Nuclear Density Functional Theory for Astrophysics

PART I – Nuclear properties and excitations

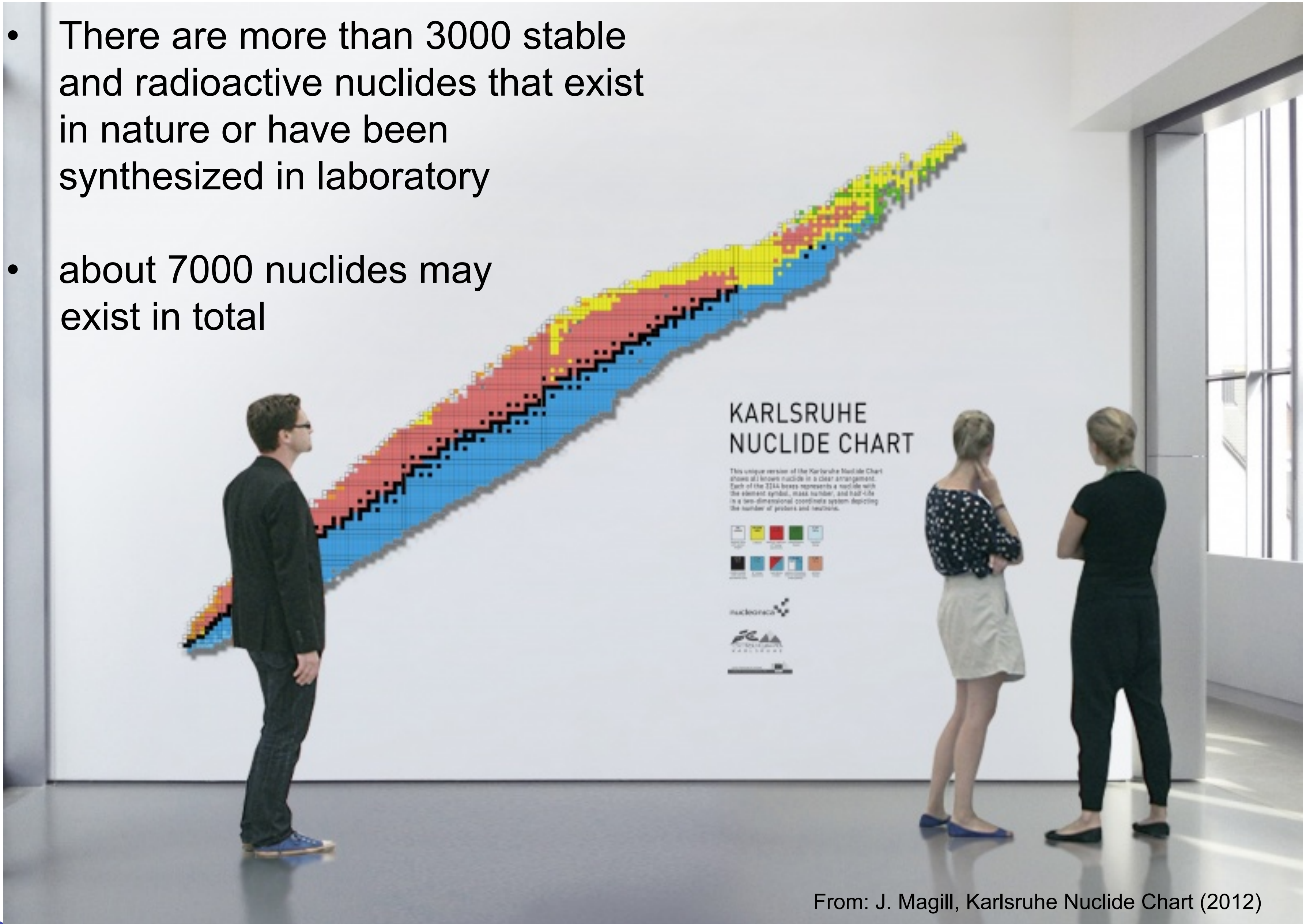
N. Paar

Department of Physics, Faculty of Science, University of Zagreb, Croatia

THE CHART OF THE NUCLIDES

- There are more than 3000 stable and radioactive nuclides that exist in nature or have been synthesized in laboratory

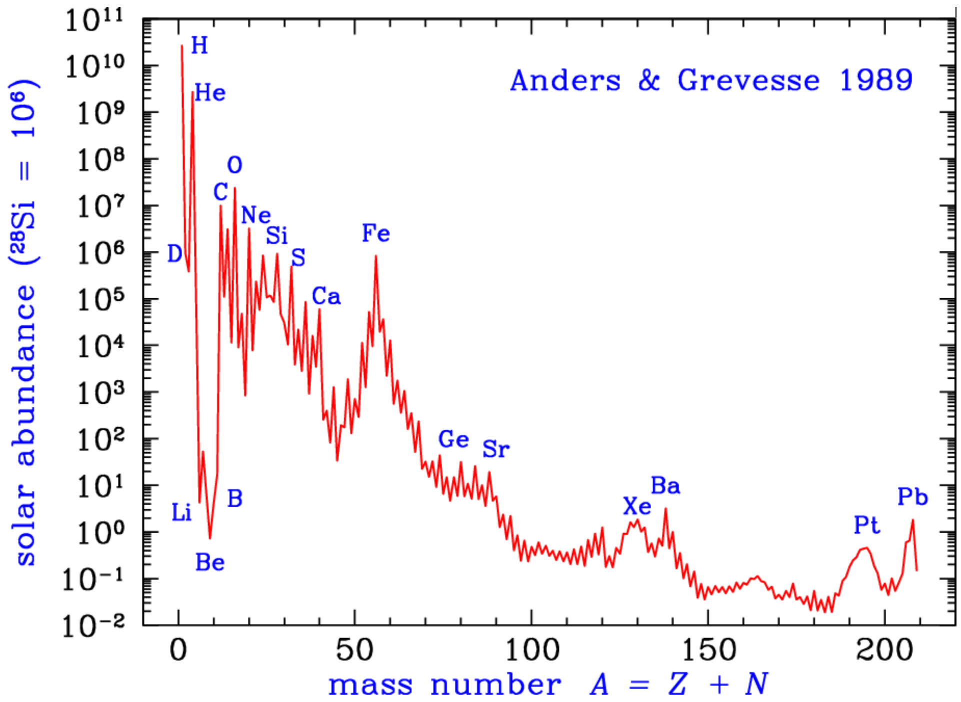
- about 7000 nuclides may exist in total



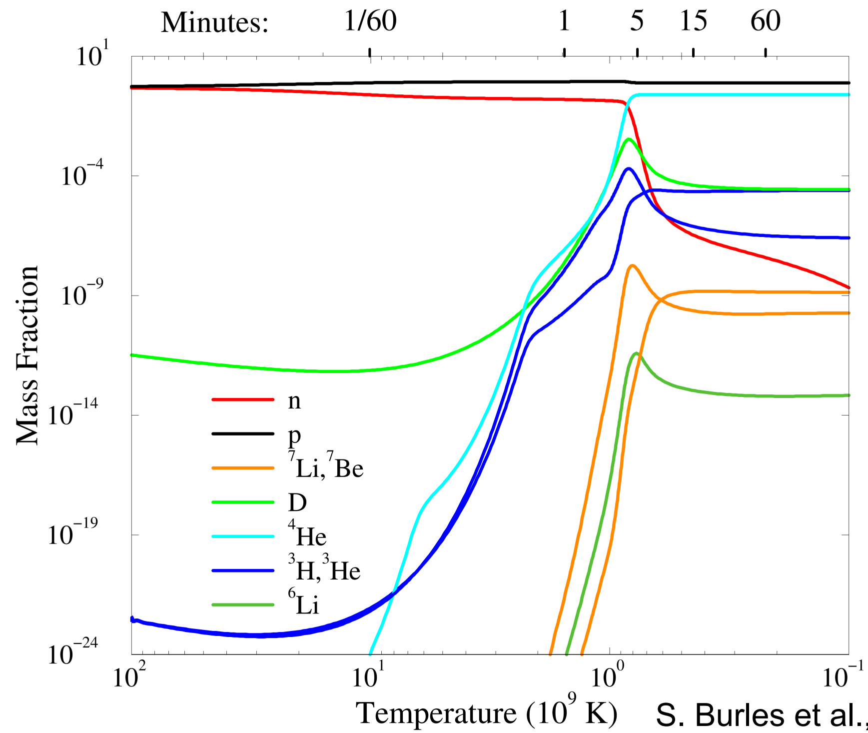
From: J. Magill, Karlsruhe Nuclide Chart (2012)

THE SYNTHESIS OF ELEMENTS IN THE UNIVERSE

How are the elements in the universe produced?



Big-bang nucleosynthesis

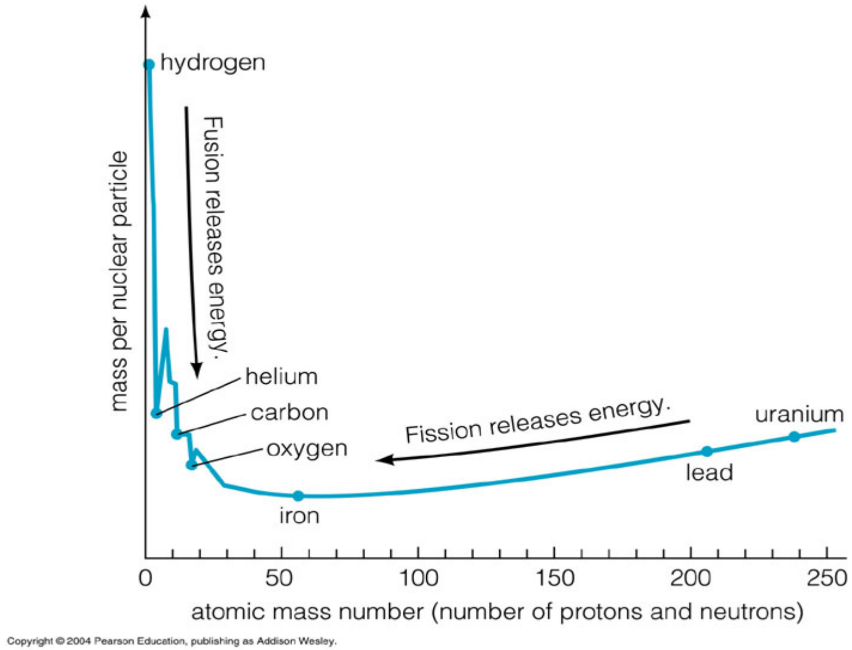


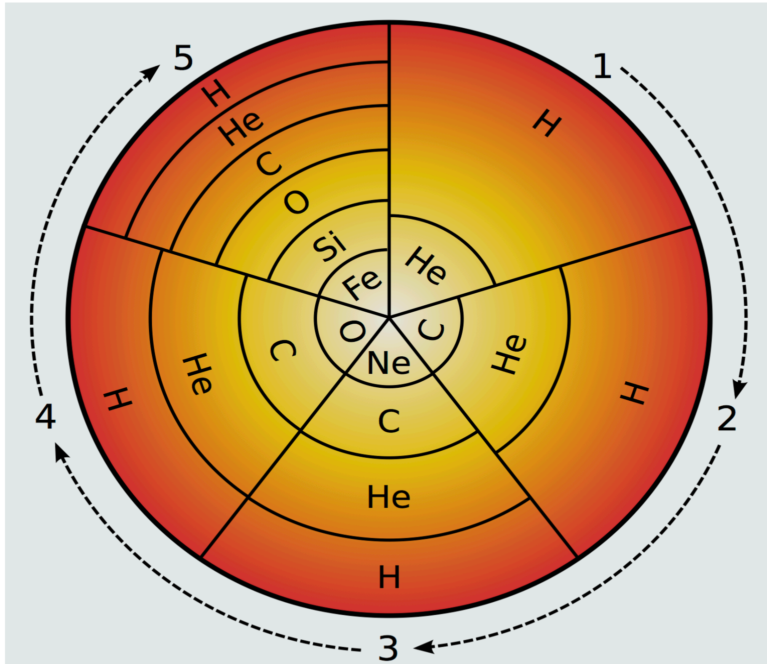
S. Burles et al., 1999.

Hydrogen – 75 %
Helium – 25 %
Deuterium and tritium – 10^{-3} %
Lithium – 10^{-8} %

Hydrostatic nucleosynthesis in stars

- Hydrogen burning continues until the fuel is spent, leading to contraction of the star and higher temperatures.
- Burning cycles with different fuel continue (depending on mass of the star) all the way to iron!

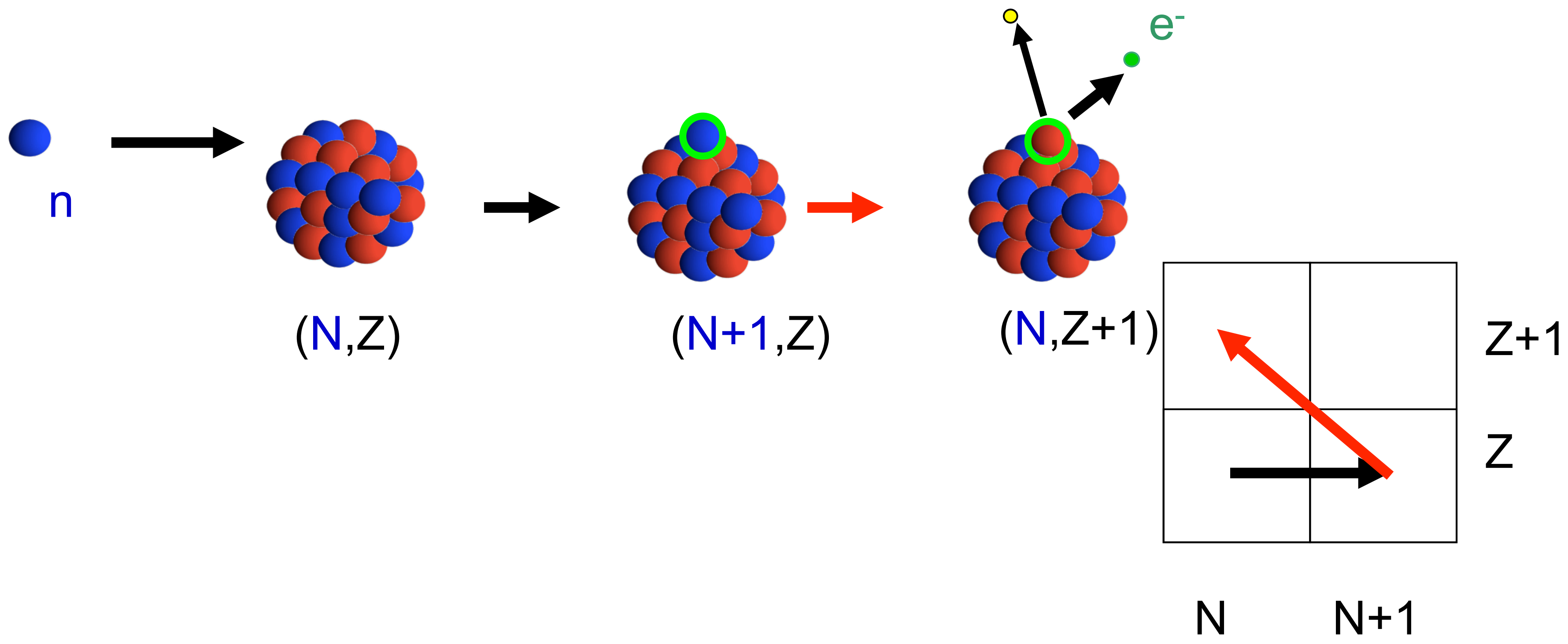




Iron and nickel have two most stable isotopes – combining them into even heavier nuclei would consume more energy than it would provide.

