

# Clustering in light neutron-rich nuclei

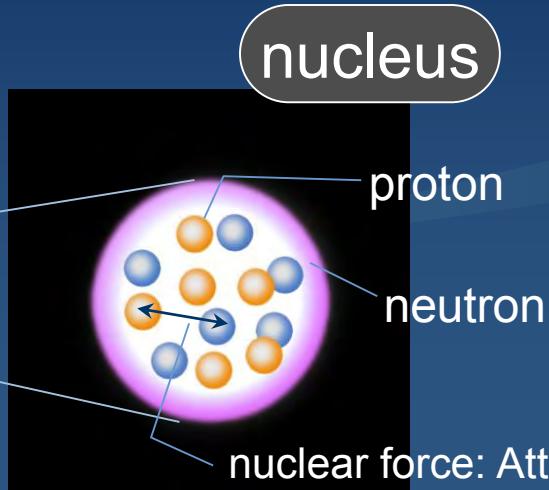
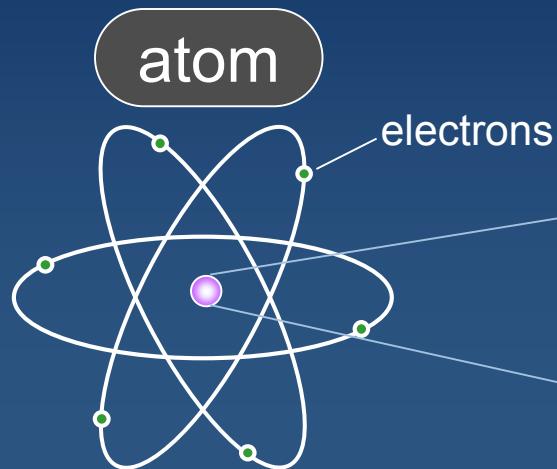
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T. Suhara (Matsue) ,Y. Taniguchi(NIMS),  
Y. Yoshida (Kyoto)

# 1. Introduction

# Nuclear system

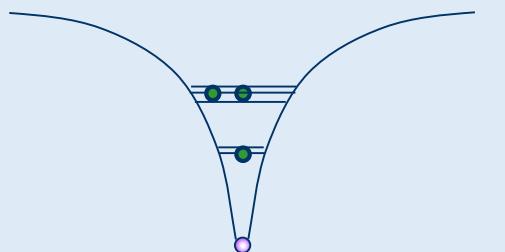


A finite quantum many-body system of protons and neutrons

## Analogy & Differences

### Electron motion

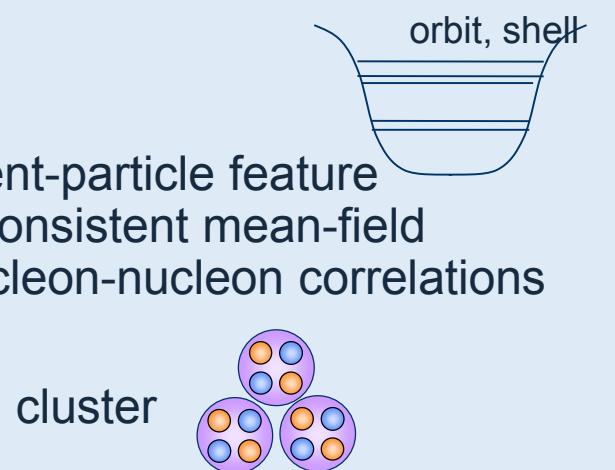
Confined by the external field



### Nucleon motion

Self-bound

1. Independent-particle feature in self-consistent mean-field
2. Strong nucleon-nucleon correlations



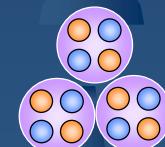
# Cluster & Mean field

Mean field, shell structure  
Independent single-particle



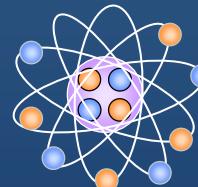
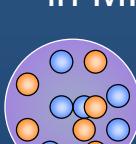
Shell structure · MF

Cluster:  
Many-body correlation



Cluster

Independent particle  
in MF



Cluster formation



many-body correlation

Cluster excitation



Developed cluster

no correlation

cluster correlation

excited states

# Cluster structures

Nuclear  
structure

$^{12}\text{C}$

high density

$\rho_0$

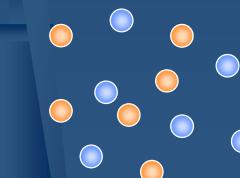
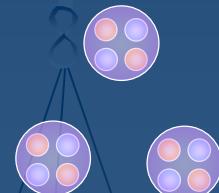
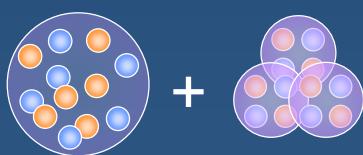
Nucleon liquid  
Fermi gas, BCS, DW

$\rho_0/3 \sim \rho_0/5$

low density

0 MeV                    10 MeV                    100 MeV      Energy

Shell & cluster corr.      cluster excitation



Nucleon gas

Cluster  
enhancement

# Cluster structures in stable and unstable nuclei

Typical cluster structures known in stable nuclei

**$^7\text{Li}$**



$\alpha + t$

**$^8\text{Be}$**



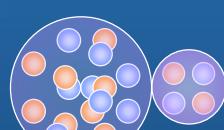
$\alpha + \alpha$

**$^{12}\text{C}$**



$3\alpha$

**$^{20}\text{Ne}$**



$^{16}\text{O} + \alpha$

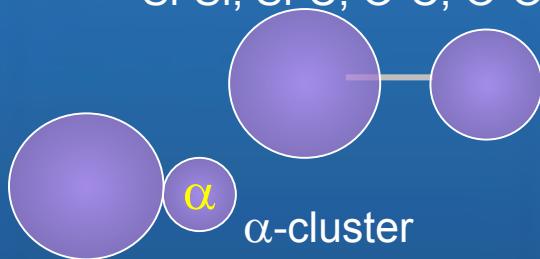
**$^{16}\text{O}^*$**



$^{12}\text{C} + \alpha$

## Heavier nuclei

Si-Si, Si-C, O-C, O-O



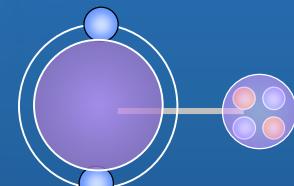
$\alpha$ -cluster

$^{36}\text{Ar}-\alpha$ ,  $^{24}\text{Mg}-\alpha$ ,  $^{28}\text{Si}-\alpha$

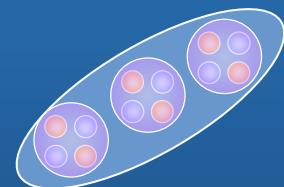
$^{40}\text{Ca}^*$ ,  $^{28}\text{Si}^*$ ,  $^{32}\text{S}^*$

## Unstable nuclei

$\alpha$ -cluster  
excitation

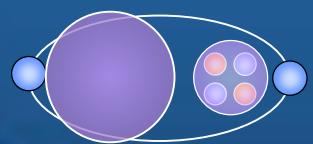


$3\alpha$  linear chain



$^{14}\text{C}^*$

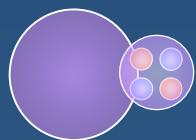
Molecular  
orbital



Be, C, O, Ne, F

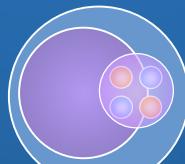
# Cluster structures in n-rich nuclei

gs cluster correlation



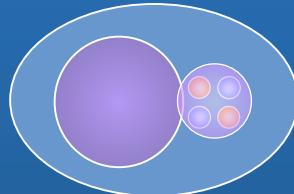
stable nuclei

cluster weakening/  
melting



n-rich nuclei

Clustering  
in deformed  
Neutron density



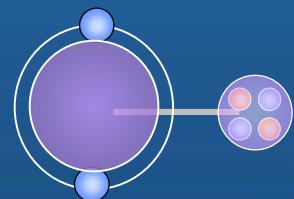
- ✓ n-skin suppression
- ✓ large deformation
- ✓ breaking of magic number  $N=8, N=20$

n-rich Be, F, Ne

Molecular orbital

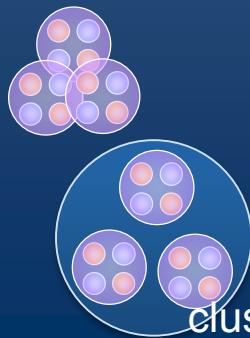


cluster excitation



n-rich C

linear chain  
in  $C^*$

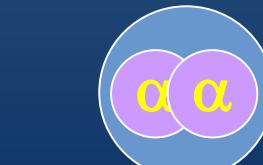
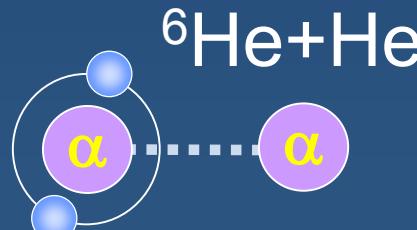


cluster gas

# Two modes in n-rich nuclei with cluster structures

$^{10}\text{Be}$

Soic et al., Freer et al., Saito et al.,  
Curtis et al., Milin et al., Bohlen et al.,  
Seya, Von Oerzten, Descouvemont et al.,  
Itagaki et al., Dote et al., K-E et al.  
Arai et al., M. Ito et al.



Be isotopes

Atomic:  
Cluster resonance

Molecular Orbital:  
 $\sigma$ -bond structure

Normal states

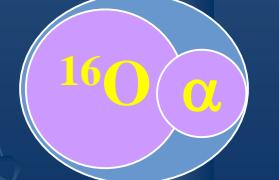
Scholz et al., Rogachev et al., Goldberg et al.,  
Ashwood et al., Yildiz et al.,  
Descouvemont, Kimura,



$^{18}\text{O} + \alpha$   
Weak coupling



Strong coupling



shell  
model-like

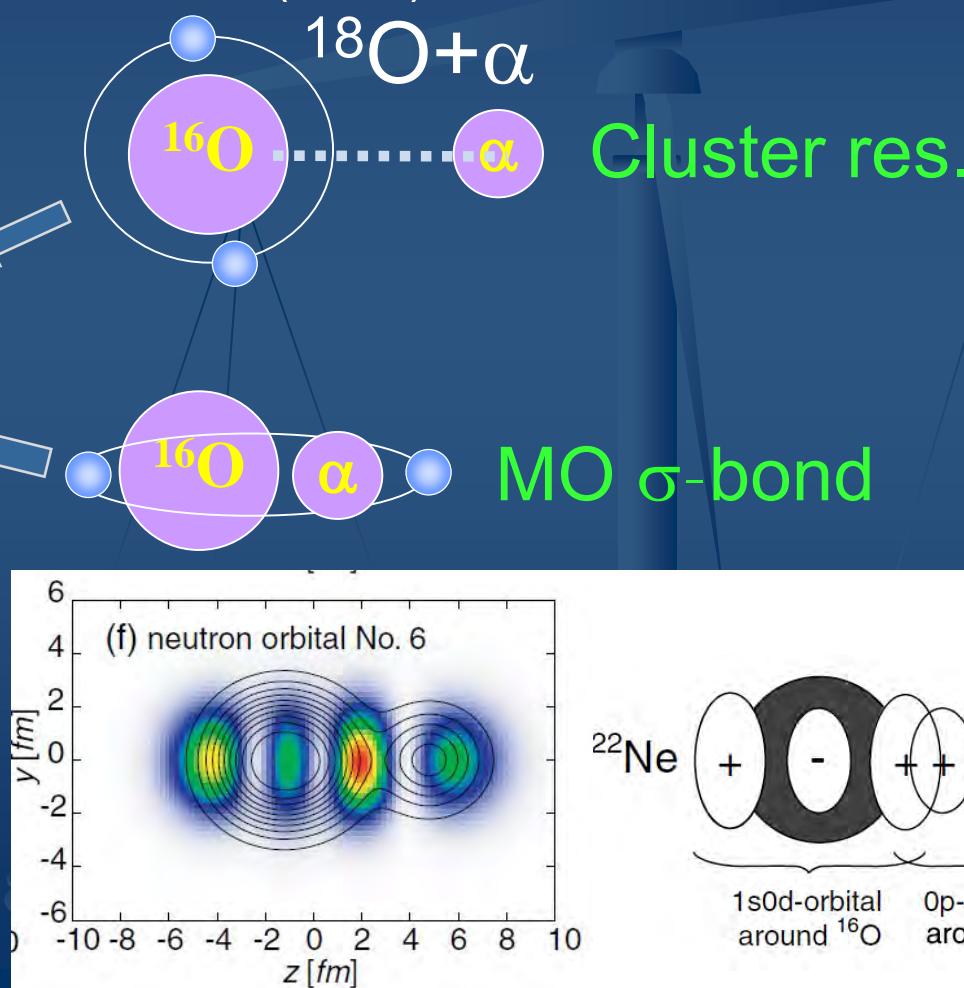
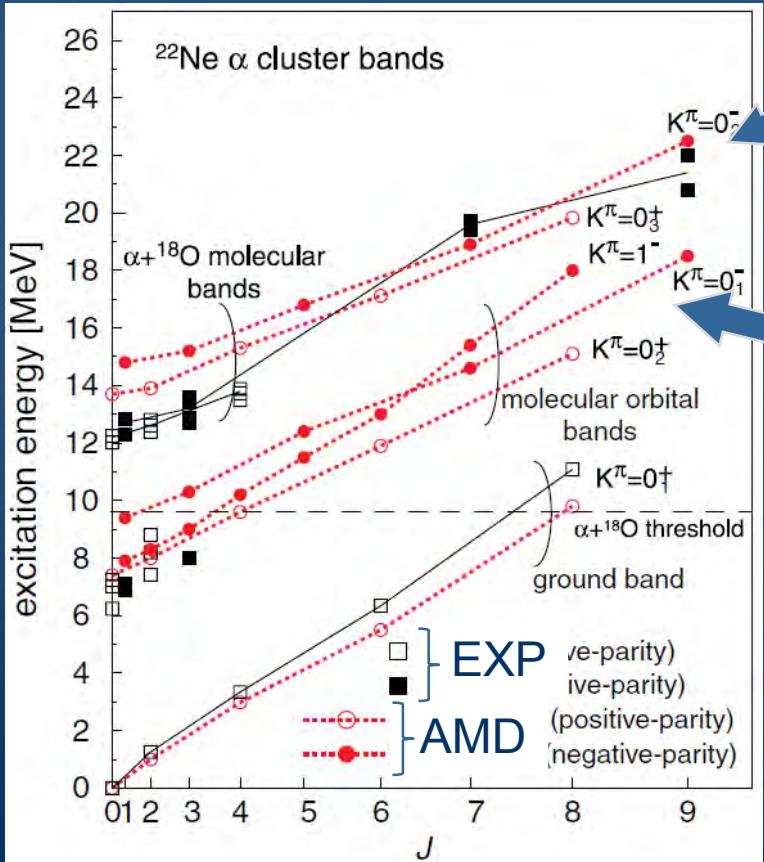
Ne, F, O isotopes

