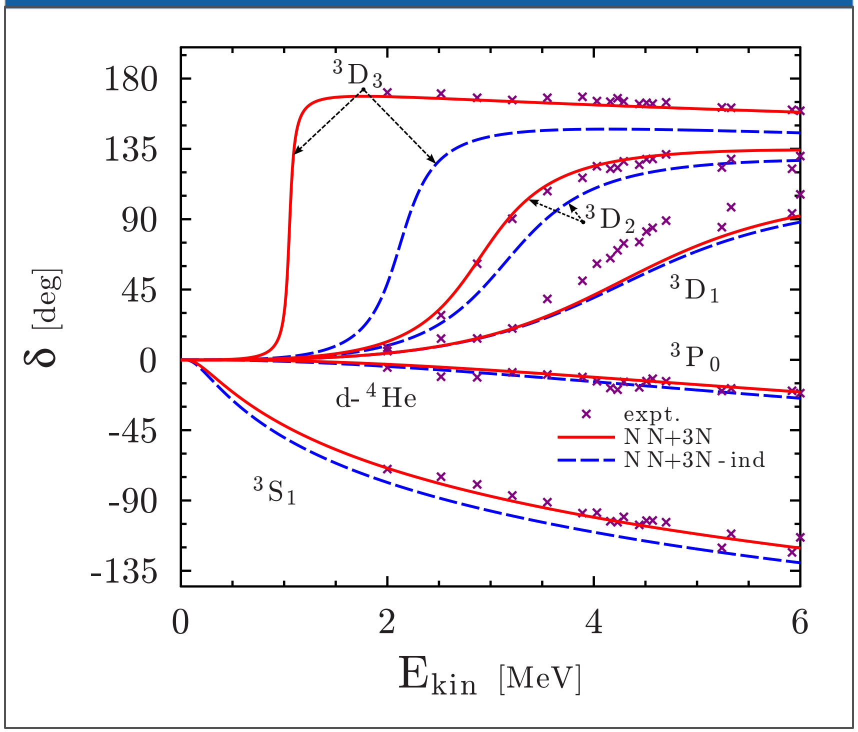

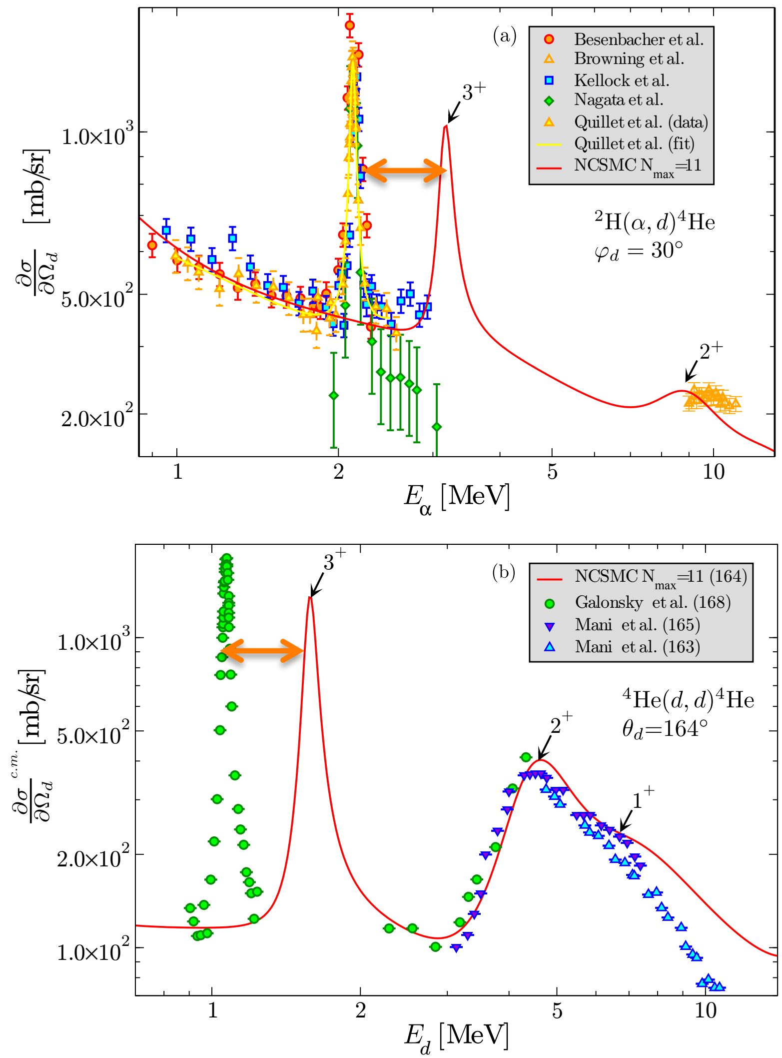

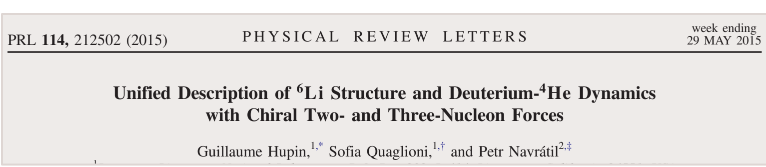

PRL 114, 212502 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

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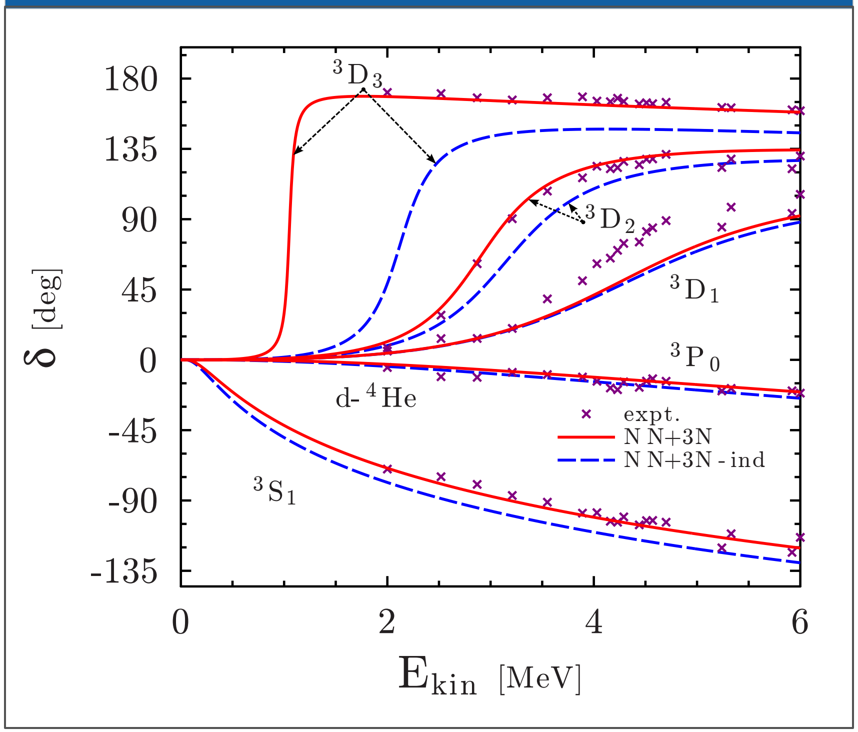

