Model-independent determination of the compositeness of near-threshold quasibound states.

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- Introduction ~weak-binding relation for bound state~

Part I

- Extension to the quasibound state
- Applications to exotic hadrons ~ $\Lambda(1405), a_0(980), f_0(980)$ ~

Part II

- Derivation without convergence of ERE
- Extended relation with the CDD pole contribution

Introduction ~exotic hadrons~

- Exotic hadrons

Hadrons which do not coincide with the predictions of the quark model.

→ More complicated internal structure can be expected.

- tetra quark, penta quark
- hadron molecule ···

→ It is important to reveal the internal structure of exotics.

e.g. ; Λ(1405)

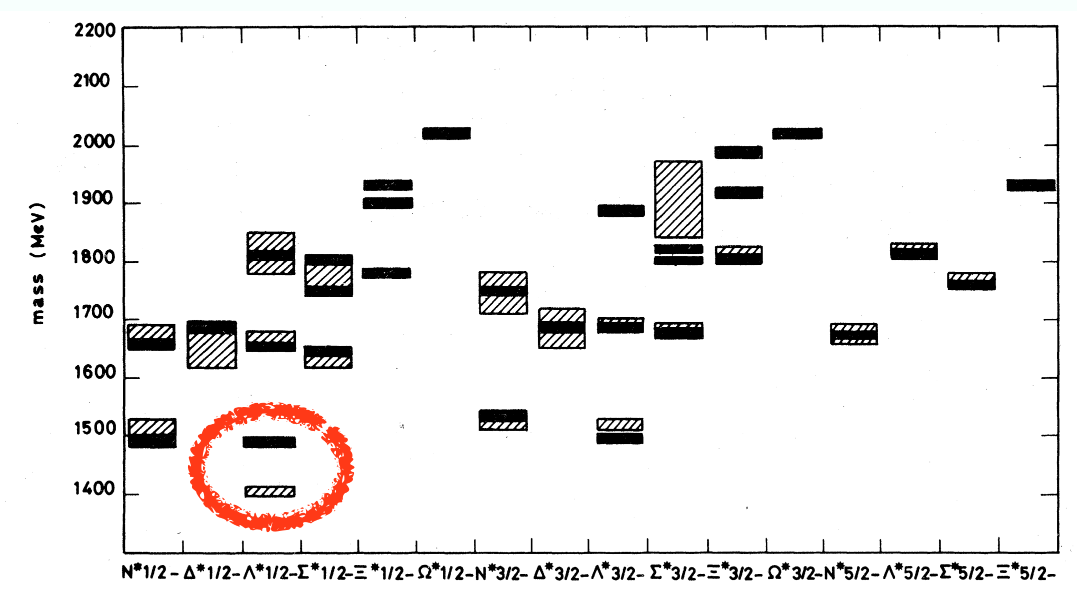
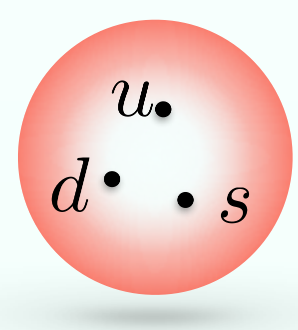


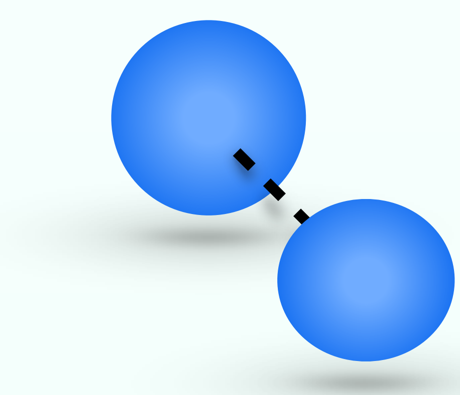
FIG. 1. Comparison of the predicted and observed spectrum of negative-parity baryons. The shaded regions corre-

N. Isgur, and G. Karl, Phys. Rev. D18, 4187 (1978)

exited Λ state(uds)