# MO bond and Cluster res. in $^{22}\text{Ne}$

Exp Scholz et al., Rogachev et al., Goldberg et al., Ashwood et al., Yildiz et al.,  
Theor: Descouvemont, Kimura,

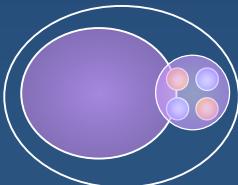
$^{22}\text{Ne}$

AMD study by Kimura, PRC75 (2007)

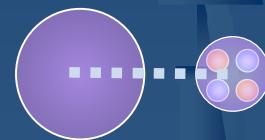


# Two kinds of cluster structure

strong coupling cluster   v.s.   weak coupling cluster

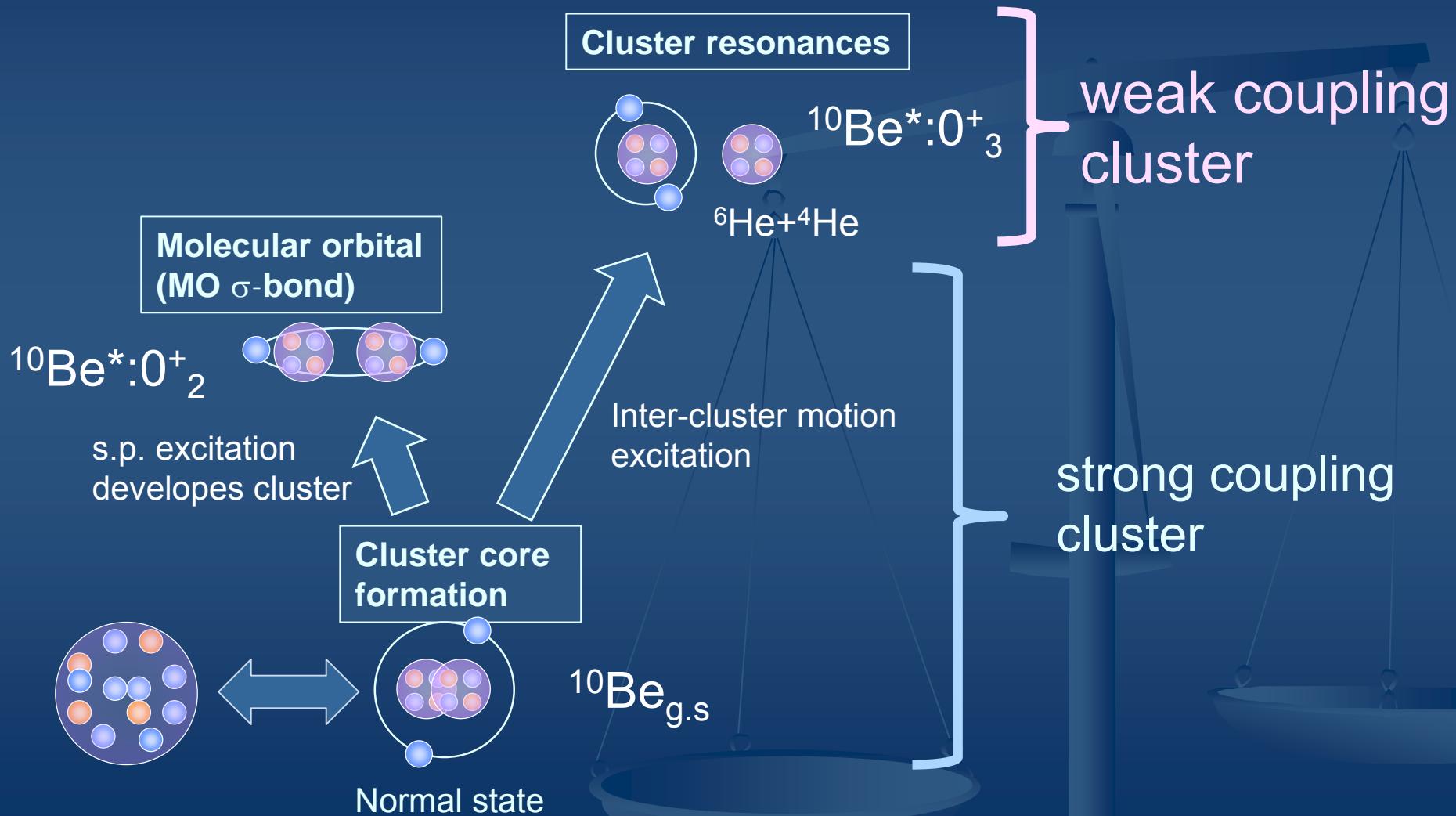


- Cluster core at surface
- Clusters are overlapping
- s.p excitation in MF
- below cluster threshold
- Indirect evidence:  
deformation, transitions,  
charge radii, s.p. config.



- Excitation of relative motion
- No overlap. far from each other
- Excited states near or  
resonances above threshold
- more direct evidence  
alpha-decay, alpha scattering

# Cluster phenomena in n-rich Be



# Rich cluster phenomena in nuclear systems

as functions of proton&neutron numbers and excitation energy

- ✓ Cluster formation/breaking in low-lying states
- ✓ MO  $\sigma$ -bond in neutron-rich nuclei
- ✓ Cluster excitation and resonances
- ✓ Many clusters : cluster gas, chain
- ✓ New types of clusters  
 $^{6,8}\text{He}+\text{He}$  in Be,  $^{10}\text{Be}+\alpha$  in  $^{14}\text{C}$ ,  $^{14}\text{C}+\alpha$  in  $^{18}\text{O}$ ,  $^{18}\text{O}+\alpha$  in  $^{22}\text{Ne}$

A theoretical method:

AMD (antisymmetrized molecular dynamics)

# 2. A theoretical model: AMD

An approach for nuclear structure to study

- cluster and mean-field aspects
- Stable and unstable nuclei
- Ground and excited states

# AMD method for structure study

## AMD wave fn.

$$\Phi = c\Phi_{\text{AMD}} + c'\Phi'_{\text{AMD}} + c''\Phi''_{\text{AMD}} + \dots$$

$$\Phi_{\text{AMD}} = \det \{\varphi_1, \varphi_2, \dots, \varphi_A\}$$

Slater det.

Gaussian

$$\varphi_i = \phi_{Z_i} \chi_i$$

$$\begin{cases} \text{spatial} & \phi_{Z_i}(r_j) \propto \exp \left[ -\nu \left( \mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}} \right)^2 \right] \\ \text{Intrinsic spins} & \chi_i = \begin{pmatrix} \frac{1}{2} + \xi_i \\ \frac{1}{2} - \xi_i \end{pmatrix} \times \begin{array}{l} p \text{ or } n \\ \text{isospin} \end{array} \end{cases}$$

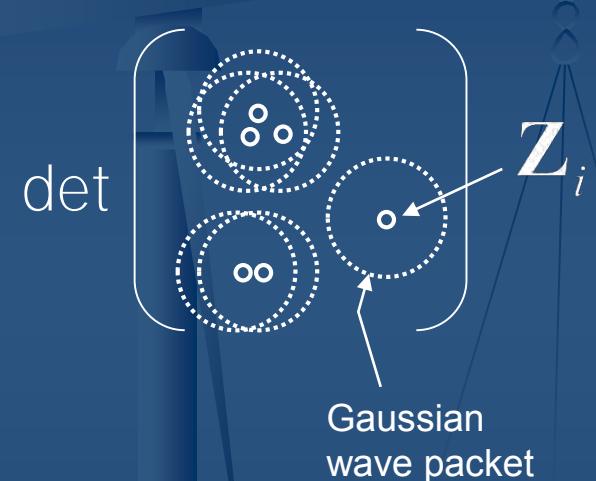
Similar to FMD wave fn.

$$\Phi_{\text{AMD}}(\mathbf{Z})$$

$$\mathbf{Z} = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A, \xi_1, \dots, \xi_A\}$$

Variational parameters:

Gauss centers, spin orientations



## Energy Variation

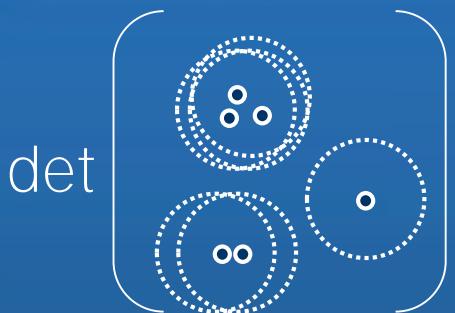
$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0$$

Model wave fn.  $\Phi$

Effective nuclear force  
(phenomenological)

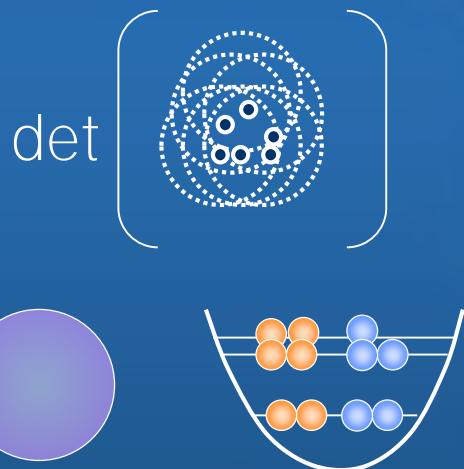
$$H^{\text{eff}} = \sum_{i=1} t_i + \sum_{i < j} v_{ij}^{\text{eff}} + \sum_{i < j < k} v_{ijk}^{\text{eff}}$$

## AMD model space



A variety of cluster st.

Cluster and MF formation/breaking

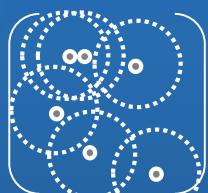
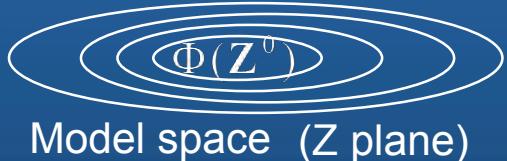


Shell structure

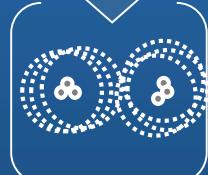
## Energy variation

### Energy surface

$$\frac{d\mathbf{Z}}{dt} = (\lambda + i\mu) \frac{1}{i\hbar} \frac{\partial E}{\partial \mathbf{Z}^*}$$



Randomly chosen  
Initial states



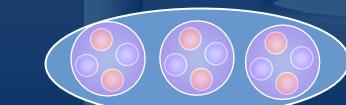
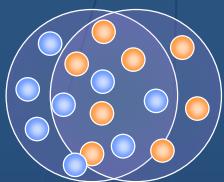
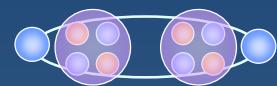
Energy minimum  
states

# 3. Some topics of cluster phenomena

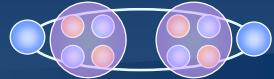
3-1. MO  $\sigma$ -bond in n-rich Be  
& Cluster resonances

3-2. Dipole resonances  $\leftrightarrow$

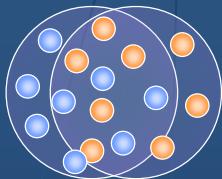
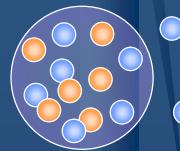
3-3. Linear chain in n-rich C



## 3-1. MO bond in n-rich Be & Cluster resonances



## 3-2. Dipole resonances ↔



## 3-3. Linear chain in n-rich C

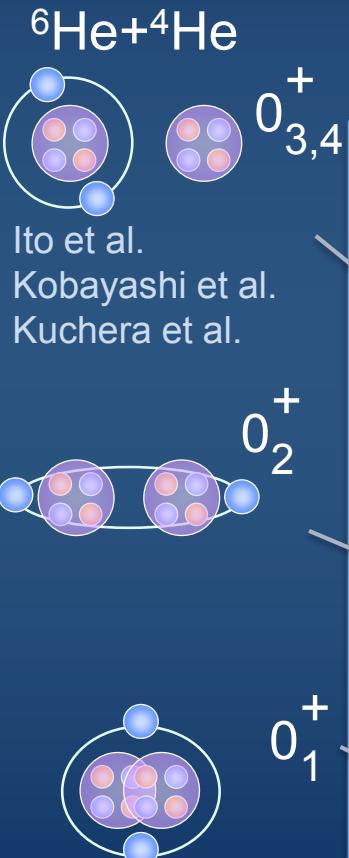


# Cluster structures in neutron-rich Be

cluster  
res.