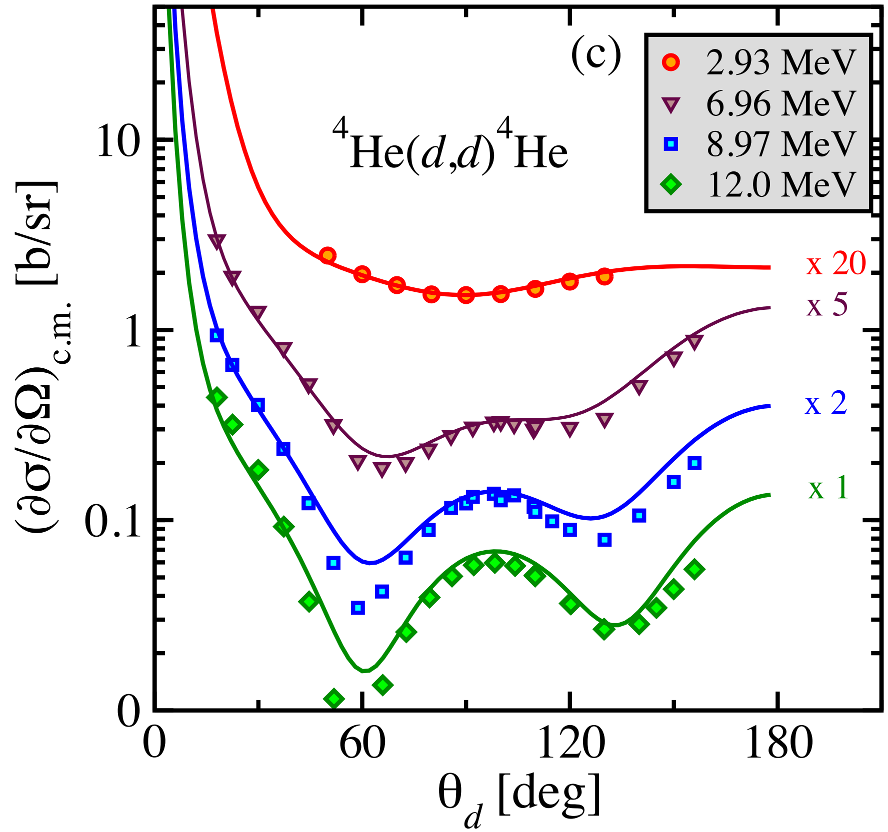

PRL 114, 212502 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

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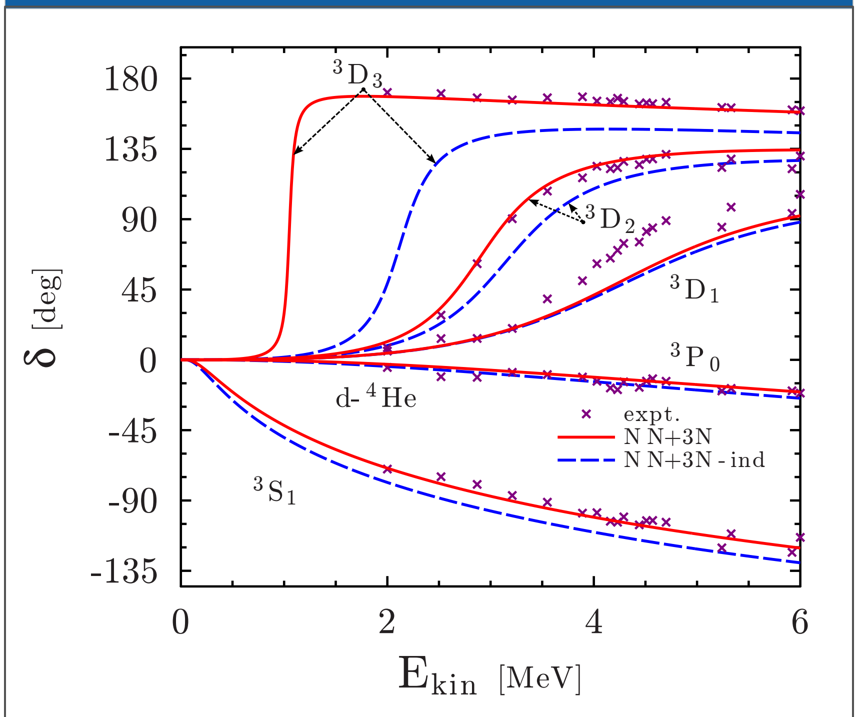
- *S*- and *D*-wave asymptotic normalization constants

d+^{4}He Scattering Phase Shifts



	NCSMC	Experiment	
C_0 [fm$^{-1/2}$]	2.695	2.91(9) [39]	2.93(15) [38]
C_2 [fm$^{-1/2}$]	−0.074	−0.077(18) [39]	
C_2/C_0	−0.027	−0.025(6)(10) [39]	0.0003(9) [41]

[38] L. D. Blokhintsev, V. I. Kukulin, A. A. Sakharuk, D. A. Savin, and E. V. Kuznetsova, Phys. Rev. C **48**, 2390 (1993).

[39] E. A. George and L. D. Knutson, Phys. Rev. C **59**, 598 (1999).

[41] K. D. Veal, C. R. Brune, W. H. Geist, H. J. Karwowski, E. J. Ludwig, A. J. Mendez, E. E. Bartosz, P. D. Cathers, T. L. Drummer, K. W. Kemper, A. M. Eiró, F. D. Santos, B. Kozlowska, H. J. Maier, and I. J. Thompson, Phys. Rev. Lett. **81**, 1187 (1998).

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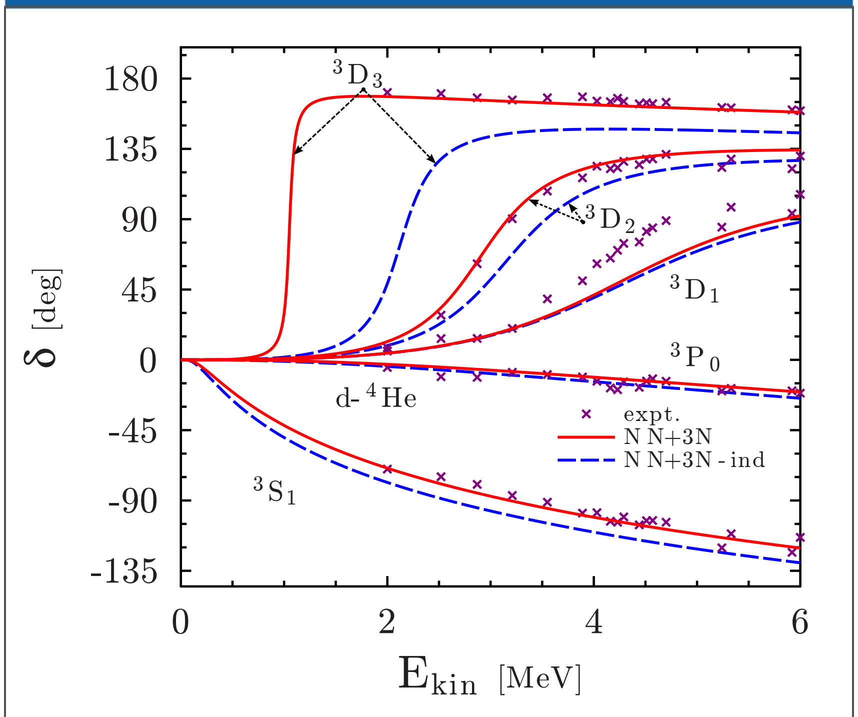

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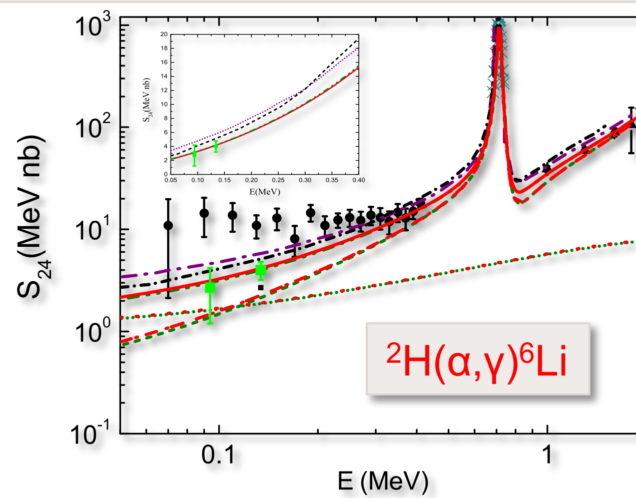
^{6}Li puzzle – too little ^{6}Li produced in BBN



A. M. Mukhamedzhanov *et al.*,1602.07395

PRL 114, 212502 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

Unified Description of ^{6}Li Structure and Deuterium-^{4}He Dynamics with Chiral Two- and Three-Nucleon Forces

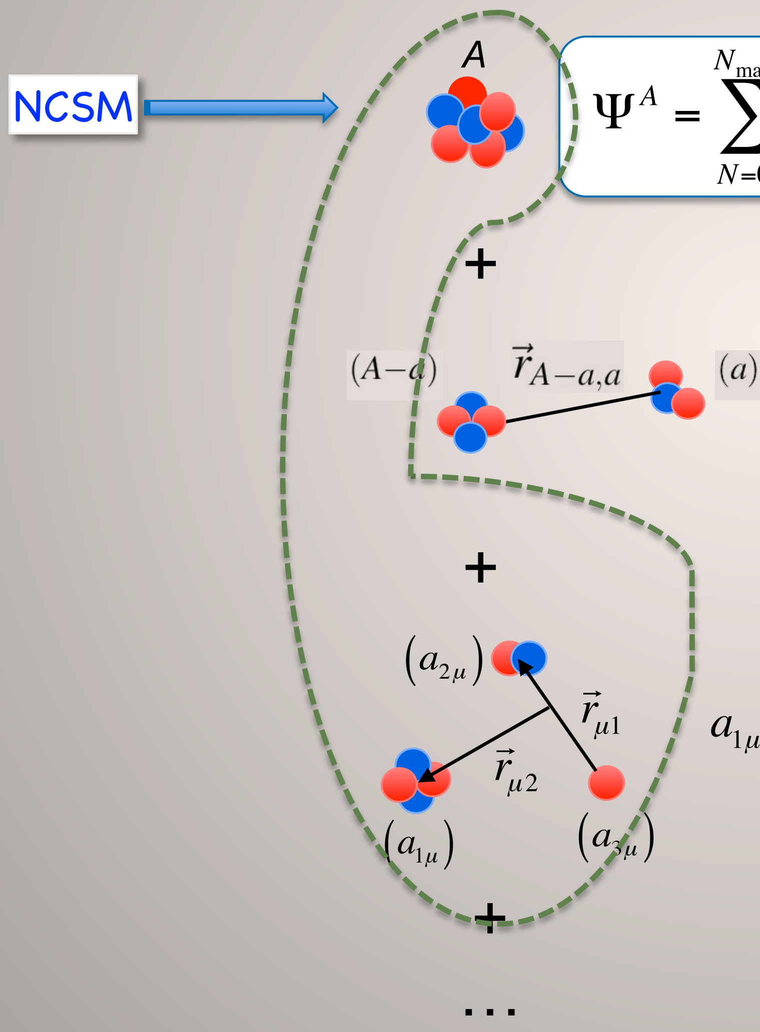
Guillaume Hupin,[1,*] Sofia Quaglioni,[1,†] and Petr Navrátil[2,‡]

