Unitary dispersive approach for the three pion light meson decays

Igor Danilkin

Thanks to **JPAC**: P. Guo, C. Fernandez-Ramırez,
V. Mathieu and A. P. Szczepaniak

MENU2016, Kyoto, July 25-30, 2016

PRiSMA

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JG|U

Table of contents

- Introduction & motivation
- Light mesons decays:

  $\eta \to 3\pi$ and light quark masses
  $\omega, \varphi \to 3\pi$

- Method (Khuri Treiman & Pasquier inversion)
- Results and outlook

Motivation

Aim

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Spectrum of QCD: complete understanding, discover new resonances, exotics …

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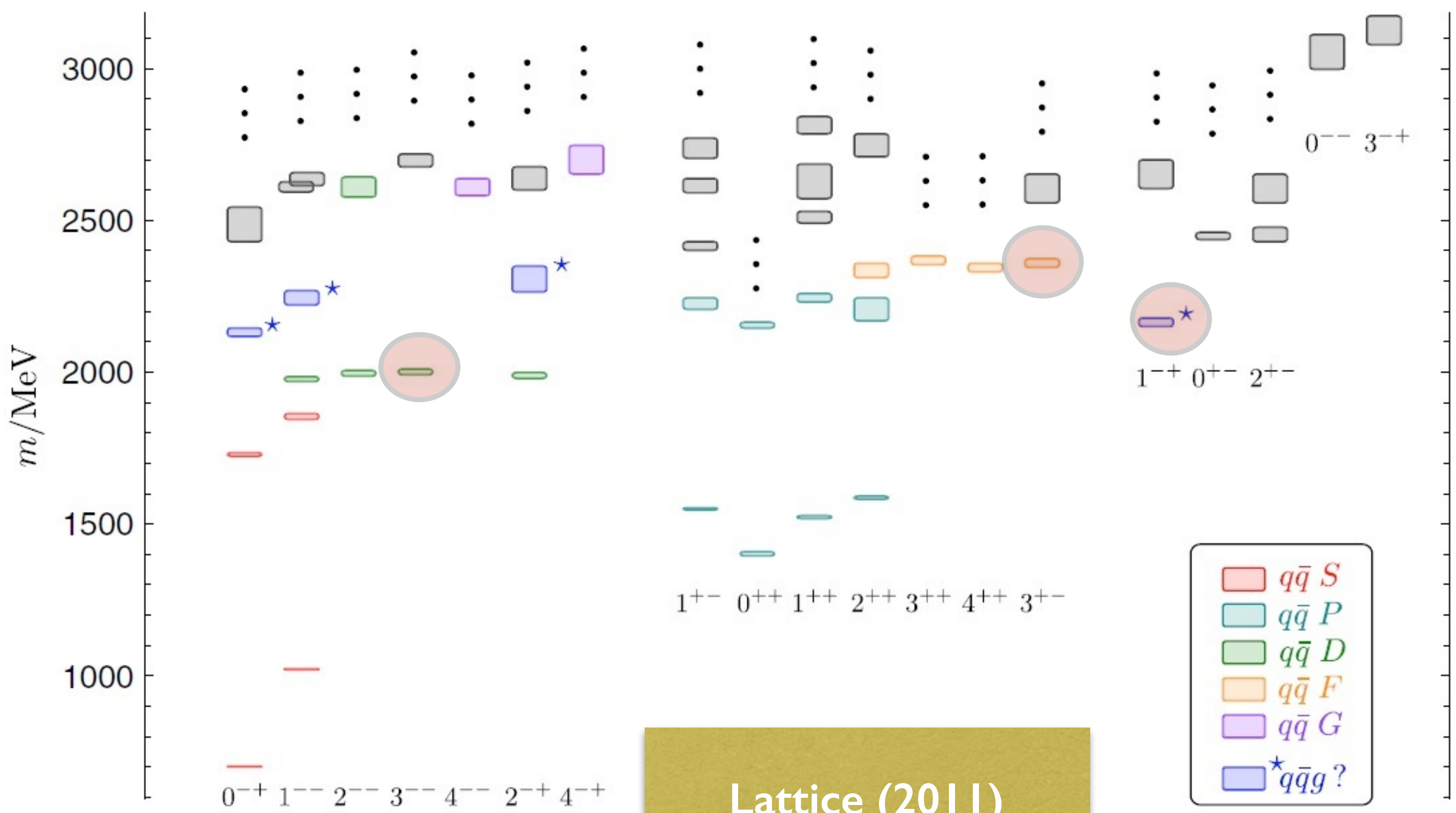


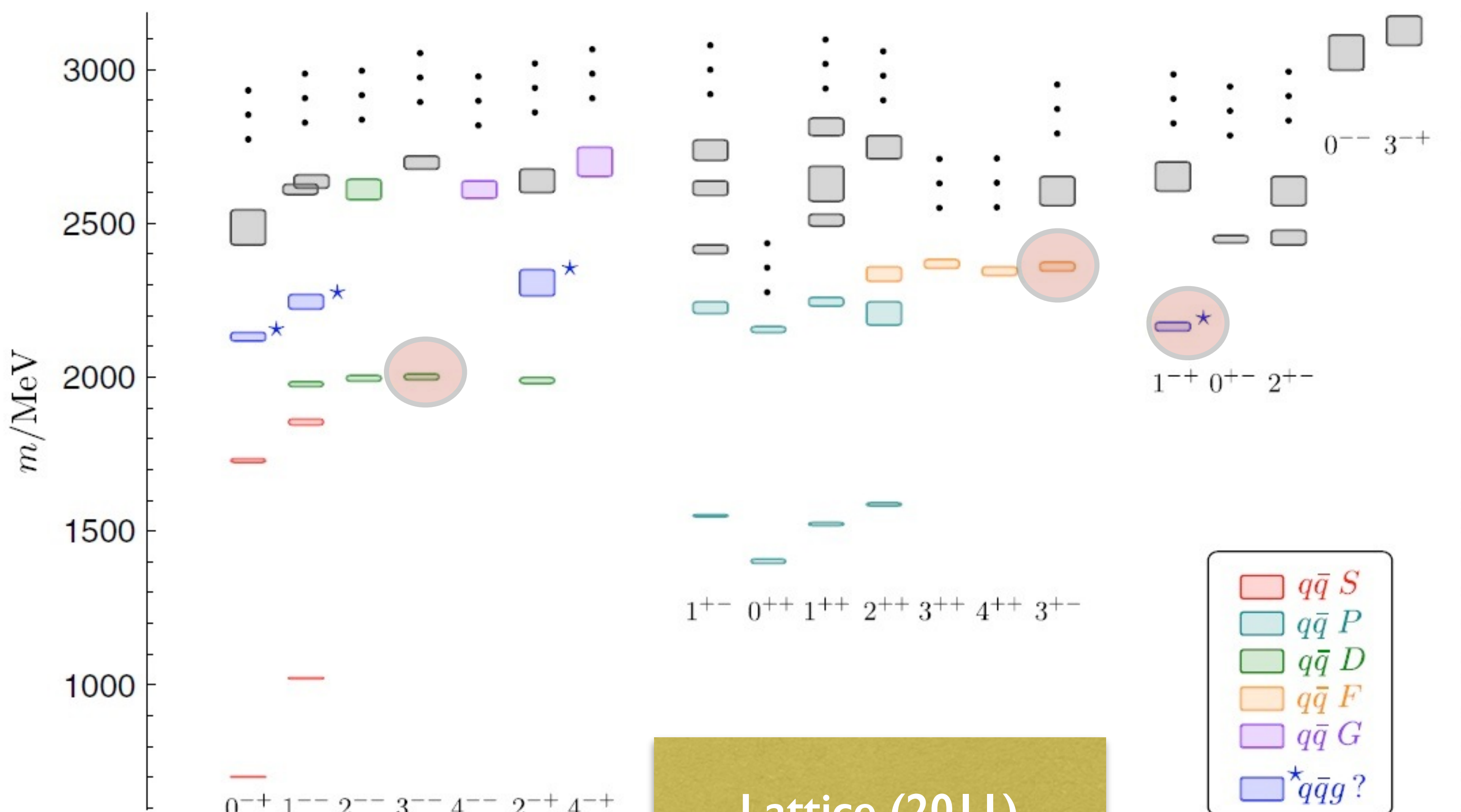
Lattice (2011)
Jozef J. Dudek

Motivation

Aim

Spectrum of QCD: complete understanding, discover new resonances, exotics …

Decay properties of the known states



Lattice (2011)
Jozef J. Dudek

- multi-body (final state) interactions are expected to play a crucial role for the hadron spectroscopy
- analysis of the precision exp. data
- test/develop/cross-check tools on conventional states → move to exotic

Motivation

ω-meson discovered in ~1960th

π p →ω p, Kp →ωΛ, e^+e^-→ 3π, pp→ ωππ, … **number of events:** 10^3 - 10^4

Dalitz plot: ω→3π

$$\frac{d^2\Gamma}{ds\,dt} \propto |\vec{p}_+ \times \vec{p}_-|^2 |F(s,t)|^2$$

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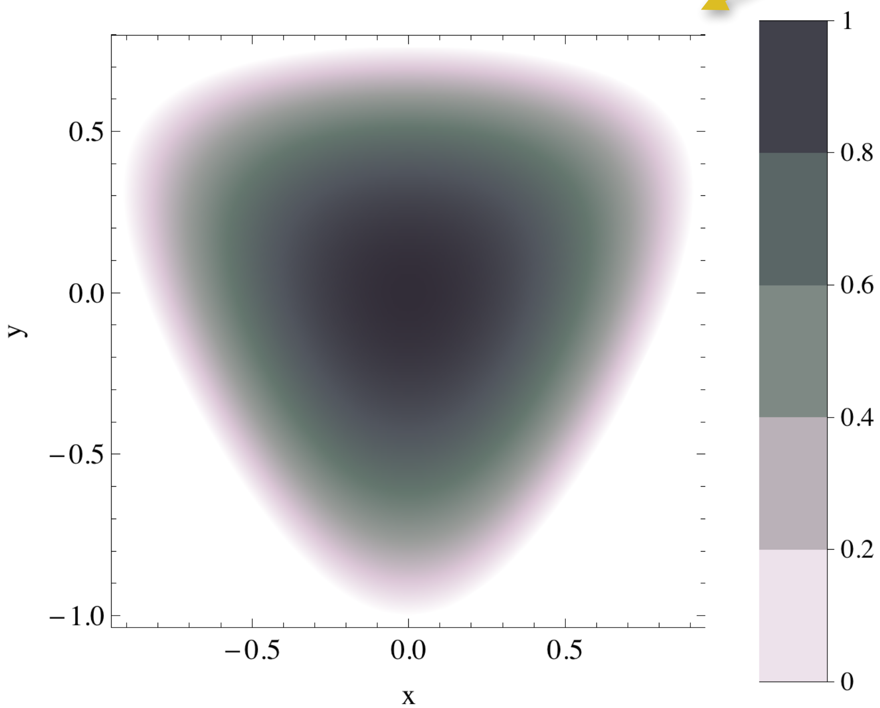

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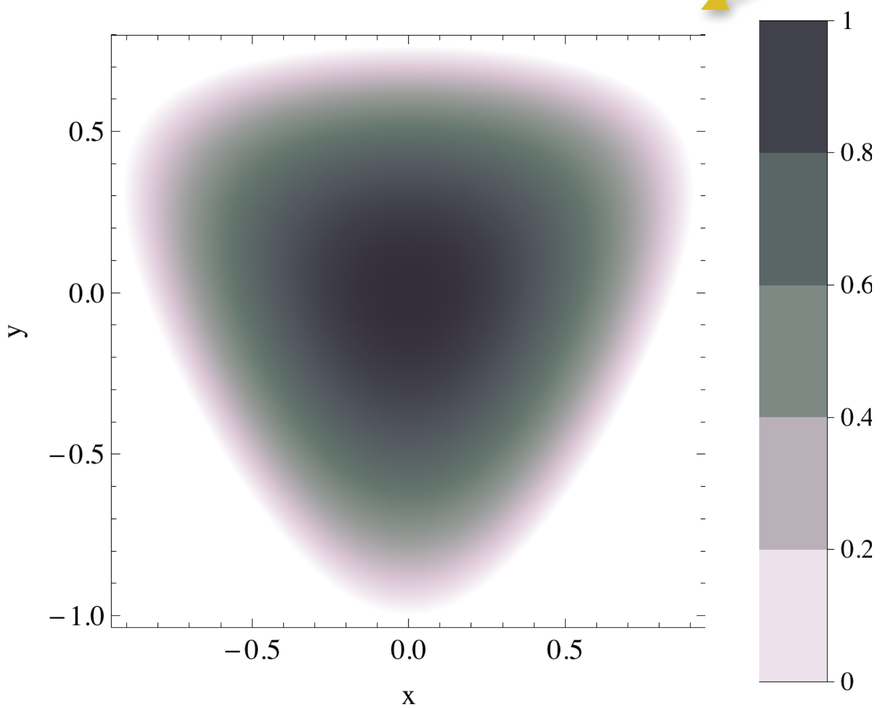

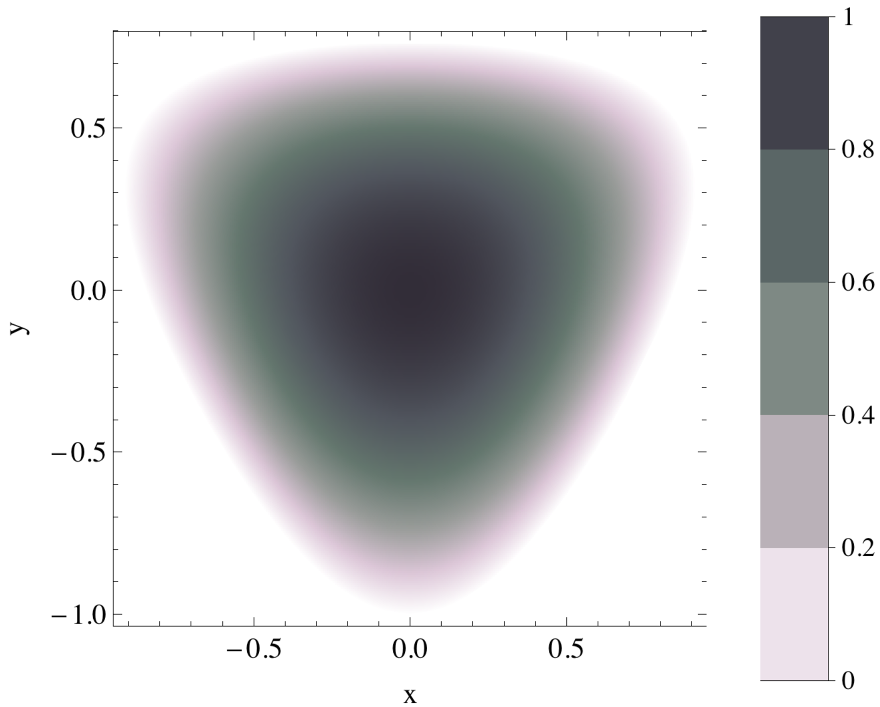

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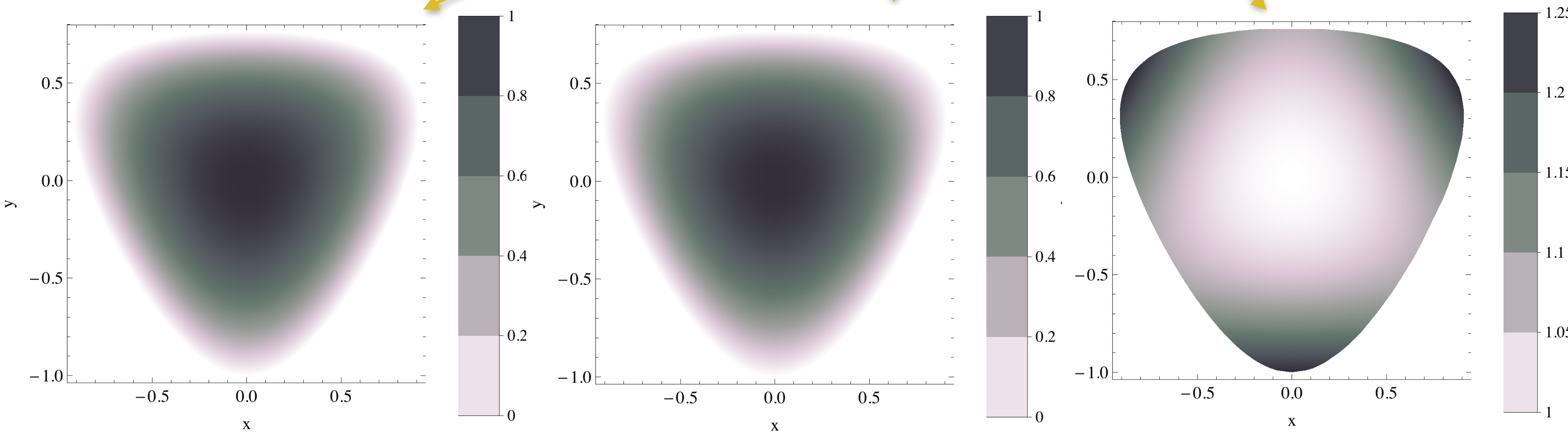

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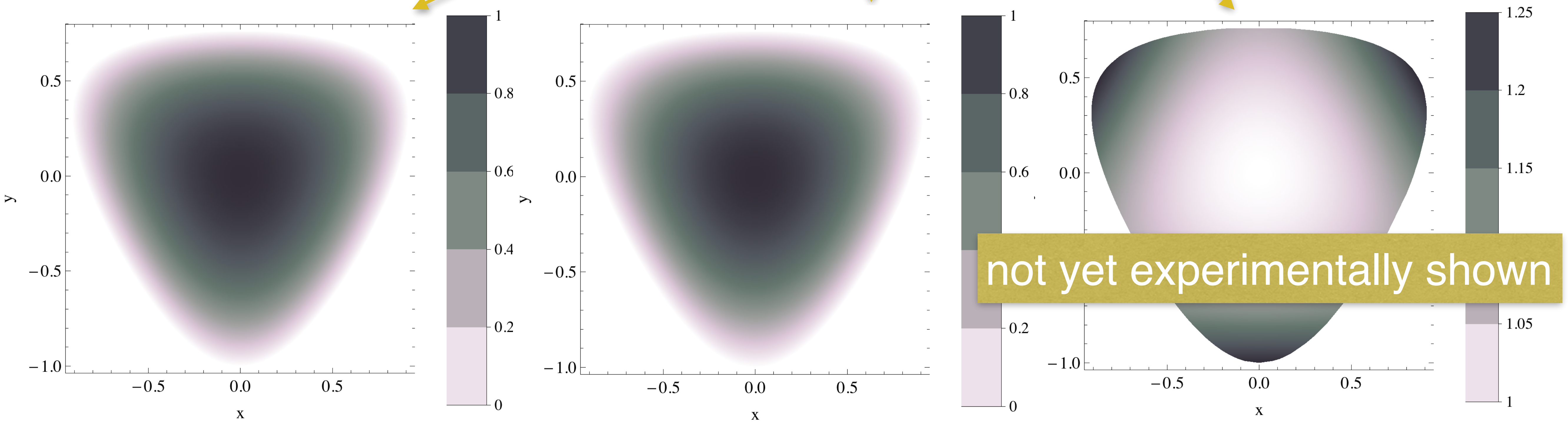