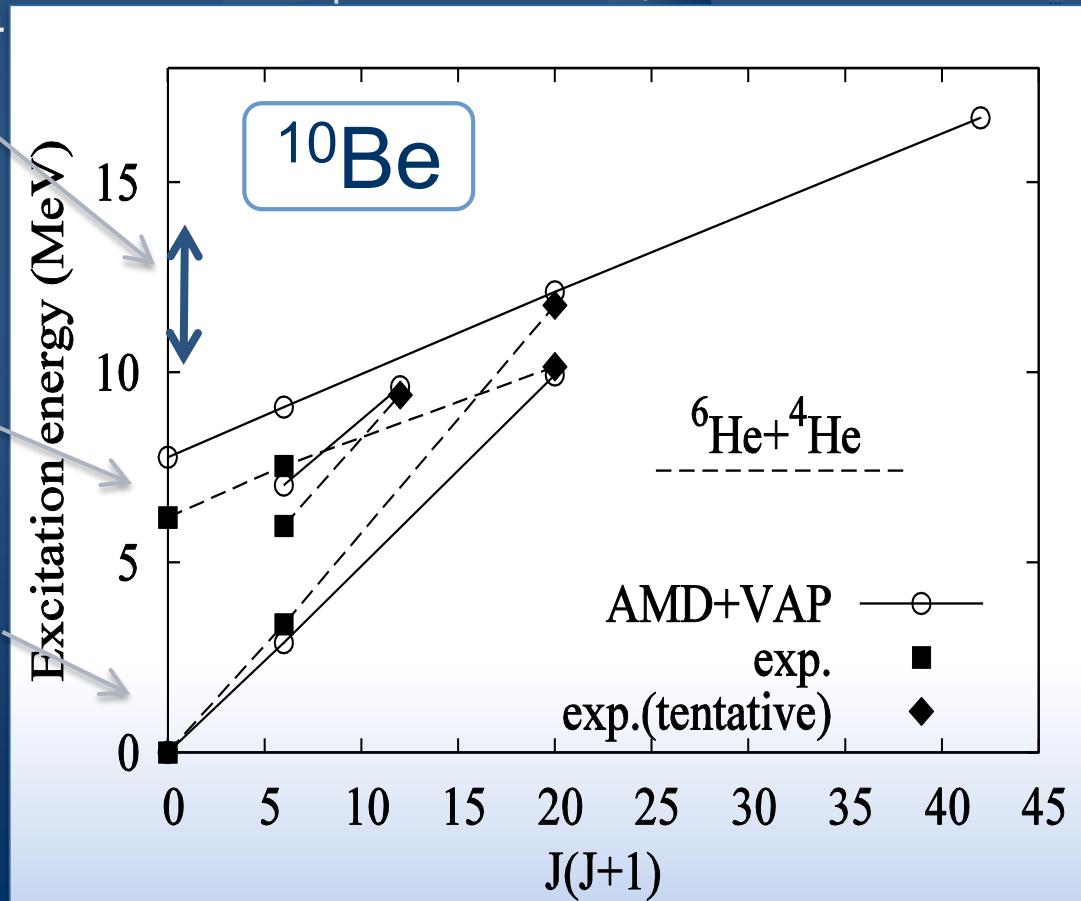
MO  $\sigma$ -bond

Normal



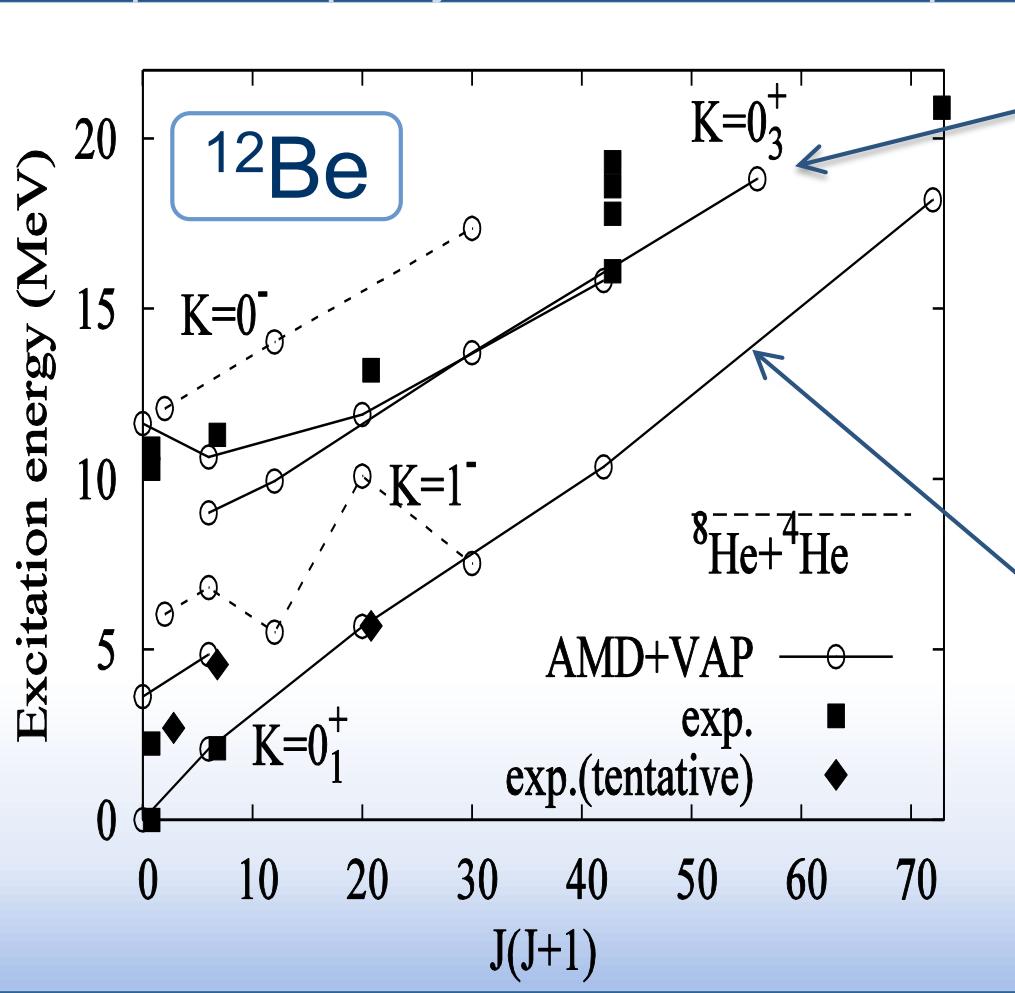
${}^{10}\text{Be}$ : energy levels

AMD calc. Y. K-E, et al. PRC (98)  
Exp: Milin et al. '05, Freer et al. '06

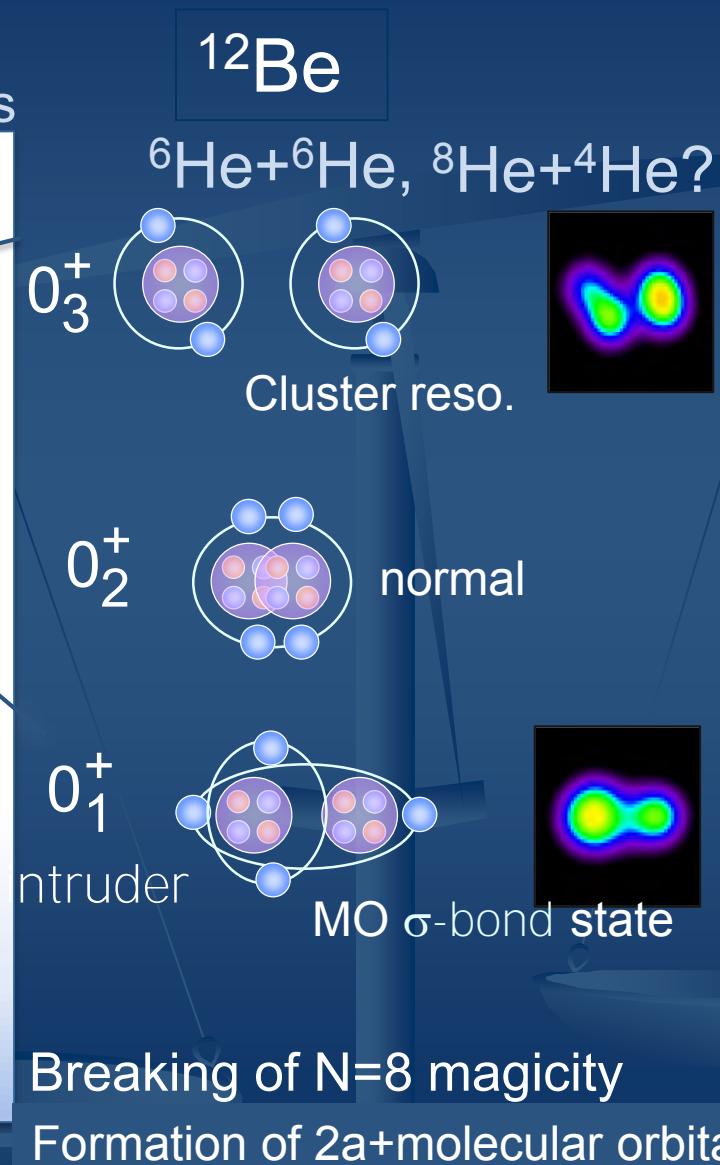


# Energy levels of $^{12}\text{Be}$

VAP calculation with AMD method  
positive parity states with normal spins

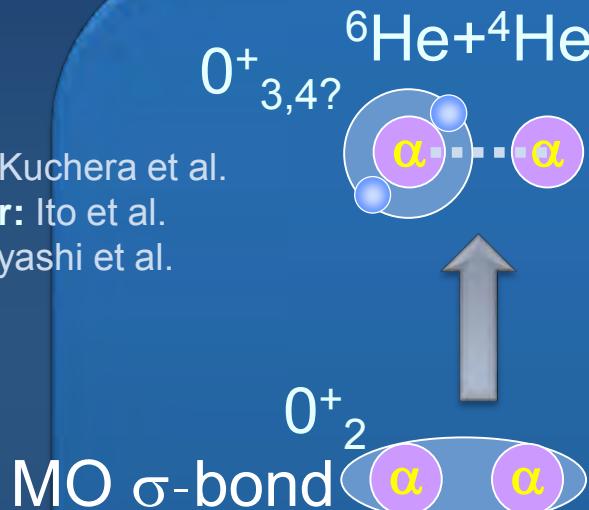


Y.K et al., PRC 68, 014319 (2003)

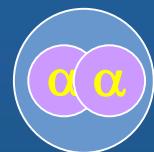


# Cluster formation, MO, and Cluster resonance

Exp: Kuchera et al.  
Theor: Ito et al.  
Kobayashi et al.



Normal



${}^{10}\text{Be}$

${}^{6,8}\text{He} + {}^{6,4}\text{He}$  cluster reso.

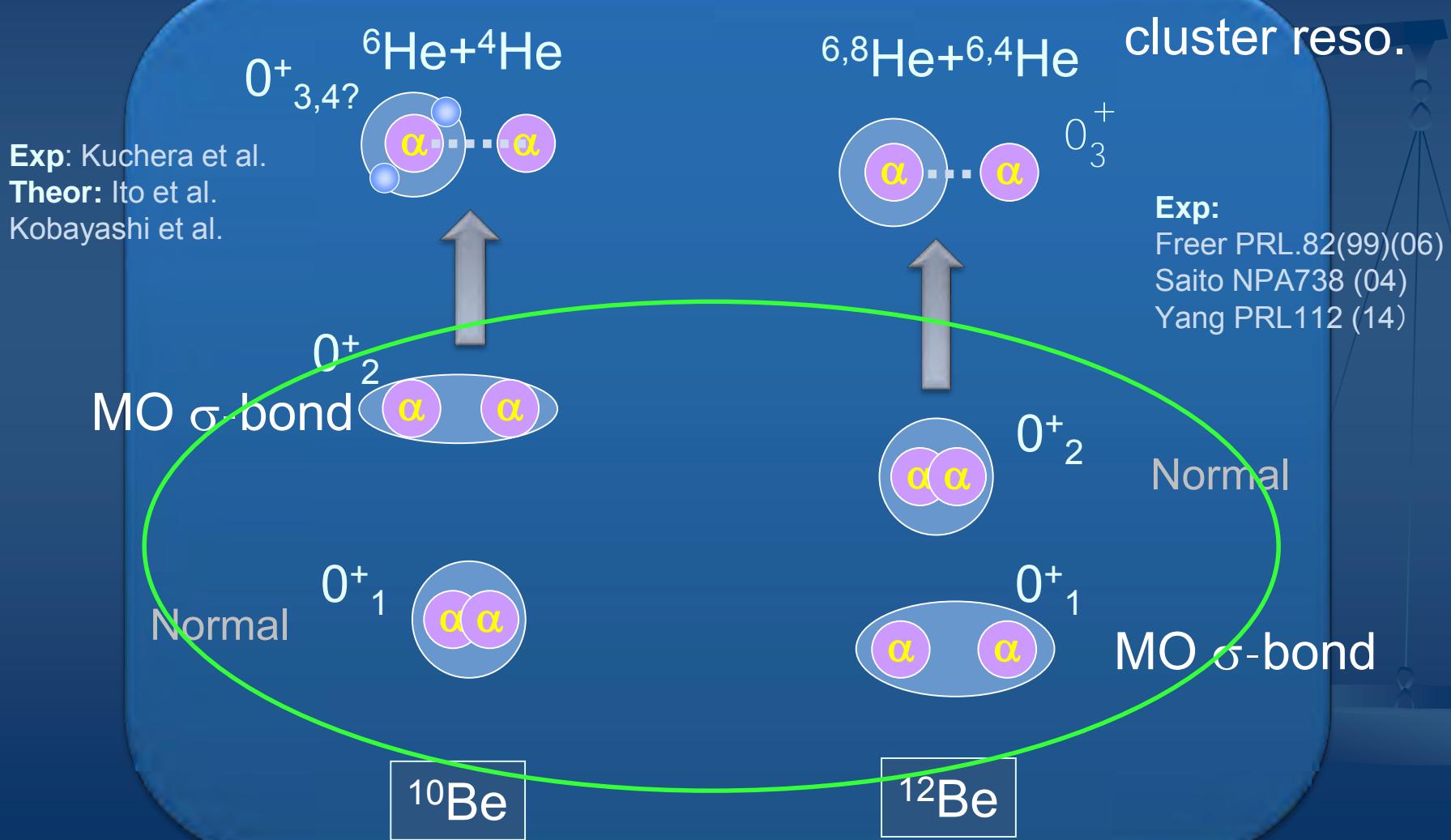
Exp:  
Freer PRL.82(99)(06)  
Saito NPA738 (04)  
Yang PRL112 (14)