Three nuclei in initial or final state

- Many astrophysics relevant reactions involve three nuclei in the initial or final state
  - $^{3}He(^{3}He,2p)^{4}He$ – major pp chain reaction
  - Three-particle fusion may bridge the A=5, 8 gaps
  - Building blocks typically α, p, n
  - triple-alpha capture $^{4}He(\alpha\alpha,\gamma)^{12}C$
  - $^{4}He(\alpha n,\gamma)^{9}Be$ – initiation of the r-process
  - $^{4}He(nn,\gamma)^{6}He$ – followed by $^{6}He(\alpha,n)^{9}Be$ could be an alternative to $^{4}He(\alpha n,\gamma)^{9}Be$ in neutron rich environments
- Experimental investigation challenging

Extending no-core shell model beyond bound states

Include more many nucleon correlations…



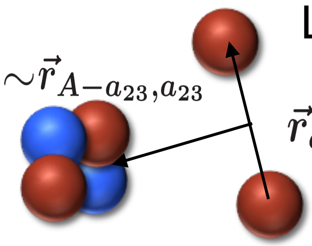
$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

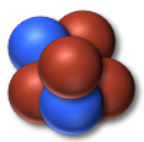
…using the Resonating Group Method (RGM) ideas

NCSMC for three-body clusters

NCSM/RGM-3B
Long range description

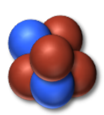


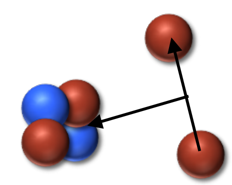
NCSM
Short range description



+

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \Big| \quad , \lambda \Big\rangle + \sum_{\nu} \int d\vec{x}\, d\vec{y}\; \gamma_{\nu}(\vec{x},\vec{y})\; \hat{A}_{\nu} \Big| \quad , \nu \Big\rangle$$

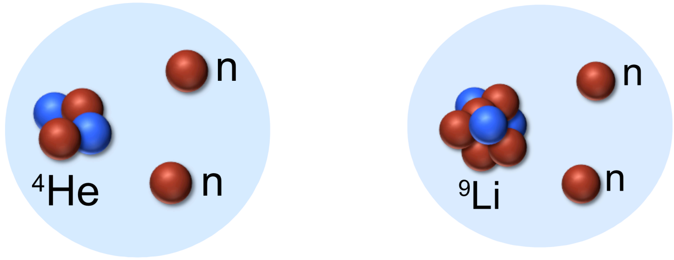




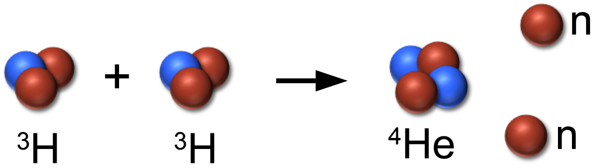
Unknowns

NCSMC for three-body clusters

- Two-neutron halo nuclei



- Transfer reactions with three-body continuum final states



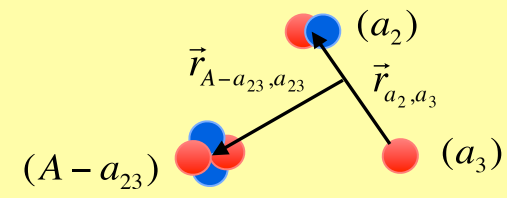
NCSMC for three-body clusters

- The starting point:

$$\Psi^{(A)}_{RGM} = \sum_{a_2 a_3 \nu} \int d\vec{x}\, d\vec{y}\; G^{(A-a_{23},a_2,a_3)}_{\nu}(x,y)$$

$$\times \hat{A}^{(A-a_{23},a_2,a_3)} \left| \Phi^{(A-a_{23},a_2,a_3)}_{\nu \vec{x}\vec{y}} \right\rangle$$

$$\rho^{5/2} \sum_{K} \chi^{(A-a_{23},a_2,a_3)}_{\nu K}(\rho)\, \phi^{\ell x \ell y}_{K}(\alpha)$$



$$\psi^{(A-a_{23})}_{\alpha_1} \psi^{(a_2)}_{\alpha_2} \psi^{(a_3)}_{\alpha_3} Y^{\ell x,\ell y}(\hat{x},\hat{y})$$

$$\times\ \delta\left(\vec{x} - \vec{r}_{a_2,a_3}\right) \delta\left(\vec{y} - \vec{r}_{A-a_{23},a_{23}}\right)$$

- Solves:

$$\sum_{a_2 a_3 \nu K} \int d\rho\; \rho^5 \left[H^{K',K}_{a'\nu',a\nu}(\rho',\rho) - E\, N^{K',K}_{a'\nu',a\nu}(\rho',\rho) \right] \rho^{-5/2} \chi^{(A-a_{23},a_2,a_3)}_{\nu K}(\rho) = 0$$

  - Where the hyperspherical coordinates are given by:

$$\rho = \sqrt{x^2 + y^2}, \quad \alpha = \arctan\left(\frac{y}{x}\right) \qquad \left(x = \rho\cos\alpha, \quad y = \rho\sin\alpha\right)$$

NCSMC for three-body clusters: ^{6}He ~ ^{4}He+n+n

C. Romero-Redondo, S. Quaglioni, P. Navratil, G. Hupin, arXiv: 1606.00066

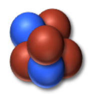
The **NCSM** 6-nucleon eigenstate compensates for the missing many-body correlations

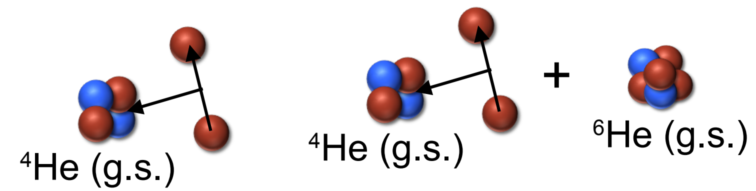
λ=1.5 fm^{-1}

SRG N^{3}LO NN potential

Experimental value
-29.269 MeV

Energy of 0$^+$ g.s.





N_{max}	NCSM	NCSM/RGM	NCSMC (0^+_1)
4	-27.70	-27.14	-28.29
6	-27.98	-28.91	-30.02
8	-28.95	-28.61	-29.69
10	-29.45	-28.70	-29.86
12	-29.66	-28.70	-29.86
Extrapolation	-29.84(4)	---	---