Nucleosynthesis of elements heavier than Fe:

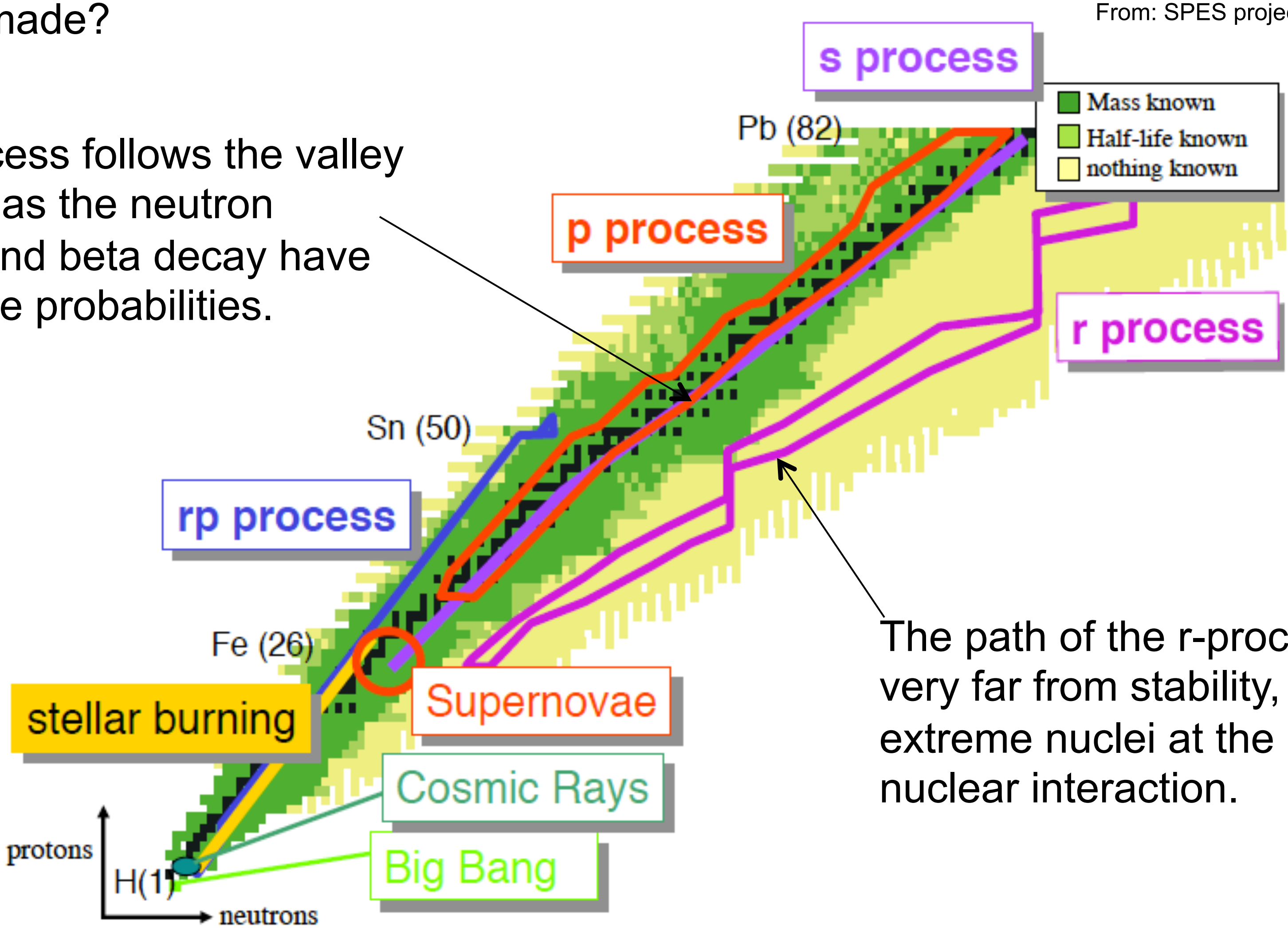
- Neutron capture and successive β^-decay



THE SYNTHESIS OF ELEMENTS IN THE UNIVERSE

- How and where are the the heaviest elements made?

The s-process follows the valley of stability as the neutron captures and beta decay have comparable probabilities.



The path of the r-process lies very far from stability, involves extreme nuclei at the limits of nuclear interaction.

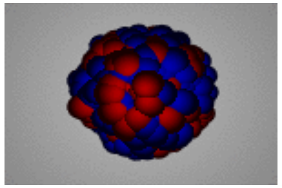
Nuclear Astrophysics
(stellar evolution, nucleosynthesis)

⇕

Nuclear physics
(strong, weak, electromagnetic forces)
nuclei and nuclear matter

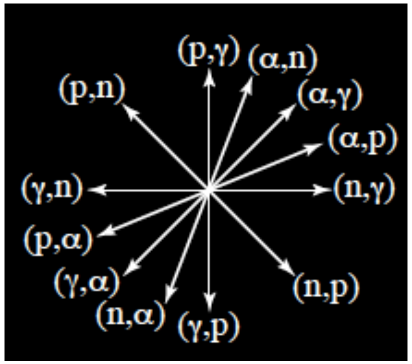
Nuclear Structure and Dynamics

- masses
- deformations
- radii
- excitations
- collective effects
- …



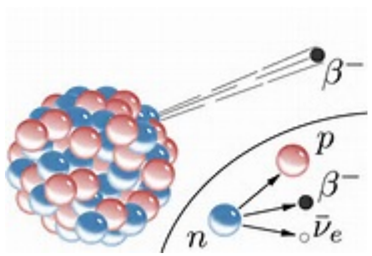
Nuclear Reactions

- thermonuclear reactions
- …



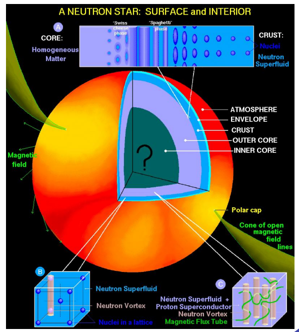
Weak processes

- $\beta^{\pm}$ decays
- delayed n emission
- electron capture
- ν-nucleus processes
- …



Nuclear Matter

- equation of state
- symmetry energy
- neutron matter
- neutron stars
- …



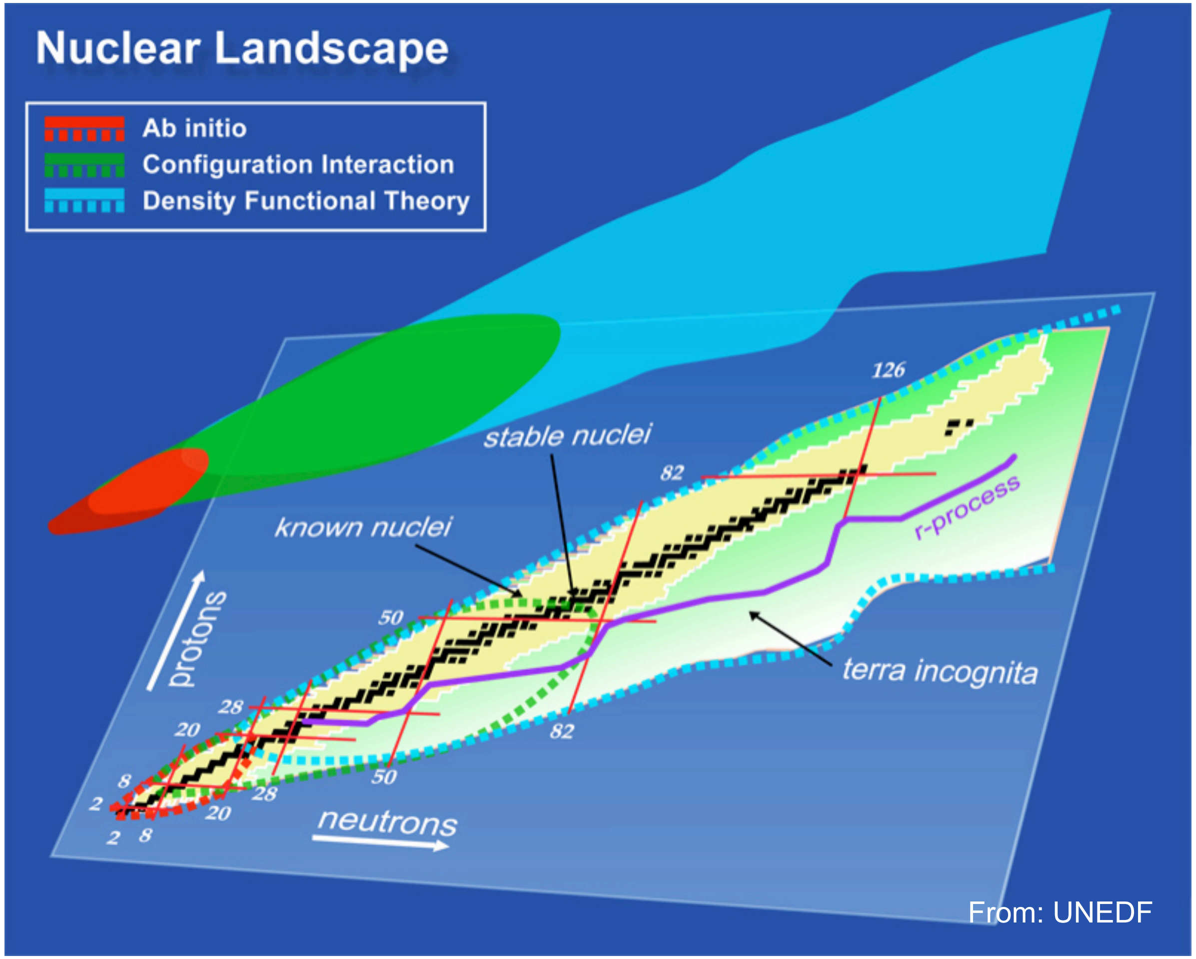
NUCLEAR THEORY

- The goal: understanding the properties of nuclei for the synthesis of elements and properties of stars

- Nuclear theory approaches (microscopic)
  - Ab initio models
  - Interacting shell model
  - energy density functionals
  - …

- Method of choice: Density functional theory allows a consistent approach to nuclear matter, finite nuclei and nuclear processes along the nuclide map

- Mainly based on mean field models (Skyrme, Gogny, relativistic)



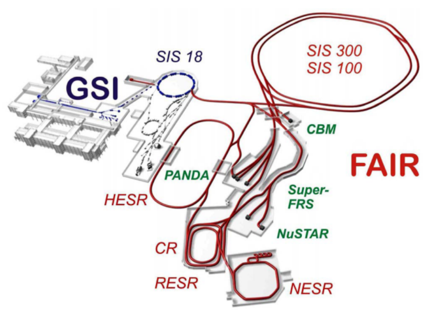
DENSITY FUNCTIONAL THEORY

NEW GENERATION RIB FACILITIES

RIBF RIKEN NiSHiNA CENTER RIKEN Nishina Center for Accelerator-Based Science

Spiral2

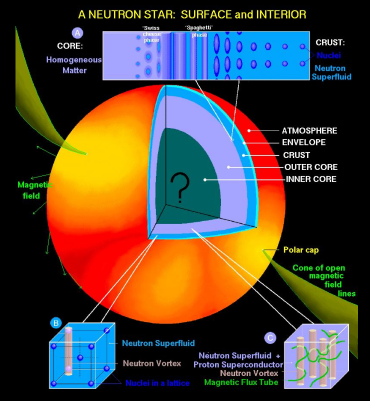
FRIB

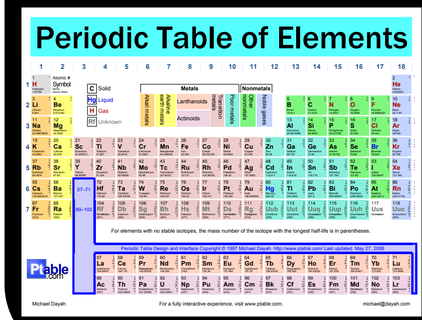


Nuclear Energy Density Functional

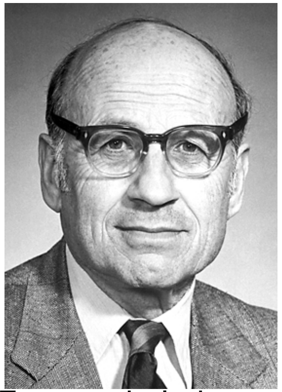
Nuclear ground state properties and excitations; EOS

Stellar nuclear processes and reactions





DENSITY FUNCTIONAL THEORY (DFT)



From: nobelprize.org

- **Walter Kohn** – The Nobel Prize in Chemistry 1998
  - ✧ "for his development of the density-functional theory"
- Kohn, Hohenberg, Sham, …
- Successful applications of DFT in chemistry and condensed-matter physics

- Within the DFT it is not necessary to account for every electron's movement. Instead, one could look at the average density of electrons in the space.
- DFT shifts the emphasis from the individual wave functions to the density

$$\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N) \Longrightarrow \rho(\vec{r})$$

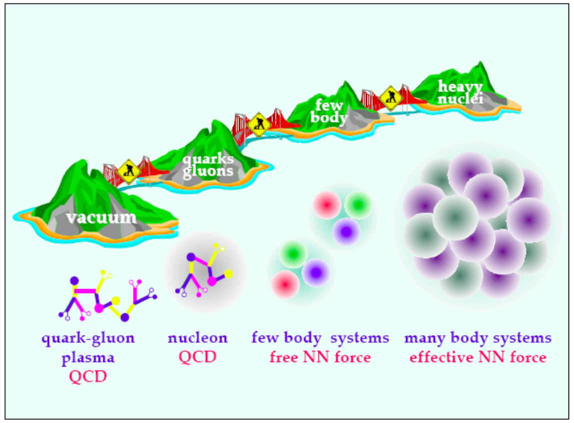
- Hohenberg-Kohn theorem – the exact energy of a quantum many body system is a functional $E(\rho)$ of the local density $\rho(\vec{r})$
- Ground state density and other ground state observables are obtained by minimizing a suitable energy functional $E(\rho)$

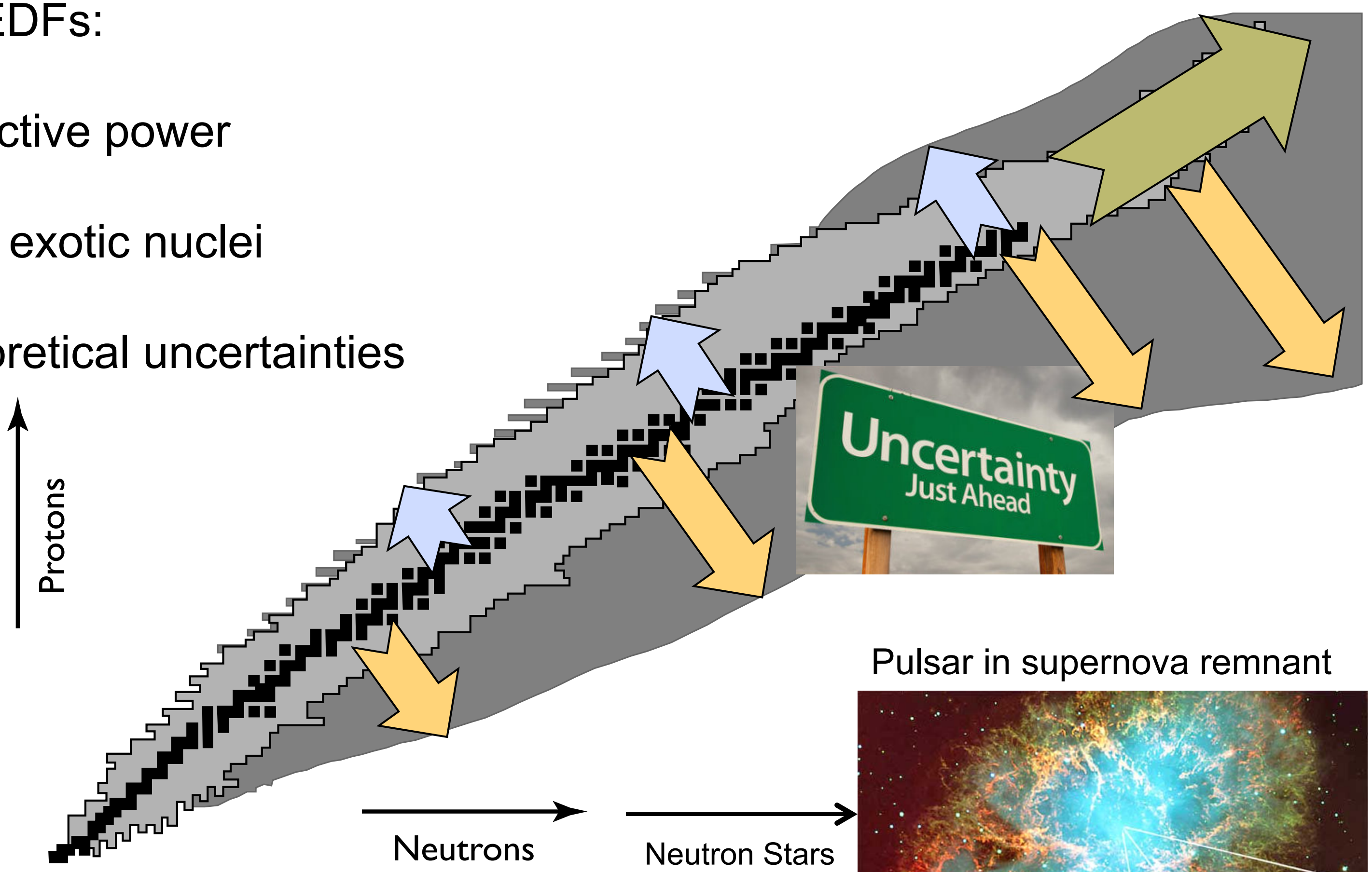
The strategy in the nuclear EDF approach:

- Establish an optimal EDF based on some effective nuclear interaction
- the nuclear energy functional is so far phenomenological and not connected to realistic NN-interaction
- Constrain the empirical parameters of the functional and its validation using available many body observables such as masses, radii, pseudo-data, etc.
- Complicated many body effects are encoded in the empirical constants
- EDF valid through the entire chart of nuclides, light and heavy, spherical and deformed
- Develop theory frameworks for applications of the functional to address various static and dynamic nuclear phenomena, processes, nuclear equation of state, neutron stars, …

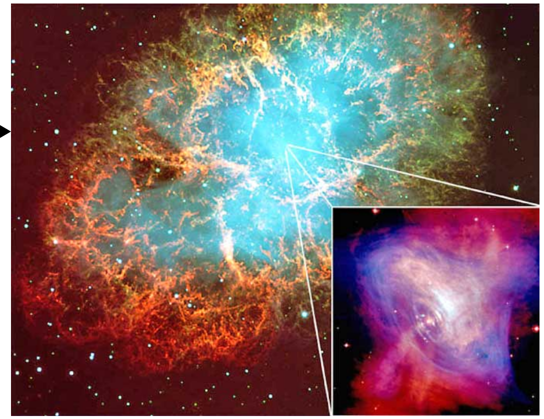
ENERGY DENSITY FUNCTIONAL (EDF)

- Challenges for the EDFs:
  - ✧ improving predictive power
  - ✧ extrapolation to exotic nuclei
  - ✧ quantifying theoretical uncertainties





Pulsar in supernova remnant

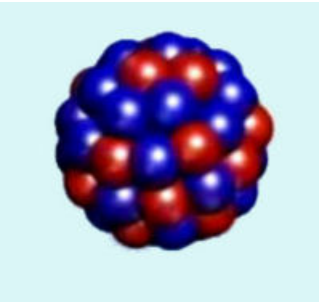


From:: Crab Nebula from VLT: FORS Team, ESO; X-ray Image (inset): NASA/CXC/ASU/J. Hester et al;
Optical Image (inset): NASA/HST/ASU/J. Hester et al.

