Three-nucleon reactions with chiral forces

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Plan:

1. Formalism and calculational scheme
2. Results for nd reactions – low energies and standard NN potentials
3. Results for nd reactions – higher energies and standard NN potentials
4. Relativistic effects
5. Chiral effective field theory potentials
6. Summary

- Semi-phenomenological models of nucleon-nucleon potentials (AV18, CDBonn,NijmI and II) provide very good descritpion of 2-nucleon bound state (deuteron) and very good description of NN scattering data
- Bound state of three nucleons (^3H, ^3He) calculated with NN interactions only is underbound → **IMPORTANT IMFORMATION**: three-nucleon Hamiltonian *H* must contain also a three-nucleon potential – a term which cannot be reduced to interactions of pairs of nucleons :

$$H = H_0 + V_{12} + V_{23} + V_{31} + V_4$$

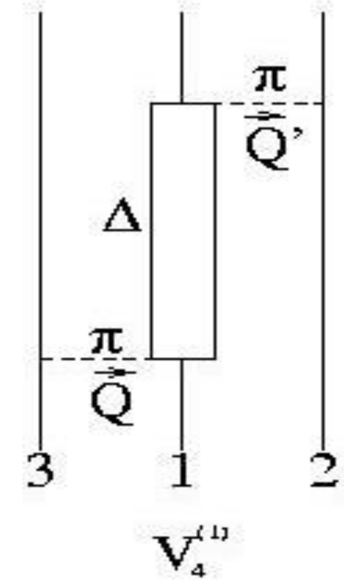
$$V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)}$$

splitting of 3N potential to three parts with proper symmetry

- **Can we describe the data in 3N continuum with such three-nucleon Hamiltonian ?**

First model of three-nucleon force (3NF) came from Japan:

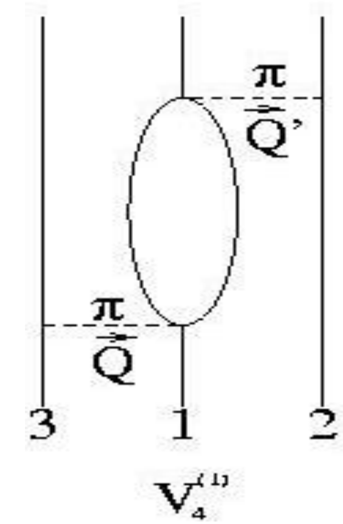
J.Fujita and H.Miyazawa, Prog. Theor. Phys. 17, 360 (1957)).



Other important 3N potential models:

- Model Tucson-Melbourne based on pion-nucleon scattering amplitude

(S.A.Coon, W.Glöckle, PRC23, 1790 (1981))



- Urbana IX 3NF model is based on 2π-exchange with Δ-resonance exctitation in intermediate state; there is also spin- and isospin-independent short-range term (B.S.Pudliner et al., PRC56, 1720 (1997))

What equations must be solved ?

Theoretical description of 3N continuum (elastic nucleon-deuteron scattering and the deuteron breakup reaction) requires solution of the following Faddeev-type equation for T|ϕ_d> state:

$$T|\phi_d\rangle = tP|\phi_d\rangle + (1+tG_0)V_4^{(1)}(1+P)|\phi_d\rangle$$
$$+ tPG_0T|\phi_d\rangle + (1+tG_0)V_4^{(1)}(1+P)G_0T|\phi_d\rangle,$$

Free 3N propagator

2N transition operator generated from L-S equation

$$t = V_{23} + V_{23}G_0^{2N}t$$

Part of V_4 symmetrical with respect to exchange of nucleons 2 and 3

$|\phi_d\rangle \equiv |\varphi_d\rangle|\vec{q}_0\rangle$ is composed of the deuteron internal wave function and the state of the relative nucleon-deuteron motion.

These equations can be solved numerically precisely for any 2N potential and 3N-force.

The transition amplitude for elastic scaterring is given by:

$$\left\langle \phi_d ' \right| U \left| \phi_d \right\rangle = \left\langle \phi_d ' \right| P G_0^{-1} \left| \phi_d \right\rangle + \left\langle \phi_d ' \right| V_4^{(1)} \left(1 + P\right) \left| \phi_d \right\rangle$$
$$+ \left\langle \phi_d ' \right| P T \left| \phi_d \right\rangle + \left\langle \phi_d ' \right| V_4^{(1)} \left(1 + P\right) G_o T \left| \phi_d \right\rangle$$

Transition amplitude for the deuteron breakup is given by:

$$\left\langle \phi_0 \right| U_0 \left| \phi_d \right\rangle = \left\langle \phi_0 \right| \left(1 + P\right) T \left| \phi_d \right\rangle$$

The singularities of operator G_0 and pole of operator t in calculations for positive energies, makes treatment of the 3N continuum much more difficult than of the 3N bound state!

Low energies (below about 30 MeV) and standard NN potentials (AV18, CDBonn, Nijm1 and 2)

ELSEVIER

Physics Reports 274 (1996) 107–285

PHYSICS REPORTS

The three-nucleon continuum: achievements, challenges and applications

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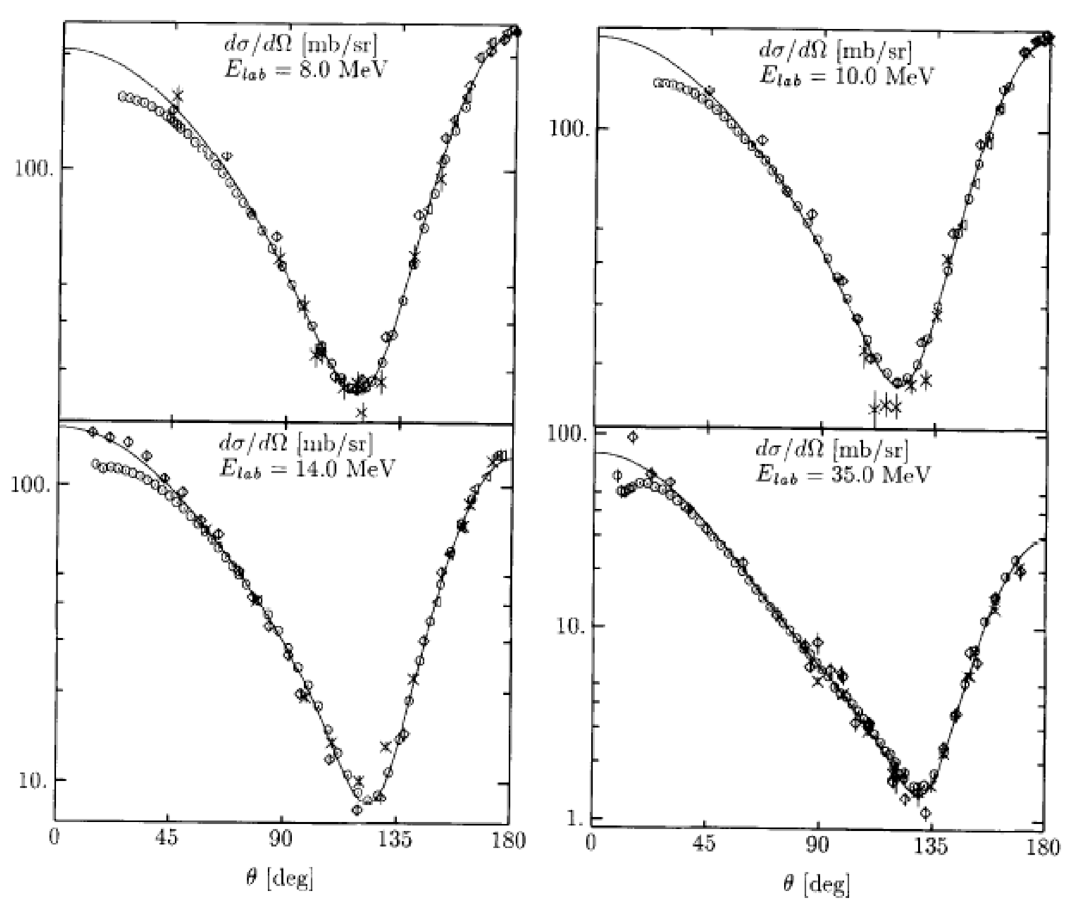


Fig. 7. Angular distributions for elastic Nd scattering. Comparison of pd and nd data. 8 MeV: nd data (○) [431], (●) [252], (×) [436], (◁) [234]; pd data (○) [416]. 10 MeV: nd data (○) (10.3 MeV) [431], (×) [24], (◁) [234]; pd data (○) [416]. 14 MeV: nd data (×) [440], (◁) [234], (○) [47]; pd data (○) [416]. 35 MeV: nd data (×) [62], (○) (36 MeV) [411]; pd data (○) [71]. The solid curve is the AV18 NN force prediction for 8, 10 and 14 MeV and the Nijm I NN force prediction for 35 MeV.

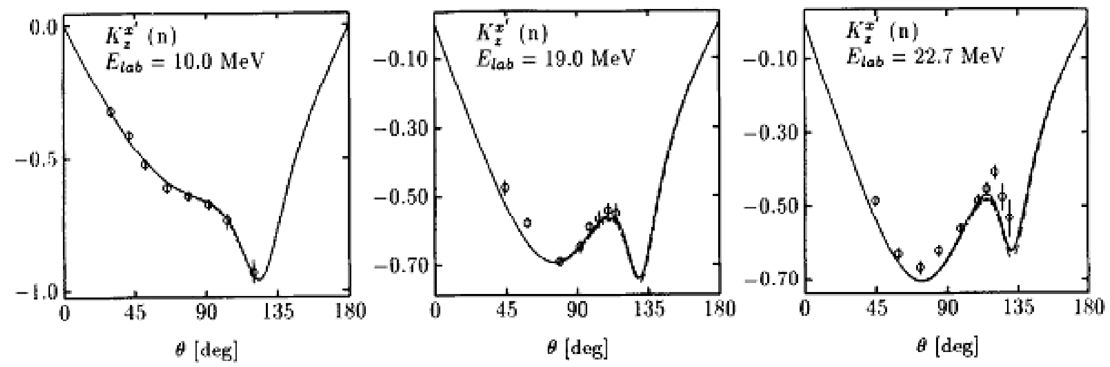


Fig. 22. The nucleon to nucleon spin transfer coefficient $K_z^{z'}$ for elastic Nd scattering. Comparison of data with NN force predictions. 10 MeV: pd data (○) [457]. 19 MeV: pd data (○) [474]. 22.7 MeV: pd data (○) [300,301,186]. The NN force predictions are (——) Nijm I, (– – –) Nijm II, (- - - - -) Nijm 93, and (······) AV18.