$\bar{K}N$ bound state

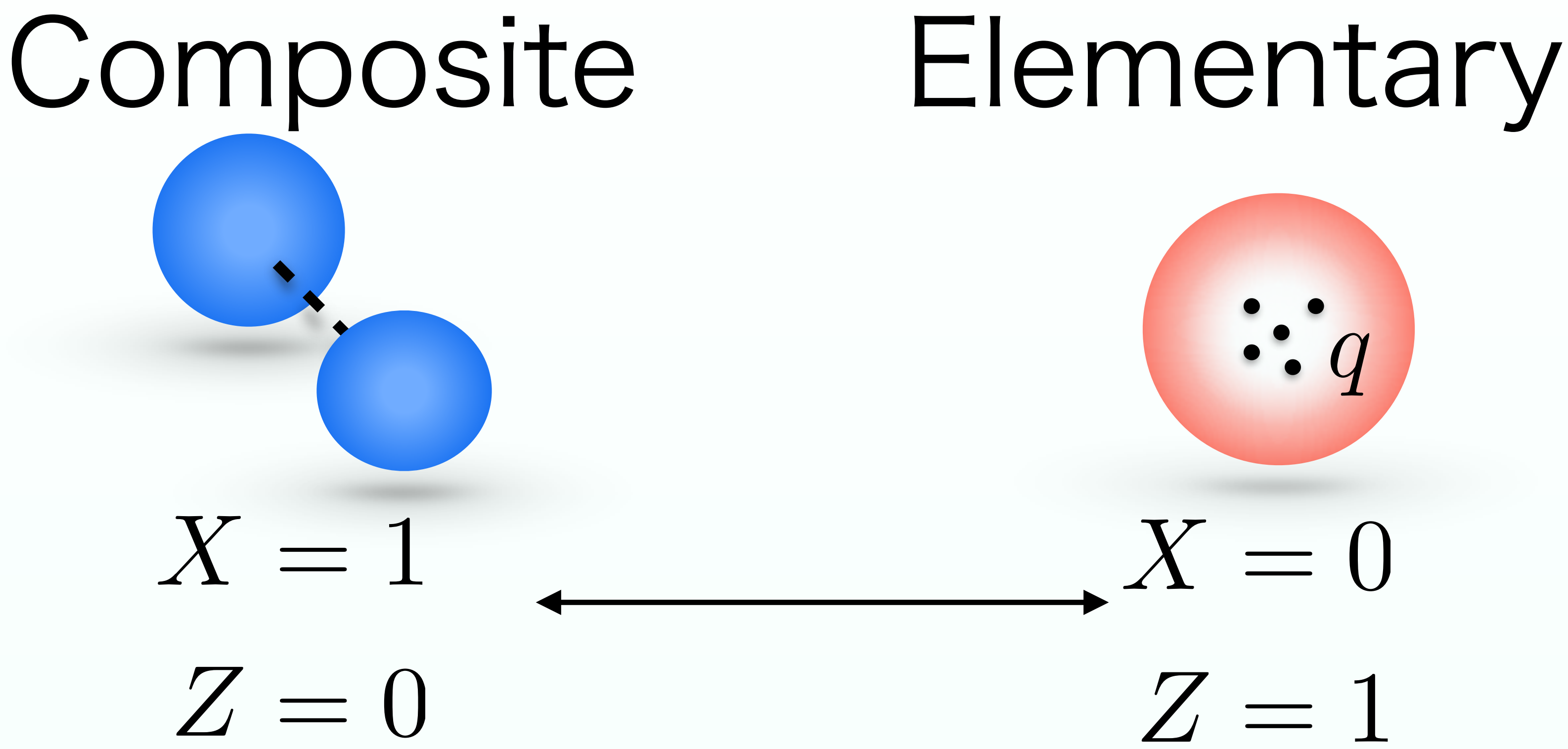


Compositeness of bound state

Previous work

Introduced to study deuteron by Weinberg.

S. Weinberg, Phys. Rev. 137, B672 (1965)



Condition

- weakly bound
- stable state
- s-wave

Output

- X ; weight of composite state $(0 < X < 1)$
- Z ; wave function renormalization $(0 < Z < 1)$
- a_0 ; scattering length
- B ; binding energy

$$R = \frac{1}{\sqrt{2\mu B}}$$

(μ ;reduced mass of scat. state)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(R_{\rm typ}/R\right)\right\}$$

typical length scale

We can extract the information of the internal structure using experimental observables.

Part I

~unstable states~

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Extension to the quasibound state.

System

Two channel scattering

- scattering channel $|\boldsymbol{p}\rangle$
- decay channel $|\boldsymbol{p}'\rangle$

$|\boldsymbol{p}\rangle$ can decay to $|\boldsymbol{p}'\rangle$.

Unstable quasibound state $|QB\rangle$ exists near $|\boldsymbol{p}\rangle$ threshold.

The interaction has a typical length scale $R_{\rm typ}$.

Effective field theory

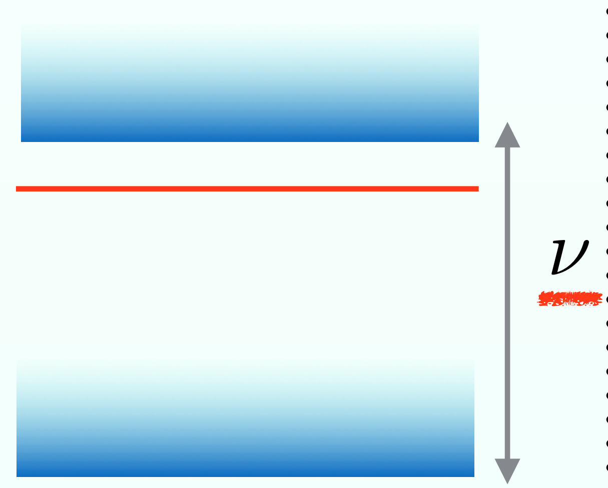
To discuss the near-threshold physics, we use following non-relativistic EFT.

Free field $H_{\rm free}$ eigenstate

$|\boldsymbol{p}\rangle$ scattering channel

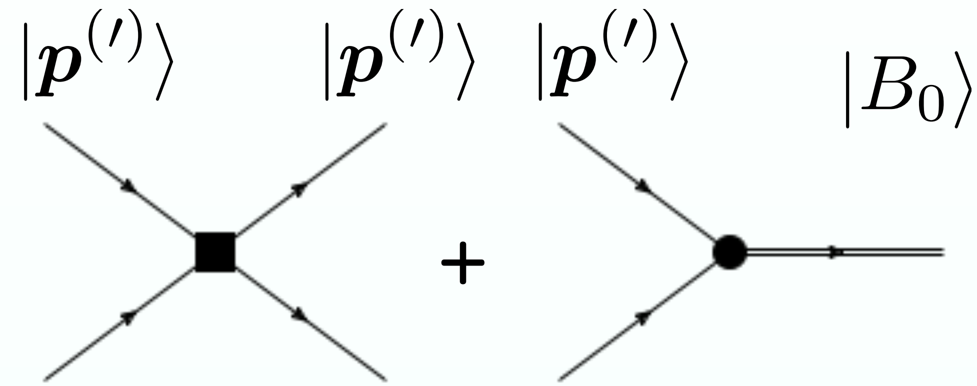
$|B_0\rangle$ discrete channel

$|\boldsymbol{p}'\rangle$ decay channel



Interaction

$\mathcal{H}_{\rm int} =$



point interaction

Eigenstate

$$H = H_{\rm free} + H_{\rm int}$$

$$H|QB\rangle = E_{QB}|QB\rangle$$

$$E_{QB} = -B - i\Gamma/2$$; complex

We consider the compositeness of $|\boldsymbol{p}\rangle$ channel ; X.

Extension to the quasibound state.

Definition of compositeness

Bound state

Bound state $|B\rangle$ is normalized with $\langle B|B\rangle = 1$

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\langle B|\boldsymbol{p}\rangle\langle\boldsymbol{p}|B\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}|\langle\boldsymbol{p}|B\rangle|^2$$

$$Z \equiv |\langle B_0|B\rangle|^2$$

- $X + Z = 1$
- $0 < X, Z < 1$

The probabilistic interpretation is guaranteed for X and Z.

