

Light-cone QCD sum rules  
for soft contribution  
to exclusive Drell-Yan process  
at J-PARC

**Kazuhiro Tanaka** (Juntendo U/KEK)

Light-cone QCD sum rules  
for soft contribution  
to exclusive Drell-Yan process  
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**1. Inclusive DY**

general aspects from theory side

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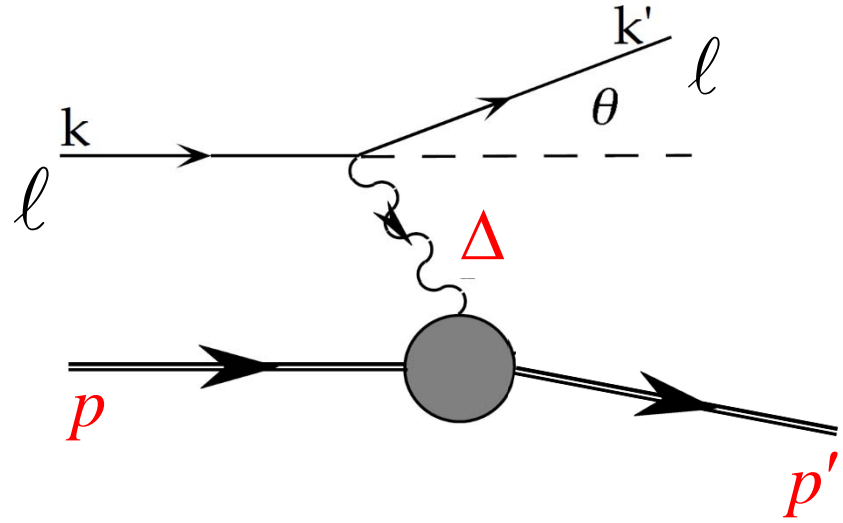
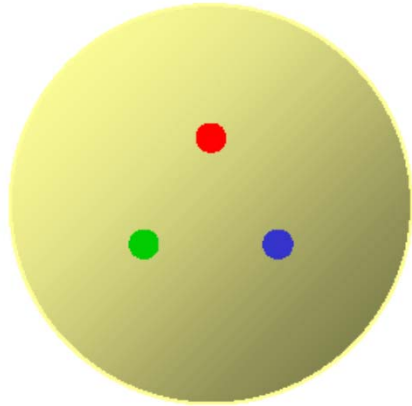
**1. Inclusive DY**

general aspects from theory side

**2. Exclusive DY**

general aspects & specific approach

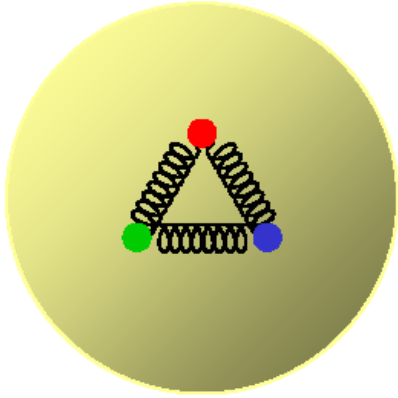
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$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

$$\langle N(\mathbf{p}') | \bar{\psi}(0) \gamma^\mu \psi(0) | N(\mathbf{p}) \rangle \sim \bar{u}(\mathbf{p}') \left\{ \gamma^\mu F_1(\Delta^2) + \frac{i\sigma^{\nu\mu} \Delta_\nu}{2M_N} F_2(\Delta^2) \right\} u(\mathbf{p})$$

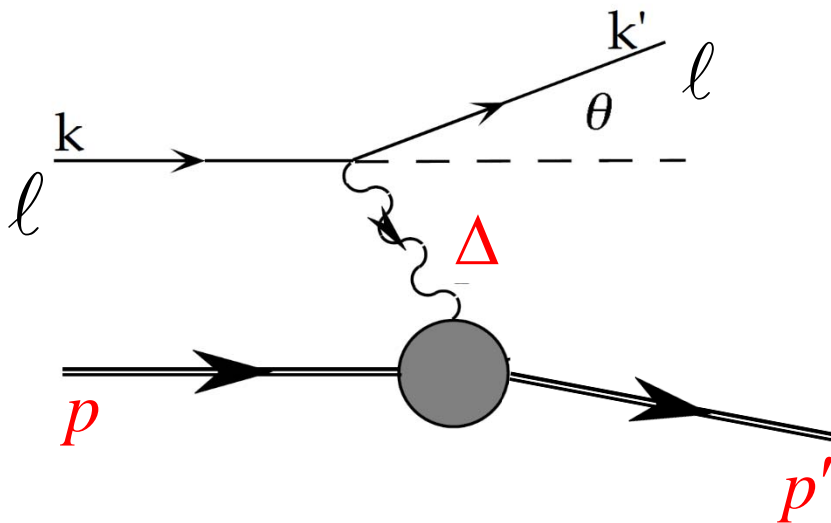
$$\Delta = \mathbf{p}' - \mathbf{p}$$

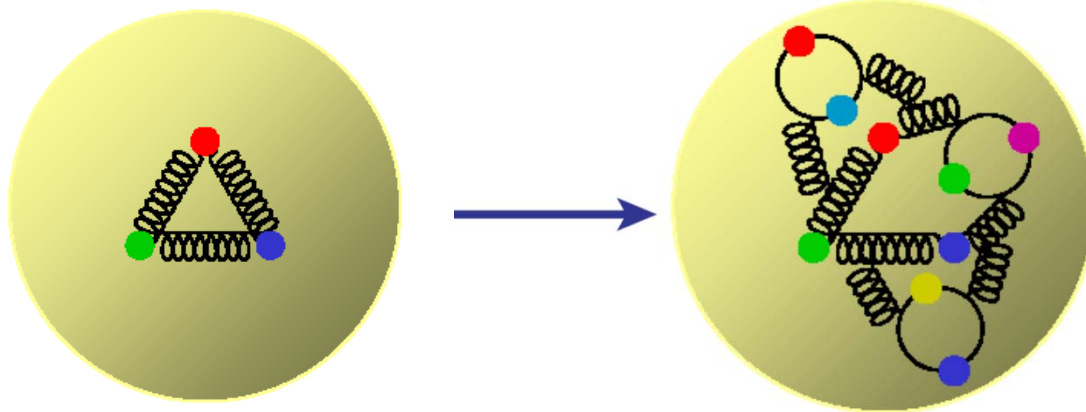


$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

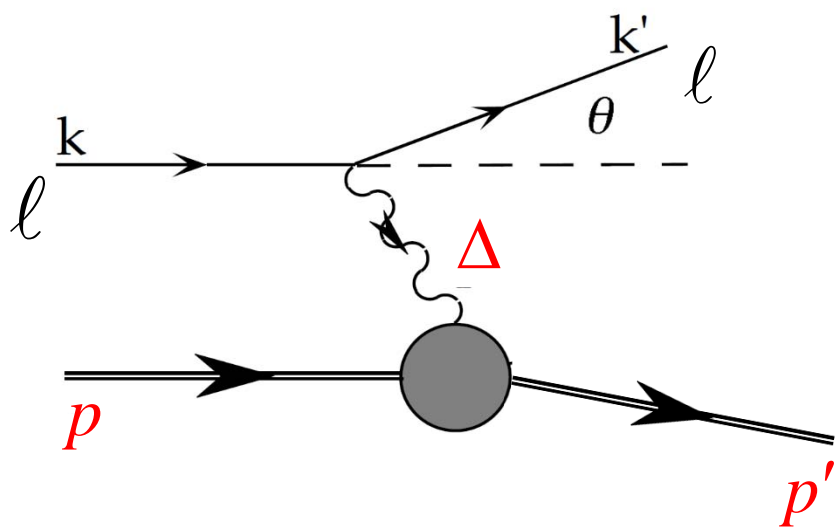
$$\langle N(\mathbf{p}') | \bar{\psi}(0) \gamma^\mu \psi(0) | N(\mathbf{p}) \rangle \sim \bar{u}(\mathbf{p}') \left\{ \gamma^\mu F_1(\Delta^2) + \frac{i\sigma^{\nu\mu} \Delta_\nu}{2M_N} F_2(\Delta^2) \right\} u(\mathbf{p})$$

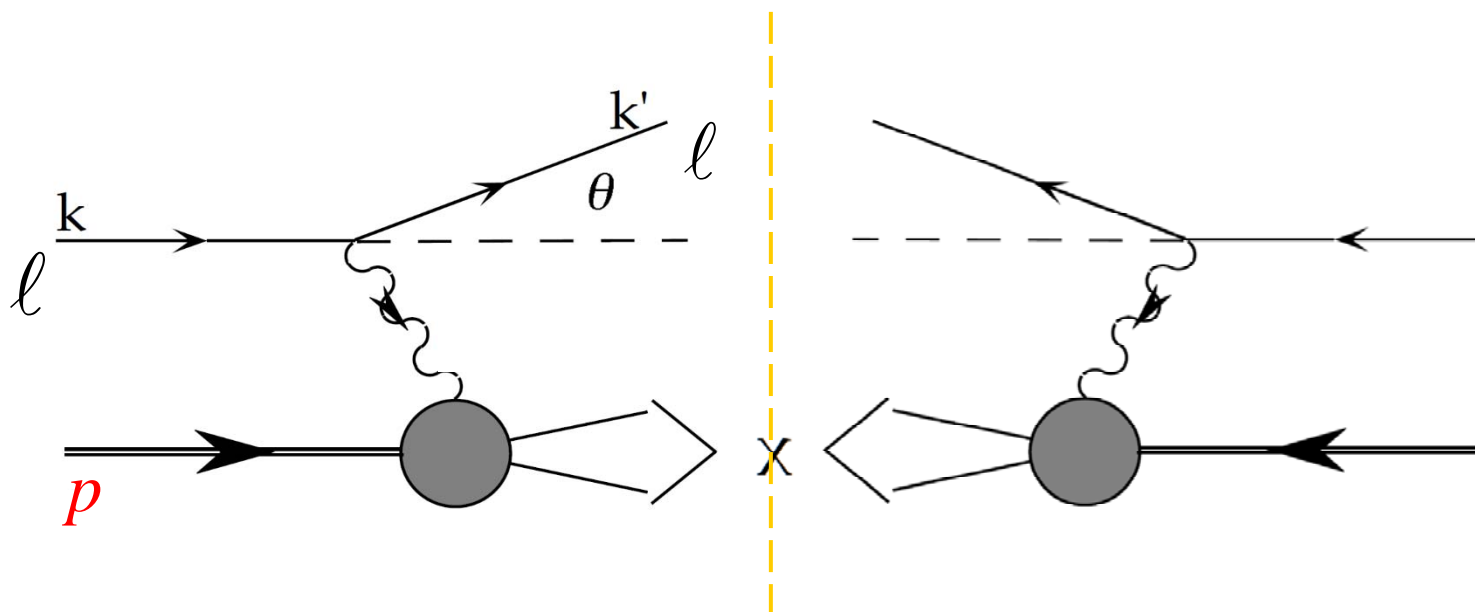
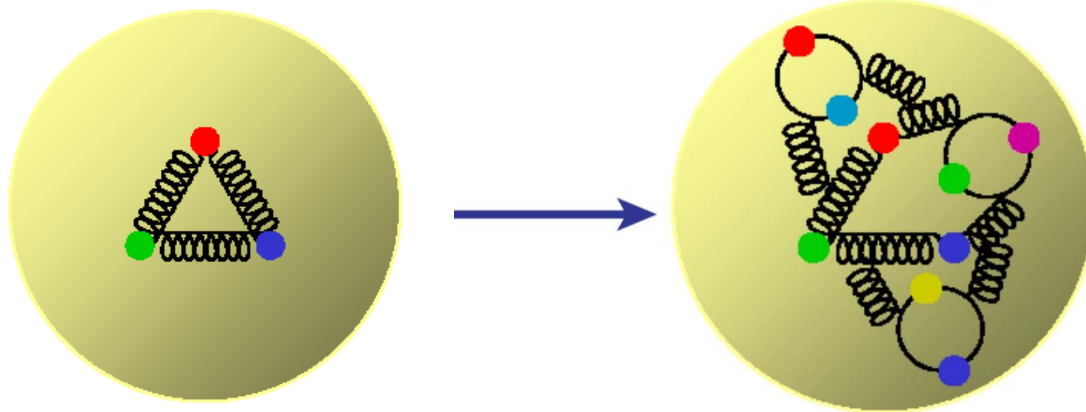
$$\Delta = \mathbf{p}' - \mathbf{p}$$

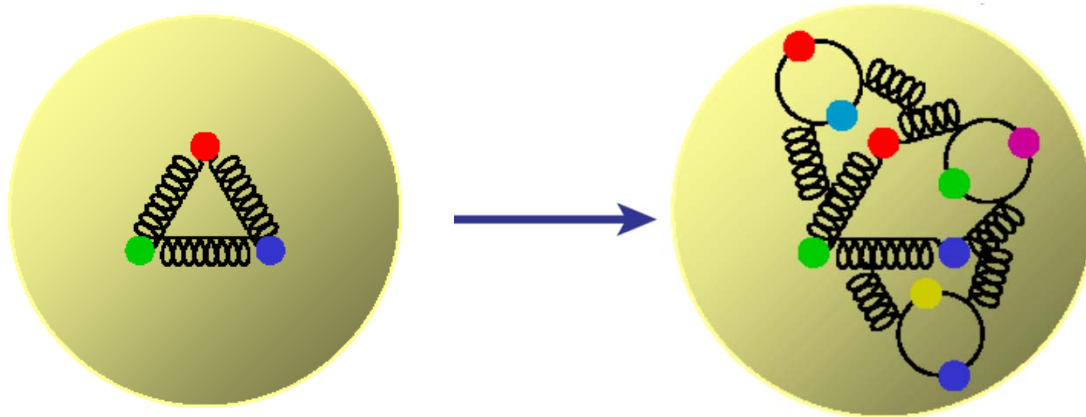




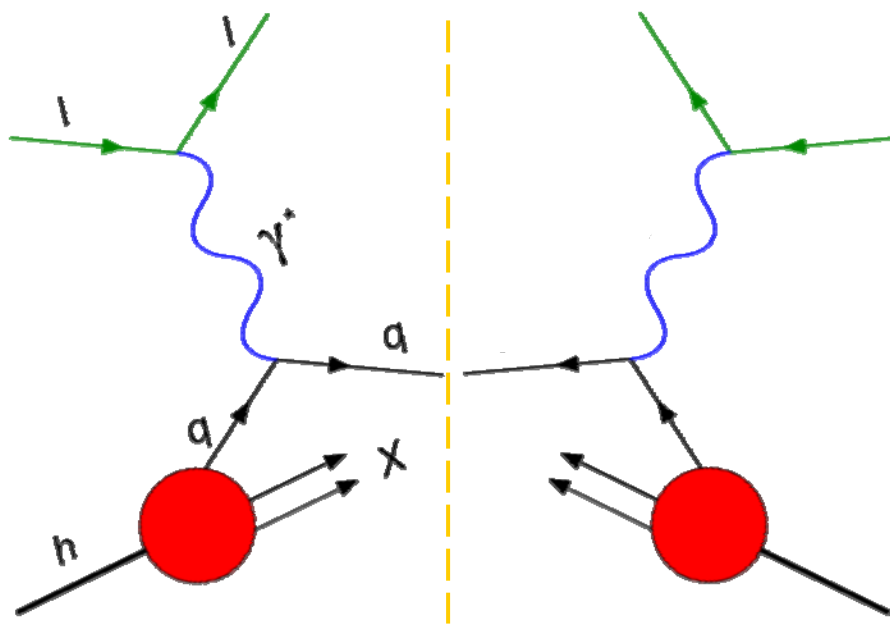
$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$



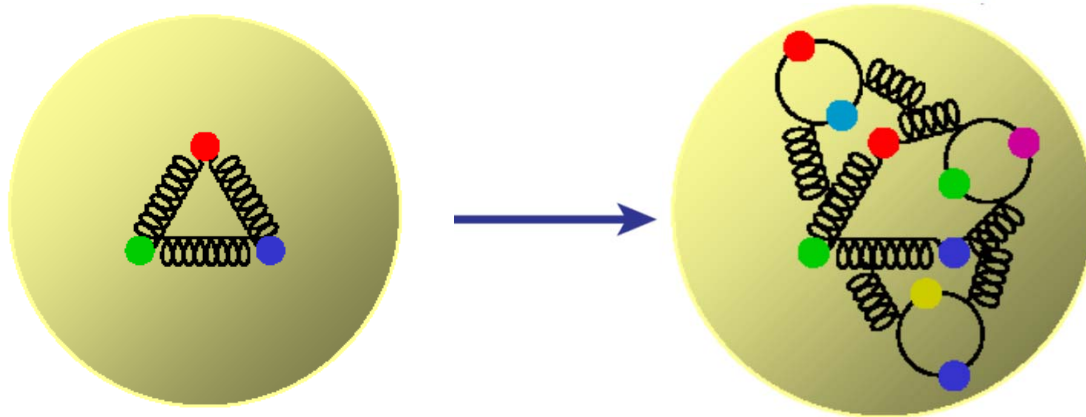




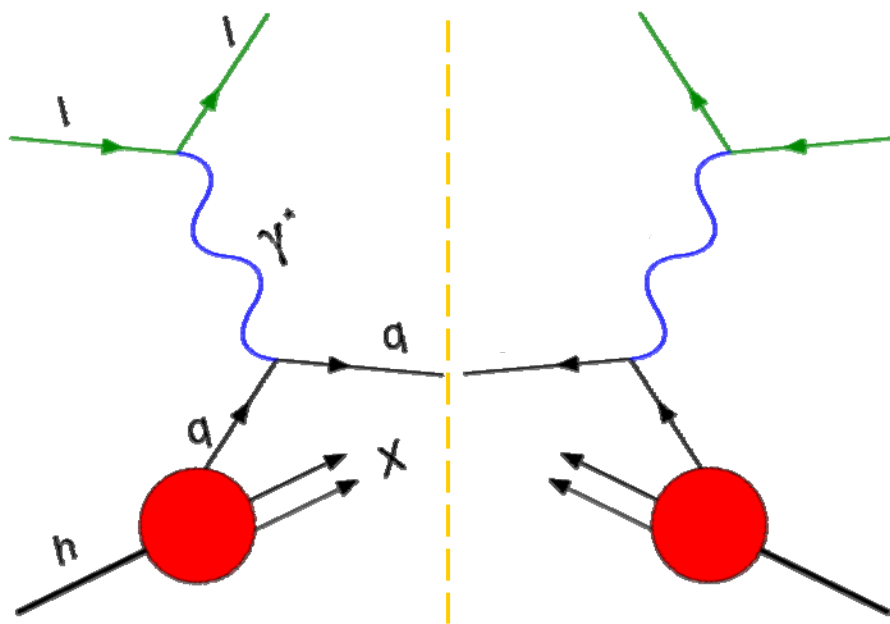
$$\langle N(p) | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$

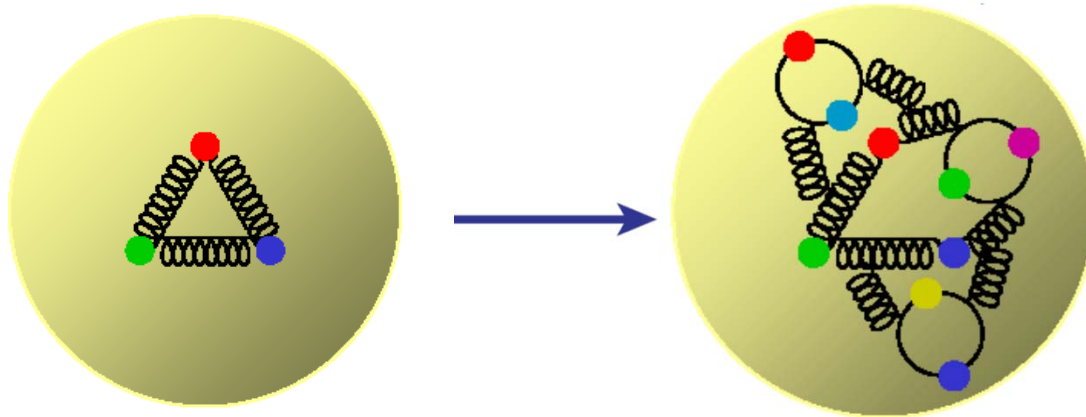




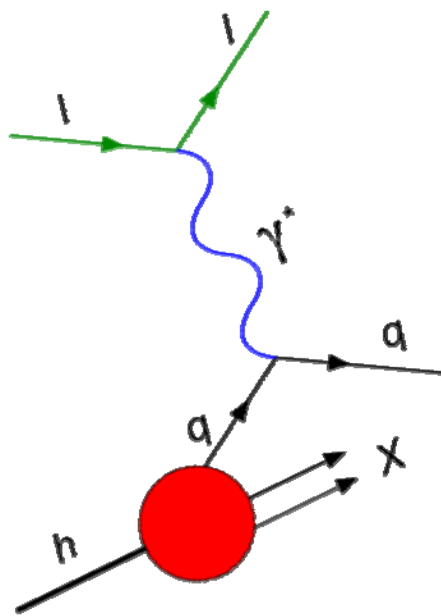


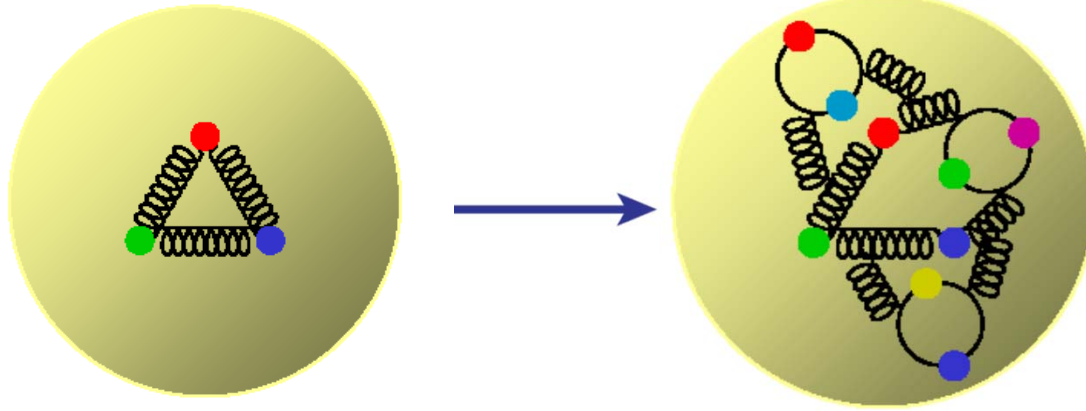
$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$



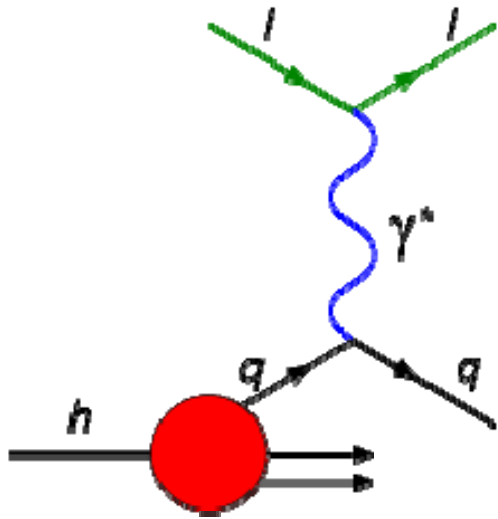


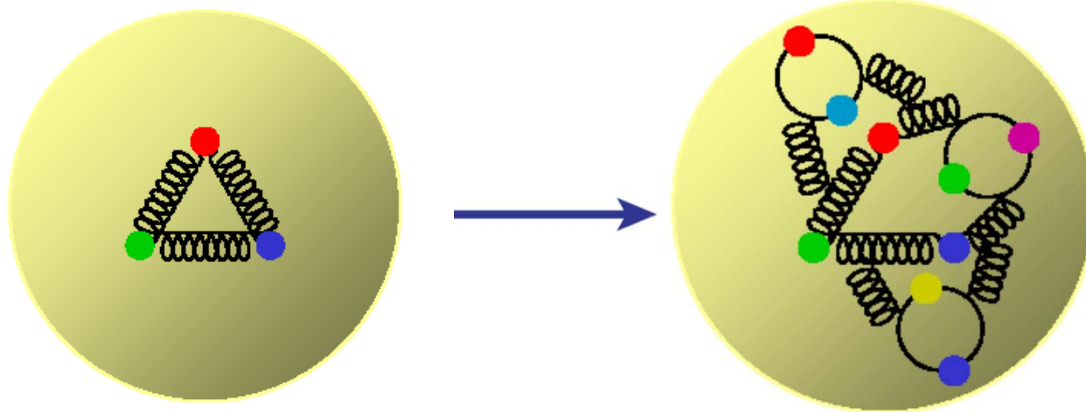
$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$



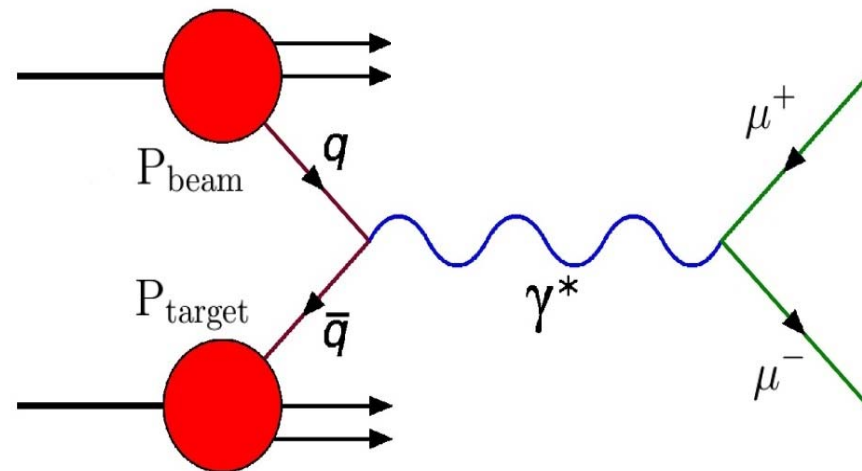
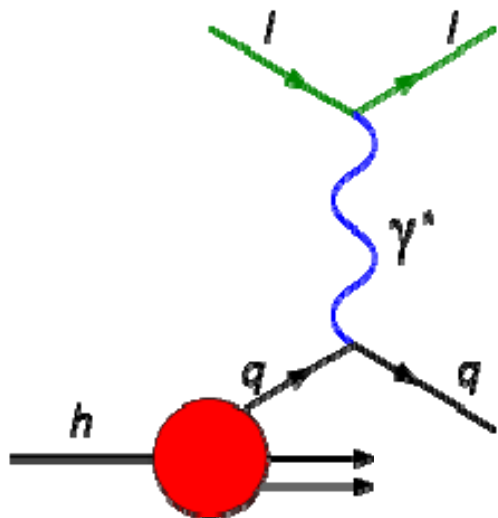


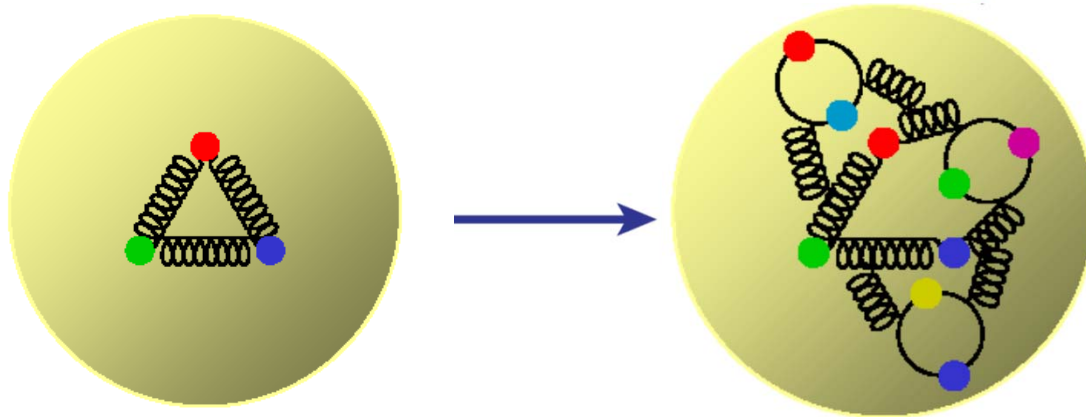
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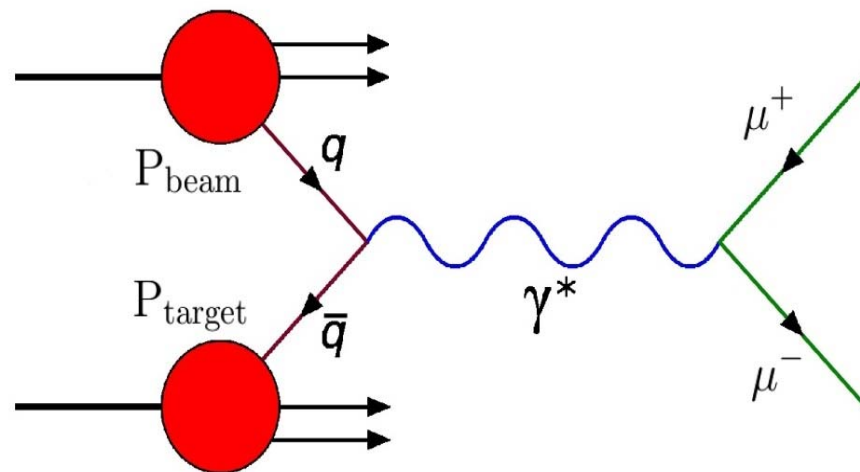


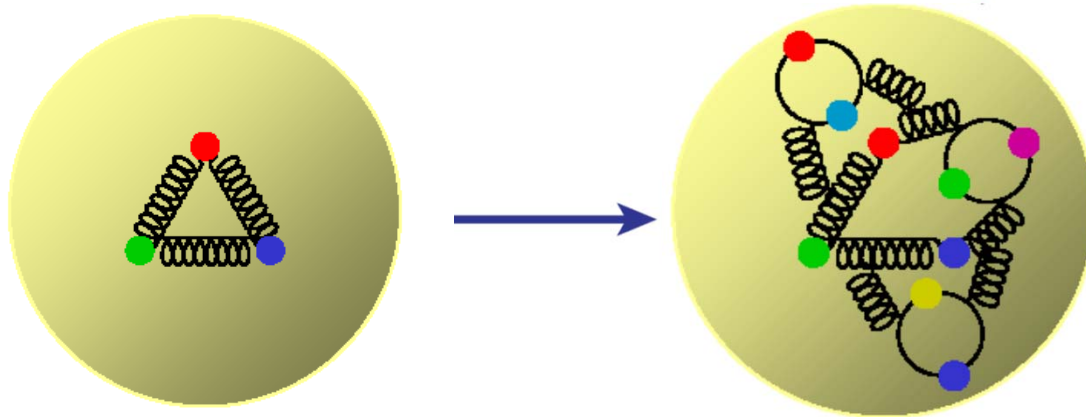
$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$





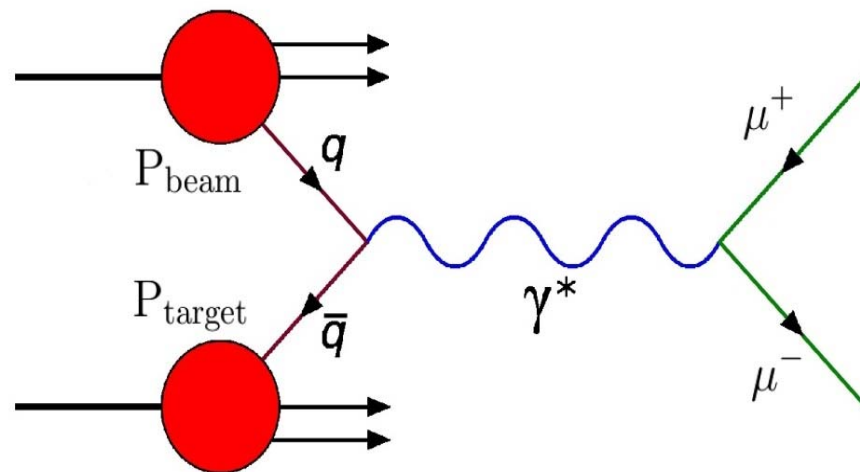
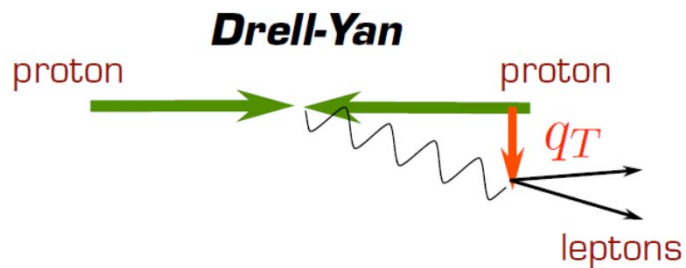
$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$

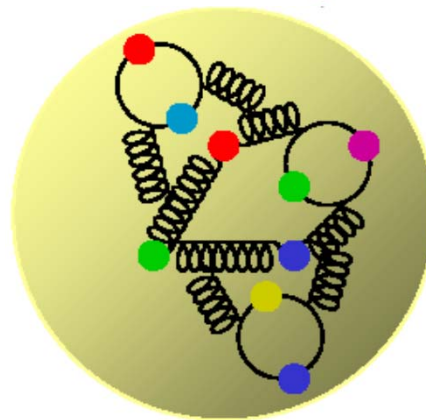
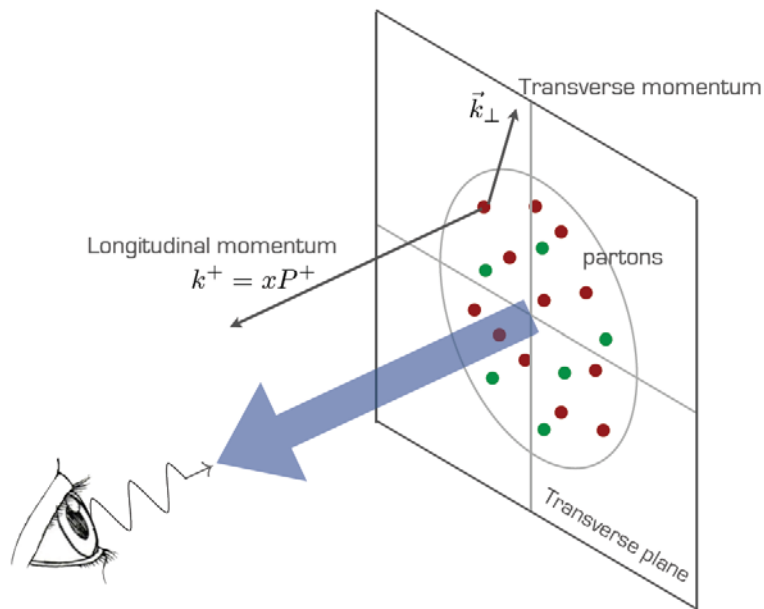




$$\int d^2 z_{\perp} e^{-ik_{\perp} \cdot z_{\perp}} \int dz^- e^{ixp^- z^-} \langle N(p) | \psi^{\dagger}(0) \psi(z^-, z_{\perp}) | N(p) \rangle$$