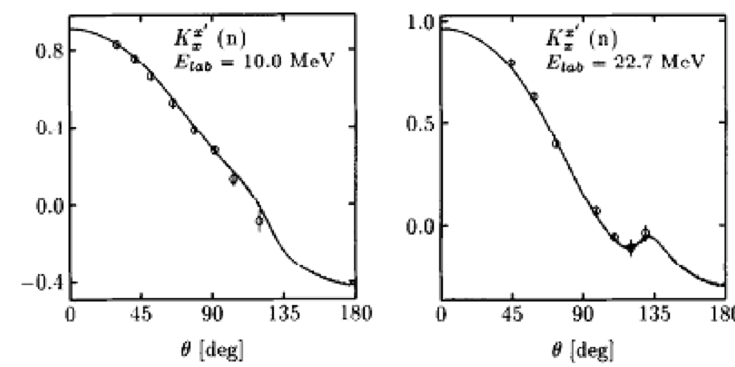
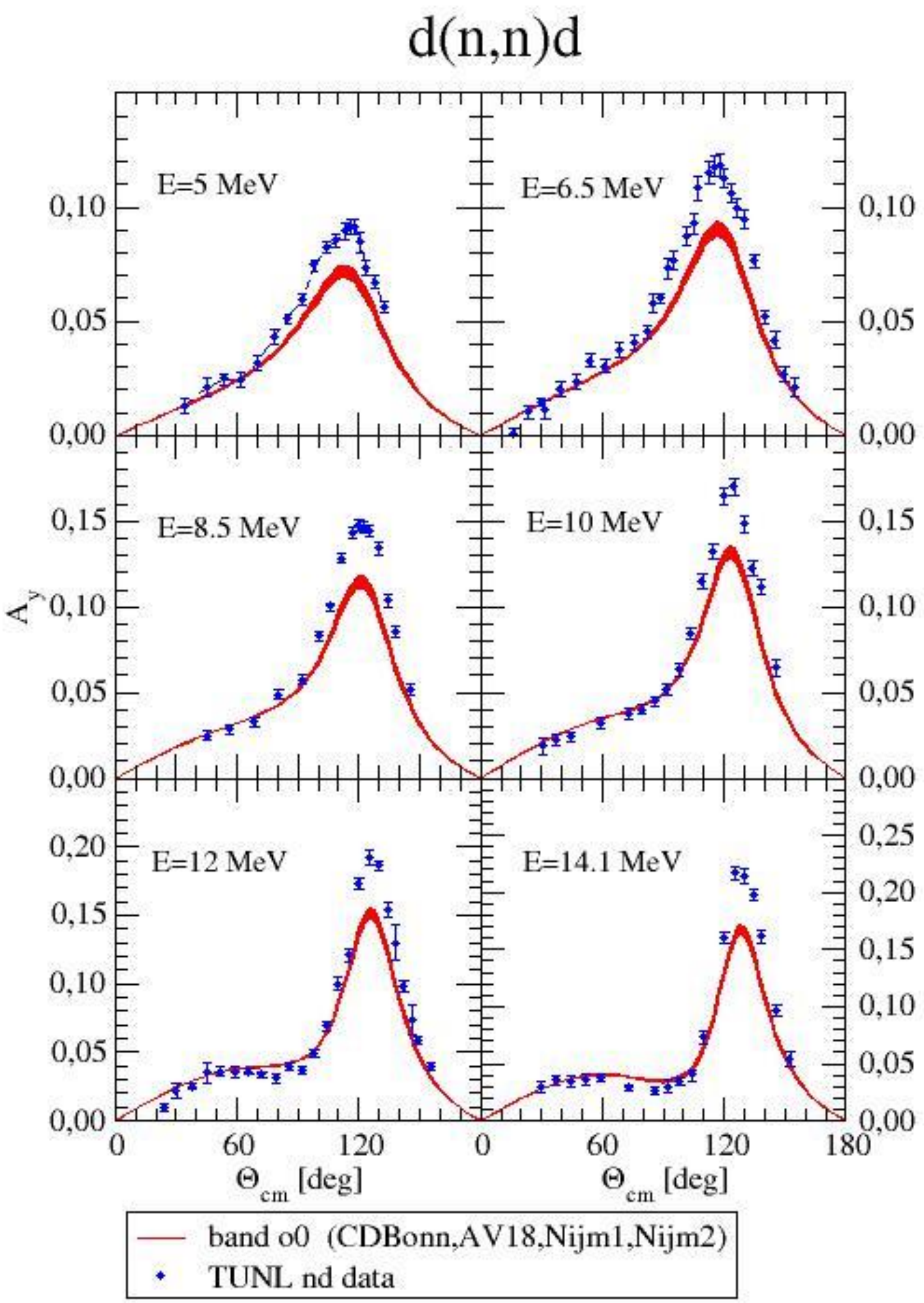
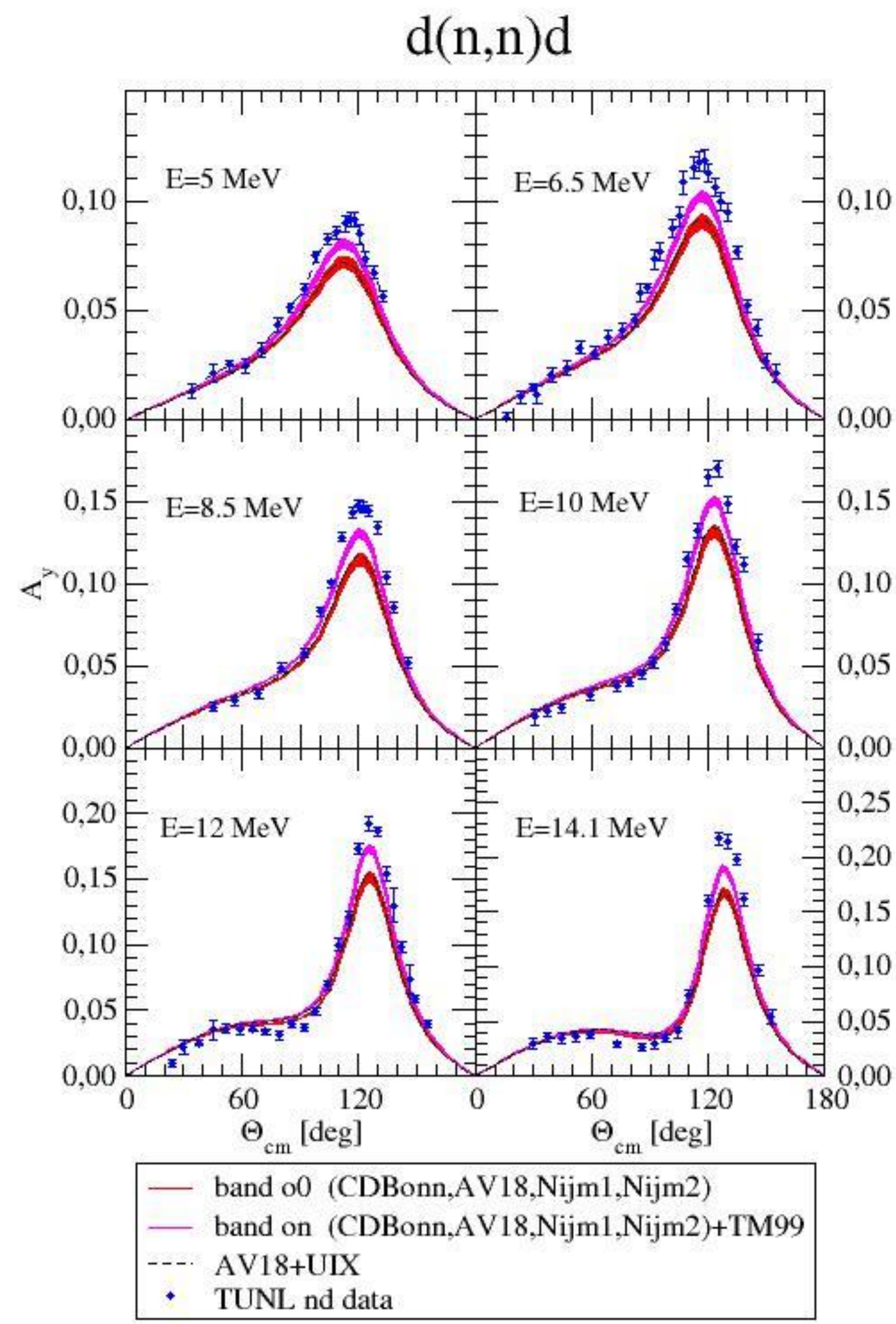


Fig. 23. The nucleon to nucleon spin transfer coefficient $K_x^{z'}$ for elastic Nd scattering. Comparison of data with NN force predictions. 10 MeV: pd data (○) [457]. 22.7 MeV: pd data (○) [300,301,186]. The NN force predictions are (——) Nijm I, (– – –) Nijm II, (- - - - -) Nijm 93, and (······) AV18.

Quite good descritption of low energy data (below about 30 MeV): it seems that at these energies one does not need a 3NF !

Exception: A_y puzzle





Remark: quite different effects of TM99 and UIX 3NF's (in spite of the fact that both are of 2π-exchange origin. That can imply that consistency in derivation of NN- and 3N-forces is important.

Higher energies: large discrepancies between data and theory based on NN forces only, starts to appear

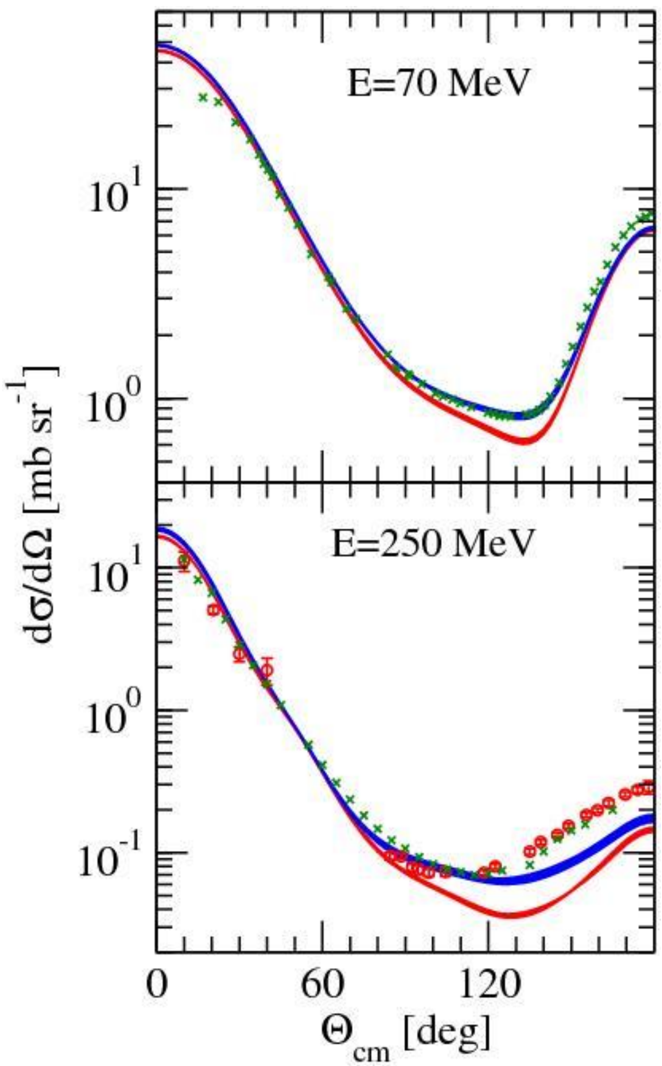
Elastic scattering d(p,p)d

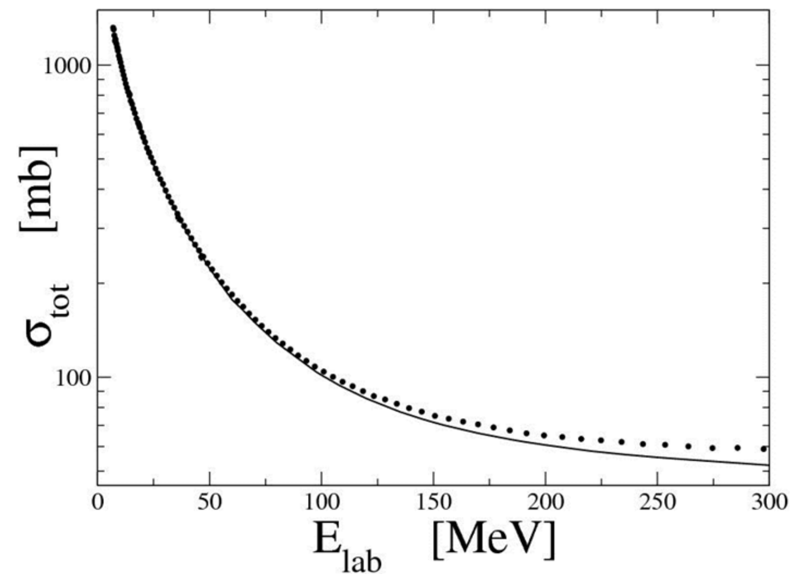
NN only (AV18, CD Bonn, Nijm1, Nijm2)