Quasibound state

To normalize unstable state, we introduce Gamow state $|\overline{QB}\rangle$.

Normalization condition becomes

$$\langle\overline{QB}|QB\rangle = \langle QB^*|QB\rangle = 1.$$

T. Berggren, Nucl. Phys. A 109 (1968)

The expectation value of the any operator becomes complex number.

- $X + Z = 1$
- ~~$0 < X, Z < 1$~~ $X, Z \in C$

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\langle\overline{QB}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|QB\rangle$$

The probabilistic interpretation is not guaranteed!

Extension to the quasibound state.

- Assuming $|E_{QB}|$ is small, we expand a_0 with respect to $1/R$.

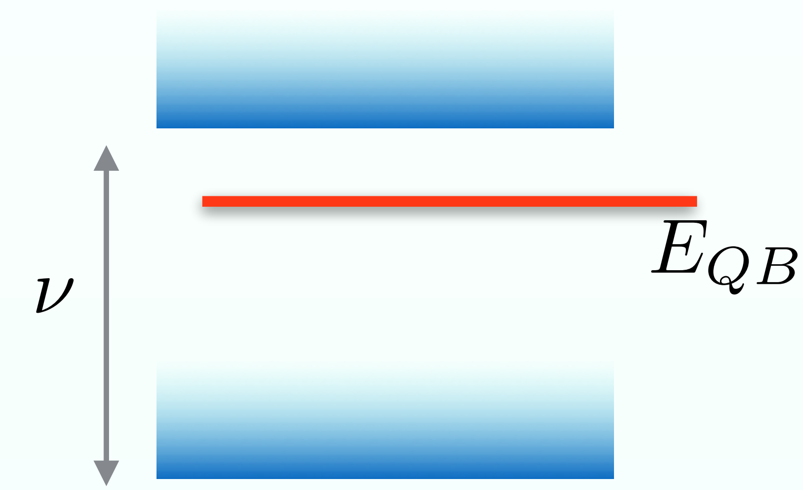
Y. Kamiya and T. Hyodo, Phys. Rev. C. 93.035203, arXiv:1509.00146

$$a_0 = R\left[\underbrace{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right)}_{\text{original}} + \underbrace{\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}_{\text{new}}\right]$$

$$R = \frac{1}{\sqrt{-2\mu E_{QB}}}$$

$$l = \frac{1}{\sqrt{2\mu\nu}}$$

If $|R_{\rm typ}/R|$ and $|l/R|^3$ are sufficiently smaller than 1, we can extract X from a_0 and E_{QB}.



- Notice
  - a_0, E_{QB}, X are all complex numbers, then above relation is established among them.
  - If the the contribution of decaying mode is neglected, the compositeness relation is same to the one for bound state.
  - The same argument is valid for the case with $\mathrm{Re}\ E_h > 0$.

Interpretation of X

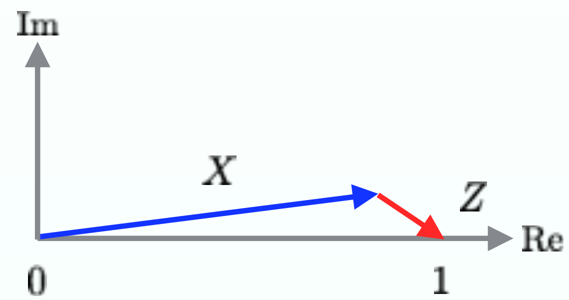
Interpretation of the complex compositeness

- There is no common interpretation of the complex X.

(1) close to bound state case

$$\begin{cases} X = 0.8 + 0.1i \\ Z = 0.2 - 0.1i \end{cases}$$

small cancellation in X+Z



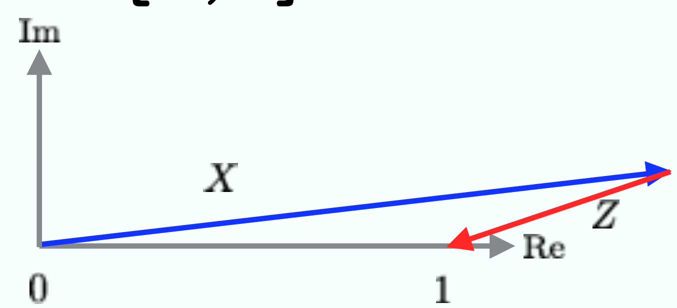
bound state case

$$\begin{cases} X = 0.8 \\ Z = 0.2 \end{cases}$$

probabilistic interpretation is available

(2-a) When real part is not in [0,1]

$$\begin{cases} X = 1.9 + 0.2i \\ Z = -0.9 - 0.2i \end{cases}$$



large cancellation in X+Z

(2-b) When imaginary part is large.

$$\begin{cases} X = 0.9 + 0.8i \\ Z = 0.1 - 0.8i \end{cases}$$

When the cancelation is small,
we can interpret the complex compositeness.

Interpretation of X

Our proposal

c.f. T. Berggren, Phys. Lett. B 33 (1979) 8

For probabilistic interpretation we define the following real quantities.