NCSMC for three-body clusters: ^{6}He ~ ^{4}He+n+n

C. Romero-Redondo, S. Quaglioni, P. Navratil, G. Hupin, arXiv: 1606.00066

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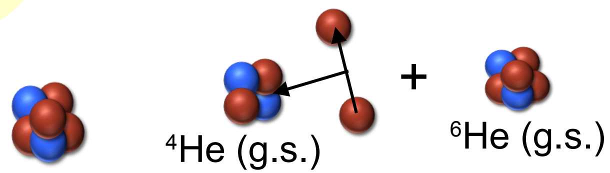
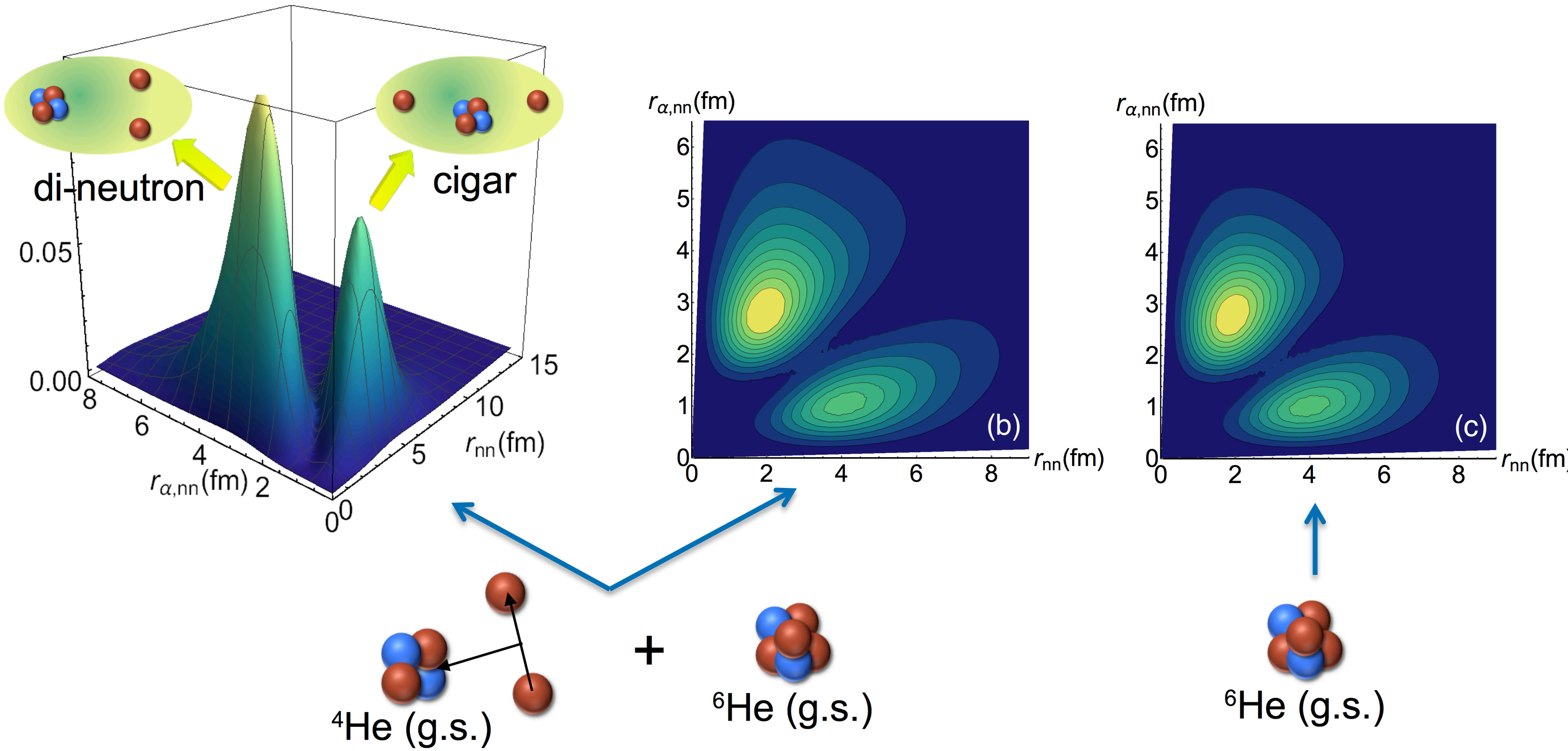


N_{max}	NCSM	NCSMC (0^+_1)
8	-26.44	-28.81
10	-27.70	-28.97
12	-28.37	-29.17
Extrapolation	-29.20(11)*	---

*D. Sääf, C. Forssén, PRC **89** 011303 (2014)

NCSMC for three-body clusters: ^{6}He ~ ^{4}He+n+n

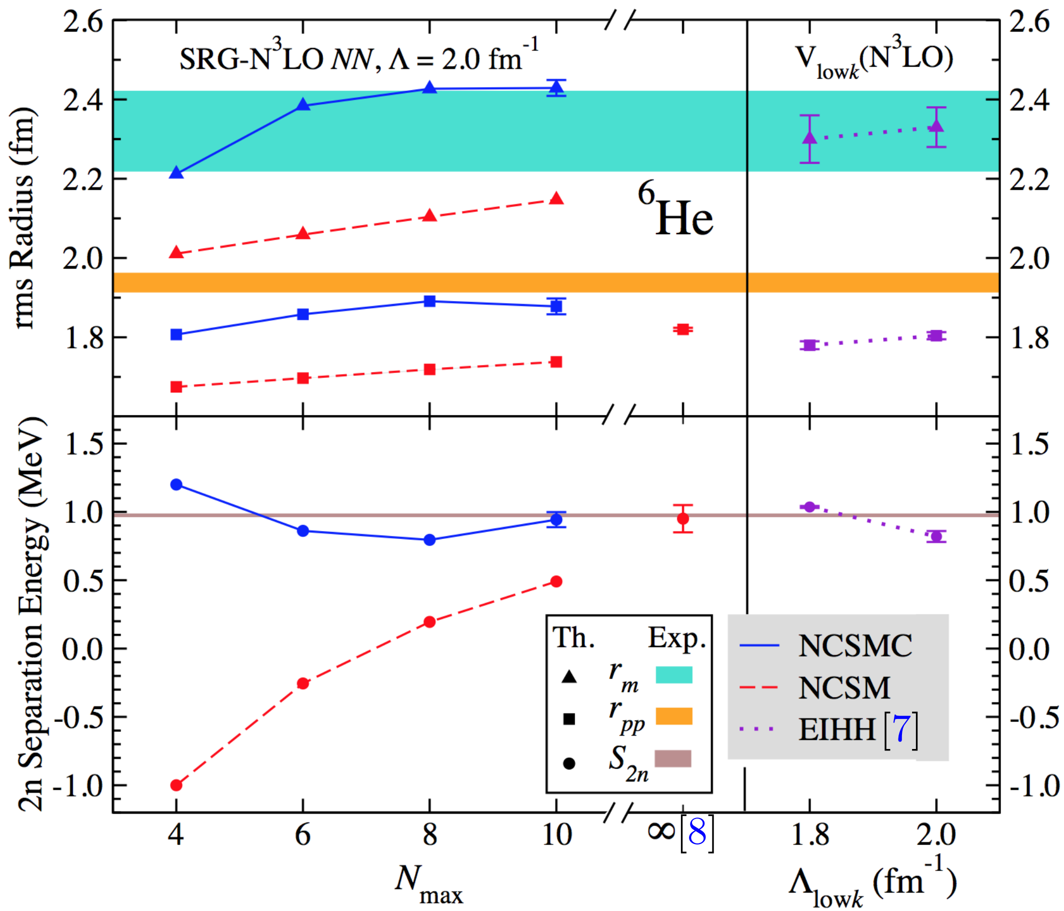
C. Romero-Redondo, S. Quaglioni, P. Navratil, G. Hupin, arXiv: 1606.00066



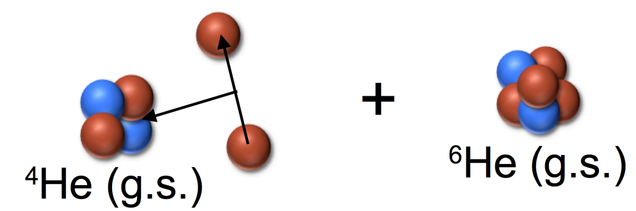
The probability distribution of the ^{6}He ground state presents two peaks corresponding to the di-neutron and cigar configurations