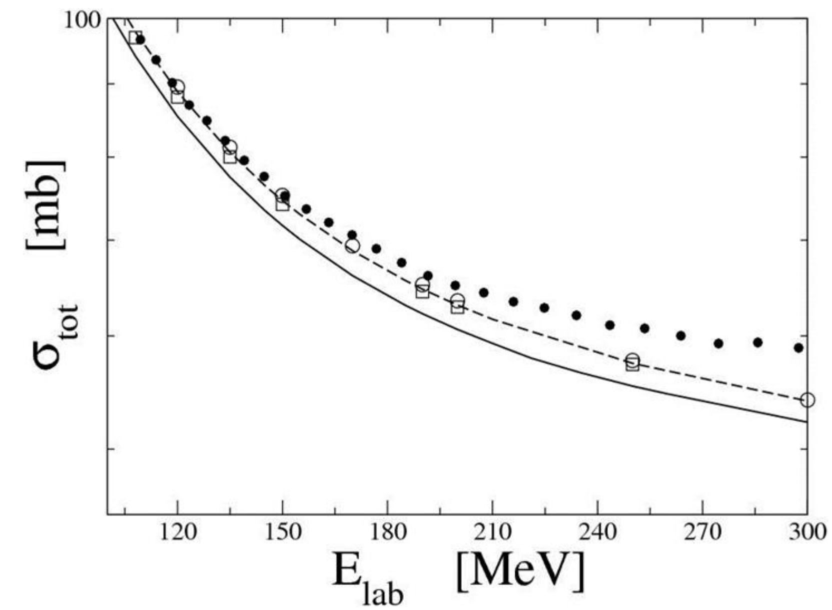
NN+3NF TM99

Total nd cross section: (W.P.Abfalterer et al. PRL 81(1998)57)

- up to ~ 50 MeV good agreement with predictions based on 2N forces
- adding 3NF provides explanation of the disagreement up to ~ 150 MeV
- at even larger energies a clear disagreement which increases with energy







data 70: K.Sekiguchi et al., PR C65, 034003 (2002)

data 250:
x nd – Y.Maeda et al., PR C76, 014004 (2007)
o pd – K.Hatanaka et al., PR C 66, 044002 (2002)

- **what is responsible for large differences between theory and data in 250 MeV cross section and in the total nd cross section even after inclusion of 2π-exchange 3NF ?**

- **it is evident, that in the applied dynamics something is wrong or missing**

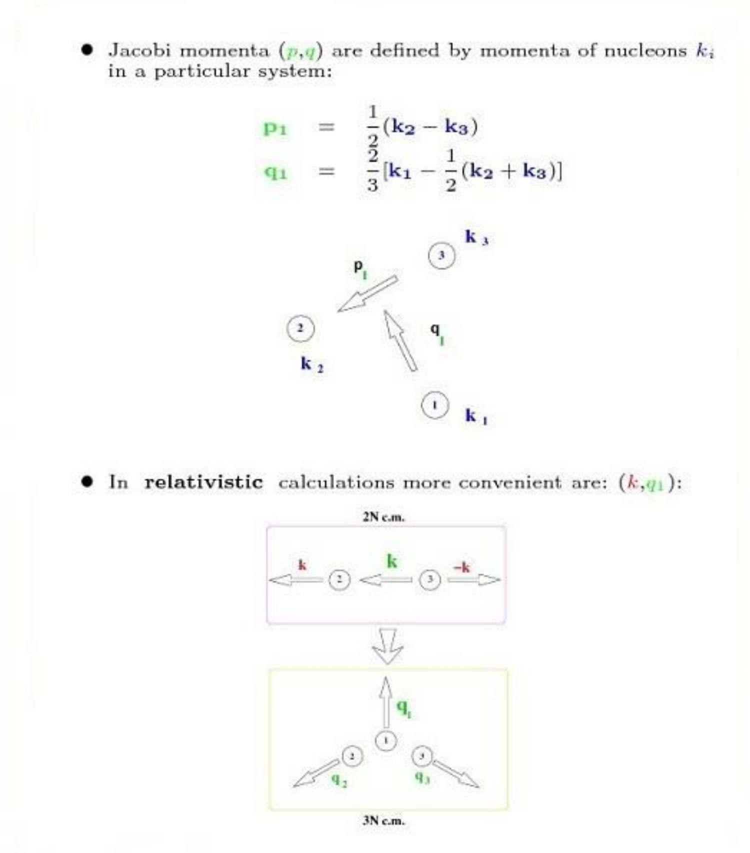
- **one possibility is that omitted short-range components of 3NF become more and more important with increasing energy**

- **however, increasing energy means also a transition to a region, where relativity could be important → relativistic Faddeev calculations**

Relativistic Faddeev calculations

(PR C71, 054001 (2005),
PR C77, 034004 (2008),
PR C83, 044001 (2011))

The formal structure of the equations remains the same but the ingredients change



- Form of the free Hamiltonian H_0 (and G_0) changes:

$$H_0 = \sqrt{\left[2\sqrt{m^2 + \vec{k}^2}\right]^2 + \vec{q}^2} + \sqrt{m^2 + \vec{q}^2}$$

- Interacting 2N subsystem (2-3) has nonzero total momentum **-q** in the 3N c.m. system, what leads to the boosted potential V:

$$V(\vec{q}) \equiv \sqrt{\left[2\sqrt{m^2 + \vec{k}^2} + v\right]^2 + \vec{q}^2} - \sqrt{\left[2\sqrt{m^2 + \vec{k}^2}\right]^2 + \vec{q}^2}$$

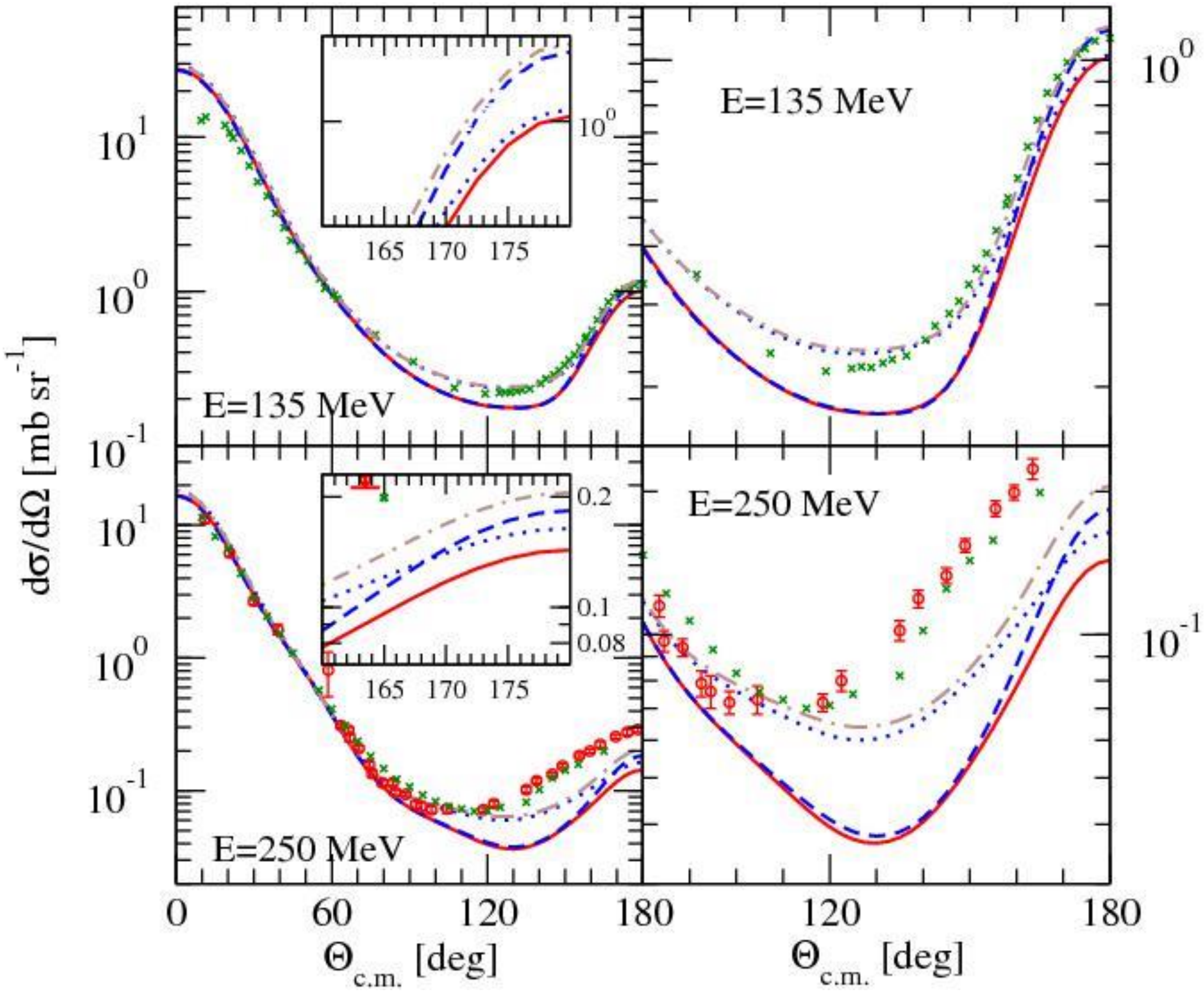
- V(**q**=0) reduces to the relativistic potential **v** defined in the 2N c.m. system. From V(**q**) the boosted t-matrix is obtained

- The Lorentz transformation from 2N to 3N c.m. is performed along the total momentum of the 2N subsystem, which in general is not parallel to momenta of these nucleons. This leads to **Wigner rotation** of spin states. When defining 3N partial wave states care must be taken about spin states

Faddeev approach with both 3NF and relativity included

elastic scattering: d(p,p)d

data 135 - K.Sekiguchi et al., Phys. Rev. Lett. 95, 162301 (2005)



only NN forces (CDBonn):

nrel solid red __

rel dashed blue ---

NN + 3NF (TM99):

nrel dotted blue …

rel dashed-dotted brown -.-

data 250:
x nd – Y.Maeda et al.,
PR C76, 014004 (2007)

o pd – K.Hatanaka et al.,
PR C 66, 044002 (2002)