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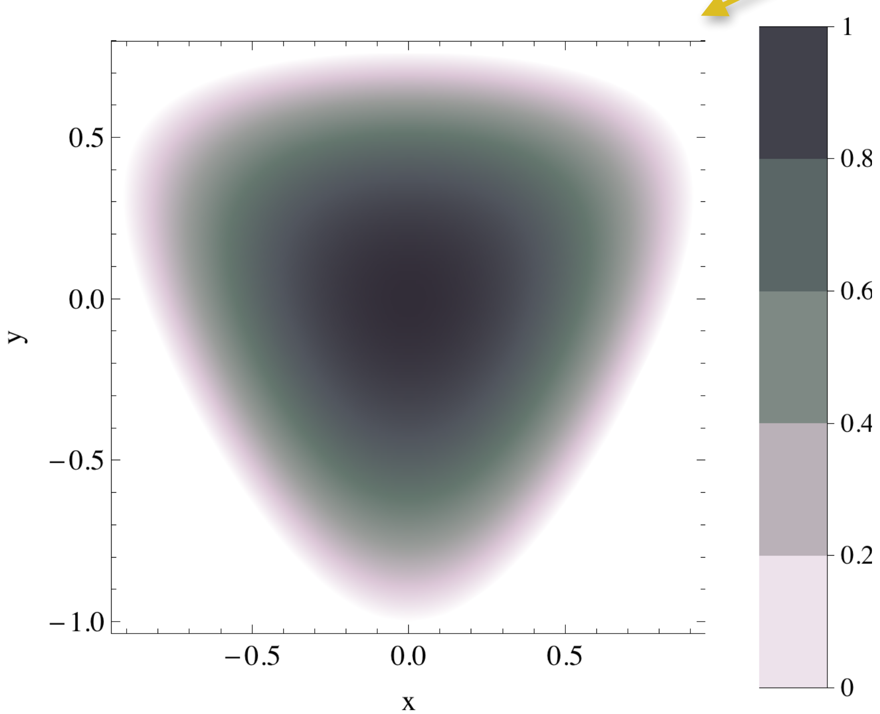
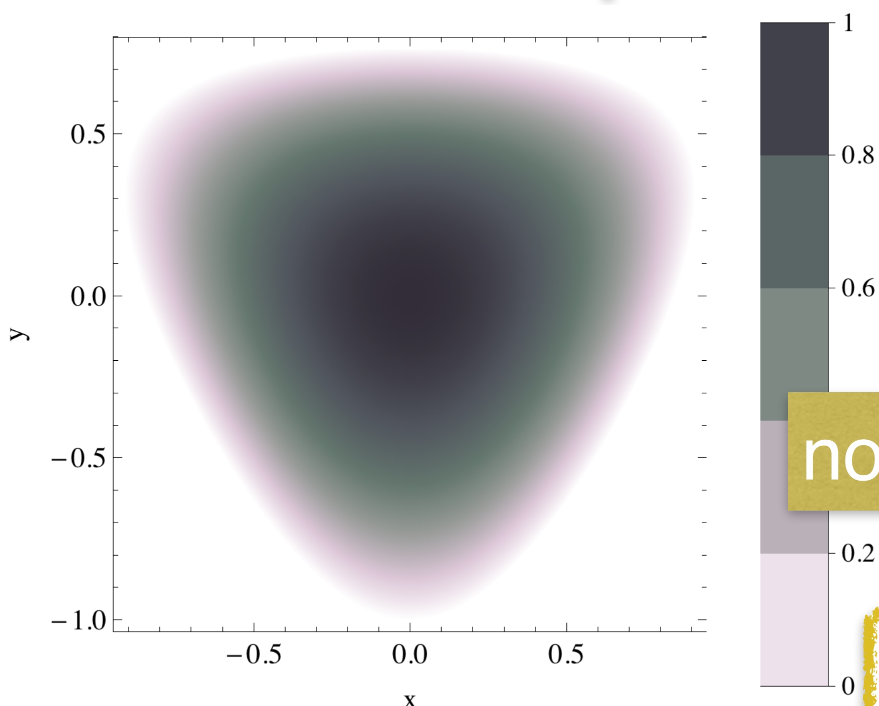
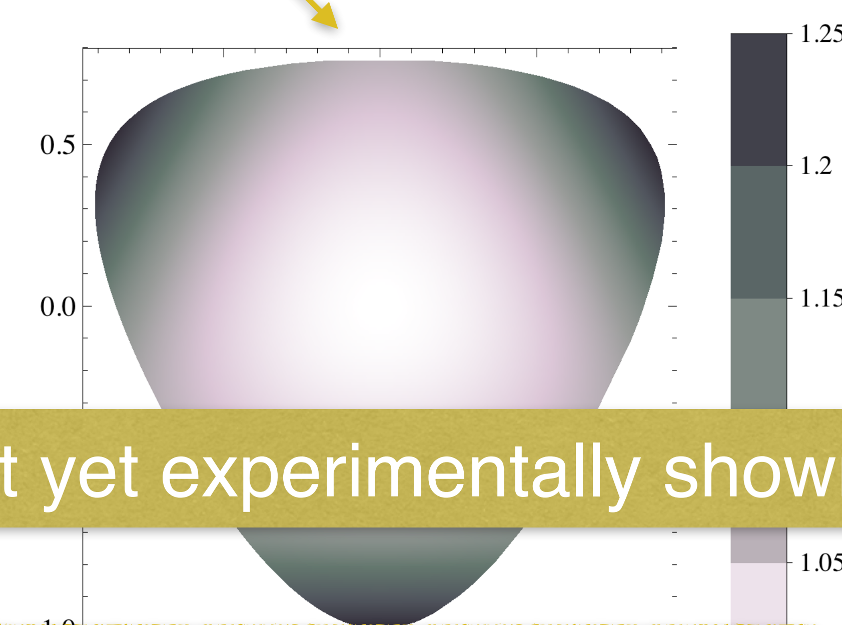




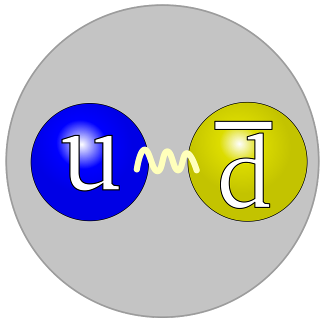


not yet experimentally shown

CLAS, WASA, KLOE
number of events: 10^6 - 10^7

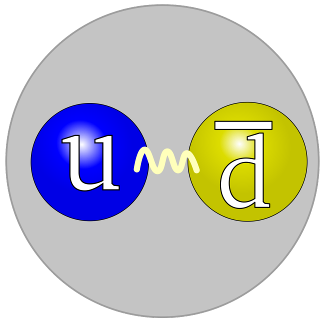
Motivation

$$\eta, \pi \sim$$



- Isospin violating decay: sensitive to quark mass difference

$$\frac{1}{Q^2} = \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2}$$

$\eta, \pi \sim$ 

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0} = 66_{[\mathrm{LO}]} + 94_{[\mathrm{NLO}]} + \ldots = 296 \pm 16\,\mathrm{eV}_{[\mathrm{Exp}]}$$

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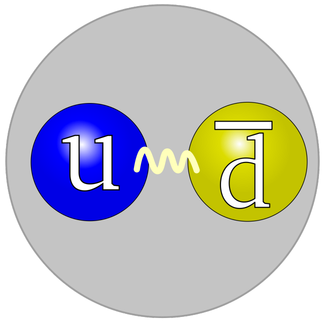
$\eta, \pi \sim$ 

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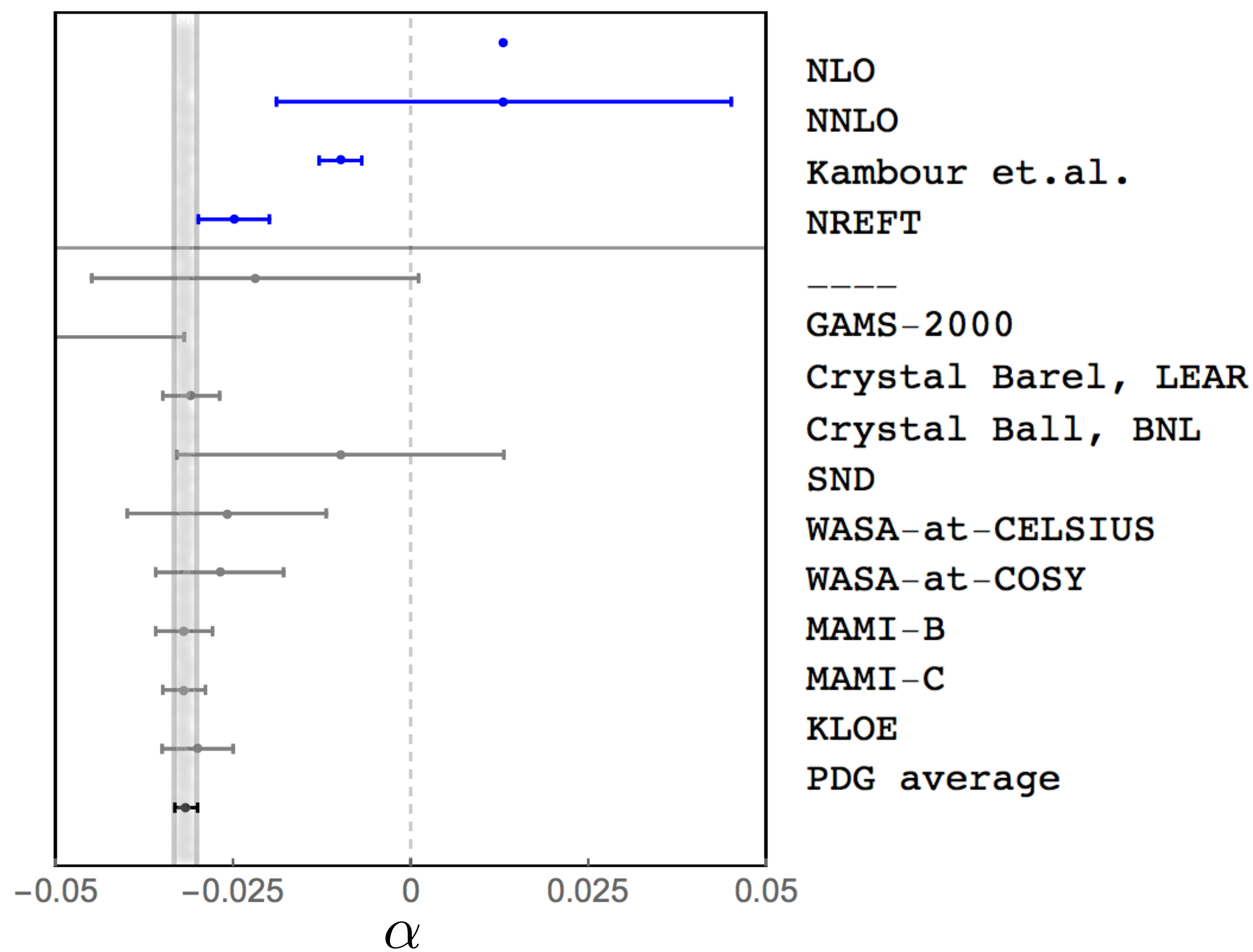
- Slope parameter puzzle for $\eta\to3\pi^0$

$$\left|A_{\eta\to3\pi^0}\right|^2 \propto 1 + 2\alpha z + \ldots$$



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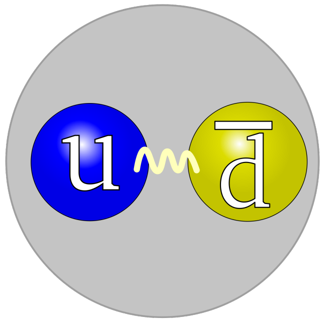
$\eta, \pi \sim$ 

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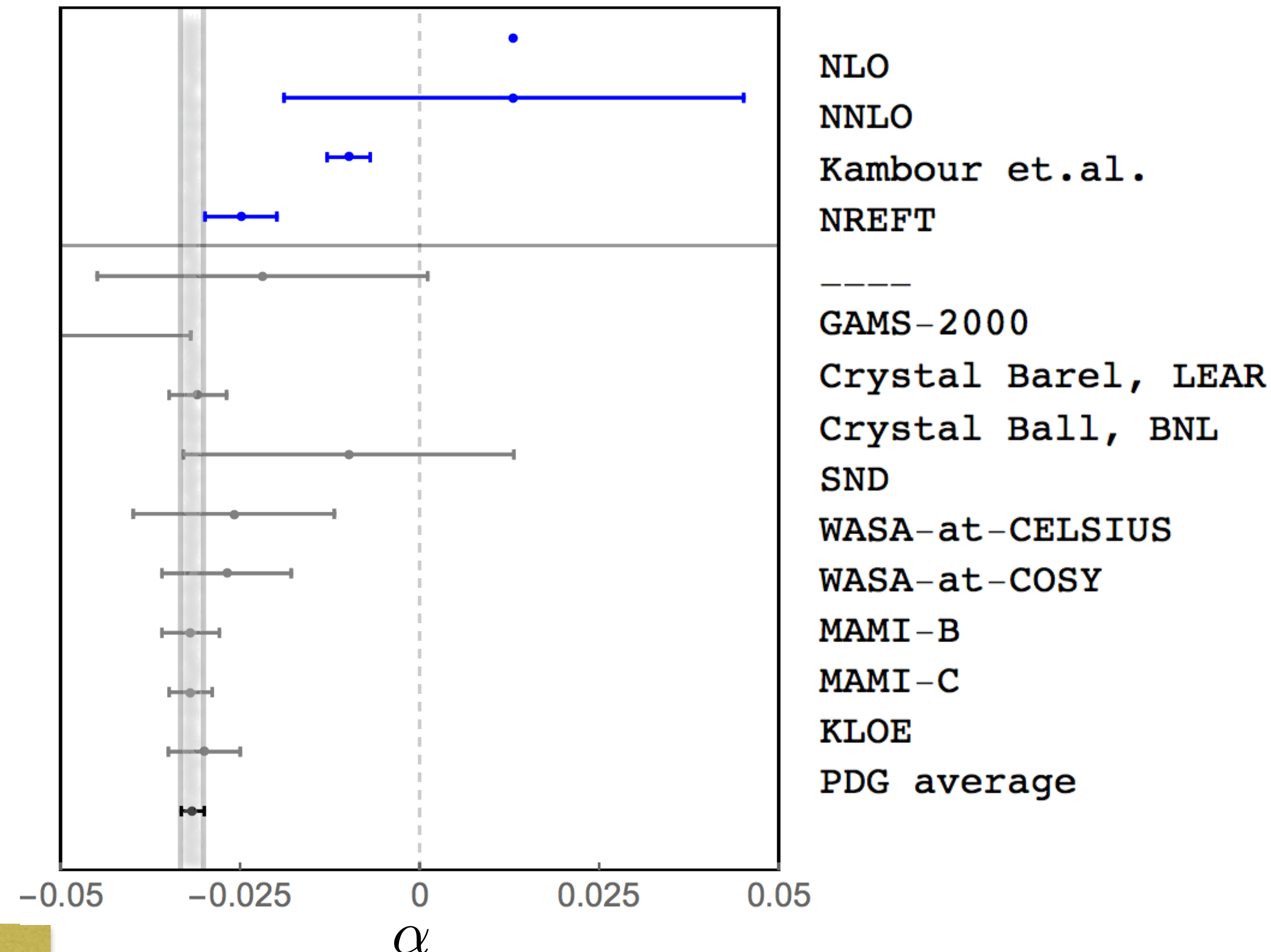
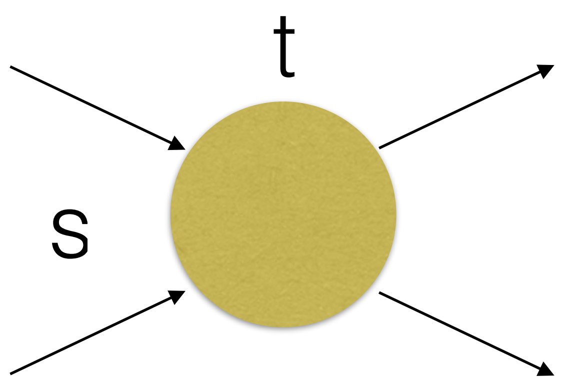
- New data on $\eta\to\pi^+\pi^-\pi^0$



WASA-at-COSY (2014) 1.2×10^7 decays

KLOE-2 (2016) 4.7×10^6 events

First principle constraints



$$A(s,t) = \sum_J^\infty (2J+1)\, P_J(z)\, f_J(s)$$

Unitarity: for small s unitarily is "simple"

$$\text{Disc}\, f_J(s) = \rho(s)\, f_J(s+)\, f_J(s-)$$

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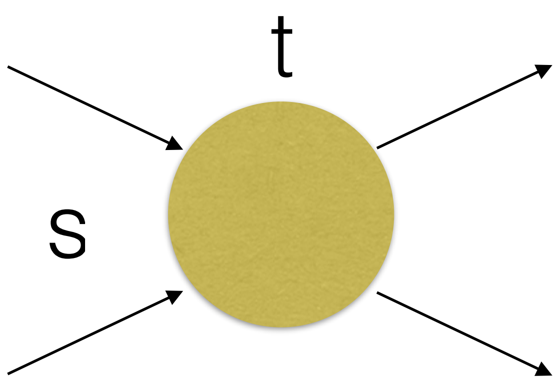

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- the same function A(s,t) should describe different processes (rotate the diagram by 90° or flip the leg)