NCSMC for three-body clusters: ^{6}He ~ ^{4}He+n+n

C. Romero-Redondo, S. Quaglioni, P. Navratil, G. Hupin, arXiv: 1606.00066

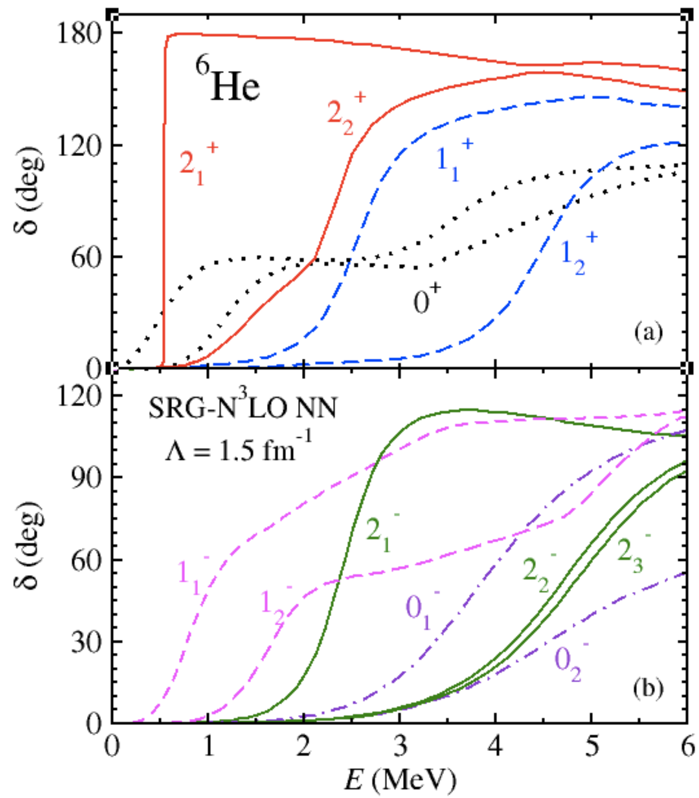


SRG N^{3}LO NN potential with λ=2 fm^{-1}

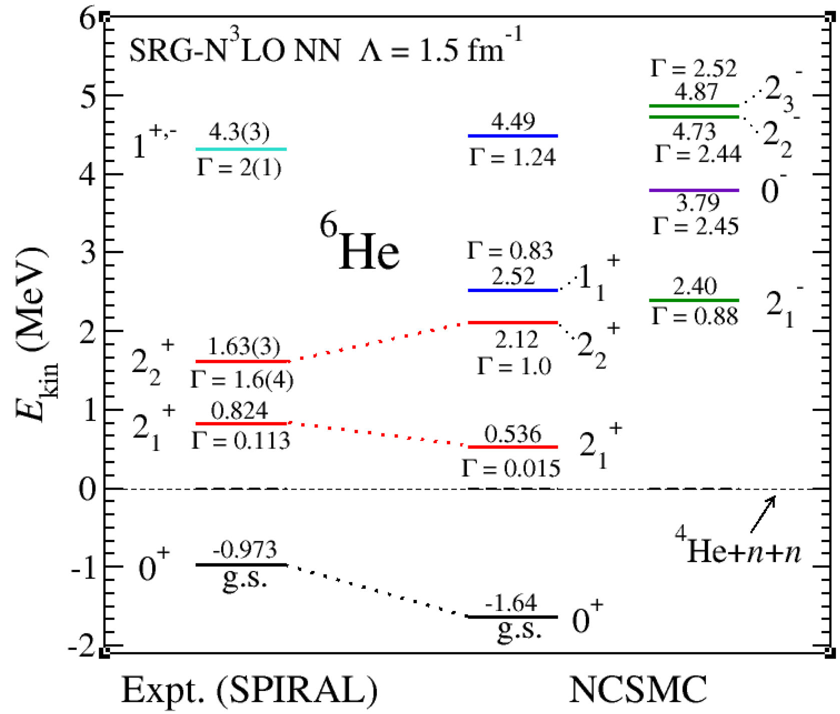


Separation energy, point proton and matter radius simultaneously consistent with experiment

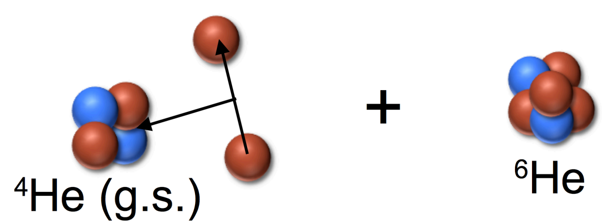
NCSMC for three-body clusters: ^{6}He ~ ^{4}He+n+n



C. Romero-Redondo, S. Quaglioni, P. Navratil, G. Hupin, arXiv: 1606.00066



Prediction of lots of low-lying resonances.
Experimental picture incomplete



Ground-state and scattering state wave functions available.
Calculation of ^{4}He(nn,γ)^{6}He in progress…