$\tilde{X}$; probability to find the scattering state in physical state

$\tilde{Z}$; probability to find the other states

U ; degree of uncertainty of the interpretation

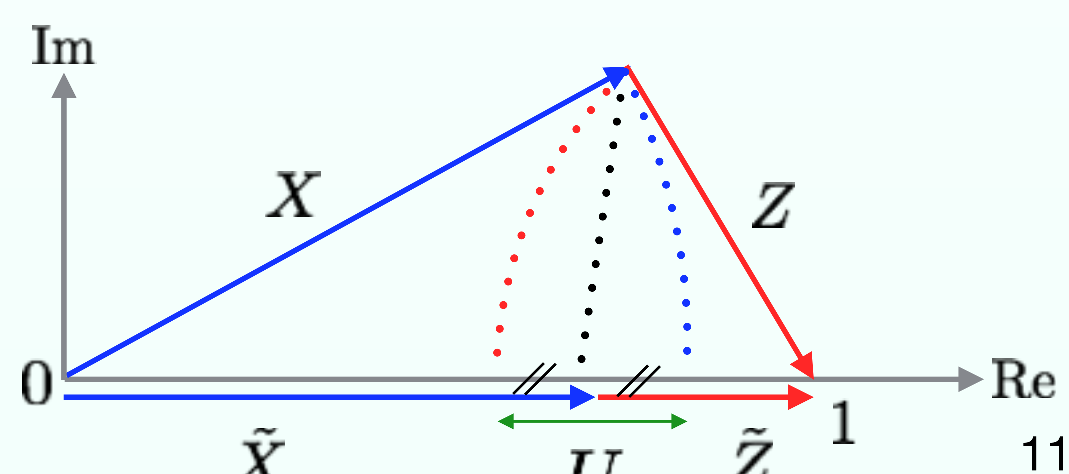
$$\tilde{X} \equiv \frac{1-|Z|+|X|}{2}$$

$$\tilde{Z} \equiv \frac{1-|X|+|Z|}{2}$$

$$U \equiv |Z|+|X|-1$$

conditions :

- $\tilde{X} + \tilde{Z} = 1$
- $0 \leq \tilde{X}, \tilde{Z} \leq 1$
- When the cancellation is 0,
  $\tilde{X} = X, \tilde{Z} = Z, U = 0$.
- U becomes large
  when the cancellation becomes large.



If U is small, we interpret $\tilde{X}$ as the probability.

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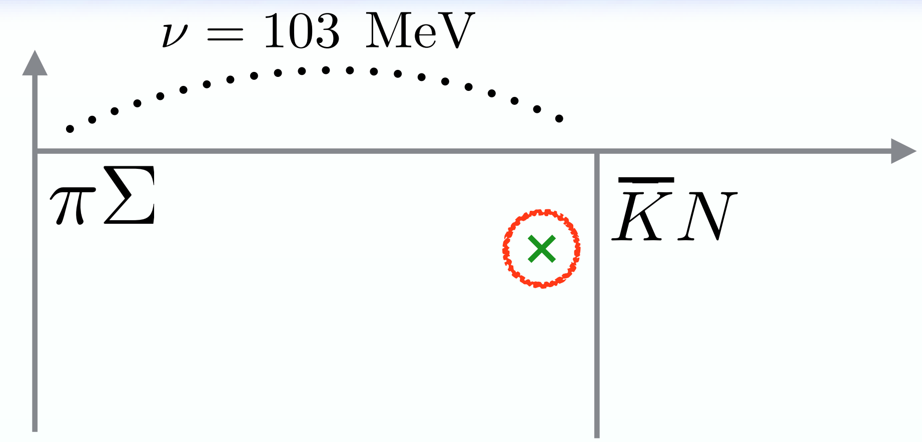
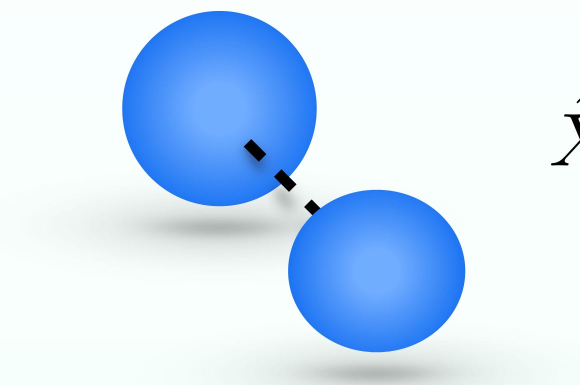
Applications to hadrons

- $\Lambda(1405)$ ($I=0$ $\bar{K}N$ scattering)

  $J^P = \frac{1}{2}^-$



$\bar{K}N$ molecule?

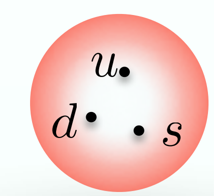


$\tilde{X} = 1$

or

other components?

$\tilde{X} = 0$



e.g.

- Λ excited states(uds)
- penta-quark state

…

Rtyp is estimated from rho meson exchange int. $(R_{\rm typ} \sim 0.25\ \text{fm})$ → $\left|\frac{R_{\rm typ}}{R}\right| \lesssim 0.17$

l is estimated from difference of the threshold energy → $\left|\frac{l}{R}\right|^3 \lesssim 0.14$

$$R = \frac{1}{\sqrt{-2\mu E_{QB}}}$$

$$l = \frac{1}{\sqrt{2\mu\nu}}$$

$$a_0 = R\left[\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right] \Rightarrow X = \frac{a_0}{2R - a_0} \Rightarrow \tilde{X}, U$$

can be neglected

Applications to hadrons

- $\Lambda(1405)$ in $I=0$ $\bar{K}N$ scattering

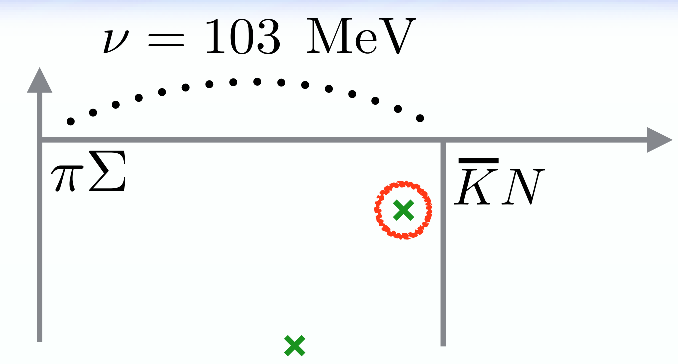
  We use E_{QB} and a_0 in the following papers.

  (1) Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A 881 98 (2012)

  (2) M. Mai and U. G. Meissner, Nucl. Phys. A 900, 51 (2013)

  (3) Z. H. Guo and J. A. Oller, Phys. Rev. C 87, 035202 (2013)

  (4)M. Mai and U.-G. Meißner, Eur. Phys. J. A 51, 30 (2015).