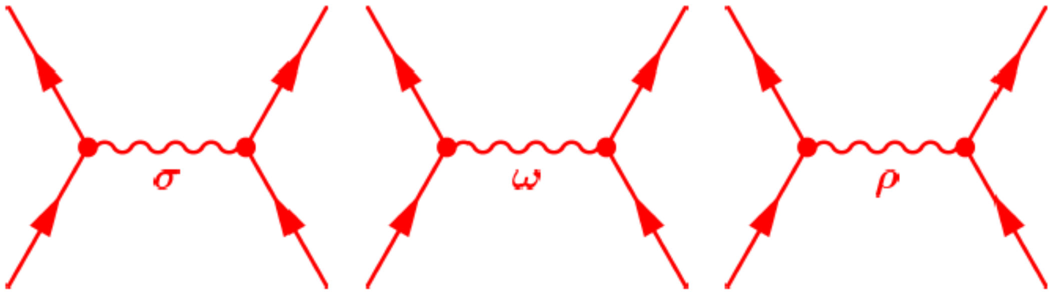
  - ✧ connecting ab-initio methods and EDFs
  - ✧ universal EDF for astrophysical models

SELF-CONSISTENT RELATIVISTIC MEAN FIELD MODEL

- The implementation of density functional theory in the relativistic framework in terms of self-consistent relativistic mean-field model
- The basis is an effective Lagrangian with relativistic symmetries



System of Dirac nucleons coupled by the exchange meson and the photon fields



sigma-meson:
attractive scalar field

omega-meson:
short-range repulsive

rho-meson:
isovector field

Extensions:

- pairing correlations (Relativistic Hartree-Bogoliubov model)

SELF-CONSISTENT RELATIVISTIC MEAN FIELD MODEL

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$$

- the Lagrangian of the free nucleon: $\mathcal{L}_N = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi$

- the Lagrangian of the free meson fields and the electromagnetic field:

$$\begin{aligned}\mathcal{L}_m &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &\quad -\frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

- minimal set of interaction terms:

$$\mathcal{L}_{int} = -\bar{\psi}\Gamma_\sigma\sigma\psi - \bar{\psi}\Gamma_\omega^\mu\omega_\mu\psi - \bar{\psi}\vec{\Gamma}_\rho^\mu\vec{\rho}_\mu\psi - \bar{\psi}\Gamma_e^\mu A_\mu\psi.$$

- with the vertices: $\Gamma_\sigma = g_\sigma, \quad \Gamma_\omega^\mu = g_\omega\gamma^\mu, \quad \vec{\Gamma}_\rho^\mu = g_\rho\vec{\tau}\gamma^\mu, \quad \Gamma_e^m = e\frac{1-\tau_3}{2}\gamma^\mu$

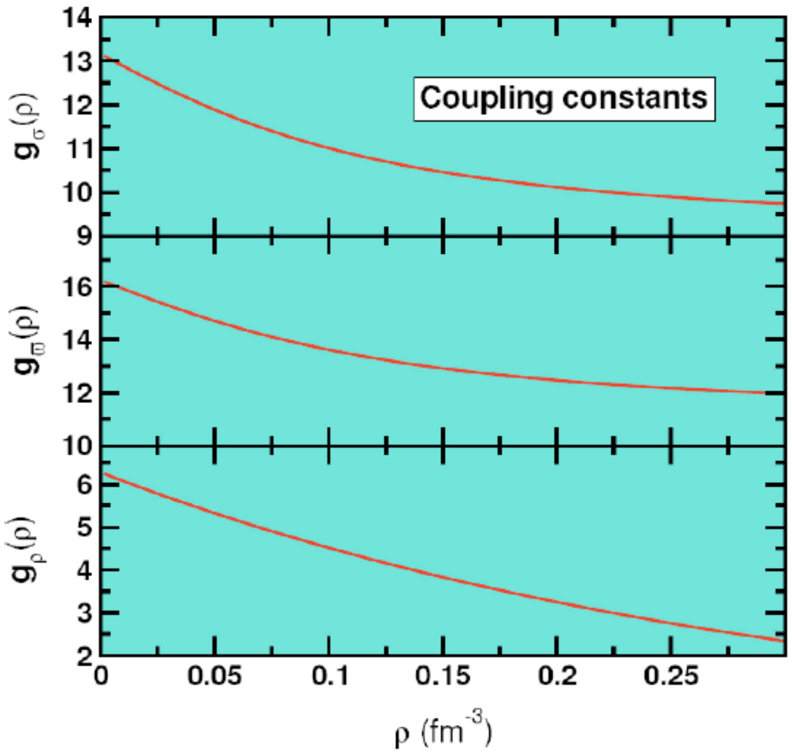
$$\partial_\mu \frac{\partial L}{\partial\left(\partial_\mu q_k\right)} - \frac{\partial L}{\partial q_k} = 0.$$

→

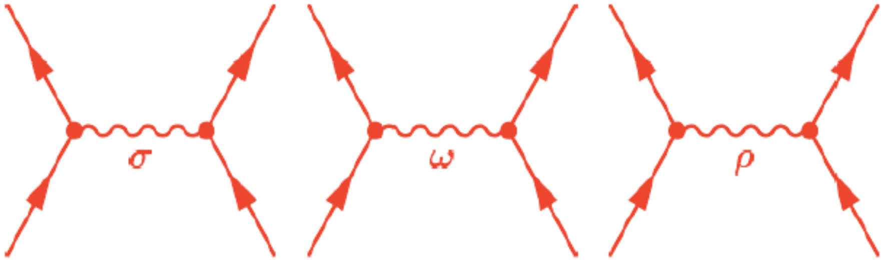
- Dirac equation (nucleons)
- Klein-Gordon eqs. (meson fields)
- Self-consistent solution

SELF-CONSISTENT RELATIVISTIC MEAN FIELD MODEL

- Effective density dependence of the model – motivated by ab-initio calculations
- Density dependent meson-nucleon couplings



Effective interactions with medium-dependent couplings:



COUPLING PARAMETERS:

$g_\sigma(\rho)$, $g_\omega(\rho)$, $g_\rho(\rho)$

SELF-CONSISTENT RELATIVISTIC MEAN FIELD MODEL

- Relativistic point coupling model

- The basis is an effective Lagrangian with four-fermion (contact) interaction terms; isoscalar-scalar, isoscalar-vector, isovector-vector, derivative term

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}(i\gamma \cdot \partial - m)\psi \\
&- \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\
&- \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\
&- \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A\frac{(1-\tau_3)}{2}\psi
\end{aligned}
$$

- many-body correlations encoded in density-dependent coupling functions that are motivated by microscopic calculations but parameterized in a phenomenological way

- Extensions: pairing correlations in finite nuclei
  - Relativistic Hartree-Bogoliubov model
  (e.g. with separable form of the pairing interaction Y. Tian, Z. Y. Ma, P. Ring, PLB 676, 44 (2009).)

T. Niksic, et al., Comp. Phys. Comm. 185, 1808 (2014).

SELF-CONSISTENT RELATIVISTIC MEAN FIELD MODEL

- Density dependence of the couplings - To establish the density dependence of the couplings one could start from a microscopic equation of state of symmetric and asymmetric nuclear matter.

$$\alpha_i(\rho) = a_i + (b_i + c_i x)e^{-d_i x} \qquad (i \equiv S, V, TV)$$

$$x = \rho/\rho_{sat}$$

- 12 model parameters:

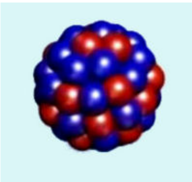
a_s, b_s, c_s, d_s	isoscalar-scalar
a_v, b_v, d_v	isoscalar-vector
b_{TV}, d_{TV}	isovector-vector
δ_s	derivative term
g_n, g_p	pairing correlations (strength parameters)

CONSTRAINING THE FUNCTIONAL

- The model parameters $\mathbf{p} = (p_1, \dots, p_n)$ are constrained directly by many-body observables using χ^2 minimization

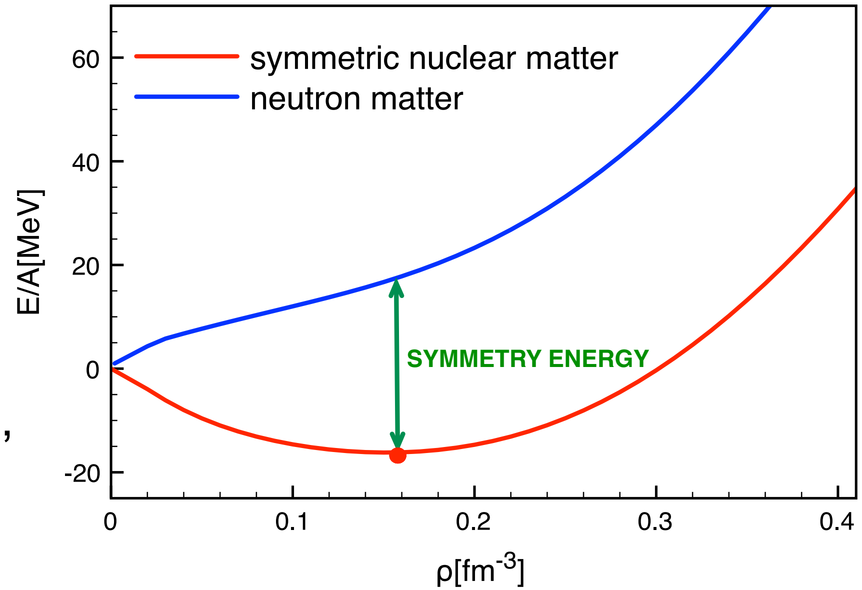
$$\chi^2(\boldsymbol{p}) = \sum_{\imath=1}^{m} \left(\frac{\mathcal{O}_\imath^{\text{theo.}}(\boldsymbol{p}) - \mathcal{O}_\imath^{\text{ref.}}}{\Delta \mathcal{O}_\imath^{\text{ref.}}} \right)^2$$

- Calculated values can be compared to experimental, observational, and pseudo-data



properties of finite nuclei – binding energies, charge radii, diffraction radii, surface thicknesses, pairing gaps, etc.,…

- nuclear matter properties – equation of state, binding energy and density at the saturation, symmetry energy J & L, incompressibility…



- Isovector channel of the EDF is weakly constrained by exp. data such as binding energies and charge radii. Possible observables for the isovector properties: *neutron radii, neutron skins, dipole polarizability, pygmy dipole strength, neutron star radii*

NUCLEAR MATTER EQUATION OF STATE AND SYMMETRY ENERGY

Nuclear matter equation of state:

$$E(\rho, x) = E_{SNM}(\rho) + E_{sym}(\rho)(1-2x)^2 + ...$$

$$\rho = \rho_n + \rho_p \ , x = \rho_p/\rho$$

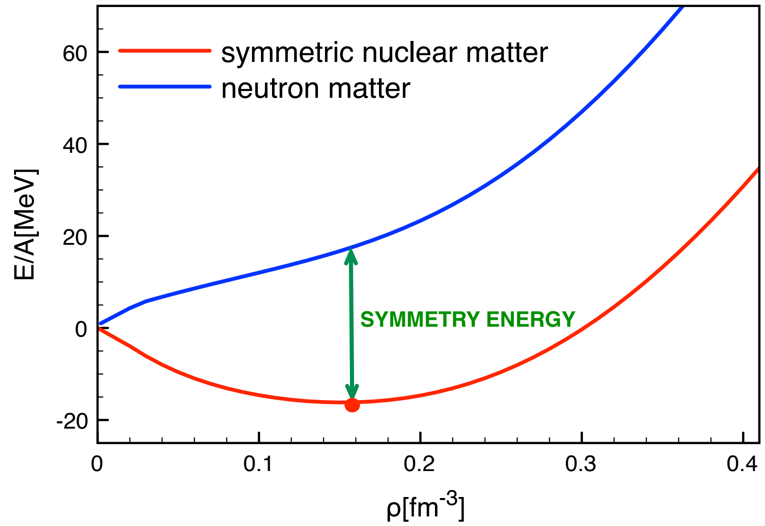
Symmetry energy term:

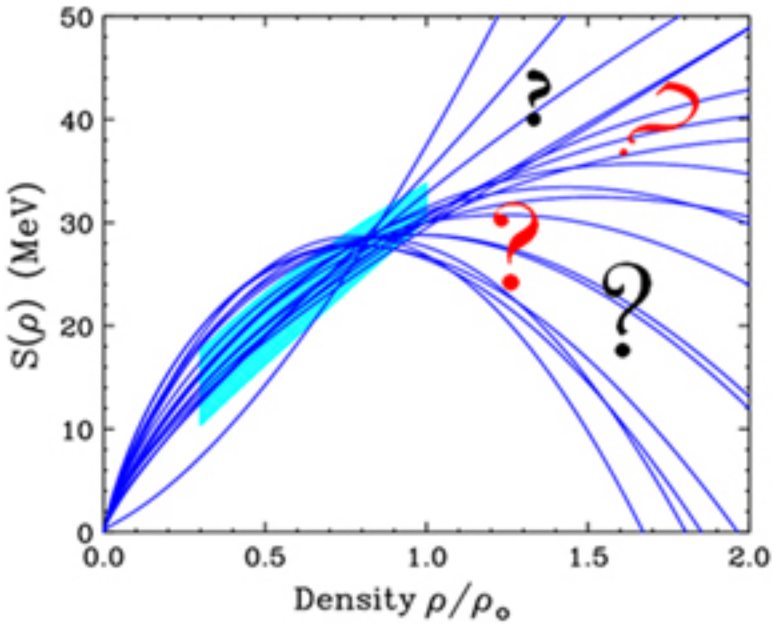
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \frac{dS_2(\rho)}{dr}|_{\rho_0}$$

J – symmetry energy at saturation density
L – slope of the symmetry energy (related to the pressure of neutron matter)

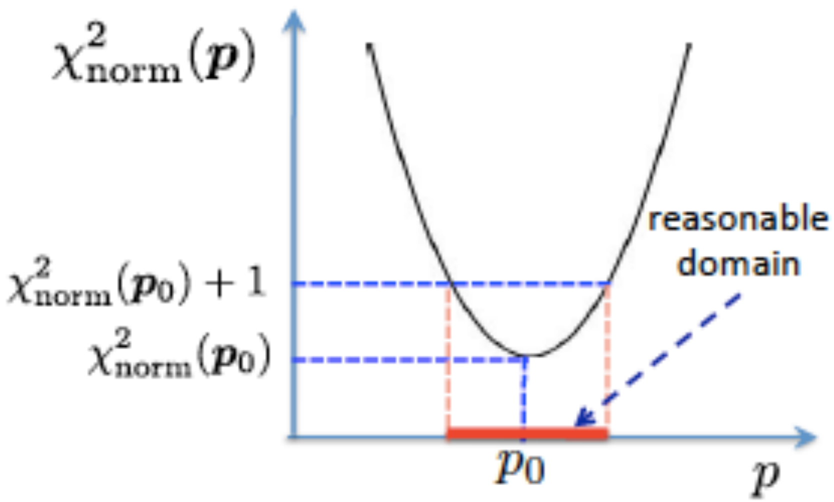


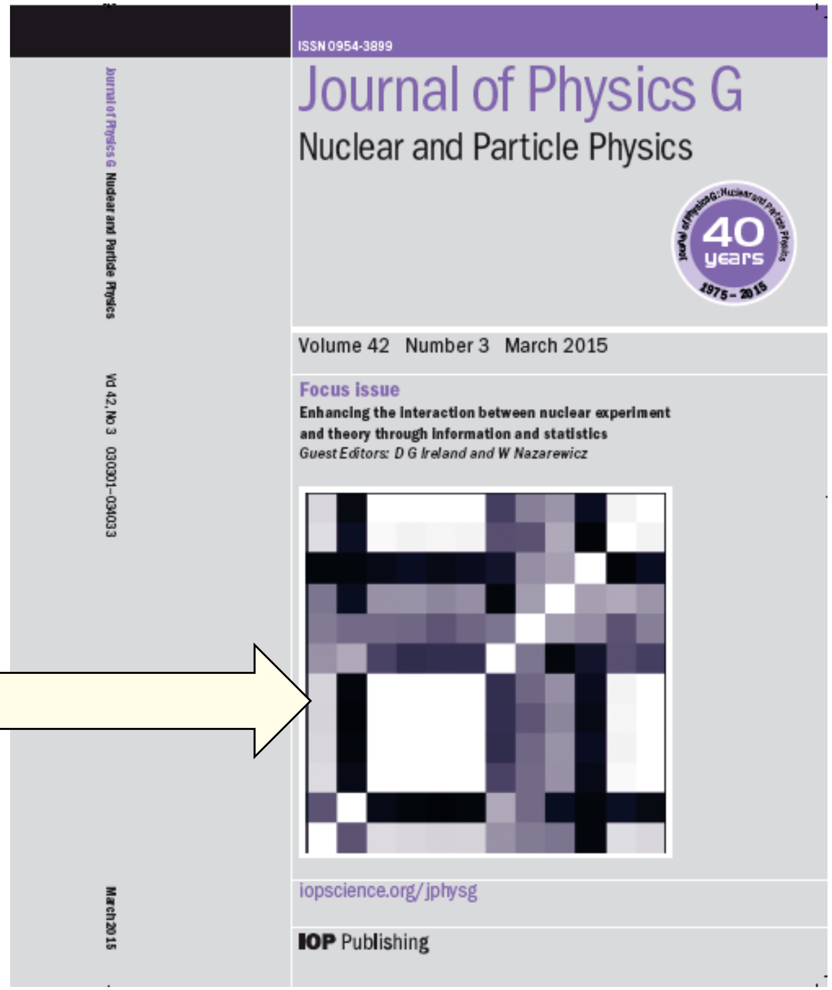


B. Tsang, NSCL

COVARIANCE ANALYSIS IN THE FRAMEWORK OF EDFs

- The quality of X^2 minimization is an indicator of the statistical uncertainty





- Correlation matrix shows the correlations between various quantities.