Solar *p-p* chain

p-p chain

$p(p,e^{+}\nu)d$

$d(p,\gamma)^{3}He$

86% | 14%

$^{3}He(^{3}He,2p)^{4}He$

$^{3}He(\alpha,\gamma)^{7}Be$

14% | 0.02%

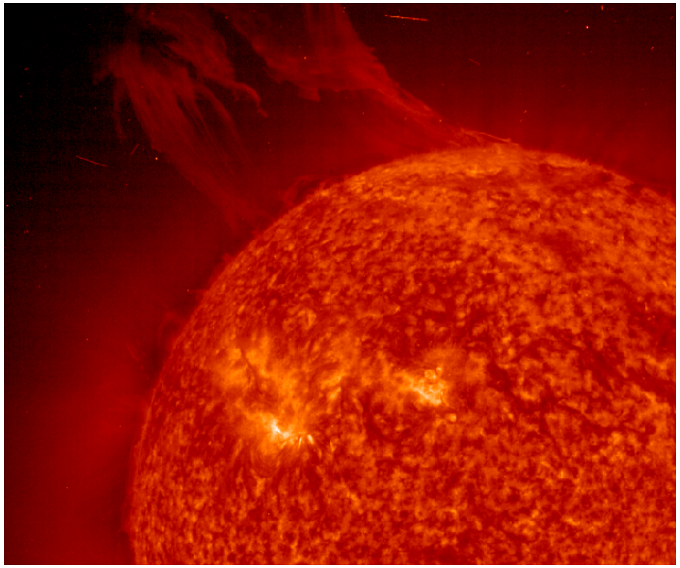
$^{7}Be(e^{-},\nu)^{7}Li$

$^{7}Li(p,\alpha)^{4}He$

$^{7}Be(p,\gamma)^{8}B$

$^{8}B(e^{+},\nu)^{8}Be$

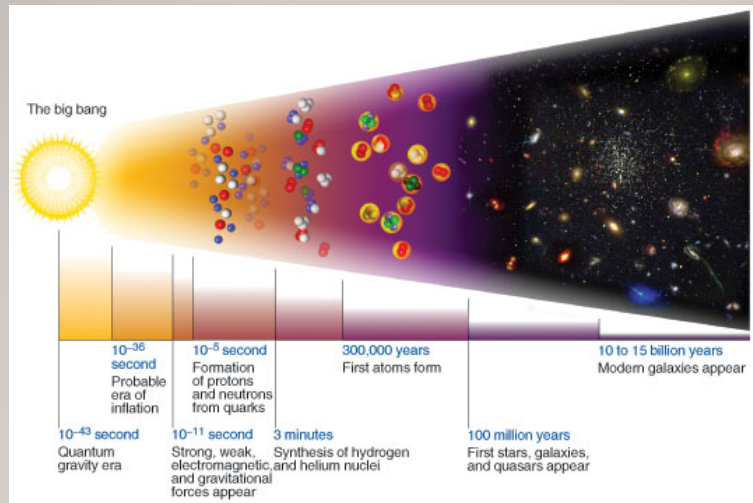
$^{8}Be(\alpha)^{4}He$

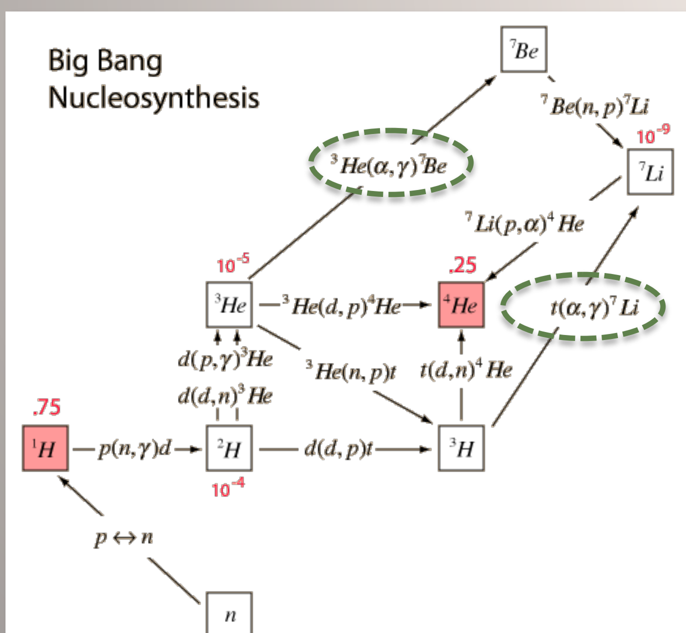


Solar neutrinos

E_{ν} < 15 MeV

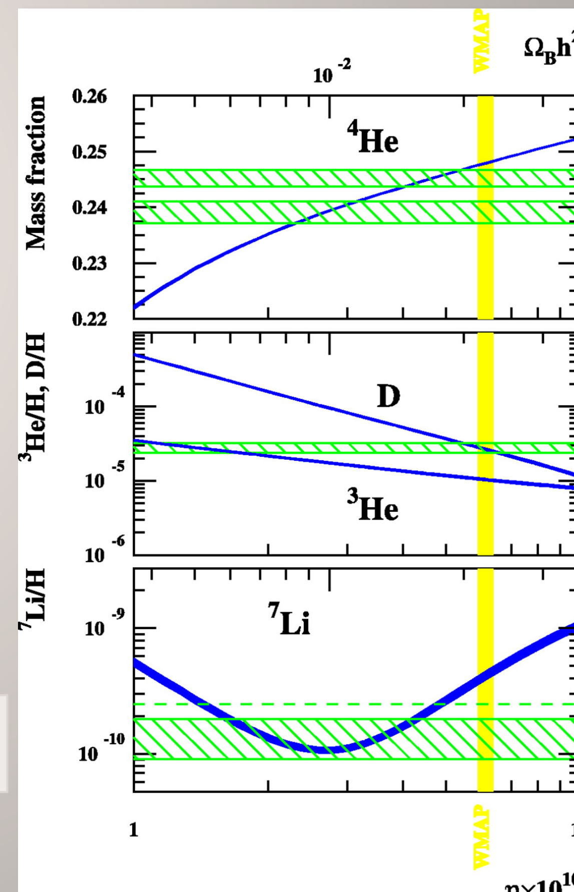
Big Bang nucleosythesis

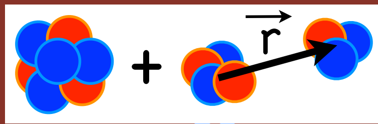




Key reactions

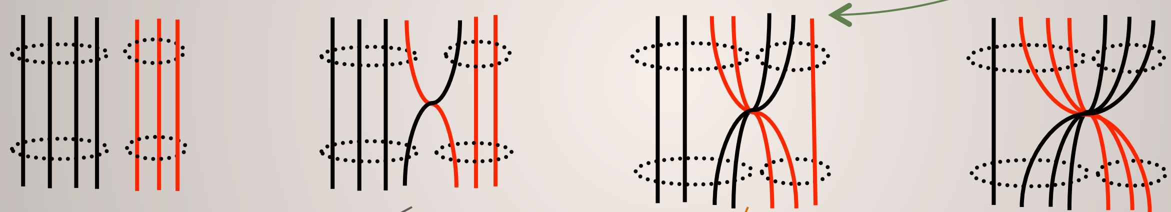
^{7}Li puzzle





^{3}He-^{4}He and ^{3}H-^{4}He scattering

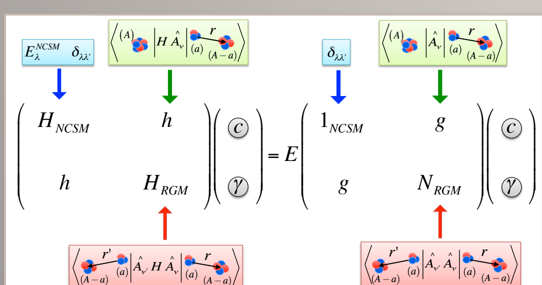
$$\left\langle \begin{matrix}(1,\dots,A\text{-}3)\\ r' \\ (A\text{-}2,A\text{-}1,A)\end{matrix} \right| 1-\sum_{i=1}^{A-3}\sum_{j=A-2}^{A}\hat{P}_{ij}+\tfrac{1}{2}\sum_{i\neq j=1}^{A-3}\left(\hat{P}_{i,A}\hat{P}_{j,A-1}+\hat{P}_{i,A}\hat{P}_{j,A-2}+\hat{P}_{i,A-1}\hat{P}_{j,A-2}\right) - \tfrac{1}{6}\sum_{i\neq j\neq k}\hat{P}_{i,A}\hat{P}_{j,A-1}\hat{P}_{k,A-2} \left| \begin{matrix}(1,\dots,A\text{-}3)\\ r \\ (A\text{-}2,A\text{-}1,A)\end{matrix} \right\rangle$$



$${}_{SD}\left\langle \psi_{\mu_1}^{(A-3)} \middle| a^+ a \middle| \psi_{\nu_1}^{(A-3)} \right\rangle_{SD}$$

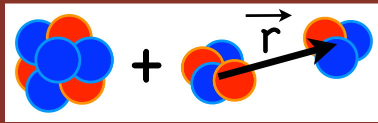
$${}_{SD}\left\langle \psi_{\mu_1}^{(A-3)} \middle| a^+ a^+ a\, a \middle| \psi_{\nu_1}^{(A-3)} \right\rangle_{SD}$$

$${}_{SD}\left\langle \psi_{\mu_1}^{(A-3)} \middle| a^+ a^+ a^+ a\, a\, a \middle| \psi_{\nu_1}^{(A-3)} \right\rangle_{SD}$$

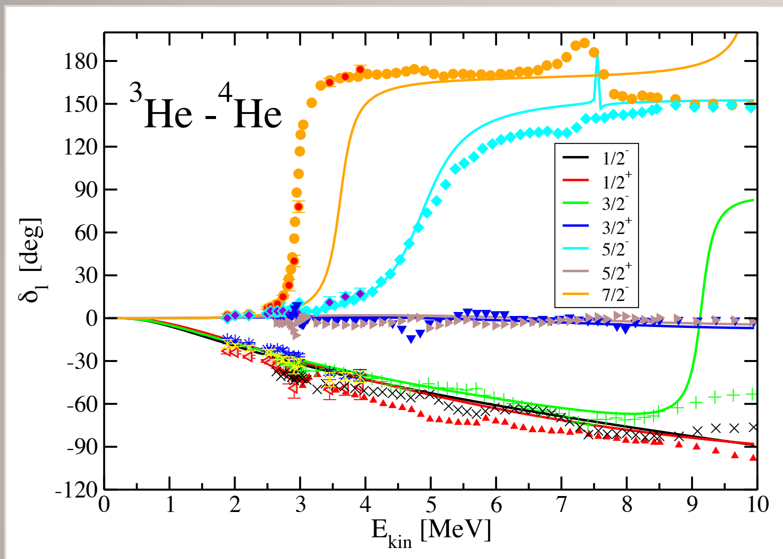


For A=7 use completeness

TRIUMF



^{3}He-^{4}He and ^{3}H-^{4}He scattering



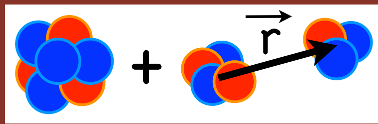
	^{7}Be		^{7}Li	
	NCSMC	Expt.	NCSMC	Expt.
$E_{3/2^-}$ [MeV]	-1.52	-1.586	-2.43	-2.467
$E_{1/2^-}$ [MeV]	-1.26	-1.157	-2.15	-1.989
r_{ch} [fm]	2.62	2.647(17)	2.42	2.390(30)
Q [e fm^2]	-6.14		-3.72	-4.00(3)
μ [μ_N]	-1.16	-1.3995(5)	+3.02	+3.256

J. Dohet-Eraly, P.N., S. Quaglioni, W. Horiuchi, G. Hupin, F. Raimondi, PLB **757**, 430 (2016)

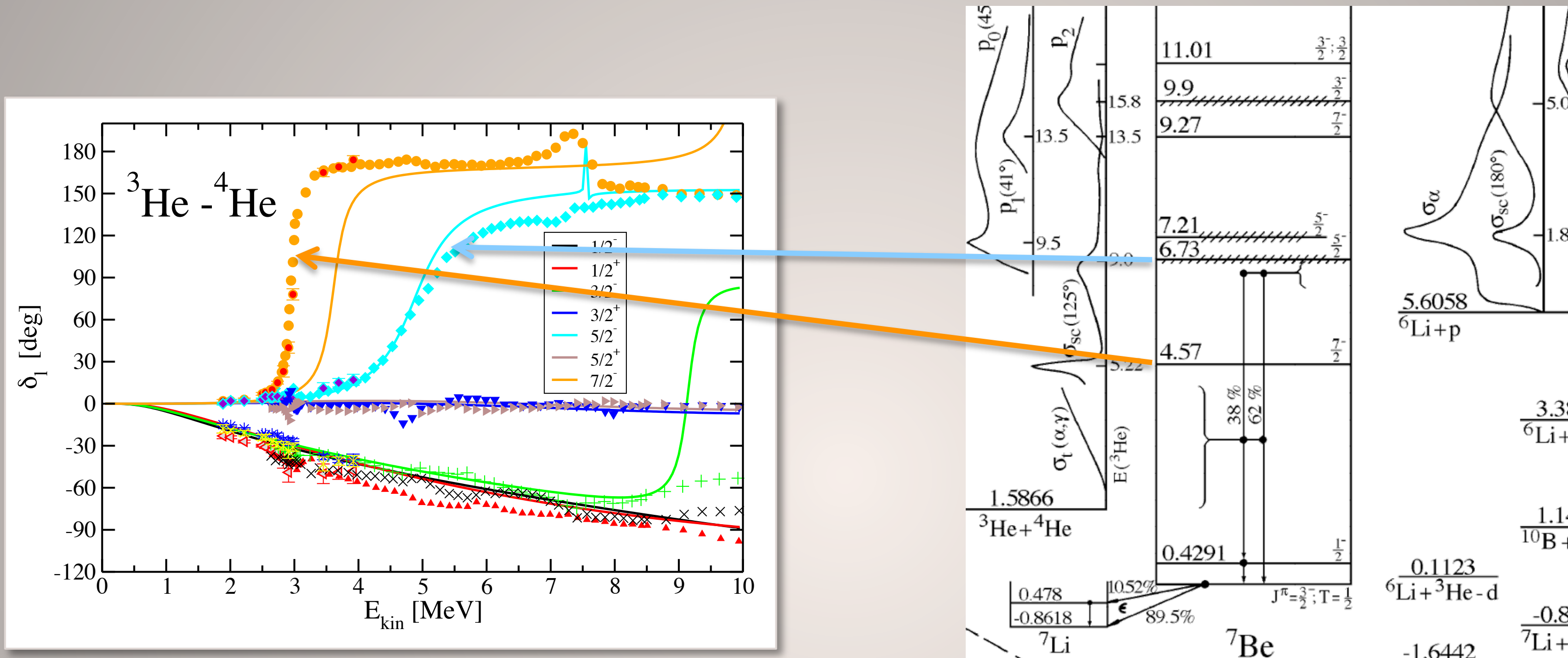
NCSMC calculations with chiral SRG-N^{3}LO *NN* potential (λ=2.15 fm^{-1})

^{3}He, ^{3}H, ^{4}He ground state, 8(π-) + 6(π+) eigenstates of ^{7}Be and ^{7}Li