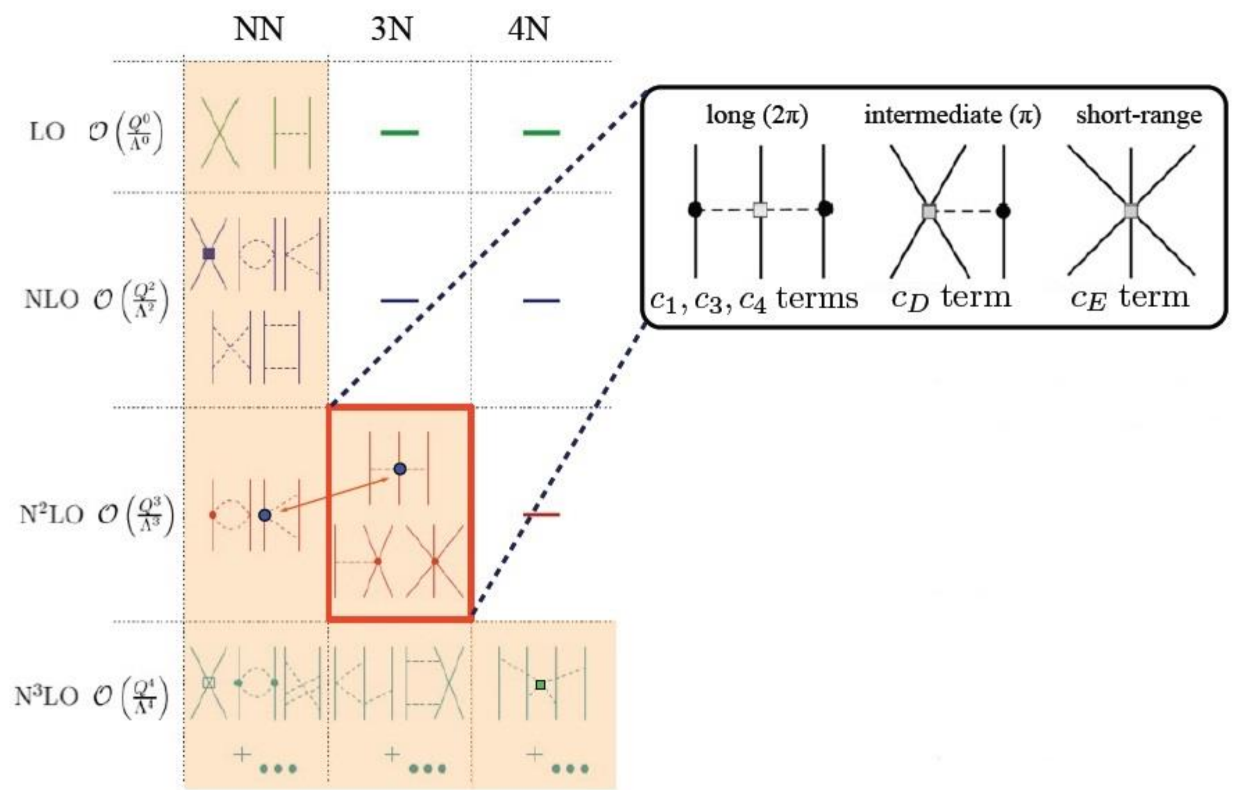
- **effects of relativity seen only in Nd elastic scattering backward cross section !**
- **relativistic effects are not responsible for large discrepancies in elastic Nd scattering cross section**
- **interplay of relativity and 3NF's leads to a slight increase of the cross section at angles larger than θ_{cm} ~ 100^o**

It follows that:

- **relativistic effects are not responsible for large discrepancies in elastic Nd scattering**

- **those discrepancies must come from neglection of short-range 3NF components which become active at higher energies**

Challenge: apply NN and 3NF's derived consistently in the framework of chiral perturbation theory



Chiral EFT for nuclear forces

perturbation in $(Q/\Lambda)^\nu$

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LENPIC (Low Energy Nuclear Physics International Collaboration): to understand nuclear structure and reactions with chiral forces

Few remarks on chiral forces:

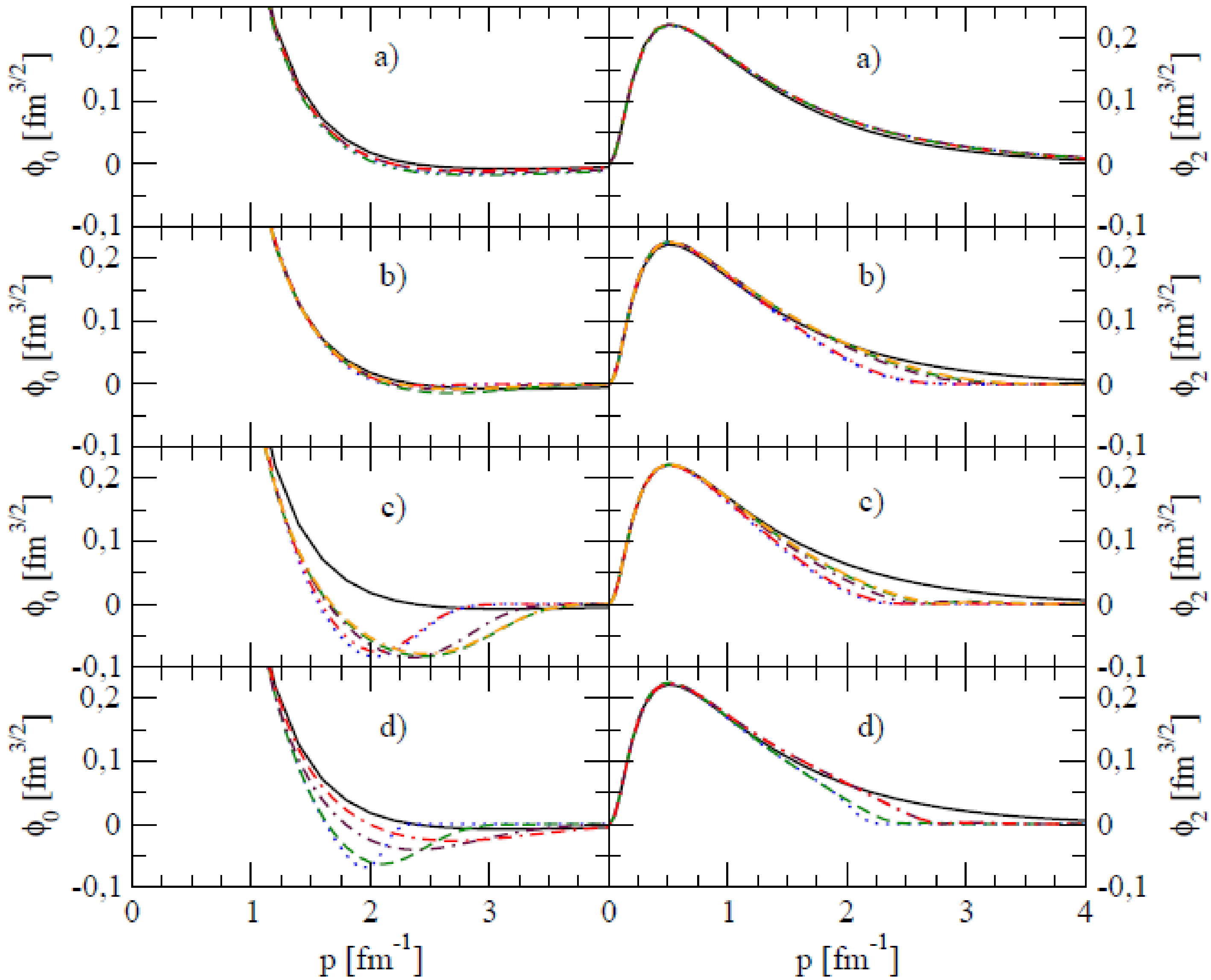
- In order to reproduce properly 2N data up to about 250 MeV N3LO order of chiral expansion is required
  - About 2 years ago: NN interaction up to N3LO, 3NF interaction up to N4LO (N3LO used in preliminary calculations at low energies)
  - nonlocal momentum space regularization has been applied:

  V → f(p',Λ) V(p',p) f(p,Λ) with $f(p,\Lambda) \equiv e^{-p^6/\Lambda^6}$

  V^4 → f(p',q',Λ) V^4(p',q',p',q) f(p,q,Λ) with $f(p,q,\Lambda) \equiv e^{-(p^2+0.75q^2)^3/\Lambda^6}$

  what leads to finite cut-off artefacts (problems when applied to higher energy Nd scattering)

- New, improved chiral force, presented by Bochum-Bonn group in 2014:
  - E. Epelbaum, H. Krebs, U.-G. Meißner, Eur. Phys. J. A51 (2015) 3,26 – up to N3LO
  - E. Epelbaum, H. Krebs, U.-G. Meißner arXiv:1412.4623 [nucl-th] – up to N4LO
  - Local regularization in the coordinate space $V_{lr}(\mathbf{r})$ → $V_{lr}(\mathbf{r})f(\mathbf{r})$ with $f(\vec{r}) \equiv \left(1 - e^{-r^2/R^2}\right)^n$
  - R=0.8–1.2 fm what corresponds to Λ=330-500 MeV
  - Such regularization preserves more long-range OPE and TPE physics
  - All LECs in the long-range part are taken from pion-nucleon scattering without fine tuning
  - Very good description of the deuteron properties, phase shifts etc.



Standard NN pot.

N^2LO - Bochum

N^3LO - Bochum

N^3LO - Idaho

FIG. 9: (color online) The momentum space deuteron wave function for different NN potentials. The S- and D-components (ϕ_0 and ϕ_2, respectively) are shown in the left and right columns, respectively. In a) wave functions for standard NN potentials are shown by different lines: AV18 - dotted (blue), CD Bonn - solid (black), Nijm 93 - dashed (maroon), Nijm I - dashed-dotted (red), and Nijm II - dashed-double-dotted (green). In b) and c) wave functions for the Bochum N^2LO and N^3LO NN potentials with different cut-off parameters from Table I are shown, respectively: (450,500) - dotted (blue) line, (600,500) - dashed (green) line, (550,600) - dashed-dotted (maroon) line, (450,700) - dashed-double-dotted (red) line, (600,700) - double-dashed-dotted (orange) line. The Idaho N^3LO wave functions for different cut-off parameters from Table II are shown in d): 414 - dotted (blue) line, 450 - dashed (green) line, 500 - dashed-dotted (maroon) line, 600 - dashed-double-dotted (red) line. For comparison in b), c) and d) also the CD Bonn wave function is shown by solid (black) line.