- J. Dobaczewski, W. Nazarewicz, P.-G. Reinhard, JPG 41, 074001 (2014)
- X. Roca Maza et al., JPG 42, 034033 (2015)

COVARIANCE ANALYSIS IN THE FRAMEWORK OF EDFs

- Assume that χ^2 is a well behaved hyper-function of the parameters around their optimal value $\mathbf{p}_0, \partial_{\mathbf{p}}\chi^2(\mathbf{p})\,|_{\mathbf{p}=\mathbf{p}_0} = 0$

- Near the minimum, χ^2 can be approximated by a Taylor expansion as an hyper-parabola in the parameter space

$$\chi^2(\boldsymbol{p}) - \chi^2(\boldsymbol{p}_0) \approx \frac{1}{2}\sum_{\imath,\jmath}^{n}(p_\imath - p_{0\imath})\partial_{p_\imath}\partial_{p_\jmath}\chi^2(p_\jmath - p_{0\jmath})$$

Curvature matrix:

$$\mathcal{M}_{ij} = \tfrac{1}{2}\partial_{p_i}\partial_{p_j}\chi^2|_{\mathbf{p}_0}$$

Covariance between two quantities A and B:

$$\overline{\Delta A\,\Delta B} = \sum_{ij}\partial_{p_i}A(\hat{\mathcal{M}}^{-1})_{ij}\partial_{p_j}B$$

- Variance $\overline{\Delta^2 A}$ and $\overline{\Delta^2 B}$ define **statistical uncertainties of each quantity.**

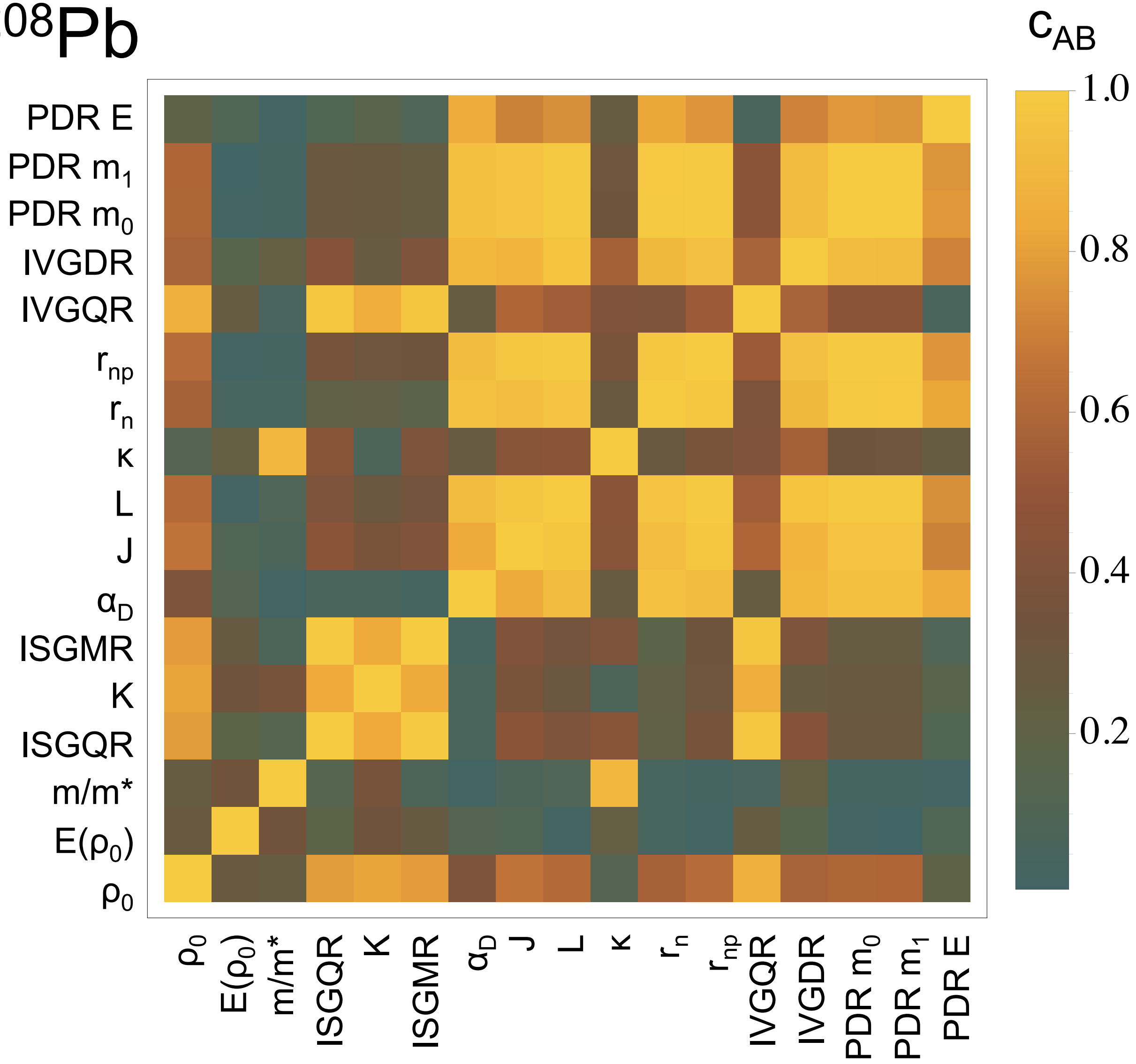
Pearson product-moment correlation coefficient provides a measure of the correlation (linear dependence) between two variables A and B.

$$c_{AB} = \frac{|\overline{\Delta A\,\Delta B}|}{\sqrt{\overline{\Delta A^2}\;\overline{\Delta B^2}}}$$

J. Dobaczewski, W. Nazarewicz, P.-G. Reinhard, JPG 41, 074001 (2014).

CORRELATIONS: NUCLEAR MATTER vs. PROPERTIES OF NUCLEI

^{208}Pb



Correlation matrix between nuclear matter properties and several quantities in ^{208}Pb (DDME-min1)

- neutron skin thickness, properties of giant resonances, pygmy strength

$c_{AB}=1$: A & B strongly correlated

$c_{AB}=0$: A & B uncorrelated

NEUTRON SKINS AND DENSITY DEPENDENCE OF THE SYMMETRY ENERGY

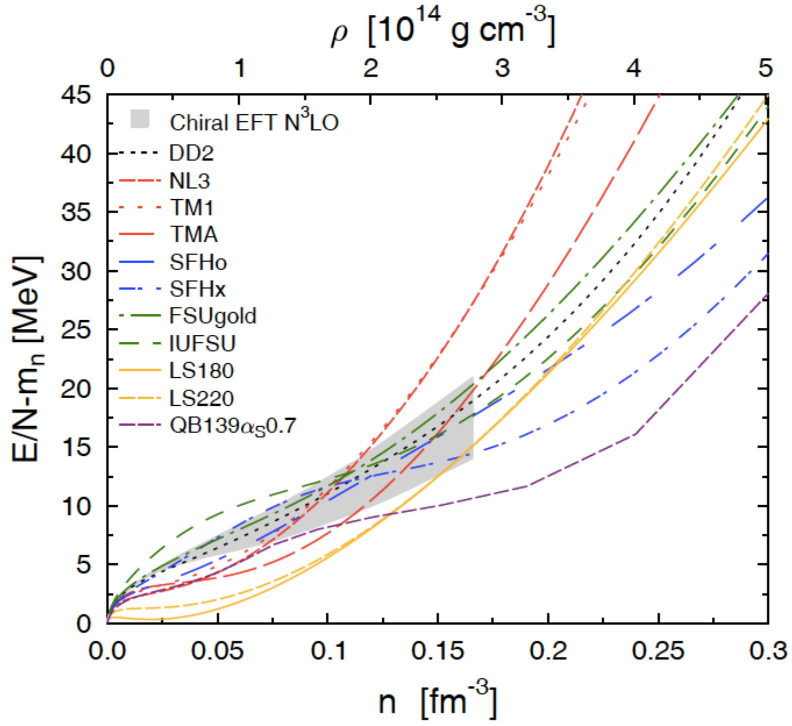
- In nuclei, neutron densities are equally important as charge densities, but more difficult to assess

- The thickness of the neutron skin $r_{np} = r_n - r_p$ depends on the pressure of neutron matter $P_{PNM} \sim L$: the size of r_{np} increases with pressure as neutrons are pushed out against surface tension

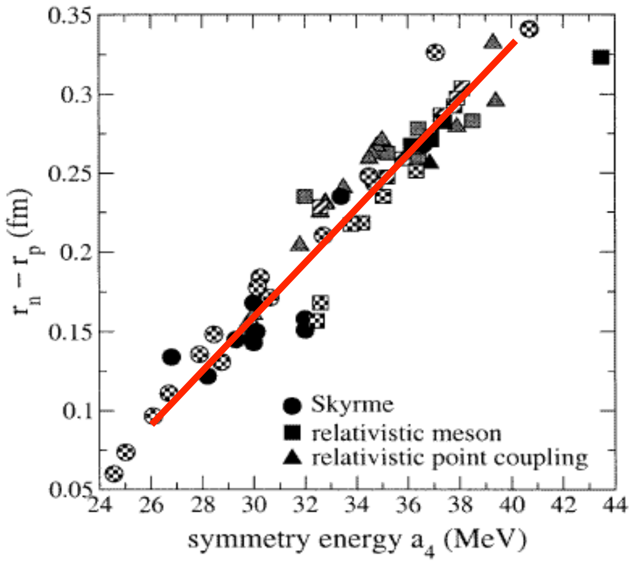
- The same pressure supports a neutron star against gravity (models with thicker neutron skins - neutron stars with larger radii)

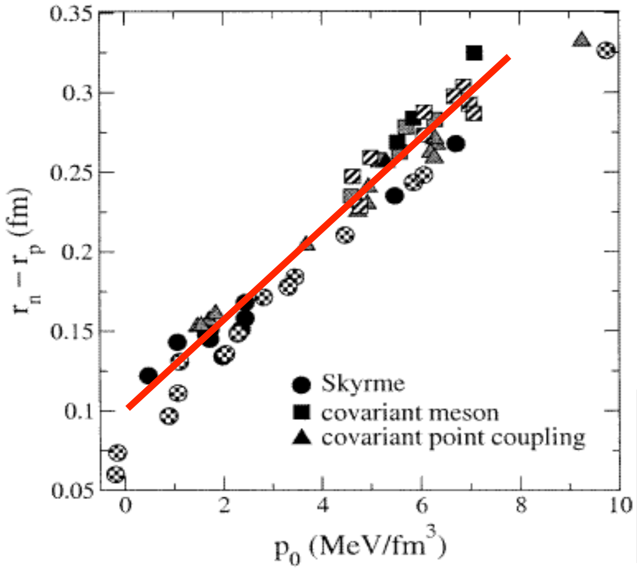
- The pressure of neutron matter $P_{PNM} \sim L$ is poorly constrained

Large theoretical uncertainties in the energy per particle as a function of the density for pure neutron matter.



T. Fischer et al. EPJ A 50, 46 (2014).

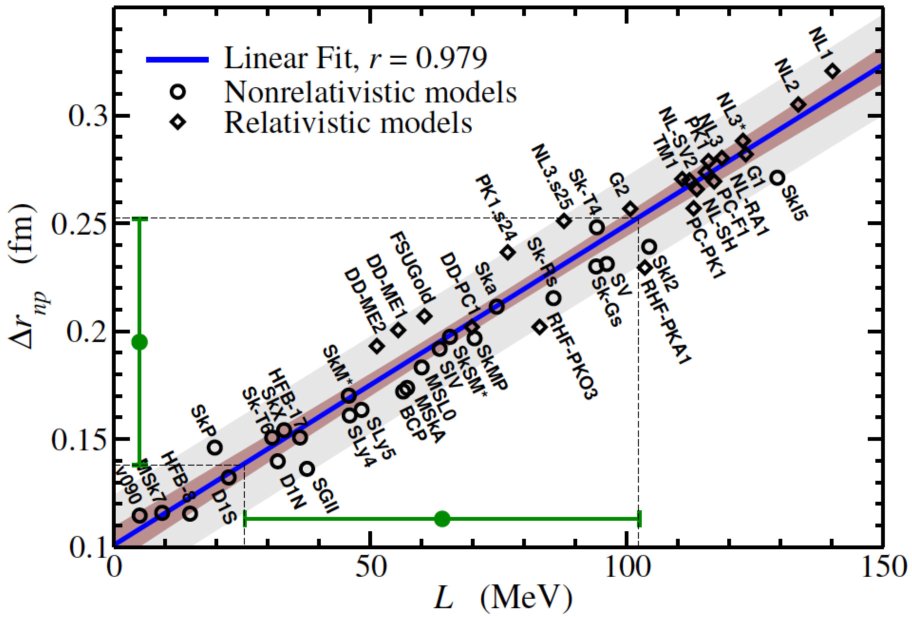




R.J.Furnstahl
Nucl. Phys. A 706, 85 (2002)

- strong <u>linear</u> correlation between neutron skin thickness and parameters **J (a_4), L (p_0)**

^{208}Pb



X. Roca-Maza et al.,
Phys. Rev. Lett. 106, 252501 (2011)

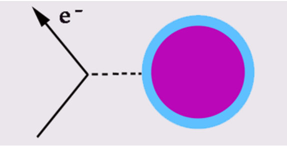
NEUTRON SKINS AND DENSITY DEPENDENCE OF THE SYMMETRY ENERGY

- An accurate measurement of the neutron radius / neutron skin thickness in ^{208}Pb may have important implications for understanding the symmetry energy and the properties of neutron stars

- Abrahamyan et al. PRL 108, 112502 (2012) parity violating electron scattering

Lead Radius Experiment (PREx) @ JLab

$$R_n - R_p = 0.33^{+0.16}_{-0.18}$$



- From nuclear collective motion:

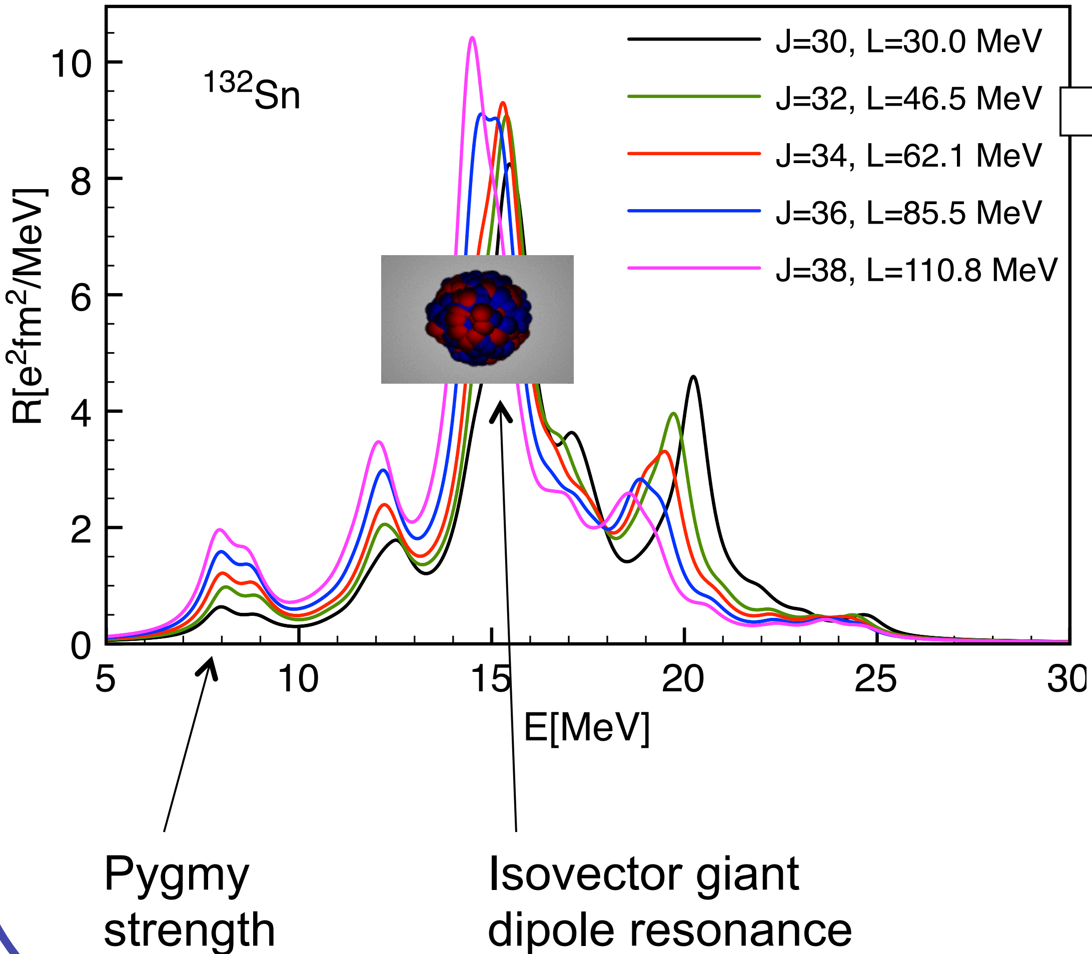
Various modes of excitation provide constraints on the neutron skin thickness, e.g.