Preliminary: N_{max}=12, hΩ=20 MeV



^{3}He-^{4}He and ^{3}H-^{4}He scattering



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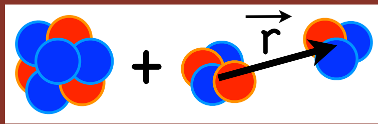
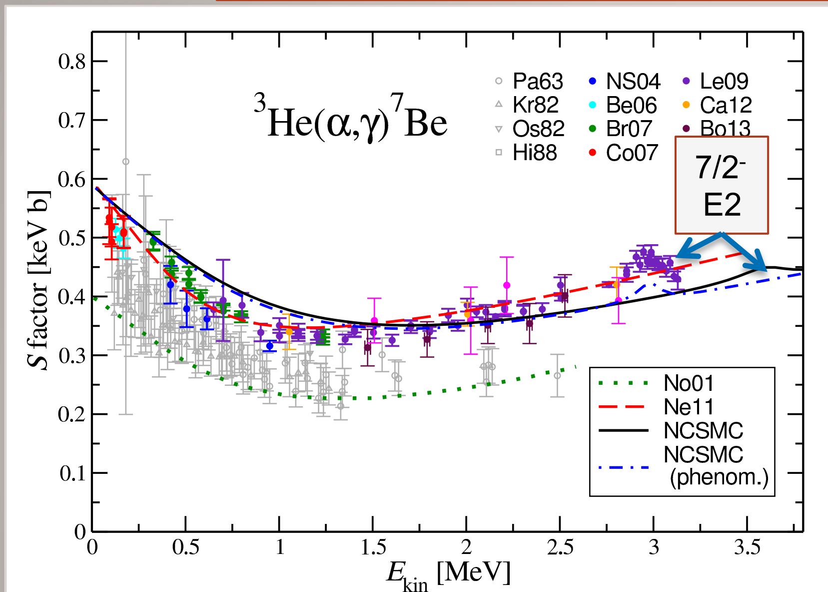
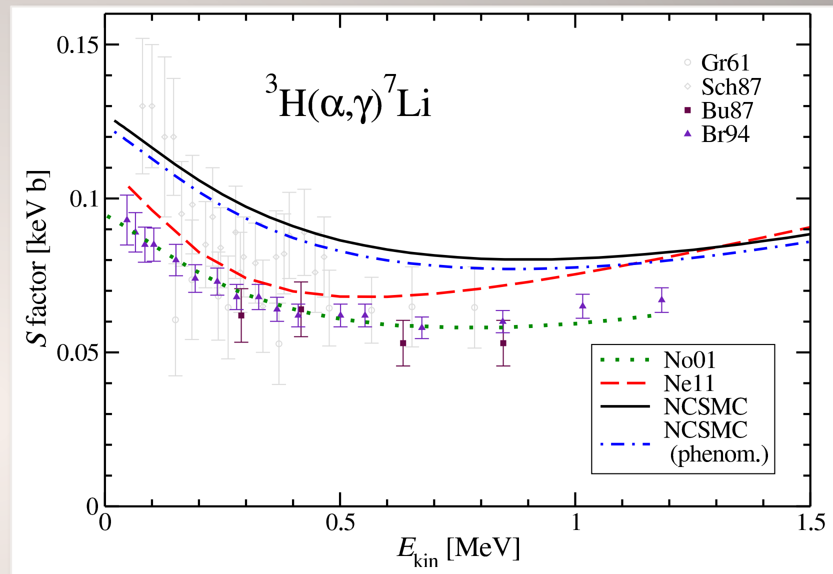
TRIUMF



^{3}He-^{4}He and ^{3}H-^{4}He capture

E1 radiative capture with small E2 contribution at 7/2$^-$ resonance





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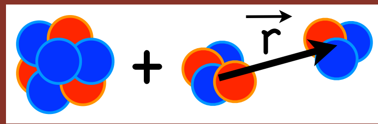
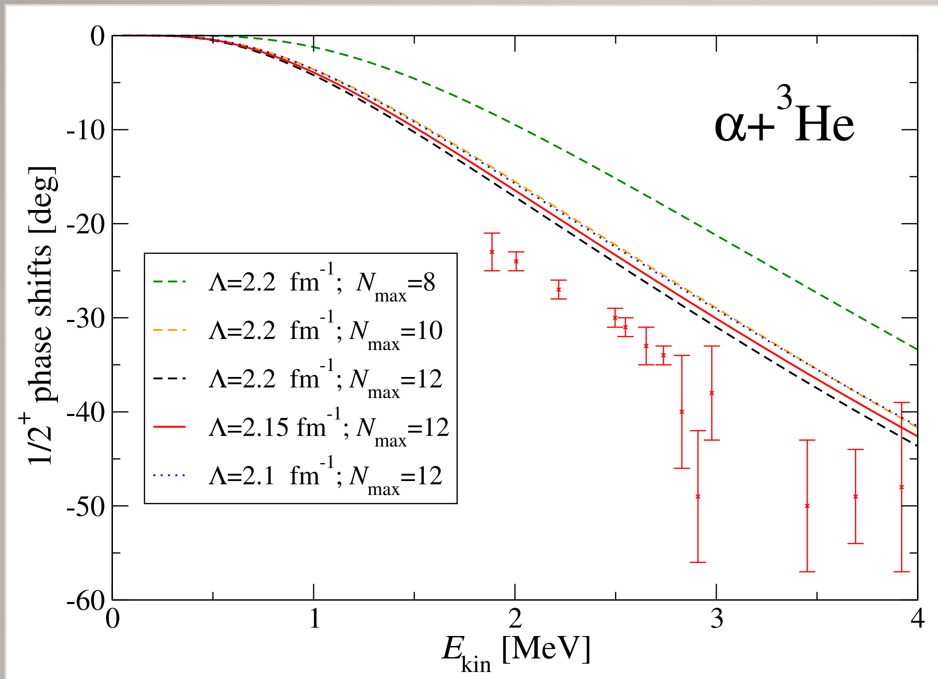
^{3}He, ^{3}H, ^{4}He ground state, 8(π-) + 6(π+) eigenstates of ^{7}Be and ^{7}Li

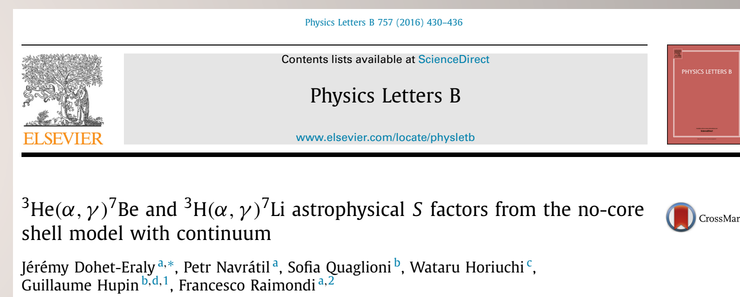
Preliminary: N_{max}=12, ħΩ=20 MeV

Theoretical calculations suggest that the most recent and precise ^{7}Be and ^{7}Li data are inconsistent



^{3}He-^{4}He S-wave phase shifts





Physics Letters B 757 (2016) 430–436

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^{3}He$(\alpha,\gamma)^7$Be and ^{3}H$(\alpha,\gamma)^7$Li astrophysical S factors from the no-core shell model with continuum

Jérémy Dohet-Eraly[a,*], Petr Navrátil[a], Sofia Quaglioni[b], Wataru Horiuchi[c], Guillaume Hupin[b,d,1], Francesco Raimondi[a,2]

J. Dohet-Eraly, P.N., S. Quaglioni, W. Horiuchi, G. Hupin, F. Raimondi, PLB **757**, 430 (2016)

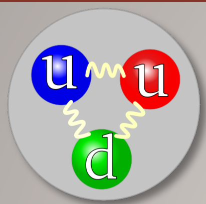
NCSMC calculations with chiral SRG-N^{3}LO *NN* potential (λ=2.15 fm^{-1})

^{3}He, ^{3}H, ^{4}He ground state, 8(π-) + 6(π+) eigenstates of ^{7}Be and ^{7}Li