MO  $\sigma$ -bond

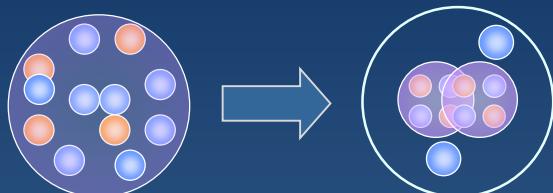
${}^{12}\text{Be}$

# Cluster formation, MO in low-energy region



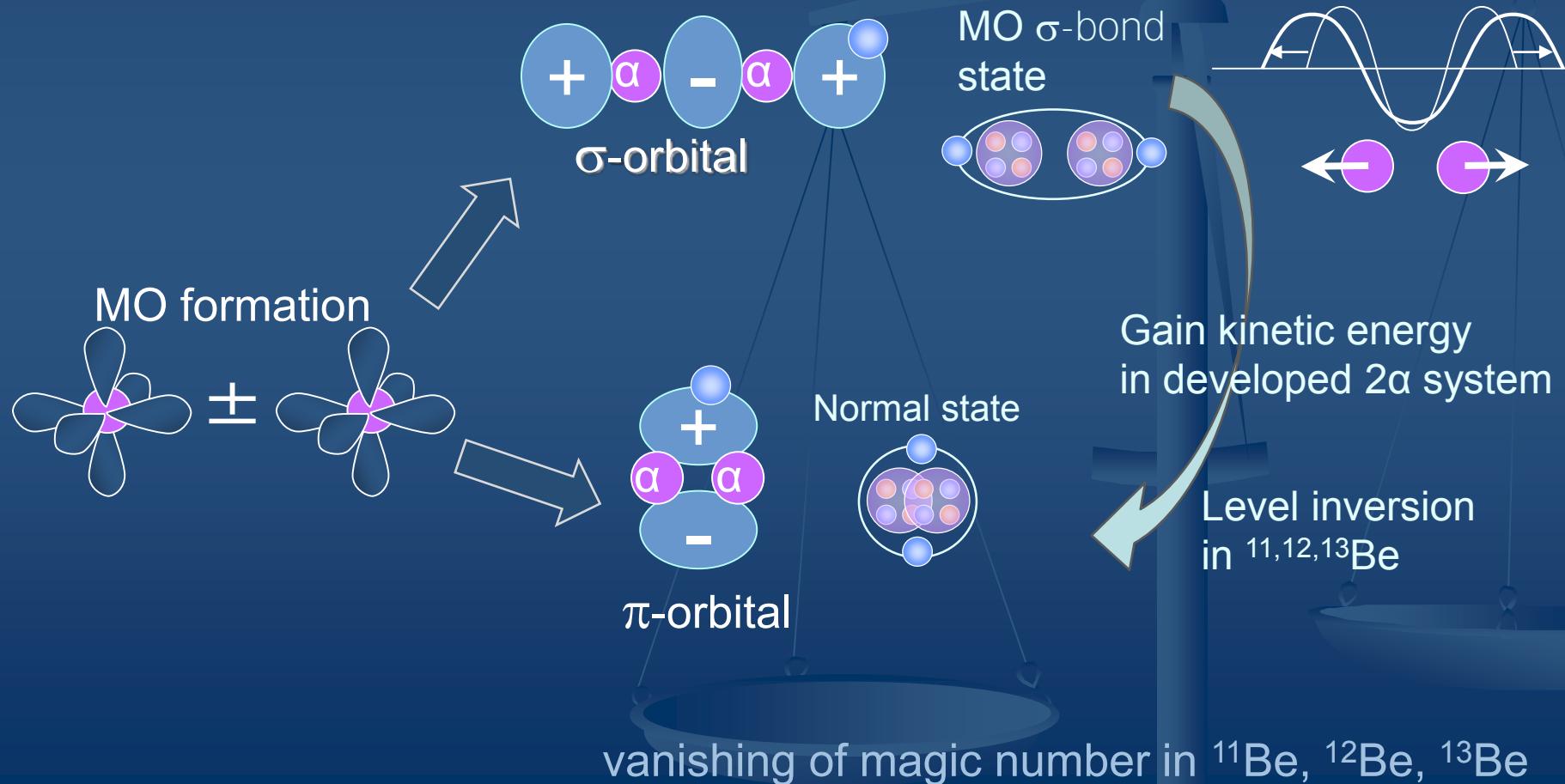
# Molecular orbital(MO) structure in Be

2 $\alpha$ -core formation

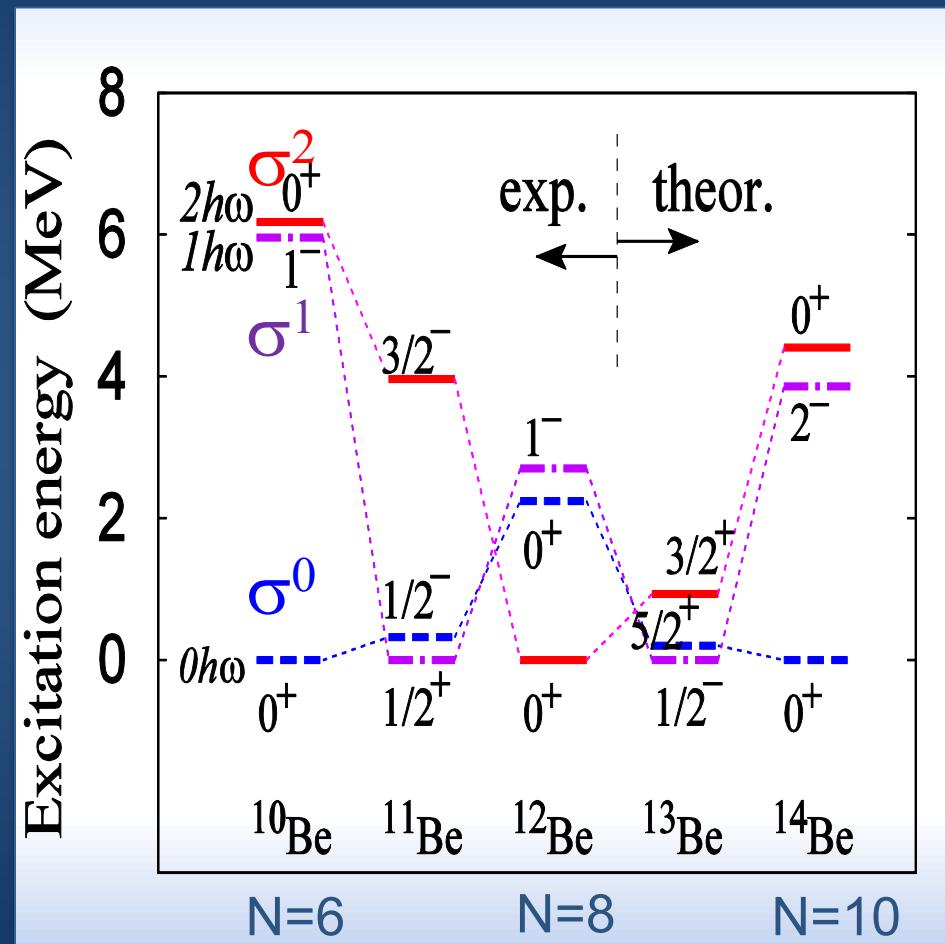
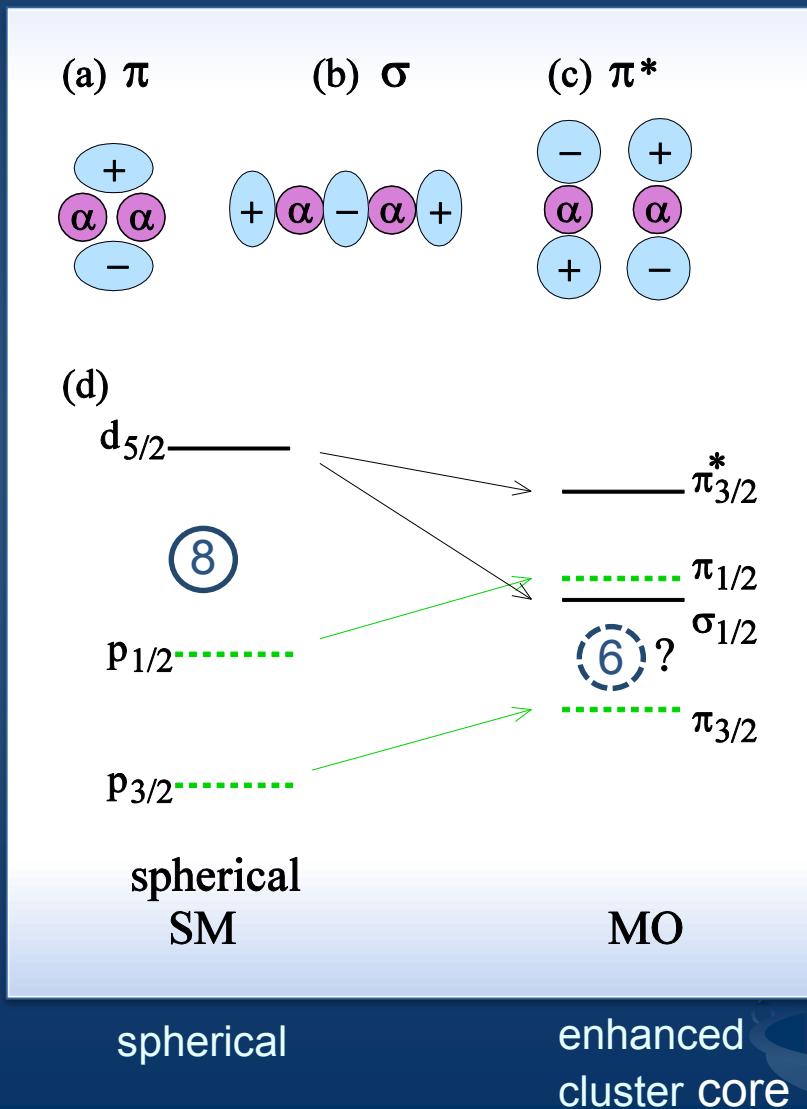


Seya PTP65(81), von Oertzen ZPA354(96)  
N. Itagaki PRC61(00), Y. K-E.. Ito PLB588(04)

MO formation

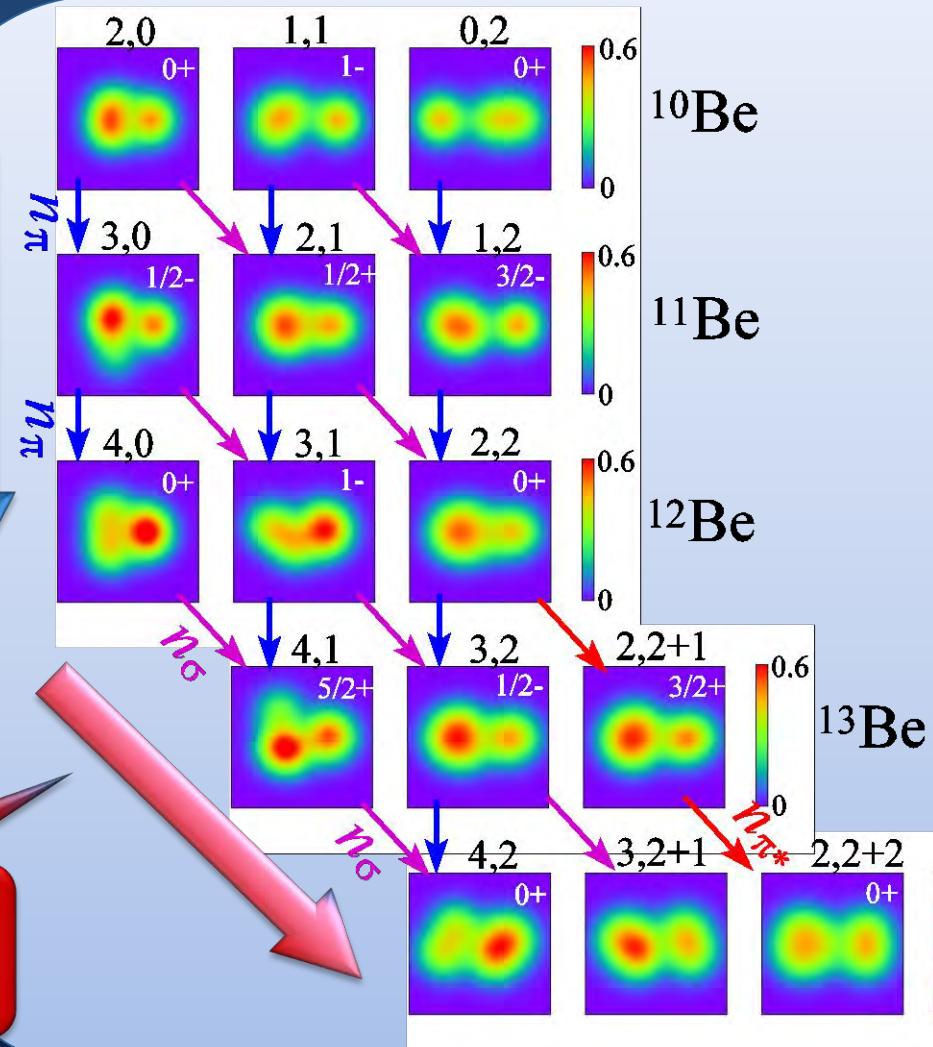


# $N=8$ magic number breaking



# cluster enhancement by $\sigma$ -orbital neutrons

$\pi$ -orbital neutrons  
suppress clustering.

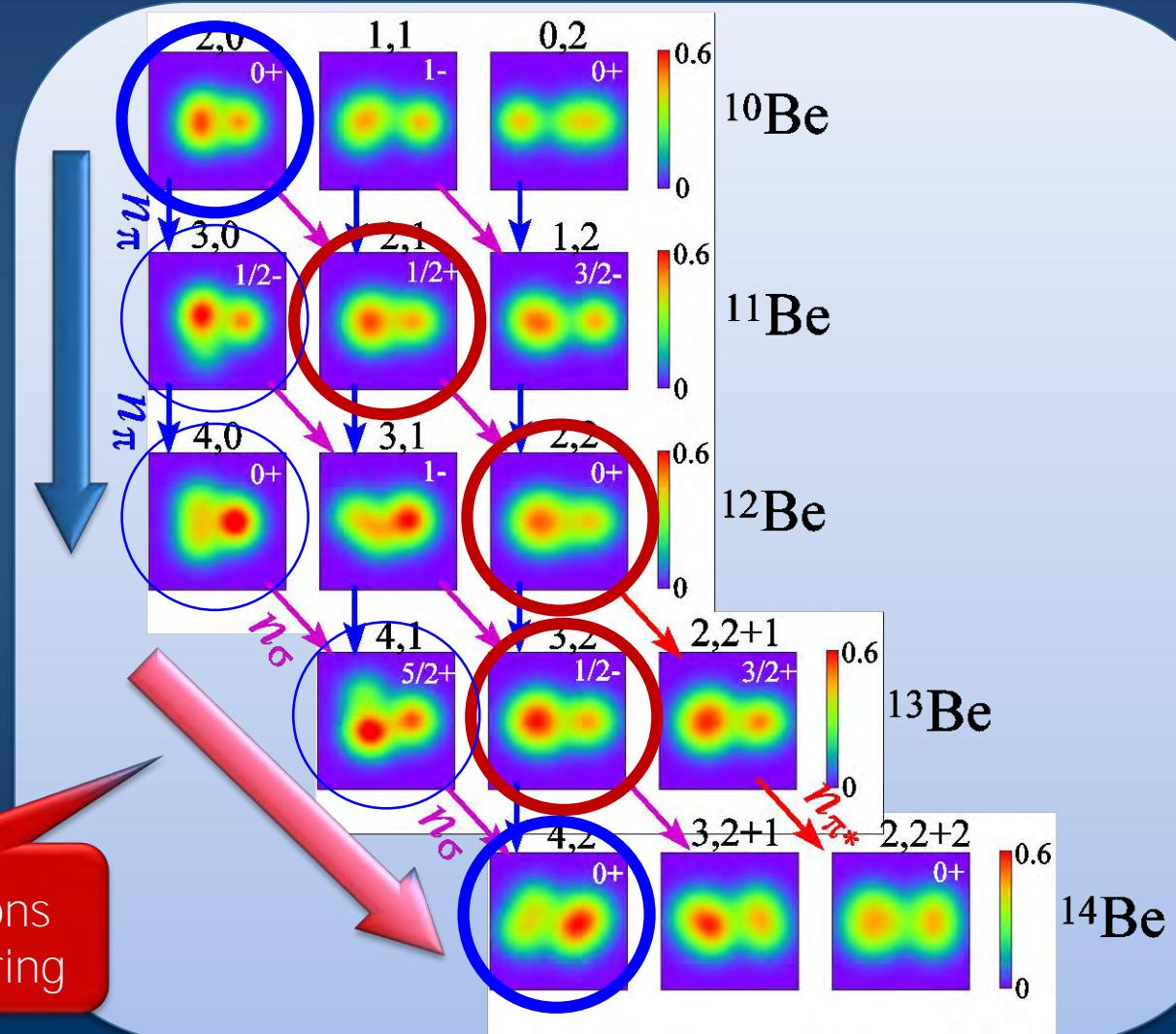


$\sigma$ -orbital neutrons  
enhance clustering

# cluster enhancement by $\sigma$ -orbital neutrons

$\pi$ -orbital neutrons  
suppress clustering.