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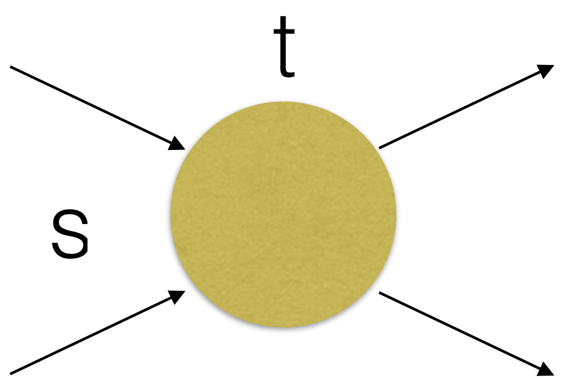

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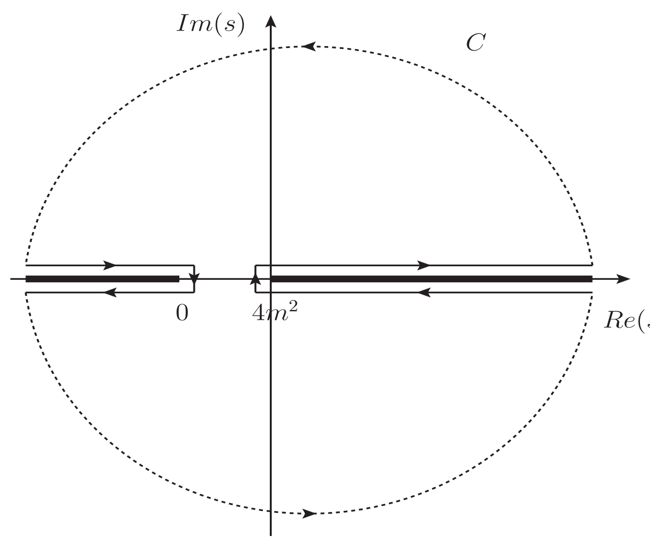
Analyticity

- relates scattering amplitude at different energies



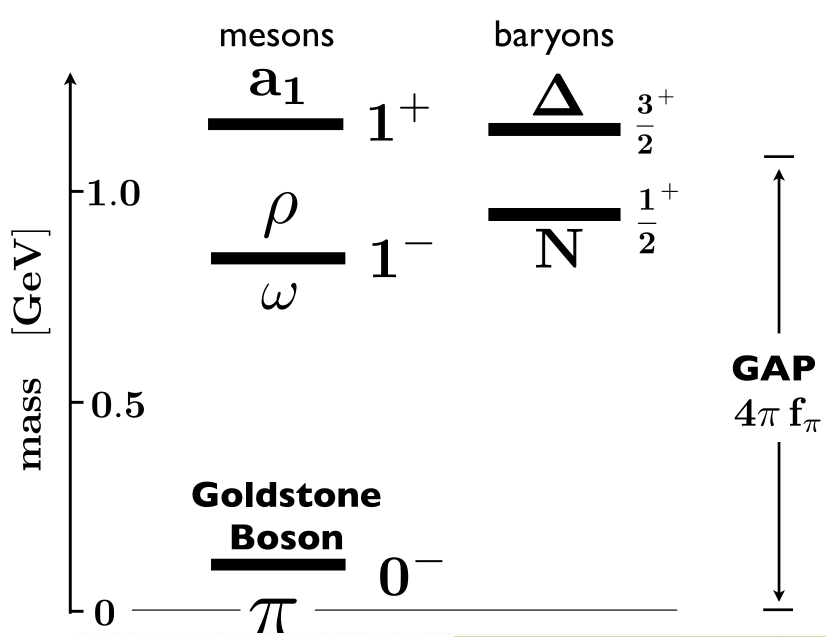
$$f_J(s) = \frac{1}{2\pi i}\int_C ds' \frac{f_J(s')}{s'-s} = \int_{-\infty}^{0} \frac{ds'}{\pi}\frac{\mathrm{Disc} f_J(s')}{s'-s} + \int_{4m^2}^{\infty} \frac{ds'}{\pi}\frac{\mathrm{Disc}\, f_J(s')}{s'-s}$$

First principle constraints

$$\mathcal{L}_{QCD} = \sum_{f=\substack{u,d,s,\\ c,b,t}} \bar{q}_f \left(i\gamma^\mu D_\mu - m_f\right) q_f - \frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\,\mu\nu}$$

- at high energies: asymptotic freedom → perturbative QCD
- at low energies: **chiral symmetry**

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$$



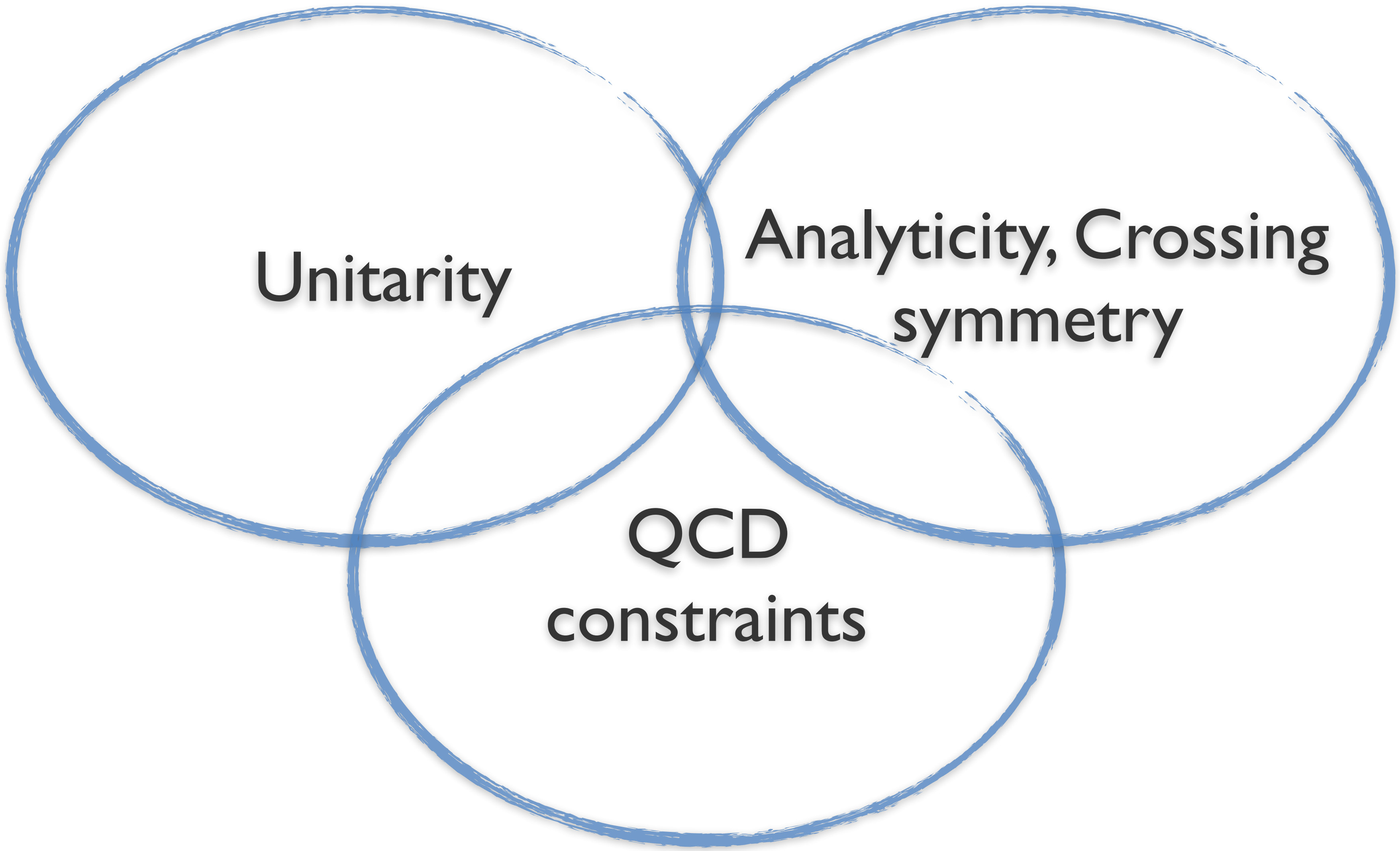
Wolfram Weise

Chiral perturbation theory (χPT)

- d.o.f. - hadrons
- expansion in mass and momenta
- Unknown coupling constants (L_i) fitted to the data

Weinberg
Gasser & Leutwyler

First principle constraints



In practice rigorous implementation of these principles is **very hard**. However, for a given reaction it is possible to kinematically isolate regions where specific processes dominate.

Roy-Steiner: $\pi\pi$, πK, πN
Ananthanarayan (2001), Buttiker (2001), Ditsche (2012), et al.

Method

- P.w. expansion

$$A(s,t,u) = \sum_{J=0}^{\infty} (2J+1)\, P_J(\cos\theta)\, f_J(s)$$

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- Reconstruction theorem:
  crossing symmetry, analyticity up to NNLO

$$A(s,t,u) = \sum_J^{J_{max}} ...a_J(s) + \sum_J^{J_{max}} ...a_J(t) + \sum_J^{J_{max}} ...a_J(u)$$

ππ scattering
Fuchs, Sazdjian, Stern (1993)

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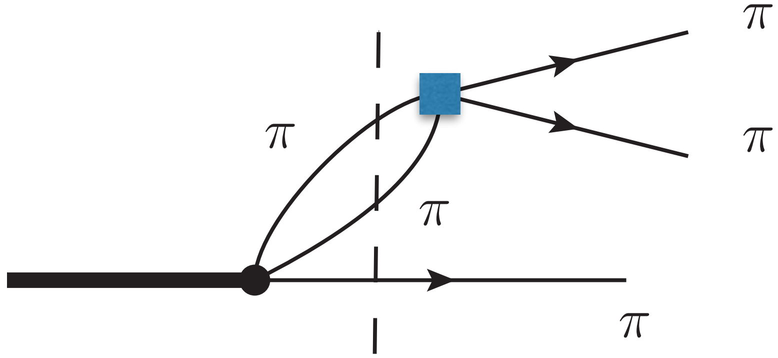
$\pi\pi$ scattering
Fuchs, Sazdjian,
Stern (1993)

- Unitarity



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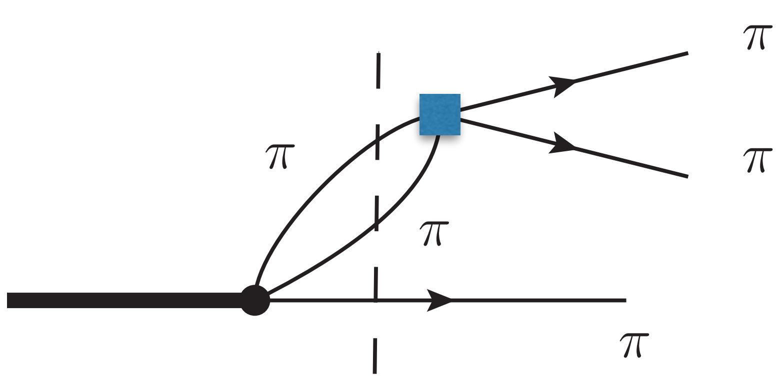
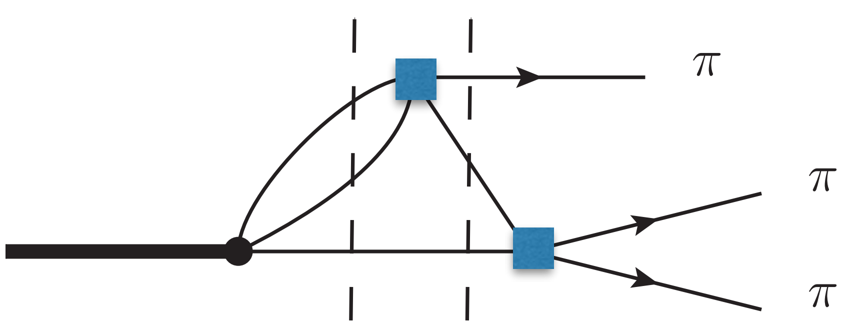
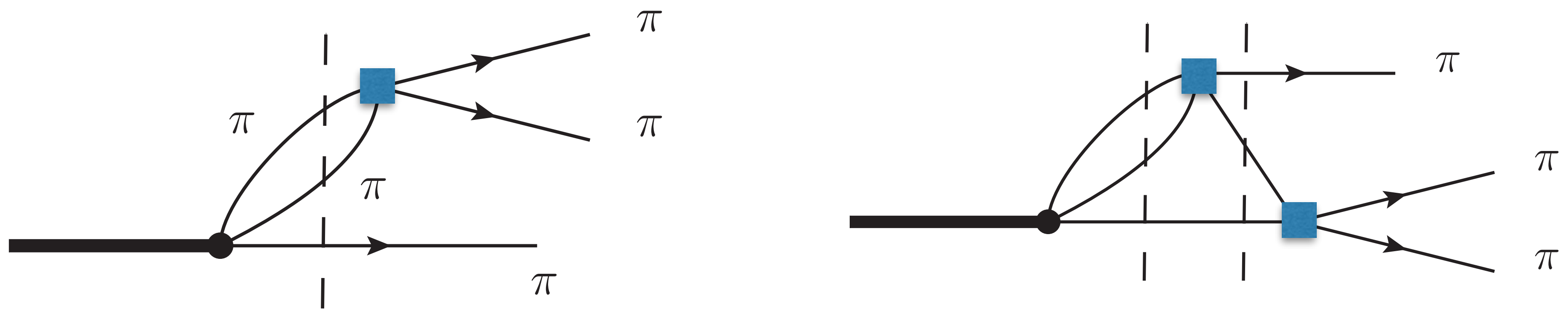




Khuri, Treiman (1960)
Aitchison (1977)

Unitarity



$$\mathrm{Disc}\, a_J(s) = t_J^*(s)\, \rho(s) \left(a_J(s) + \int_{-1}^{+1} \frac{d\cos\theta}{2} ... a_J(t) \right)$$

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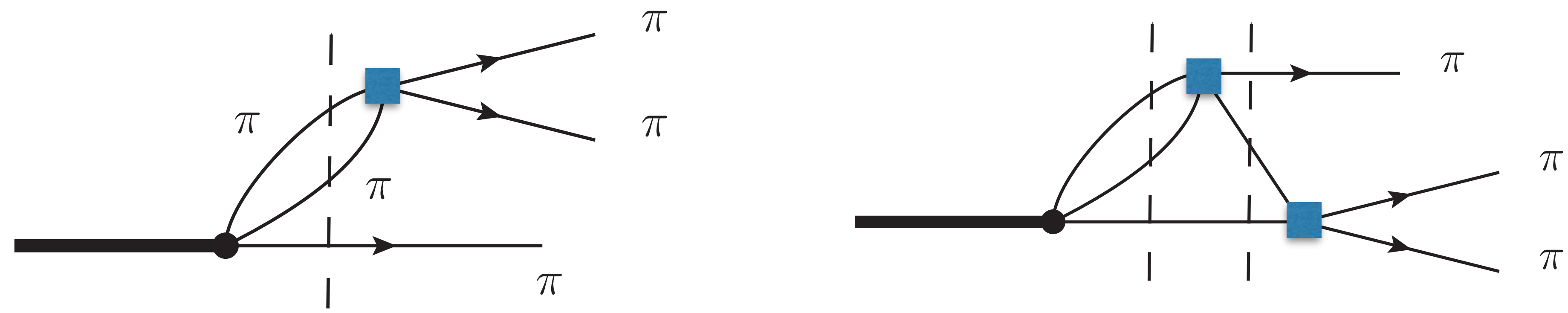

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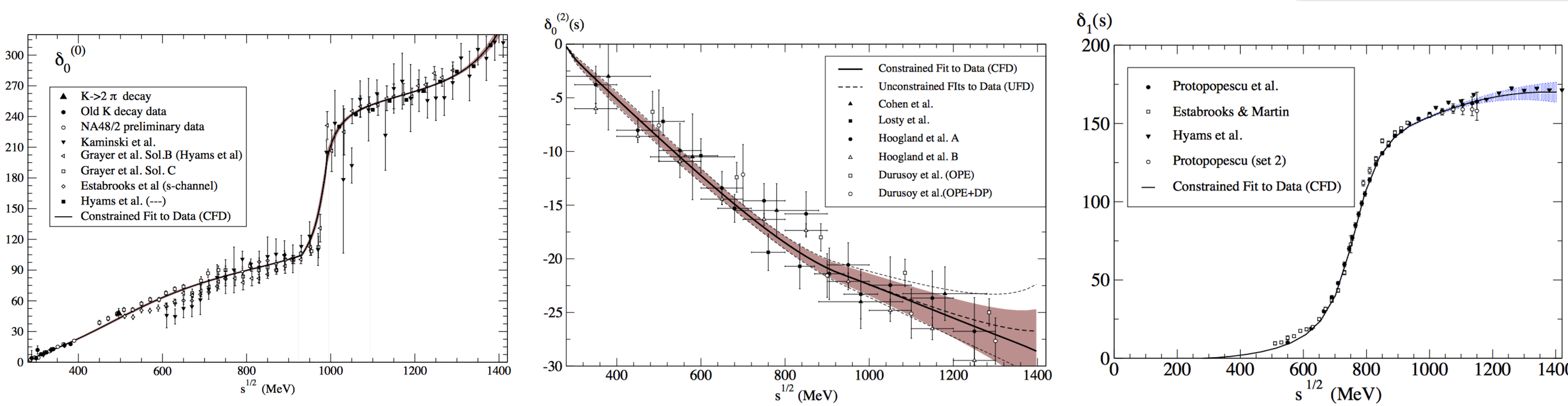
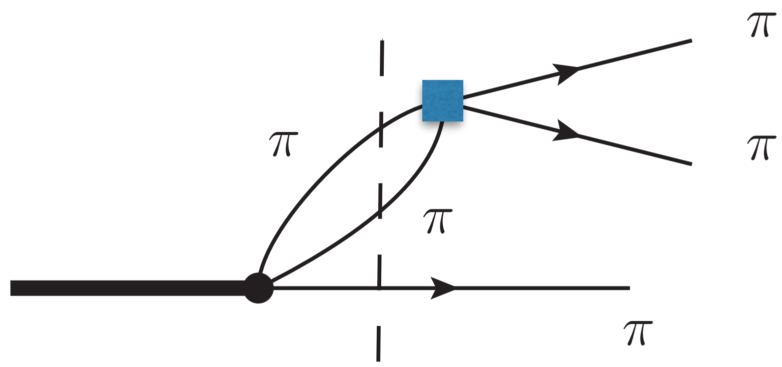
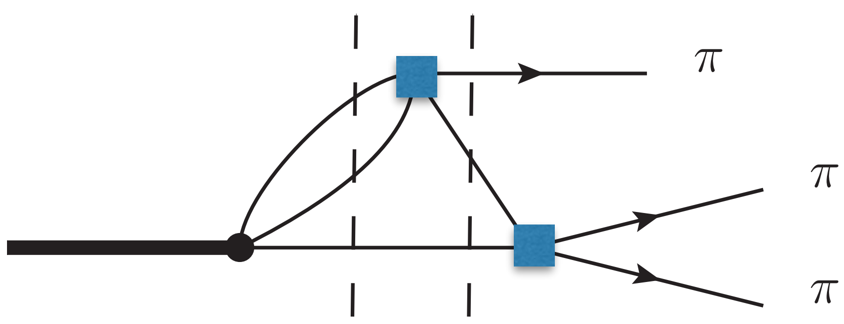
input

Roy analysis (2011)
R. Garcia-Martin
at.al.