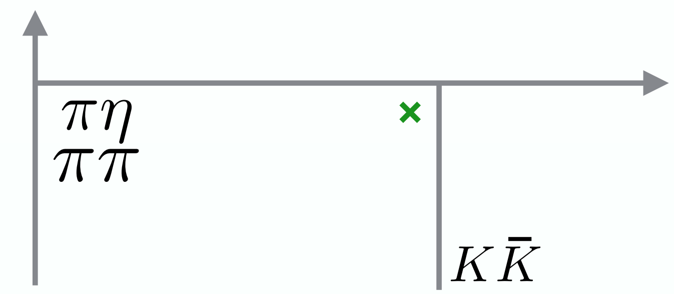
Ref.	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	-10-i26	1.39-i0.85	1.3+i0.1	1.0	0.5
(2)	-4-i8	1.81-i0.92	0.6+i0.1	0.6	0.0
(3)	-13-i20	1.30-i0.85	0.9-i0.2	0.9	0.1
(4)-1	2-i10	1.21-i1.47	0.6+i0.0	0.6	0.0
(4)-2	- 3-i12	1.52-i1.85	1.0+i0.5	0.8	0.6

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- $\tilde{X}$ is close to 1.

→ $\Lambda(1405)$: $\bar{K}N$ composite dominance

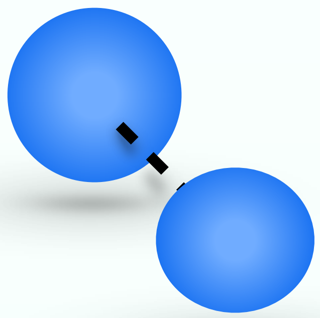
Applications to hadrons

- $a_0(980)$ 、$f_0(980)$ ($K\bar{K}$ scattering)
  $(I=1)$ $(I=0)$
  $J^{PC}=0^{++}$



$K\bar{K}$ molecule ?

J. D. Weinstein and N. Isgur, PRD 41 (1990)



$\tilde{X}=1$

or

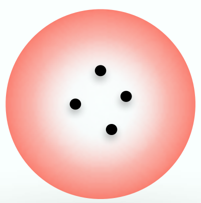
other components?

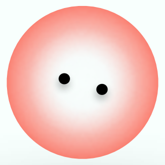
e.g.

$\tilde{X}=0$

- tetra quark state
  R. L. Jaffe, PRD 15 (1977)
- $q\bar{q}$ meson state

…





$$\left|\frac{R_{\rm typ}}{R}\right|\lesssim 0.17 \qquad \left|\frac{l}{R}\right|^3\lesssim 0.04$$

$$a_0=R\left[\frac{2X}{1+X}+\mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right)+\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right] \Rightarrow X=\frac{a_0}{2R-a_0} \Rightarrow \tilde{X}, U$$

can be neglected

Applications to hadrons

- $a_0(980)$ in $K\overline{K}$ scattering

  We determine E_{QB} and a_0 from Flatte parameters which are obtained experimental analyses.

c. f. : V. Baru et al. Phys. Lett. B 586, 53 (2004)

T. Sekihara and S. Kumano, Phys. Rev. D 92, 034010 (2015)

(1) G. S. Adams et al. [CLEO Collaboration], Phys. Rev. D 84, 112009 (2011)

(2) F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 681, 5 (2009)

(3) D. V. Bugg, Phys. Rev. D 78, 074023 (2008)

(4) S. Teige et al. [E852 Collaboration], Phys. Rev. D 59, 012001 (1999)

Set	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	31-i70	-0.03-i0.53	0.2-i0.2	0.3	0.1
(2)	3-i25	0.17-i0.77	0.2-i0.2	0.2	0.1
(3)	9-i36	0.05–i0.63	0.2-i0.2	0.2	0.1
(4)	15-i29	-0.13-i0.52	0.1-i0.4	0.1	0.1

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- $\tilde{X}$ is close to 0.

→ $a_0(980)$: small $K\bar{K}$ fraction

Applications to hadrons

c. f. T. Sekihara and S. Kumano, Phys. Rev. D 92, no. 3, 034010 (2015)

- $f_0(980)$ in $K\overline{K}$ scattering

  We determine E_{QB} and a_0 from Flatte parameters which are obtained experimental analysis.

(1) T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 84, 052012 (2011)

(2) F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 634, 148 (2006)

(3) A. Garmash et al. [Belle Collaboration], Phys. Rev. Lett. 96, 251803 (2006)

(4) M. Ablikim et al. [BES Collaboration], Phys. Lett. B 607, 243 (2005)

(5) J. M. Link et al. [FOCUS Collaboration], Phys. Lett. B 610, 225 (2005)

(6) M. N. Achasov et al., Phys. Lett. B 485, 349 (2000)

Ref.	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	19-i30	0.02-i0.95	0.3-0.3	0.4	0.2
(2)	-6 -i10	0.84-i0.85	0.3-i0.1	0.3	0.0
(3)	-8 -i28	0.64-i0.83	0.4-i0.2	0.4	0.1
(4)	10-i18	0.51-i1.58	0.7-i0.3	0.6	0.1
(5)	-10-i29	0.49-i0.67	0.3-i0.1	0.3	0.0
(6)	10-i7	0.52-i2.41	0.9-i0.2	0.9	0.1

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- Values of $\tilde{X}$ are not consistent.

More precise analysis is needed.

Part II

~CDD pole contribution~

CDD pole and weak-binding relation

CDD (Castillejo Dalitz Dyson) pole(E_c) and internal structure

CDD pole : $f(E_c) = 0$

L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).