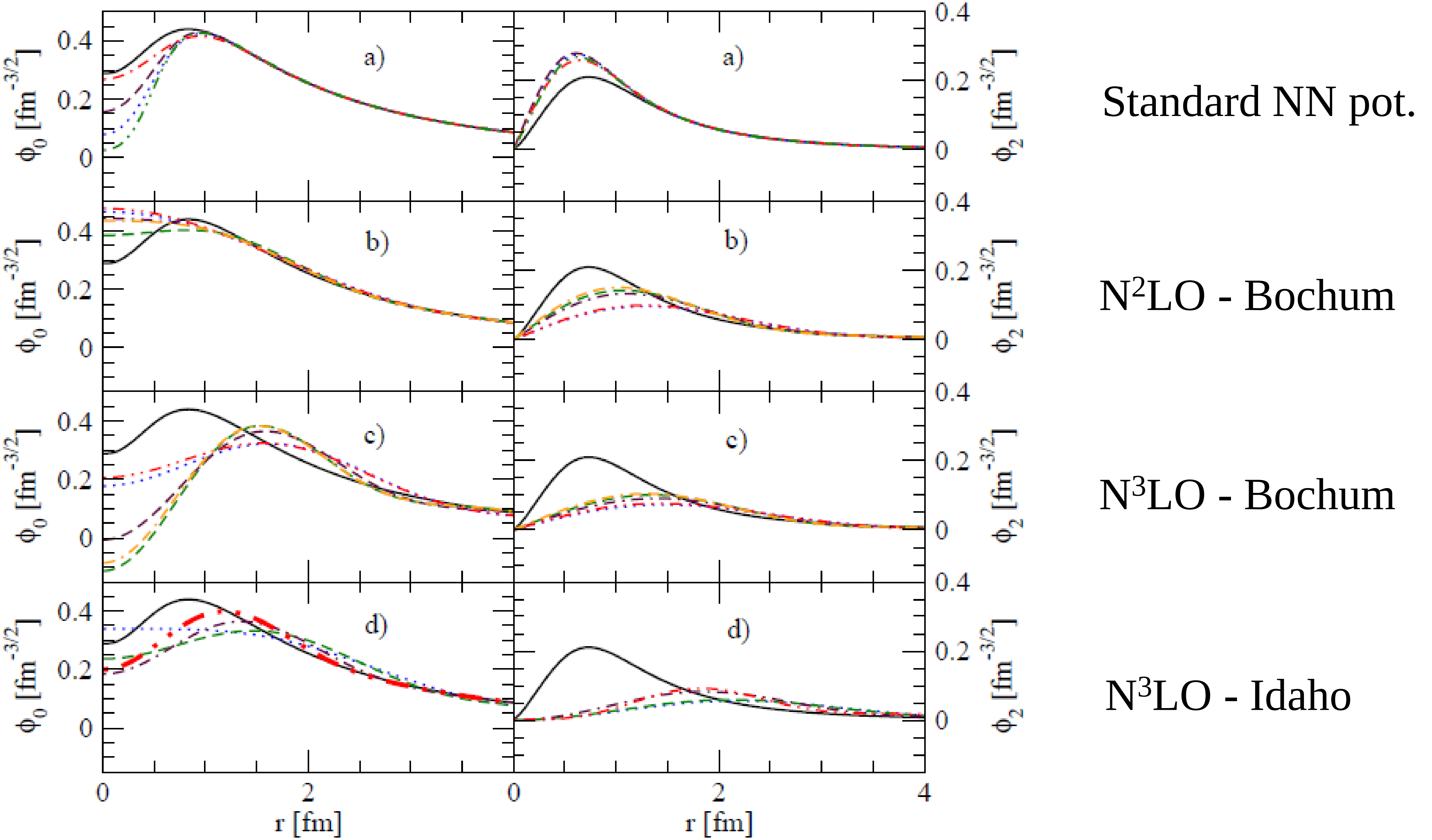
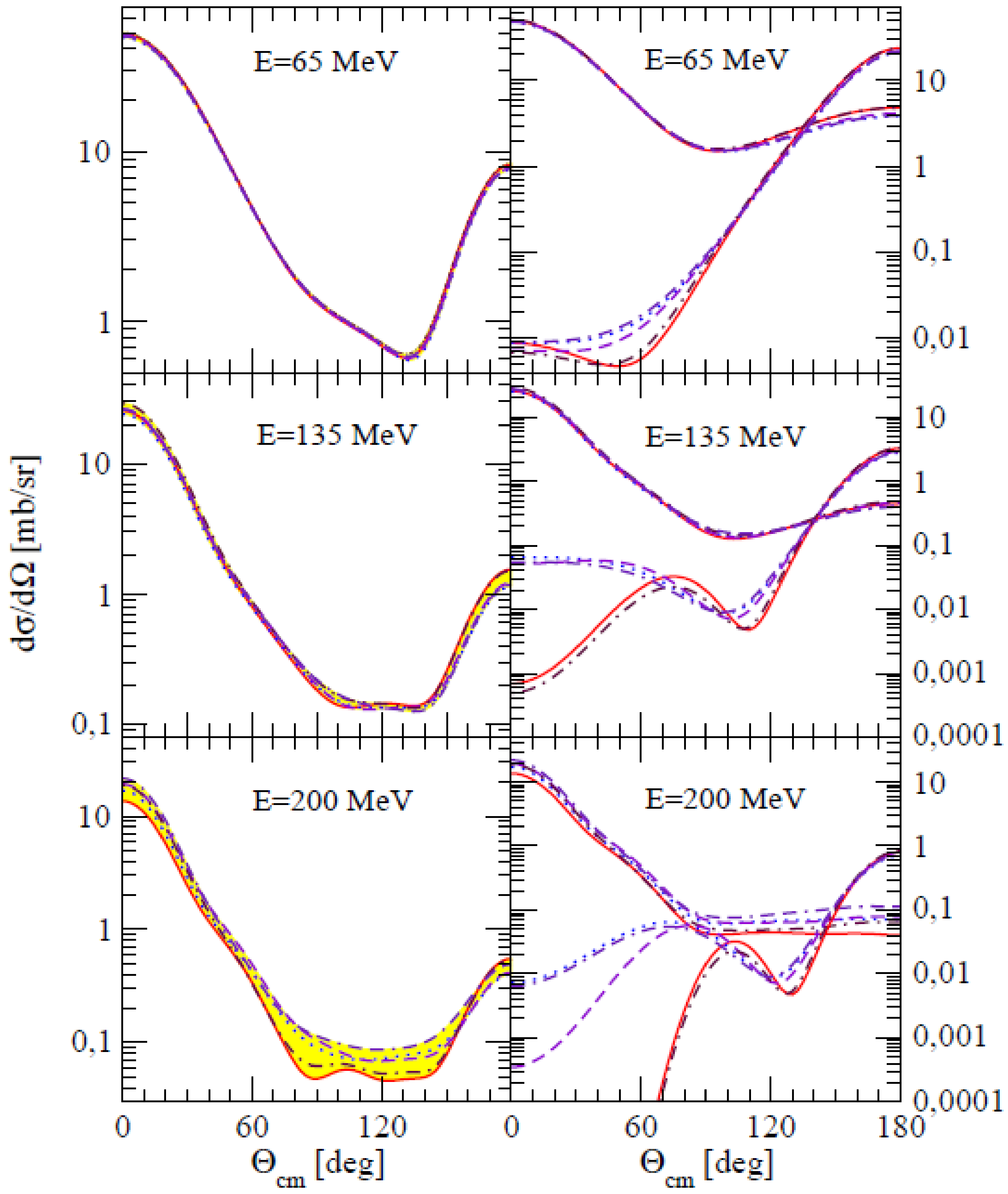


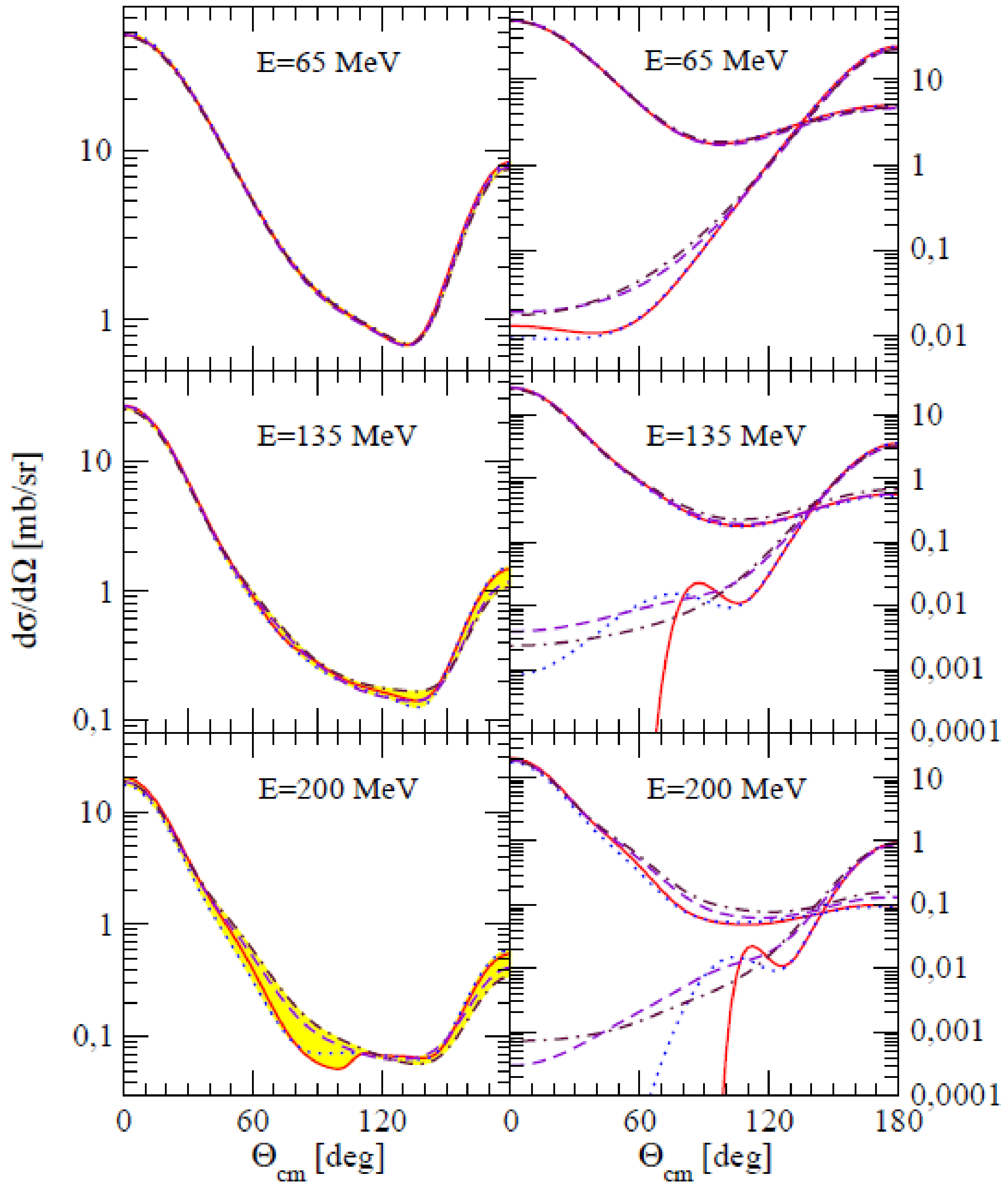
FIG. 10: (color online) The coordinate space deuteron wave function for different NN potentials. For explanation of lines see Fig.9.