Pasquier inversion





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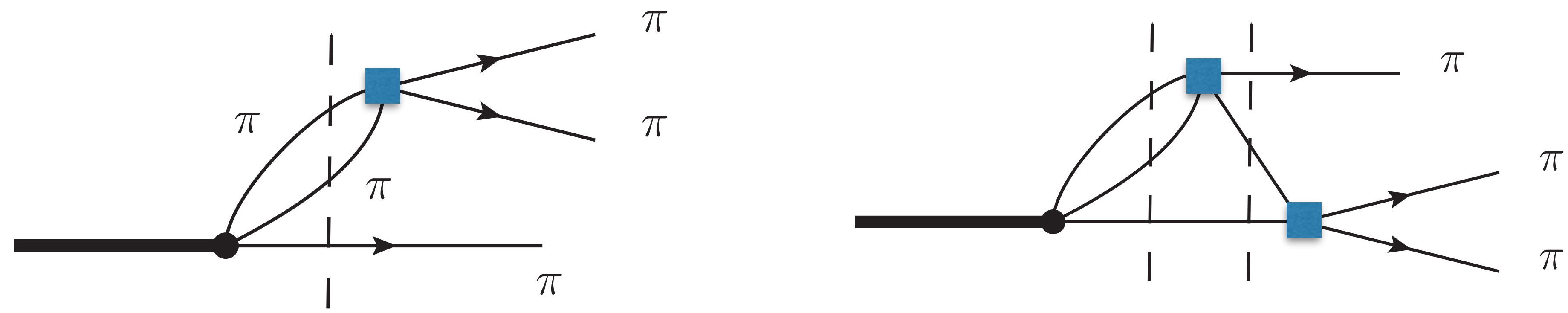

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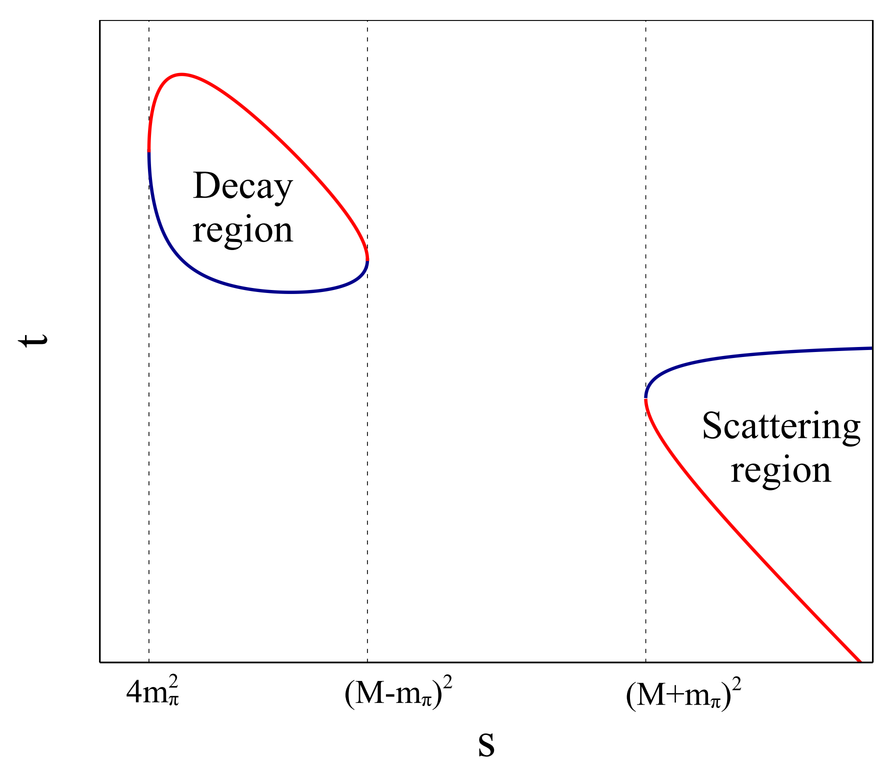
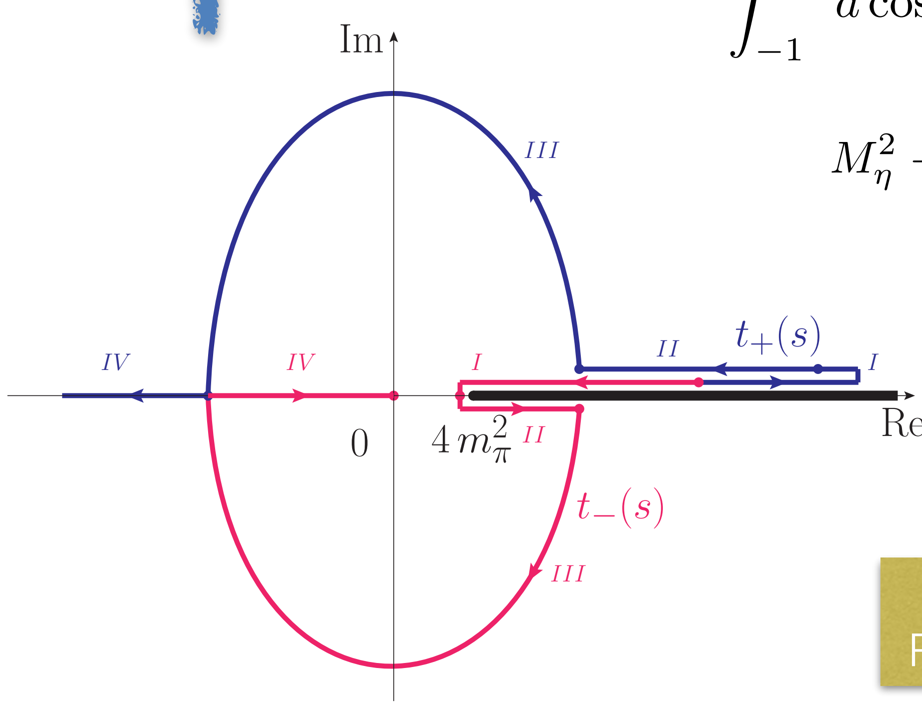


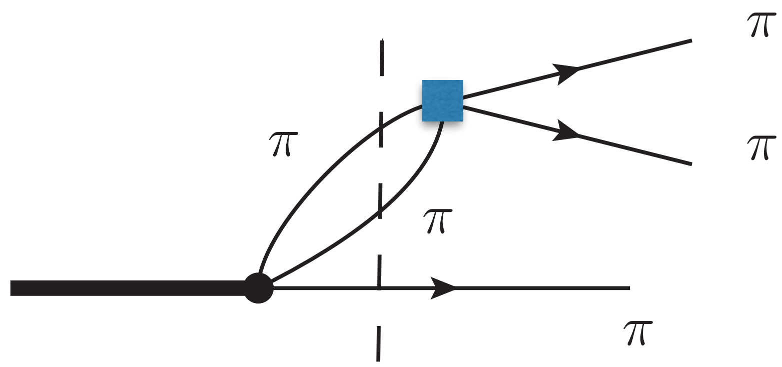
$$\int_{-1}^{+1} d\cos\theta \to \int_{t_-(s)}^{t_+(s)} dt$$

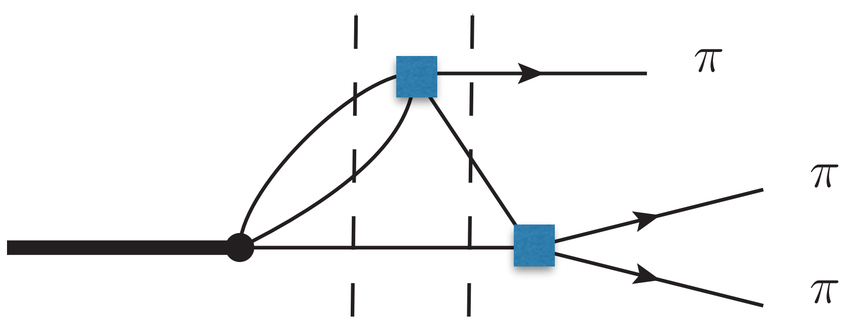
$$M_\eta^2 \to M_\eta^2 + i\,\epsilon$$



Bronzan, Kacser
PR132,2703 (1963)

Pasquier inversion





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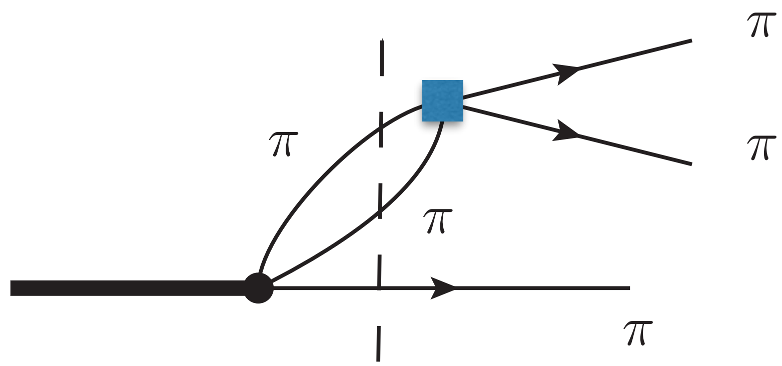

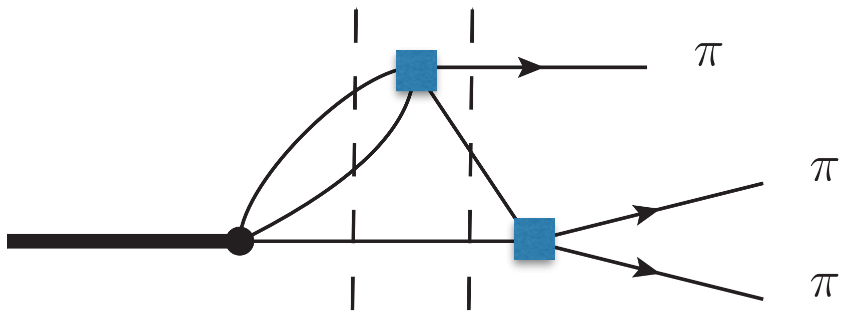

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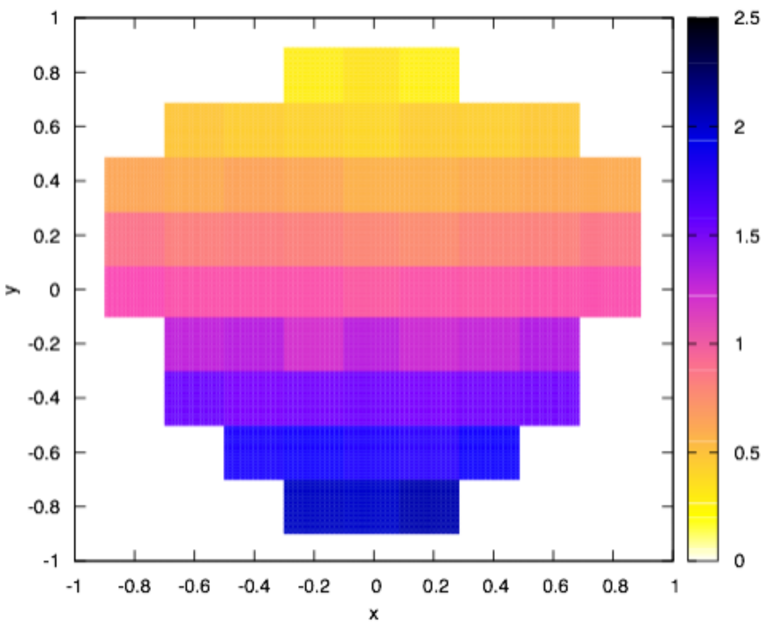
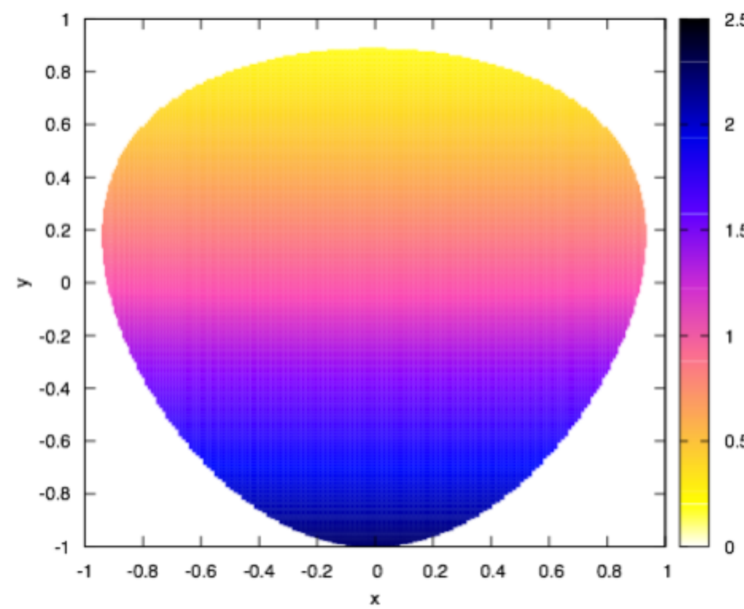
$$\int_{4\,m_\pi^2}^{\infty} \frac{ds'}{\pi} \int_{t_-(s)}^{t_+(s)} dt\ \ldots \quad \longrightarrow \quad = \int_{\Gamma'} dt \int_{s_-(t)}^{s_+(t)} ds' \ldots$$

Pasquier et al.
Phys. Rev. 170, 1294

P. Guo, I.D.,
A. Szczepaniak
EPJA 2015

$\eta \to \pi^+ \pi^- \pi^0$ (WASA-at-COSY fit)



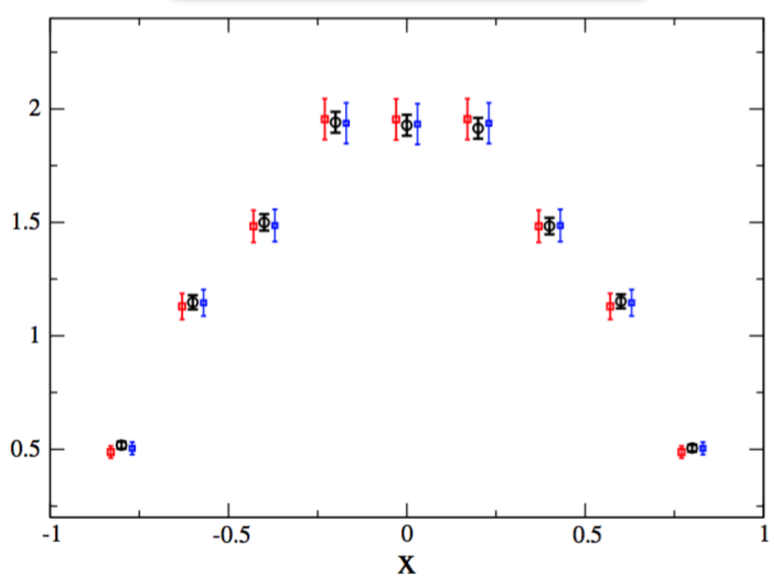


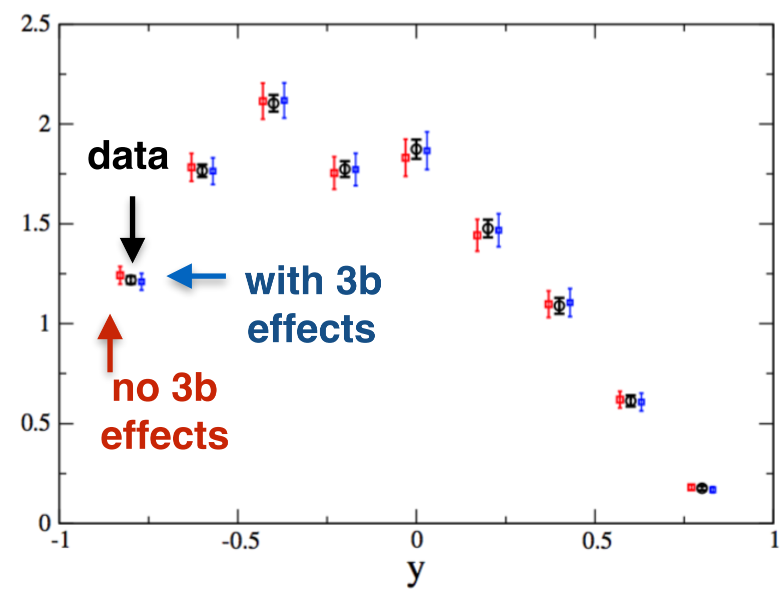
WASA-at-COSY
PRC90 4 045207
1.2×10^7 decays