G. F. Chew and S. C. Frautschi, Phys. Rev. 124, 264 (1961).

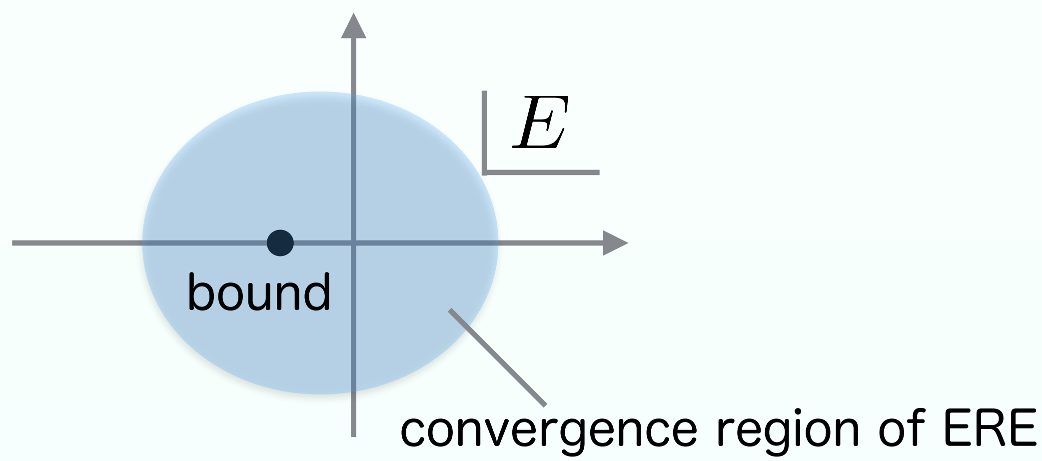
・represents the contribution from outside model

V. Baru et al, Eur. Phys. J. A44, 93 (2010), 1001.0369.
T. Hyodo, Phys. Rev. Lett. 111 (2013) 132002.
Z.-H. Guo and J. A. Oller, Phys. Rev. D93, 054014 (2016), 1601.00862.

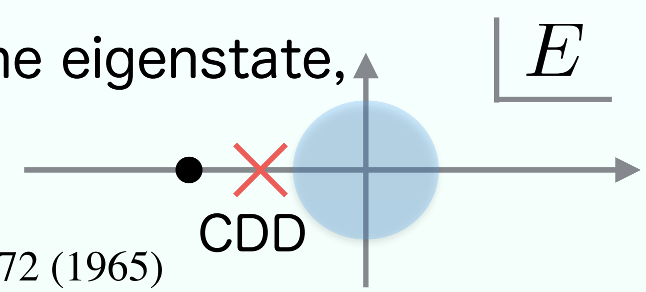
Condition of the weak-binding relation

In the derivation of the relation
we assume that effective range expansion (ERE)
work well at the pole of eigenstate.

$$f(E) = \left[-\frac{1}{a_0} + \frac{r_e}{2}p^2 - ip\right]^{-1} \quad \text{(s-wave)}$$



When the CDD pole lies near the threshold and ERE fails to describe the eigenstate,
the weak-binding relation is not available.



S. Weinberg, Phys. Rev. 137, B672 (1965)

If CDD pole lies near the threshold, we cannot use
the previous weak-binding relation to study internal structure.

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Derivation without convergence of ERE

For simplicity, we consider the stable bound state case.

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} |\langle \boldsymbol{p}|B\rangle|^2$$

- Compositeness

T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015), 1411.2308.

$$H|B\rangle = E_B|B\rangle \quad (E_B < 0)$$

$|B\rangle$: bound state

$$X = -g^2 G'(E_B)$$

coupling constant between $|\boldsymbol{p}\rangle$ and $|B\rangle$.

$$G(E) = \frac{1}{2\pi^2}\int_0^\Lambda p^2 dp \frac{1}{E - p^2/(2\mu) + i0^+}$$: loop function

- The leading term of the $G'(E_B)$ is cutoff independent.

T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015), 1411.2308.

$$G'(E_B) = \frac{\mu}{4\pi E_B R}\left\{1 + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)\right\}$$

R_{typ} : typical length scale of int. $(\sim 1/\Lambda)$

$$R \equiv 1/\sqrt{-2\mu E_B}$$

Derivation without convergence of ERE

- Compositeness

T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015), 1411.2308.

$$X = -g^2 G'(E_B)$$

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} |\langle \boldsymbol{p}|B\rangle|^2$$

$$H|B\rangle = E_B|B\rangle \quad (E_B < 0)$$

$|B\rangle$: bound state

If we approximate g^2 with ERE

$$f(E) = [p\cot\delta - ip]^{-1}$$

$$-\frac{1}{a_0} + \frac{r_e}{2}p^2 + \mathcal{O}(R_{\rm eff}^3 p^4)$$

$$g^2 = -\lim_{E\to E_B} \frac{2\pi}{\mu}(E - E_B) f(E)$$

$R_{\rm eff}$: range scale characterizing ERE

$$g^2 = \frac{2\pi}{\mu^2} \frac{1}{R - r_e + R\mathcal{O}((R_{\rm eff}/R)^3)}$$

$$X = \frac{1}{1 - \frac{r_e}{R} + \mathcal{O}\left(\left(\frac{R_{\rm eff}}{R}\right)^3\right)} \left[1 + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right]$$

equivalent to the original Weinberg's relation.

In this approximation, the CDD pole contribution is dropped out from the weak-binding relation.

To include the CDD pole contribution, a better approximation for g^2 is needed .

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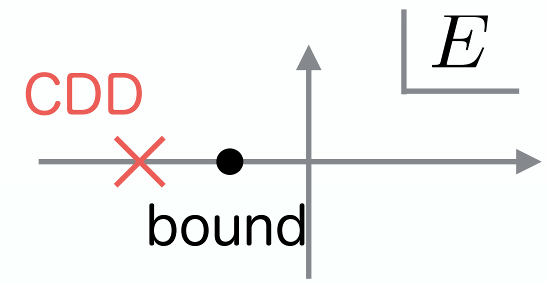
Extended relation with the CDD pole contribution

- To take account of the contribution of CDD pole



$$X = -g^2 G'(E_B)$$

$$f(E) = [p \cot \delta - ip]^{-1}$$

taking CDD pole contribution

$$p \cot \delta \to \frac{b_0 + b_1 p^2}{1 + c_1 p^2} + \mathcal{O}(R_{\text{Padè}}^5 p^6)$$

Pade approximant

Y. Kamiya, T. Hyodo, arXiv:1607.01899 [hep-ph].

$$\Rightarrow \quad X = \left[1 - \frac{4R(a_0 - R)^2}{a_0^2 r_e} + \mathcal{O}\left(\left(\frac{R_{\text{Padè}}}{R}\right)^5\right)\right]^{-1} \left(1 + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right)$$

Even when the ERE does not describe the bound state,
we can estimate the compositeness using experimental observables.