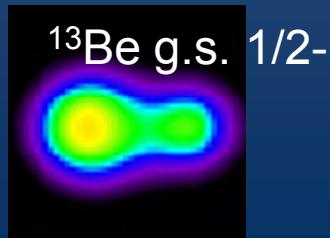
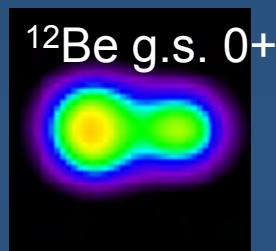
$\sigma$ -orbital neutrons  
enhance clustering



# Experimental probes for magic number breaking in Be

Y.K-E.PRC (03),(12) , Ito PRL(08) Dufour NPA(10)

Fortune PRC(06), Blanchon PRC(10)



➤ deformation in  $^{12}\text{Be}(\text{gs})$

Inelastic scat. life time:  
Iwasaki PLB481(00),  
Imai PLB673(09)

➤ intruder config. in  $^{12}\text{Be}(\text{gs})$

1n-knockout reac.:  
Navin PRL85(00),  
Pain PRL96(06)

➤  $^{12}\text{Be}(0_2^+)$  with p-shell config.

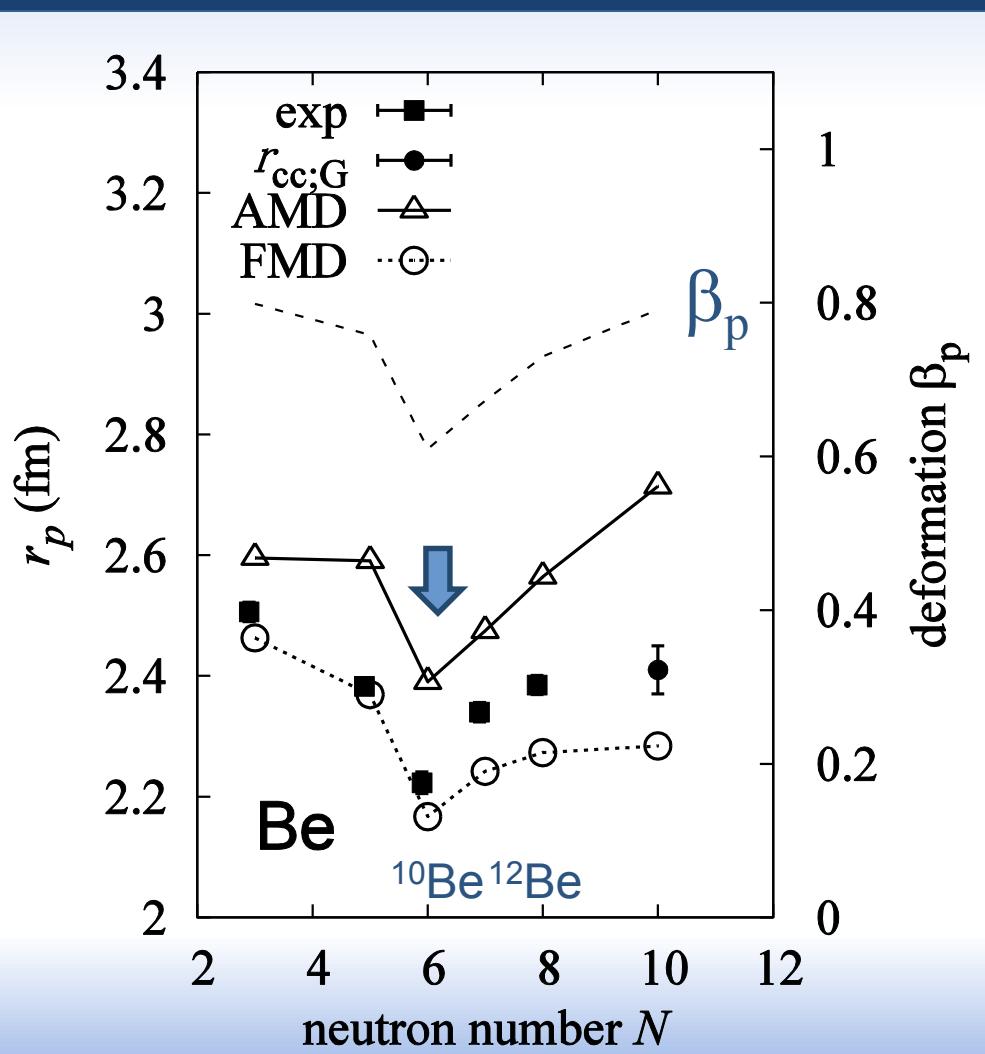
Shimoura PLB654 (07)

B(GT) with charge ex.:  
Meharchand PRL108 (12)

➤ abnormal parity of  $^{13}\text{Be}(\text{gs})$  now on discussion

Kondo et al. PLB690 (10)

# proton radii along isotope chain



Change of clustering (deformation)  
Is reflected in proton radii(charge radii)



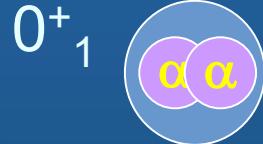
Point-proton radius is minimum  
at  $N=6$  ( $^{10}\text{Be}$ )

# Cluster resonances

Exp: Kuchera et al.  
Theor: Ito et al.  
Kobayashi et al.

MO bond

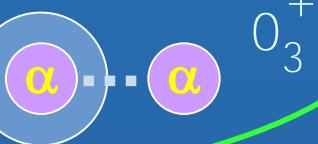
Normal



$^{10}\text{Be}$

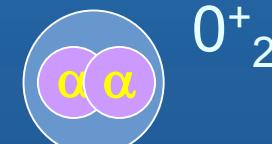


cluster reso.

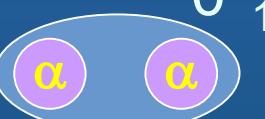


Exp:  
Freer PRL.82(99)(06)  
Saito NPA738 (04)  
Yang PRL112 (14)

Normal

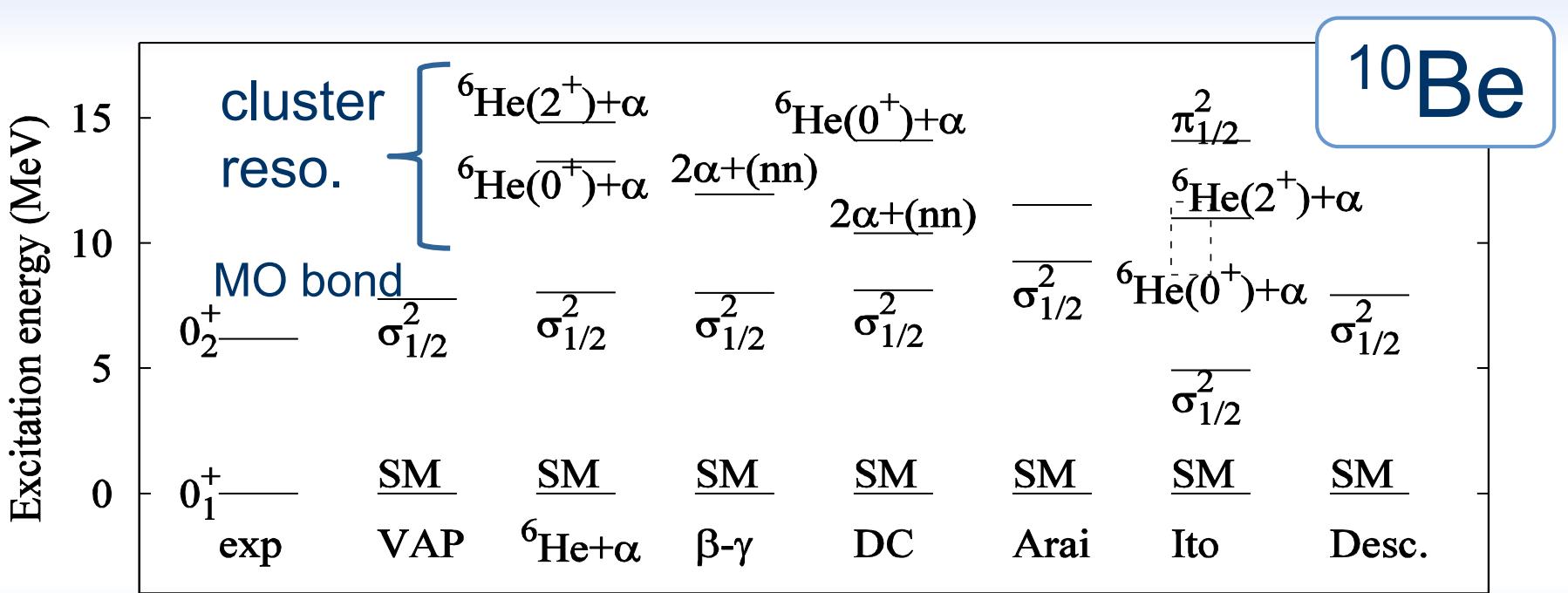


MO bond



$^{12}\text{Be}$

# Theoretical predictions of cluster resonances in $^{10}\text{Be}$



**Exp.**

Kuchera et al. PRC 84, 054615 (2011).

**Theor.**

Y. K-E. et al. PRC60, 064304 (1999)

T. Suhara and Y. K-E. et al., PTP123, 303 (2010)

F. Kobayashi and Y. K-E., Phys. Rev. C 86, 064303 (2012)

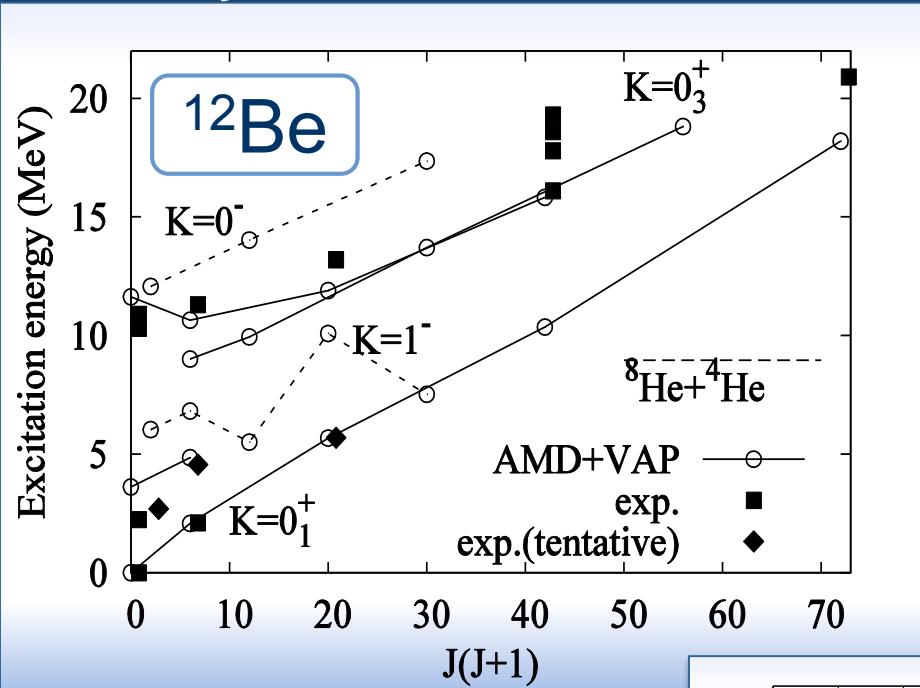
Arai et al., PRC69, 014309 (2004).

Ito et al., PLB636, 293 (2006).

P. Descouvemont et al., NPA 699 (2002) 463

# Theoretical predictions and observed states in $^{12}\text{Be}$

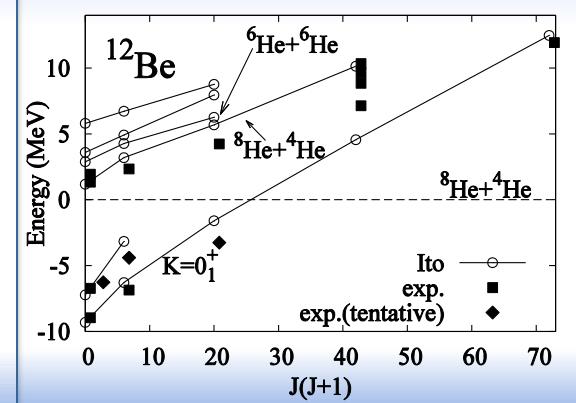
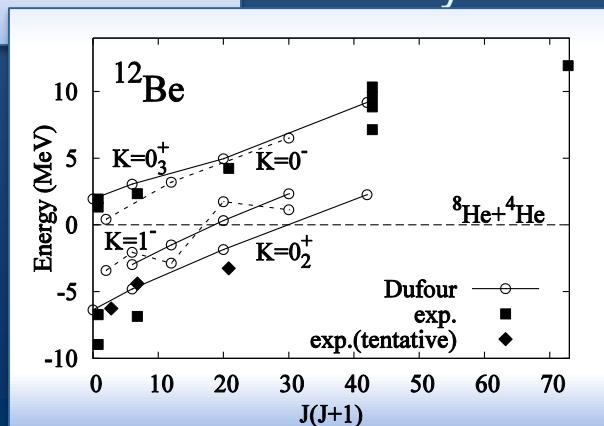
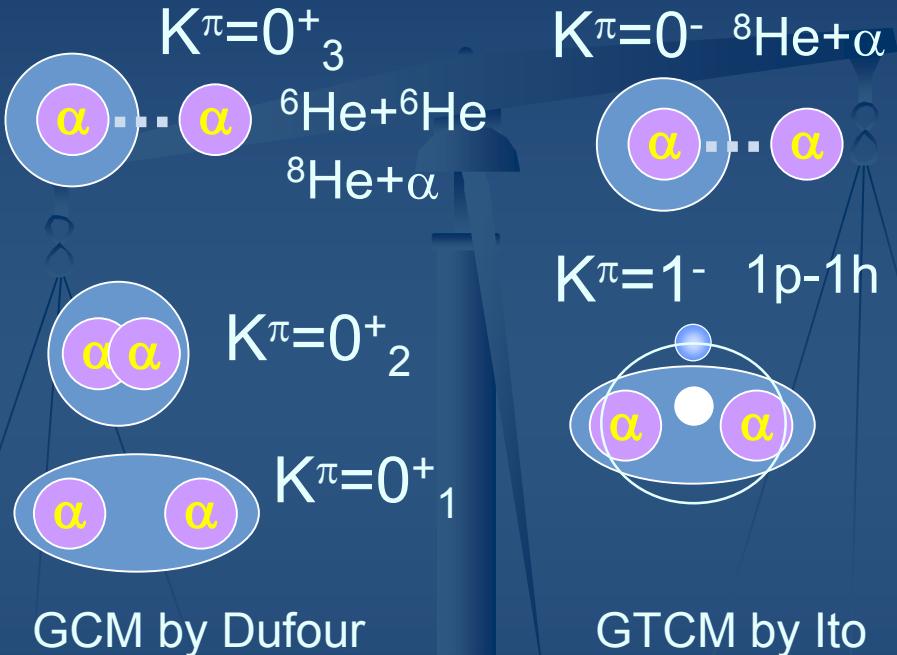
AMD by K-E.