P. Guo et al (JPAC)
PRD92 5 054016

Fit to WASA

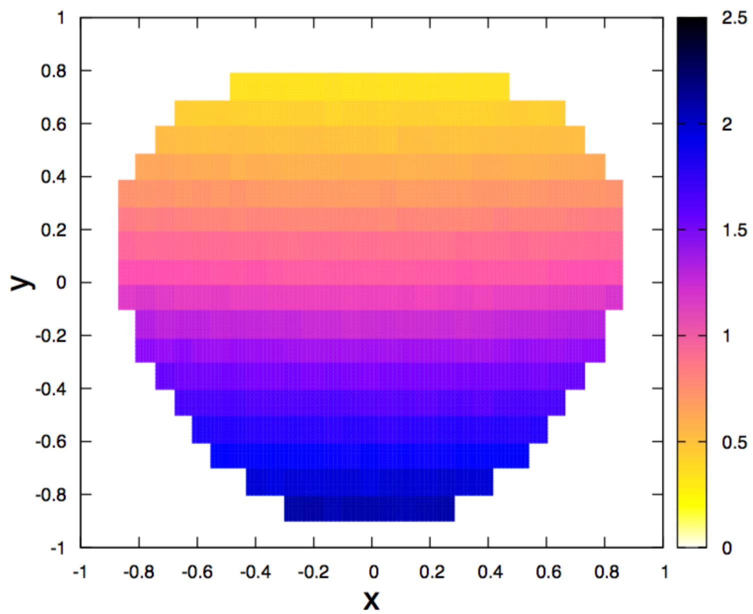
$\chi^2/d.o.f.$	*no 3b*	*with 3b*
(L,I)=(0,0), (1,1) 1 real par.	*1,45*	*0,96*
(L,I)=(0,0),(1,1), (0,2) 2 real par.	*0,94*	*0,9*

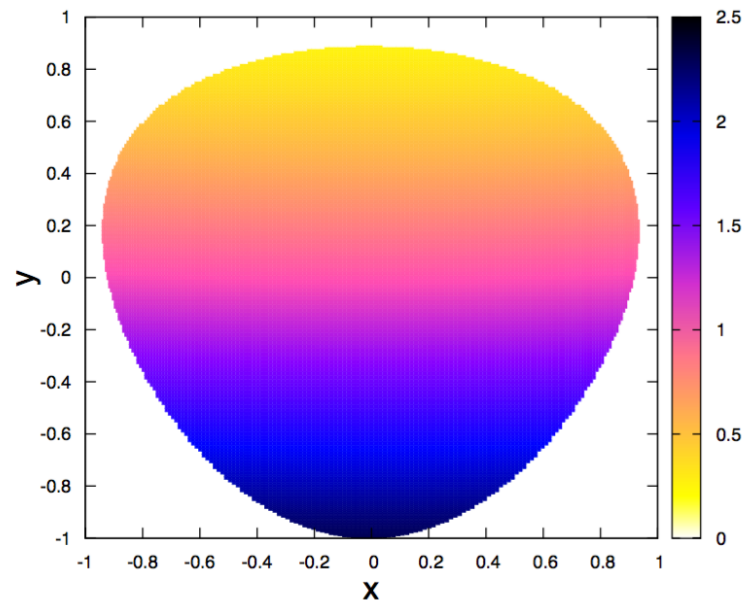




$$x \propto (t-u)$$
$$y \propto (s_c - s)$$

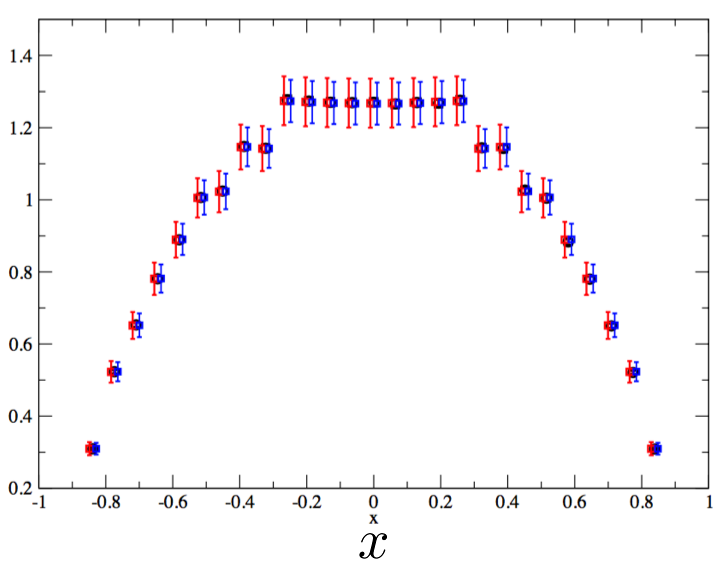
$\eta \to \pi^+ \pi^- \pi^0$ (KLOE-2 fit)

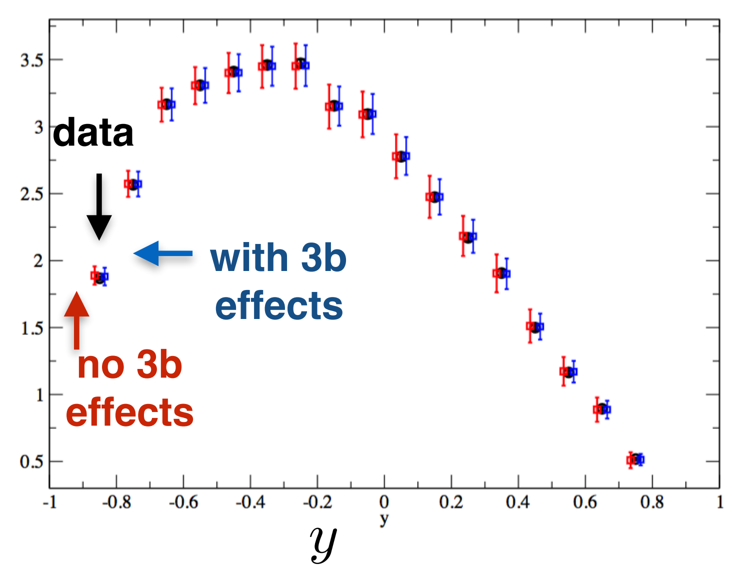




KLOE-2
arXiv:1601.06985
4.7×10^6 events

(preliminary)



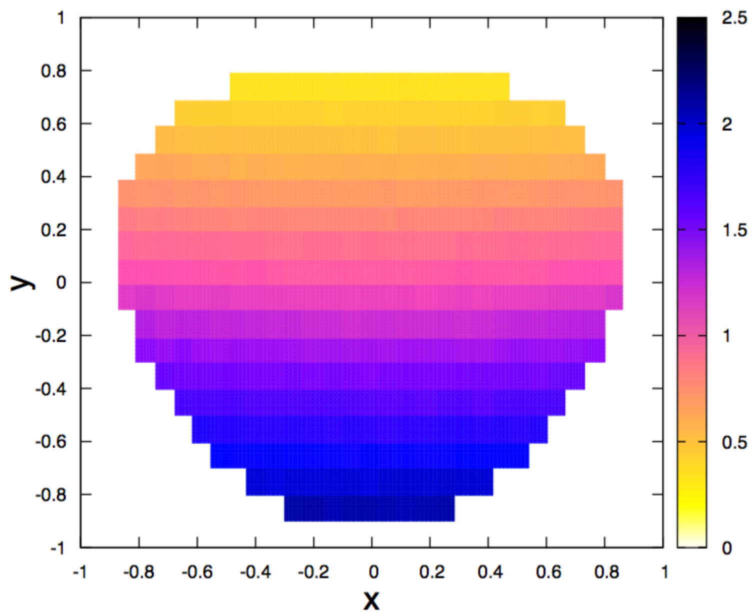


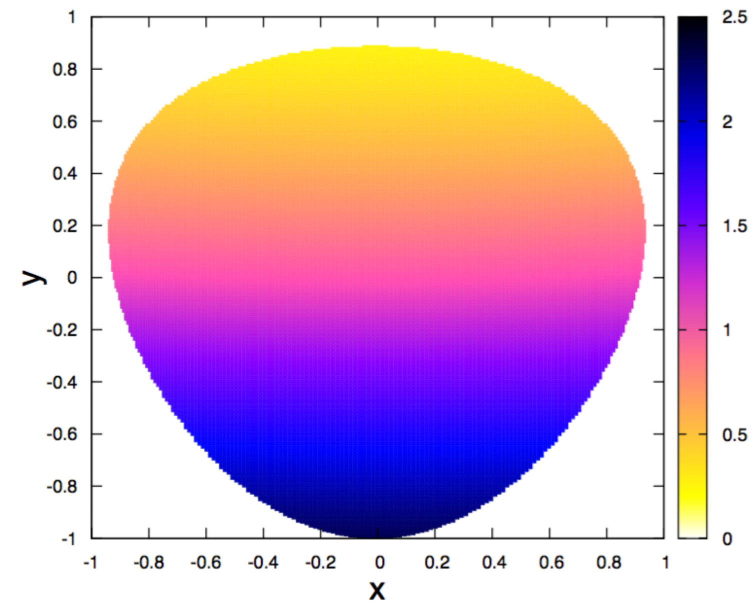
$$x \propto (t-u)$$
$$y \propto (s_c - s)$$

Fit to KLOE-2

$\chi^2/d.o.f.$	no 3b	with 3b
(L,I)=(0,0), (1,1) 1 real par.	10,4	2,61
(L,I)=(0,0),(1,1),(0,2) 2 real par.	1,21	1,29

$\eta \to \pi^+ \pi^- \pi^0$ (KLOE-2 fit)





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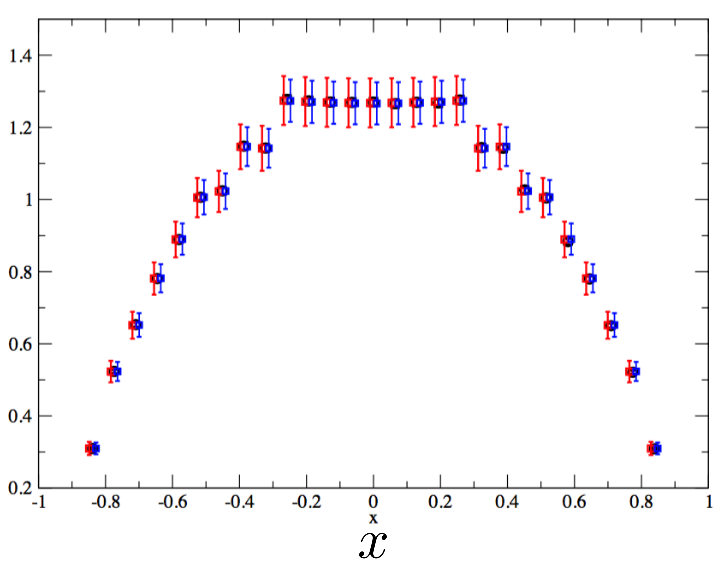

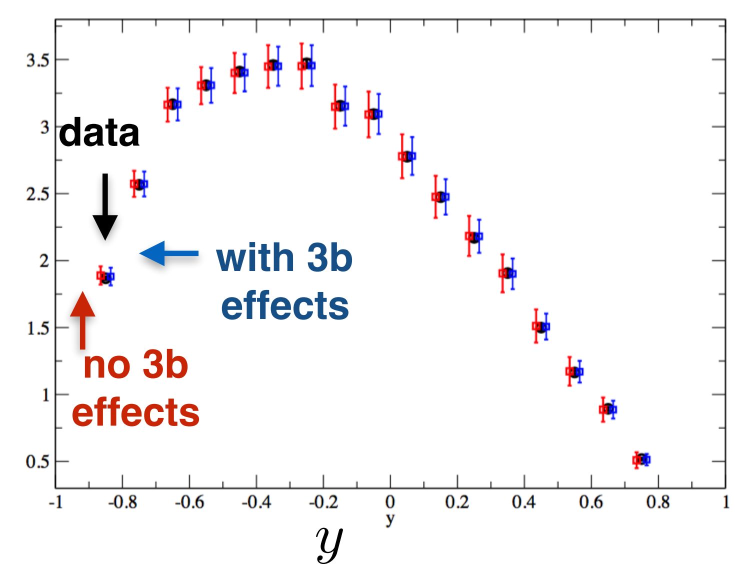

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Fit to KLOE-2

$\chi^2/d.o.f.$	*no 3b*	*with 3b*
(L,I)=(0,0), (1,1) 1 real par.	*10,4*	*2,61*
(L,I)=(0,0),(1,1), (0,2) 2 real par.	*1,21*	*1,29*

Combined fit: WASA & KLOE-2

$\chi^2/d.o.f.$	*no 3b*	*with 3b*
(L,I)=(0,0), (1,1) 1 real par.	*9,5*	*1,64*
(L,I)=(0,0),(1,1), (0,2) 2 real par.	*1,54*	*1,61*

$\eta \to 3\pi^0$

- Dalitz plot expansion:

$$|A_{\eta\to\pi^+\pi^-\pi^0}|^2 \propto 1 + a\,y + b\,y^2 + d\,x^2 + f\,y^3 + \ldots$$

$$|A_{\eta\to 3\pi^0}|^2 \propto 1 + 2\alpha z + \ldots$$

$\eta \to 3\pi^0$

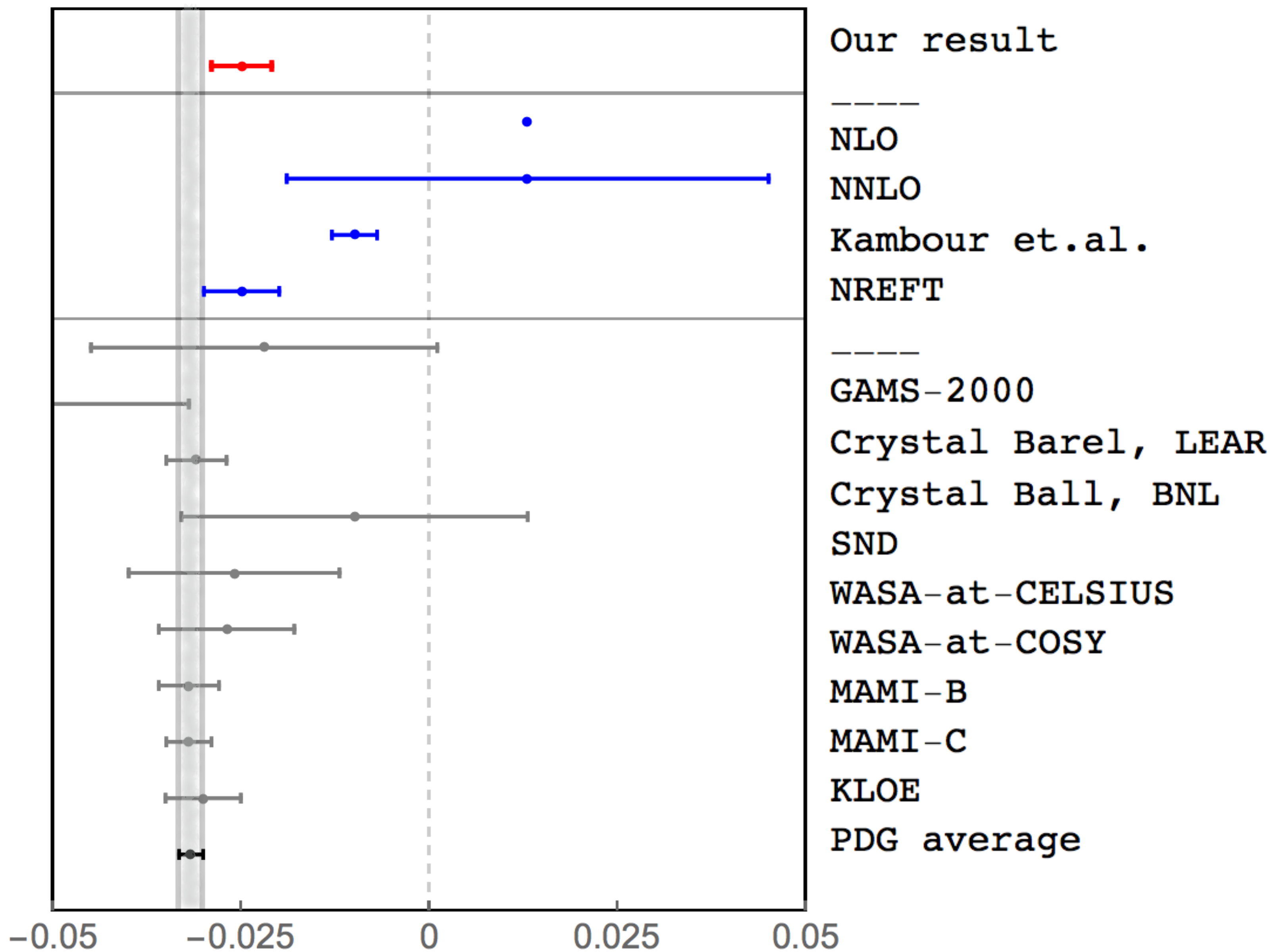
- Dalitz plot expansion:

$$|A_{\eta\to\pi^+\pi^-\pi^0}|^2 \propto 1 + a\,y + b\,y^2 + d\,x^2 + f\,y^3 + \ldots$$

$$|A_{\eta\to 3\pi^0}|^2 \propto 1 + 2\alpha z + \ldots$$

- Prediction:

$$\begin{aligned} \alpha &= -0.025 \pm 0.004 \\ \alpha^{\mathrm{PDG}} &= -0.0317 \pm 0.0016 \end{aligned}$$