Extended relation with the CDD pole contribution

Verification with model

We compare the effectiveness of the estimation using the previous and extended weak-binding relation.

exact compositeness in this model

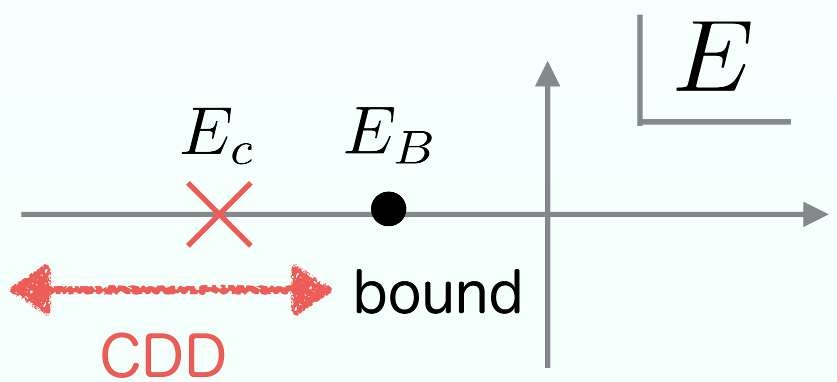
$$X_{\mathrm{exact}} = -g^2 G'(E_B)$$

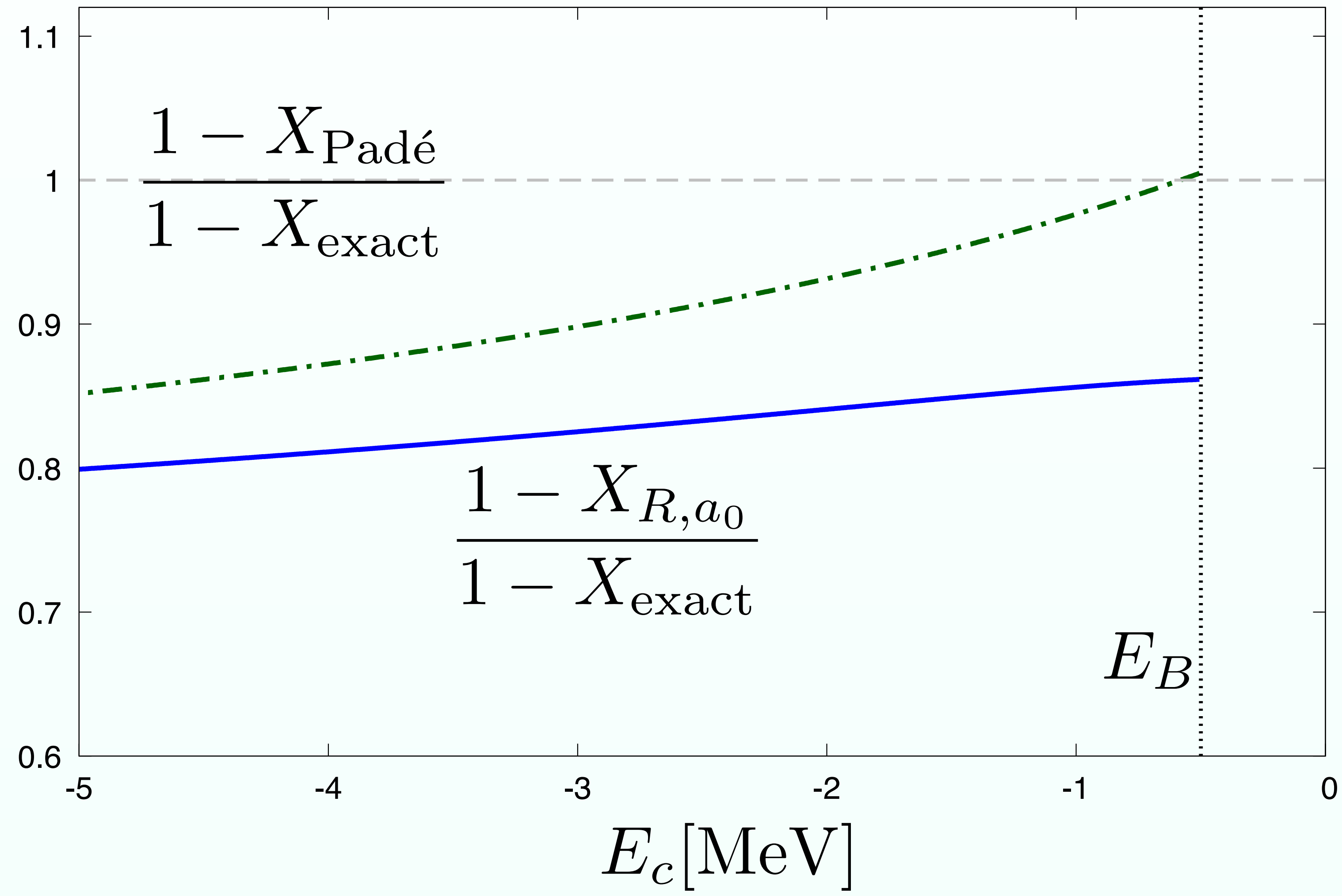
original relation

$$X_{R,a_0} = \frac{a_0}{2R - a_0}$$

extended relation

$$X_{\mathrm{Padè}} = \left[1 - \frac{4R(a_0 - R)^2}{a_0^2 r_e}\right]^{-1}$$





The estimation of the compositeness is improved, when the CDD pole lies near the threshold.