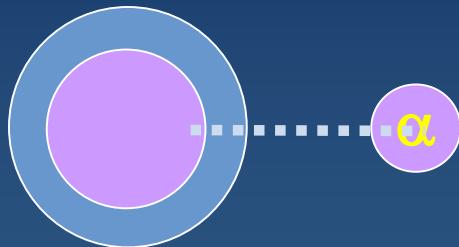
Exp. Freer PRL.82(99)(06)  
Saito NPA738 (04)  
Yang PRL112 (14)

Theor.

Y. K-E. et al. PRC68 (03)  
Ito et al., PRC85 (12)  
Dufour et al., NPA836 (10) 242



# $\alpha$ -cluster states in n-rich nuclei



Cluster resonances

New states discovered and suggested at  
 $E_x = \text{several} \sim 20 \text{ MeV}$   
in  $\alpha$ -decay,  $\alpha$ -transfer,  $\alpha$ -scattering

$^{6,8}\text{He} + \alpha$  in  $\text{Be}^*$

Exp: Soic et al., Freer et al., Saito et al., Curtis et al., Milin et al., Bohlen et al.,  
Theor: Seya, von Oertzen, Descouvemont et al., Itagaki et al., K-E et al.,  
Arai et al., M. Ito et al.

$^{10}\text{Be} + \alpha$  in  $^{14}\text{C}^*$

Exp Soic '04, von Oertzen '04, Price '07, Haigh '08, Fritsch '16, Yamaguchi  
Theor: Suhara '10

$^{14}\text{C} + \alpha$  in  $^{18}\text{O}^*$

Exp Scholz et al., Rogachev et al., Goldberg et al., Ashwood et al., Yildiz et al.,  
Theor: Descouvemont, Kimura,

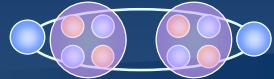
$^{18}\text{O} + \alpha$  in  $^{22}\text{Ne}^*$

Exp Scholz '72, Rogachev '01, Goldberg '04, Ashwood '06, Yildiz et al.,  
Theor: Descouvemont '88, Kimura '07

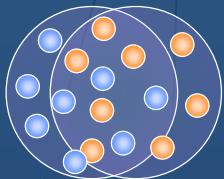
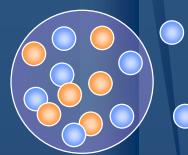
Universal phenomena?

Further experimental and theoretical studies are requested.

3-1. MO bond in n-rich Be  
& Cluster resonances



3-2. Dipole resonances ↔



3-3. Linear chain in n-rich C

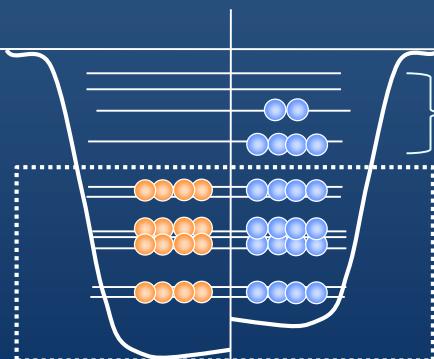


# E1(Dipole) resonances

Separation of LEDR  
From GDR

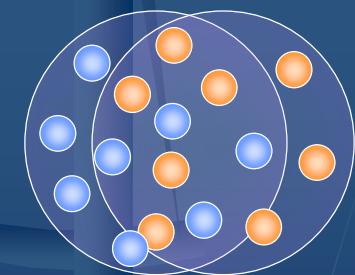
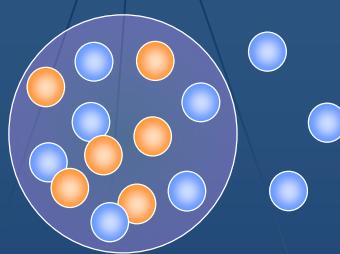
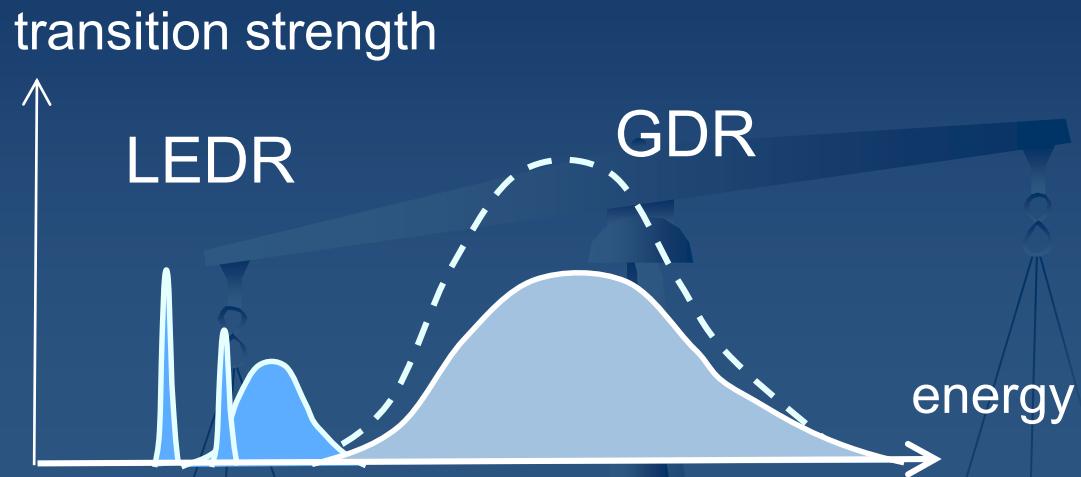


New excitation modes



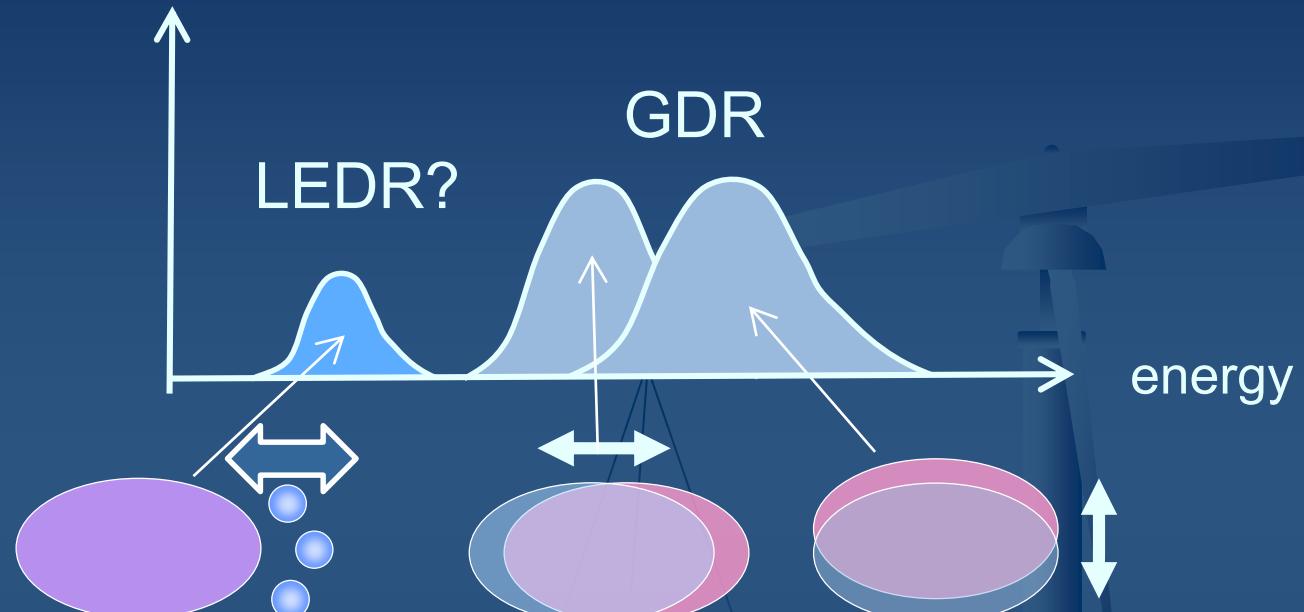
valence  
neutrons  
core

Core+Xn



LEDR: low-energy dipole resonance  
PDR: collective LEDR

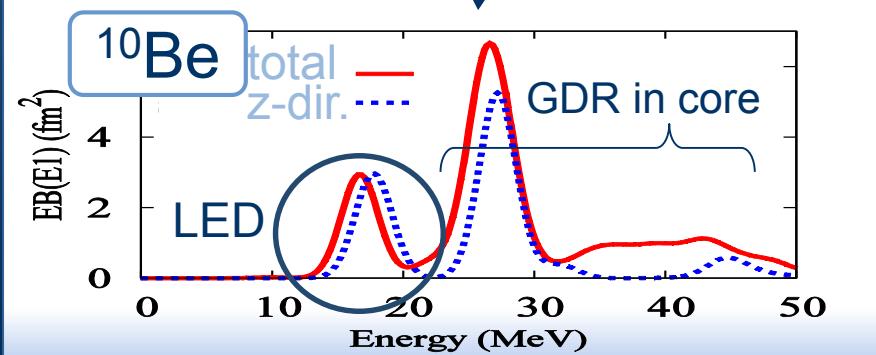
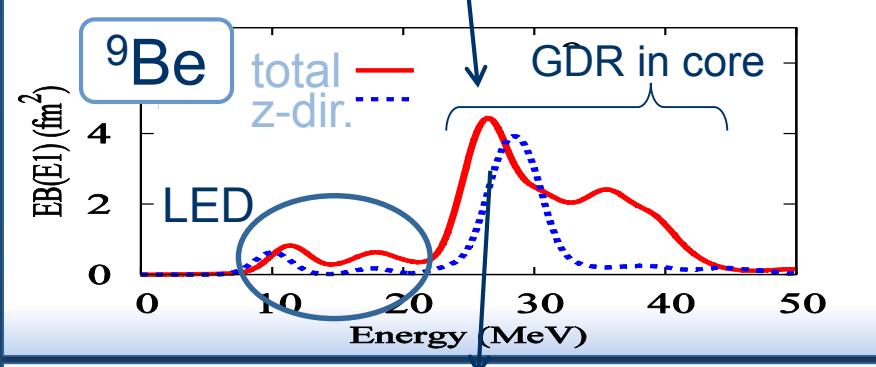
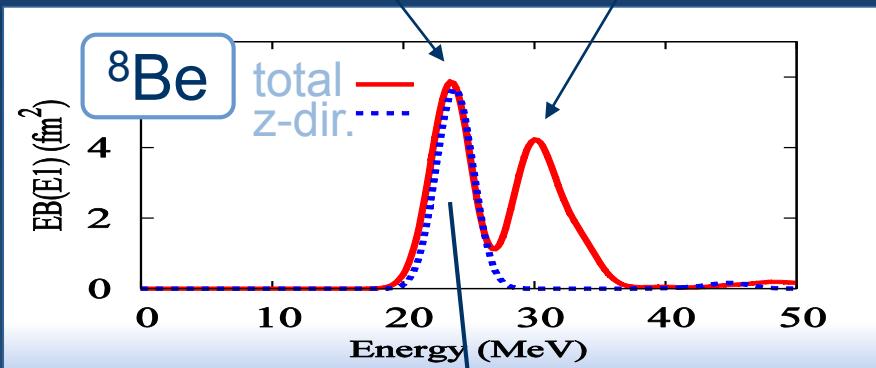
# deformed neutron-rich nuclei



Valence neutron modes  
against core

Two peak structure  
in prolate deformation

# B(E1) of Be isotopes calculated by shifted AMD



smearing factor 2MeV

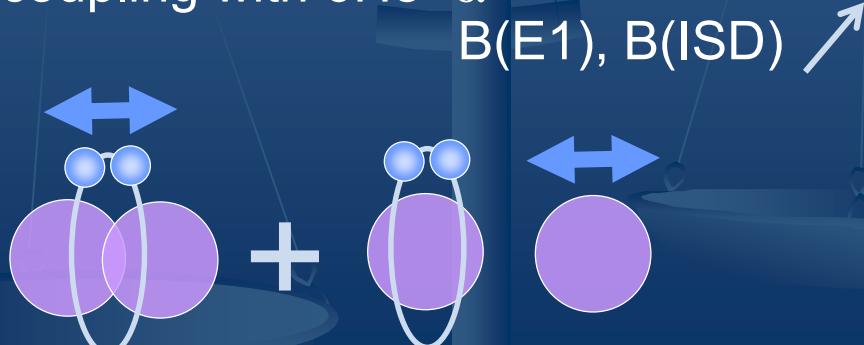
■ GDR in  ${}^8\text{Be}$  core  
two peaks in prolate state



Lower peak  
not affected

higher peak  
broadened

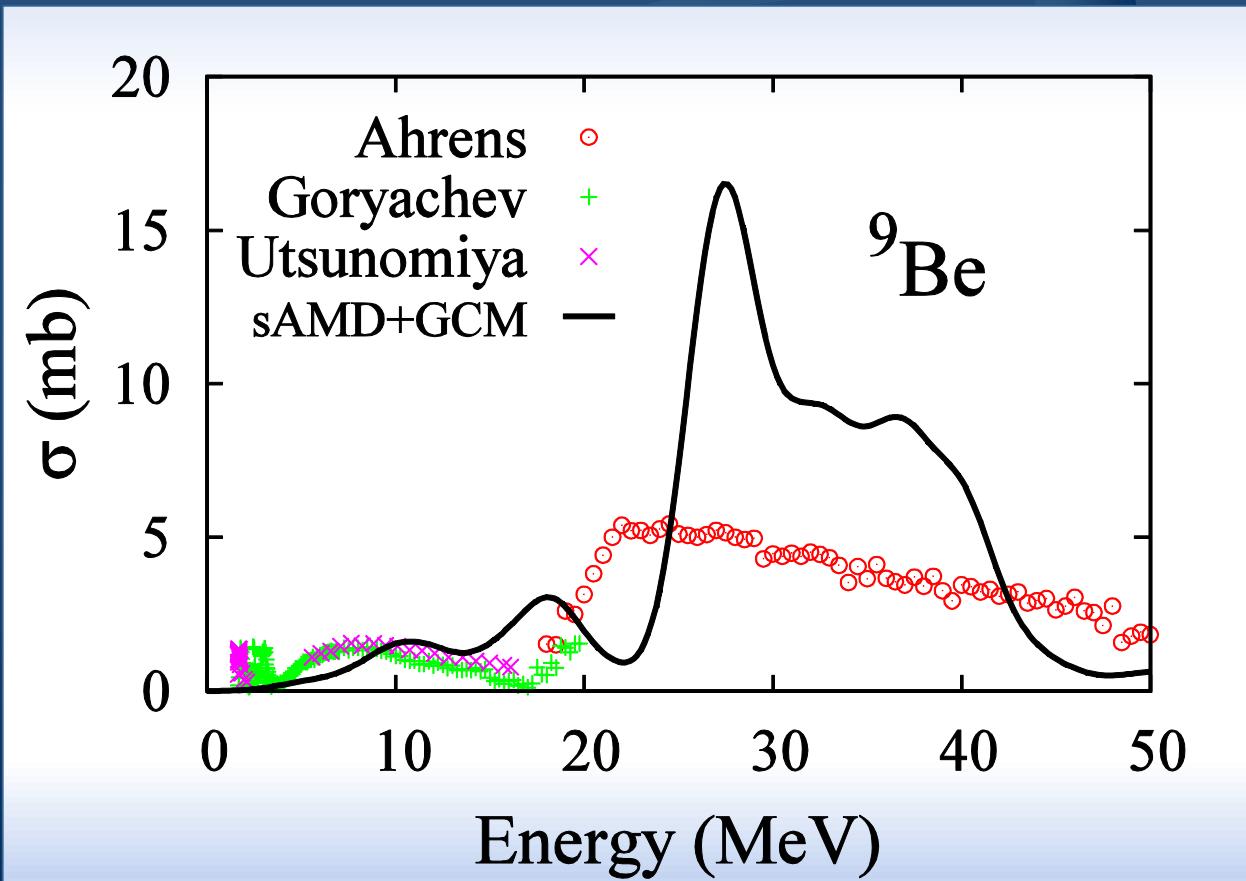
■ LEDR:  
Coherent two-neutron motion  
coupling with  ${}^6\text{He} + \alpha$