Matching to ChPT

We described Dalitz distribution normalised to unity at x=y=0

$$|A_{\eta\to\pi^+\pi^-\pi^0}|^2 \propto 1 + a\,y + b\,y^2 + d\,x^2 + f\,y^3 + \ldots$$

$$\Gamma^{exp}_{\eta\to\pi^+\pi^-\pi^0} \propto \int |A_{\eta\to\pi^+\pi^-\pi^0}|^2 \propto \boxed{\frac{N^2}{Q^4}} \qquad \frac{1}{Q^2} = \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2}$$

fix overall normalisation

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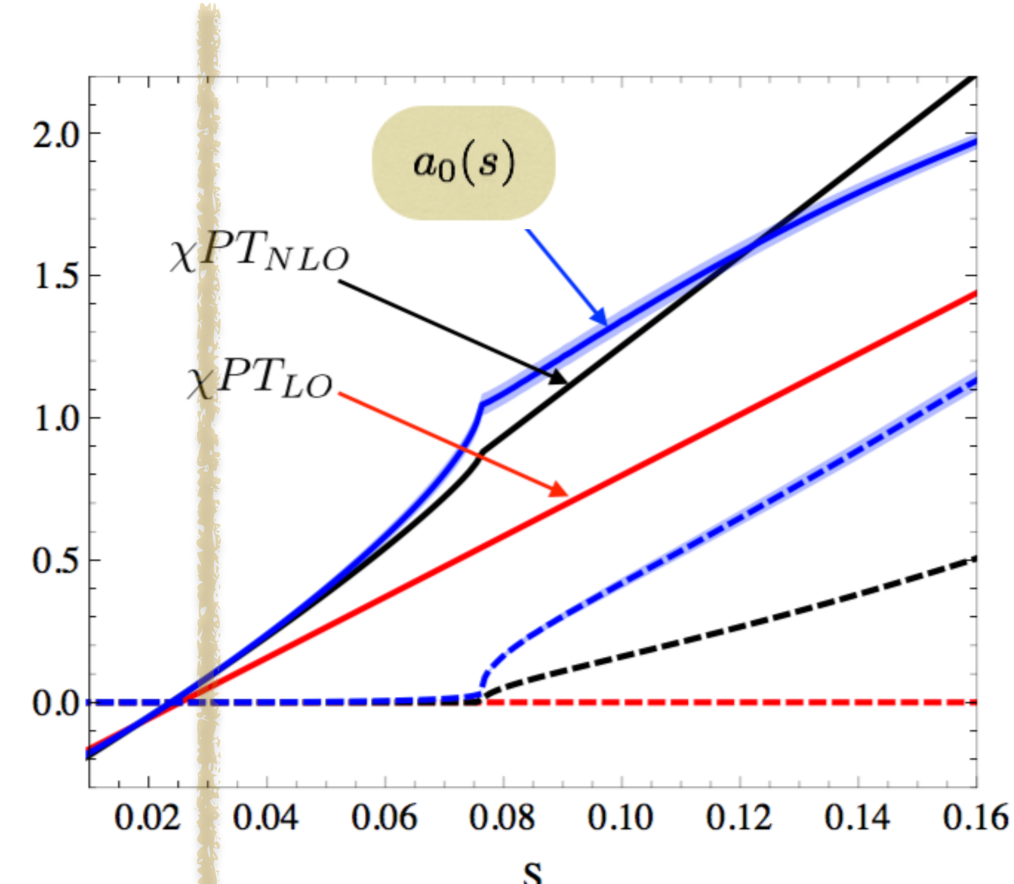
fix overall normalisation

Matching to ChPT

$$A(s,t,u) = \sum_J^{J_{max}} \ldots a_J(s) + \sum_J^{J_{max}} \ldots a_J(t) + \sum_J^{J_{max}} \ldots a_J(u)$$

$$A^{\chi PT}(s,t,u) = -\frac{1}{Q^2}\frac{m_K^2(m_K^2 - m_\pi^2)}{3\sqrt{3}\,m_\pi^2 f_\pi^2}\left(\sum_J^{J_{max}} \ldots a_J^{\chi PT}(s) + \ldots\right)$$

Match individual *(I, J)* components of the full amplitude near Adler zero $s=4/3\ m_\pi^2$



Matching point

Q-value predictions

Quark mass double ratio:

$$\frac{1}{Q^2} = \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2}$$

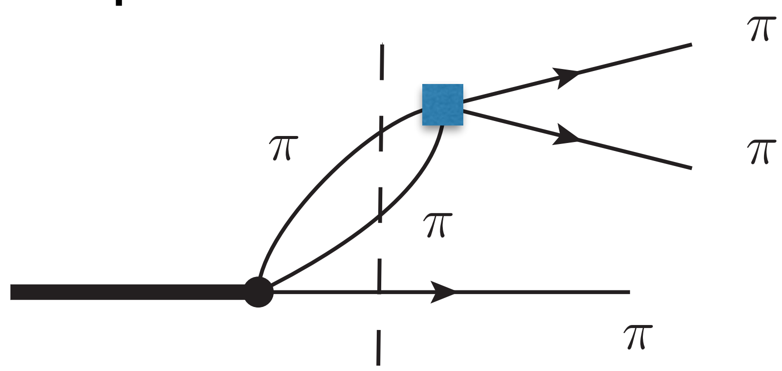
	Q
Our result (fit to WASA@COSY)	21.4 ± 0.4
Our result (fit to KLOE-2)	21.7 ± 0.4
Our result (combined fit)	21.6 ± 0.4
Lattice, FLAG, 2016 ($N_f = 2+1$)	22.5 ± 0.8
Lattice, FLAG, 2016 ($N_f = 2+1+1$)	22.2 ± 1.6
NLO	20.1
NNLO	22.9
Dispersive (Kambor *et al.*)	22.4 ± 0.9
Dispersive (Kampf *et al.*)	23.1 ± 0.7
Dispersive (Colangelo *et al.*, prelim.)	21.5 ± 0.5

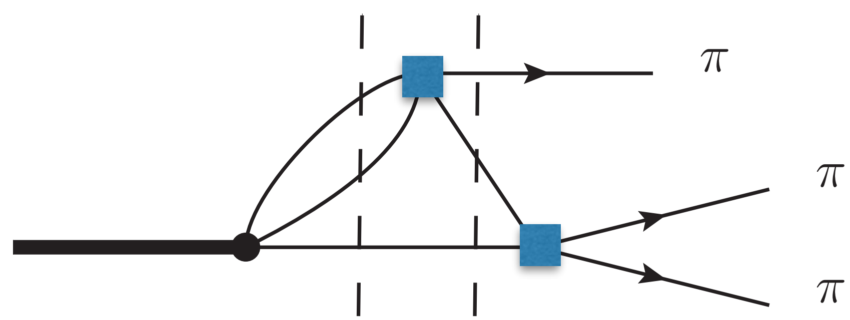
Quark masses

$$\hat{m} = 3.42 \pm 0.09 \text{ MeV} \qquad m_u = 2.04 \pm 0.14 \text{ MeV}$$
$$m_s = 93.8 \pm 0.24 \text{ MeV} \longrightarrow m_d = 4.80 \pm 0.08 \text{ MeV}$$

Lattice, FLAG, (Nf=2+1), 2014

ω/φ is spin 1 particle:



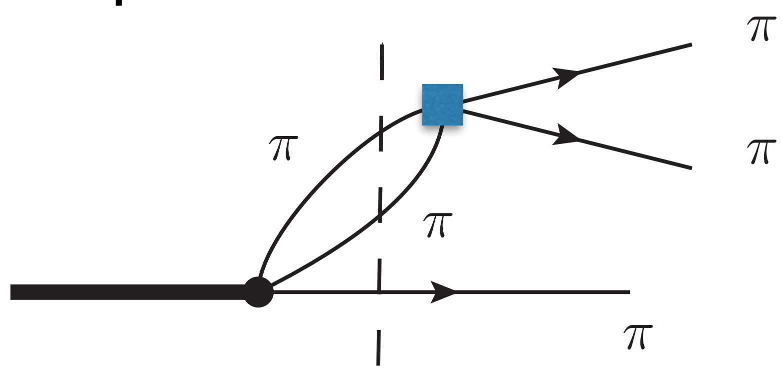


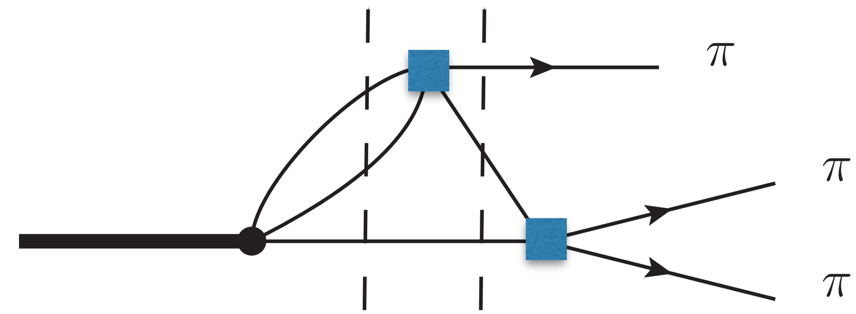
$$\mathrm{Disc}\, a_J(s) = t_J^*(s)\,\rho(s)\left(a_J(s) + \int_{t_-(s)}^{t_+(s)} dt \ldots a_J(t)\right)$$

$$a_J(s) = \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, a_J(s)}{s' - s}$$

$\omega,\ \varphi \to 3\pi$

ω/φ is spin 1 particle:





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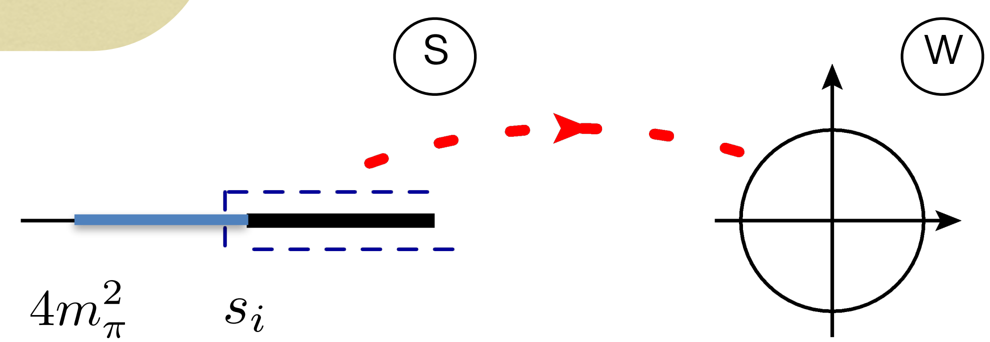
inelastic contributions parametrize with a conformal mapping expansion $\sum_{i=0}^{N} C_i\, \omega(s)^i$

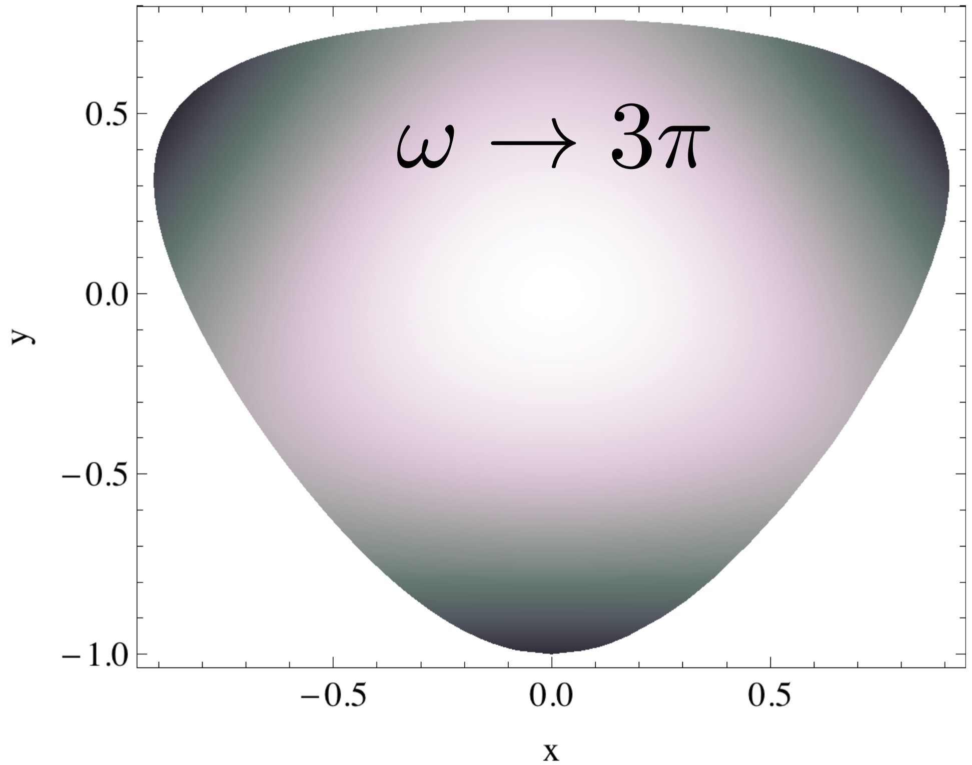
$$= \int_{4m_\pi^2}^{s_i} \ldots + \int_{s_i}^{\infty} \ldots$$



Coefficients C_i play the role of subtraction constants in conventional approach

Niecknig, Kubis, Schneider (2012)

$\omega,\ \varphi \to 3\pi$

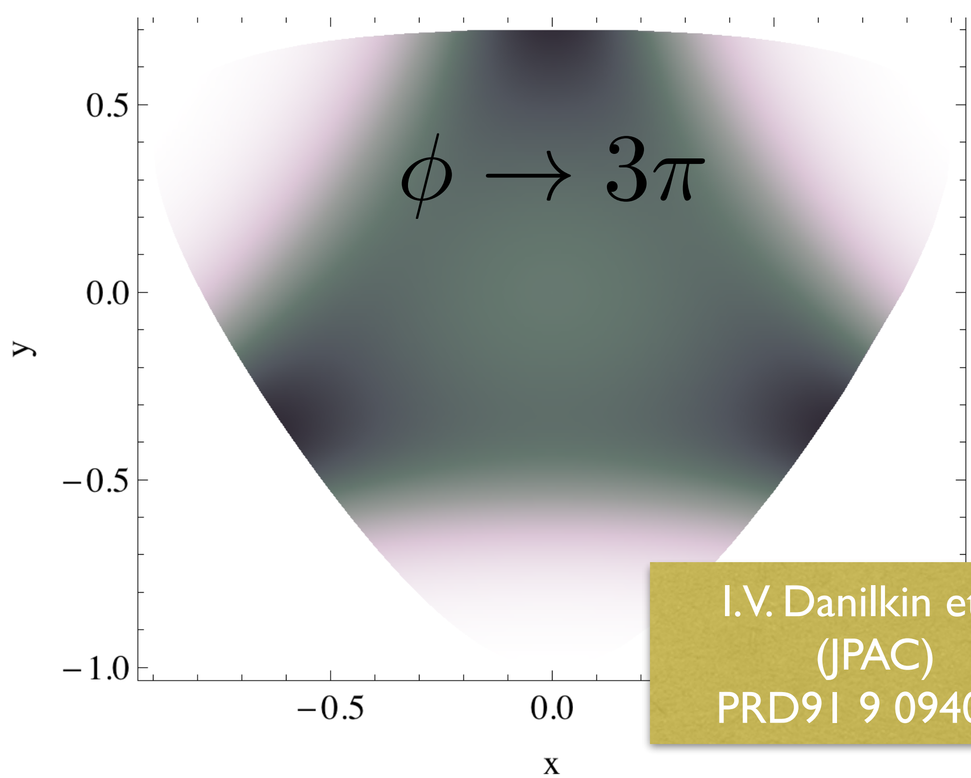


$$\frac{d^2\Gamma}{ds\,dt} \propto |\vec{p}_+ \times \vec{p}_-|^2 |A(s,t)|^2$$

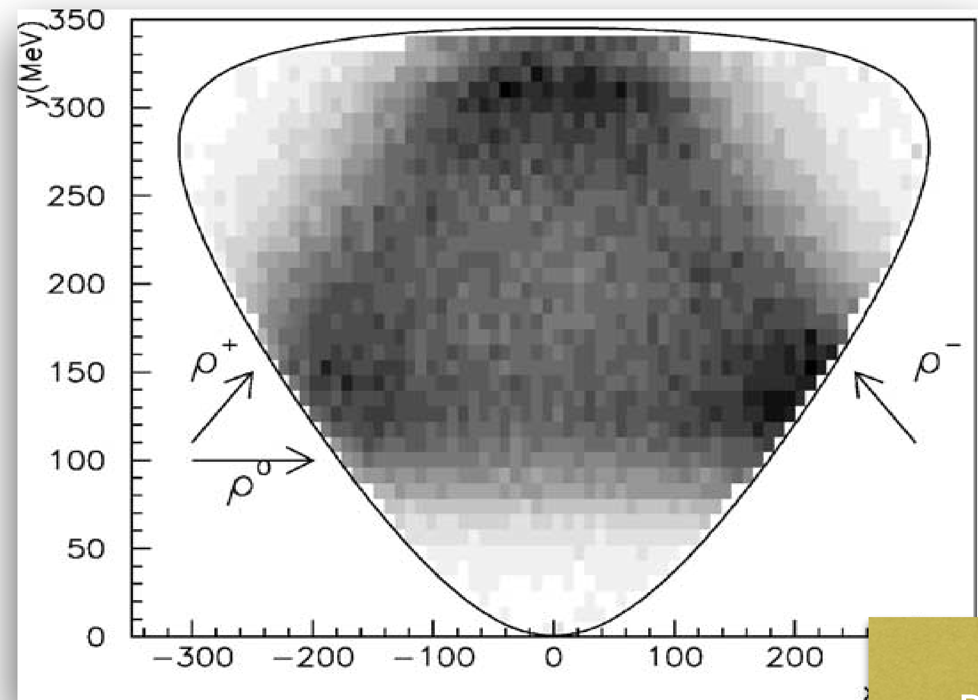
Carlos Salgado, Volker Crede, Chris Zeoli, etc.