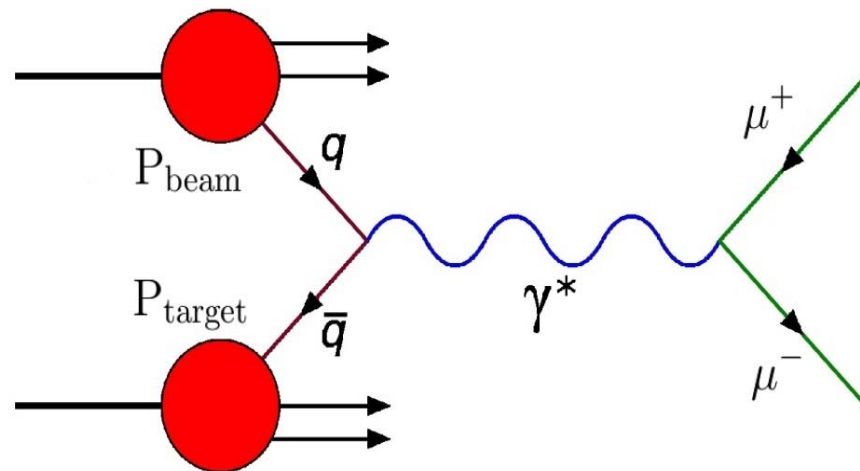
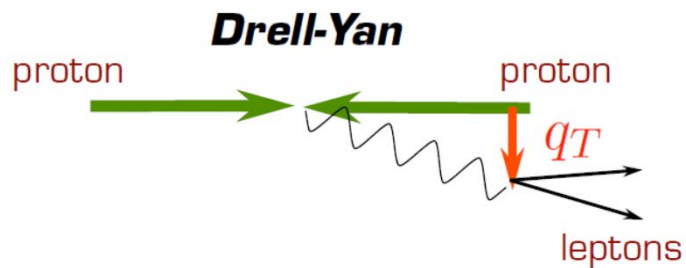
TMD

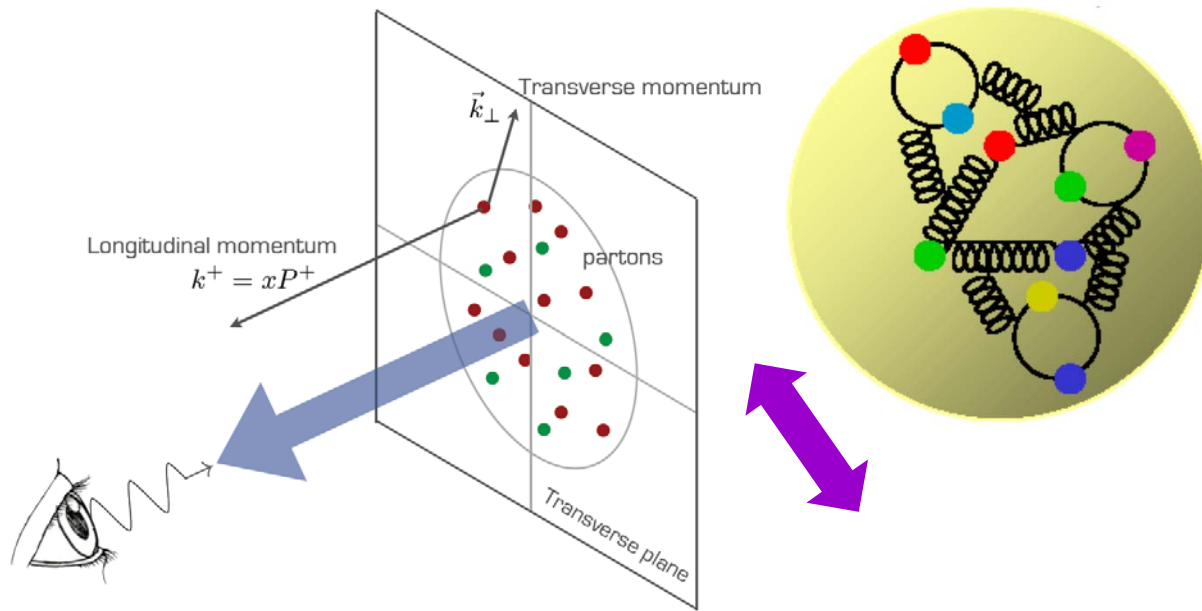




$$\int d^2 z_{\perp} e^{-i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \int dz^{-} e^{ixp z^{-}} \langle N(p) | \psi^{\dagger}(0) \psi(z^{-}, \mathbf{z}_{\perp}) | N(p) \rangle$$

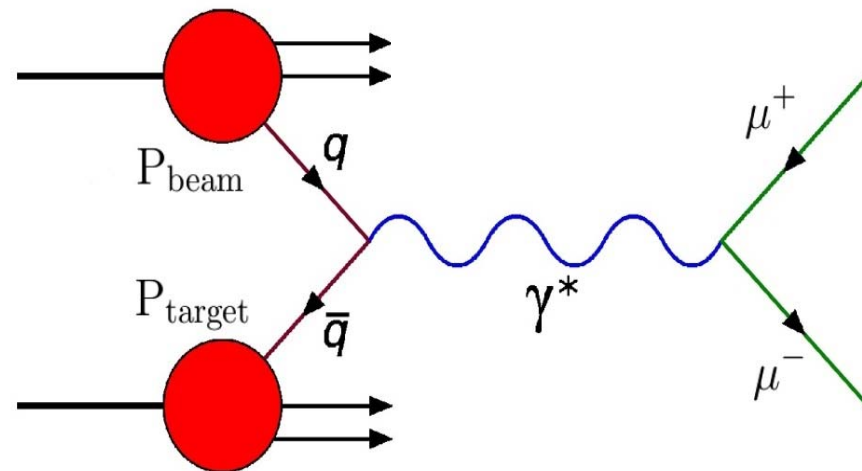
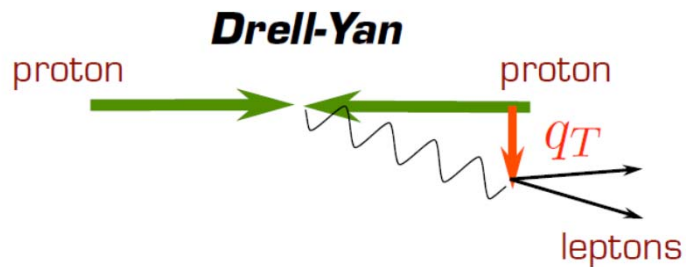
TMD





$$\int d^2 z_{\perp} e^{-i\vec{k}_{\perp} \cdot \vec{z}_{\perp}} \int dz^{-} e^{ixpz^{-}} \langle N(p) | \psi^{\dagger}(0) \psi(z^{-}, z_{\perp}) | N(p) \rangle$$

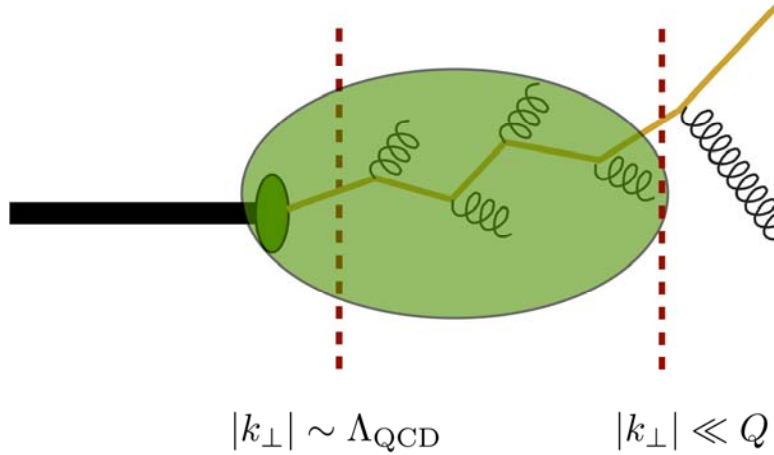
TMD





“intrinsic”  
transverse  
momentum

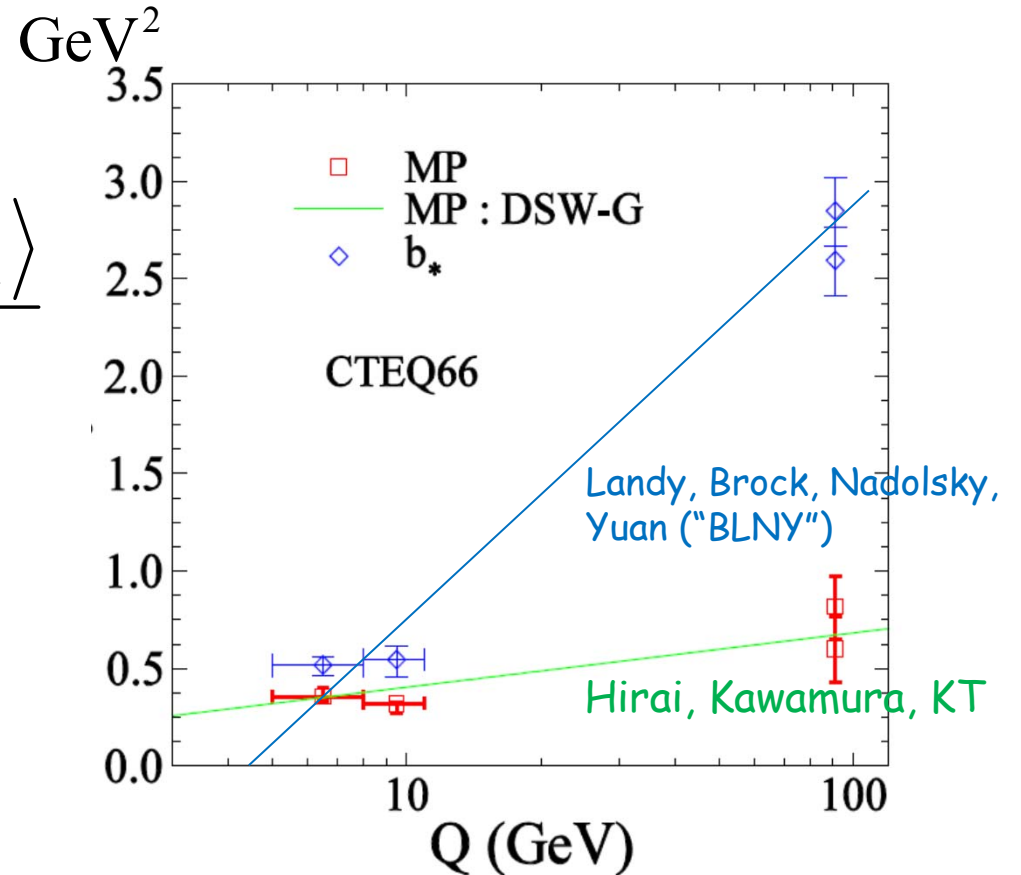
soft and collinear  
gluon radiation



$Q$ : dilepton mass

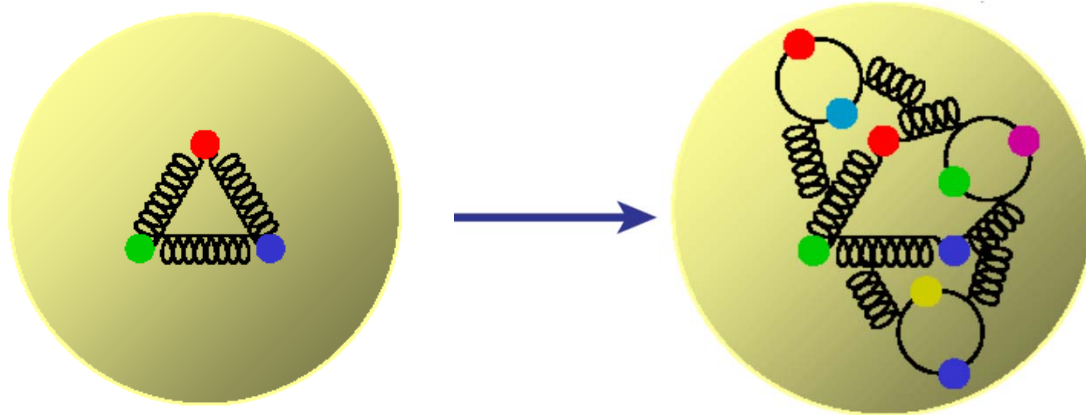
Global fit

$$\frac{\langle k_{\perp}^2 \rangle}{2}$$



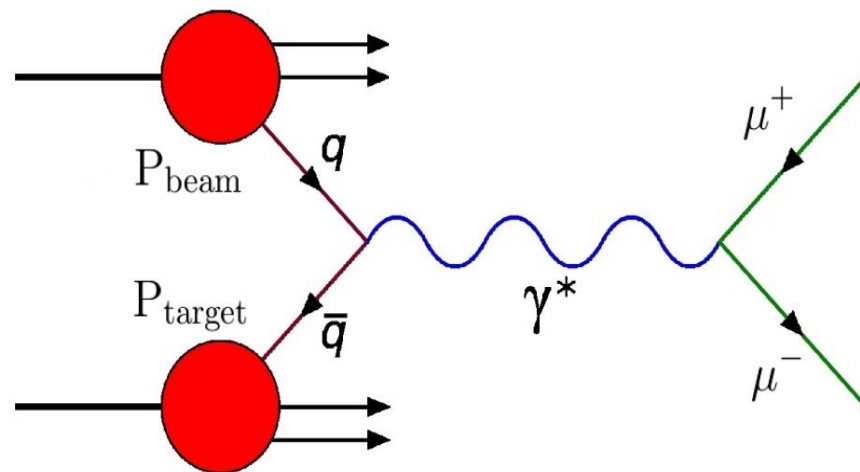
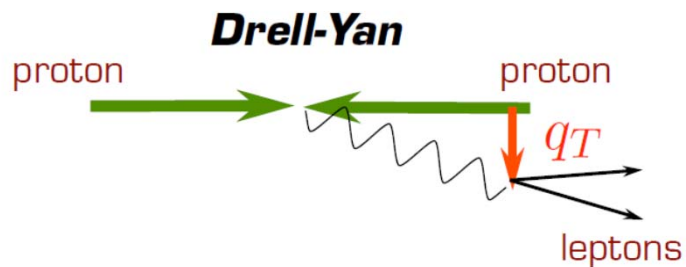
Experimental data sets

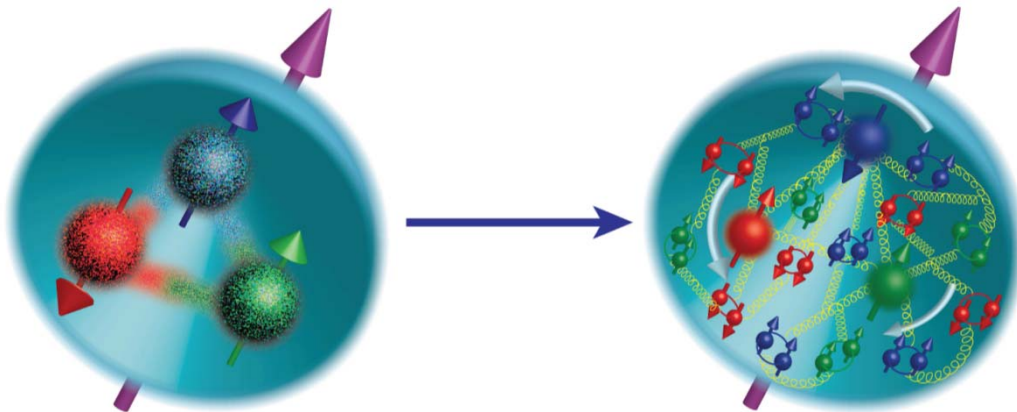
	Exp	$\sqrt{s}$ (GeV)	Target	Q range (GeV)	# of data ( $p_T < 22$ GeV)
DY	R209	62	P-P	5.0 - 8.0	5
	R209	62	P-P	8.0 - 11.0	5
Z <sup>0</sup>	CDF run-0	1800	P-Pbar	75 - 105	7
	CDF run-1	1800	P-Pbar	66 - 116	33
	D0 run-1	1800	P-Pbar	75 - 105	15



$$\int d^2 z_{\perp} e^{-ik_{\perp} \cdot z_{\perp}} \int dz^- e^{ixp z^-} \langle N(p) | \psi^{\dagger}(0) \psi(z^-, z_{\perp}) | N(p) \rangle$$

TMD





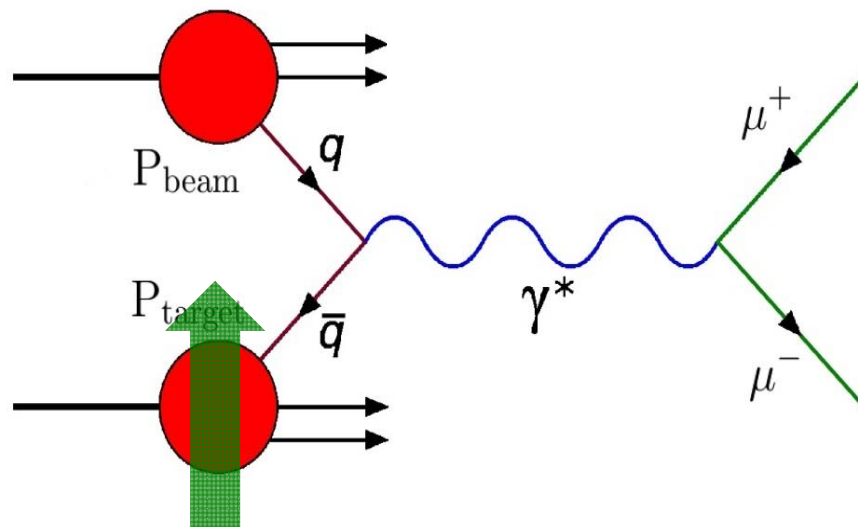
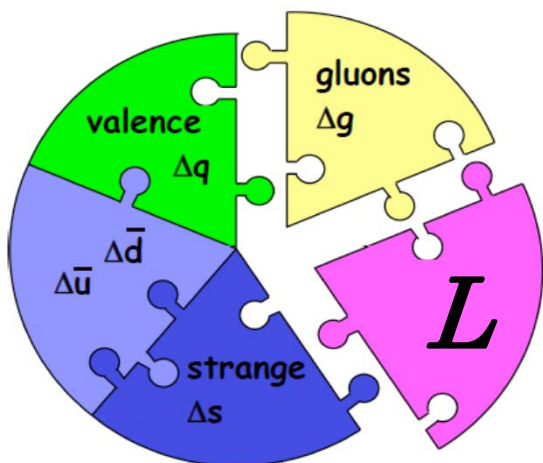
## Sivers

$$f_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p, S_\perp) | \psi^\dagger(0) \psi(z^-, z_\perp) | N(p, S_\perp) \rangle$$



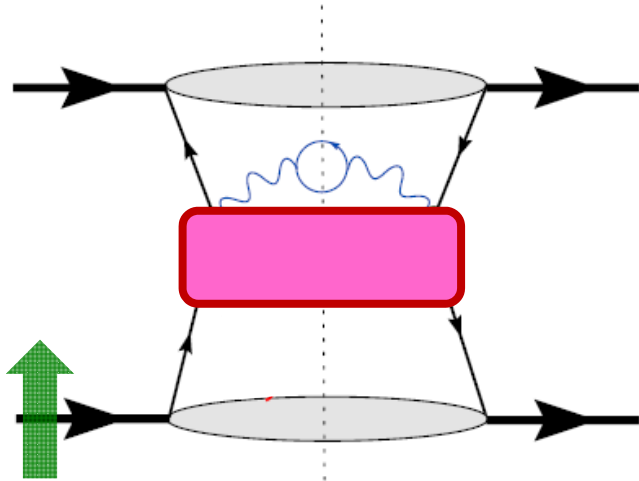
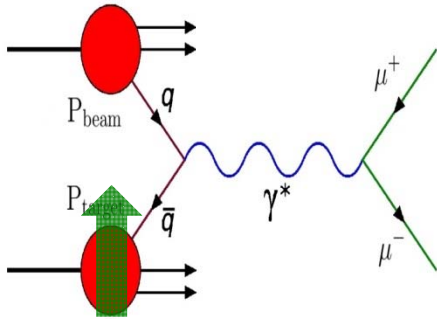
$L_q$

$$S = \frac{1}{2} = L + \frac{1}{2} \Delta \Sigma + \Delta G$$



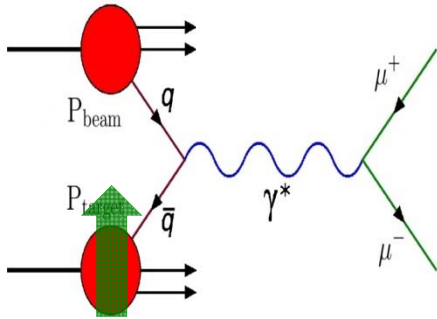
$$f_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p, \mathbf{S}_\perp) | \psi^\dagger(0) \psi(z^-, \mathbf{z}_\perp) | N(p, \mathbf{S}_\perp) \rangle$$

**Sivers**

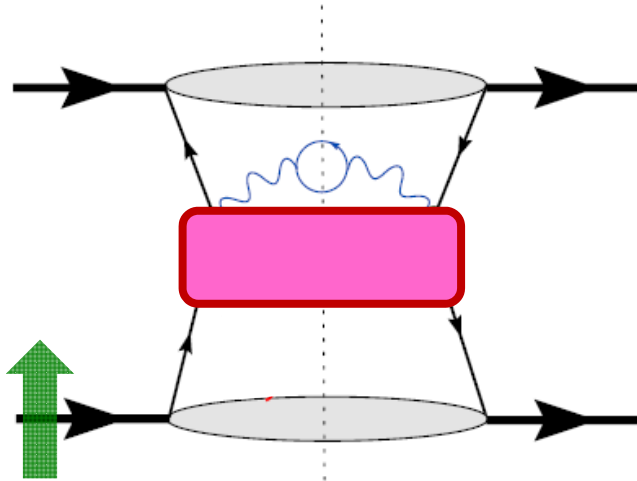


$$f_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p, \mathbf{S}_\perp) | \psi^\dagger(0) \psi(z^-, z_\perp) | N(p, \mathbf{S}_\perp) \rangle$$

**Sivers**

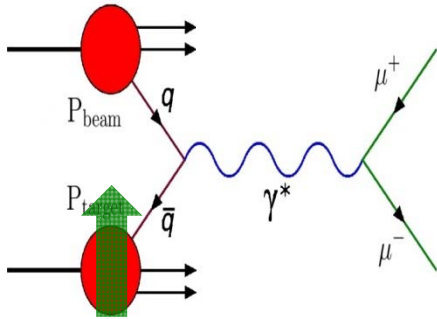


*"T-odd"*

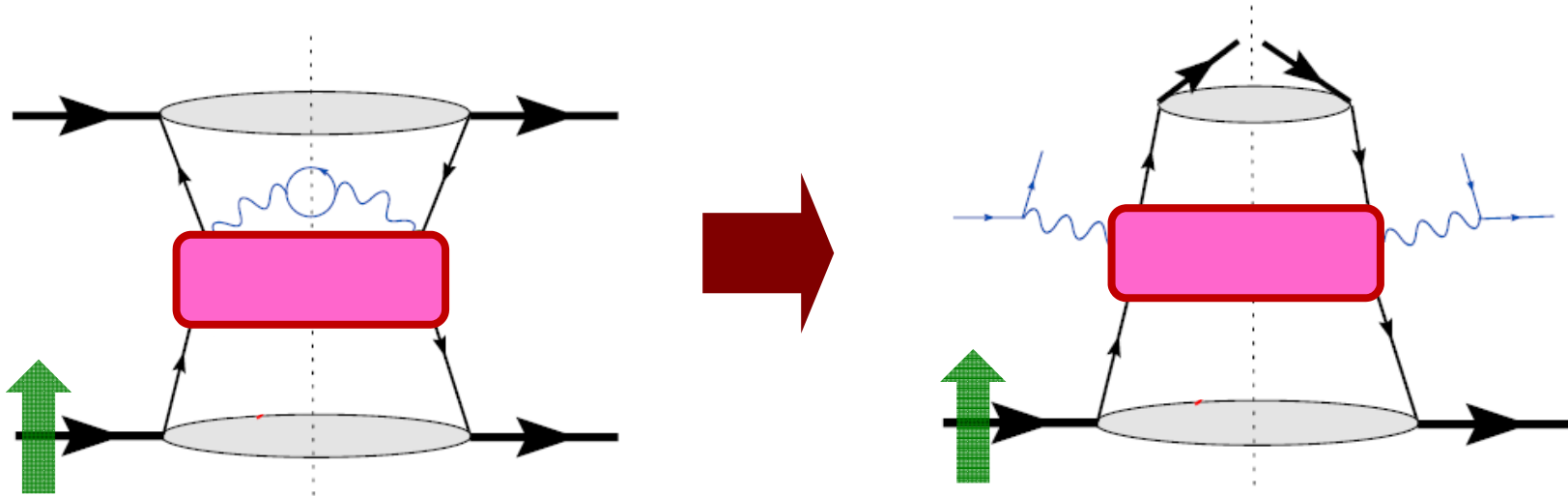


$$f_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p, \mathbf{S}_\perp) | \psi^\dagger(0) \psi(z^-, \mathbf{z}_\perp) | N(p, \mathbf{S}_\perp) \rangle$$

**Sivers**



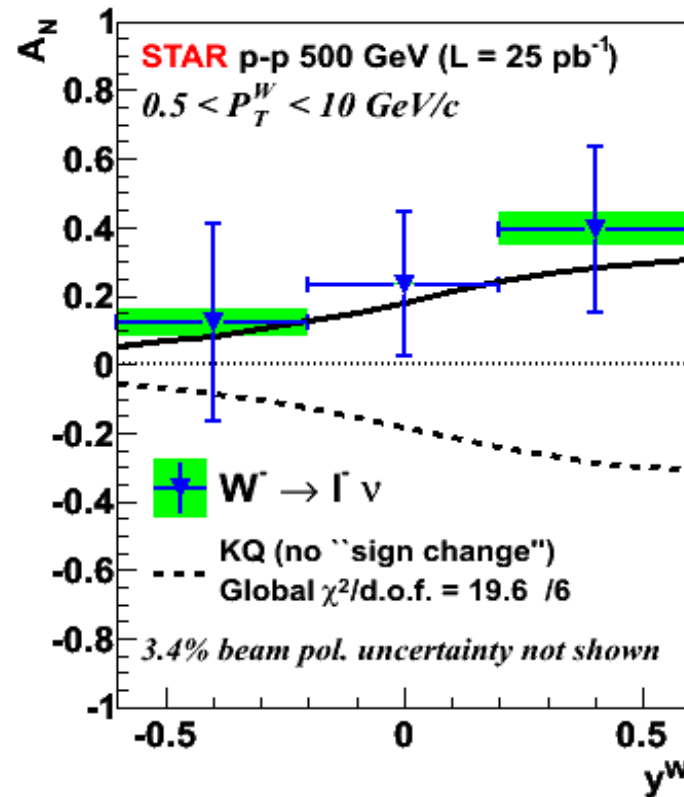
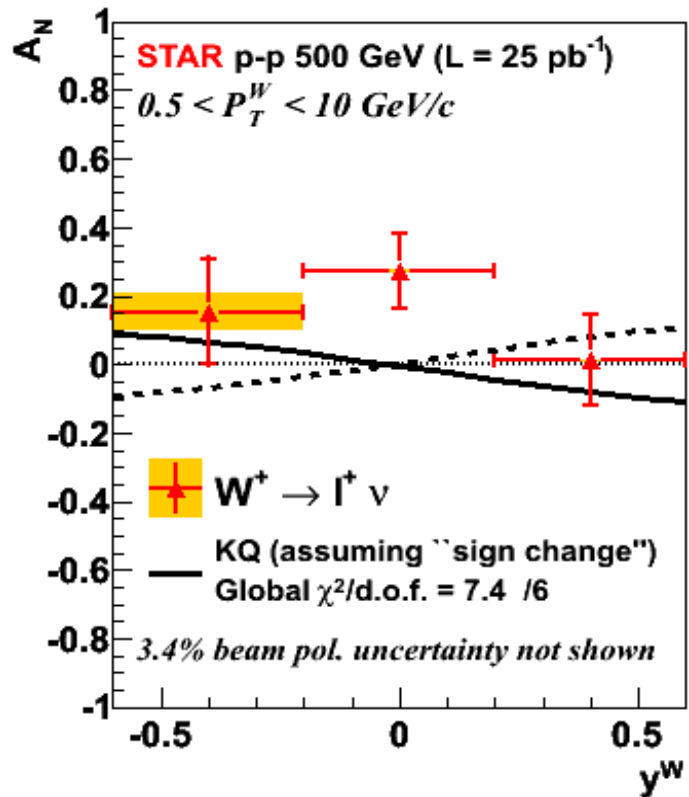
**"T-odd"**



$$f_1^\perp(x, k_\perp) \Big|_{DY} = - f_1^\perp(x, k_\perp) \Big|_{SIDIS}$$

# W production $p^\uparrow + p \rightarrow W^\pm$ at RHIC

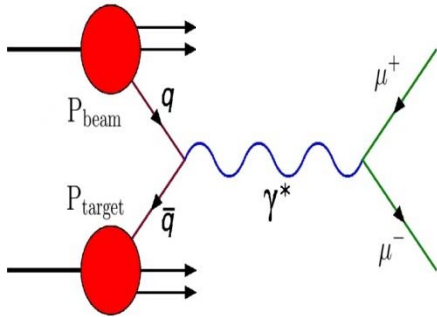
Run-2009  
(PRL116, 132301)



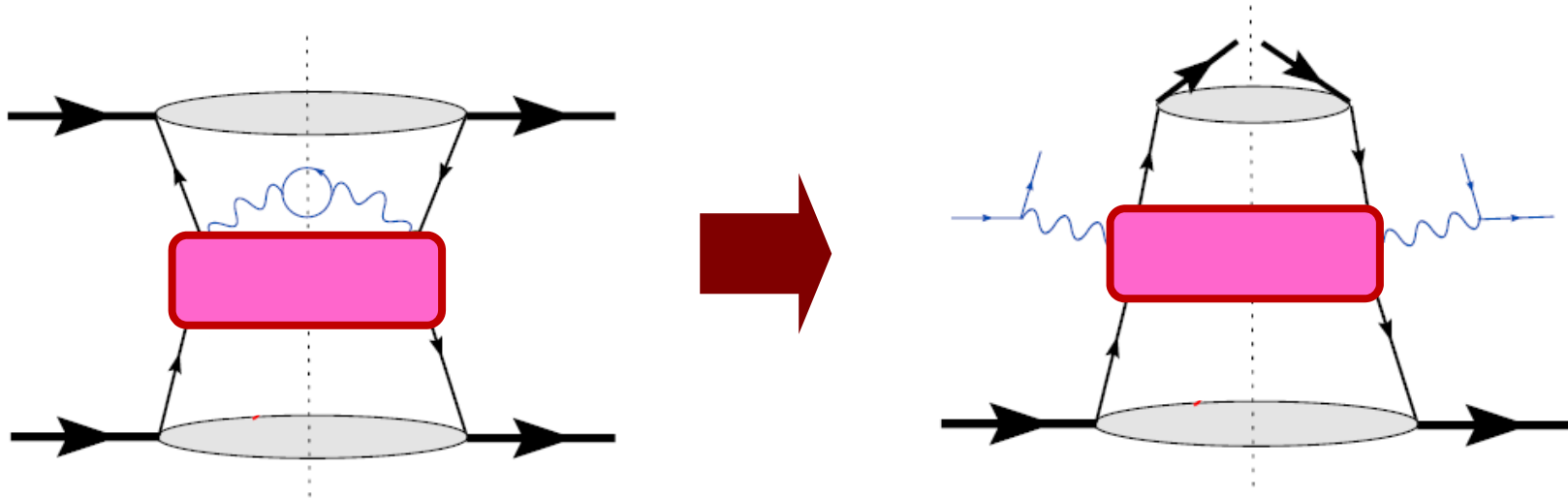
$$f_1^\perp(x, k_\perp) \Big|_{DY} = - f_1^\perp(x, k_\perp) \Big|_{SIDIS}$$

$$h_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p) | \psi^\dagger(0) \gamma^\perp \psi(z^-, z_\perp) | N(p) \rangle$$

Boer-Mulders



"*T-odd*"

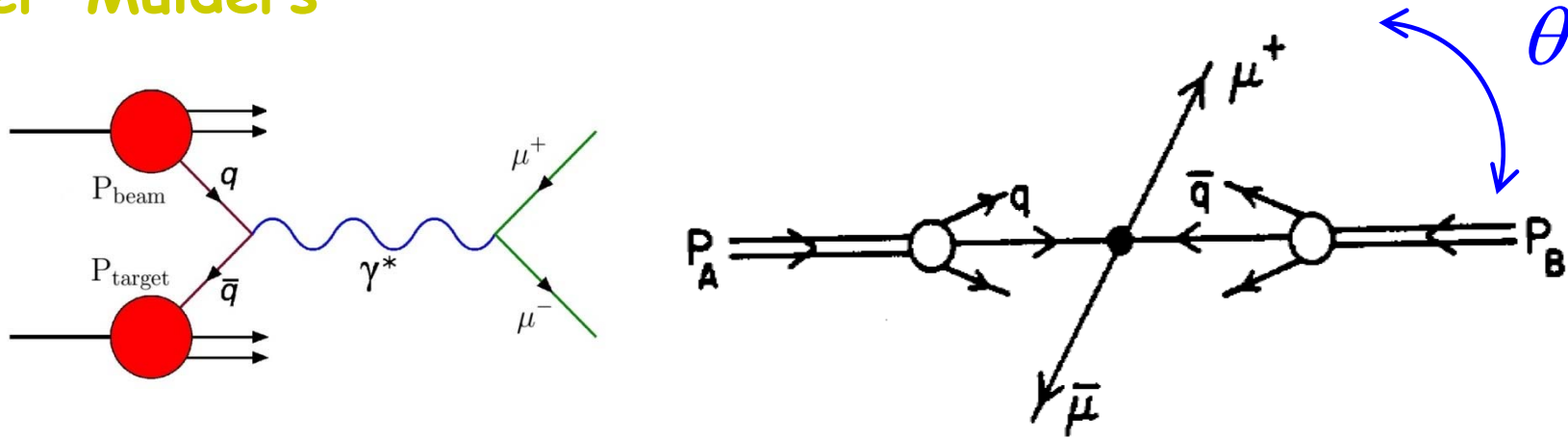


$$h_1^\perp(x, k_\perp) \Big|_{DY} = - h_1^\perp(x, k_\perp) \Big|_{SIDIS}$$



$$h_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p) | \psi^\dagger(0) \gamma^\perp \psi(z^-, z_\perp) | N(p) \rangle$$