  - Pygmy dipole resonances: A. Carbone et al., PRC 81, 041301(R) (2010)
    A. Klimkiewitz et al., PRC 76, 051603(R) (2007)
  - Dipole polarizability: A. Tamii et al., PRL 107, 062502 (2011)
    D.M. Rossi et al., PRL 111, 242503 (2013)
  - Anti-analog GDR: A. Krasznahorkay et al., PLB 720, 428 (2013)
  - Quadrupole resonances: S.S. Henshaw, M.W. Ahmed, G. Feldman et al, PRL 107, 222501 (2011)
  - …

- Other approaches: pion photoproduction Taubert et al. PRL 112, 242502 (2014), etc.

CONSTRAINING THE SYMMETRY ENERGY

- Isovector dipole transition strength – calculations are based on the same set of energy density functionals which vary the symmetry energy properties



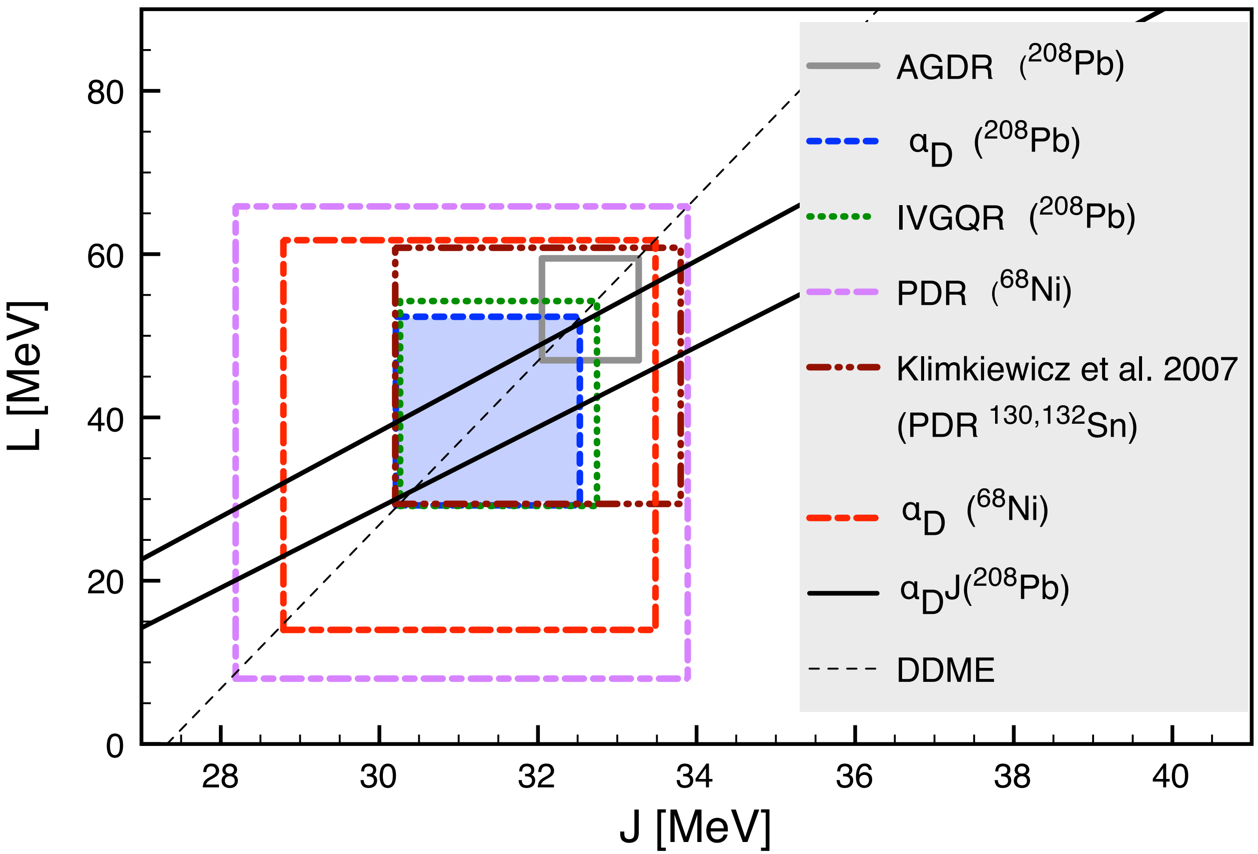
- Isovector giant dipole resonance
- Pygmy dipole strengths
- Dipole polarizability ($\alpha_D \sim m_{-1}$)

- The transition strength is sensitive on the properties of symmetry energy (J,L)

➔ Dipole response can be used to constrain effective nuclear interactions (isovector channel)

- There are exp. data available on the dipole response in nuclei !

CONSTRAINING THE SYMMETRY ENERGY



- The same set of DD-ME interactions used in the analysis based of various giant resonances and pygmy strengths (consistent theory !)
- Excellent agreement, except for the AGDR – new measurements are needed for the AGDR

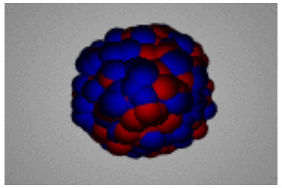
Exp. data for various excitations:

- α_D (^{208}Pb) ➔ A. Tamii et al., PRL 107, 062502 (2011) – update A. Tamii et al. (2015) (no q-deuteron).
- α_D (^{68}Ni) ➔ K. Boretzky, D. Rossi, T. Aumann, et al., (2015).
- PDR (^{68}Ni) ➔ O. Wieland, A. Bracco, F. Camera et al., PRL 102, 092502 (2009).
- (^{130,132}Sn) A. Klimkiewicz et al., PRC 76, 051603(R) (2007).
- IVGQR (^{208}Pb) ➔ S. S. Henshaw, M. W. Ahmed G. Feldman et al, PRL 107, 222501 (2011).
- AGDR (^{208}Pb) ➔ A. Krasznahorkay et al., arXiv:1311.1456 (2013)

THE NUCLEAR MATTER INCOMPRESSIBILITY

- Nuclear matter incompressibility $K_{nm} = 9\rho_0^2 \frac{d^2}{d\rho^2}\frac{E}{A}|_{\rho=\rho_0}$

ISGMR



- It can be determined from the energies of compression mode in nuclei: Isoscalar Giant Monopole Resonance (ISGMR)

- ISGMR energies are extracted from inelastic scattering of α-particles

$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$ (for nuclei)

- Strategy to reach K_{nm} (nuclear matter) :

1) Build the energy density functional (EDF), each parameterization corresponds to a K_{nm}

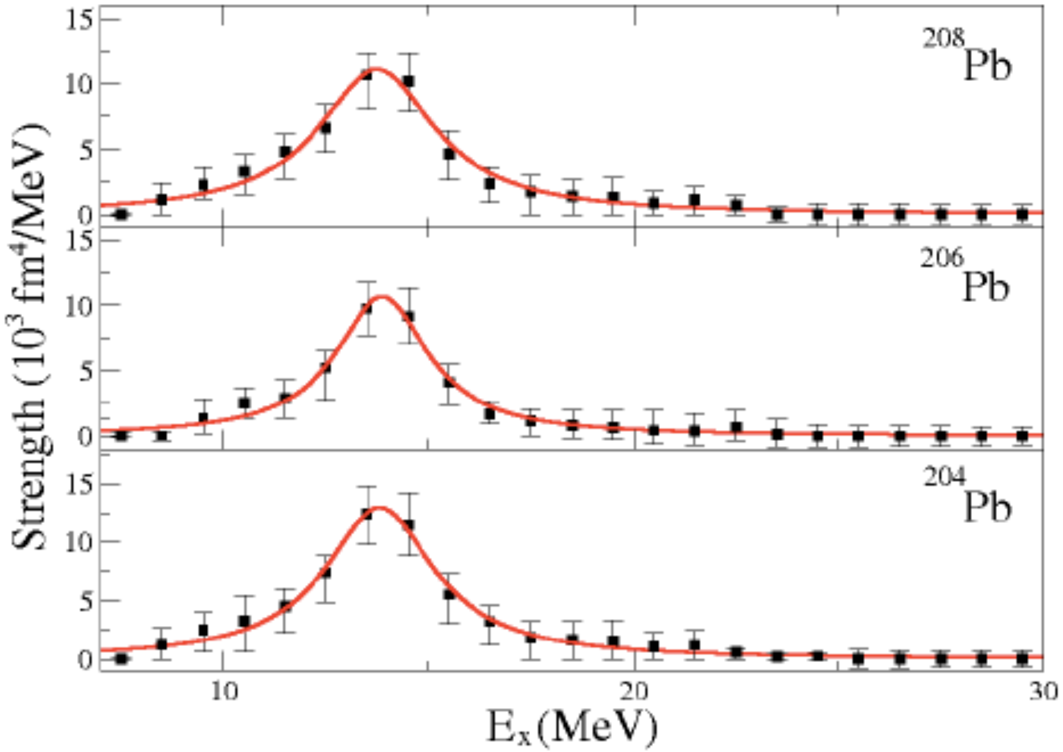
2) Calculate ISGMR excitation energy using the same EDF (e.g., RPA)

3) The K_{nm} value associated with the EDF that best describes the experimental ISGMR energy is considered as the "correct" one.

THE NUCLEAR MATTER INCOMPRESIBILITY

Using inelastic α scattering the strength distributions of the isoscalar giant monopole resonances (ISGMR) have been measured in nuclei

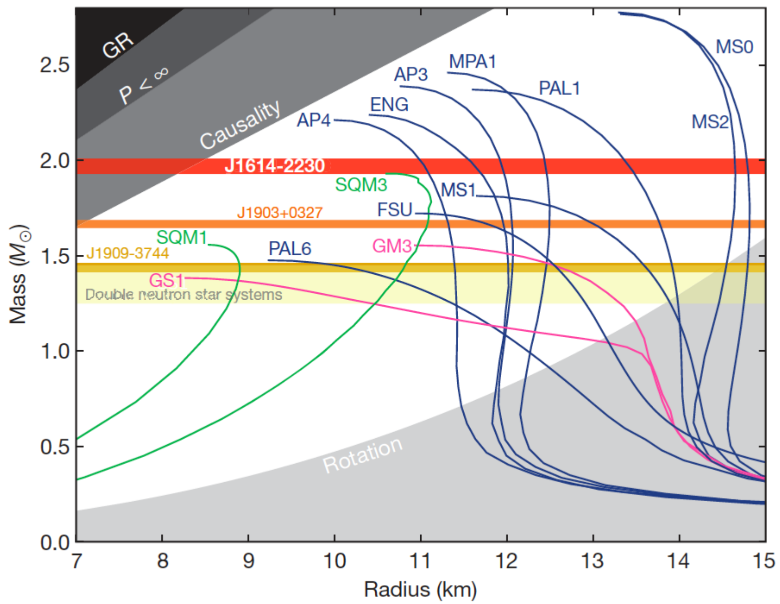
e.g., D. Patel et al., Phys. Lett. B 726, 178 (2013)



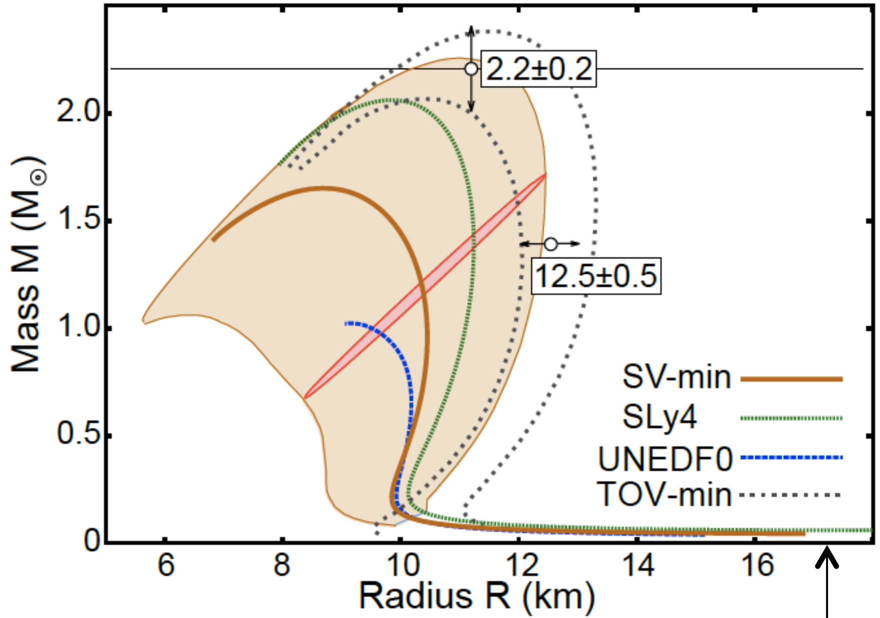
Target	E_{ISGMR} (MeV)				$\Gamma_{ISGMR}(MeV)$	$\sqrt{m_1/m_{-1}}$ (MeV)		$E_{ISGMR}A^{1/3}$ (MeV)	
	This work	RCNP-U[a]	Texas A&M[b]	KVI[c]	This work	This work	Pairing+MEM[d]	This work	Pairing+MEM[d]
^{204}Pb	13.8±0.1	-	-	-	3.3±0.2	13.7±0.1	13.4	81.2±0.6	78.9
^{206}Pb	13.8±0.1	-	-	14.0 ±0.3	2.8±0.2	13.6±0.1	13.4	81.5±0.6	79.1
^{208}Pb	13.7±0.1	13.5±0.2	13.91±0.11	13.9±0.3	3.3±0.2	13.5±0.1	14.0	81.2±0.6	82.9

NEUTRON STAR PROPERTIES

- Mass-radius relations of cold neutron stars for different EOS – observational constraints on the neutron star mass rule out many models for EOS.



P. B. Demorest et al., Nature 467, 1081 (2010)



- Building EDFs for finite nuclei and neutron stars

  Wei-Chia Chen and J. Piekarewicz, PRC 90, 044305 (2014).

  J. Erler. C.J. Horowitz, W. Nazarewicz et al., PRC 87, 044320 (2013).

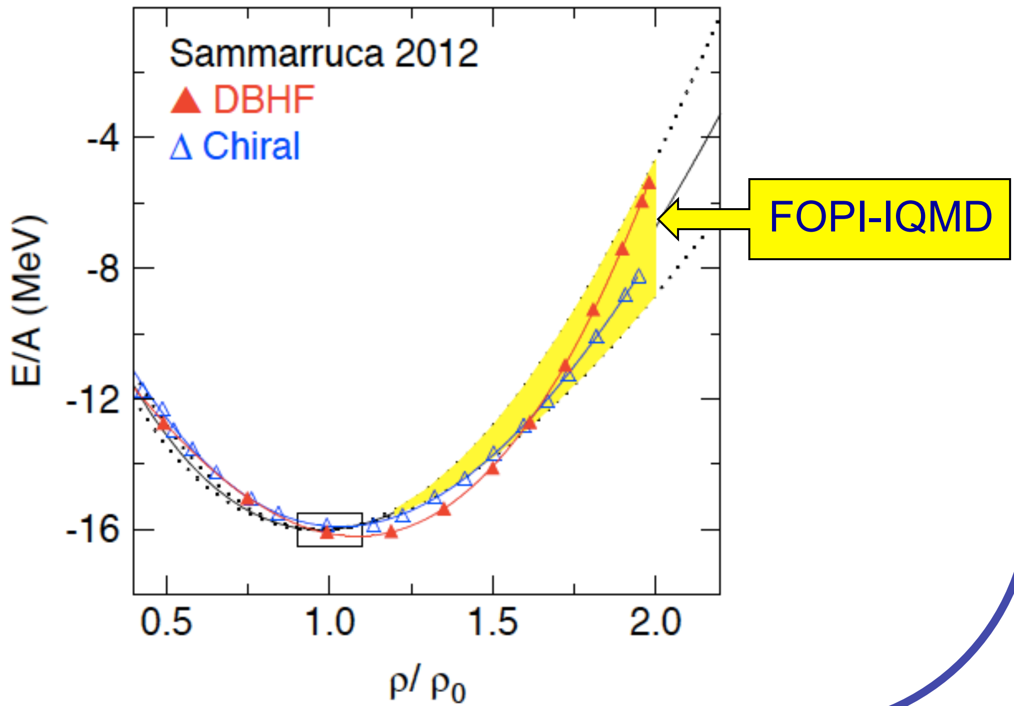
- Constraints on the maximal neutron star mass from observation:

  J. Antoniadis, P. C. C. Freire, N. Wex et al. Science 340, 448 (2013) ➔ 2.01(4) Msun

  P. B. Demorest et al., Nature 467, 1081 (2010) ➔1.97(4) Msun

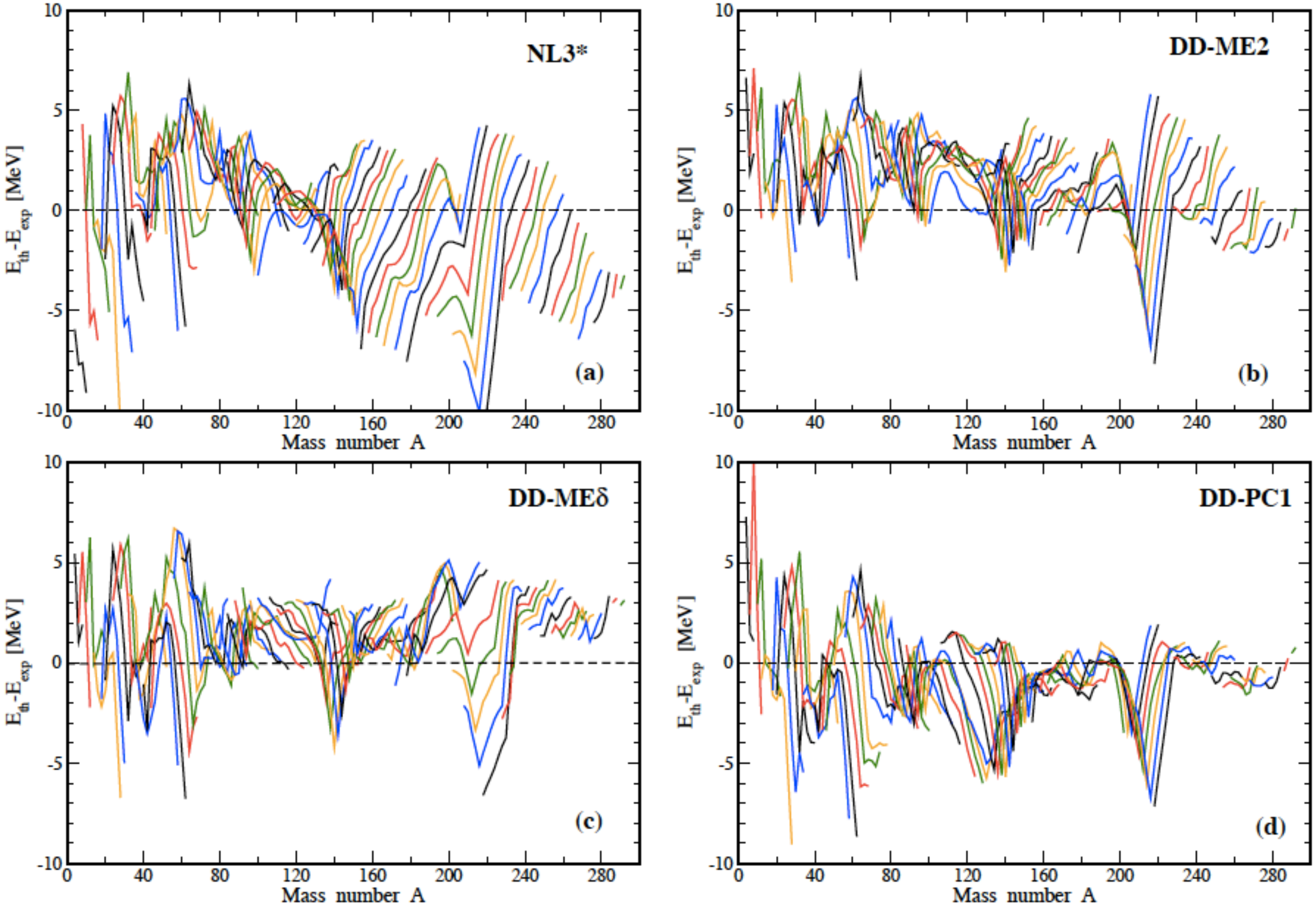
CONSTRAINTS ON THE NUCLEAR EOS BEYOND SATURATION

- The knowledge on the nuclear matter equation of state (EOS) beyond the saturation density ρ_0 is limited
- Some constraints on the EOS are possible from heavy ion collisions
- The FOPI (GSI) detector data on elliptic flow in Au+Au collisions between 0.4 and 1.5A GeV were used to establish empirical constraints on the nuclear EOS
  A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, Ch. Hartnack, arXiv:1501.05246 (2015).
- FOPI-IQMD (transport code) provides limits to the symmetric nuclear matter EOS up to $2\rho_0$



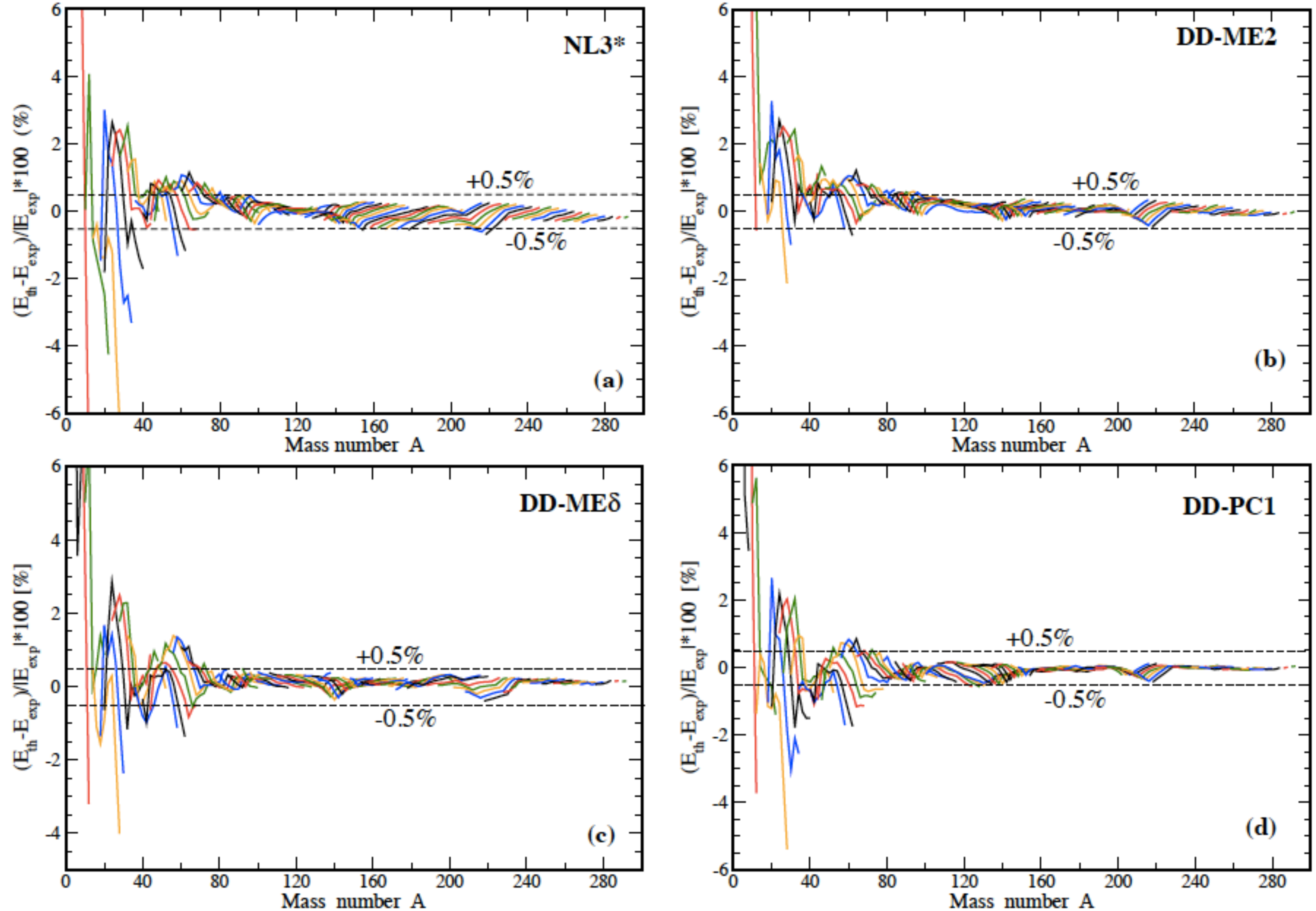
NUCLEAR MASSES

- Description of experimental data on masses: DD-ME2, DD-PC1, DD-MEδ, NL3*



S. E. Agbemava et al., PRC 89, 054320 (2014)

- Relative accuracies in description of the masses

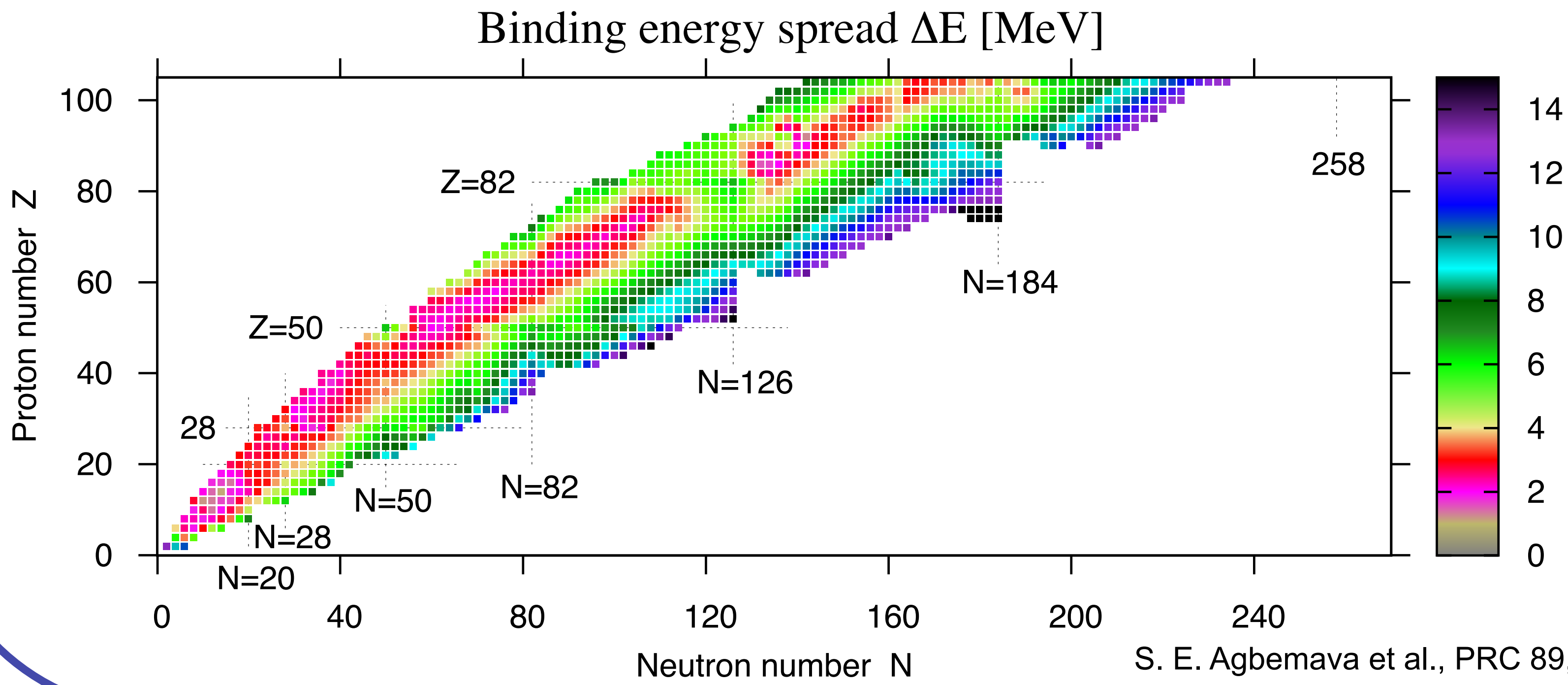


S. E. Agbemava et al., PRC 89, 054320 (2014)

Systematical model uncertainties – limitations of the model, deficient parametrizations, wrong assumptions, and missing physics due to our lack of knowledge.

- These uncertainties are difficult to estimate – systematic errors can be estimated e.g., by exploring the spread - difference between maximal and minimal values obtained for a set of EDFs: DD-ME2, DD-PC1, DD-MEδ, NL3*

$$\Delta E(Z,N) = |E_{max}(Z,N) - E_{min}(Z,N)|$$



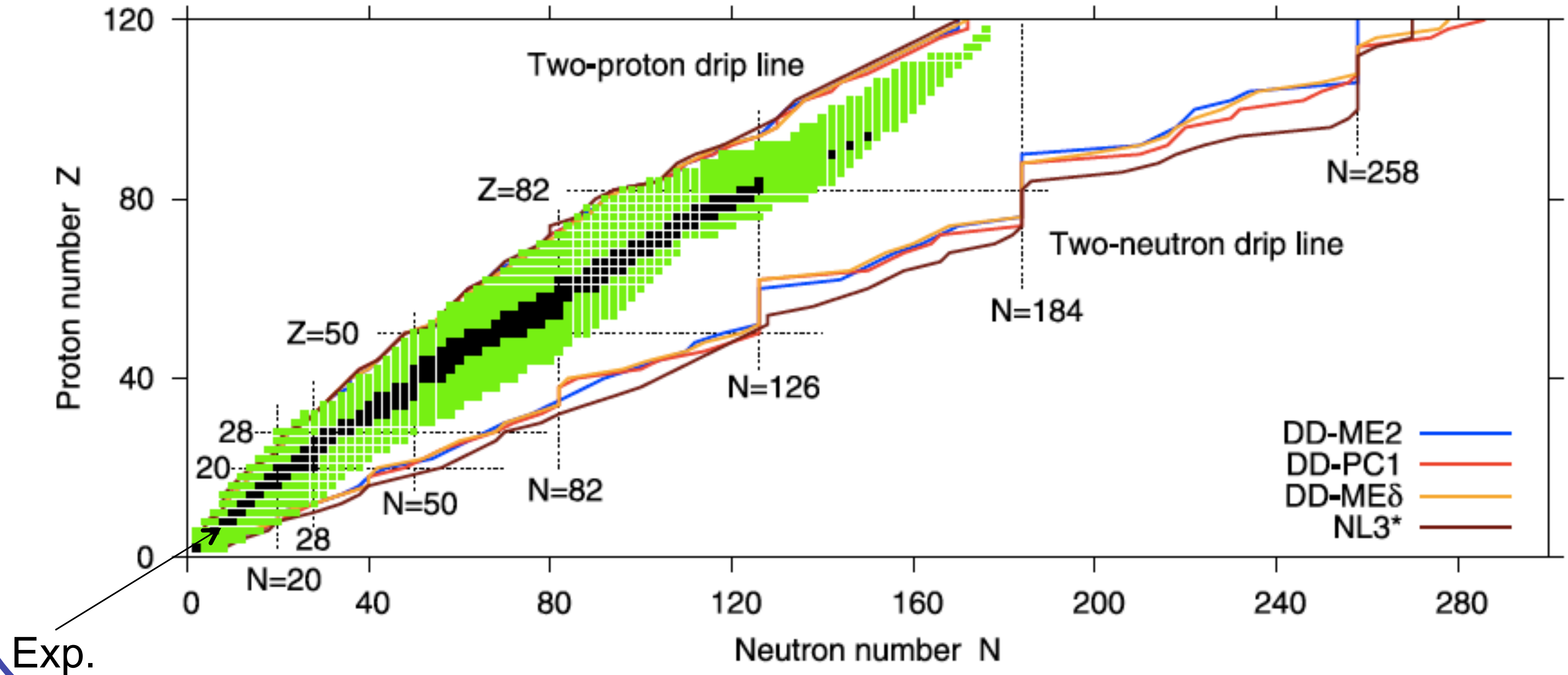
S. E. Agbemava et al., PRC 89, 054320 (2014)

NUCLEAR LANDSCAPE WITH RELATIVISTIC EDFs

- Limits of nuclear landscape – two-proton and two-neutron drip lines calculated with different relativistic EDFs: DD-ME2, DD-PC1, DD-MEδ, NL3*

$$S_{2n} = B(Z, N-2) - B(Z, N)$$
$$S_{2p} = B(Z-2, N) - B(Z, N)$$

drip lines: $\begin{cases} S_{2n} \leq 0 \\ S_{2p} \leq 0 \end{cases}$

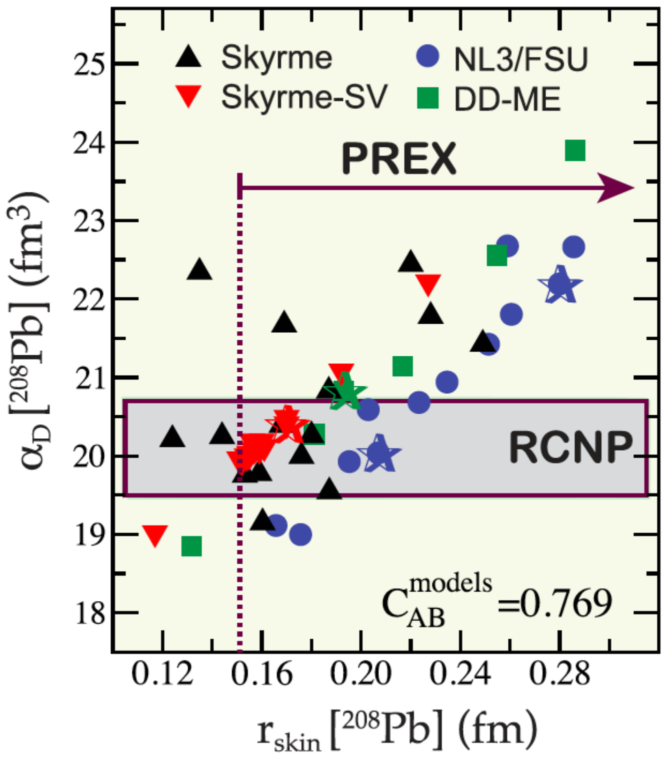


A. V. Afanasjev et al., PLB 726, 680 (2013)

SYSTEMATIC UNCERTAINTIES : NEUTRON SKIN THICKNESS

- Electric dipole polarizability α_D and neutron skin thickness (r_{skin}) for ^{208}Pb using both nonrelativistic and relativistic EDFs:

$$\alpha_D = \frac{8\pi}{9} e^2 m_{-1}$$



By using only models consistent with measured α_D (48 EDFs → 25 EDFs), systematic model uncertainty in r_{skin} is reduced.