ω→3π: fit event by event g12 CLAS data in progress

φ→3π: $\chi^2/d.o.f.$ = 1.11 (no 3b)
= 1.09 (with 3b)

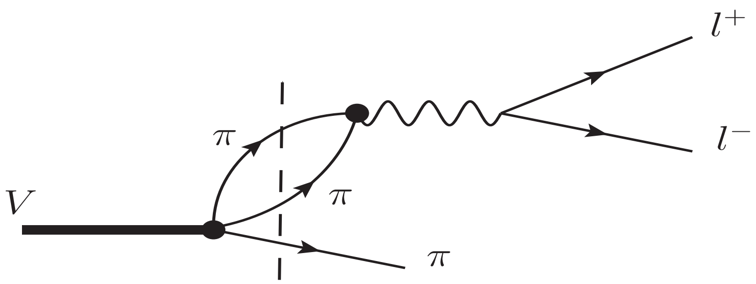


I.V. Danilkin et al (JPAC) PRD91 9 094029



KLOE Coll. PLB 561 55-60

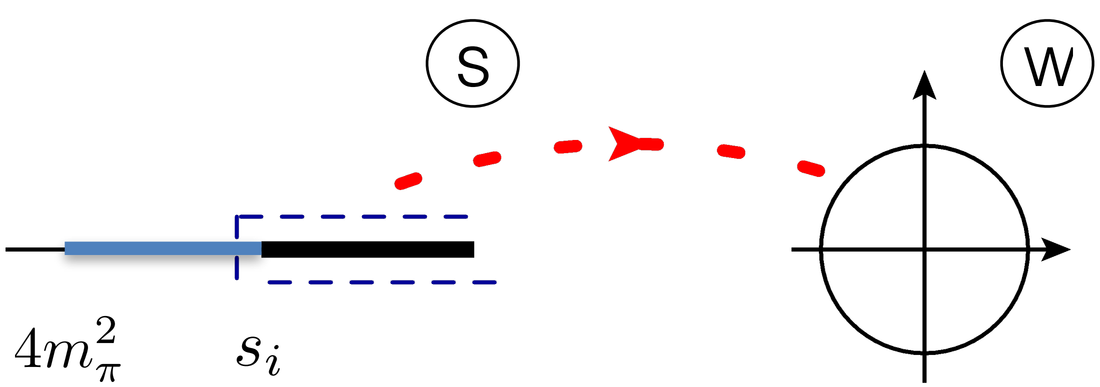
Discontinuity relation: $\omega/\varphi \to \pi^0 \gamma^*$

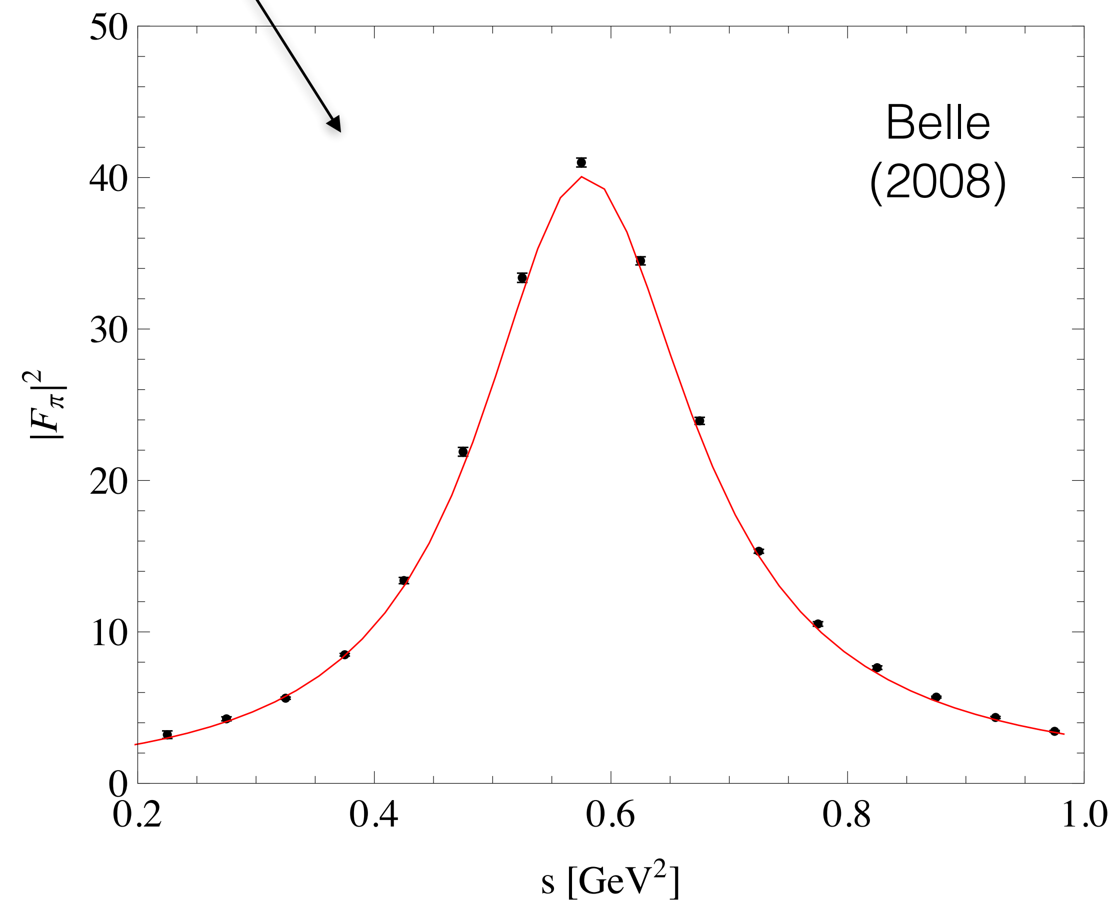


pion vector form factor $\qquad$ $\omega/\varphi \to 3\pi$

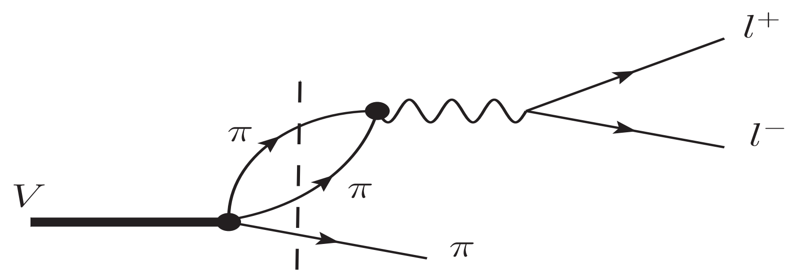
$$\mathrm{Disc}\, f_{V\pi}(s) = \frac{\rho^3(s)\, s}{128\,\pi}\, F_\pi^*(s) \int_{-1}^{1} dz' (1 - z'^2)\, F(s, t', u')$$

$$f_{V\pi}(s) = \int_{4m_\pi^2}^{s_i} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, f_{V\pi}(s')}{s' - s} + \sum_{k=0}^{N} b_k\, (\omega(s))^k$$

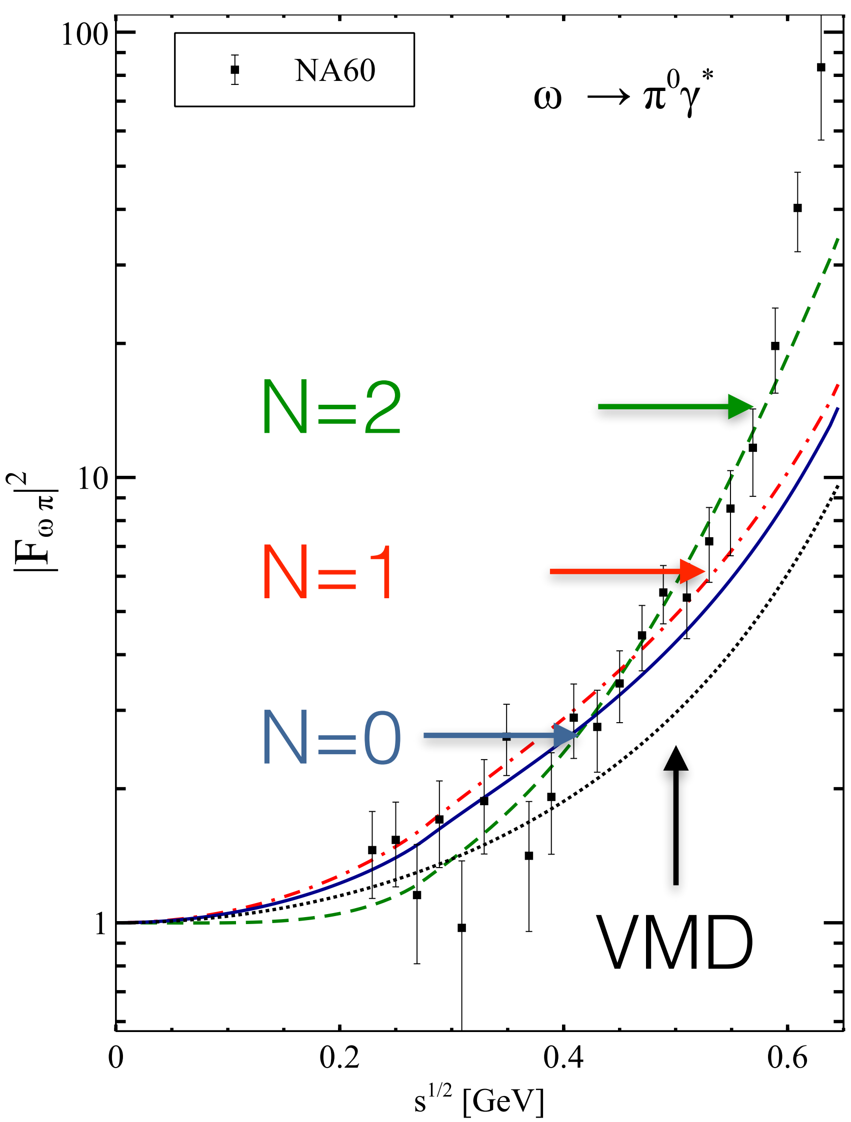


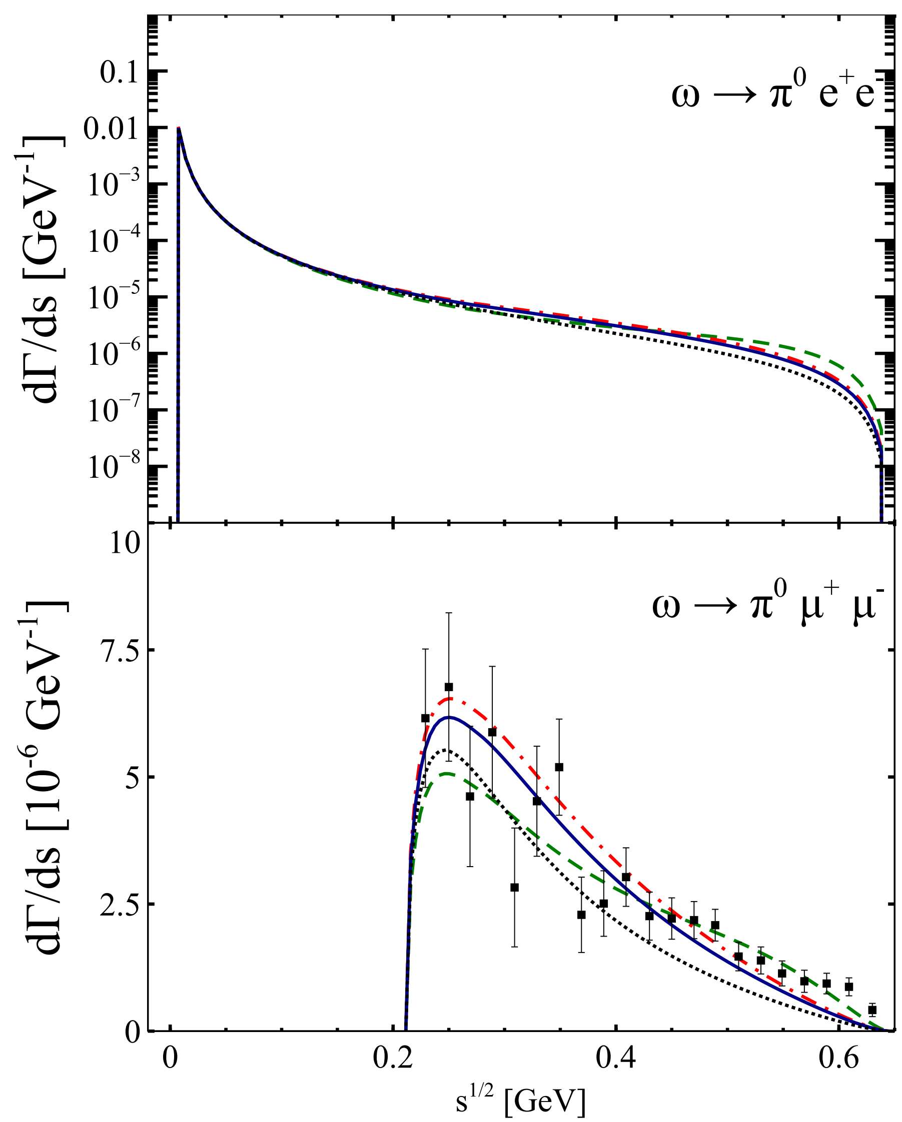


$\omega \to \pi^0 \gamma^*$



$$f_{V\pi}(s) = \int_{4m_\pi^2}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc}\, f_{V\pi}(s')}{s' - s} + \sum_{k=0}^{N} b_k \left(\omega(s)\right)^k$$





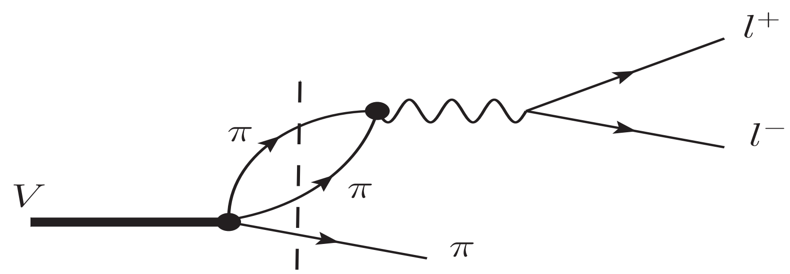
b_0 fixed from $\Gamma_{exp}(\omega \to \pi\gamma)$

NA60: Nature of the steep rise?

N=0 curve is consistent with [*preliminary*] **MAMI** data

Exp. analysis of $\varphi \to \pi\gamma$ is very important

$\varphi \to \pi^0 \gamma^*$



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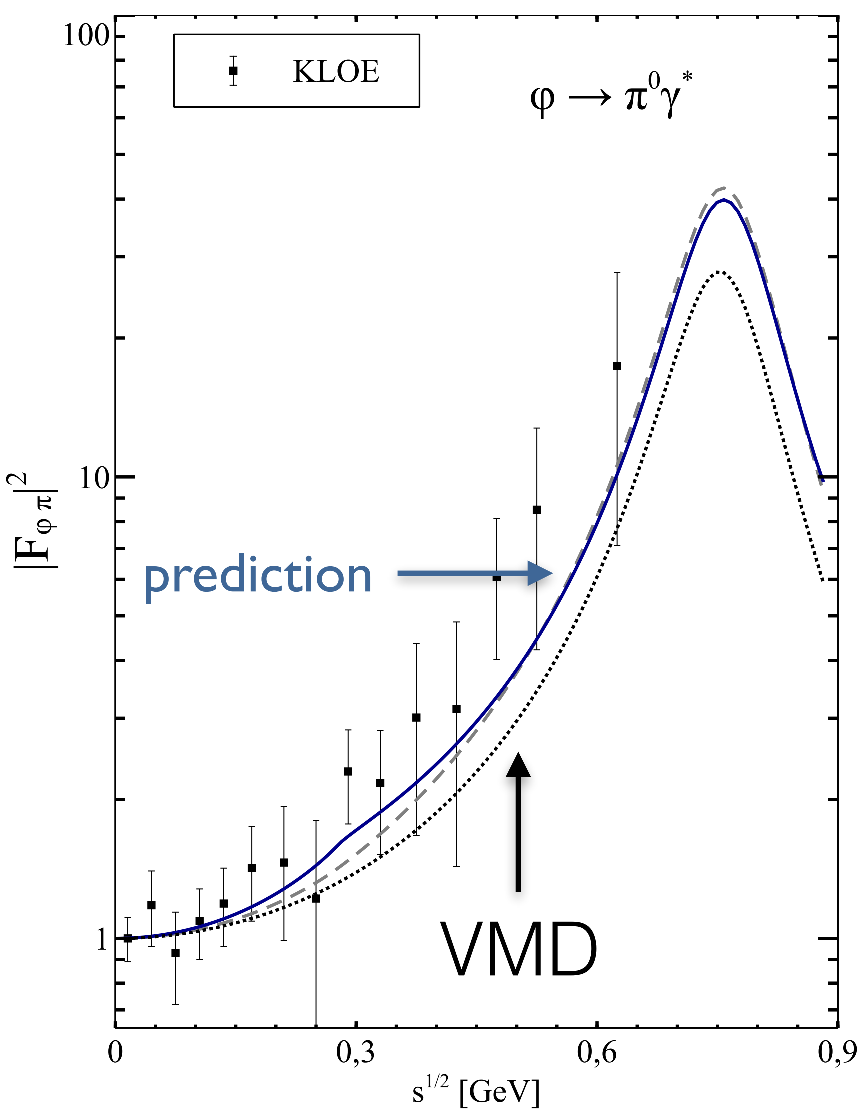
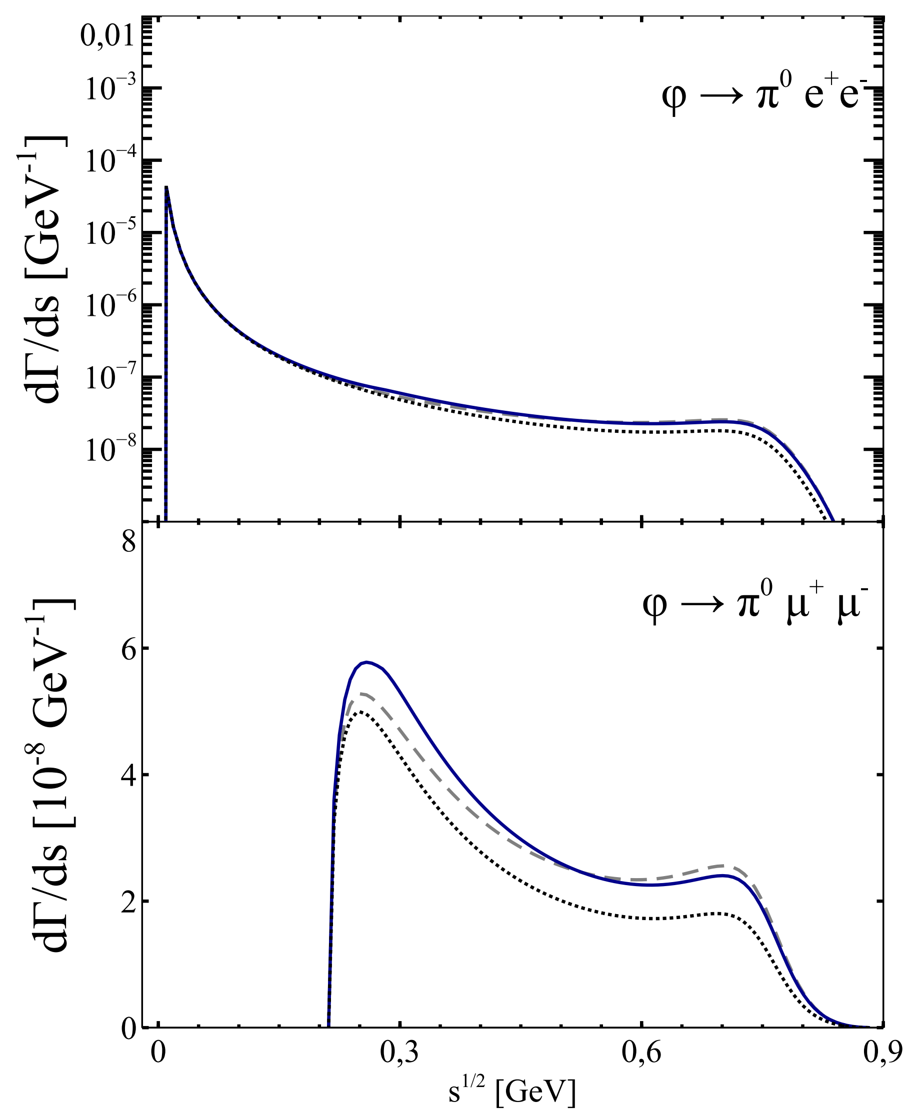




b_0 fixed from $\Gamma_{exp}(\varphi \to \pi\gamma)$

Grey: no 3b effects

Our prediction [2014] is consistent with new **KLOE** data [2016]

Summary and Outlook

- Fundamental principles (unitarity, analyticity and crossing symmetry) are very important: dispersion relations allow to take into account all rescattering effects

- 3b effects are not negligible

- Upcoming high statistic data from *CLAS, WASA-at-COSY*

- Extend to more complicated cases like: J^P(*arbitrary spin*) $\to 3\pi$, $N^* \to N\pi\pi$, $D \to K\pi\pi$, ...