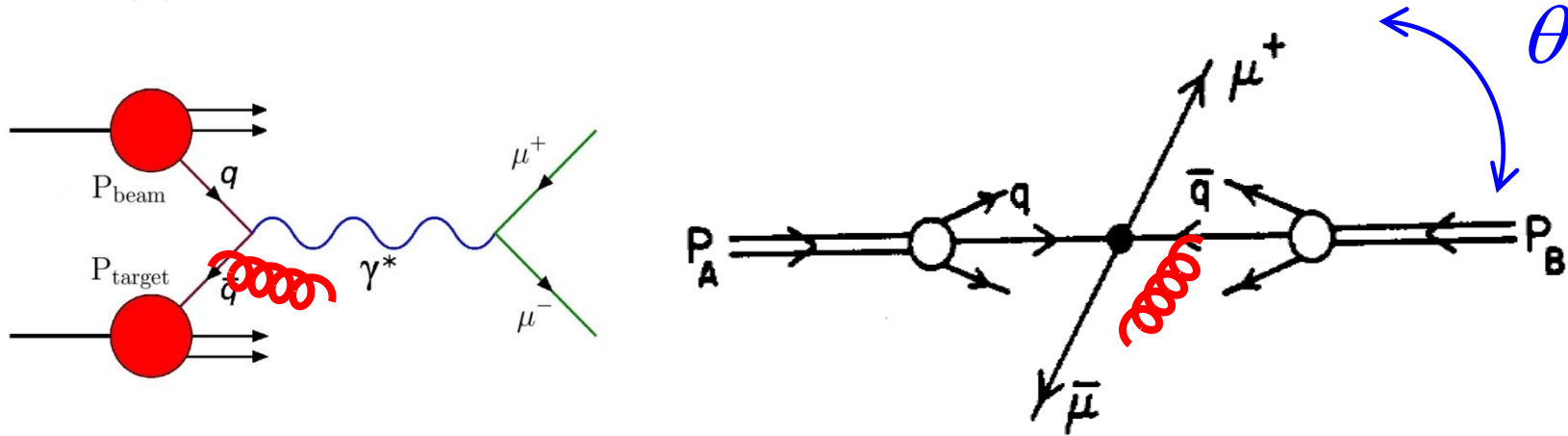
## Boer-Mulders



$$\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

$$h_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixpz} \langle N(p) | \psi^\dagger(0) \gamma^\perp \psi(z^-, z_\perp) | N(p) \rangle$$

## Boer-Mulders



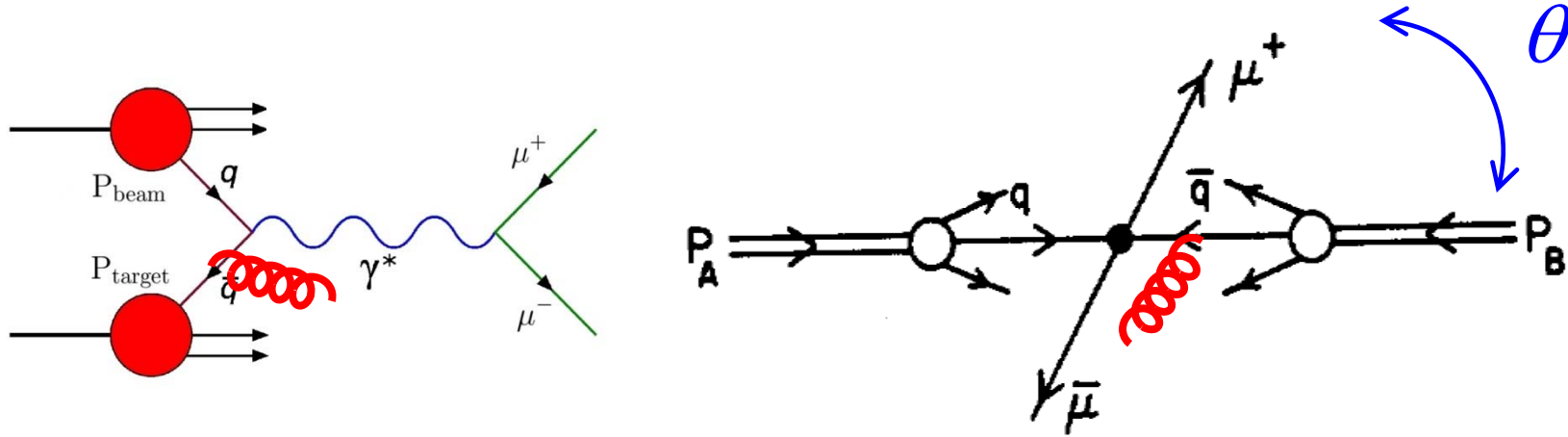
$$\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

$$1 - \lambda, \mu, \nu \propto \alpha_S q_T$$

$$1 - \lambda - 2\nu = 0 \quad \text{Lam-Tung relation ('78)}$$

$$h_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p) | \psi^\dagger(0) \gamma^\perp \psi(z^-, z_\perp) | N(p) \rangle$$

## Boer-Mulders



$$\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

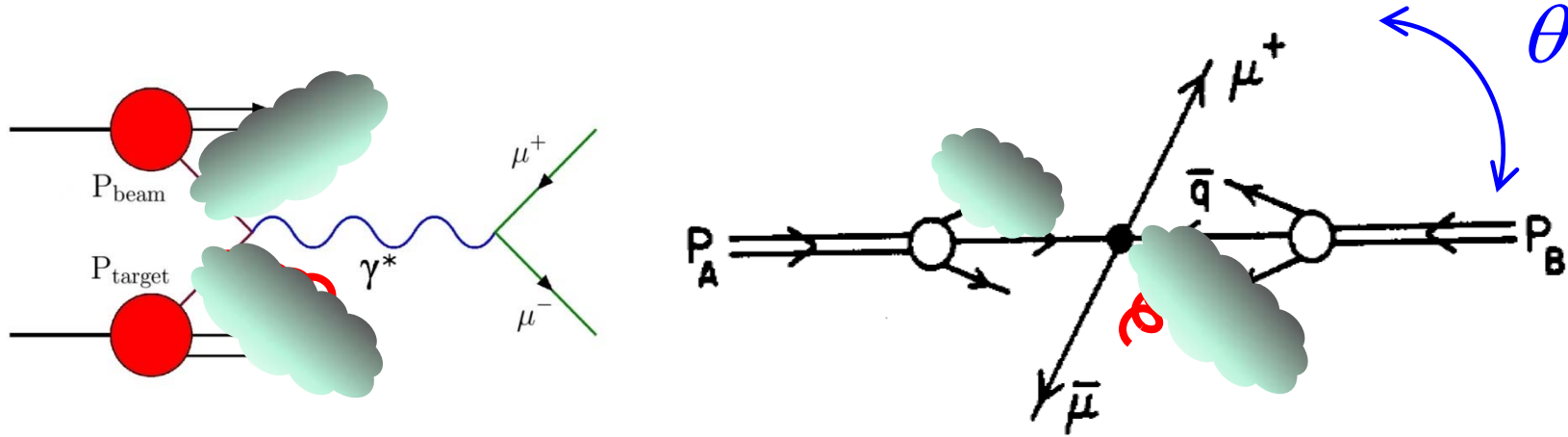
$$\nu \gg \alpha_S q_T$$

$$1 - \lambda - 2\nu \neq 0 \quad \text{Lam-Tung relation ('78)}$$

$$h_1^\perp(x, k_\perp)$$

$$h_1^\perp(x, k_\perp) \sim \int d^2 z_\perp e^{-ik_\perp \cdot z_\perp} \int dz^- e^{ixp z^-} \langle N(p) | \psi^\dagger(0) \gamma^\perp \psi(z^-, z_\perp) | N(p) \rangle$$

## Boer-Mulders

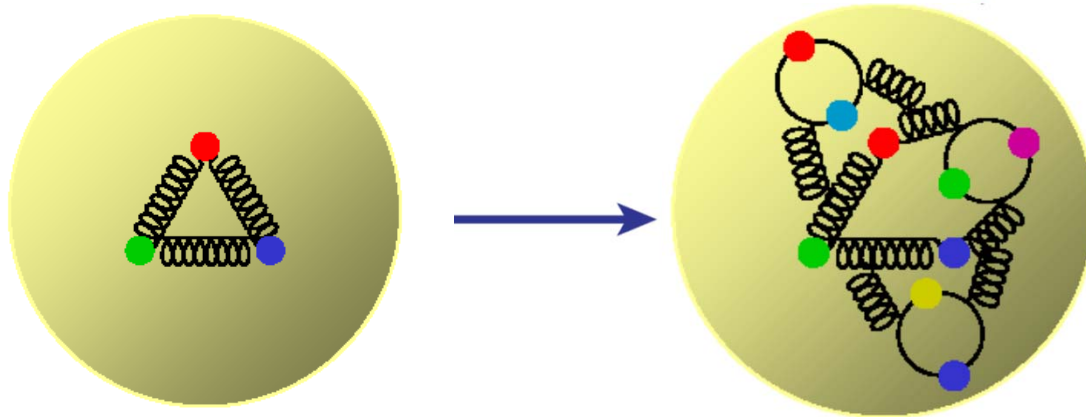


$$\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{v}{2} \sin^2 \theta \cos 2\phi$$

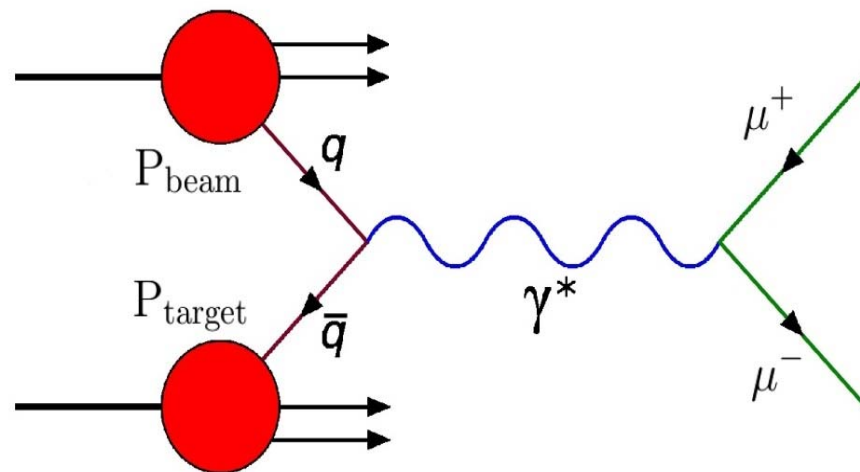
$$v \gg \alpha_S q_T$$

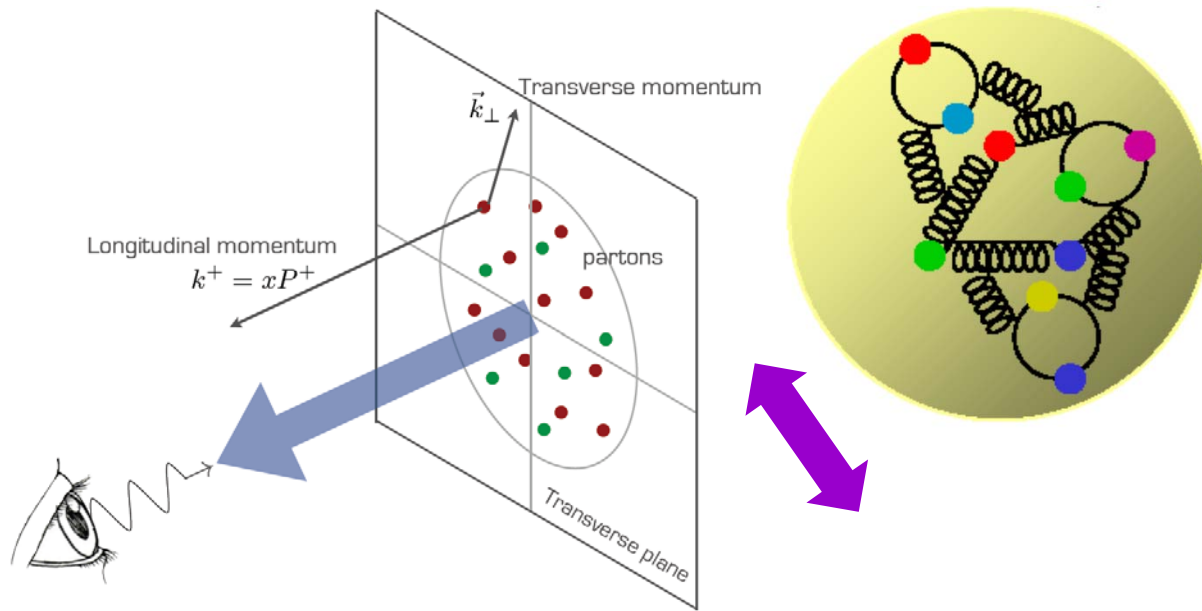
$$1 - \lambda - 2v \neq 0 \quad \text{Lam-Tung relation ('78)}$$

$h_1^\perp(x, k_\perp)$  and/or novel nonpert. mechanisms



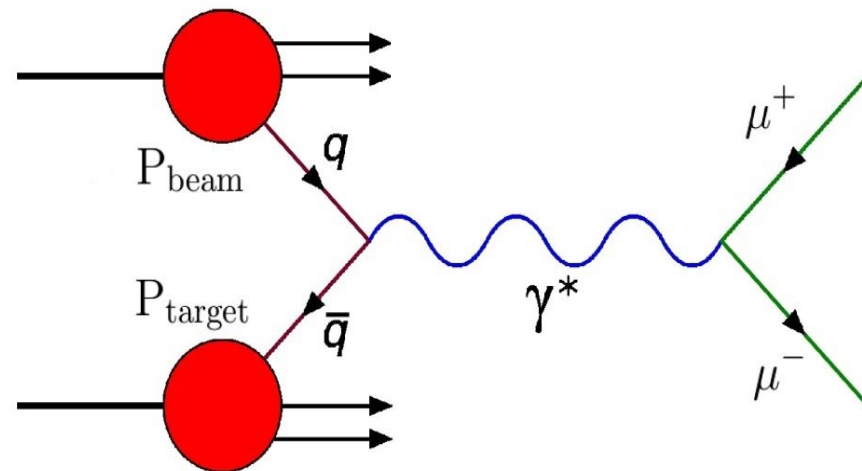
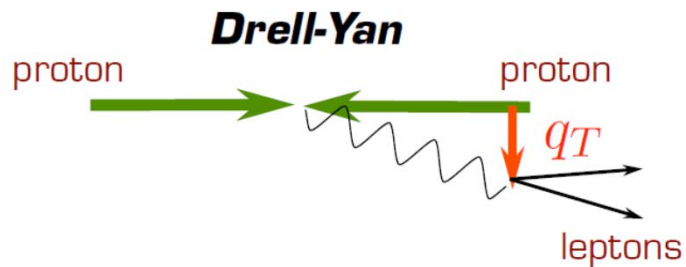
$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$

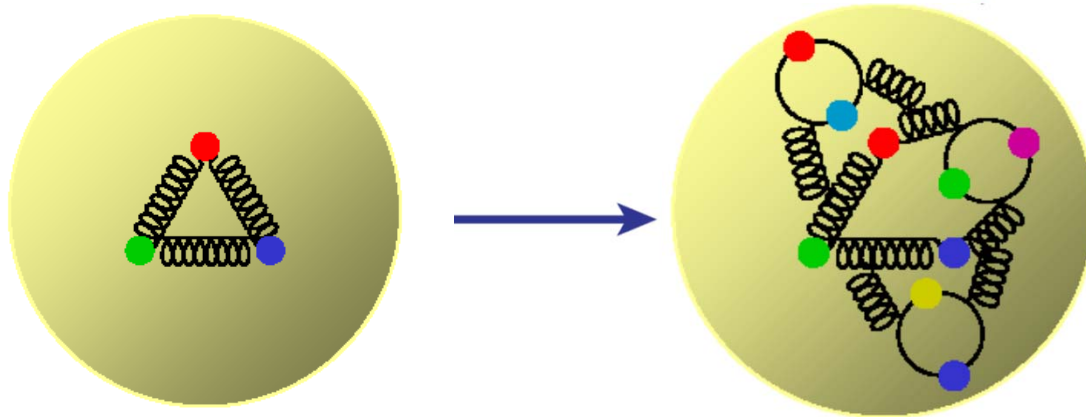




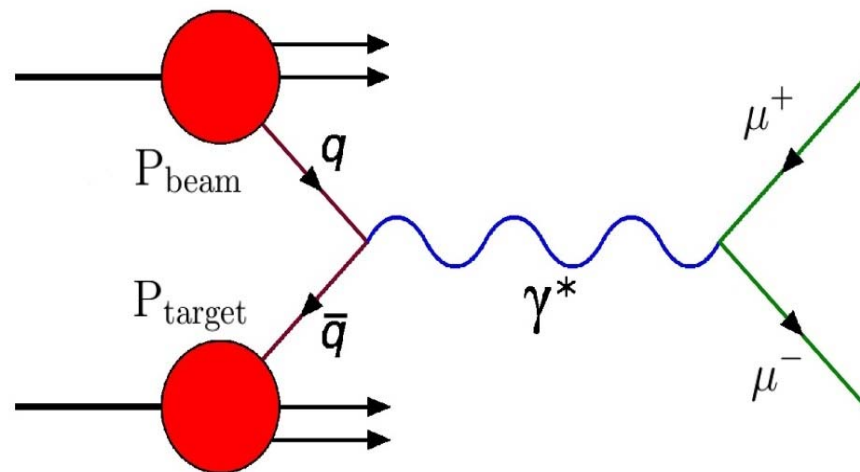
$$\int d^2 z_{\perp} e^{-i\vec{k}_{\perp} \cdot \vec{z}_{\perp}} \int dz^{-} e^{ixp z^{-}} \langle N(p) | \psi^{\dagger}(0) \psi(z^{-}, z_{\perp}) | N(p) \rangle$$

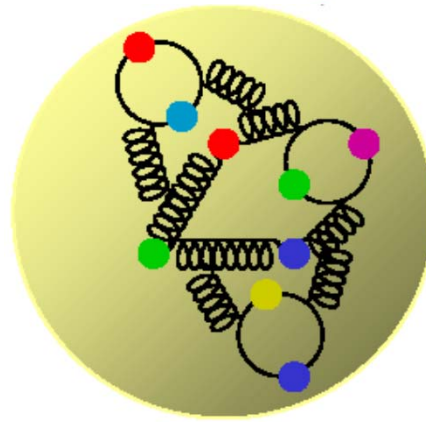
TMD



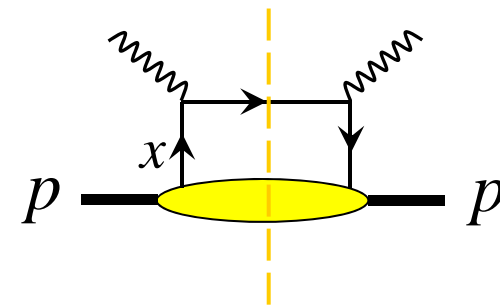


$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$

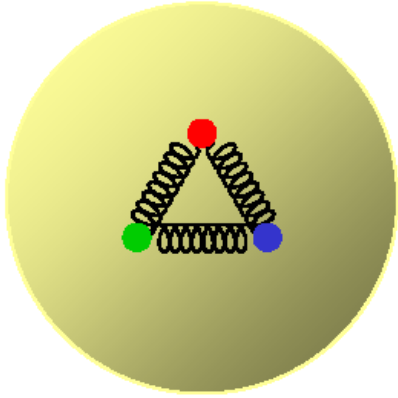




$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(\mathbf{p}) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



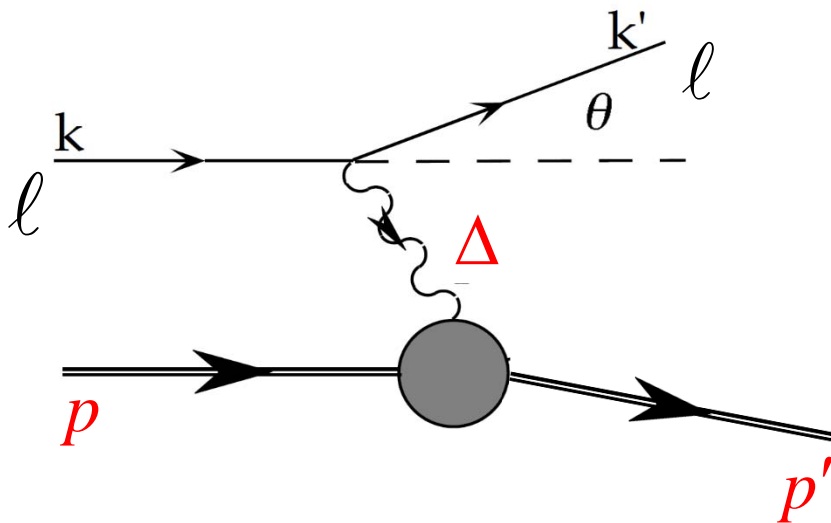


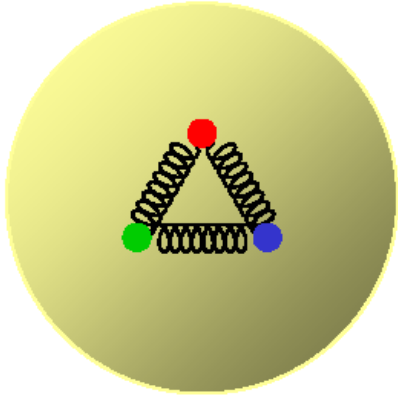


$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

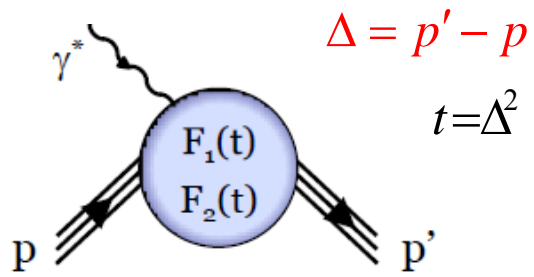
$$\langle N(\mathbf{p}') | \bar{\psi}(0) \gamma^\mu \psi(0) | N(\mathbf{p}) \rangle \sim \bar{u}(\mathbf{p}') \left\{ \gamma^\mu F_1(\Delta^2) + \frac{i\sigma^{\nu\mu} \Delta_\nu}{2M_N} F_2(\Delta^2) \right\} u(\mathbf{p})$$

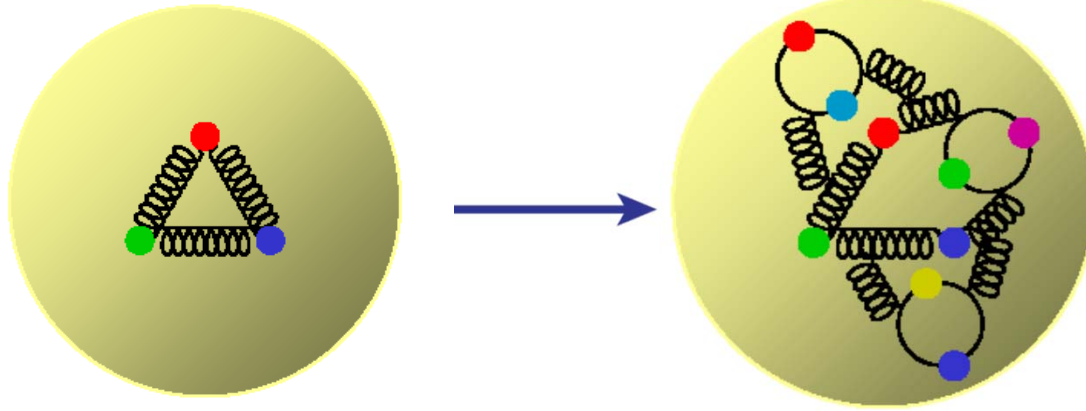
$$\Delta = \mathbf{p}' - \mathbf{p}$$





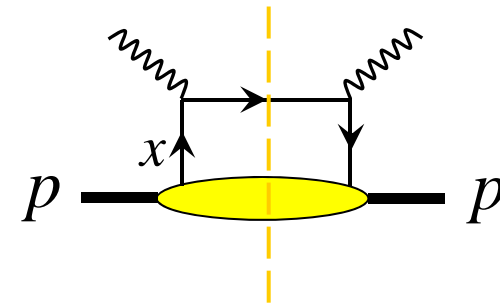
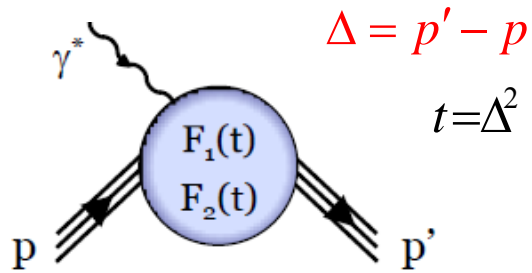
$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

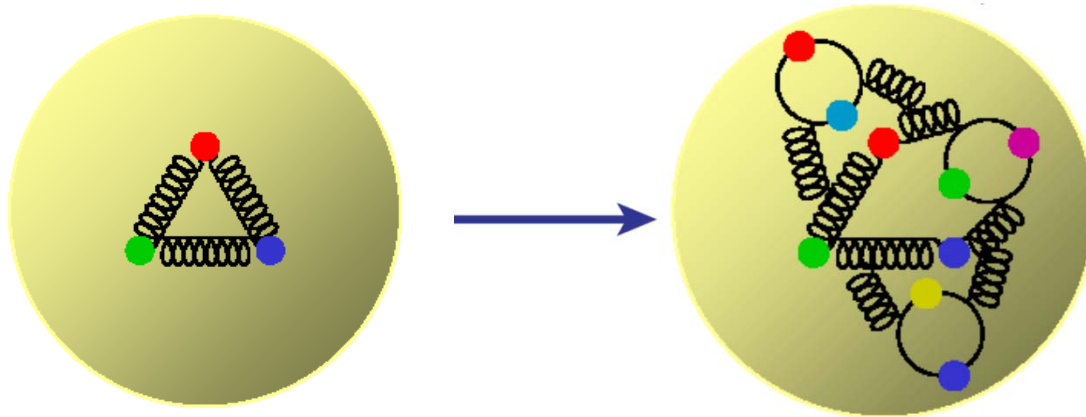




$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

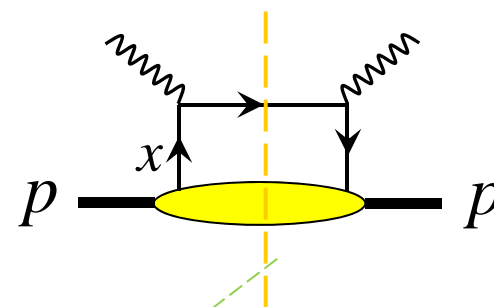
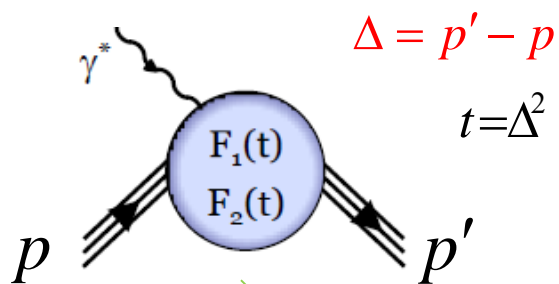
$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(\mathbf{p}) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



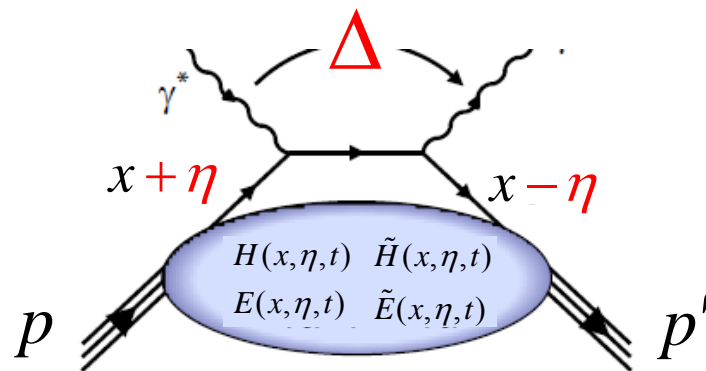


$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

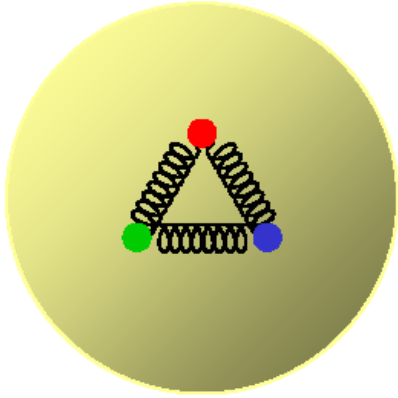
$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(\mathbf{p}) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



**GPD**



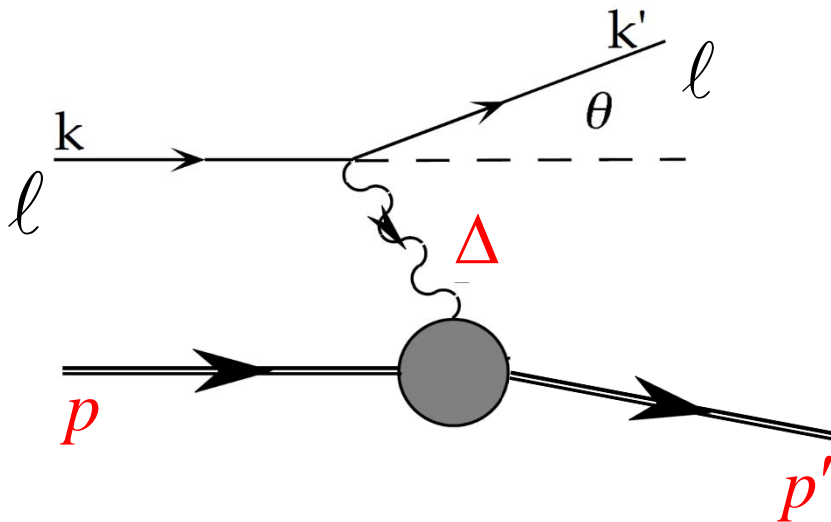
$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta})\mathbf{p}\mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

$$\langle N(\mathbf{p}') | \bar{\psi}(0) \gamma^\mu \psi(0) | N(\mathbf{p}) \rangle \sim \bar{u}(\mathbf{p}') \left\{ \gamma^\mu F_1(\Delta^2) + \frac{i\sigma^{\nu\mu} \Delta_\nu}{2M_N} F_2(\Delta^2) \right\} u(\mathbf{p})$$

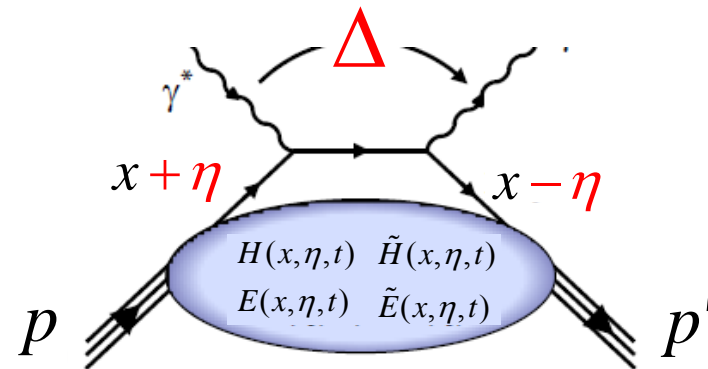
$$\Delta = \mathbf{p}' - \mathbf{p}$$



$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

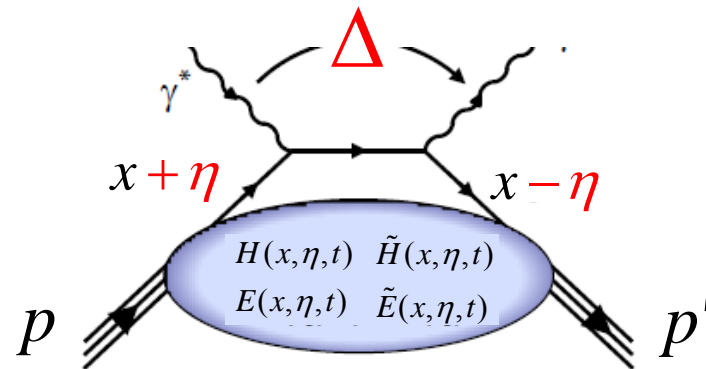
$$\int dz^- e^{i(x+\eta)Pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$

$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)Pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$

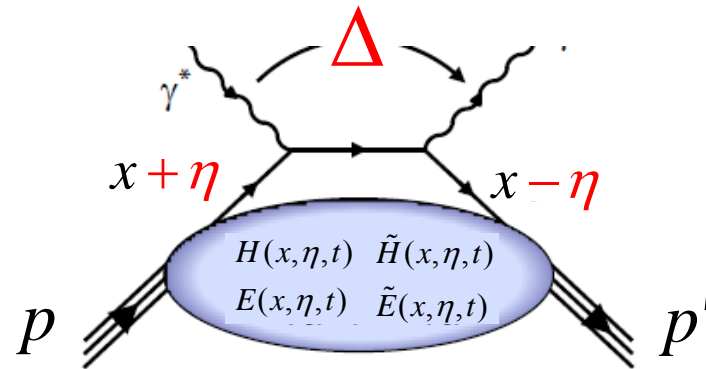
$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

$$J_q = \frac{1}{2} \int_{-1}^1 dx x (H(x, \eta, 0) + E(x, \eta, 0))$$

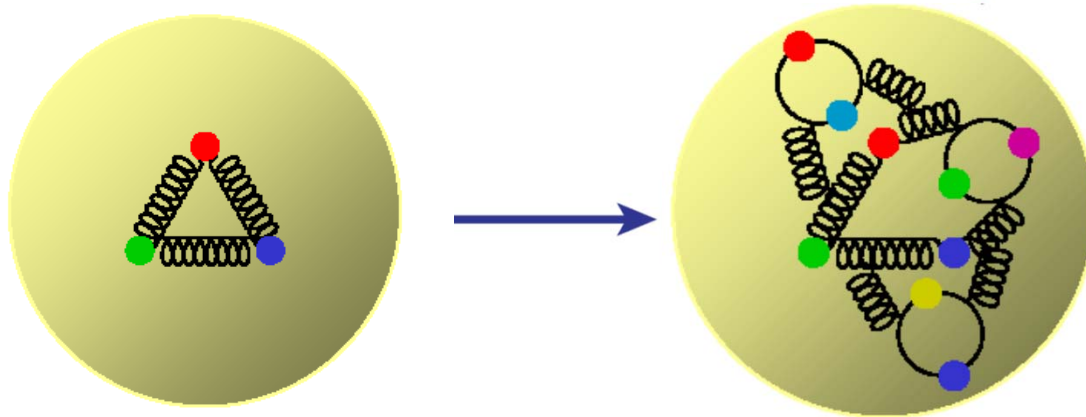
GPD



$$-2\eta\bar{P} = \Delta$$

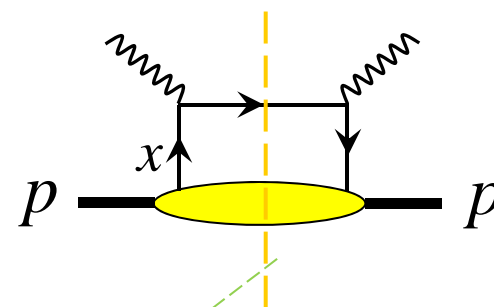
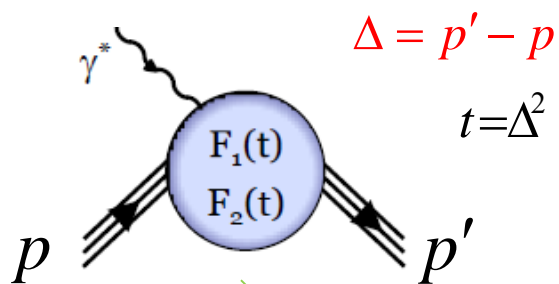
$$\int dz^- e^{i(x+\eta)Pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



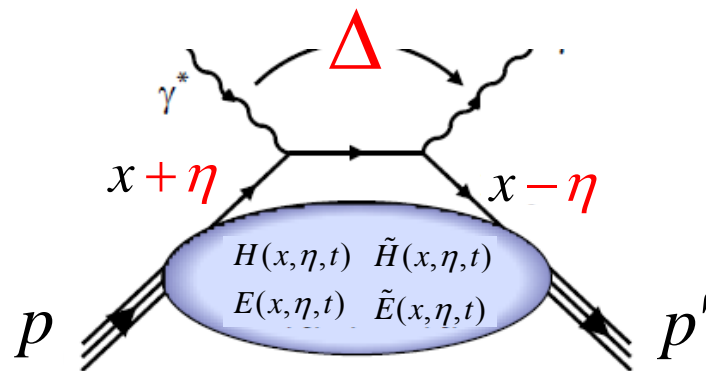


$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

$$\int d\mathbf{z}^- e^{i\mathbf{x}p\mathbf{z}^-} \langle N(\mathbf{p}) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



**GPD**



$$-2\eta\bar{P} = \Delta$$

$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta})\mathbf{p}\mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

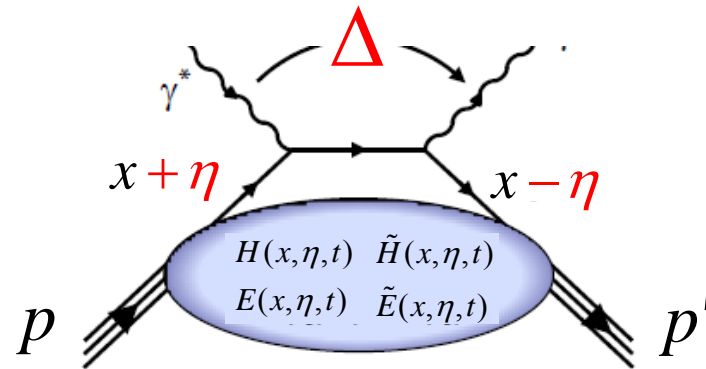
$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

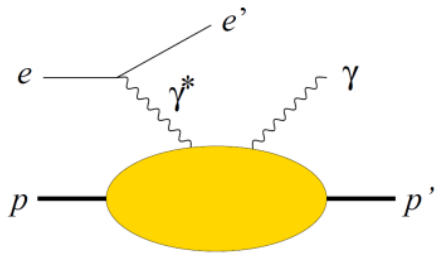
$$J_q = \frac{1}{2} \int_{-1}^1 dx x (H(x, \eta, 0) + E(x, \eta, 0))$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)Pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



$$\bar{P} = \frac{p + p'}{2}$$

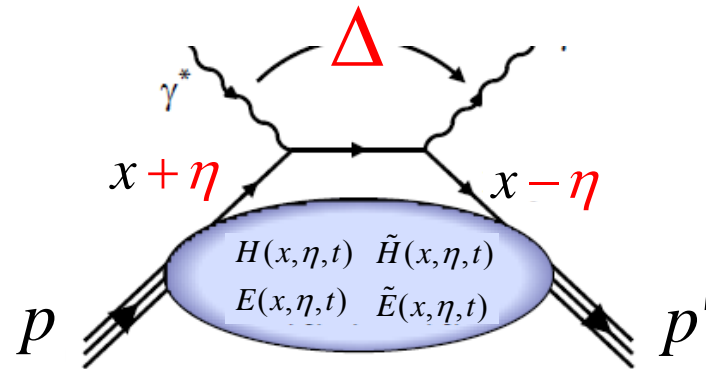
JLab, HERMES, COMPASS, ...