N^3LO Bochum

$$U = PT + PG_0^{-1}$$

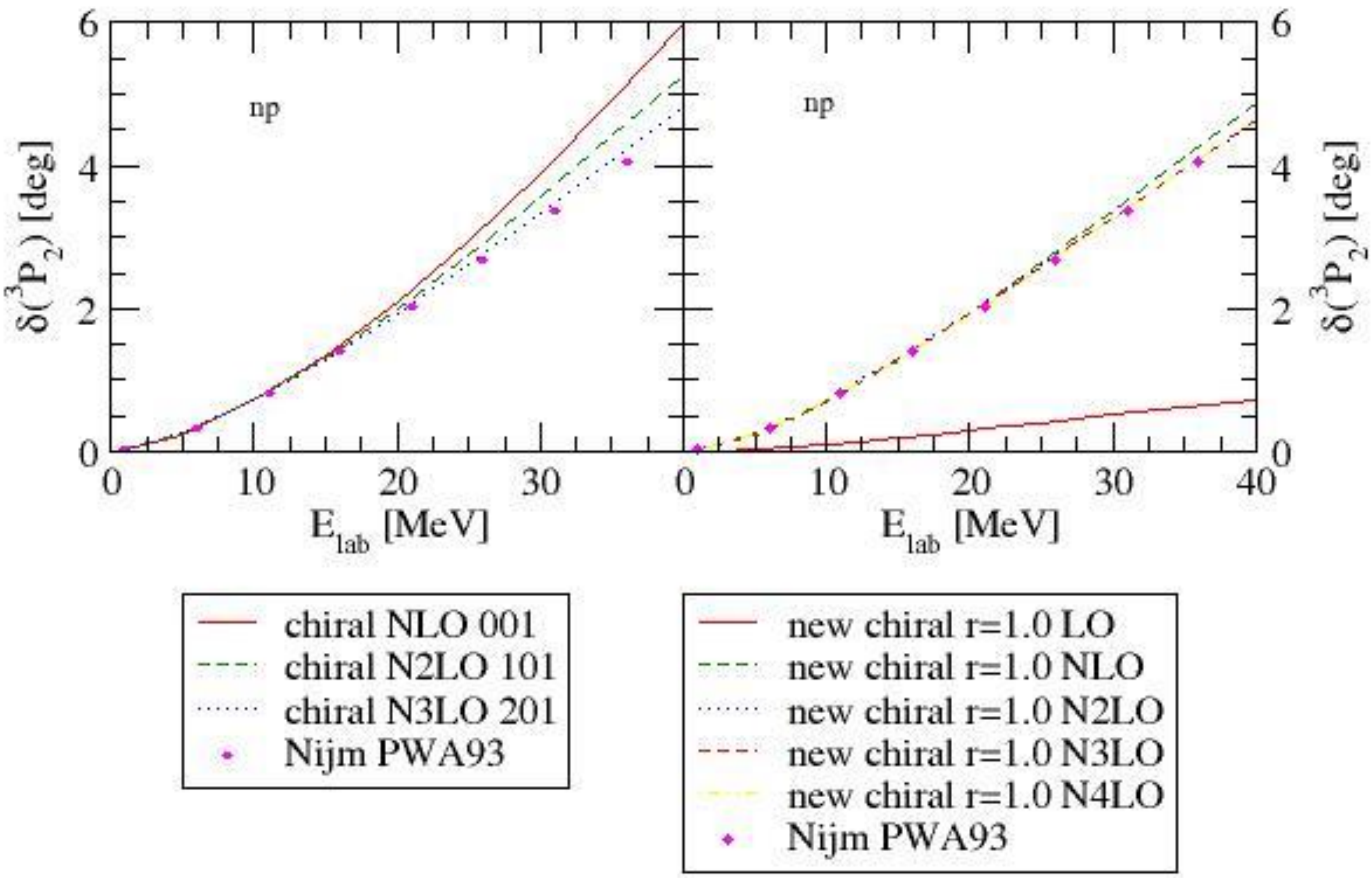
FIG. 12: (color online) The same as in Fig.11 but for the Bochum N^3LO NN potentials with different cut-off values of Table I. See Fig.11 for the description of lines and bands.



N³LO Idaho

FIG. 13: (color online) The nd elastic scattering angular distributions at 65 MeV, 135 MeV and 200 MeV of incoming neutron lab. energy calculated with the Idaho N^3LO NN potentials for different cut-off values of Table II. In the left part the nd elastic scattering cross sections are shown. In the right part "cross sections" resulting only from the PT term in the elastic scattering transition amplitude are shown (lines peaked at forward angles) together with "cross sections" based on exchange-term PG_0^{-1} only (lines peaked at backward angles). Different lines correspond to different cut-off parameters from Table II: 414 - solid (red), 450 - dotted (blue), 500 - dashed (violet), 600 - dashed-dotted (maroon). In the left part light-shaded (yellow) band shows scatter of predictions for different cut-off values.

Nonlocal regularization Local regularization



Estimation of theoretical uncertainties (E.Epelbaum et al., arXiv:1412.4623 [nucl-th], arXiv:1505.07218 [nucl.th]):

X(p) - observable and $X^{(i)}, i = 0, 2, 3, \ldots$ prediction at order $Q^{(i)}$ in the chiral expansion

Order-$Q^{(i)}$ correction :

$$\Delta X^{(2)} = X^{(2)} - X^{(0)}$$

$$\Delta X^{(i)} = X^{(i)} - X^{(i-1)} \qquad i \geq 3$$

The chiral expansion for X takes the form:

$$X^{(i)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(i)}$$

Size of corrections is expected to be $\Delta X^{(i)} = O(Q^{(i)} X^{(0)})$

Quantitative estimates of the theoretical uncertainty $\delta X^{(i)}$ of the prediction $X^{(i)}$ is made using the expected and actual sizes of higher-order contributions:

$$\delta X^{(0)} = Q^2 \left| X^0 \right|$$

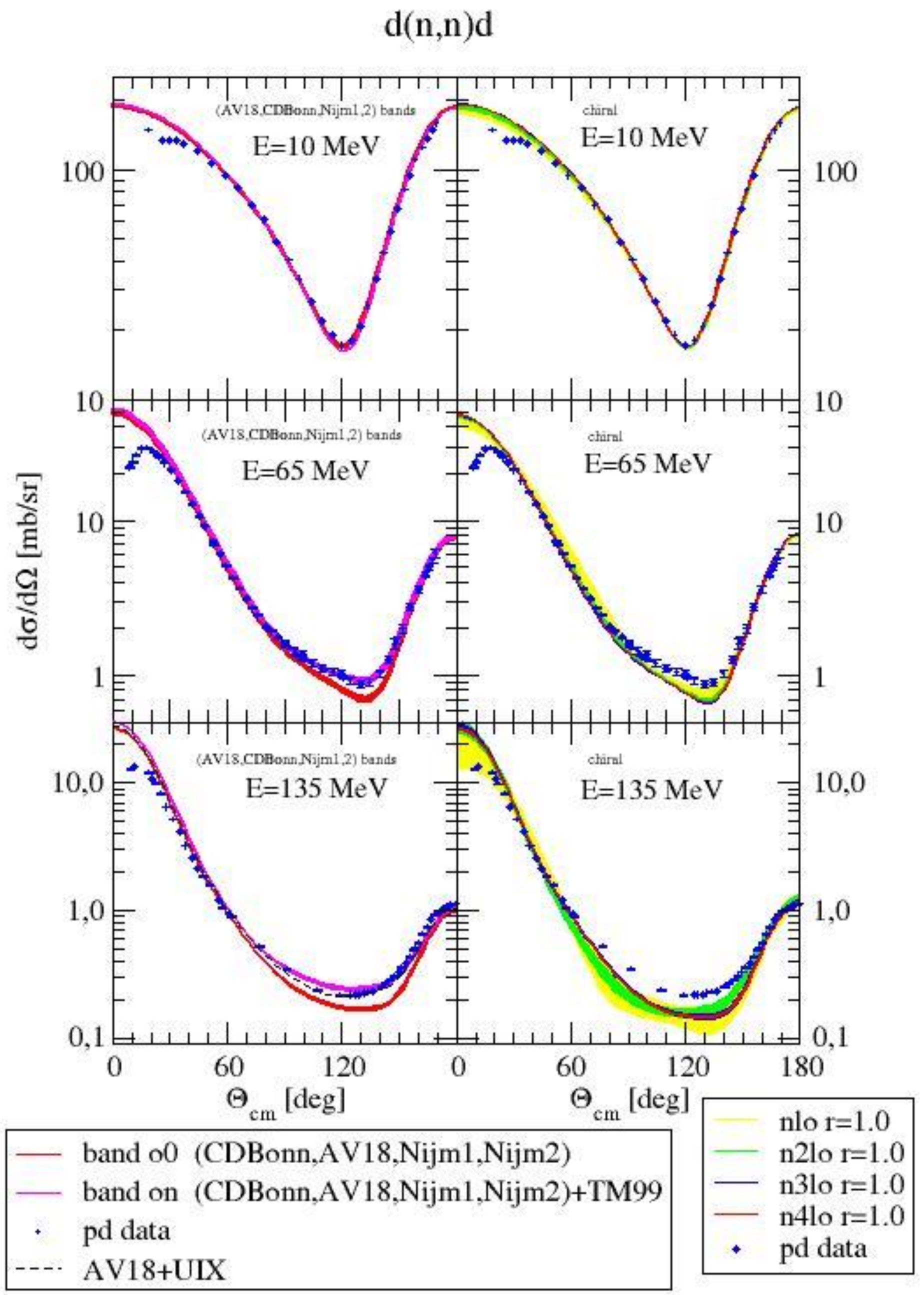
$$\delta X^{(2)} = \max\left(Q^3 \left| X^0 \right|, Q^1 \left| \Delta X^{(2)} \right|\right)$$

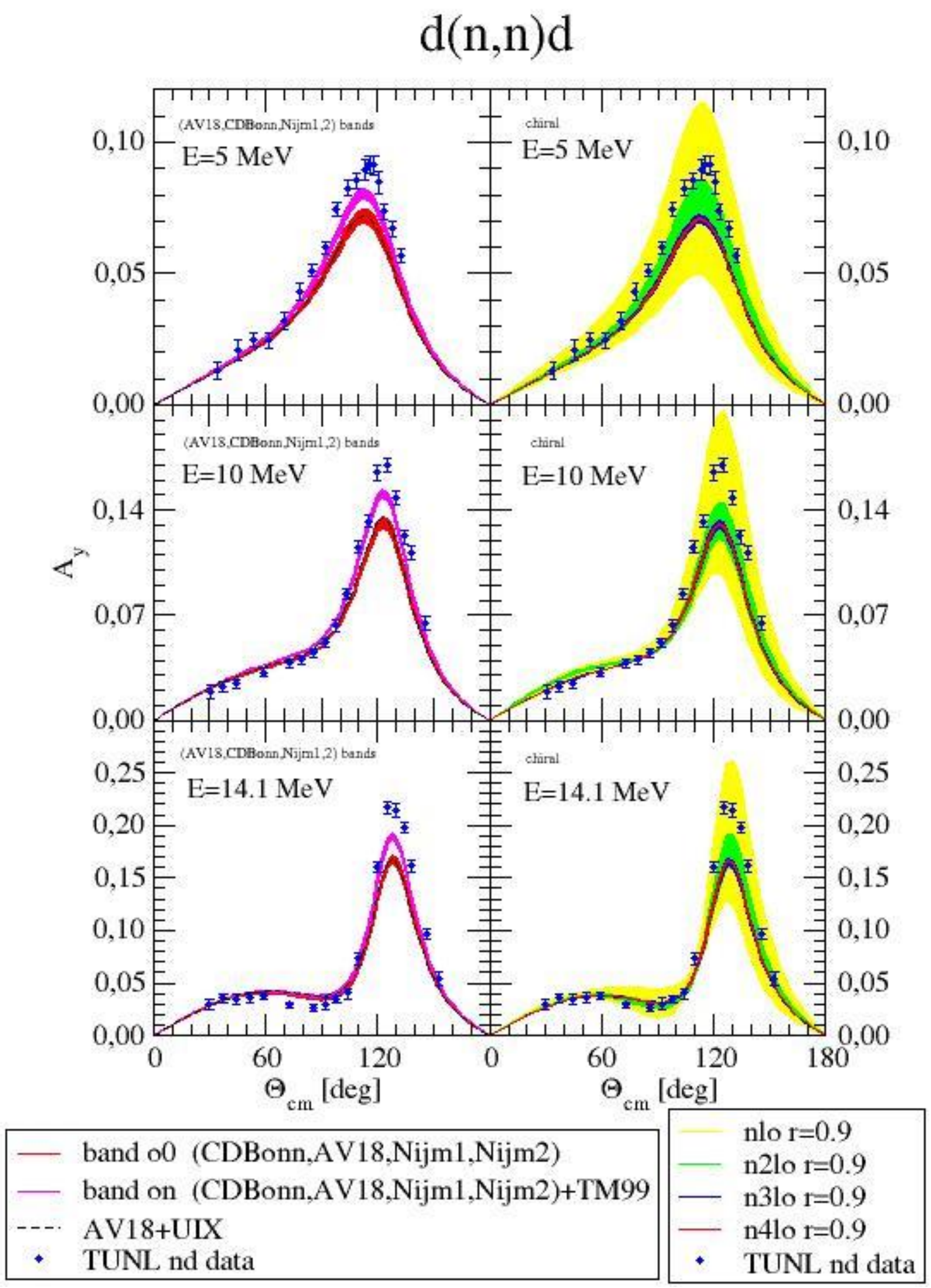
$$i \geq 3: \ \delta X^{(i)} = \max\left(Q^{i+1} \left| X^{(0)} \right|, Q^{i-1} \left| \Delta X^{(2)} \right|, Q^{i-2} \left| X^{(3)} \right|\right)$$

$$\delta X^{(2)} \geq Q \delta X^{(0)}, \qquad \delta X^{(i \geq 3)} \geq Q \delta X^{(i-1)}$$

$Q = \max(p/\Lambda_b, m_\pi / \Lambda_b)$ with $\Lambda_b = 600, 500$ and 400 MeV for regulator R=0.8-1.0 fm, R=1.1 fm and R=1.2 fm, respectively.