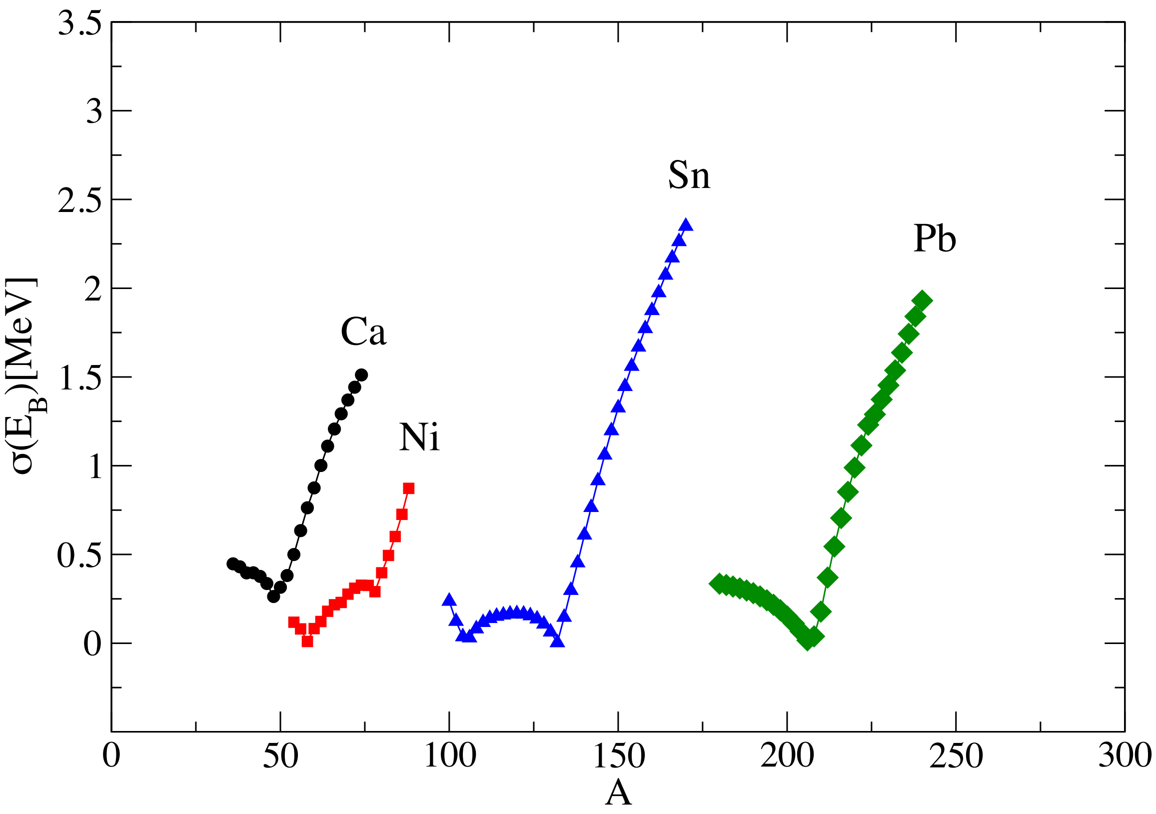
Model averaged value:

$r_{skin}(^{208}\text{Pb}) = (0.168 \pm 0.022)$ fm

J. Piekarewicz,et al., PRC 85, 041302 (R) (2012)

STATISTICAL UNCERTAINTIES IN THE NUCLEAR BINDING ENERGIES

- The evolution of statistical uncertainties of the nuclear binding energies within isotope chains (RNEDF1)



- The quality of X^2 minimization of the EDF parameters to exp. data is an indicator of the statistical uncertainty

- curvature matrix

$$\mathcal{M}_{ij} = \frac{1}{2}\partial_{p_i}\partial_{p_j}\chi^2|_{\mathbf{p}_0}$$

- Constraint on the dipole polarizability / symmetry energy improves the isovector properties of the EDF

Towards a universal relativistic nuclear energy density functional for astrophysical applications – RNEDF1 (N.P., M. Hempel et al. 2016)

The strategy to constrain the functional (relativistic point coupling model)

- Adjust the properties of 72 spherical nuclei to exp. data (binding energies (Δ=1 MeV), charge radii (0.02 fm), diffraction radii (0.05 fm), surface thickness (0.05 fm))
- Improve description of open-shell nuclei by adjusting the pairing strength parameters to empirical paring gaps (n,p) (0.14 MeV)
- constrain the symmetry energy $S_2(\rho_0)=J$ (2%) from exp. data on dipole polarizability (^{208}Pb) A. Tamii et al., PRL 107, 062502 (2011) + update (2015).
- constrain the nuclear matter incompressibility K_{nm} (2%) from exp. data on ISGMR modes (^{208}Pb); D. Patel et al., PLB 726, 178 (2013).

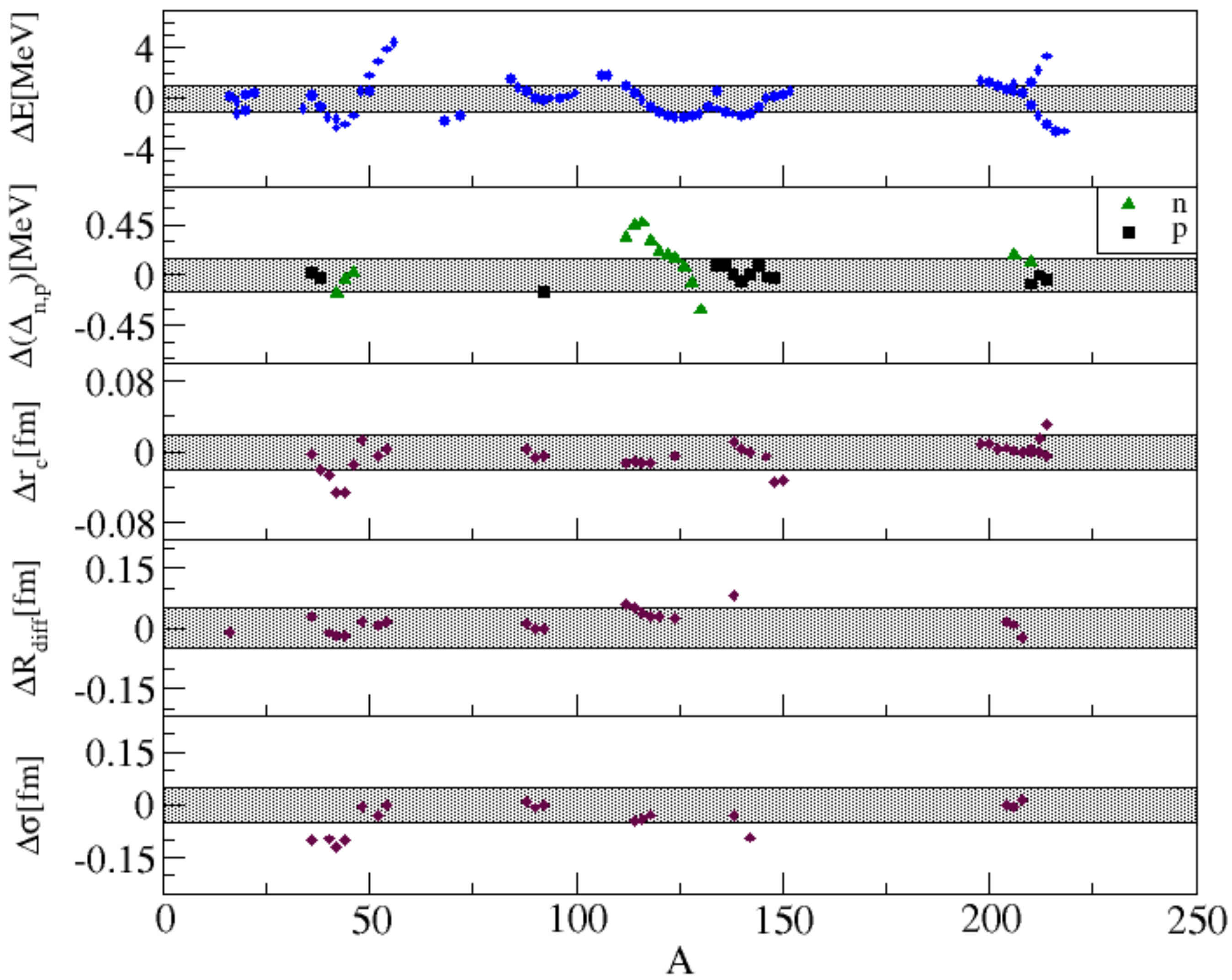
… the strategy to constrain the functional

- constrain the equation of state using the saturation point (ρ_0) and point at twice the saturation density ($2\rho_0$) from heavy ion collisions (FOPI-IQMD) (10%)
  A. Le Fevre et al., arXiv:1501.05246v1 (2015)

- constrain the maximal neutron star mass by solving the Tolman-Oppenheimer-Volkov (TOV) equations and using observational data (slightly larger value $M_{max}=2.2M_{\odot}$(5%); J. Erler. et al., PRC 87, 044320 (2013).)
  J. Antoniadis, et al. Science 340, 448 (2013); P. B. Demorest et al., Nature 467, 1081 (2010)

- the fitting protocol is supplemented by the covariance analysis
  – calculation of the curvature matrix, correlations, statistical uncertainties

RNEDF1: DEVIATIONS FROM THE EXP. DATA

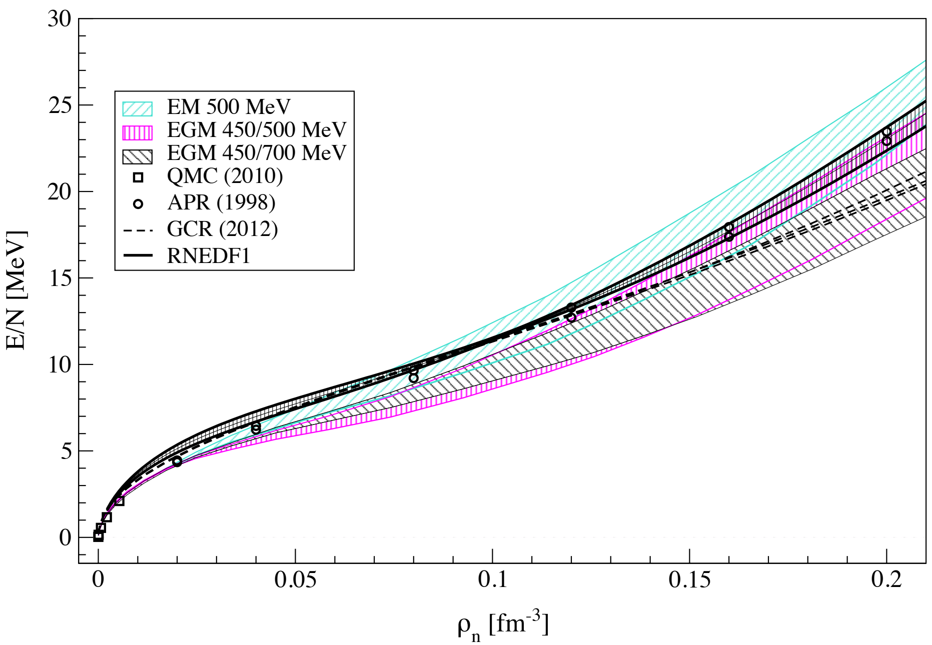


- binding energies
- pairing gaps
- charge radii
- diffraction radii
- surface thickness

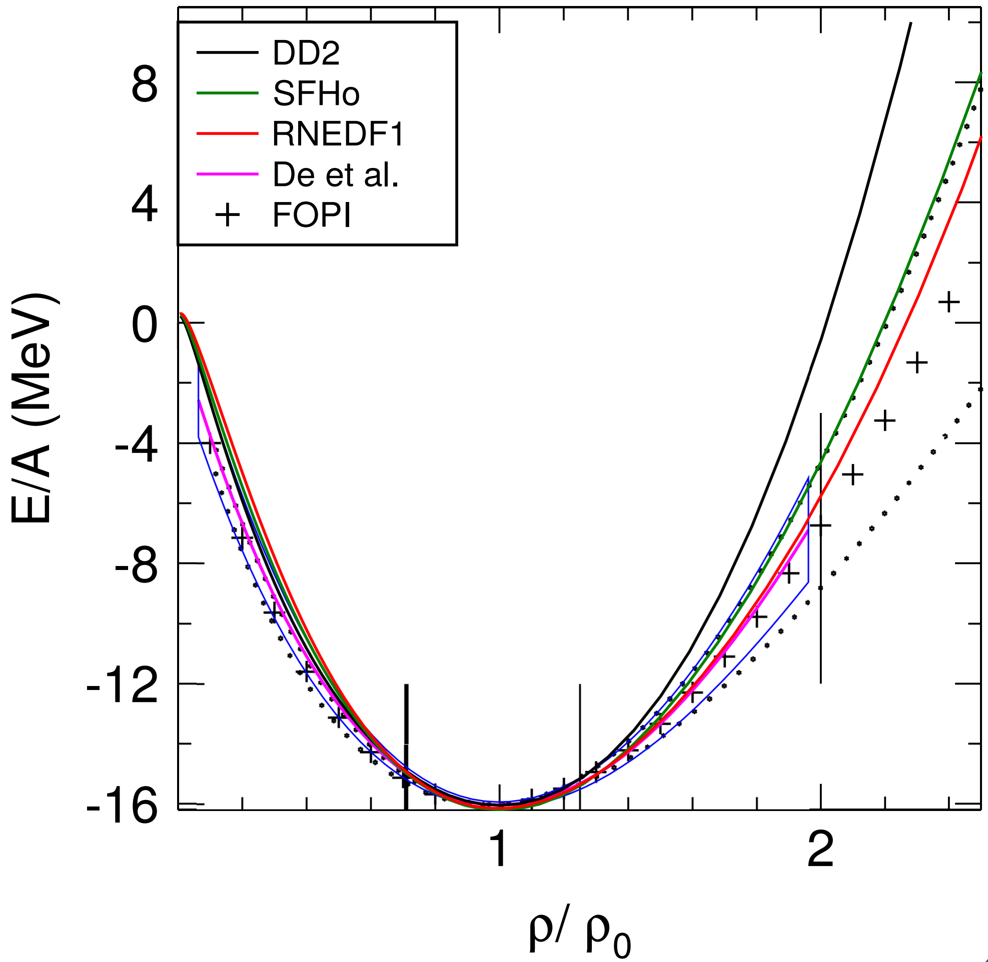
RNEDF1: NUCLEAR MATTER PROPERTIES

- Neutron matter - comparison

RNEDF1 vs. chiral EFT (I. Tews et al. 2013)

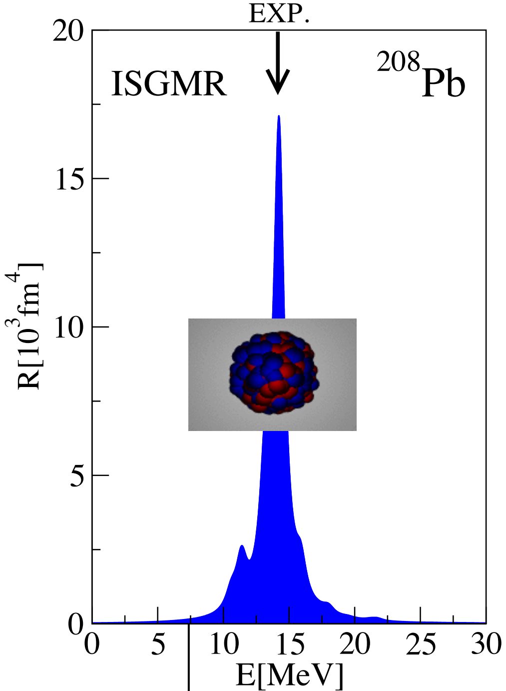


SYMMETRIC NUCLEAR MATTER

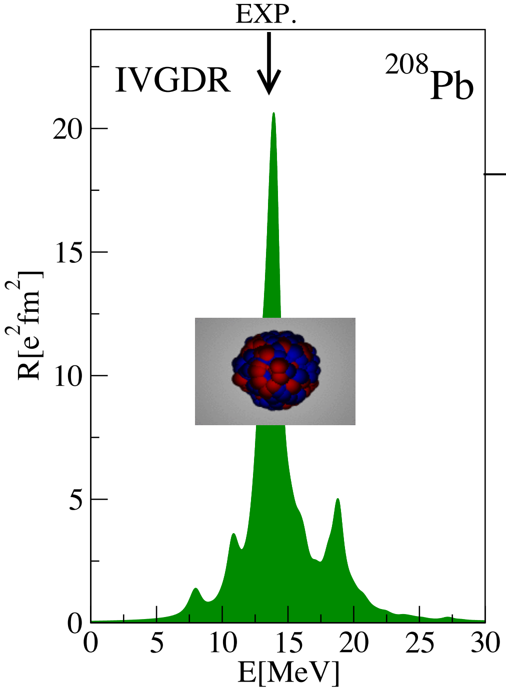


RNEDF1: COMPRESSIBILITY, SYMMETRY ENERGY

ISOSCALAR GIANT MONOPOLE RESONANCE



ISOVECTOR GIANT DIPOLE RESONANCE



Dipole polarizability:
α_D= (19.68 ± 0.21) fm^3

Exp.
α_D= (19.6 ± 0.6) fm^3

A.Tamii et al., PRL 107, 062502 (2011). + update (2015).