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

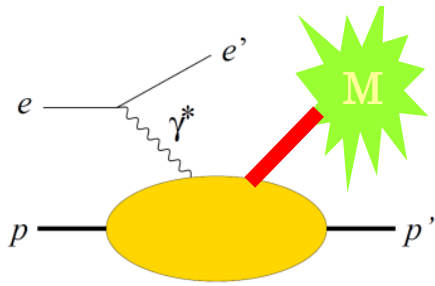
$$J_q = \frac{1}{2} \int_{-1}^1 dx x (H(x, \eta, 0) + E(x, \eta, 0))$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)Pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



$$\bar{P} = \frac{p + p'}{2}$$

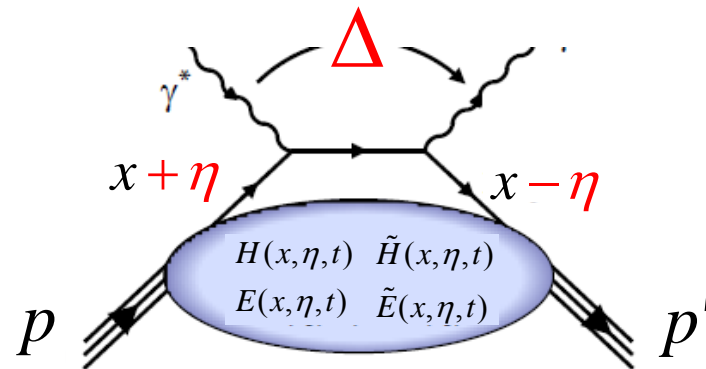
JLab, HERMES, COMPASS, ...

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

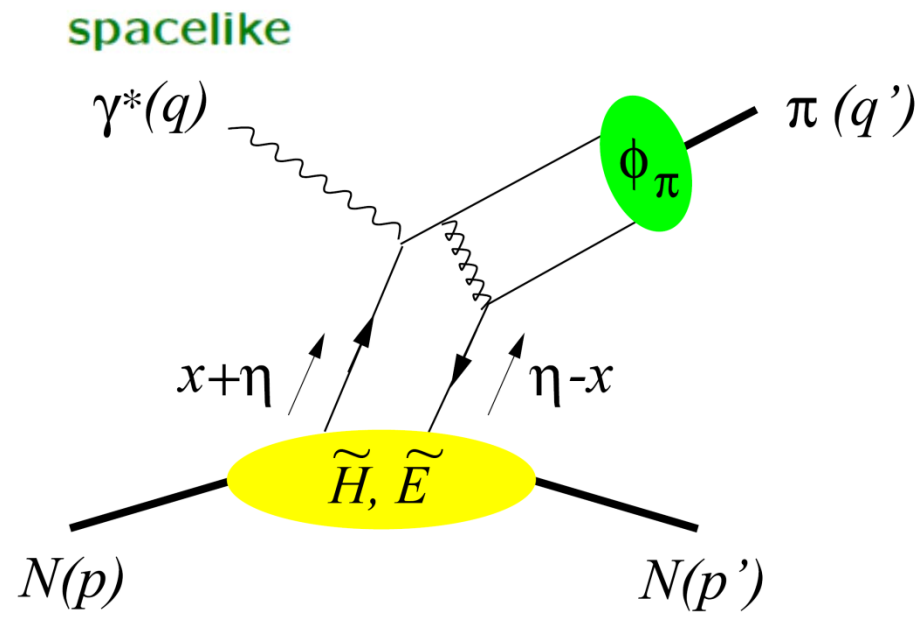
$$J_q = \frac{1}{2} \int_{-1}^1 dx x (H(x, \eta, 0) + E(x, \eta, 0))$$

GPD



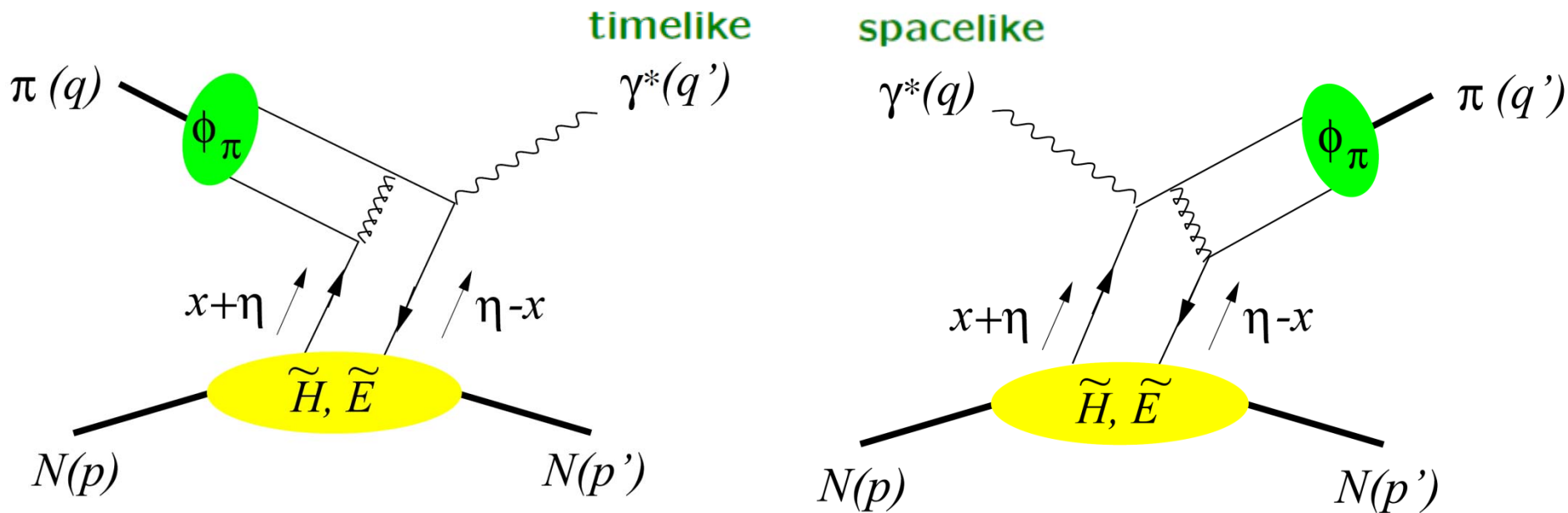
$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)Pz} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



**DVMP@JLab**

# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions



**DVMP@JLab**

# High momentum beam line at J-PARC

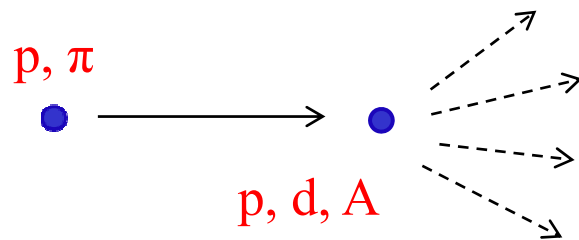
- Primary beam (proton)

$$E = 30\text{GeV} (\rightarrow 50\text{GeV?})$$

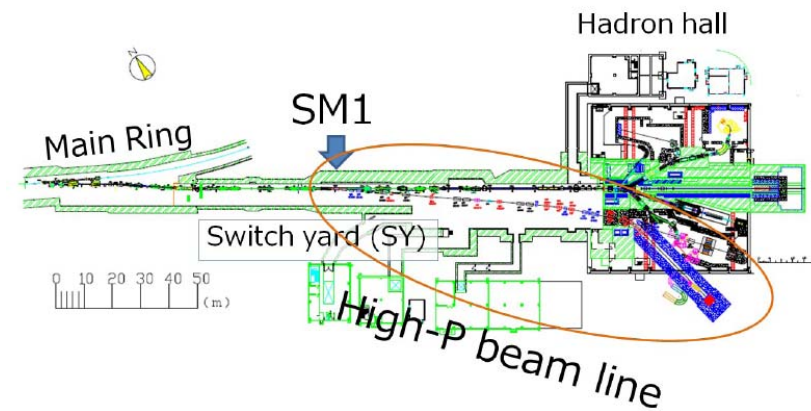
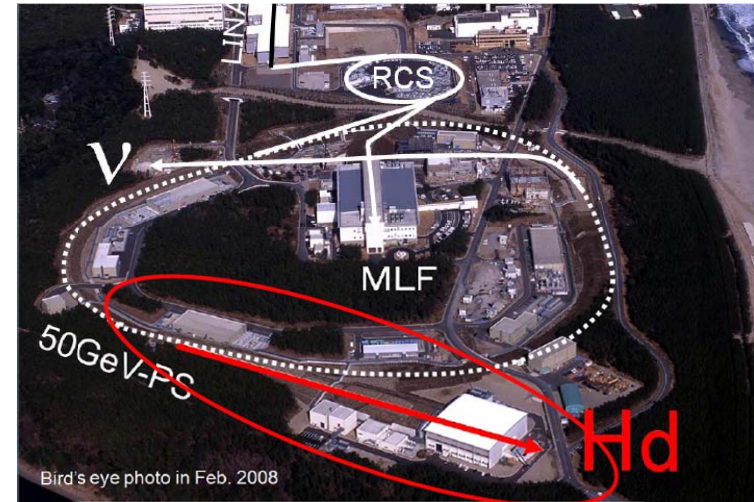
$$L = 10^{35} \text{cm}^{-2}\text{s}^{-1}$$

- Secondary beam (pion)

$$E = 15\text{-}20\text{GeV}$$



## Hadron Facility at J-PARC



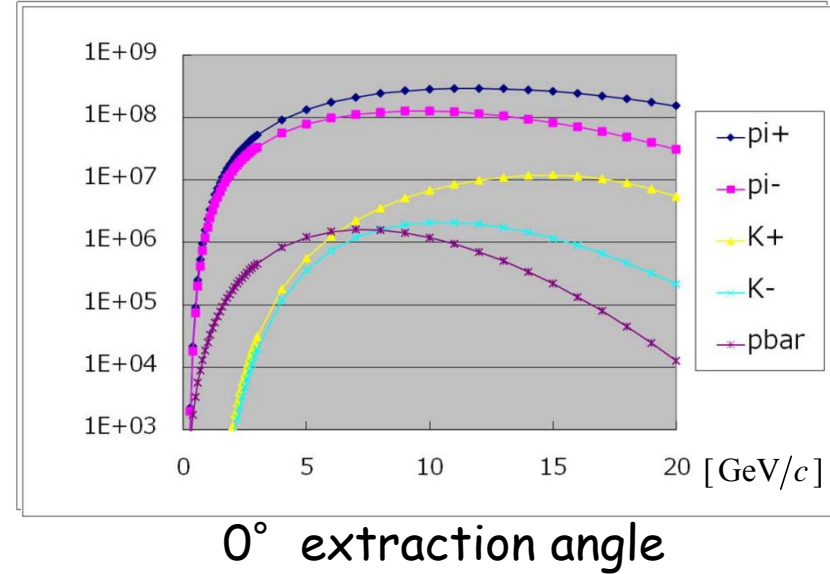
# High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

high intensity

beam loss limit @ SM1:15kW

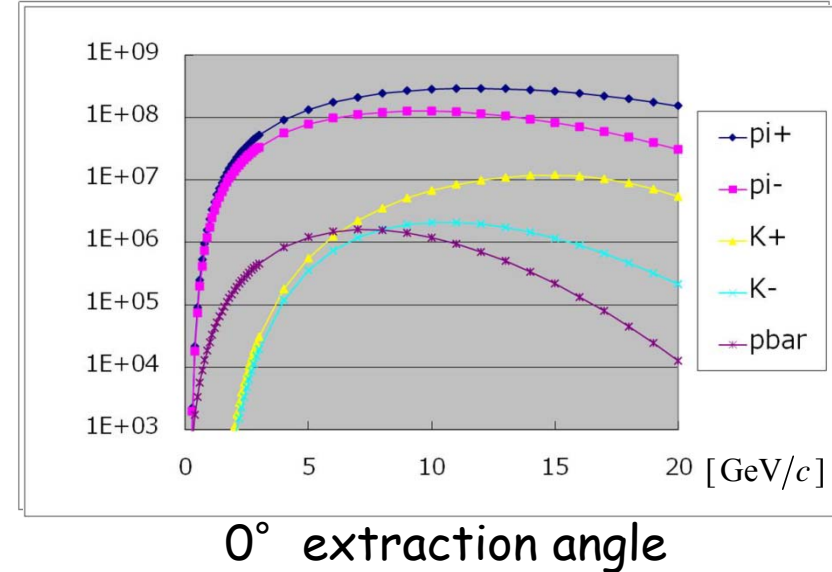
(limited by the thickness of the tunnel wall)





beam loss limit @ SM1:15kW

(limited by the thickness of the tunnel wall)



## High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

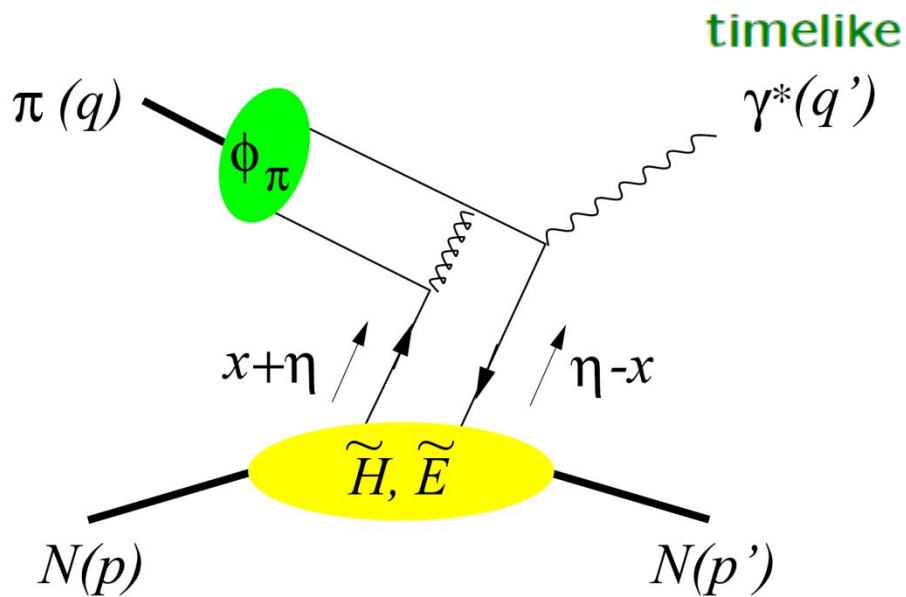
high intensity

not too high energy

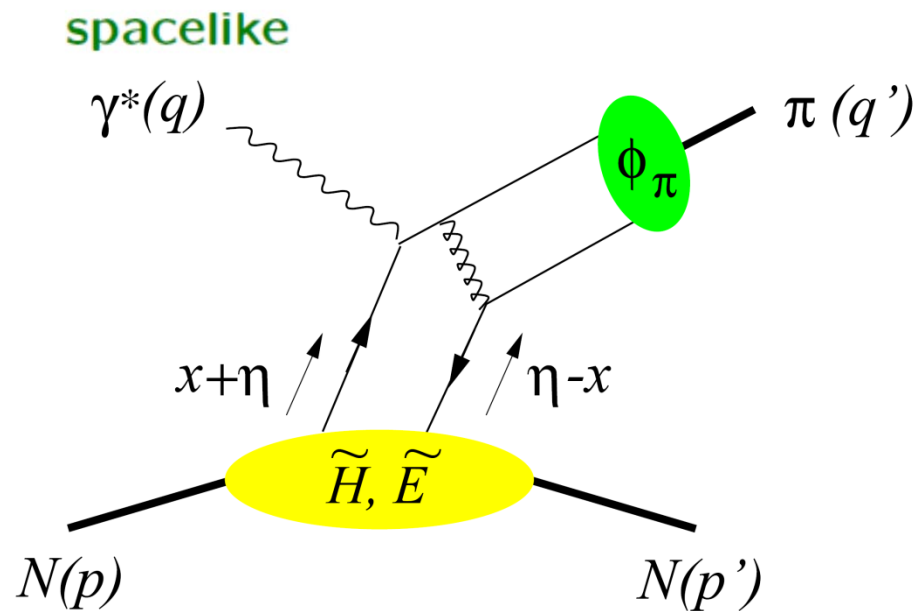
$$d\sigma \sim 1/s^a$$

best suited to study meson-induced  
hard exclusive processes

# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions

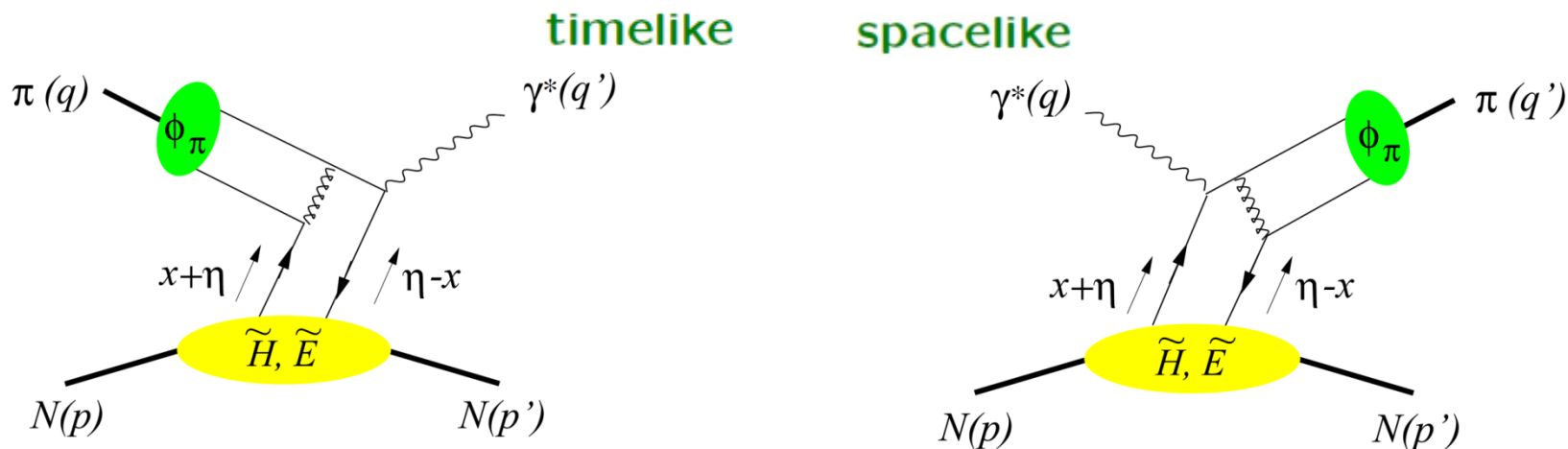


@J-PARC



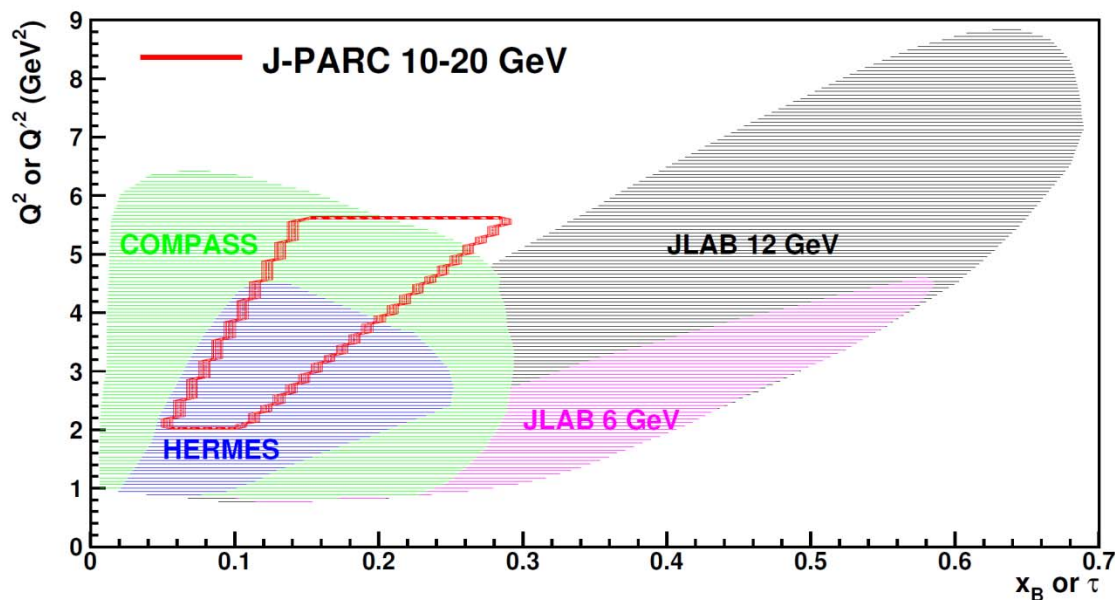
DVMP@JLab

# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions



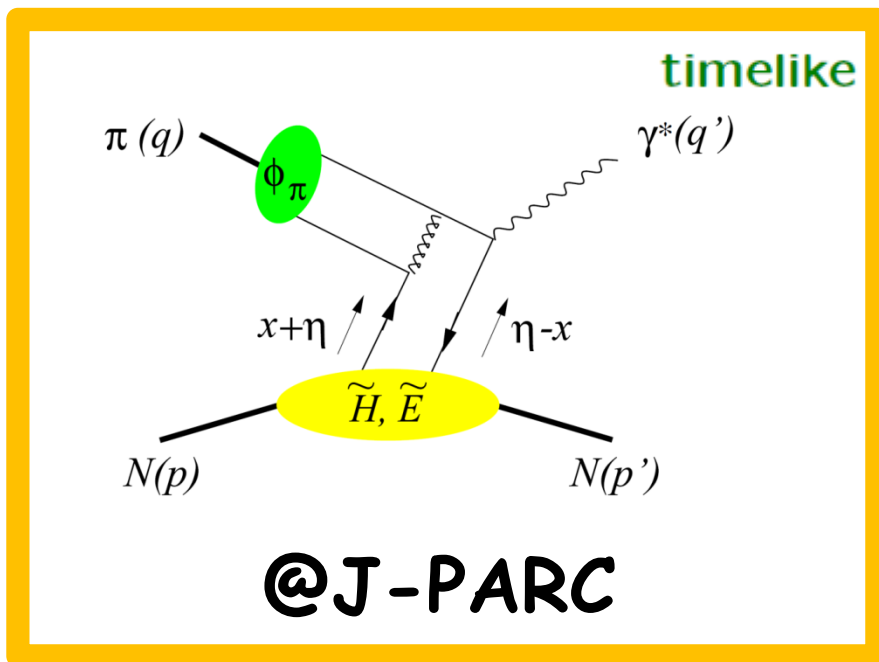
**@J-PARC**

**DVMP@JLab**

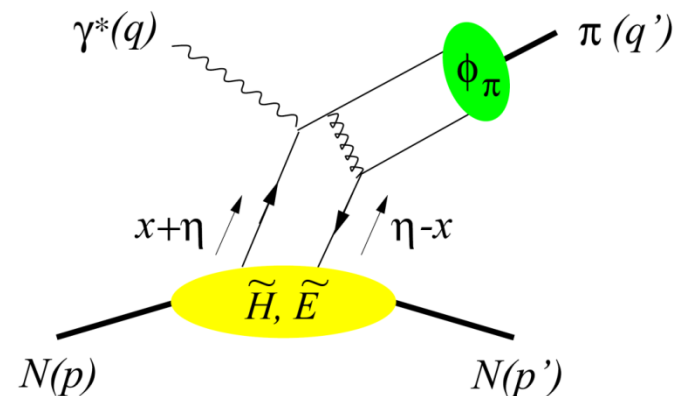


T. Sawada et al.,  
PRD93, 114034

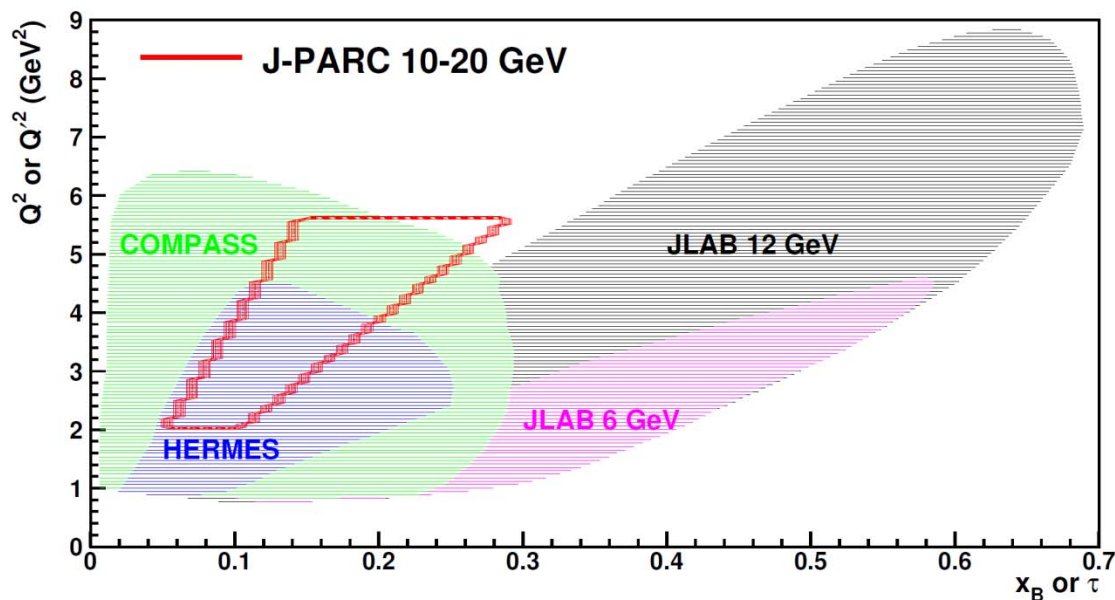
# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions



**spacelike**



**DVMP@JLab**

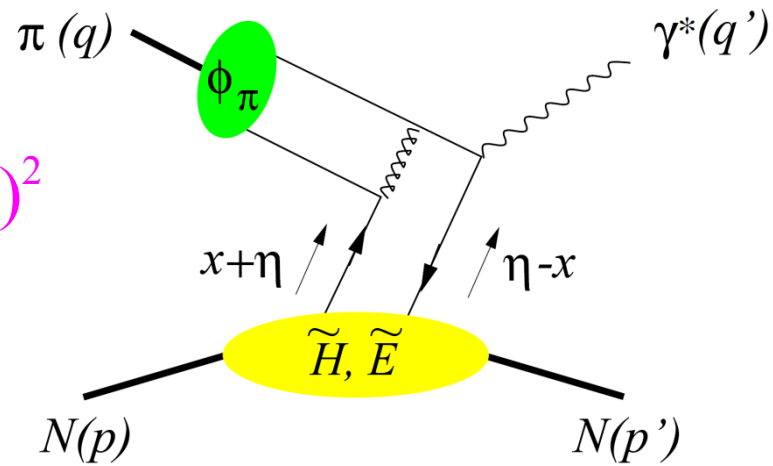


T. Sawada et al.,  
PRD93, 114034

# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

$$\text{small } t = \Delta^2 = (q - q')^2$$

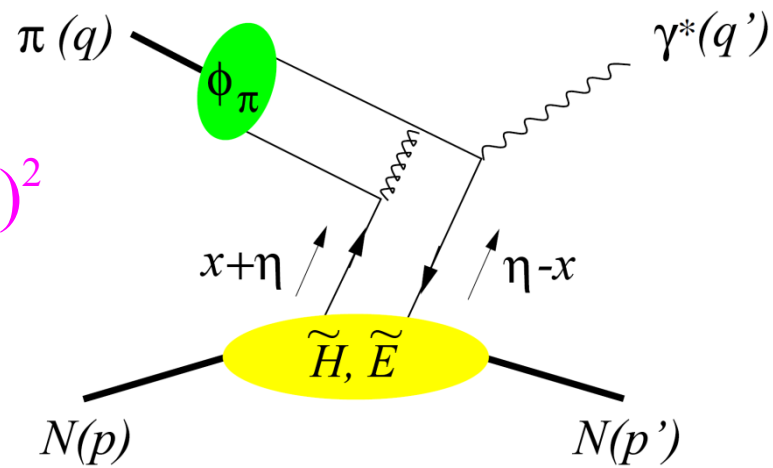


# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

“exclusive limit of DY”

small  $t = \Delta^2 = (q - q')^2$



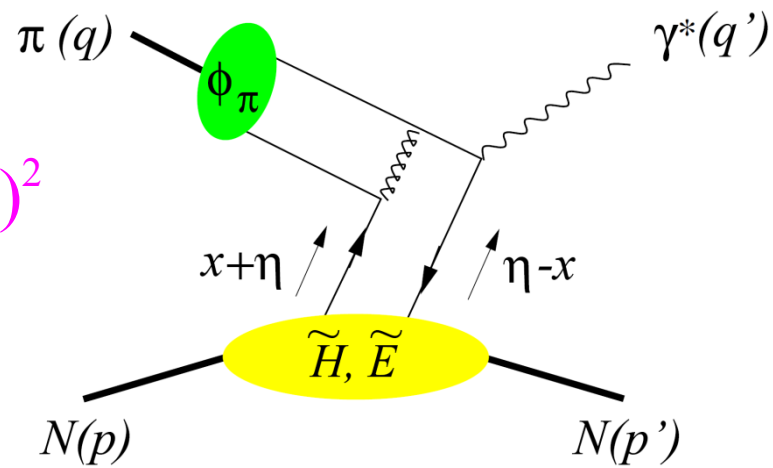
# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

Berger, Diehl, Pire, PLB523(2001)265

“exclusive limit of DY”

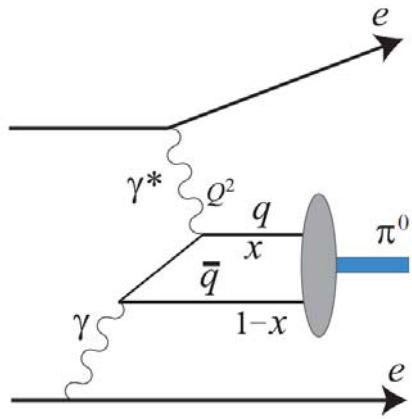
small  $t = \Delta^2 = (q - q')^2$



# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

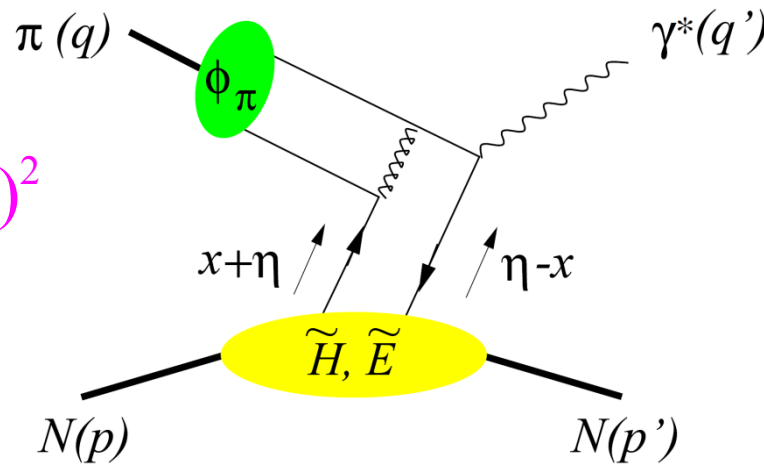
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

“exclusive limit of DY”

small  $t = \Delta^2 = (q - q')^2$



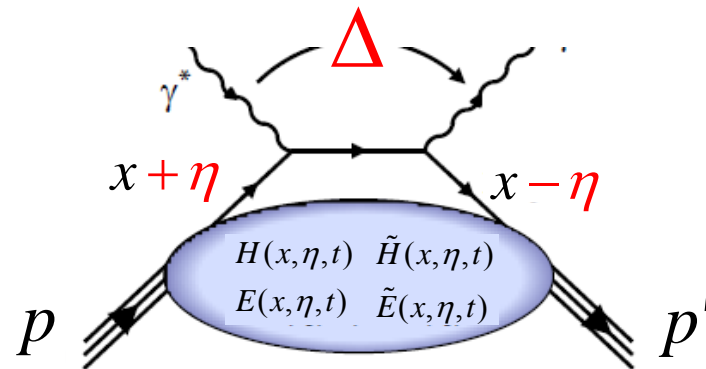


$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

GPD



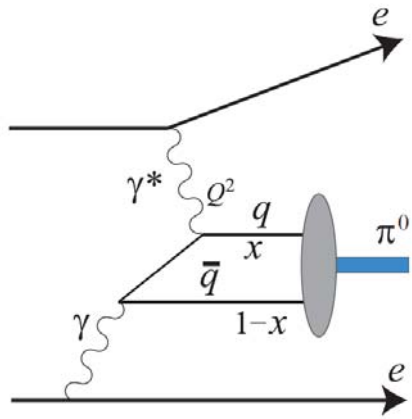
$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$

# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

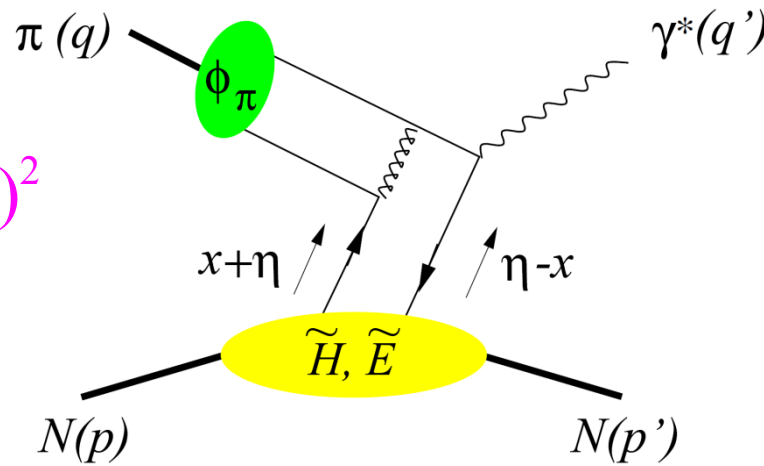
Berger, Diehl, Pire, PLB523(2001)265



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“exclusive limit of DY”

small  $t = \Delta^2 = (q - q')^2$



Bjorken variable

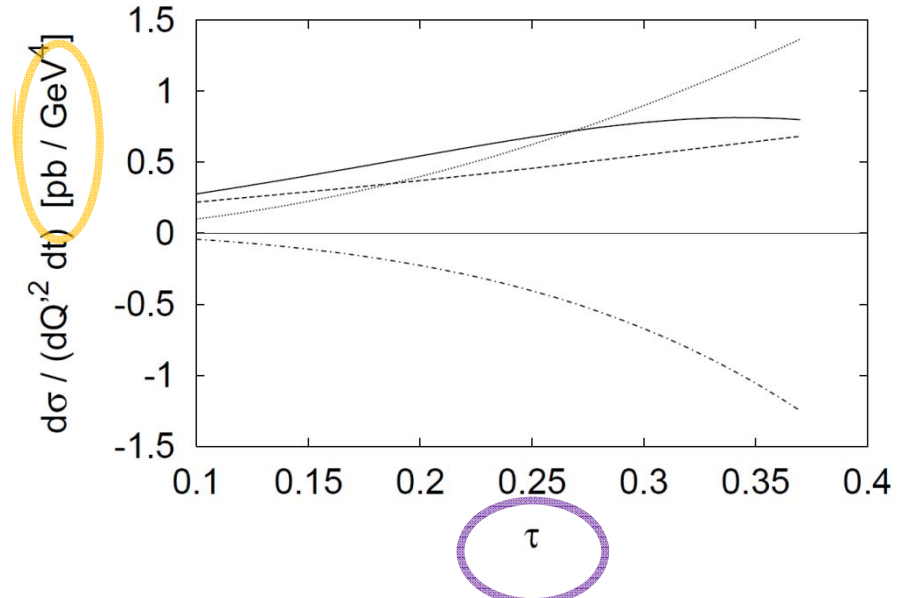
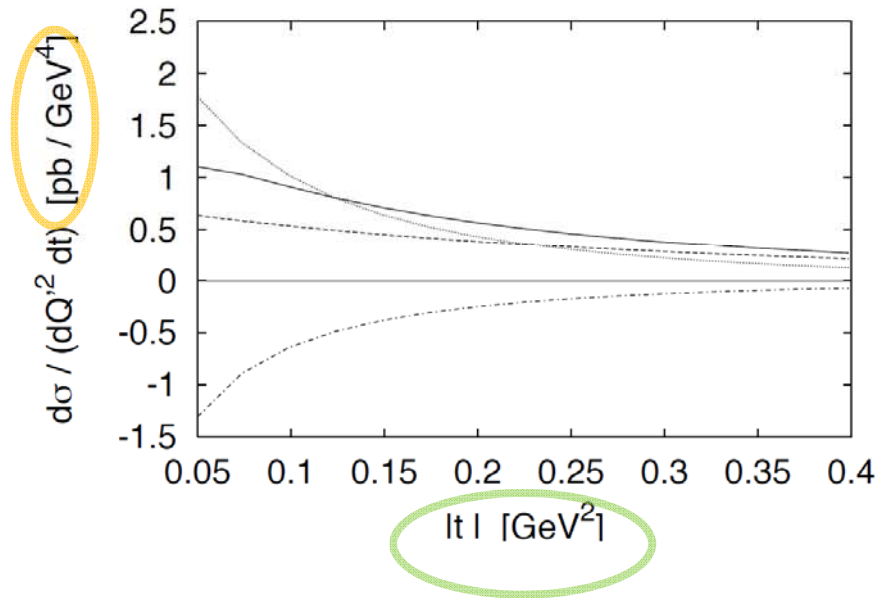
$$\tau = \frac{Q'^2}{s-M^2}$$

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$$Q'^2 = 5 \text{ GeV}^2$$

$$\tau = 0.2$$

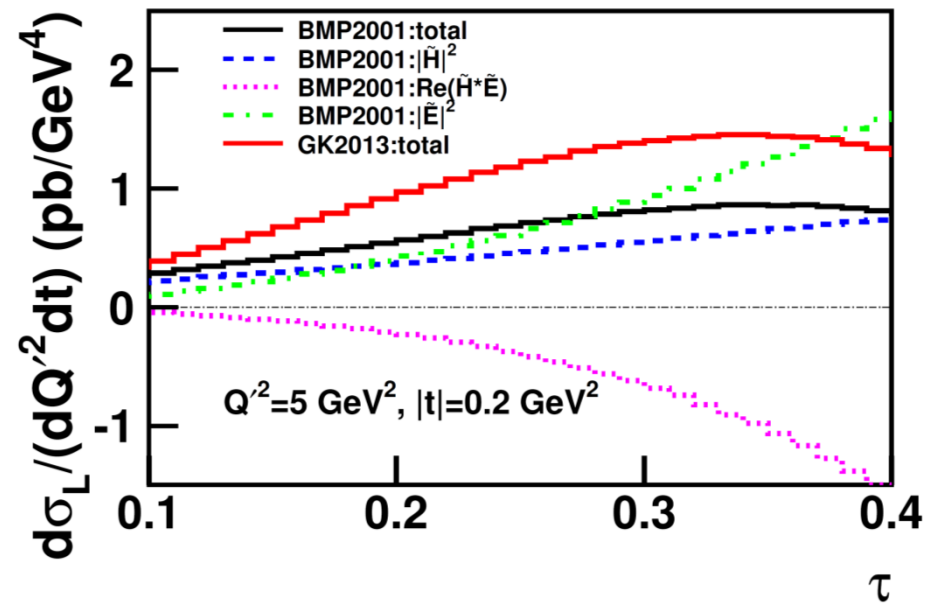
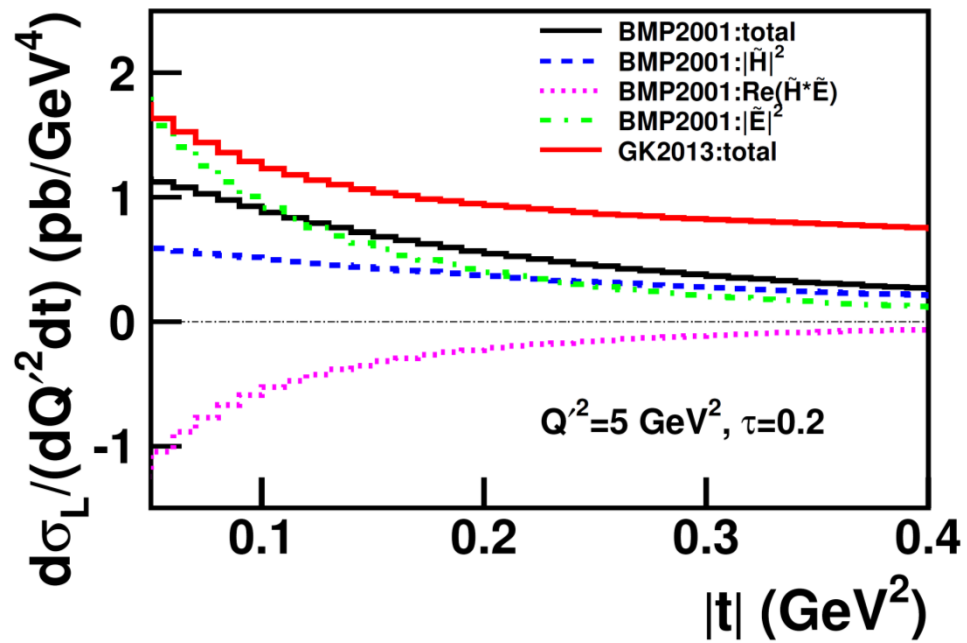
$$|t| = 0.2 \text{ GeV}^2$$



(dashed) =  $|\tilde{\mathcal{H}}|^2$  ; (dash-dotted) =  $\text{Re}(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$  ; (dotted) =  $|\tilde{\mathcal{E}}|^2$

$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \rightarrow \gamma^* n) = \frac{4\pi\alpha_{\text{em}}^2 \tau^2}{27 Q'^8} f_\pi^2 \left[ (1-\eta^2) |\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\tilde{\mathcal{H}}^{du*} \tilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\tilde{\mathcal{E}}^{du}|^2 \right]$$

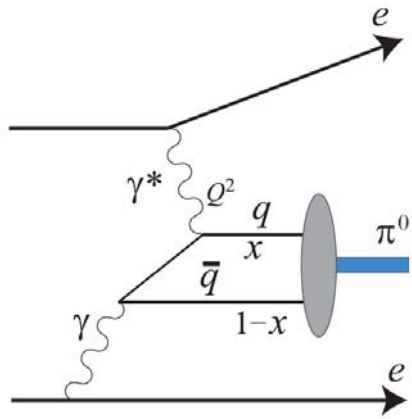
$$\tilde{\mathcal{H}}^{du} = \frac{8\alpha_s}{3} \int_0^1 du \frac{\phi_\pi(u)}{4u(1-u)} \int_{-1}^1 dx \left( \frac{e_d}{-\eta - x - i\epsilon} - \frac{e_u}{-\eta + x - i\epsilon} \right) (\tilde{H}^d(x, \eta, t) - \tilde{H}^u(x, \eta, t))$$



# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

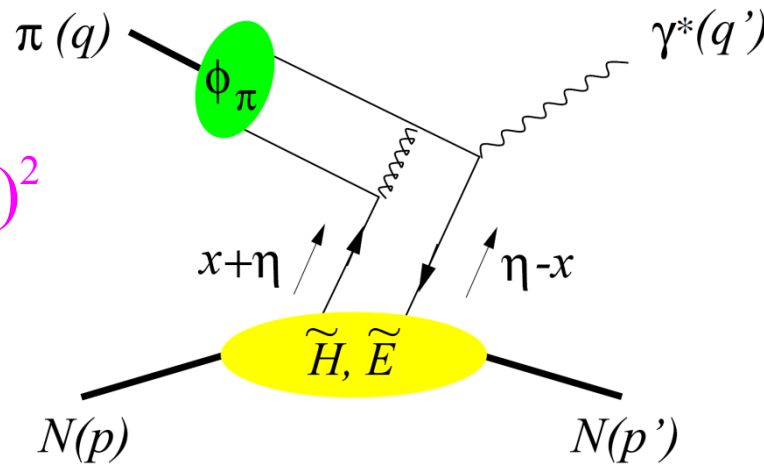
Berger, Diehl, Pire, PLB523(2001)265



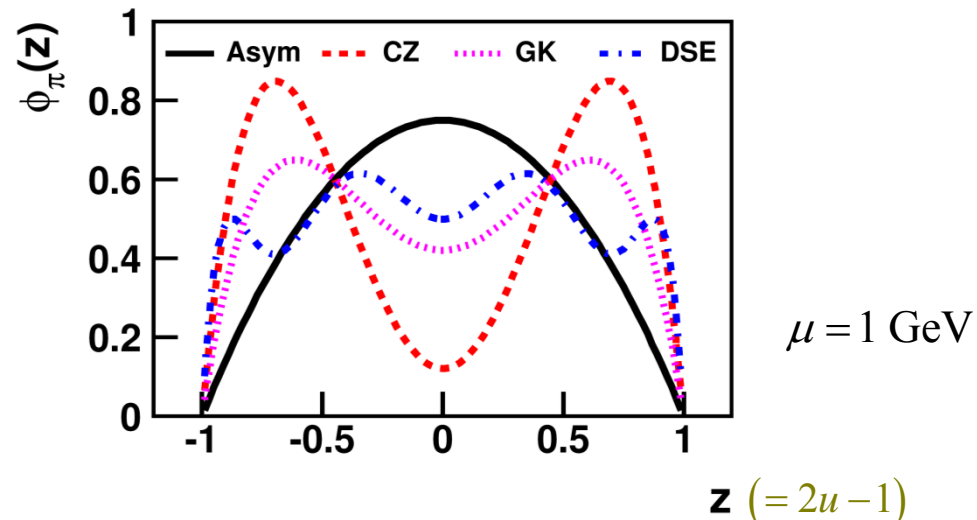
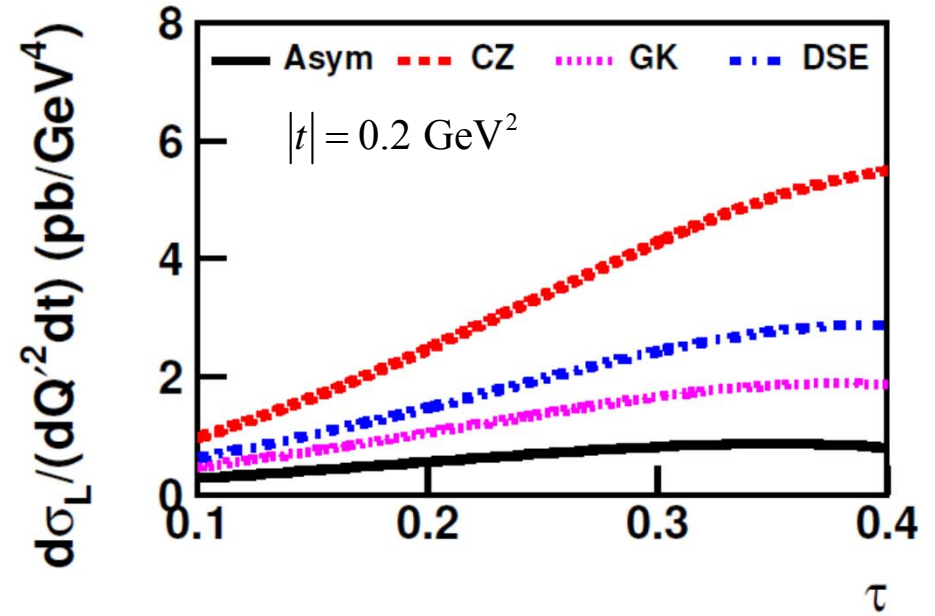
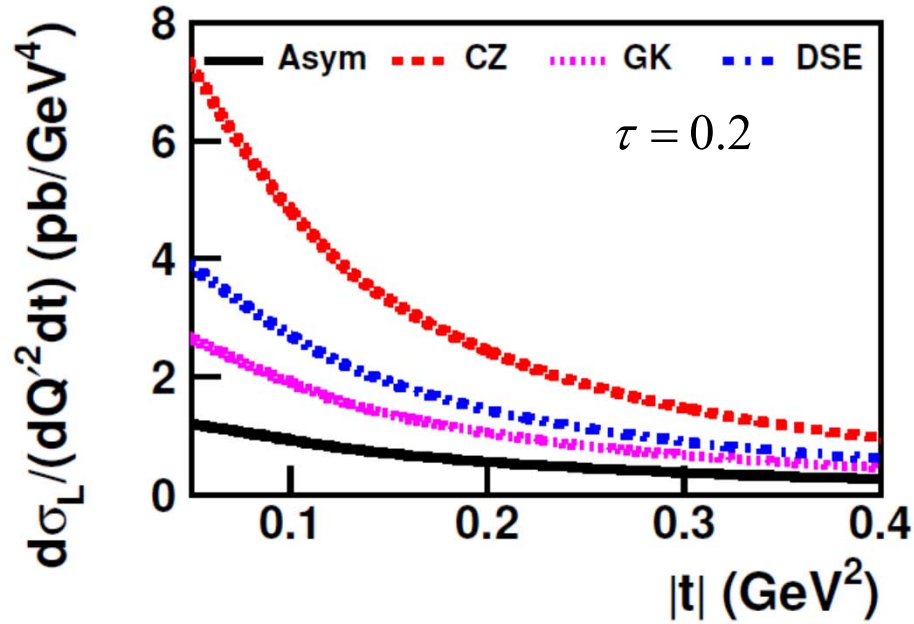
@Belle, Babar

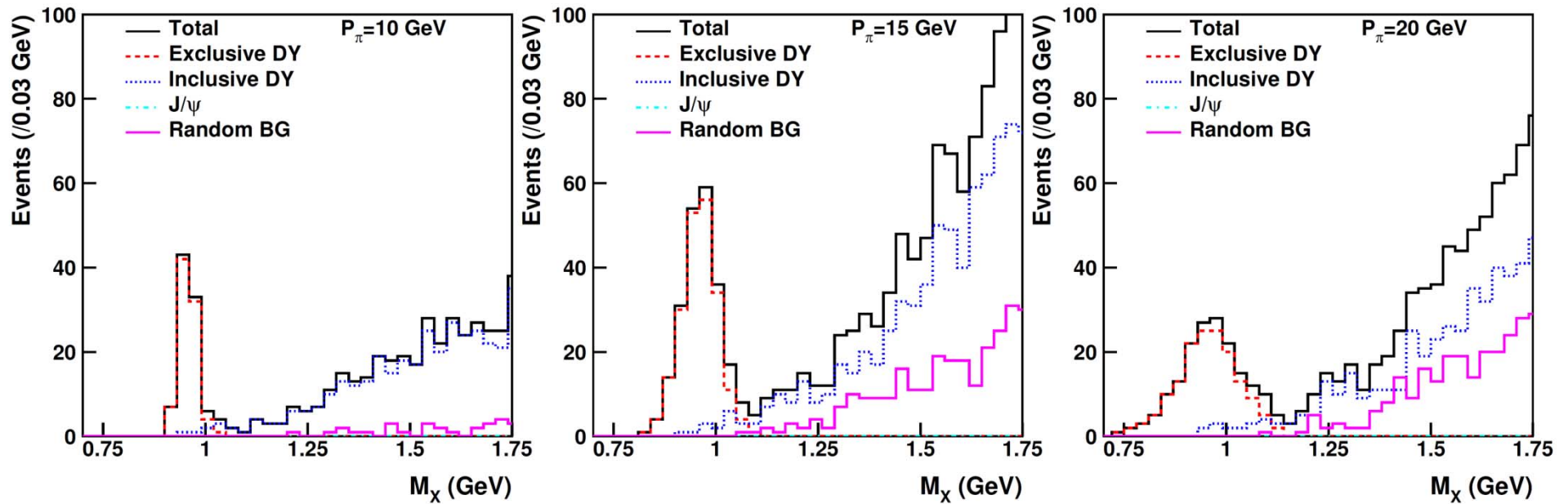
“exclusive limit of DY”

small  $t = \Delta^2 = (q - q')^2$



$$Q'^2 = 5 \text{ GeV}^2$$





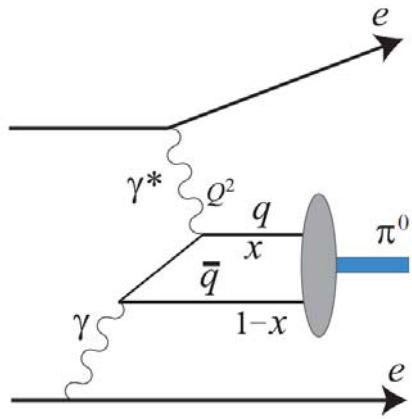
## feasibility with E50 spectrometer at J-PARC

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,  
PRD93, 114034

# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

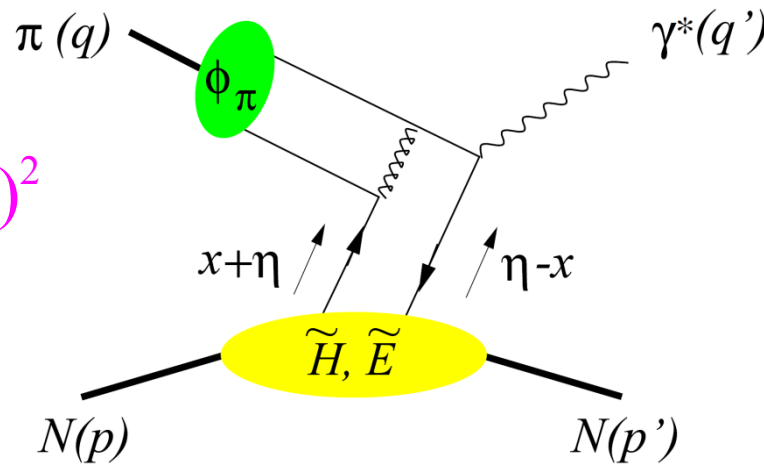
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

"exclusive DY"

small  $t = \Delta^2 = (q - q')^2$

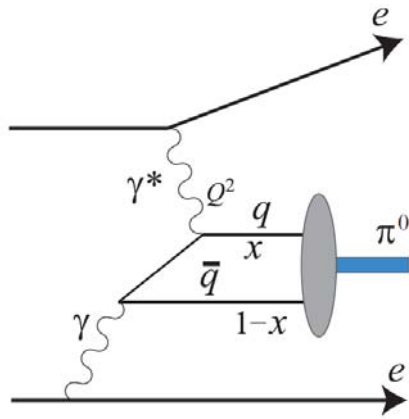




# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

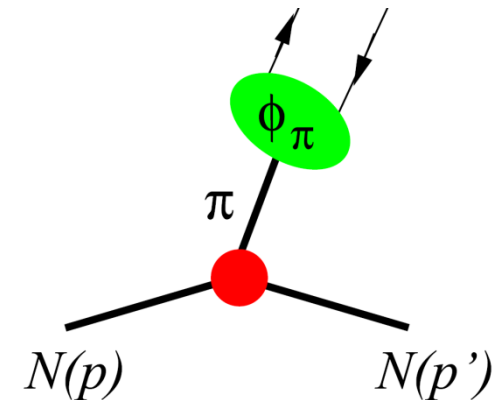
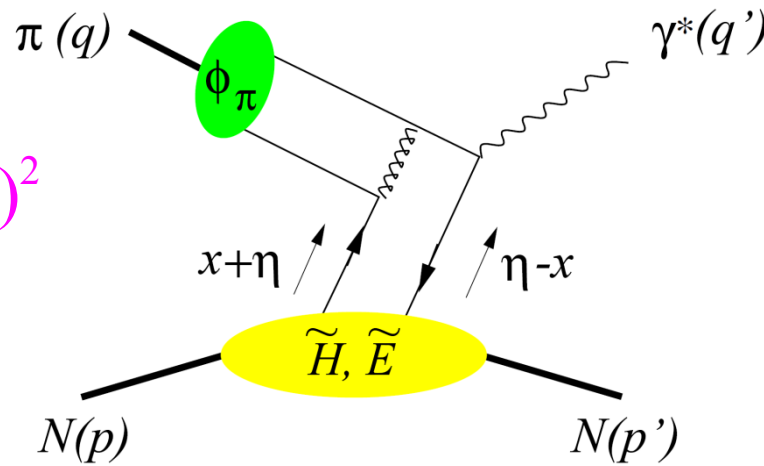
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“exclusive DY”

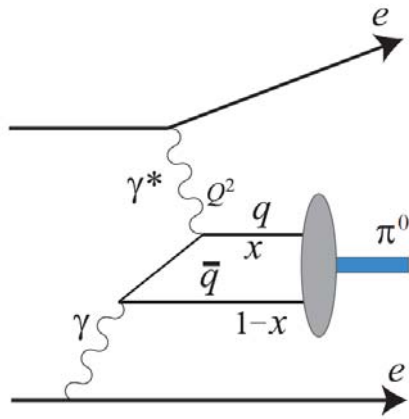
small  $t = \Delta^2 = (q - q')^2$



# Exclusive lepton pair production in $\pi N$ scattering

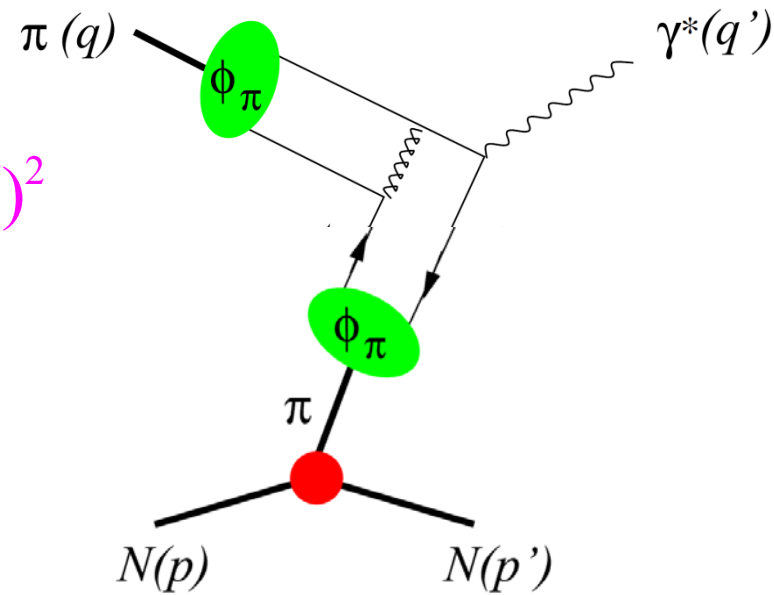
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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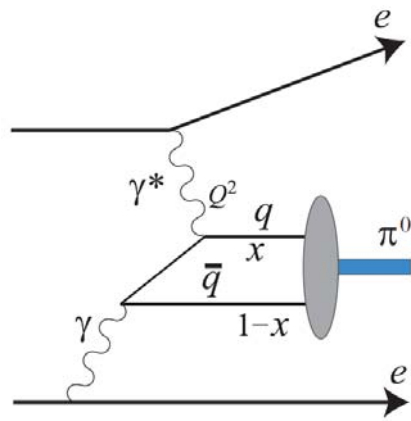
small  $t = \Delta^2 = (q - q')^2$



# Exclusive lepton pair production in $\pi N$ scattering

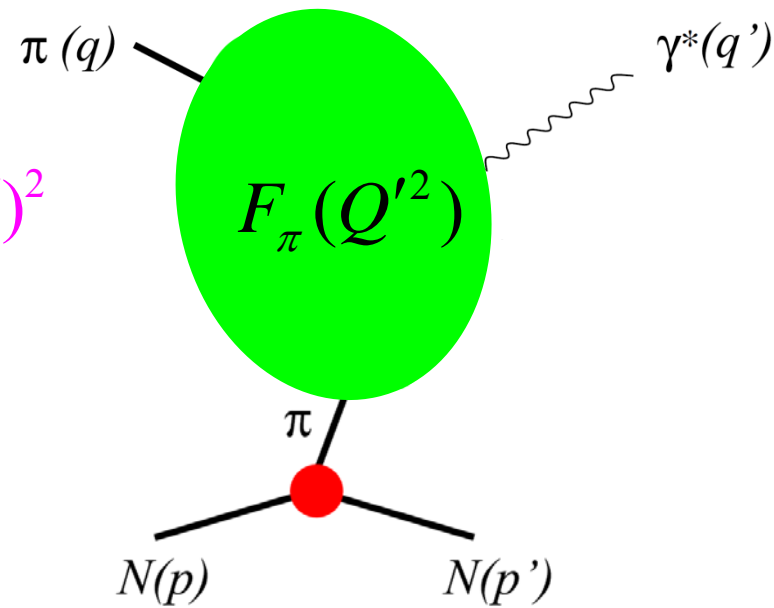
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

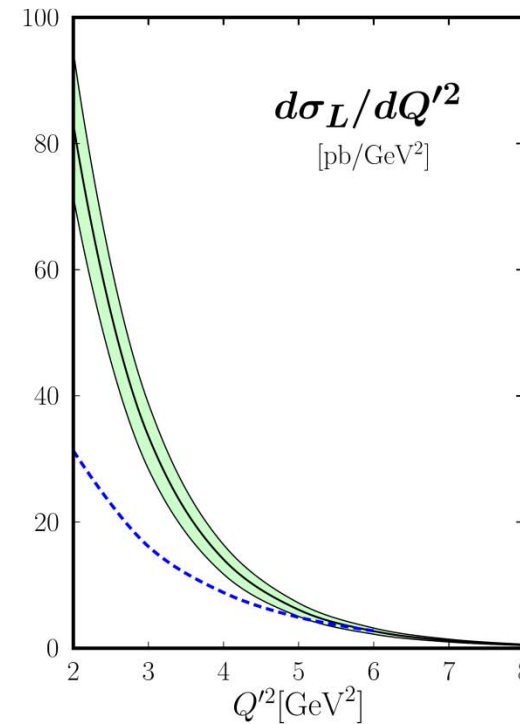
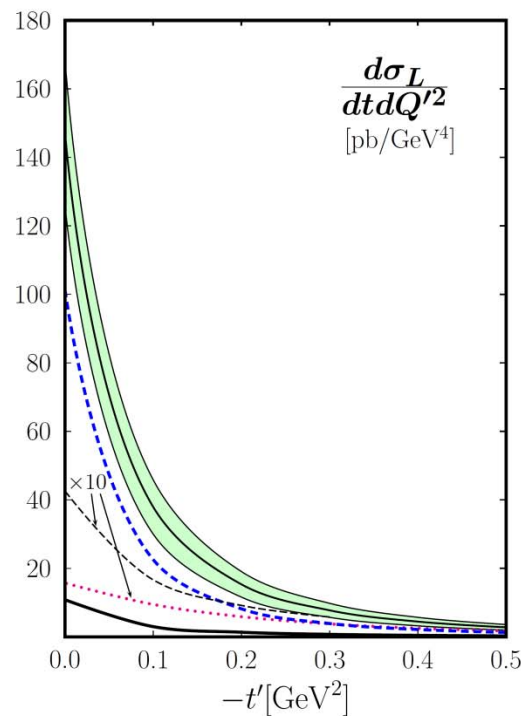
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small  $t = \Delta^2 = (q - q')^2$





$Q'^2 = 4 \text{ GeV}^2$  and  $s = 20(30) \text{ GeV}^2$       solid (blue dashed) lines with error bands  
 solid (dotted, dashed) line: interference,  $|\langle \tilde{H}^{(3)} \rangle|^2$ , leading-twist contribution

time-like pion FF:  $Q'^2 |F_\pi(Q'^2)| = 0.88 \pm 0.04 \text{ GeV}^2$  (CLEO, BaBar,  $J/\Psi \rightarrow \pi^+ \pi^-$ )

phase from disp. rel. Belicka et al(11) for  $Q'^2 < 7 \text{ GeV}^2$

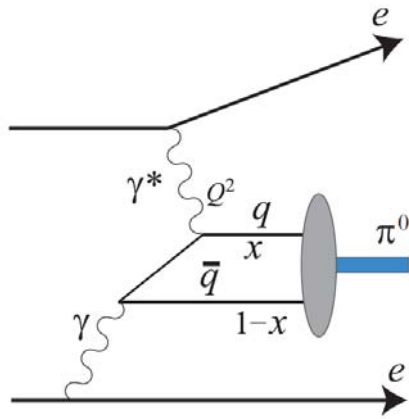
$$\delta = 182.6^\circ + 11.2^\circ(Q'^2 - 2 \text{ GeV}^2) - 1.67^\circ(Q'^2 - 2 \text{ GeV}^2)^2$$

for  $Q'^2 \geq 8.9 \text{ GeV}^2$ :  $\delta = 180^\circ$ , pQCD result

# Exclusive lepton pair production in $\pi N$ scattering

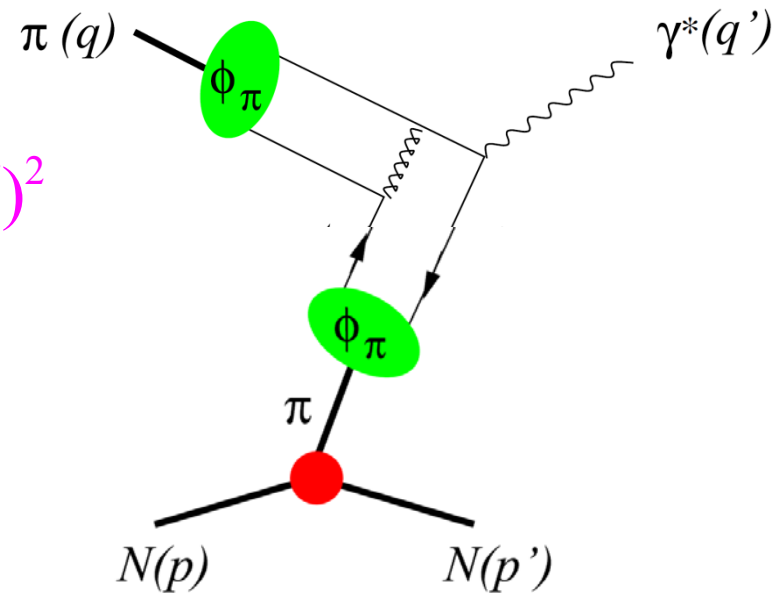
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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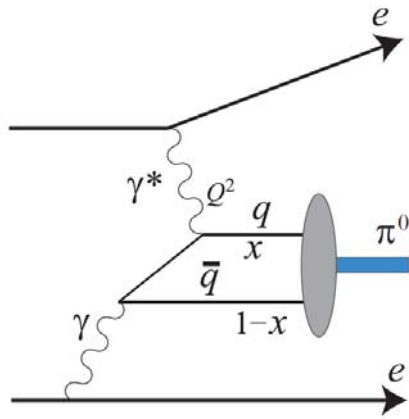
small  $t = \Delta^2 = (q - q')^2$