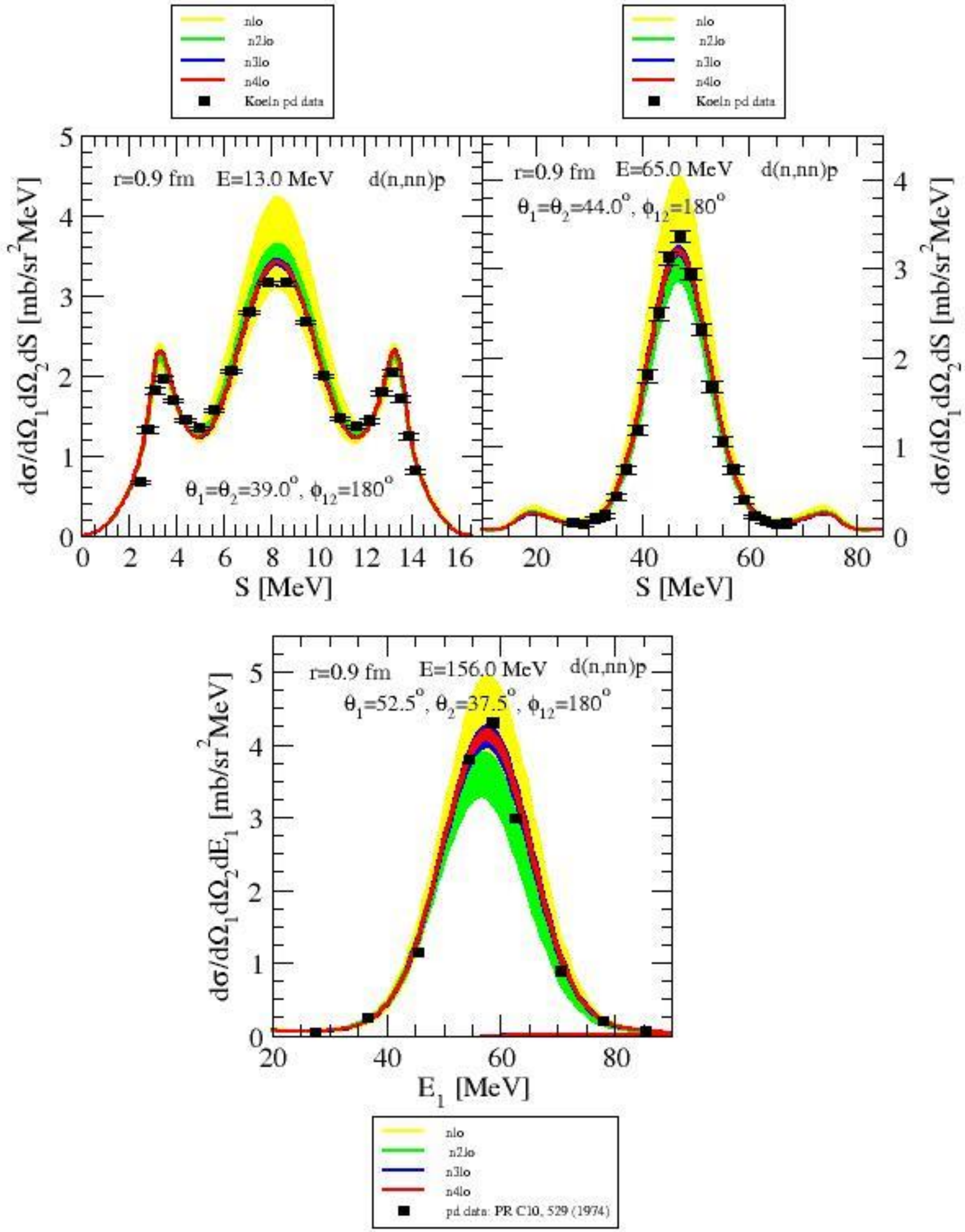
- NN developed up to N4LO: E.Epelbaum et al. arXiv:1412.4623 [nucl-th]
- Novel way of quantifying the theoetical uncertainty due to the truncation of the chiral expansion: E.Epelbaum et al. arXiv:1412.0142 [nucl-th]



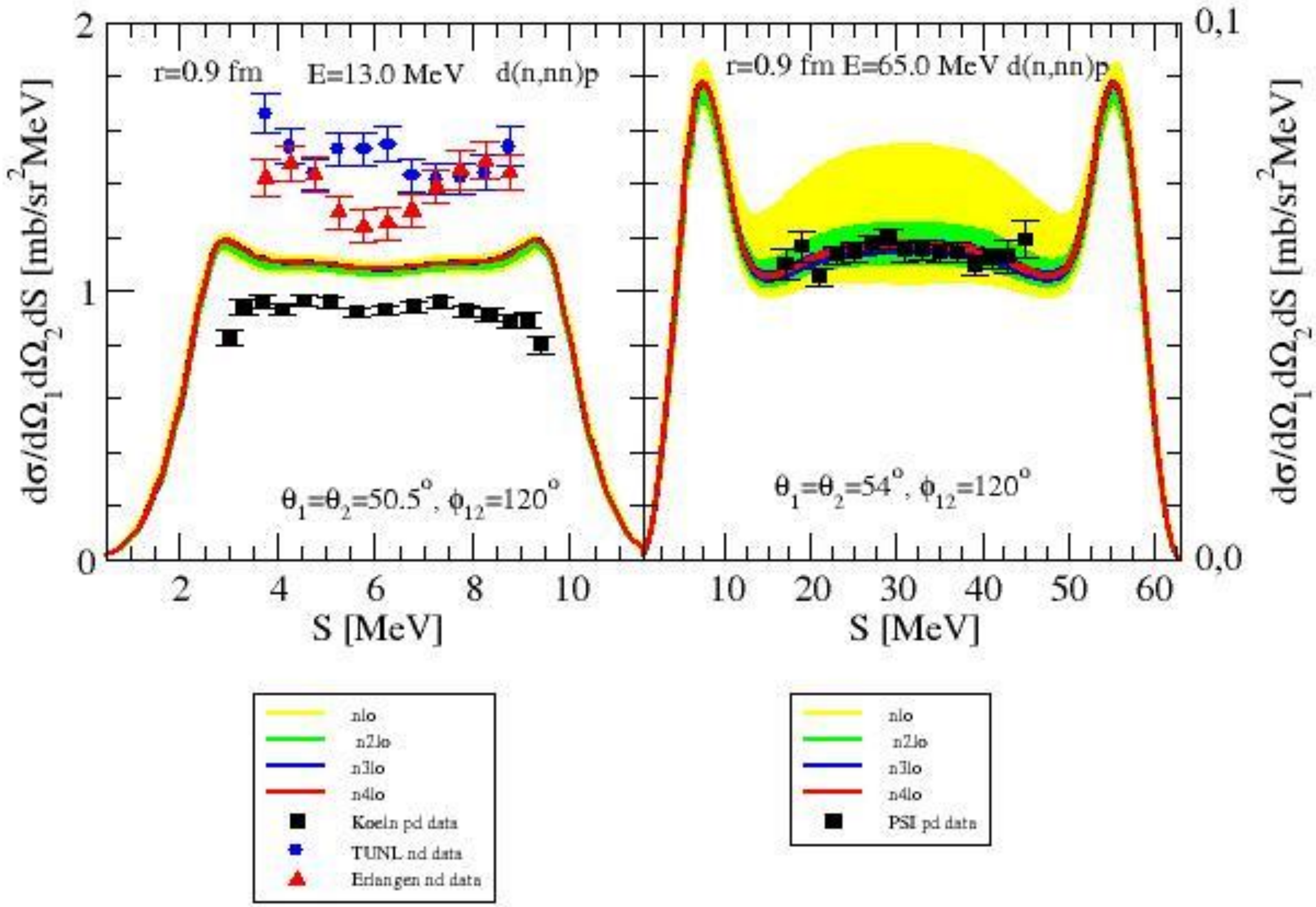


- Theoretical uncertainty grows with energy and decreases with increasing order: one thus expects precise predictions starting from N3LO
- For many observables the results at N2LO and higher orders differ from data well outside the range of quantified observables, thus providing a clear evidence for missing three-nucleon forces

Exclusive breakup reaction: quasi free pp scattering

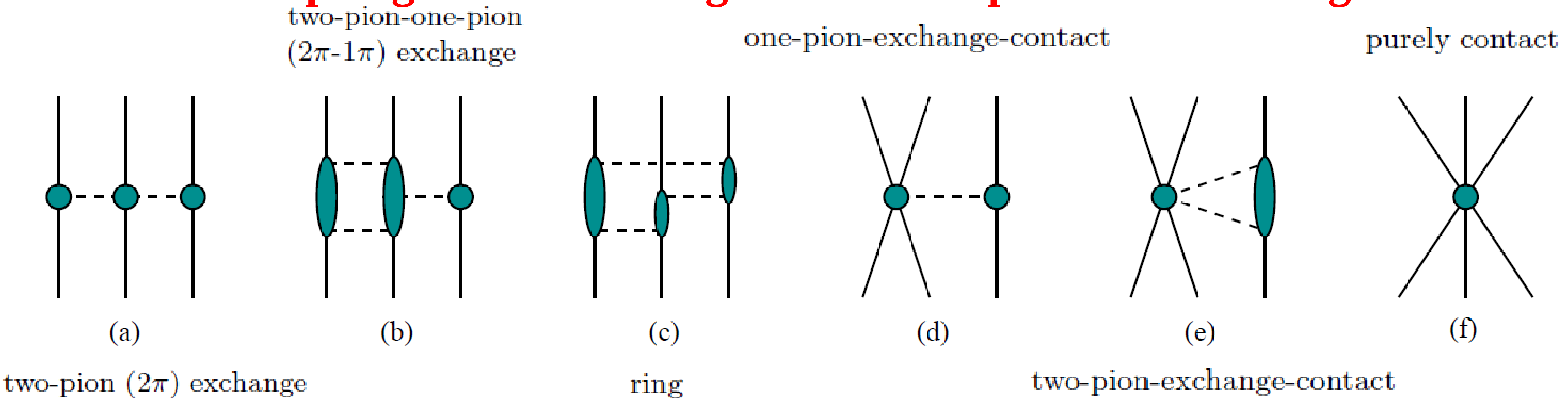


Exclusive breakup reaction: symmetric space-star configuration



Big challenge: application of full N3LO chiral force: NN + 3NF

Various topologies contributing to the 3NF up to and including N^4LO



- ❑ N^2LO: (a) + (d) + (f) (E.Epelbaum et al., PR C66, 064001 (2002))

- ❑ N^3LO: (a) + (b) + (c) + (d) + (e) + (f) + rel
  V.Bernard et al., PR C77, 064004 (2008) - long range contributions (a), (b), (c)
  V.Bernard et al., PR C84, 054001 (2011) - short range terms (e)
  and leading relativistic corrections

N^3LO contributions do not involve any unknown low energy constants !