Extended relation with the CDD pole contribution

Verification with model

We compare the validity of the estimation using the previous and extended weak-binding relation.

exact compositeness in this model

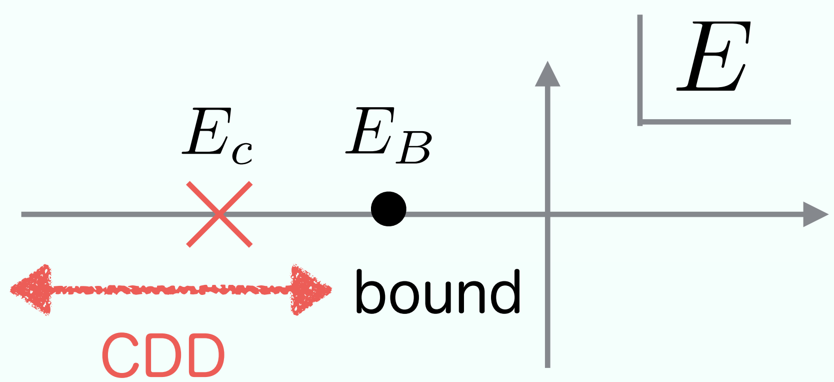
$$X_{\mathrm{exact}} = -g^2 G'(E_B)$$

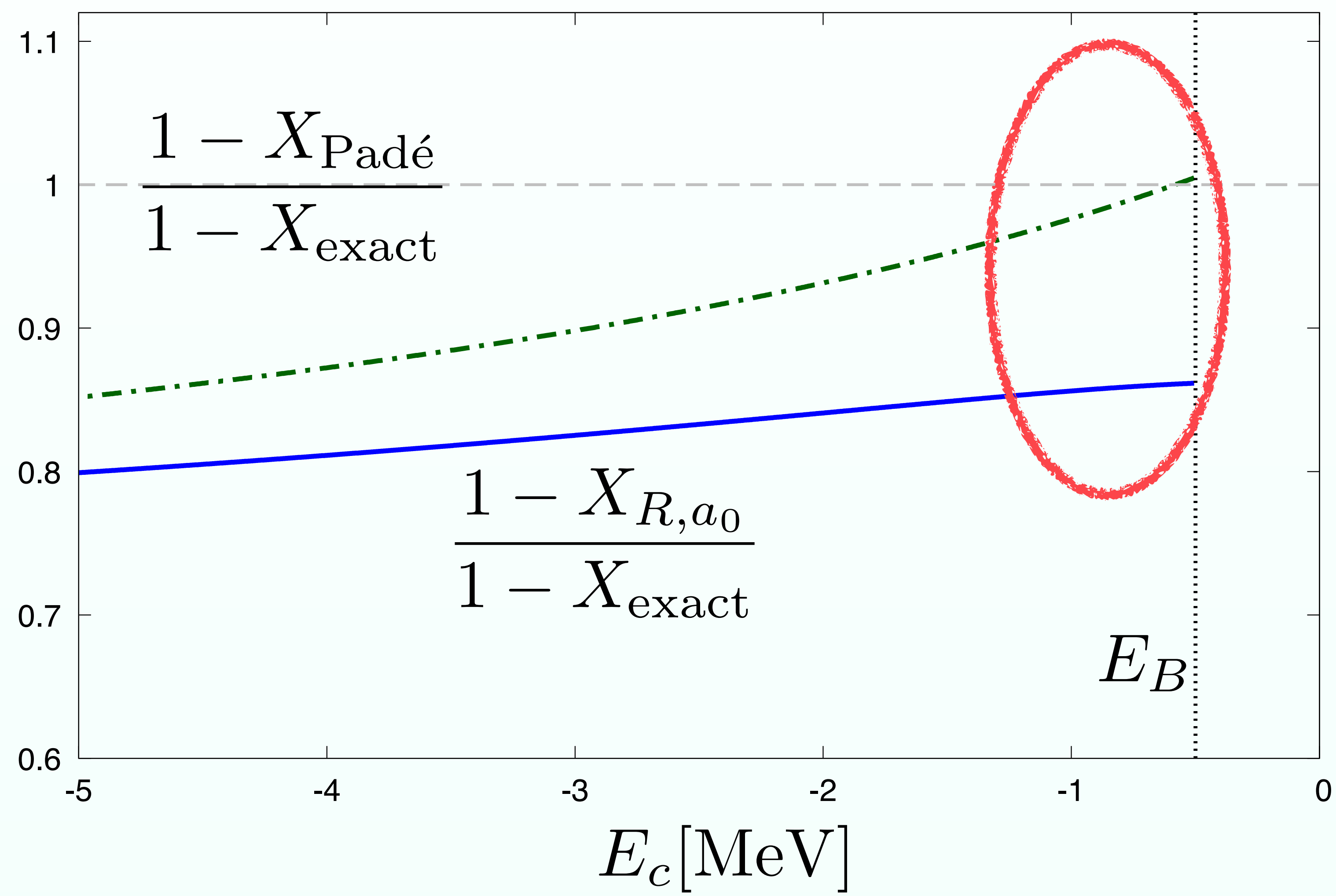
original relation

$$X_{R,a_0} = \frac{a_0}{2R - a_0}$$

extended relation

$$X_{\mathrm{Padè}} = \left[1 - \frac{4R(a_0 - R)^2}{a_0^2 r_e}\right]^{-1}$$





The estimation of the compositeness is improved, when the CDD pole lies near the threshold.

Conclusions

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Y. Kamiya and T. Hyodo, Phys. Rev. C. 93.035203, arXiv:1509.00146 [hep-ph].

Y. Kamiya, T. Hyodo, arXiv:1607.01899 [hep-ph].

Part I

- We extend the weak-binding relation to quasi-bound states and we propose an interpretation of complex X introducing real quantities $\tilde{X}$ and U.
- We apply the method to hadrons and discuss the internal structures.

→ $\Lambda(1405)$: $\bar{K}N$ composite dominance

$a_0(980)$: not $K\bar{K}$ dominance

Part II

- With the Pade approximant, we take into account the contribution of the near-threshold CDD pole and derive the extended weak-binding relation.

- We numerically examine the validity of the estimation with the generalized weak-binding relation.