# Exclusive lepton pair production in $\pi N$ scattering

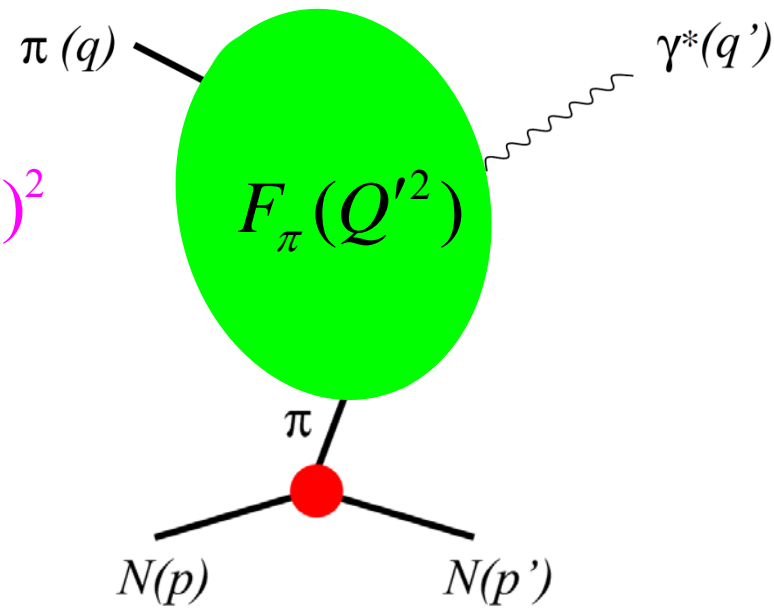
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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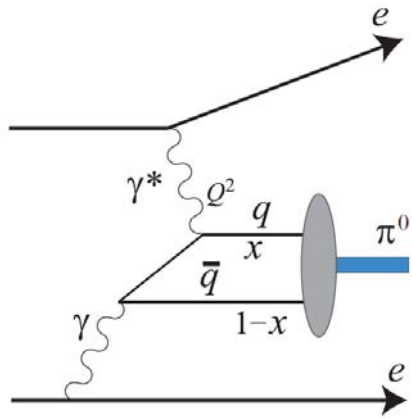
small  $t = \Delta^2 = (q - q')^2$



# Exclusive lepton pair production in $\pi N$ scattering

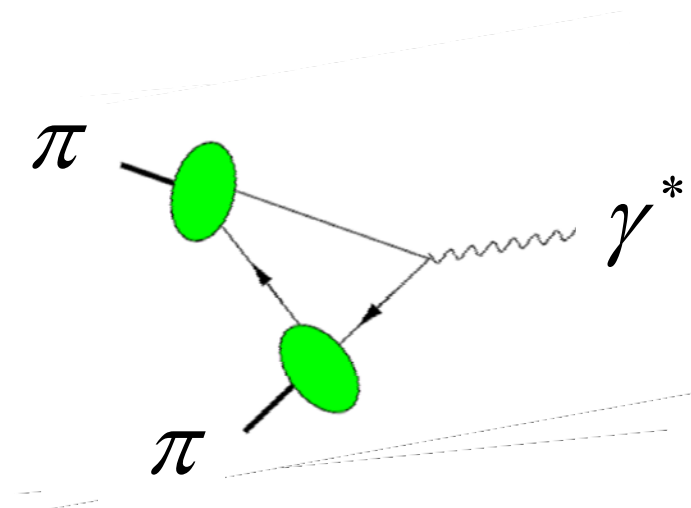
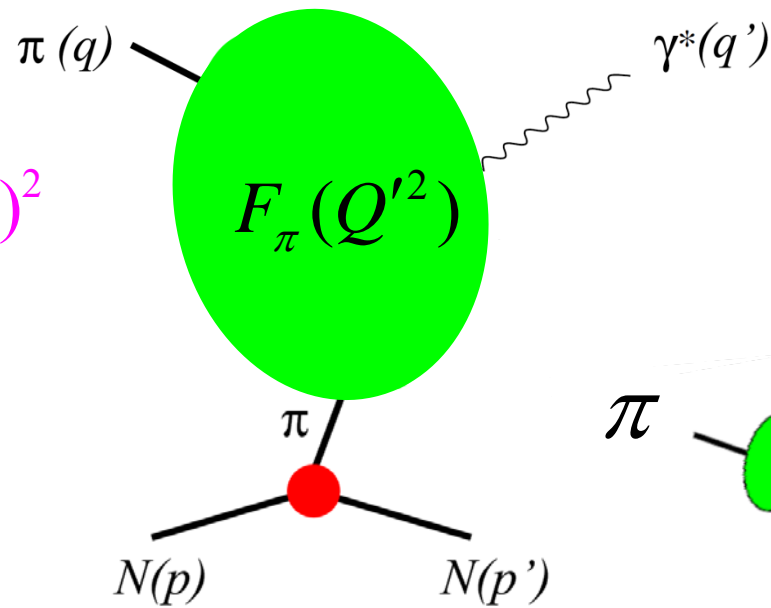
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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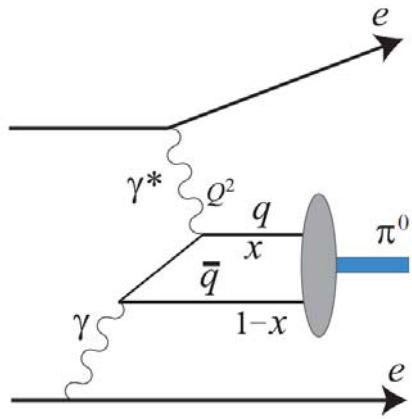
small  $t = \Delta^2 = (q - q')^2$



# Exclusive lepton pair production in $\pi N$ scattering

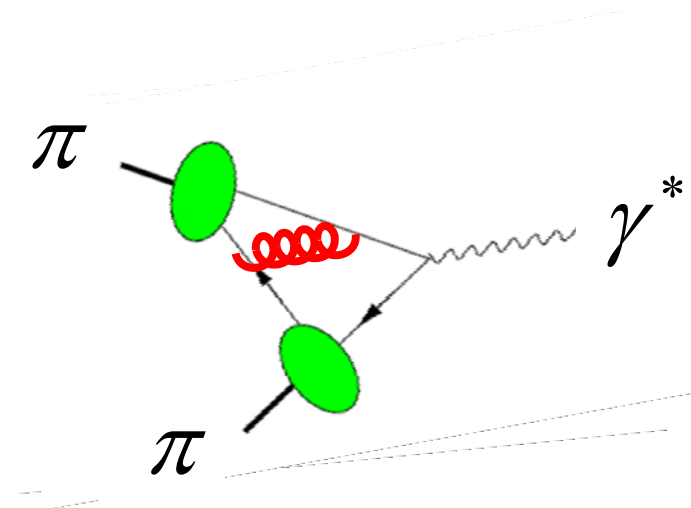
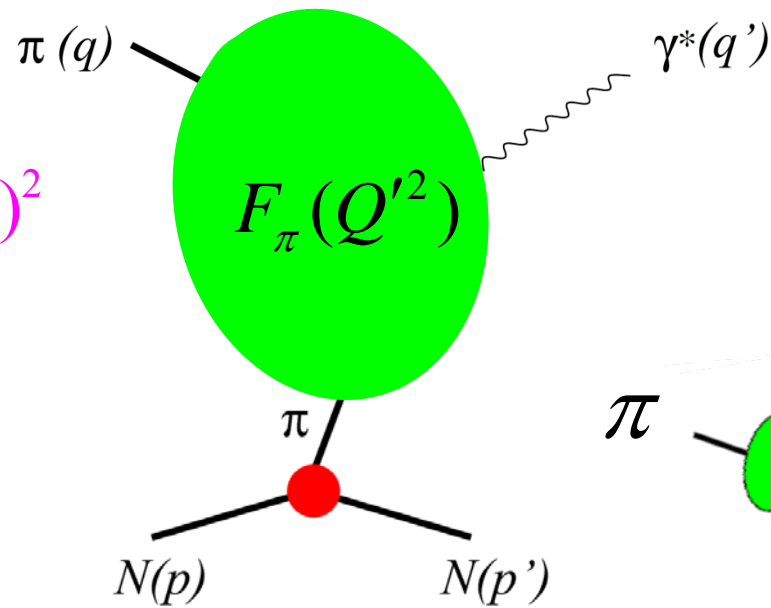
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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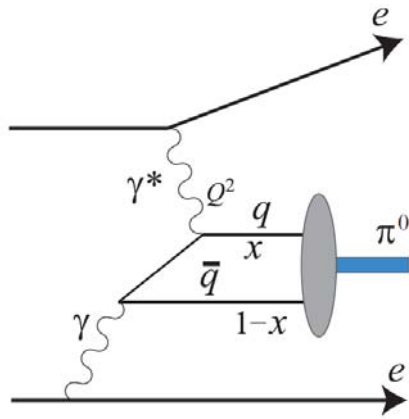




# Exclusive lepton pair production in $\pi N$ scattering

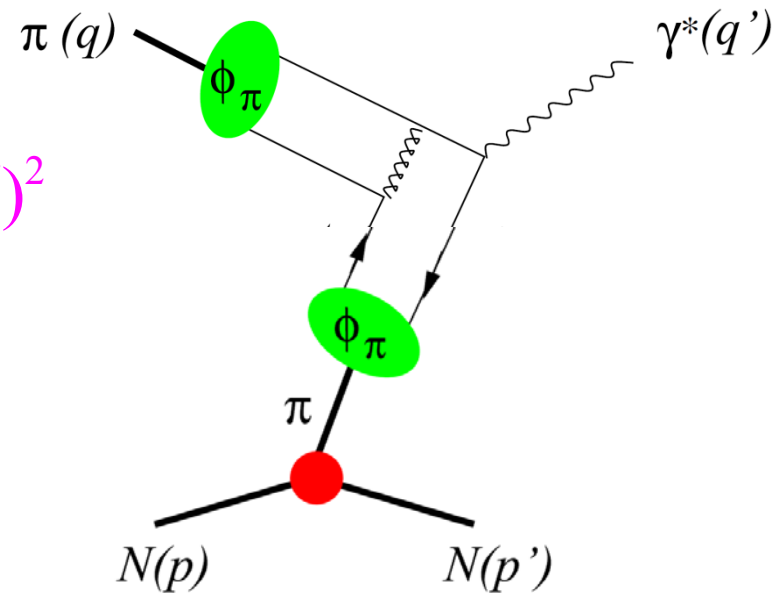
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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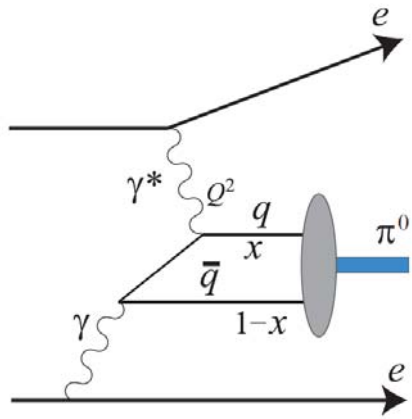
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# Exclusive lepton pair production in $\pi N$ scattering

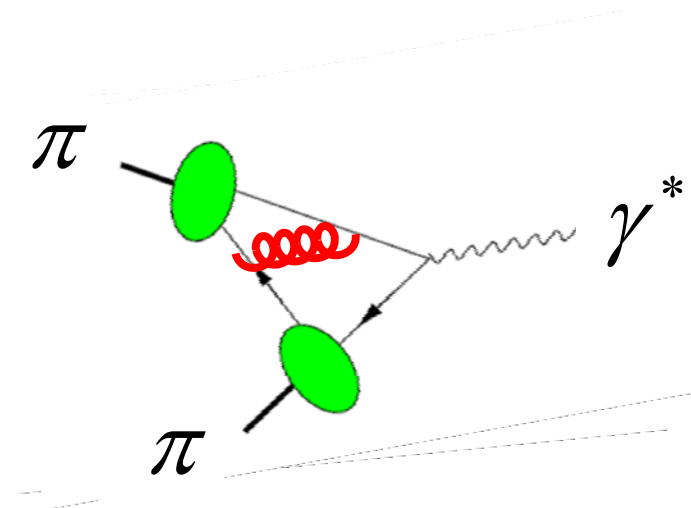
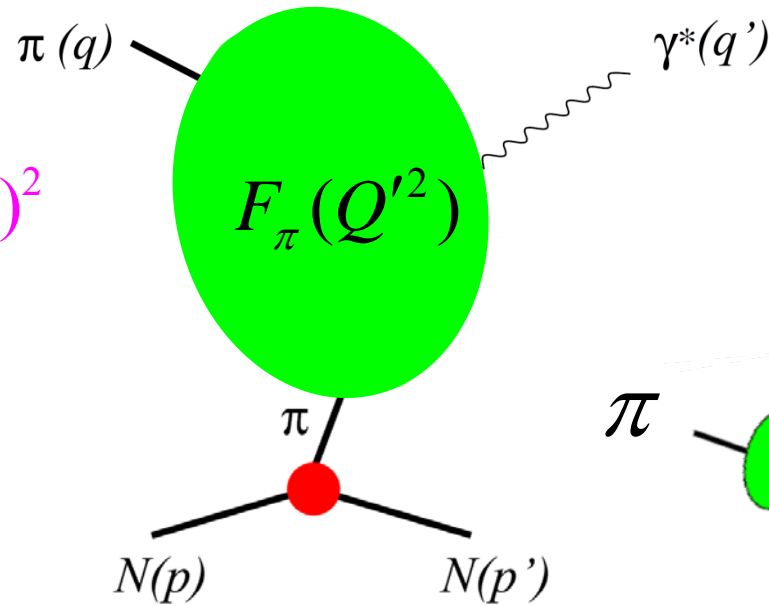
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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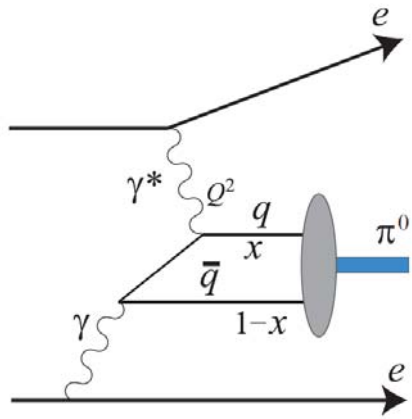
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# Exclusive lepton pair production in $\pi N$ scattering

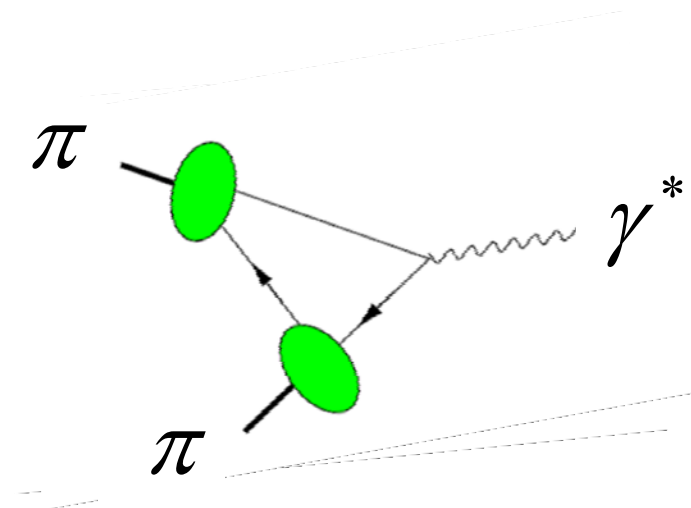
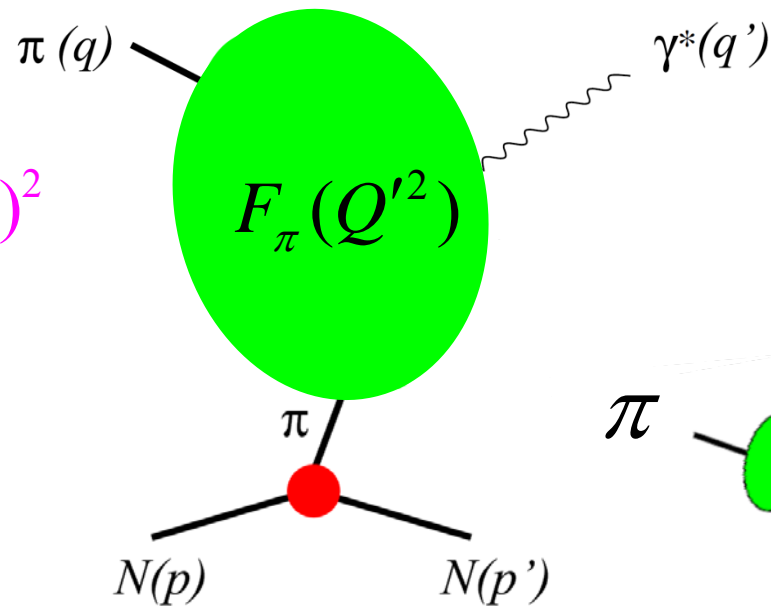
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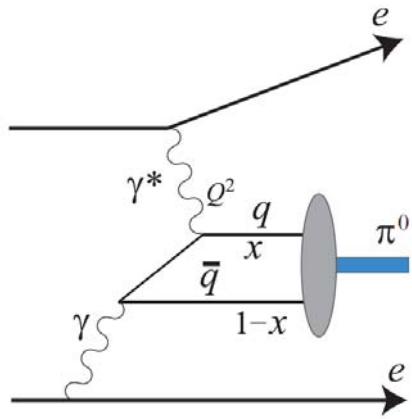
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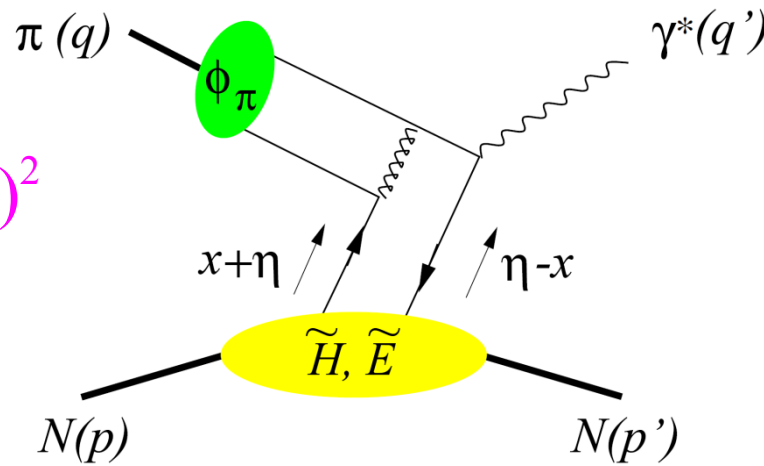
Berger, Diehl, Pire, PLB523(2001)265

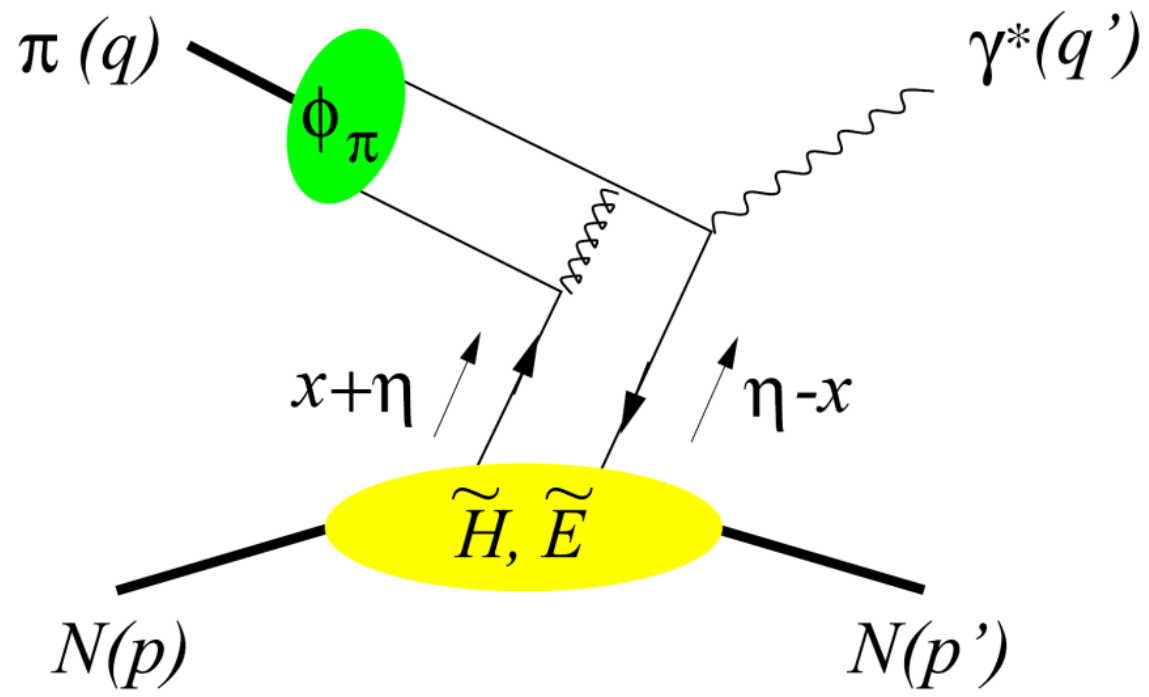


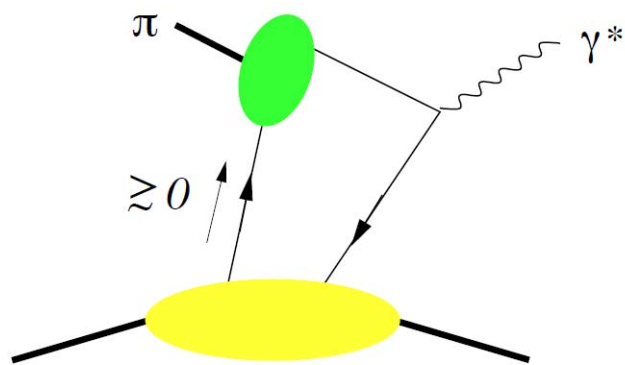
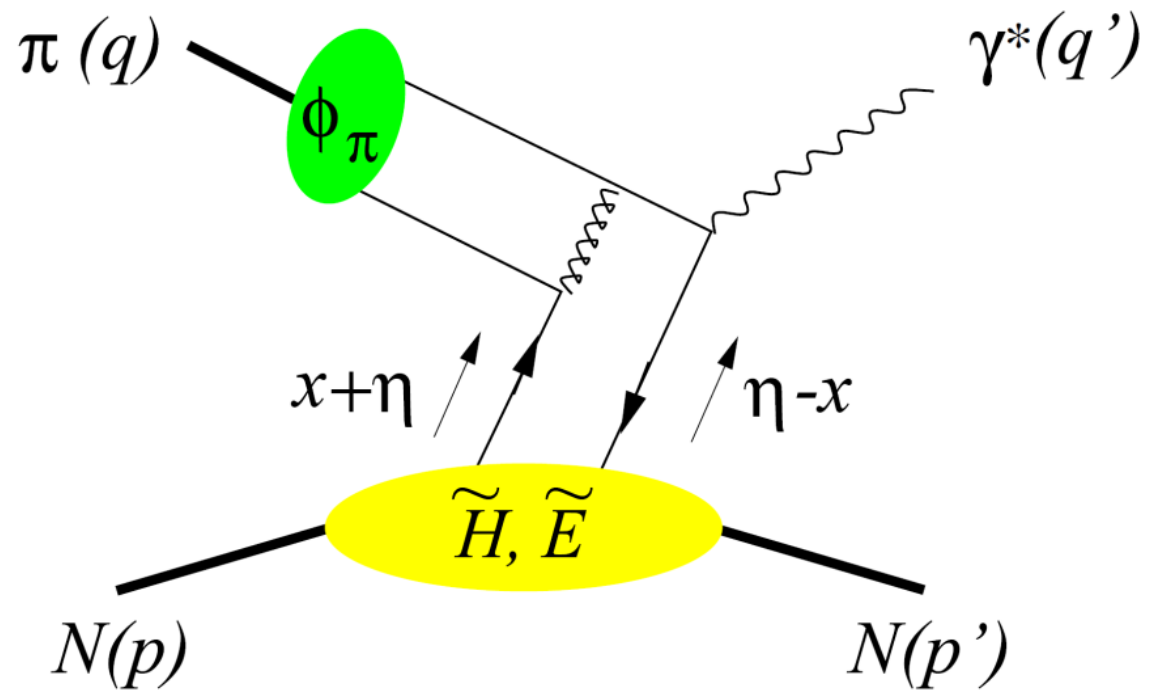
@Belle, Babar

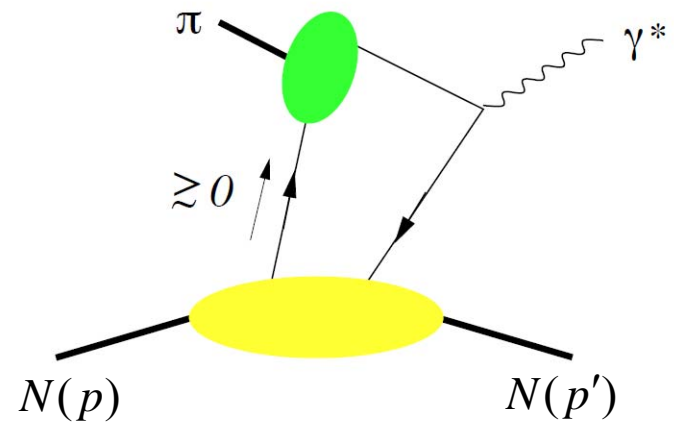
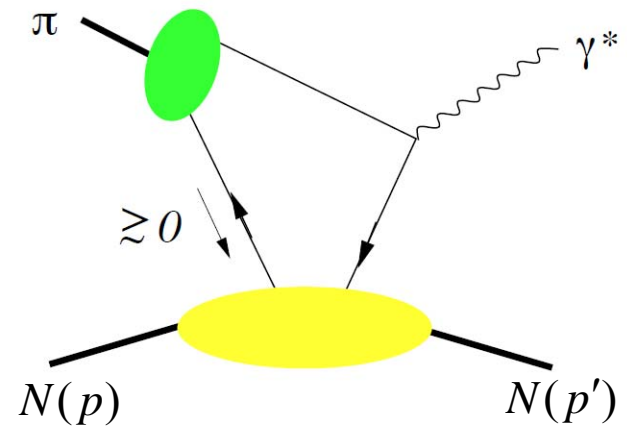
"exclusive DY"

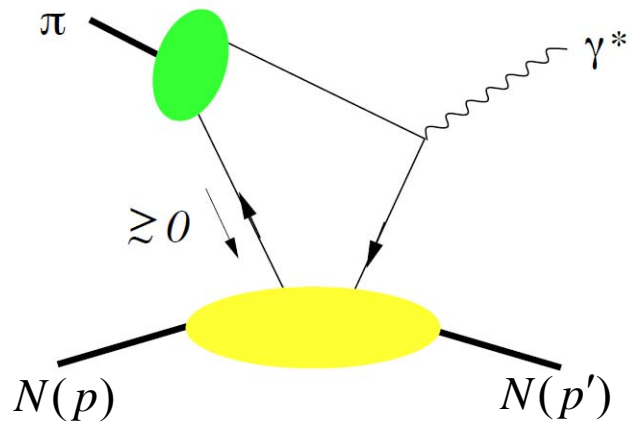
small  $t = \Delta^2 = (q - q')^2$



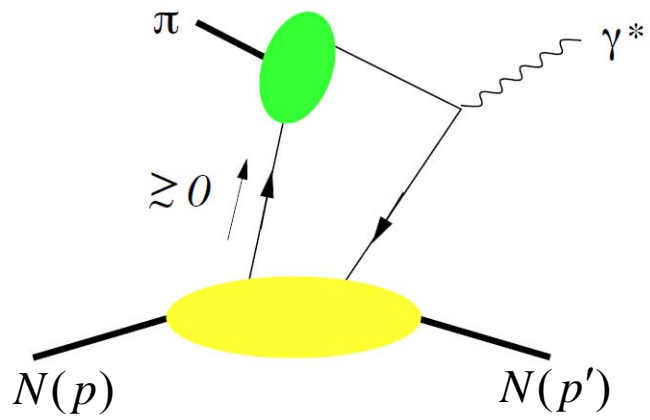




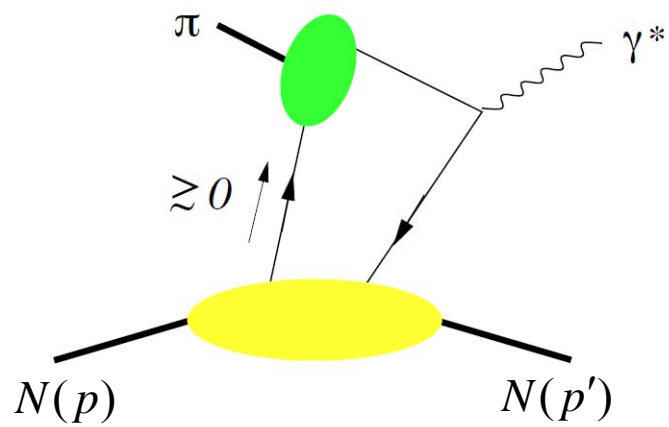
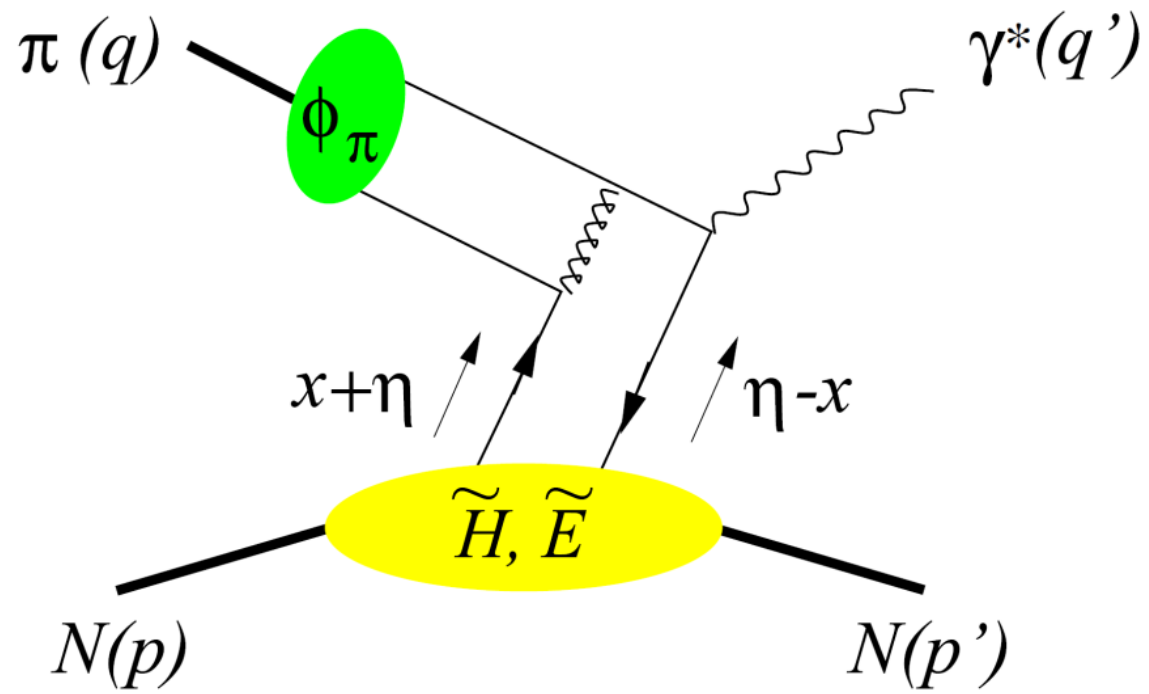


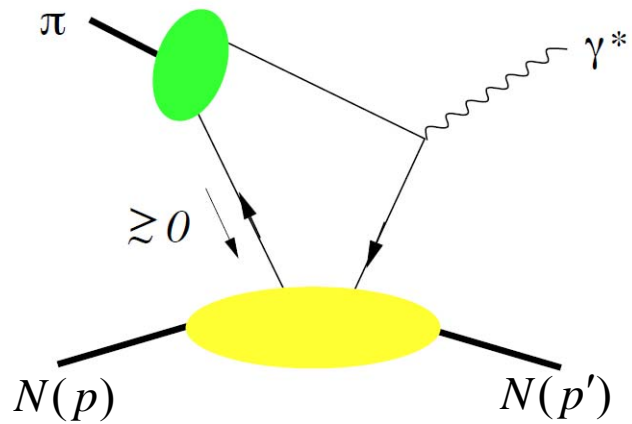


“Feynman mechanism”

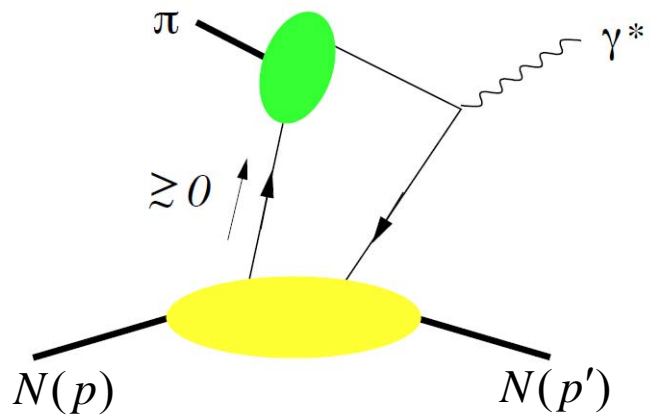


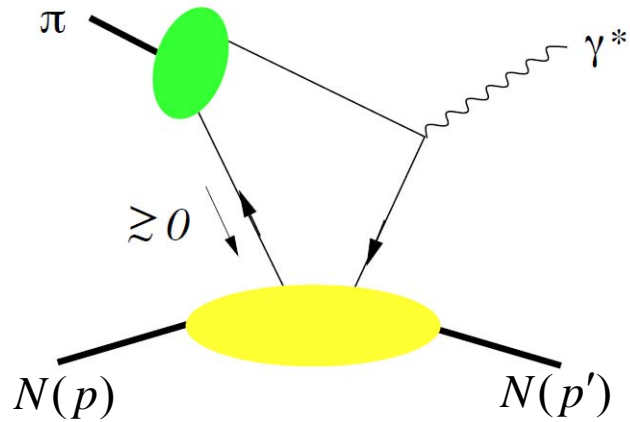






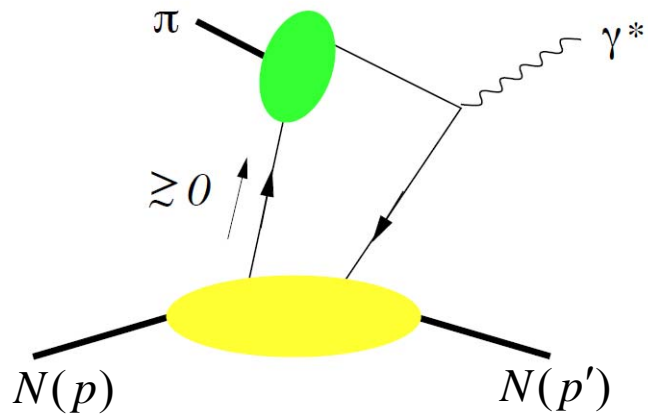
“Feynman mechanism”



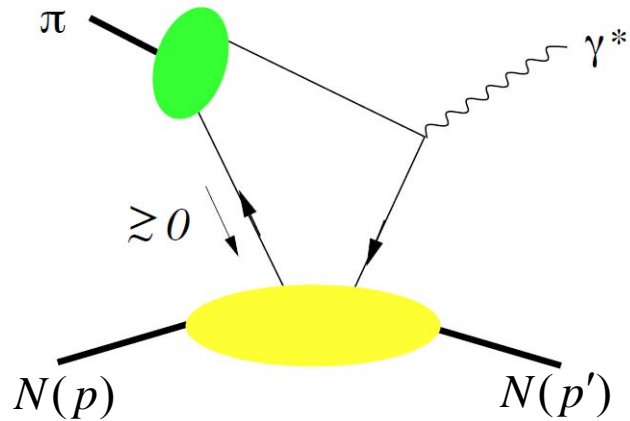


lower order in  $\alpha_s$

“Feynman mechanism”