Preliminary: N_{max}=12, hΩ=20 MeV

NCSMC calculations with chiral NN+3N forces in preparation

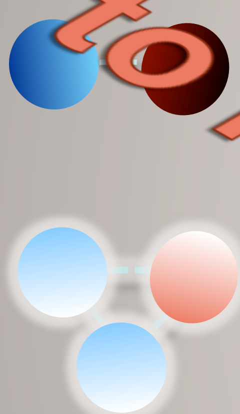
From QCD to nuclei



Low-energy QCD

NN+3N interactions from chiral EFT

…or accurate meson-exchange potentials



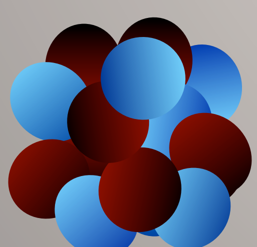
Unitary/similarity transformations

Identity or SRG or … or UCOM … Softens NN, induces 3N

$$H|\Psi\rangle = E|\Psi\rangle$$

Many-Body methods

NCSM, NCSM/RGM, CCM, GFMC, HH, Nuclear Lattice EFT…



Nuclear structure and reactions

Ultimate goal:

to nuclear

Applications to nuclear astrophysics

$^{12}C(\alpha,\gamma)^{16}O$

TRIUMF

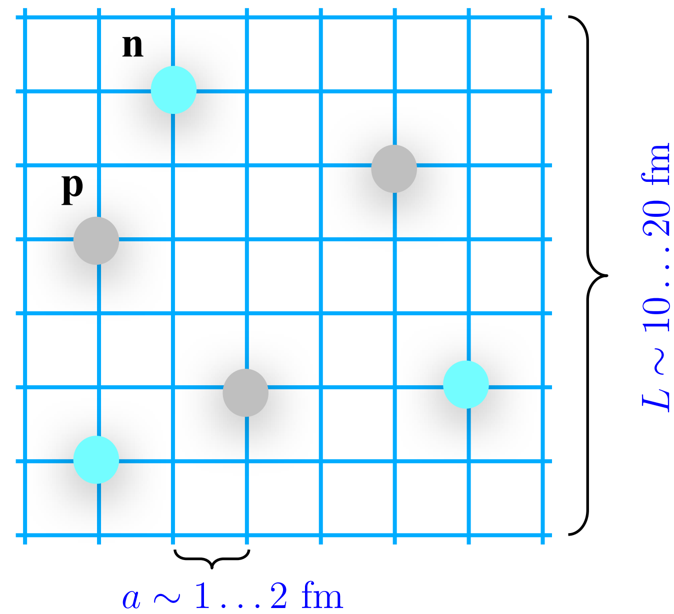
Nuclear Lattice Effective Field Theory Calculations

E. Epelbaum, H. Krebs, T. Lahde, D. Lee,U.-G. Meissner

Discretized version of chiral EFT for nuclear dynamics

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derivable within in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Borasoy, E.E., Krebs, Lee, Meißner, Eur. Phys. J. A31 (07) 105,
Eur. Phys. J. A34 (07) 185,
Eur. Phys. J. A35 (08) 343,
Eur. Phys. J. A35 (08) 357,
E.E., Krebs, Lee, Meißner, Eur. Phys. J A40 (09) 199,
Eur. Phys. J A41 (09) 125,
Phys. Rev. Lett 104 (10) 142501,
Eur. Phys. J. 45 (10) 335,
Phys. Rev. Lett. 106 (11) 192501



Physics Letters B 732 (2014) 110–115
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Lattice effective field theory for medium-mass nuclei

CrossMark

Timo A. Lähde[a,*], Evgeny Epelbaum[b], Hermann Krebs[b], Dean Lee[c], Ulf-G. Meißner[a,d,e], Gautam Rupak[f]

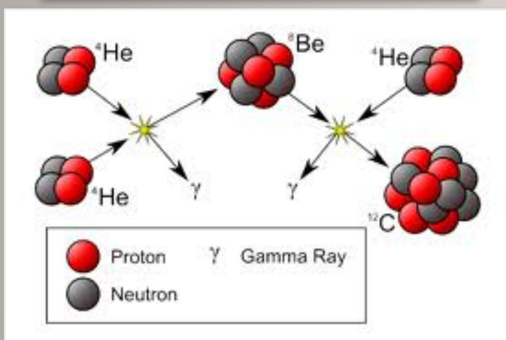
Ground states of alpha nuclei from ^{4}He to ^{28}Si

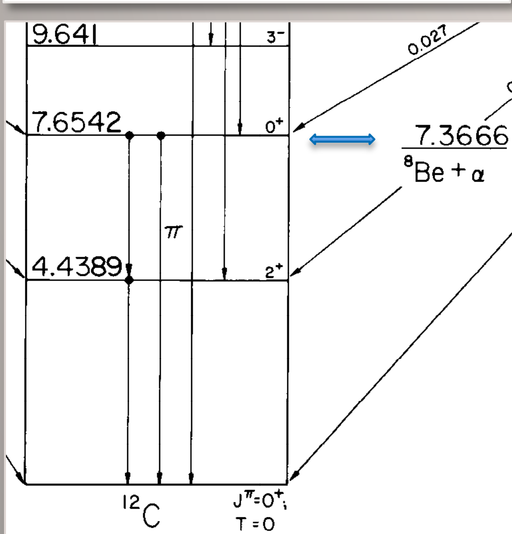
Nuclear Lattice Effective Field Theory Calculations

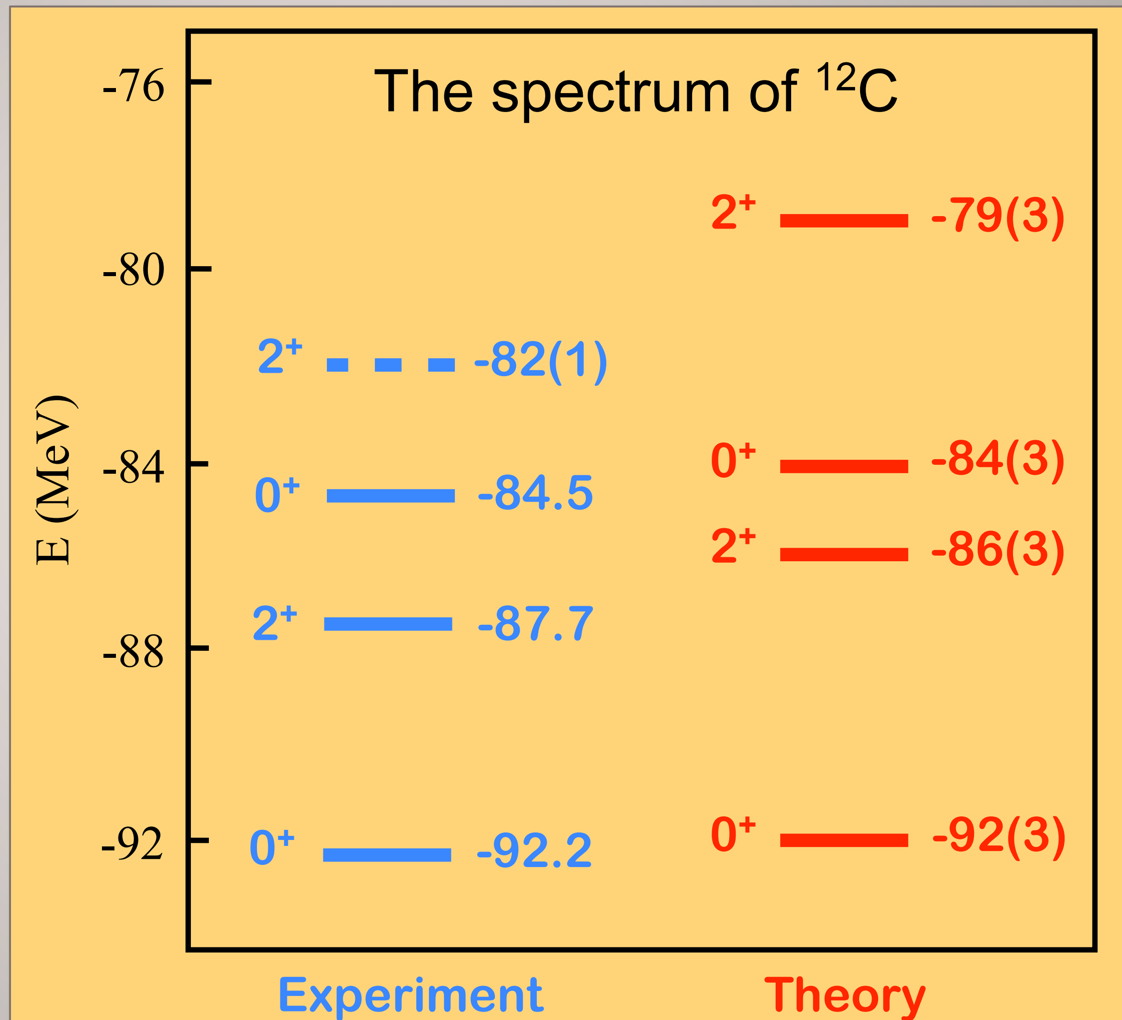
E. Epelbaum, H. Krebs, T. Lahde, D. Lee,U.-G. Meissner

The Hoyle state

^{12}C production in the Universe







Epelbaum, Krebs, Lee, Meissner, PRL 106, 192501 (2011)

TRIUMF

Conclusions and Outlook

Ab initio calculations of nuclear structure and reactions with predictive power becoming feasible beyond the latest nuclei.

Ab initio structure calculations can even reach (selected) medium & medium-heavy mass nuclei

These calculations make the connection between the low-energy QCD, many-body systems, and **nuclear astrophysics**.

Thank you!

NCSMC and NCSM/RGM collaborators

Sofia Quaglioni (LLNL)

Jeremy Dohet-Eraly, Angelo Calci (TRIUMF)

Guillaume Hupin (CEA/DAM)

Carolina Romero-Redondo (LLNL)

Francesco Raimondi (Surrey)

Wataru Horiuchi (Hokkaido)

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28th Indian-Summer School of Physics

AB INITIO METHODS IN NUCLEAR PHYSICS

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Fermionic Molecular Dynamics Approach
Self-consistent Green's Function Approach

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