- ISGMR energy determines the nuclear matter incompressibility: **K_{nm}=232.4 MeV**

- IVGDR – α_D constrains the symmetry energy of the EDF

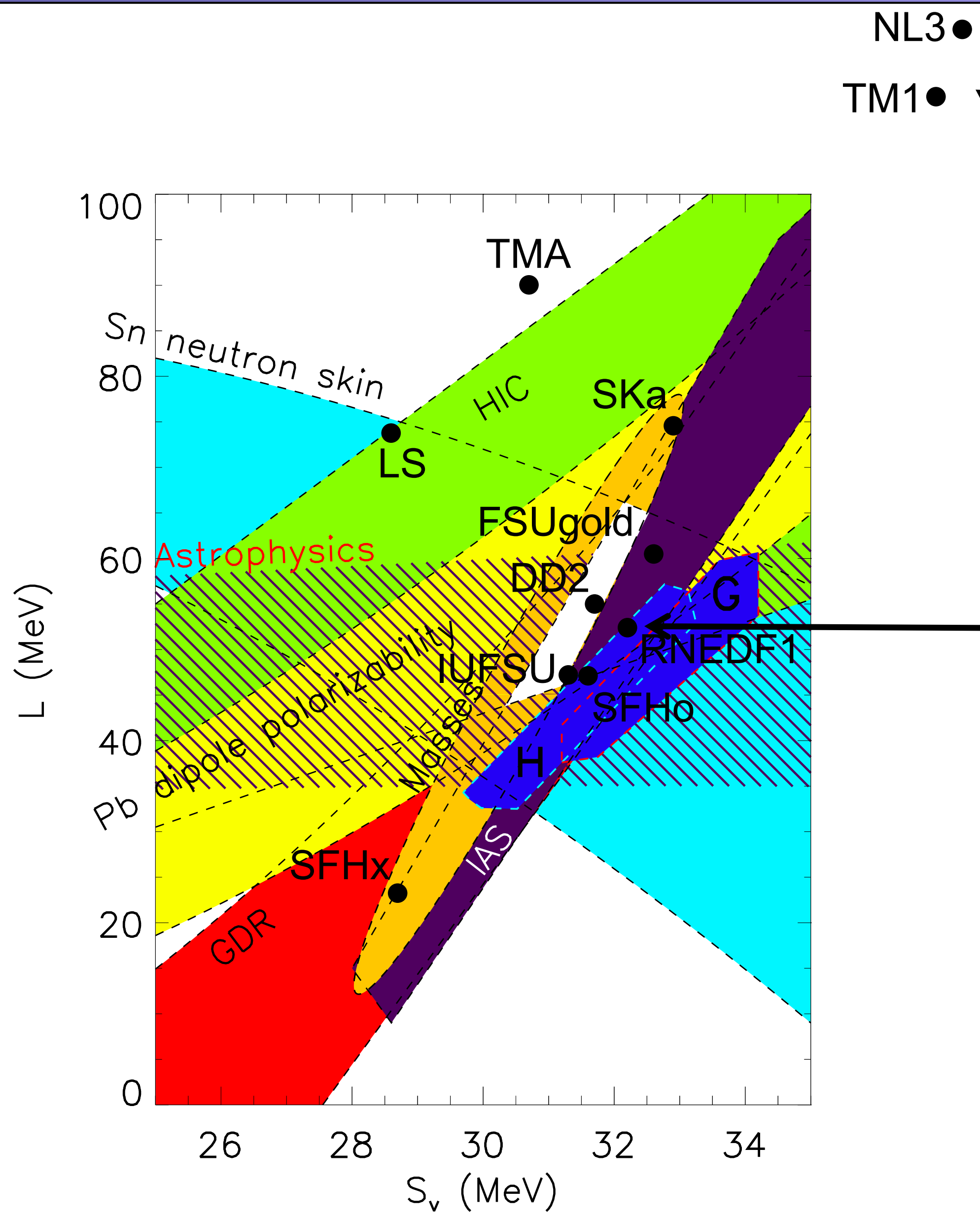
J = 31.89 MeV
L = 51.48 MeV

ISGMR:
E (Exp.) = (13.91 ± 0.11) MeV (TAMU)
E (Exp.) = (13.7 ± 0.1) MeV (RCNP)

- Lattimer & Lim, ApJ. 771, 51 (2013)

J = 29.0–32.7 MeV
L = 40.5–61.9 MeV

THE SYMMETRY ENERGY



- Many functionals are inconsistent with the observational constraints

- RNEDF1:

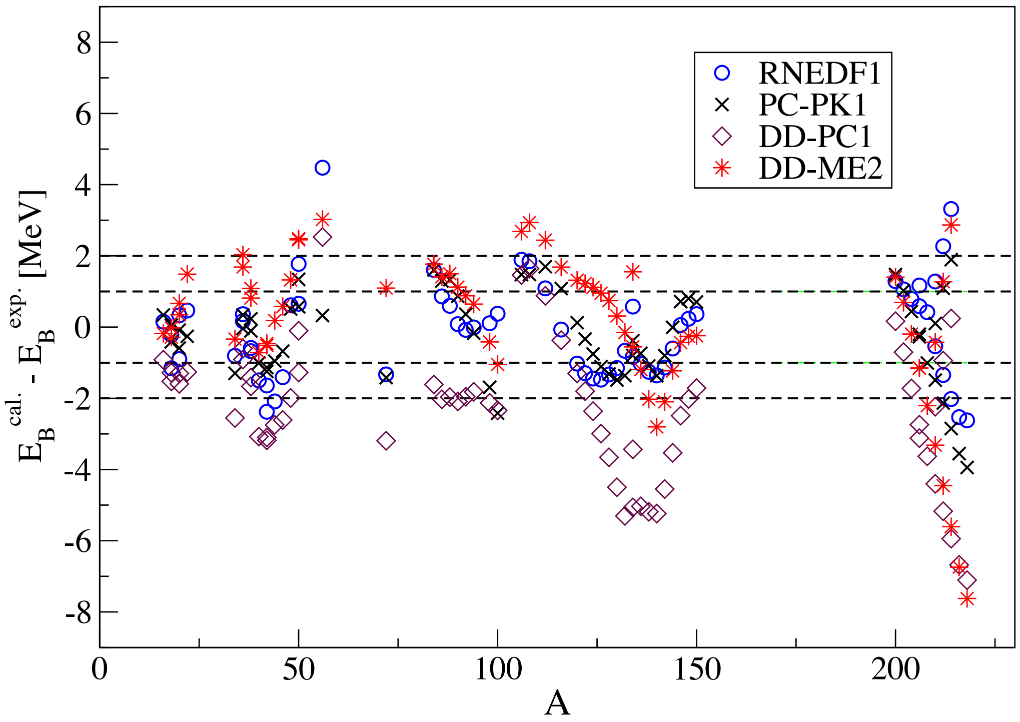
  J = 31.89 MeV
  L = 51.48 MeV

- Lattimer & Lim, ApJ. 771, 51 (2013)

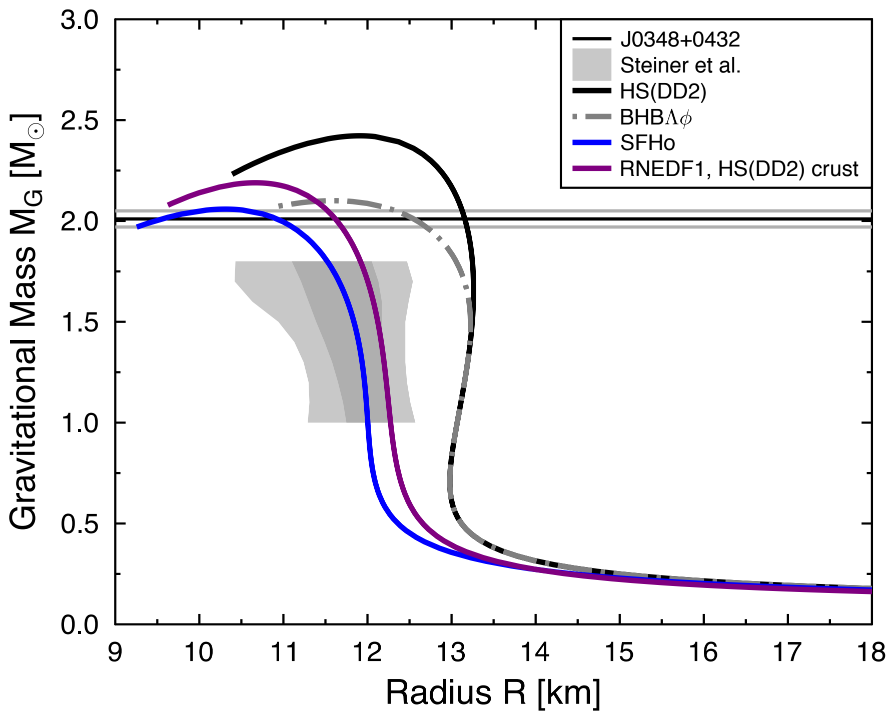
  J = 29.0–32.7 MeV
  L = 40.5–61.9 MeV

MASSES: FROM FINITE NUCLEI TOWARD THE NEUTRON STAR

Nuclear binding energies (calc. – exp.)
- Including deformation (axial symmetry)

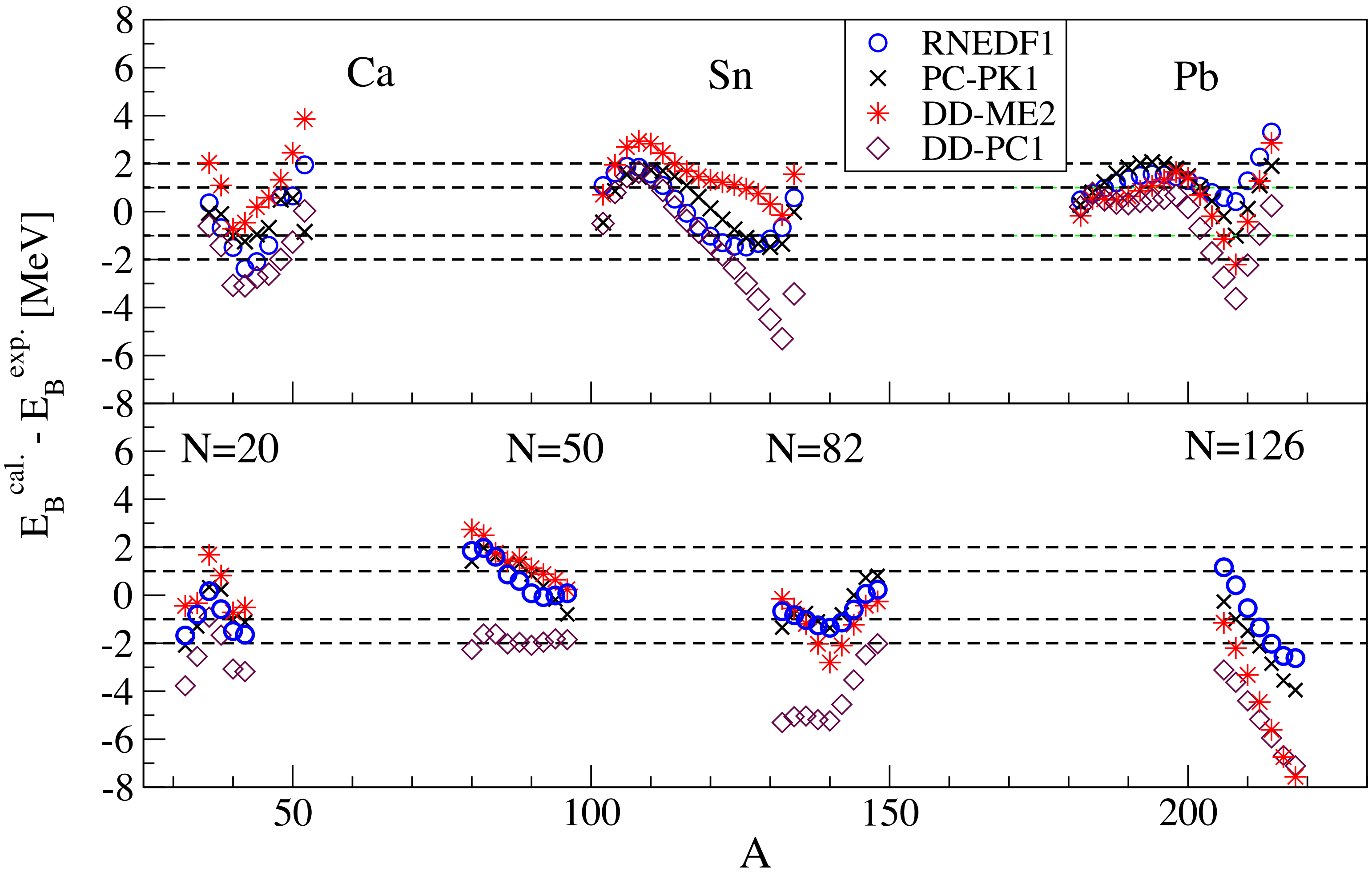


Neutron star
mass-radius relationship

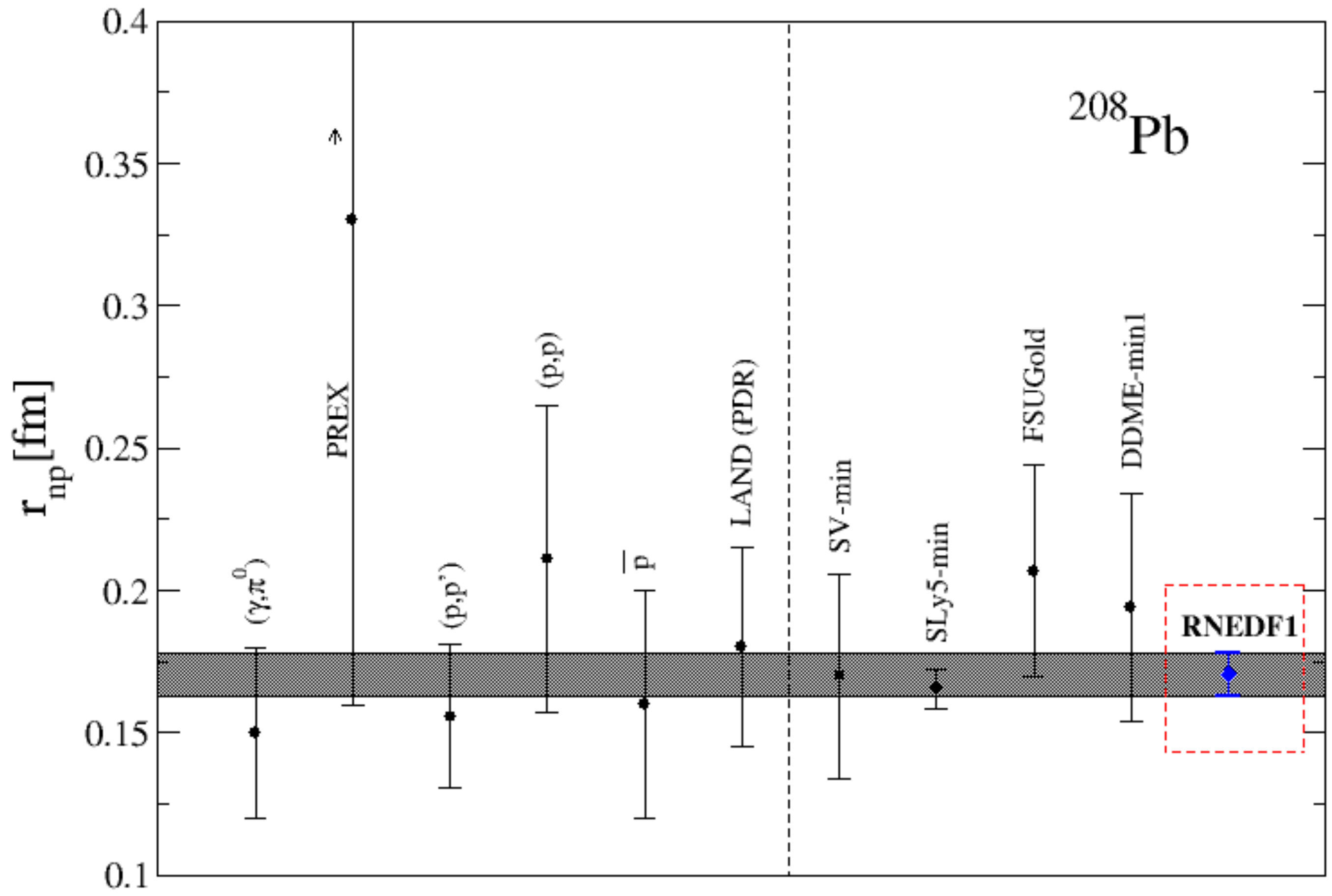


- PC-PK1: P.W.Zhao, Z. P. Li, J. M. Yao, and J. Meng, PRC 82, 054319 (2010)
  - (K=238 MeV, J=35.6, L = 113 MeV)

RNEDF1: ISOTOPE AND ISOTONE CHAINS



RNEDF1: NEUTRON SKIN THICKNESS IN ^{208}Pb



(γ,π^0): C.M. Tarbert et al., PRL 112, 242502 (2014)
PREX: S. Abrahamyan et al., PRL. 108, 112502 (2012)
(p,p'): A. Tamii et al., PRL 107, 062502 (2011)
(p,p) J. Zenihiro et al., PRC 82, 044611 (2010)
Antipr. at.: B. Kłos et al., Phys. Rev. C 76, 014311 (2007).
LAND (PDR): A. Klimkiewicz et al., PRC 76, 051603 (2007).

SV-min: P.G. Reinhard et al.
SLy5-min: X. Roca-Maza, G. Colò et al.
FSUGold: J. Piekarewicz et al.
DDME-min1: N.P. et al.