- The codes are available for downloading as well as in an interactive form online
http://www.indiana.edu/~jpac/

thank you!

spare slides

$\eta \to \pi^+ \pi^- \pi^0$

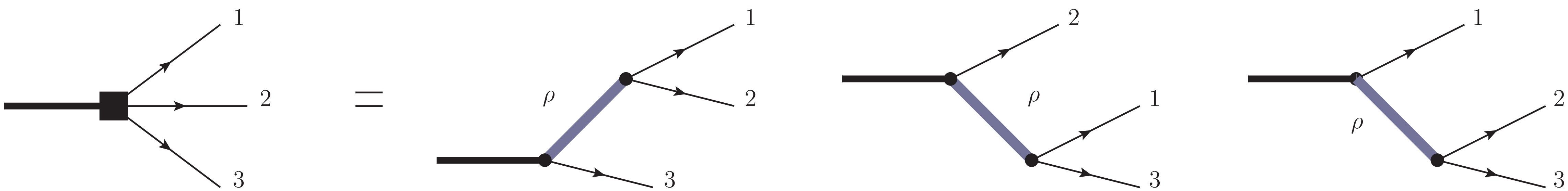
$$|A_{\eta\to\pi^+\pi^-\pi^0}|^2 \propto 1 + a\,y + b\,y^2 + d\,x^2 + f\,y^3 + \ldots$$

	a	b	d	f	g
KLOE-2 [16]	$-1.095 \pm 0.003^{+0.003}_{-0.002}$	$0.145 \pm 0.003 \pm 0.005$	$0.081 \pm 0.003^{+0.006}_{-0.005}$	$0.141 \pm 0.007^{+0.007}_{-0.008}$	$-0.044 \pm 0.009^{+0.012}_{-0.013}$
WASA-at-COSY [17]	-1.144 ± 0.018	$0.219 \pm 0.019 \pm 0.037$	$0.086 \pm 0.018 \pm 0.018$	0.115 ± 0.037	–
KLOE [16]	$-1.090 \pm 0.005^{+0.008}_{-0.019}$	$0.124 \pm 0.006 \pm 0.010$	$0.057 \pm 0.006^{+0.007}_{-0.016}$	$0.14 \pm 0.01 \pm 0.02$	–
CBarrel [14]	-1.22 ± 0.07	0.22 ± 0.11	0.06 ± 0.04 (fixed)	–	–
Layter *et al.* [49]	-1.080 ± 0.014	0.03 ± 0.03	0.05 ± 0.03	–	–
Gormley *et al.* [50]	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04	–	–
Theory (combined fit)	-1.075 ± 0.028	0.155 ± 0.006	0.084 ± 0.002	0.101 ± 0.003	-0.074 ± 0.003

NLO [22]	-1.371	0.452	0.053	0.027	–
NNLO [23]	-1.271 ± 0.075	0.394 ± 0.102	0.055 ± 0.057	0.025 ± 0.160	–
Kambor *et al.* [24]	-1.16	$0.24 \ldots 0.26$	$0.09 \ldots 0.10$	–	–
NREFT [30]	-1.213 ± 0.014	0.308 ± 0.023	0.050 ± 0.003	0.083 ± 0.019	-0.039 ± 0.002

$\omega/\varphi \rightarrow 3\pi$

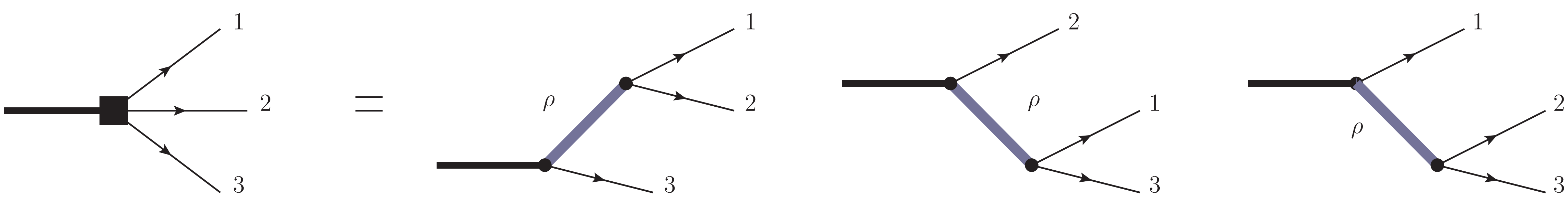
Typical Data analysis



3 BW + background term

$\omega/\varphi \to 3\pi$

Typical Data analysis



3 BW + background term

Shortcomings: unitarity is not satisfied

In view of the upcoming high statistic data (CLAS, KLOE, ..)
→ precision amplitude analysis is needed

Need to take into account final state interactions in a systematic way

(Generalized) isobar decomposition

Niecknig, Kubis
Schneider (2012)

ω/ϕ is spin 1 particle:

$$H = i\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon^{\mu}(p,\lambda)\,p_1^{\nu}\,p_2^{\alpha}\,p_3^{\beta}\,F(s,t,u)$$

$$= \sum_{J=1,3,\ldots}^{\infty} (2J+1)\,d^{J}_{\lambda 0}(\theta_s)\,f^{J}(s)$$

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p.w. expansion for $\mathcal{F}(s,t,u)$

$$F(s,t,u) = \sum_{J=1,3,\ldots}^{\infty} (p(s)\,q(s))^{J-1}\,P'_J(z_s)\,F_J(s)$$

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remove
kin. singular. !!!

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Truncate the partial waves

$$\sum_{J}^{\infty} \rightarrow \sum_{J}^{J_{max}}$$

(Generalized) isobar decomposition

Niecknig, Kubis
Schneider (2012)

ω/ϕ is spin 1 particle:

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remove
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Truncate the partial waves

$$\sum_J^{\infty} \to \sum_J^{J_{max}}$$

So-called reconstruction theorem:

$$F(s,t,u) = \sum_{J=1,3,\ldots}^{J_{max}} \ldots F_J(s) + \sum_{J=1,3,\ldots}^{J_{max}} \ldots F_J(t) + \sum_{J=1,3,\ldots}^{J_{max}} \ldots F_J(u)$$

ππ scattering
Fuchs, Sazdjian, Stern (1993)

(Generalized) isobar decomposition

Niecknig, Kubis
Schneider (2012)

ω/ϕ is spin 1 particle:

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$$= \sum_{J=1,3,\ldots}^{\infty} (2J+1)\,d^J_{\lambda 0}(\theta_s)\,f^J(s)$$

remove
kin. singular. !!!

p.w. expansion for $\mathcal{F}(s,t,u)$

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Truncate the partial waves

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ππ scattering
Fuchs, Sazdjian, Stern (1993)

$$F(s,t,u) = \sum_{J=1,3,\ldots}^{J_{max}} \ldots F_J(s) + \sum_{J=1,3,\ldots}^{J_{max}} \ldots F_J(t) + \sum_{J=1,3,\ldots}^{J_{max}} \ldots F_J(u)$$

crossing, analyticity

Dispersion relation

Integral equation

$$F(s) = \int_{4\,m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,F(s')}{s'-s}$$

$$\mathrm{Disc}\,F(s) = t^*(s)\,\rho(s)\left(F(s) + \hat{F}(s)\right) + \mathrm{inelastic}\,\theta(s > s_i)$$

Dispersion relation

Integral equation

$$F(s) = \int_{4\,m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, F(s')}{s' - s}$$

$$\mathrm{Disc}\, F(s) = t^*(s)\,\rho(s)\,\left(F(s) + \hat{F}(s)\right) + \mathrm{inelastic}\,\theta(s > s_i)$$

For practical reason we decompose

$$F(s) = \Omega(s)\,G(s)$$

Omnes (1958)

$$\mathrm{Disc}\,\Omega(s) = \rho(s)\,t^*(s)\,\Omega(s) + \mathrm{inelastic}\,\theta(s > s_i)$$

Dispersion relation

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Solve integral equation for $G(s)$

$$G(s) = \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, G(s')}{s' - s}$$

$$\mathrm{Disc}\, G(s) = \frac{\rho(s)\, t^*(s)\, \hat{F}(s)}{\Omega^*(s)} + \text{inelastic}\, \theta(s > s_i)$$

Dispersion relation

Integral equation

$$F(s) = \int_{4\,m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,F(s')}{s'-s}$$

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Solve integral equation for $G(s)$

$$G(s) = \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,G(s')}{s'-s} = \int_{4m_\pi^2}^{s_i} \ldots + \int_{s_i}^{\infty} \ldots$$

$$\mathrm{Disc}\,G(s) = \frac{\rho(s)\,t^*(s)\,\hat{F}(s)}{\Omega^*(s)} + \mathrm{inelastic}\,\theta(s>s_i)$$

Dispersion relation

Integral equation

$$F(s) = \int_{4\,m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,F(s')}{s'-s}$$

$$\mathrm{Disc}\,F(s) = t^*(s)\,\rho(s)\left(F(s)+\hat{F}(s)\right) + \text{inelastic}\,\theta(s>s_i)$$

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$$F(s) = \Omega(s)\,G(s)$$

Omnes (1958)

$$\mathrm{Disc}\,\Omega(s) = \rho(s)\,t^*(s)\,\Omega(s) + \text{inelastic}\,\theta(s>s_i)$$

parametrize with conformal mapping expansion

Solve integral equation for $G(s)$

$$G(s) = \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,G(s')}{s'-s} = \int_{4m_\pi^2}^{s_i} \ldots + \int_{s_i}^{\infty} \ldots$$

$$\mathrm{Disc}\,G(s) = \frac{\rho(s)\,t^*(s)\,\hat{F}(s)}{\Omega^*(s)} + \text{inelastic}\,\theta(s>s_i)$$

Dispersion relation

Integral equation: $$G(s) = \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, G(s')}{s'-s} = \int_{4m_\pi^2}^{s_i} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, G(s')}{s'-s} + \sum_{k=0}^{\infty} a_k \left(\omega(s)\right)^k$$

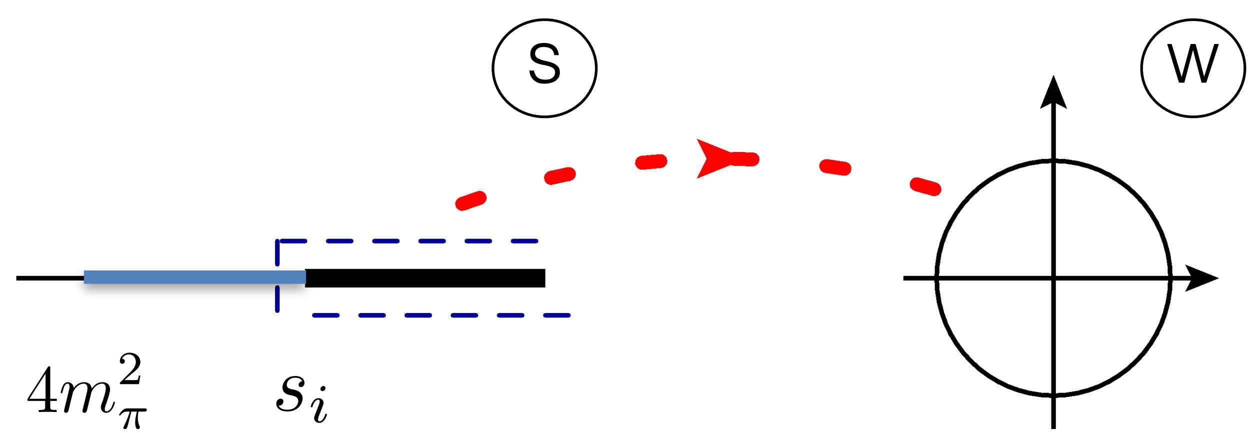
Dispersion relation

Integral equation: $$G(s) = \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, G(s')}{s'-s} = \int_{4m_\pi^2}^{s_i} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, G(s')}{s'-s} + \sum_{k=0}^{\infty} a_k \left(\omega(s)\right)^k$$

w(s) is the **conformal map of inelastic contributions**

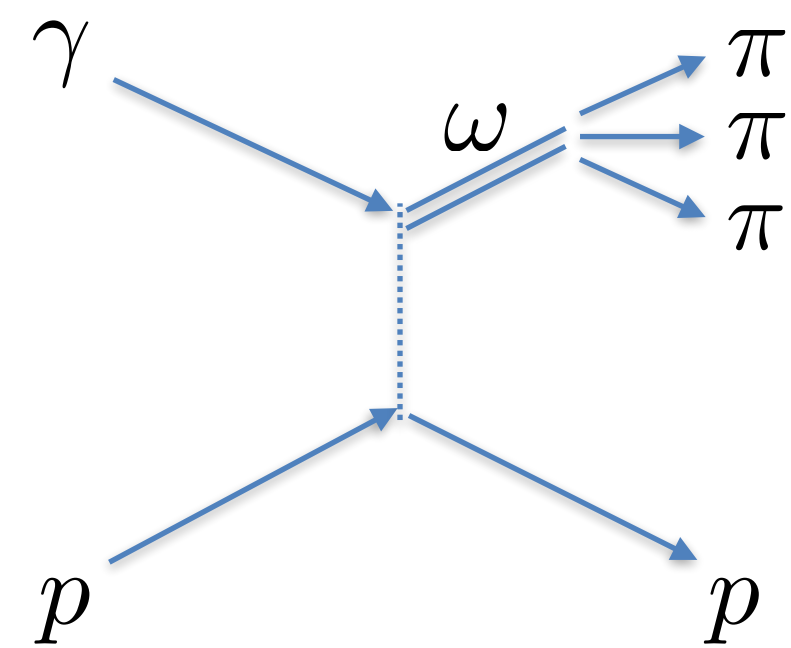
$$\omega(s) = \frac{\sqrt{s_i} - \sqrt{s_i - s}}{\sqrt{s_i} + \sqrt{s_i - s}}$$

$$s_i = 1\,\mathrm{GeV}^2$$



Yndurain (2002)

On going fitting of CLAS g12 data

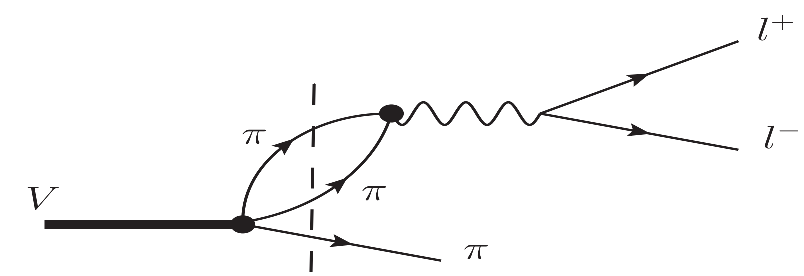


- Data: g12 experiment
- Reaction: $\gamma p \to p\,\omega \to p\,\pi^+\pi^-\pi^0$
- Incoming photon energies:
  - 1.1 - 3.8 GeV (Florida group: Volker Crede, Chris Zeoli, …)
  - >3.6 GeV (JLab: Carlos Salgado, …)
- Files: data, reconstructed Monte Carlo, generated Monte Carlo
- 4-vector format: <pi+ 4-vec: Px, Py, Pz, E;>, …
  incl: Q-value: likelihood for event being signal

AmpTools
Brought to you by: mashephe

$\omega/\varphi \to \pi^0 \gamma^*$

$$\langle \pi^0(p_0)\, l^+(p_+)\, l^-(p_-)\,|\,T\,|\,V(p_V,\lambda)\rangle =$$
$$(2\pi)^4\,\delta(p_V - p_0 - p_+ - p_-)\, H_{V\pi}$$



$$H_{V\pi} = \epsilon^\mu(p_V,\lambda)\, f_{V\pi}(s)\, \epsilon_{\mu\nu\alpha\beta}\, p_0^\nu\, q^\alpha$$
$$\frac{\imath e^2}{s}\, \bar{u}(p_-,\lambda_-)\, \gamma^\beta\, v(p_+,\lambda_+),$$

$$F_{V\pi}(s) = \frac{f_{V\pi}(s)}{f_{V\pi}(0)}$$

$$\frac{1}{\Gamma_{V\to\pi\gamma}}\frac{d\Gamma}{ds} = \frac{e^2}{12\,\pi^2}\,|F_{V\pi}(s)|^2\sqrt{1-\frac{4\,m_l^2}{s}}\left(1+\frac{2\,m_l^2}{s}\right)$$
$$\frac{1}{s}\left[\left(1+\frac{s}{M^2-m^2}\right)^2 - \frac{4\,M^2 s}{(M^2-m^2)^2}\right]^{3/2}$$

Branching ratios

$$\mathcal{B}^{th}(\omega \to \pi^0 e^+ e^-) = 7.8 \cdot 10^{-4}$$
$$\mathcal{B}^{exp}(\omega \to \pi^0 e^+ e^-) = (7.7 \pm 0.6) \cdot 10^{-4}$$

$$\mathcal{B}^{th}(\omega \to \pi^0 \mu^+ \mu^-) = 0.96 \cdot 10^{-4}$$
$$\mathcal{B}^{exp}(\omega \to \pi^0 \mu^+ \mu^-) = (1.3 \pm 0.4) \cdot 10^{-4}$$

$$\mathcal{B}^{th}(\phi \to \pi^0 e^+ e^-) = 1.45 \cdot 10^{-5}$$
$$\mathcal{B}^{exp}(\phi \to \pi^0 e^+ e^-) = (1.12 \pm 0.28) \cdot 10^{-5}$$

Prediction $\mathcal{B}^{th}(\phi \to \pi^0 \mu^+ \mu^-) = 3.9 \cdot 10^{-6}$