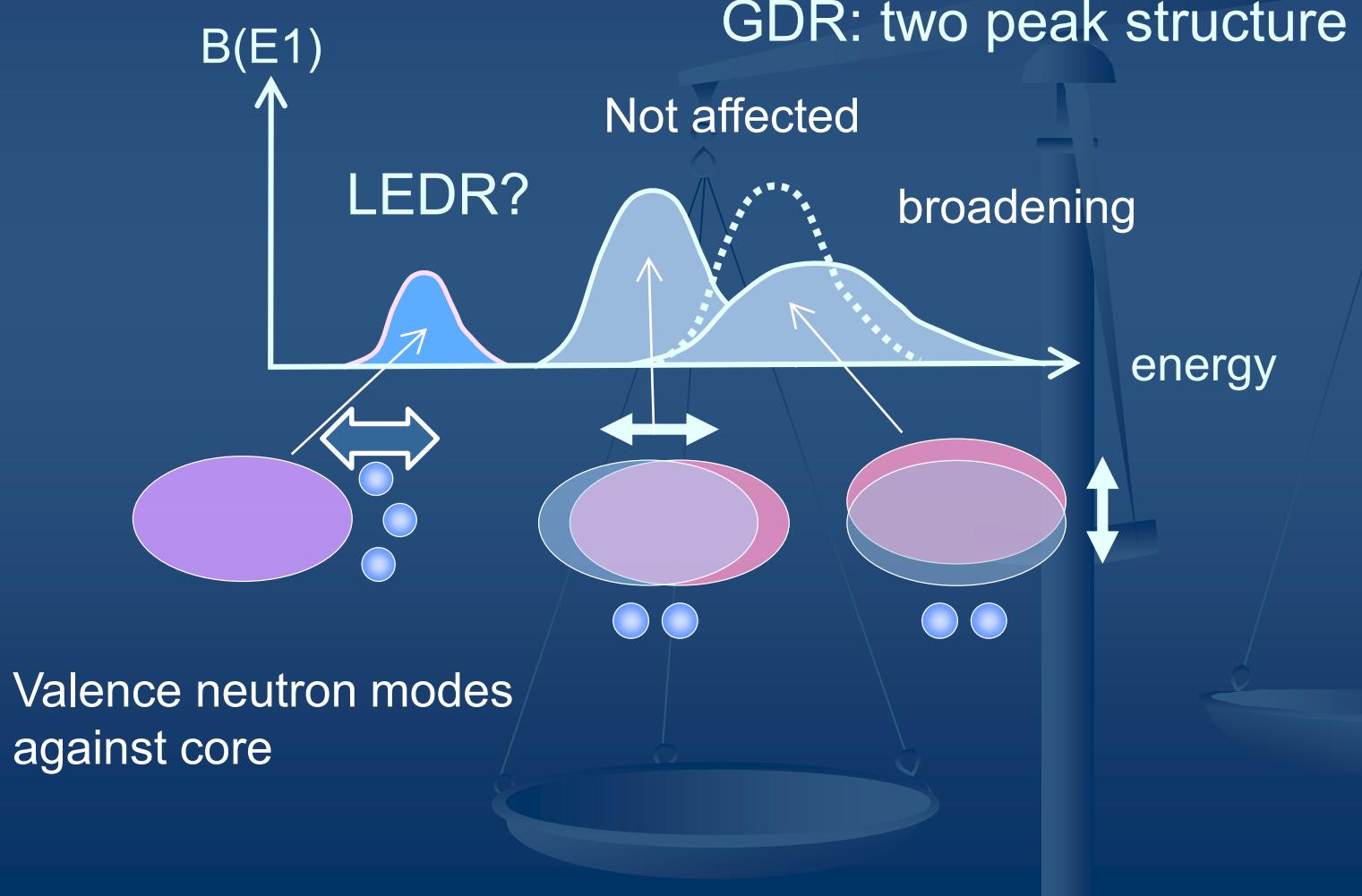
# B(E1) in ${}^9\text{Be}$ compared with experimental data

Photonuclear cross section v.s.  
sAMD+ $\alpha$ GCM calc.

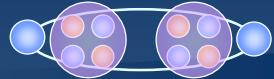
Ahrens et al.(1975)  
Goryachev et al. (1992)  
Utsunomiya et al.,(2015)



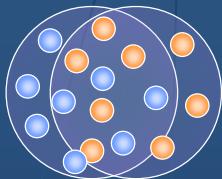
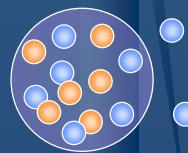
# Roles of excess neutrons in deformed neutron-rich nuclei



3-1. MO bond in n-rich Be  
& Cluster resonances



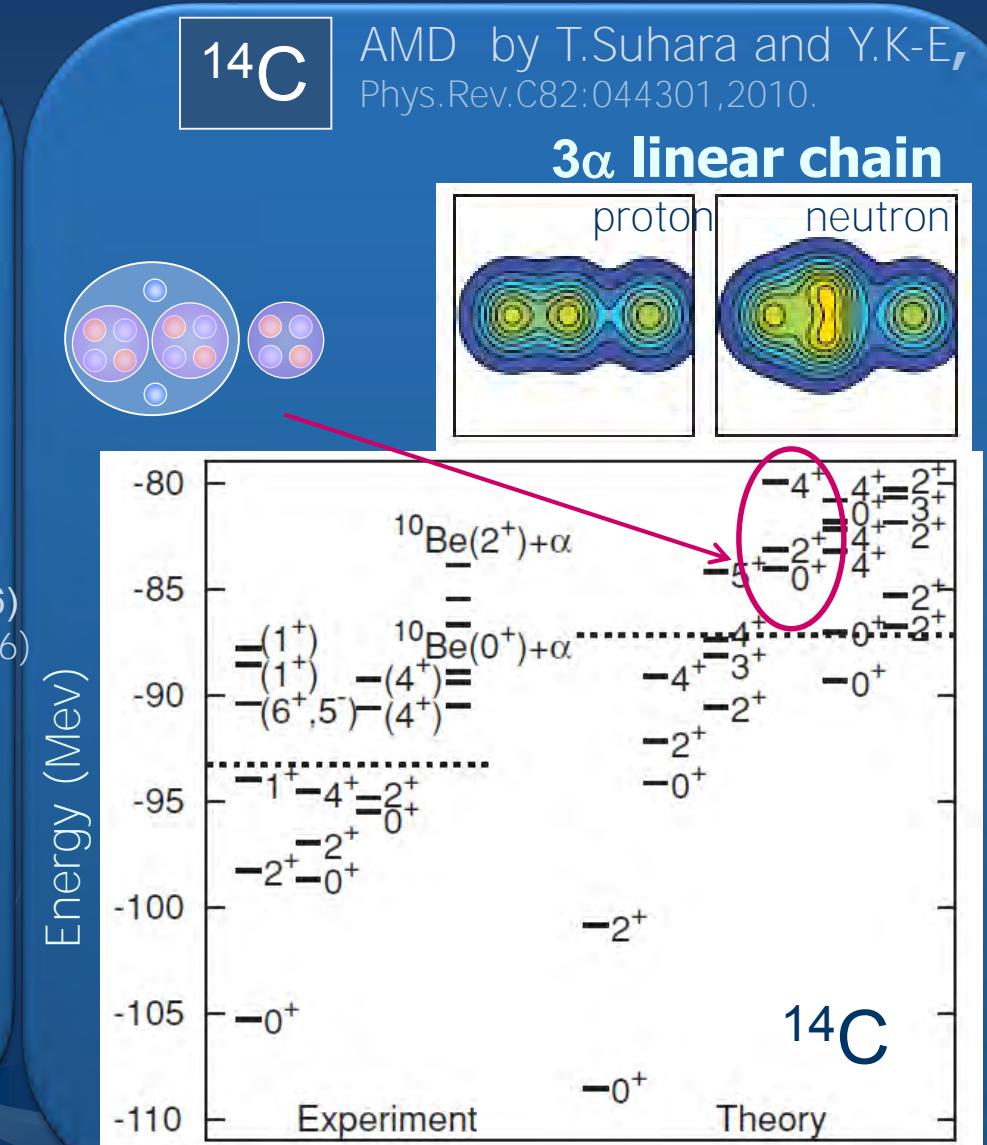
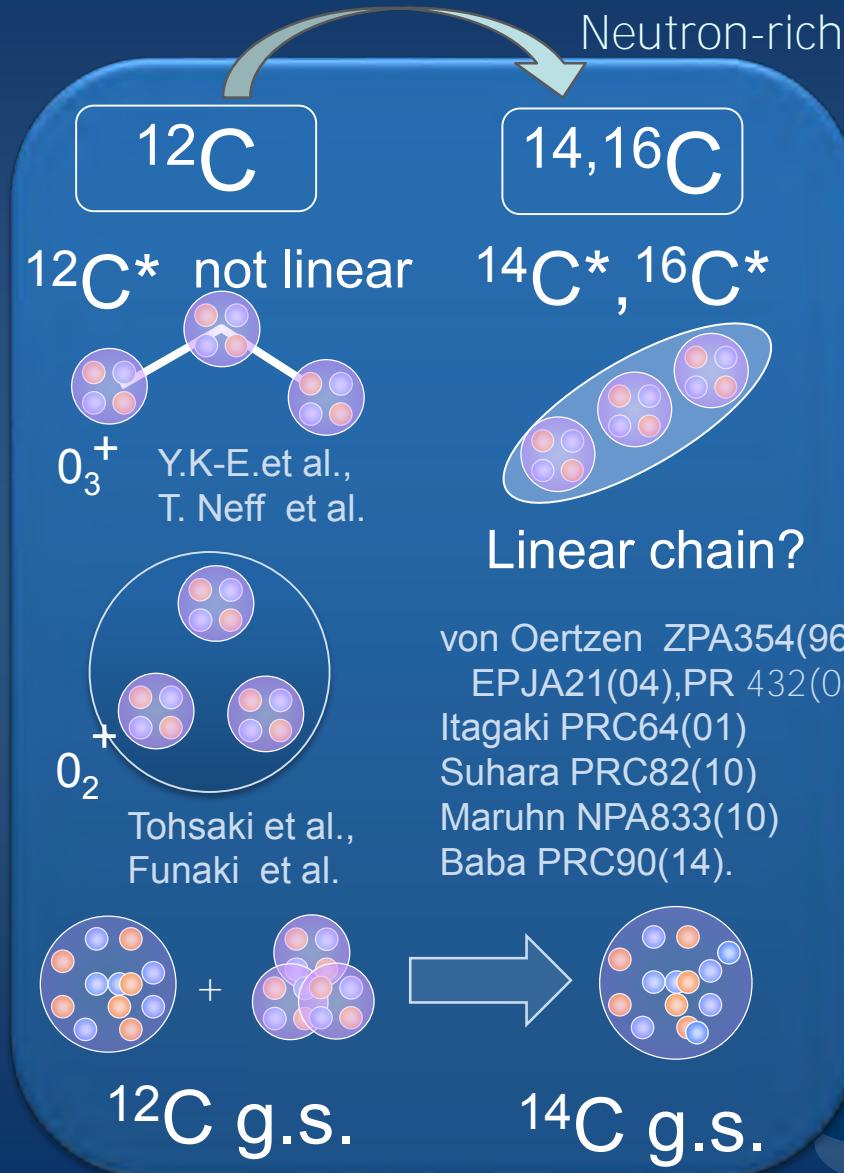
3-2. Dipole resonances ↔



3-3. Linear chain in n-rich C



# Linear chain state in $^{14}\text{C}^*$



# Experimental searching for linear chain states

- UK group: up reactions

Price et al. PRC75(2007), 014305.  $^{10}\text{Be} + \text{a}$  break up

Haigh et al. PRC78 (2008) 014319.  $^{10}\text{Be} + \text{a}$  break up

Freer et al. PRC90(2014) 054324.  $^{10}\text{Be} + \text{a}$  scattering

- US group:

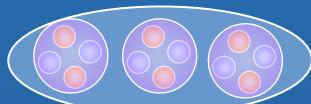
Fritsch et al., PRCC93 (2016) 014321.  $^{10}\text{Be} + \text{a}$  scattering.

- Jpn group:

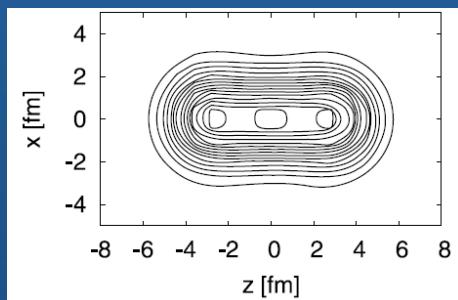
Yamaguchi et al., New results are coming soon !