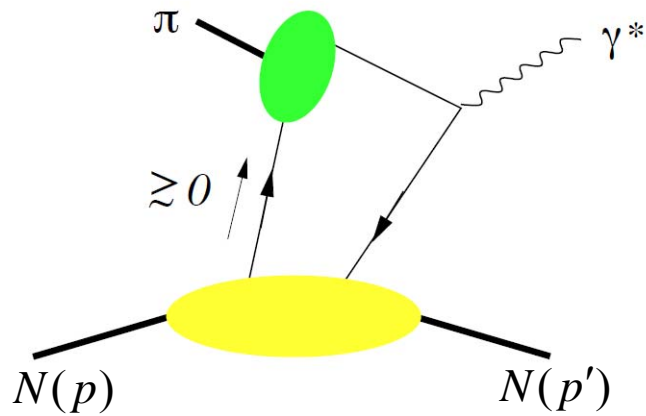


# “nonfactorizable” mechanism



lower order in  $\alpha_s$

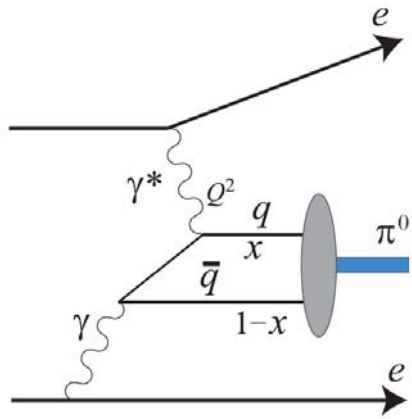
“Feynman mechanism”



# Exclusive lepton pair production in $\pi N$ scattering

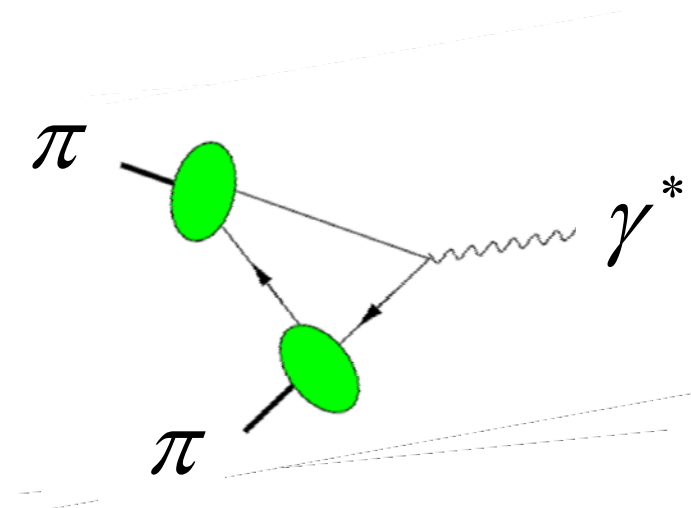
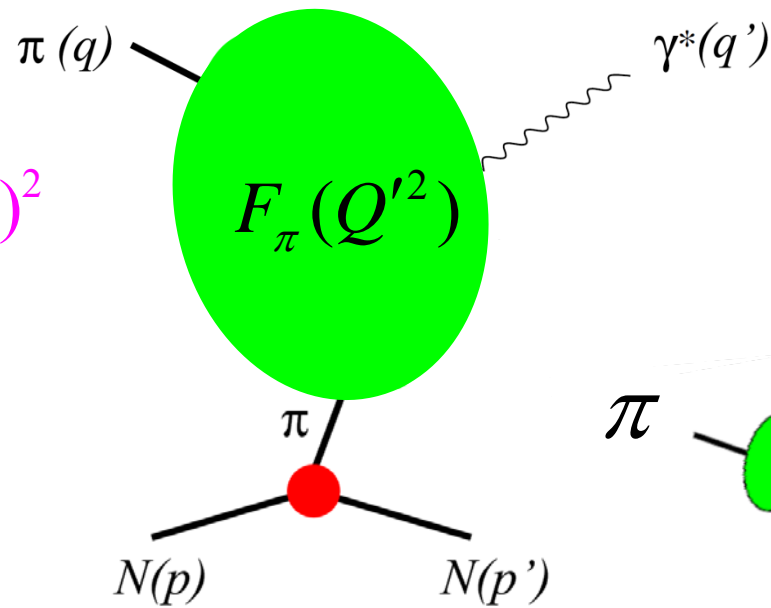
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

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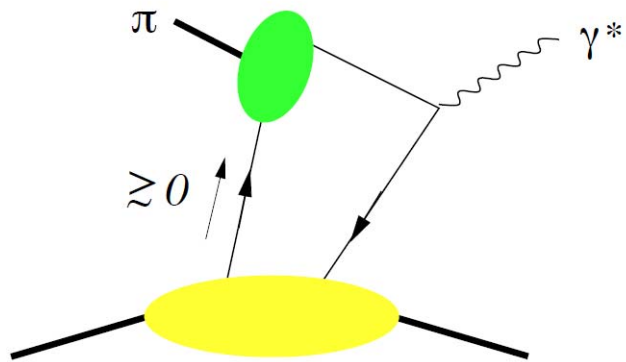
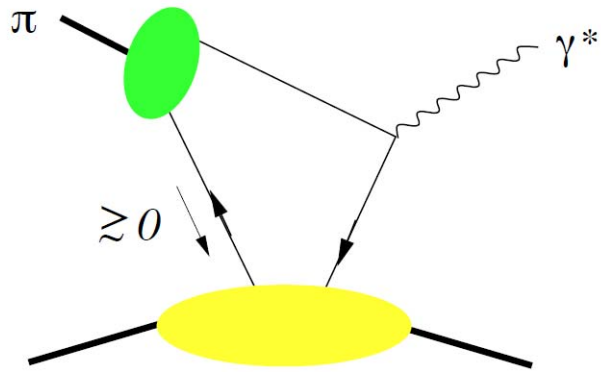


@Belle, Babar

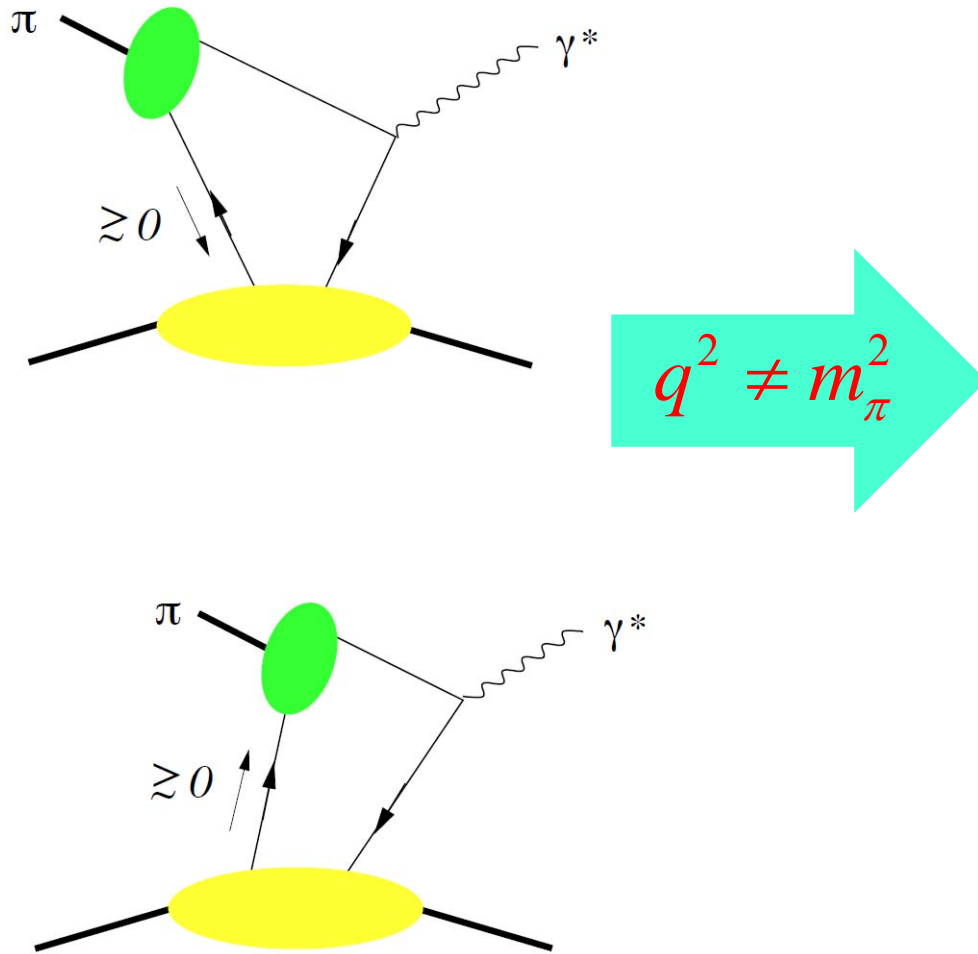
small  $t = \Delta^2 = (q - q')^2$



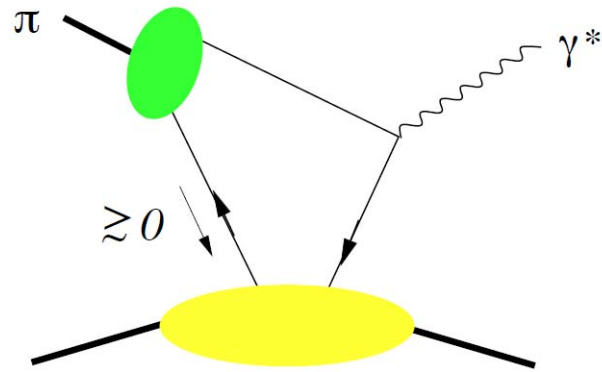
# "nonfactorizable" mechanism



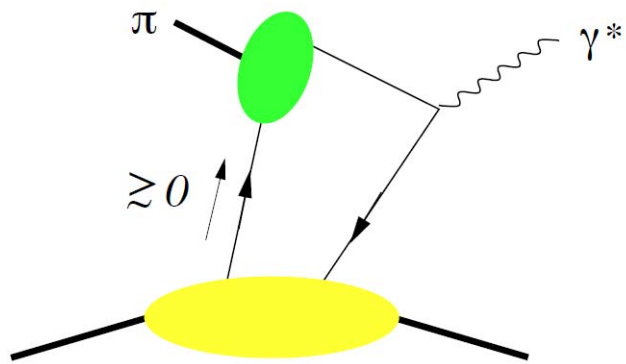
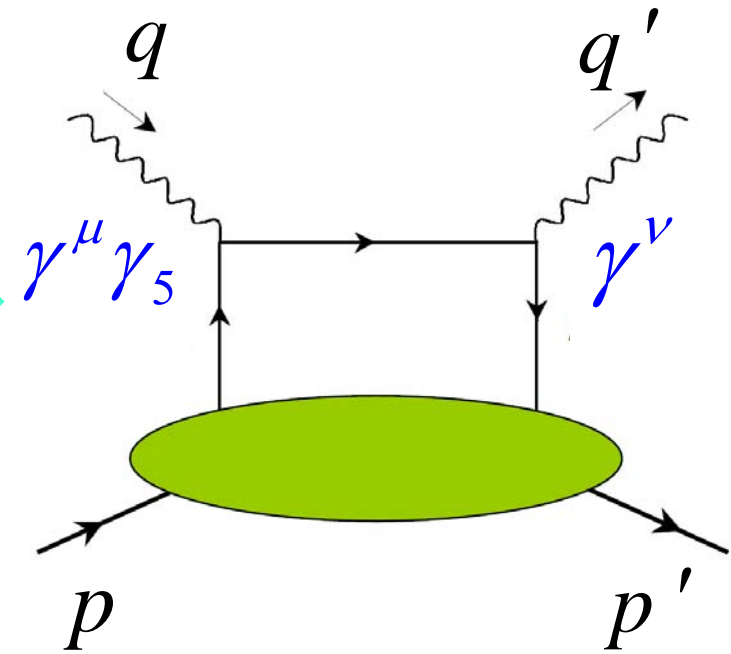
# "nonfactorizable" mechanism



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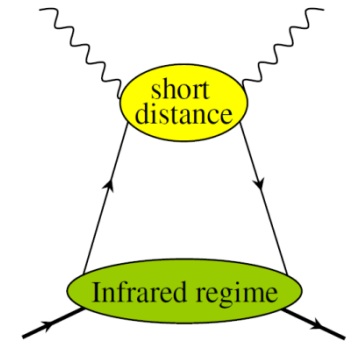
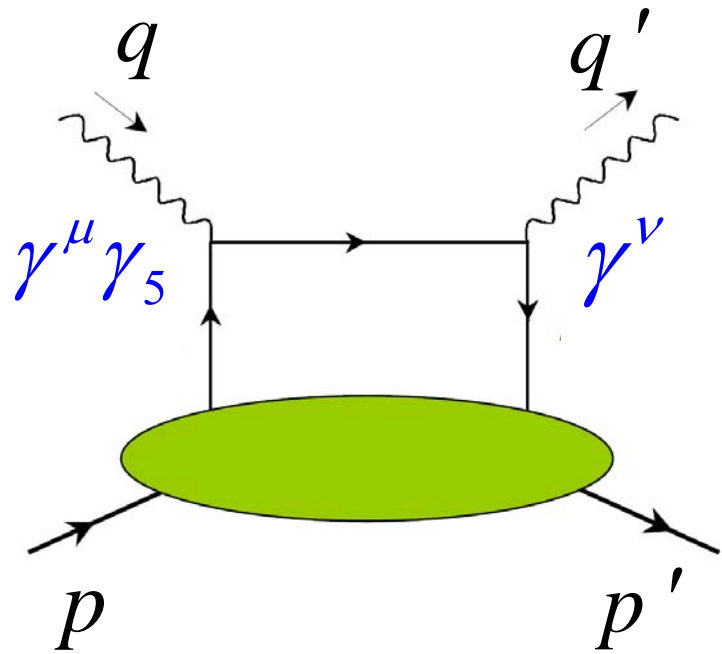
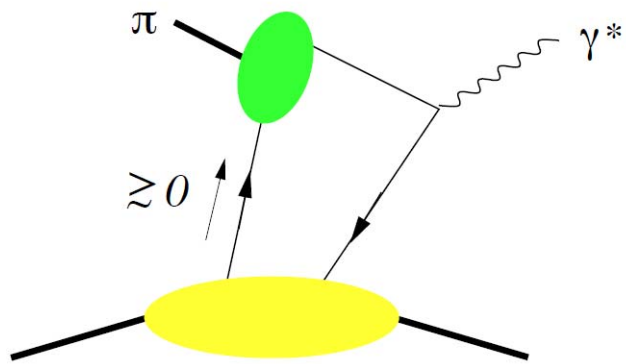
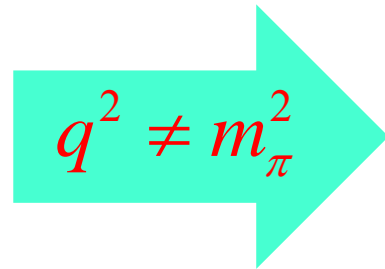
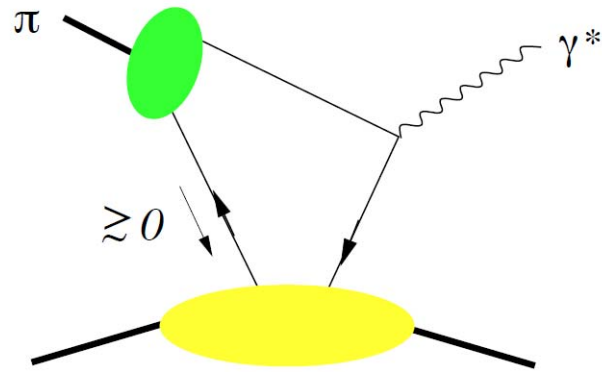


$q^2 \neq m_\pi^2$

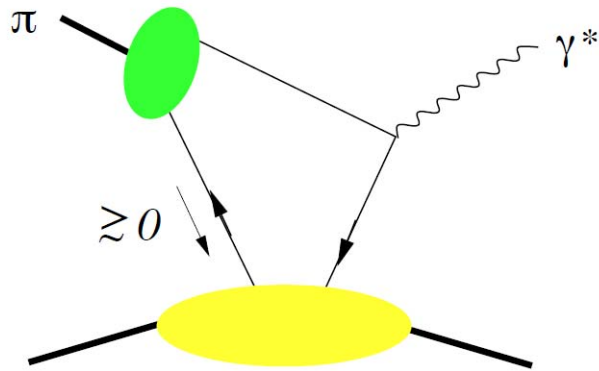




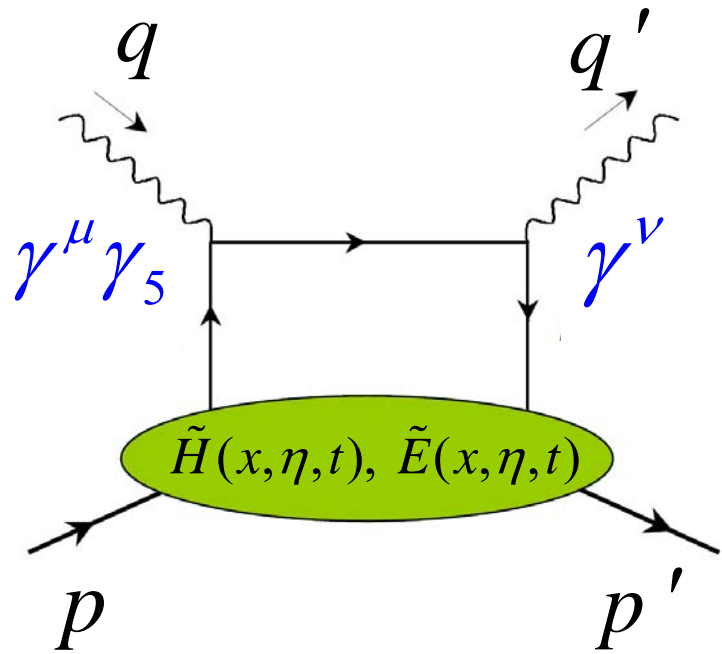
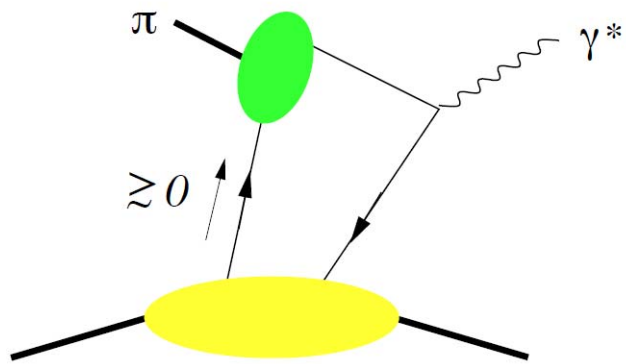
# "nonfactorizable" mechanism



# "nonfactorizable" mechanism



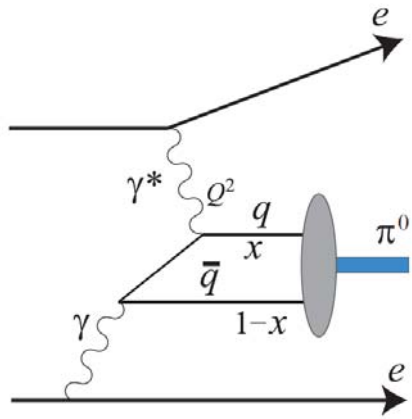
$q^2 \neq m_\pi^2$



# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

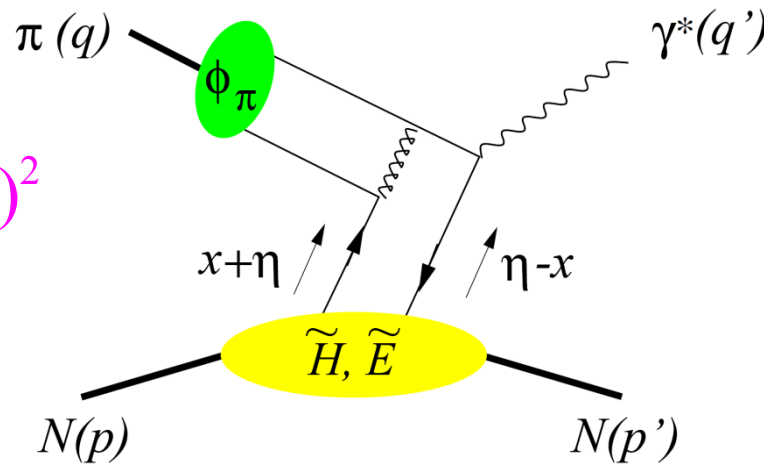
Berger, Diehl, Pire, PLB523(2001)265



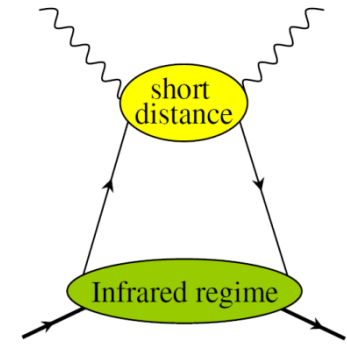
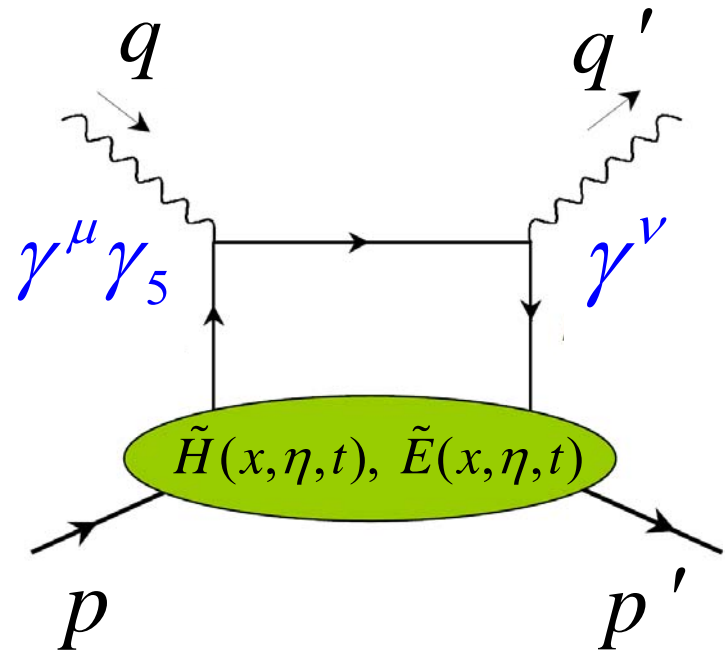
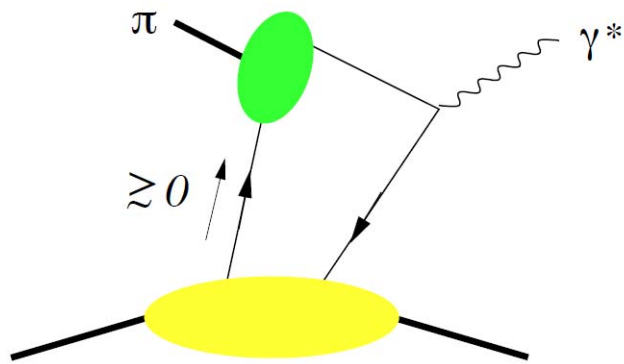
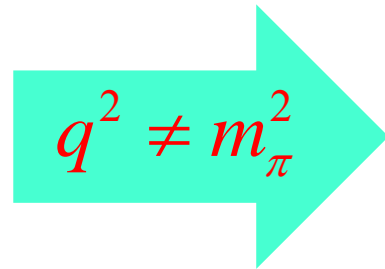
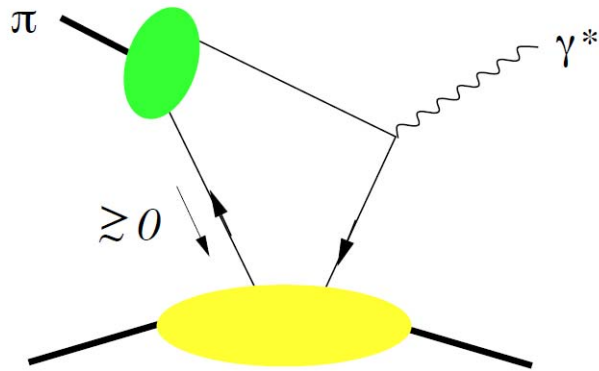
@Belle, Babar

"exclusive DY"

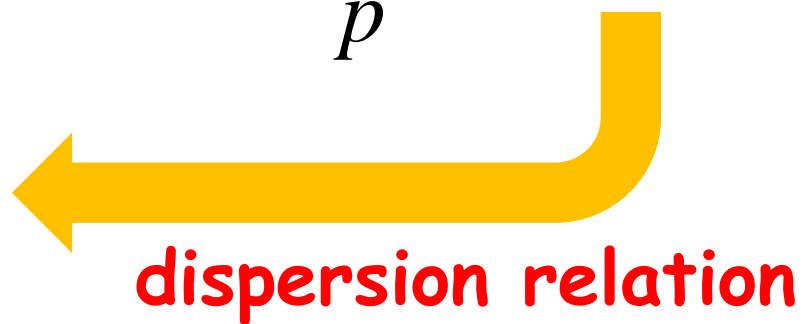
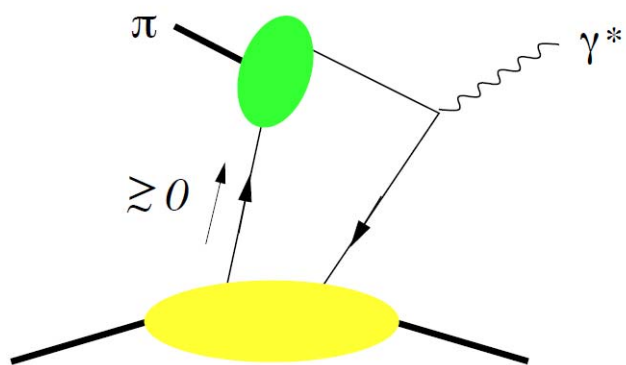
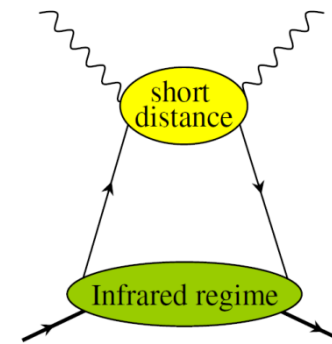
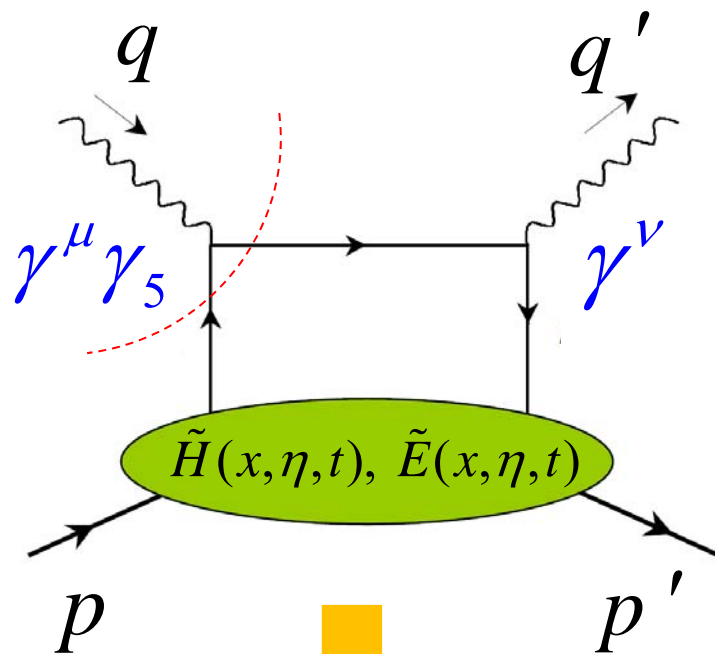
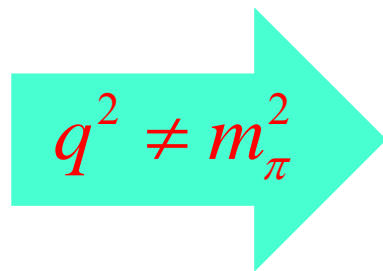
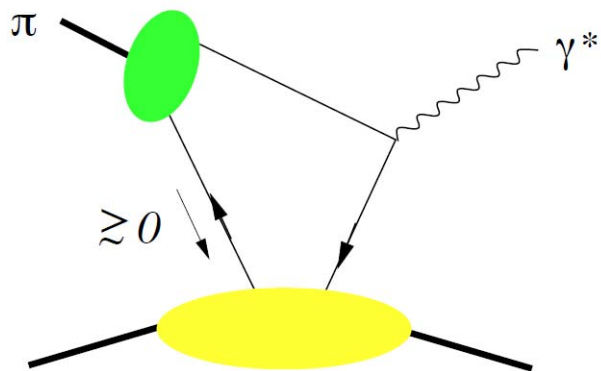
small  $t = \Delta^2 = (q - q')^2$



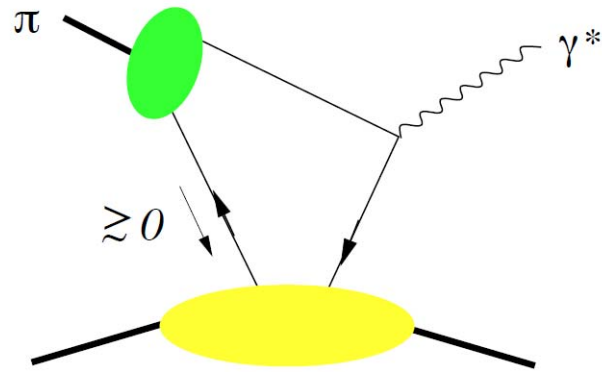
# "nonfactorizable" mechanism



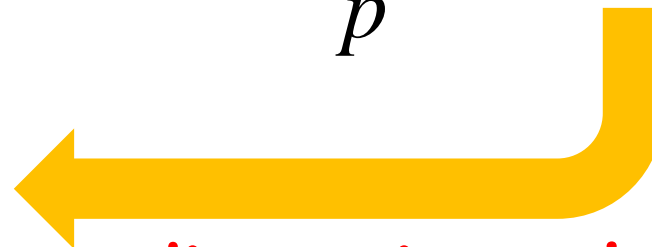
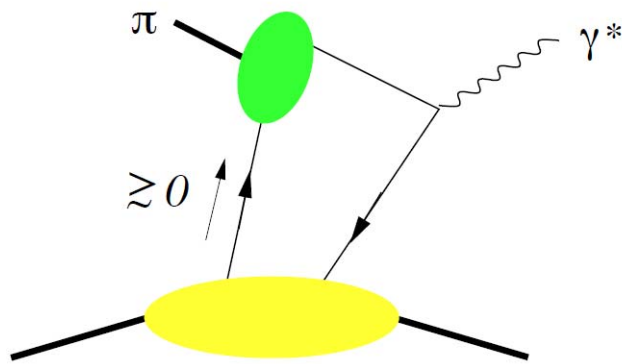
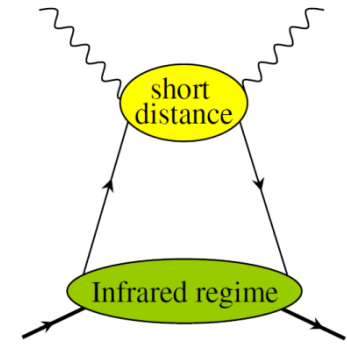
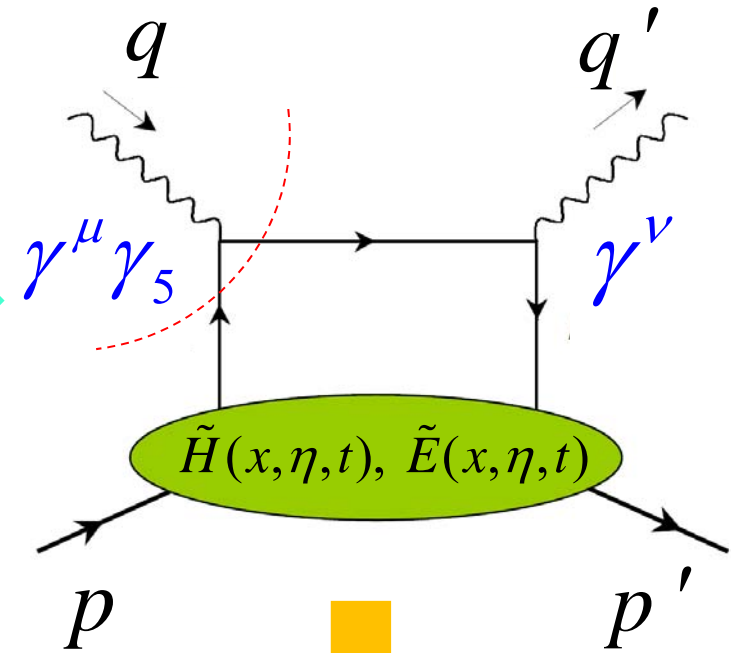
# "nonfactorizable" mechanism



# "nonfactorizable" mechanism

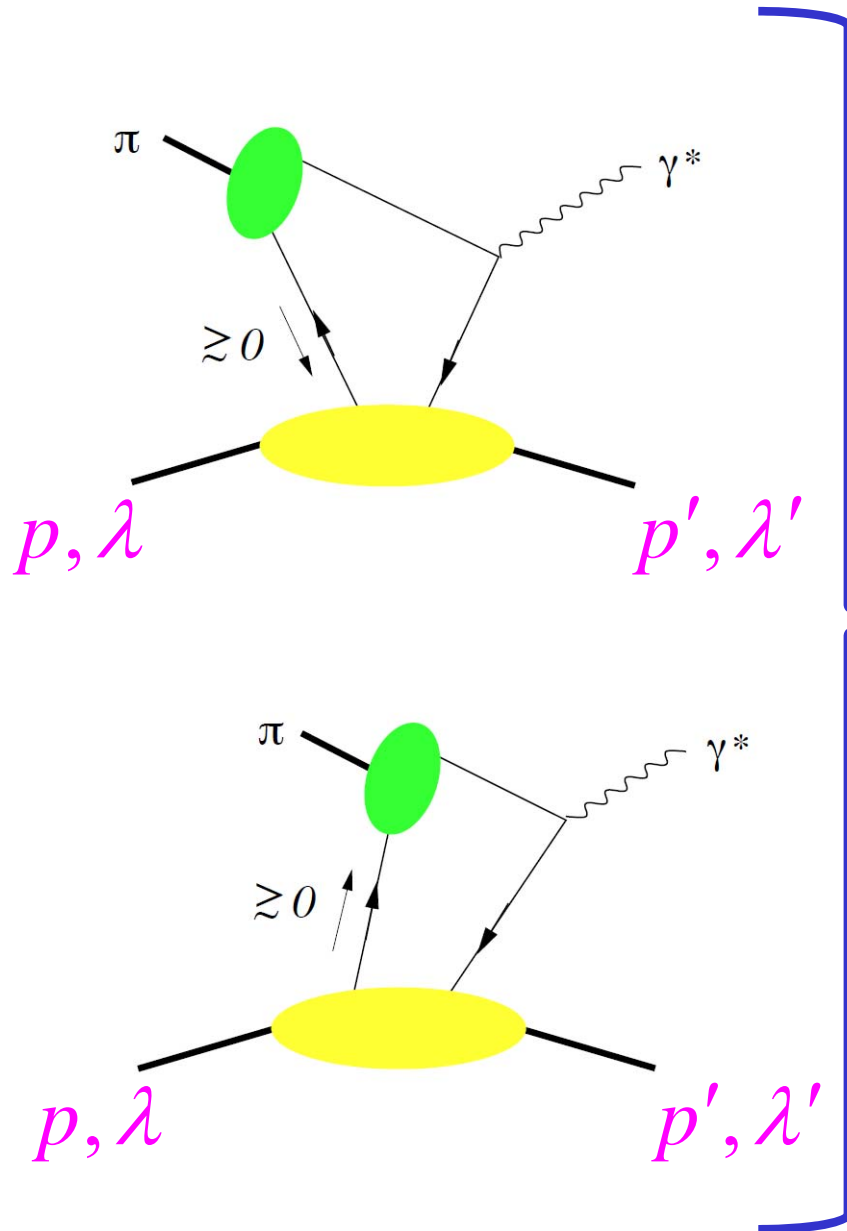


$q^2 \neq m_\pi^2$



dispersion relation  
quark-hadron duality

# "nonfactorizable" mechanism



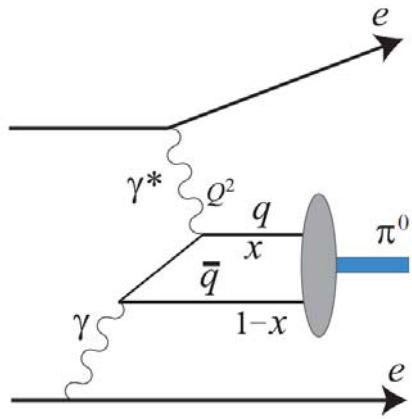
$$\begin{aligned}
 &= g_v^- \int_{\eta}^{x_0} dx e^{-\frac{x-\eta Q'^2}{x+\eta M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\
 &\times \left[ e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \\
 &\times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots
 \end{aligned}$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

# Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

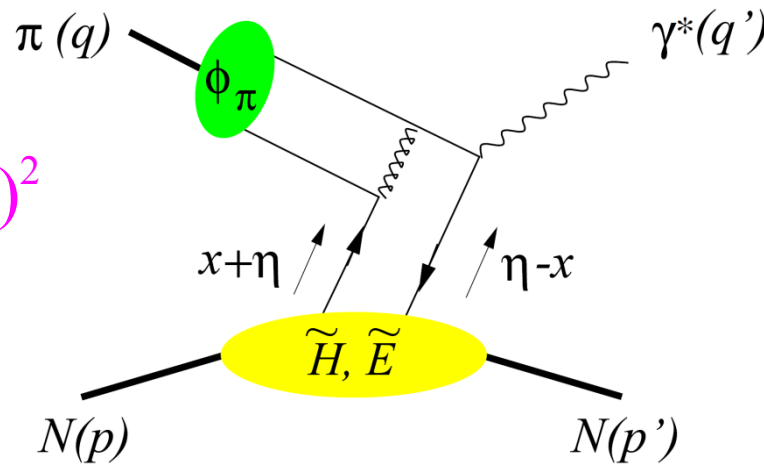
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

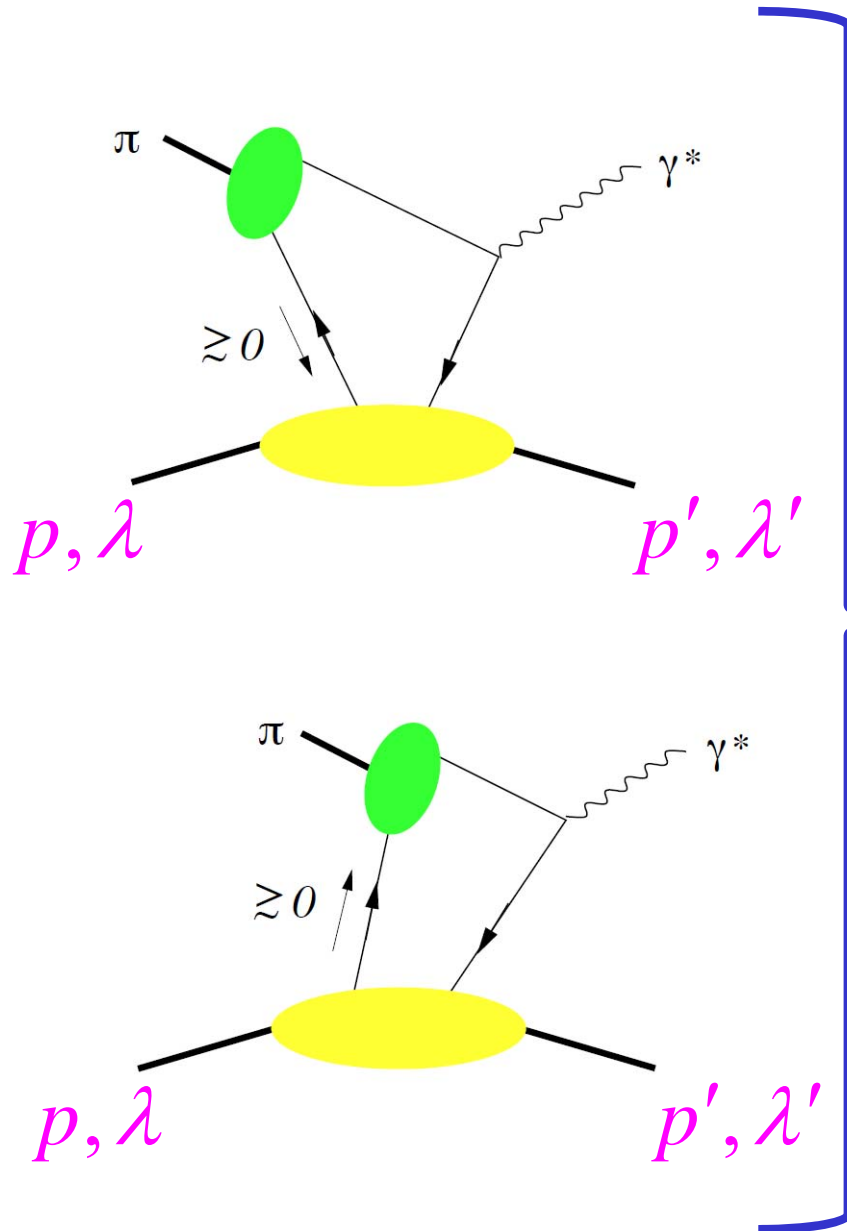
"exclusive DY"

small  $t = \Delta^2 = (q - q')^2$





# "nonfactorizable" mechanism

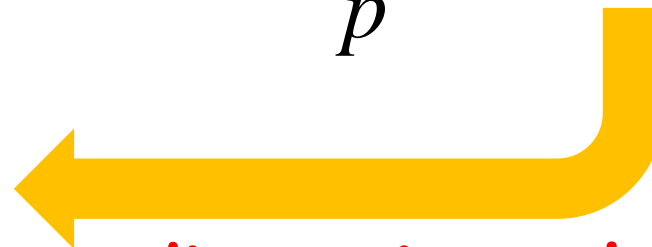
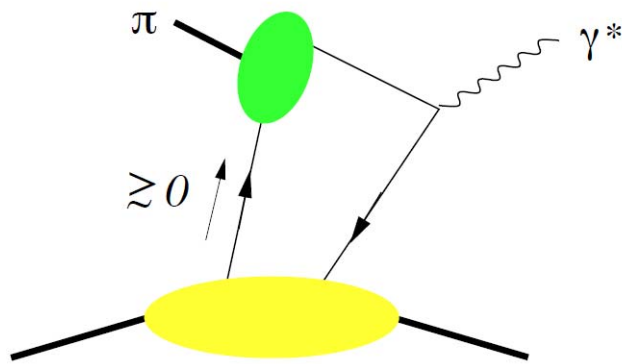
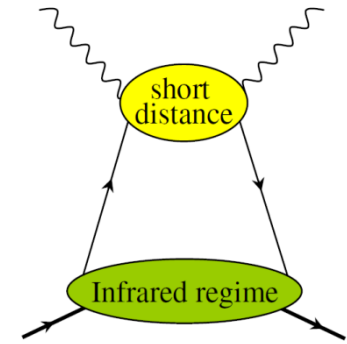
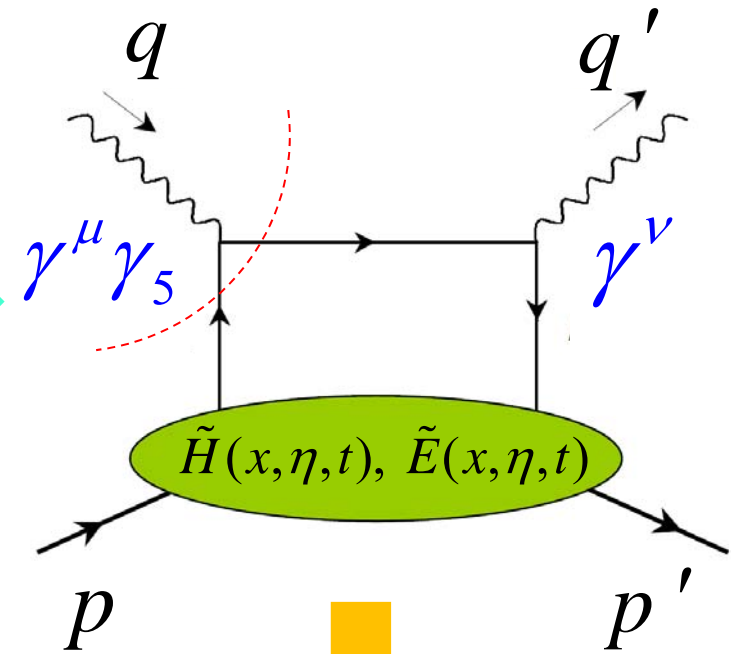
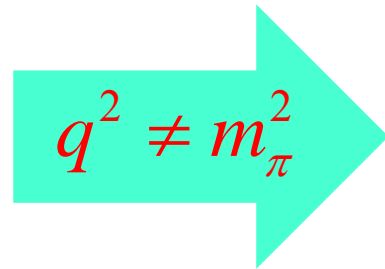
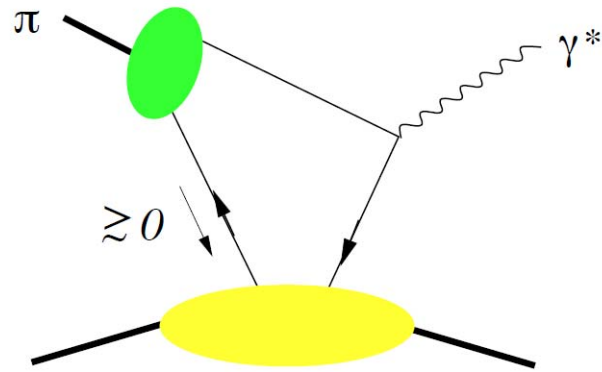


$$= g_v^- \int_{\eta}^{x_0} dx e^{-\frac{x-\eta Q'^2}{x+\eta M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\ \times \left[ e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right]$$

$$\times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots$$

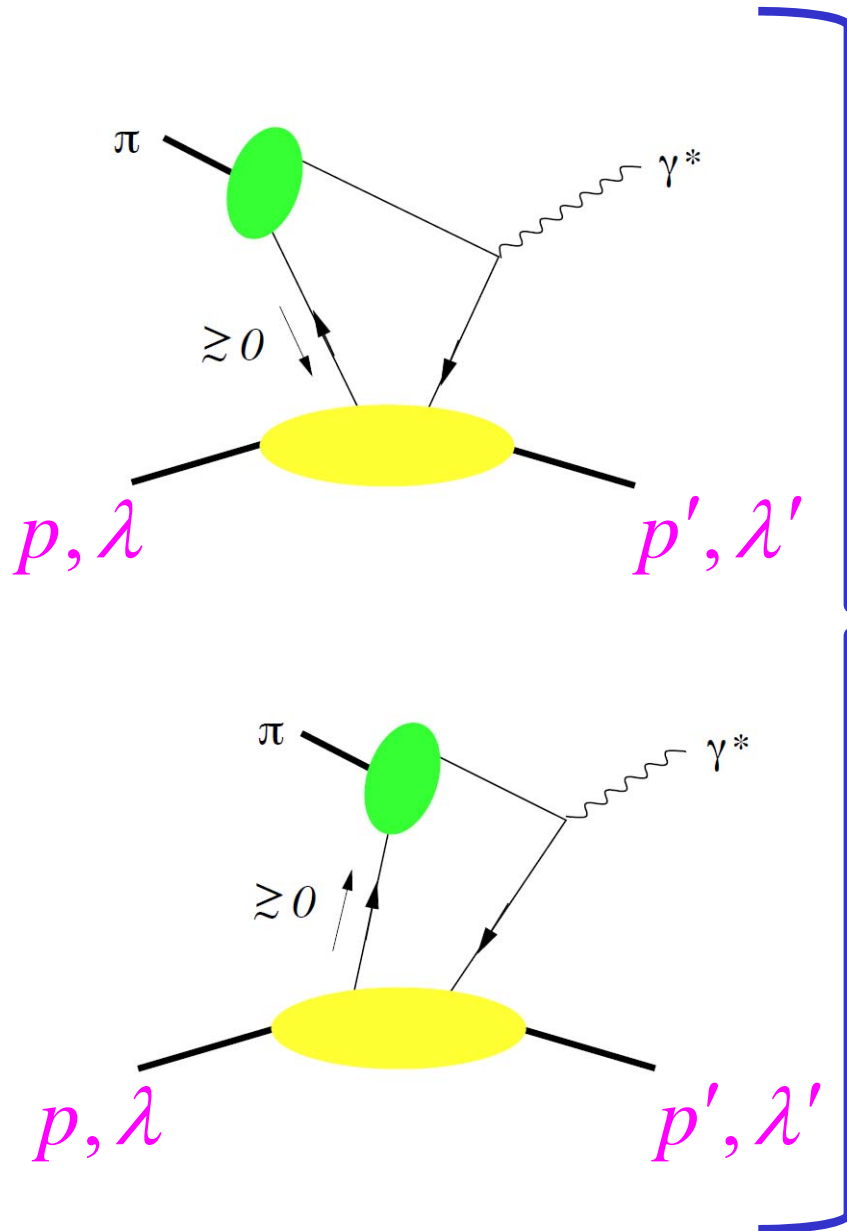
$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

# "nonfactorizable" mechanism



dispersion relation  
quark-hadron duality

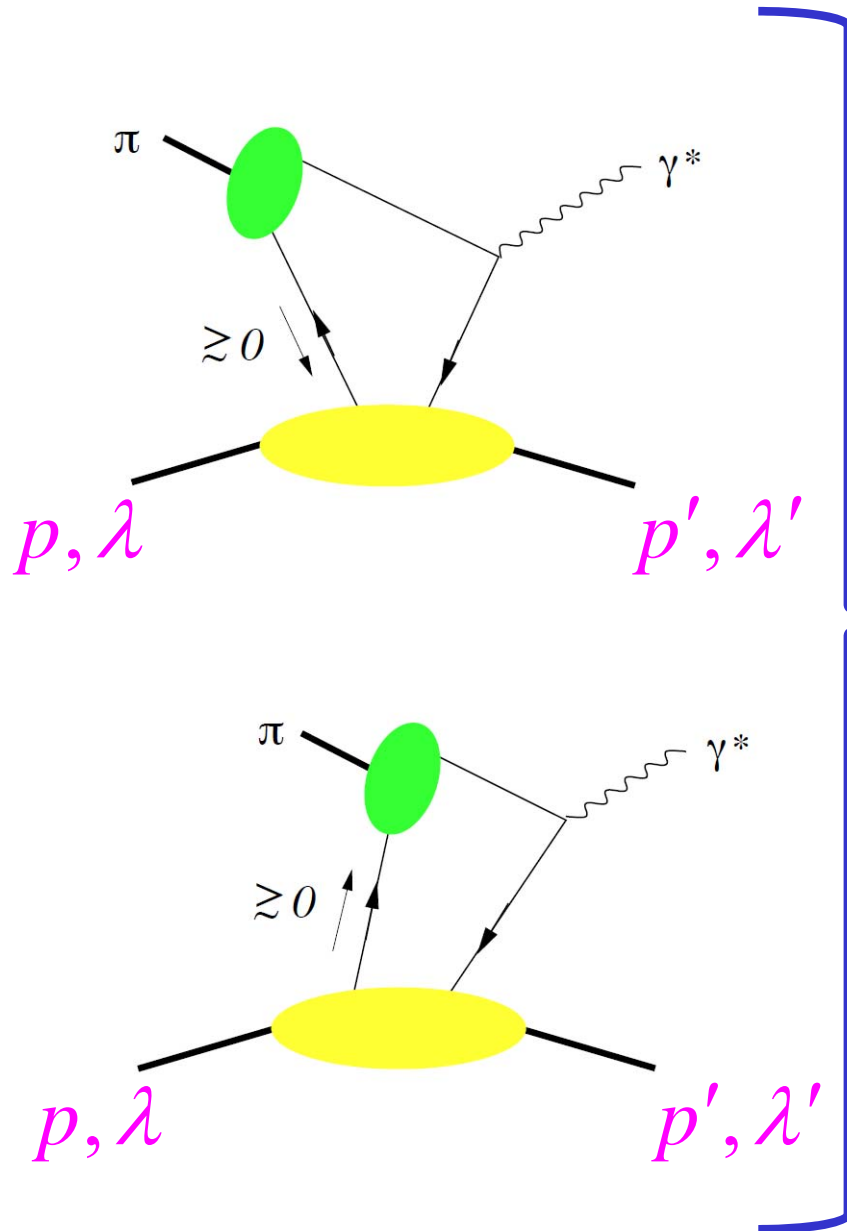
# "nonfactorizable" mechanism



$$\begin{aligned}
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 \end{aligned}$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

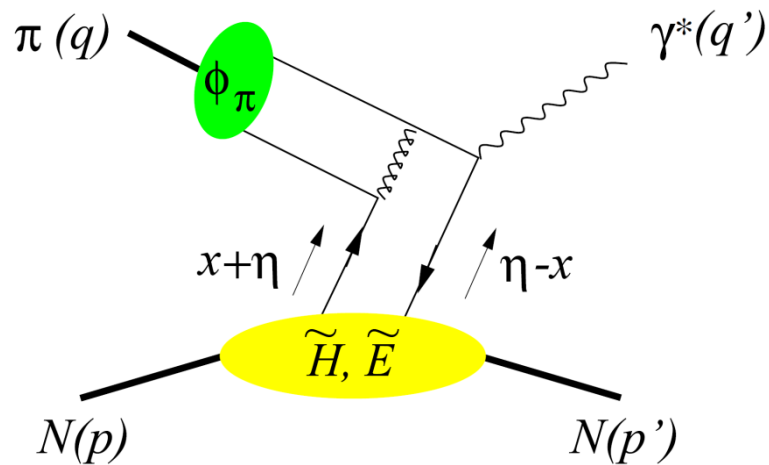
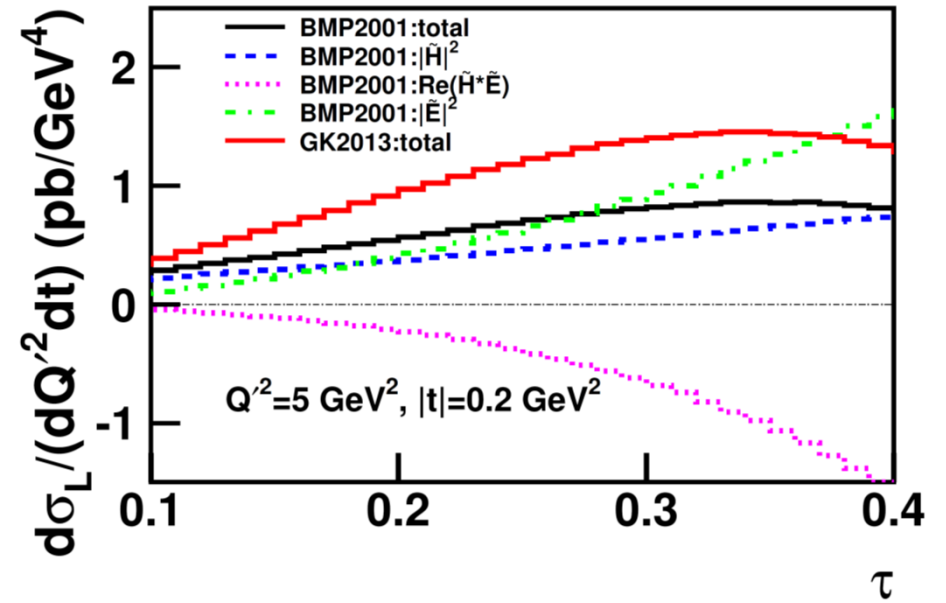
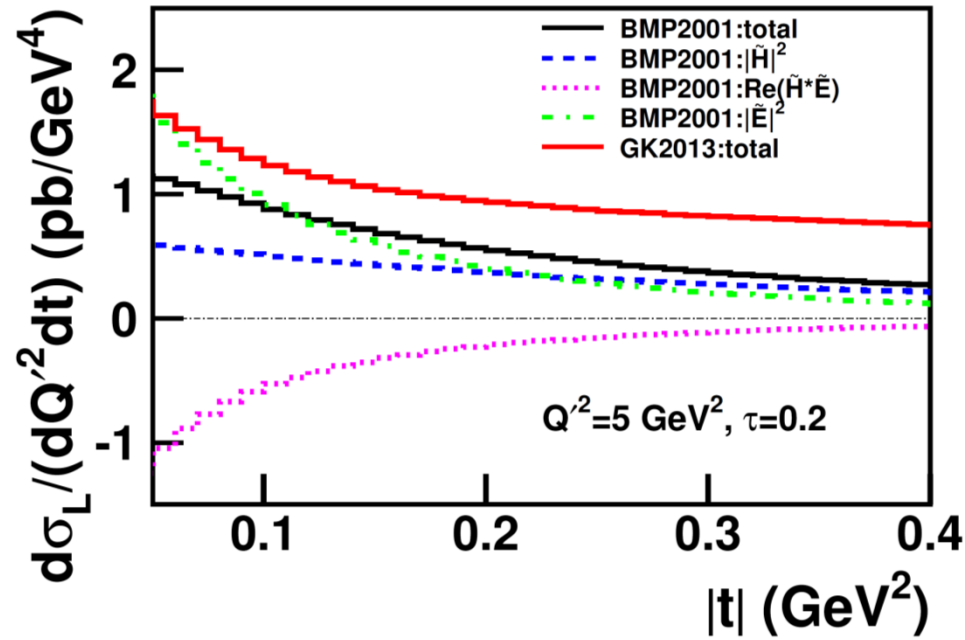
# "nonfactorizable" mechanism

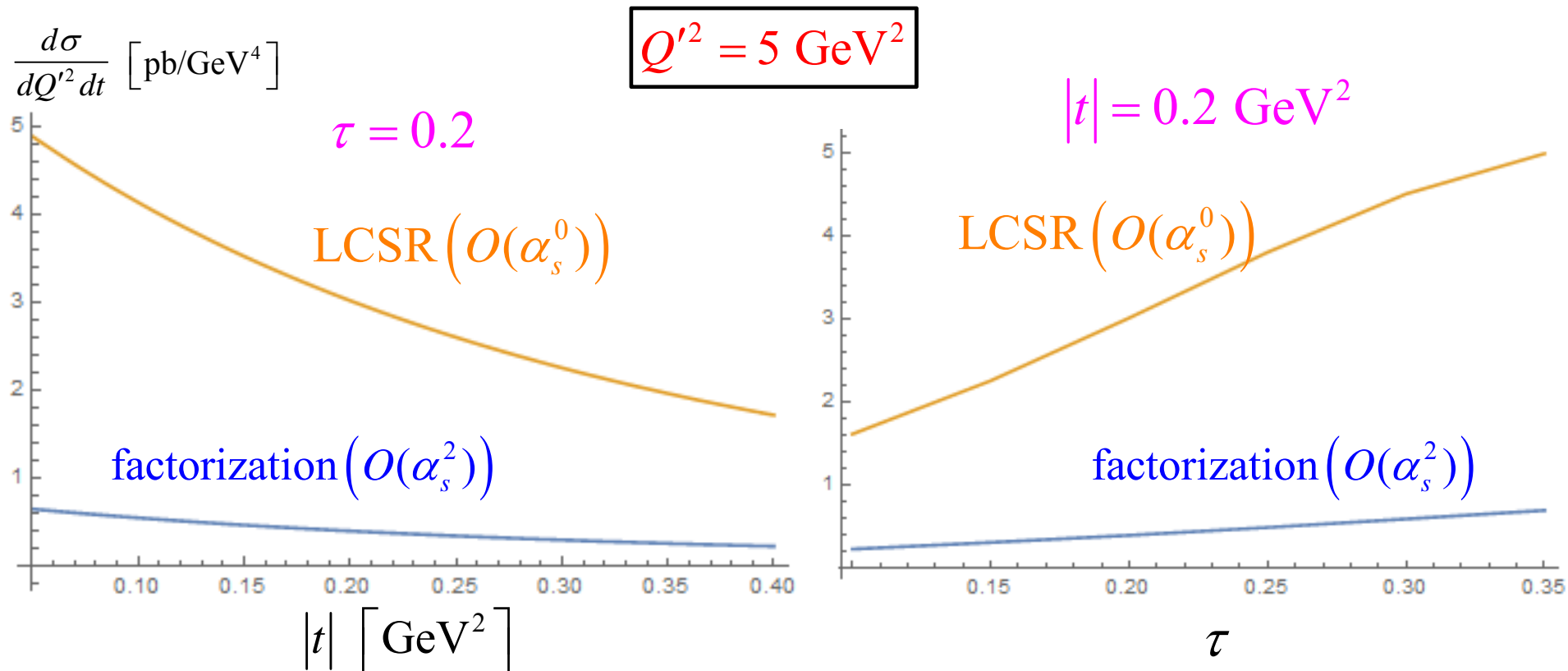


"Light-cone QCD SR (LCSR)"

$$\begin{aligned}
 &= g_v^- \int_{\eta}^{x_0} dx e^{-\frac{x-\eta Q^2}{x+\eta M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\
 &\times \left[ e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \\
 &\times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots
 \end{aligned}$$

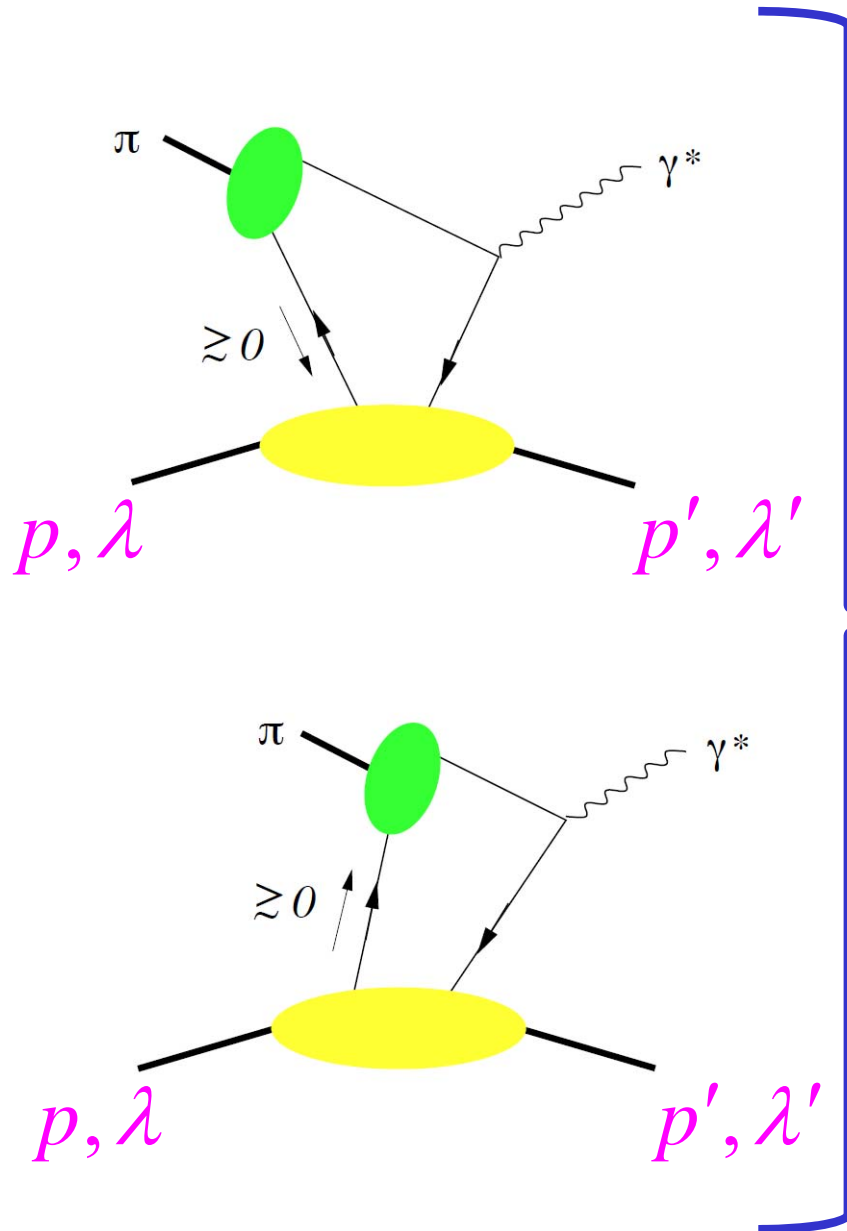
$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$





$$\frac{d\sigma}{dQ'^2 dt} (\pi^- p \rightarrow \gamma^* n) = \frac{4\pi\alpha_{\text{em}}^2}{27} \frac{\tau^2}{Q'^8} f_\pi^2 \left[ (1-\eta^2) |\widetilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\widetilde{\mathcal{H}}^{du*} \widetilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\widetilde{\mathcal{E}}^{du}|^2 \right]$$

# "nonfactorizable" mechanism



"Light-cone QCD SR (LCSR)"

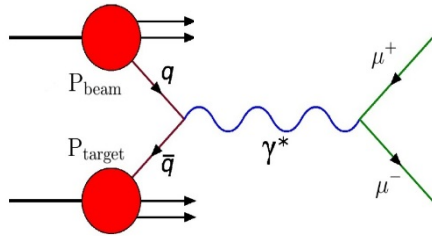
$$\begin{aligned}
 &= g_v^- \int_{\eta}^{x_0} dx e^{-\frac{x-\eta Q'^2}{x+\eta M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\
 &\times \left[ e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \\
 &\times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots
 \end{aligned}$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

# Summary

## inclusive DY

$$q(x), \bar{q}(x)$$



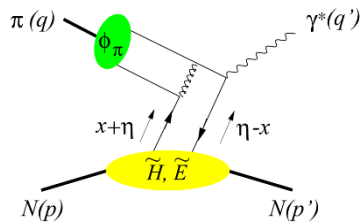
TMD

$$\langle k_{\perp}^2 \rangle$$

$$\text{Sivers}|_{DY} = - \text{Sivers}|_{SIDIS} L_q$$

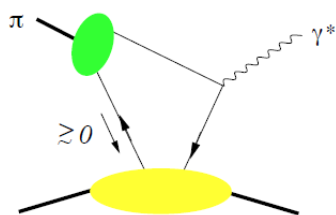
BM & angular distribution

## exclusive DY GPDs



feasibility study with J-PARC E50 spectrometer

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT  
PRD93, 114034



LCSR for nonfactorizable amplitude

SNM > factorization

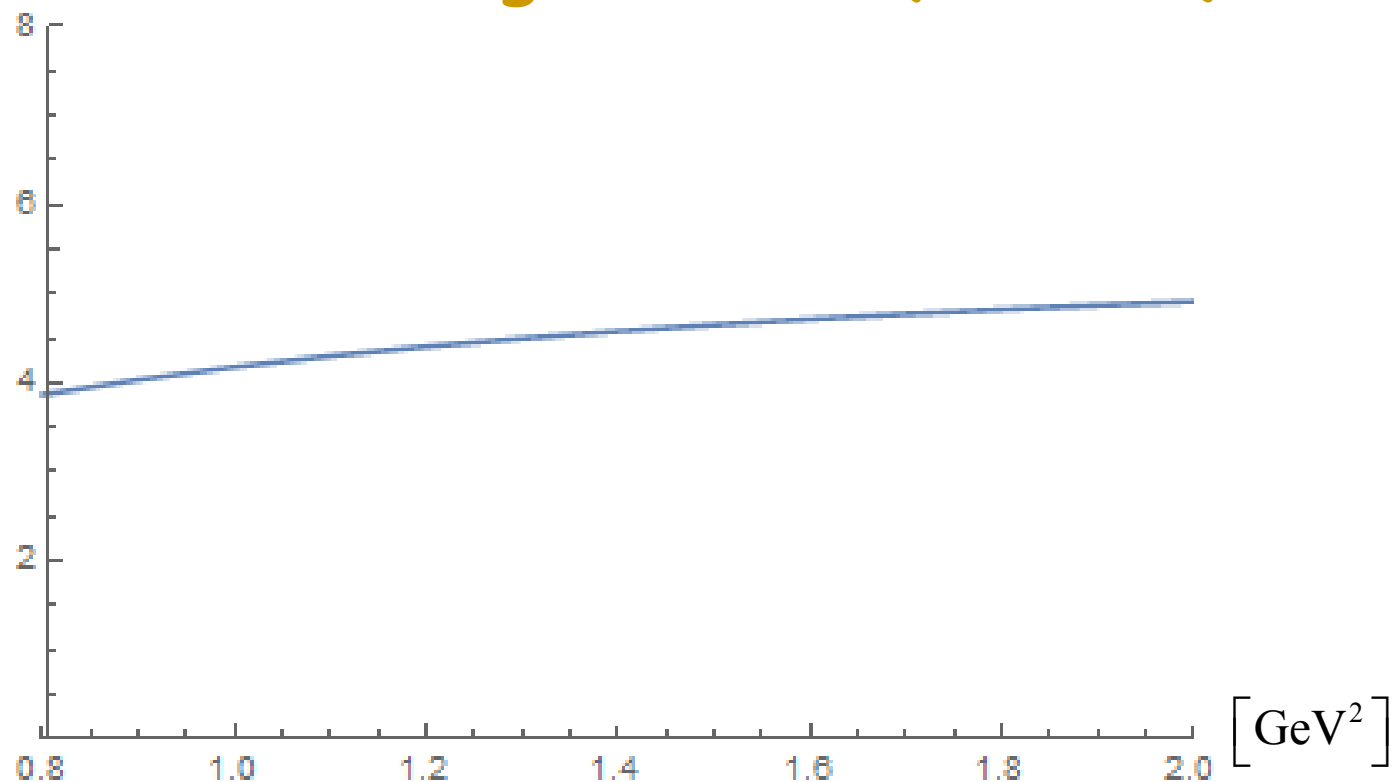
*interplay of soft/hard QCD mechanism*





$\tilde{\mathcal{F}}_{LCSR}^{du}(\eta, t)$ 

“Light-cone QCD SR (LCSR)”



$M_B^2$  (Borel parameter)

$$M_v^{\lambda', \lambda}(\pi^- p \rightarrow \gamma^* n) = -ie \frac{1}{(p+p')^+} \bar{u}(p', \lambda') \left[ \gamma^+ \gamma_5 \tilde{\mathcal{F}}_{LCSR}^{du}(\eta, t) + \gamma_5 \frac{(p' - p)^+}{2M} \tilde{\mathcal{E}}_{LCSR}^{du}(\eta, t) \right] u(p, \lambda)$$