# Linear chain in n-rich C

Meta stable  
for bending motion

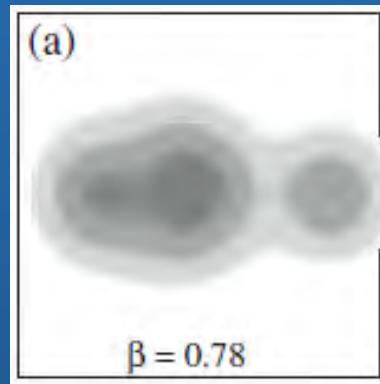


- ✓ MO model for  $^{14,16}\text{C}$   
Itagaki et al. PRC64(01)
- ✓ HF calc. for  $^{16,20}\text{C}$   
by Maruhn et al. NPA833(10)



Stretching effect  
in rotation

AMD by Y.K-E,  
PR432(2006)

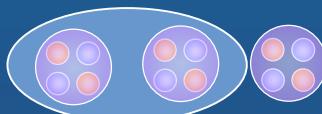


$^{15}\text{C}^*(19/2^-)$   
 $\beta=0.78$

$^{15}\text{C}^*(19/2^-)$   
 $\beta=0.78$

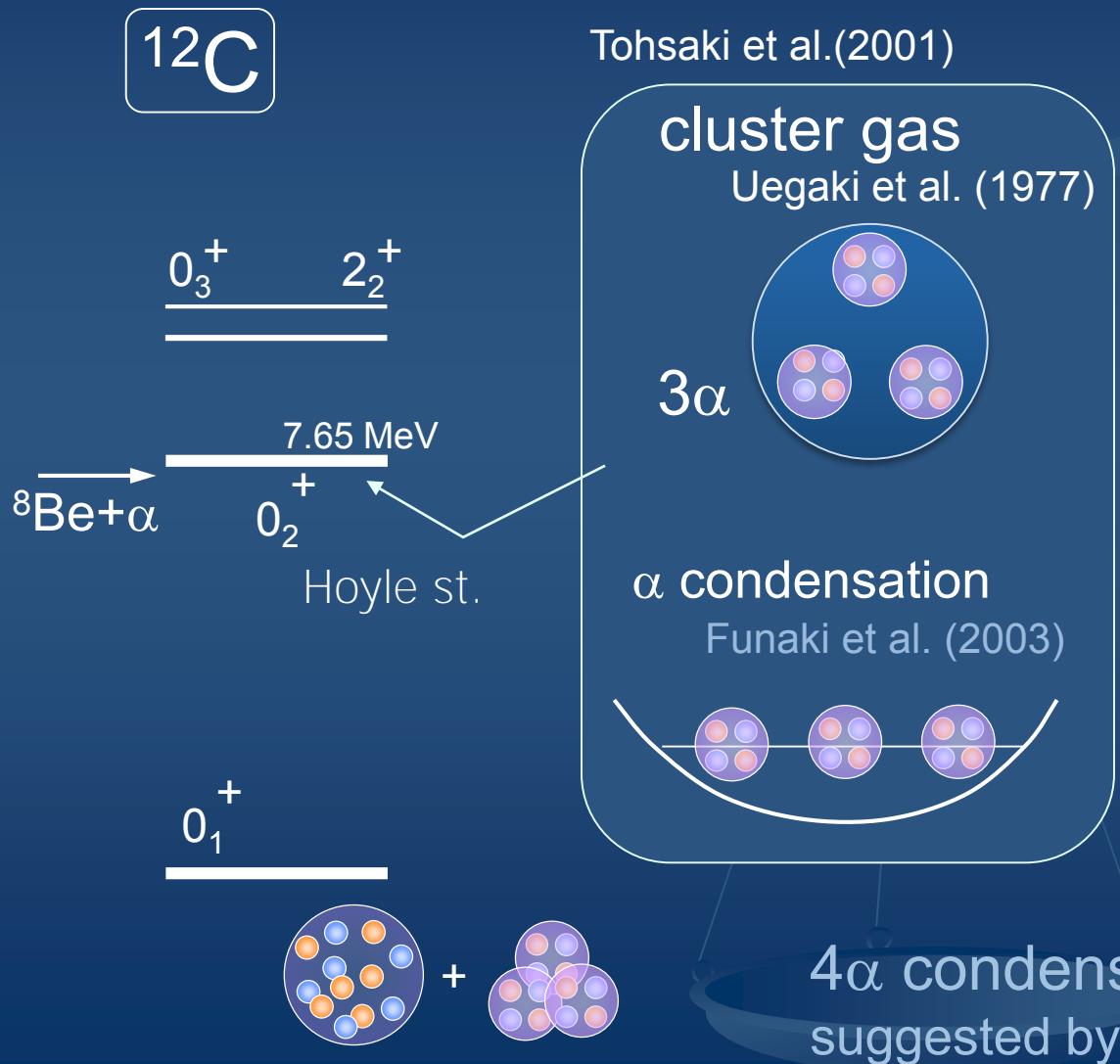
suggested  
to be a  
yrast state

$^{11}\text{Be}+\alpha$



Largely deformed  $^{11}\text{Be}$   
with MO-bond

# Cluster gas states in excited states



**Dilute cluster gas**

**Bosonic behavior:**  
 $\alpha$  particles condensate in the same orbit.

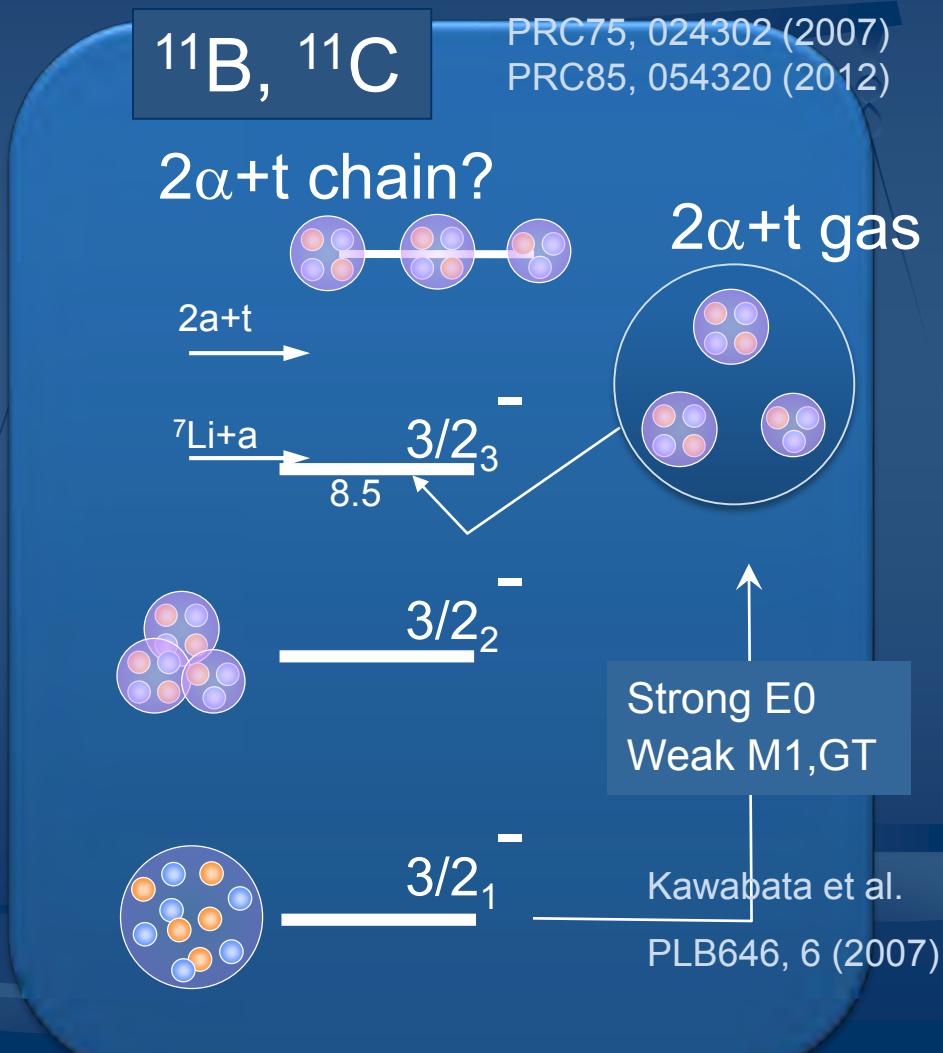
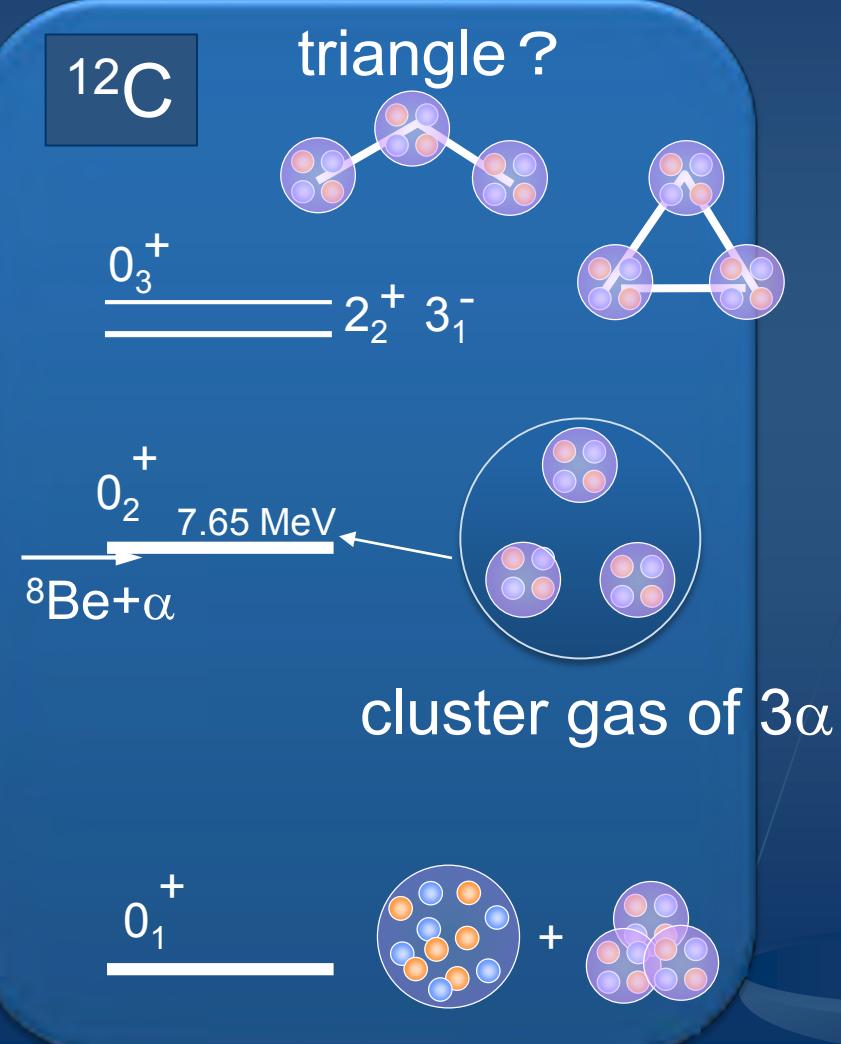
**BEC in nuclear matter**  
Roepke et al., PRL(1998)

$\updownarrow$

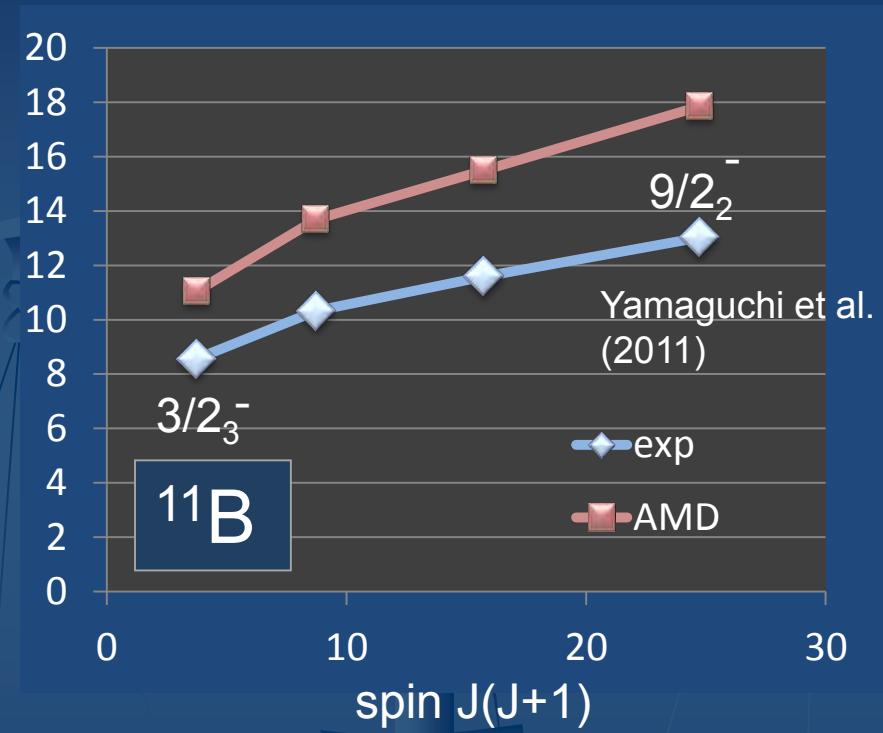
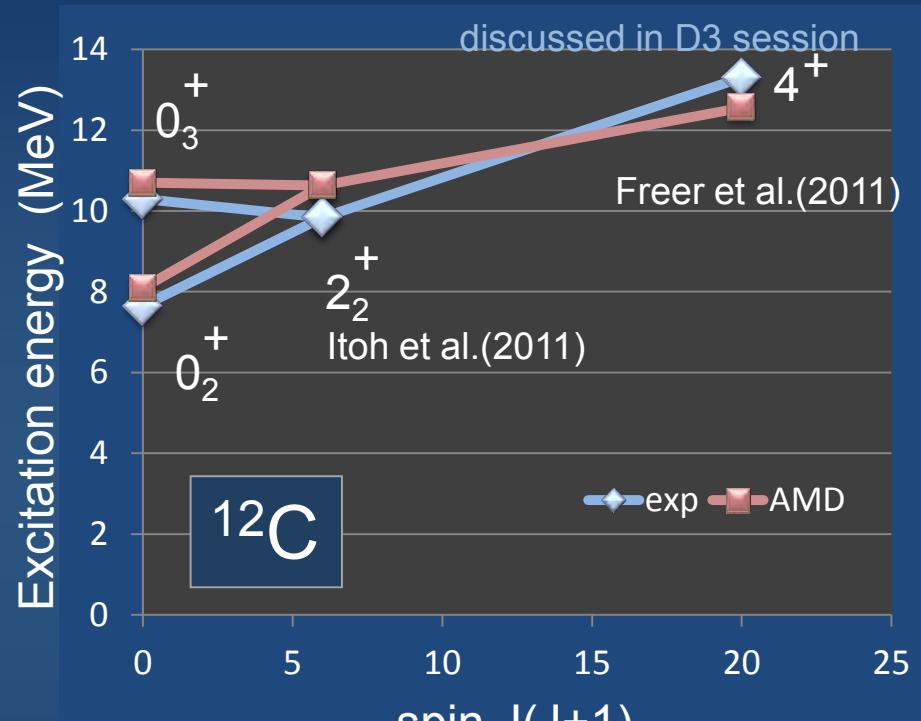
$^{16}\text{O}$

# $2\alpha+t$ cluster in $^{11}\text{B}(3/2^-_3)$

AMD by Y.K-E., Suhara



# Rotational band from cluster gas



rotation of  $3\alpha$ ,  $4\alpha$  gas  
Ohkubo et al., PLB684(2010)  
Funaki et al. PTPS196 (2012)

# Summary

Rich cluster phenomena in n-rich nuclei  
as function of proton and neutron numbers and  
excitation energy

- ✓ Cluster formation/breaking in low-lying states
- ✓ valence neutrons: MO Bond, new types of clusters
- ✓ Cluster excitation and resonances
- ✓ Many clusters : cluster gas, chain

Independent particle motion v.s. many-body correlations  
single particle excitation(mode) v.s. cluster excitation

# Systematic study in a wide region

Excitation energy

Low density

proton number

multidimension

- \* proton number
- \* neutron number
- \* excitation energy
- \* density

Neutron number