The full N^3LO 3NF depends on two parameters c_D and c_E coming with (d) and (f) terms, respectively. They are adjusted to two chosen 3N observables.

- ❑ N^4LO (longest range contributions): (a) + (b) + (c) + (d) + (e) + (f) (H.Krebs et al., arXiv:1203.0067)

Chiral 3N potential in N^2LO order:

E.Epelbaum, Prog.Part.Nucl.Phys. 57, 654 (2006)

$$V_{123} = V_{2\pi}^{(3)} + V_{1\pi,cont}^{(3)} + V_{cont}^{(3)}$$

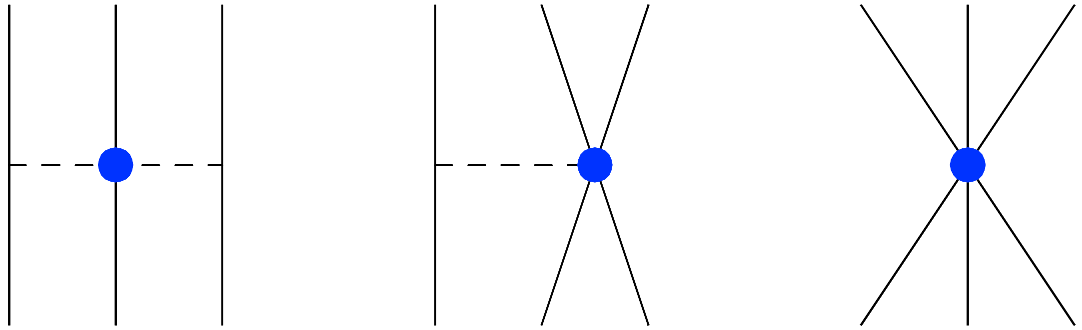
$$V_{2\pi}^{(3)} = \sum_{i\neq j\neq k} \frac{1}{2}\left(\frac{g_A}{2F_\pi}\right)^2 \frac{(\vec{\sigma}_i \circ \vec{q}_i)(\vec{\sigma}_j \circ \vec{q}_j)}{(\vec{q}_i^{\,2} + M_\pi^2)(\vec{q}_j^{\,2} + M_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

$$\vec{q}_i \equiv \vec{p}_i\,' - \vec{p}_i$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-\frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2}\vec{q}_i \circ \vec{q}_j\right] + \sum_\gamma \frac{c_4}{F_\pi^2}\varepsilon^{\alpha\beta\gamma}\tau_k^\gamma \vec{\sigma}_k \circ \left[\vec{q}_i \times \vec{q}_j\right]$$

$$V_{1\pi,cont}^{(3)} = -\sum_{i\neq j\neq k} \frac{g_A}{8F_\pi^2} D \frac{\vec{\sigma}_j \circ \vec{q}_j}{\vec{q}_j^{\,2} + M_\pi^2}\left(\vec{\tau}_i \circ \vec{\tau}_j\right)\left(\vec{\sigma}_i \circ \vec{q}_j\right)$$

$$V_{cont}^{(3)} = \frac{1}{2}\sum_{j\neq k} E\left(\vec{\tau}_j \circ \vec{\tau}_k\right)$$



Two free parameters : D i E

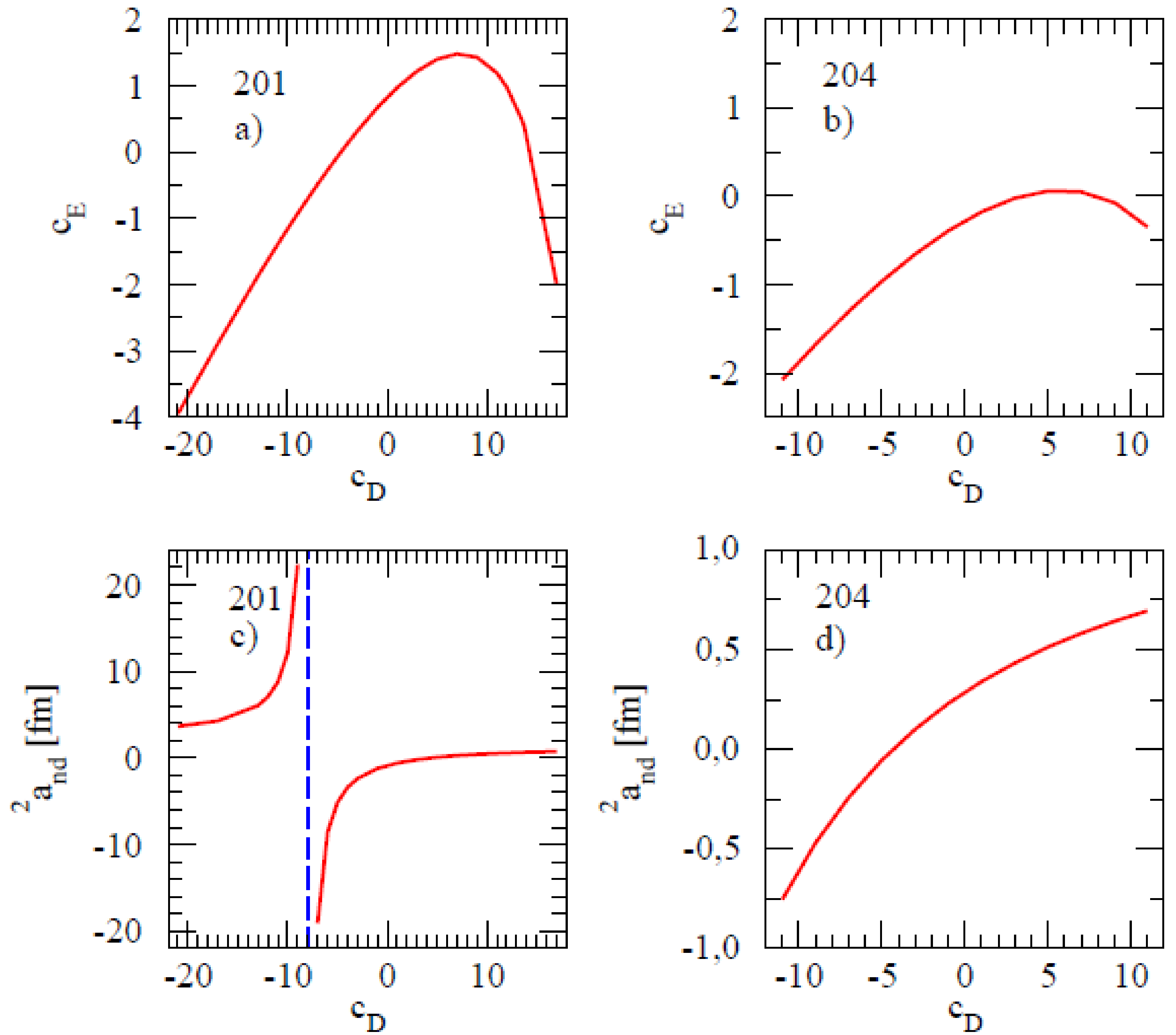
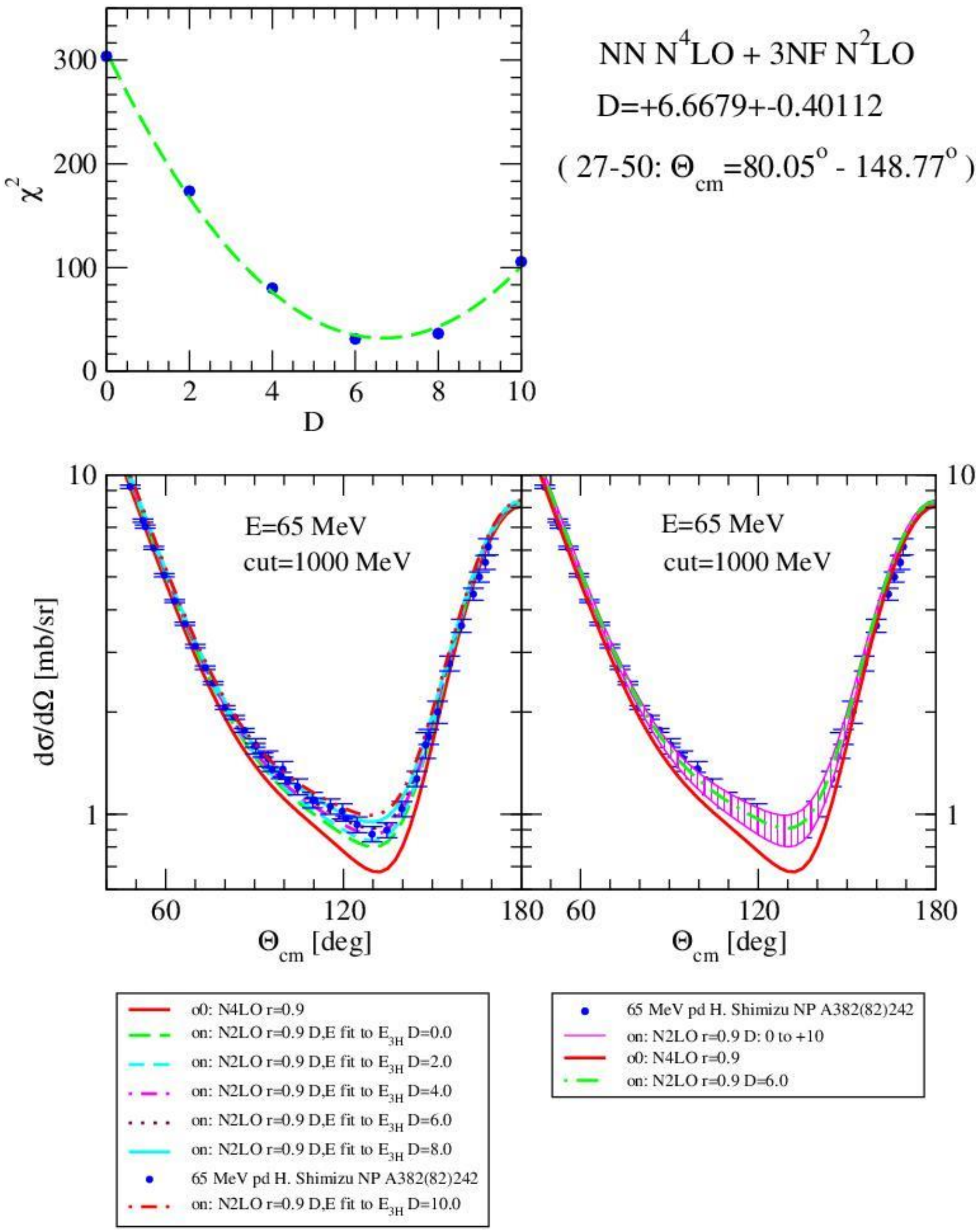
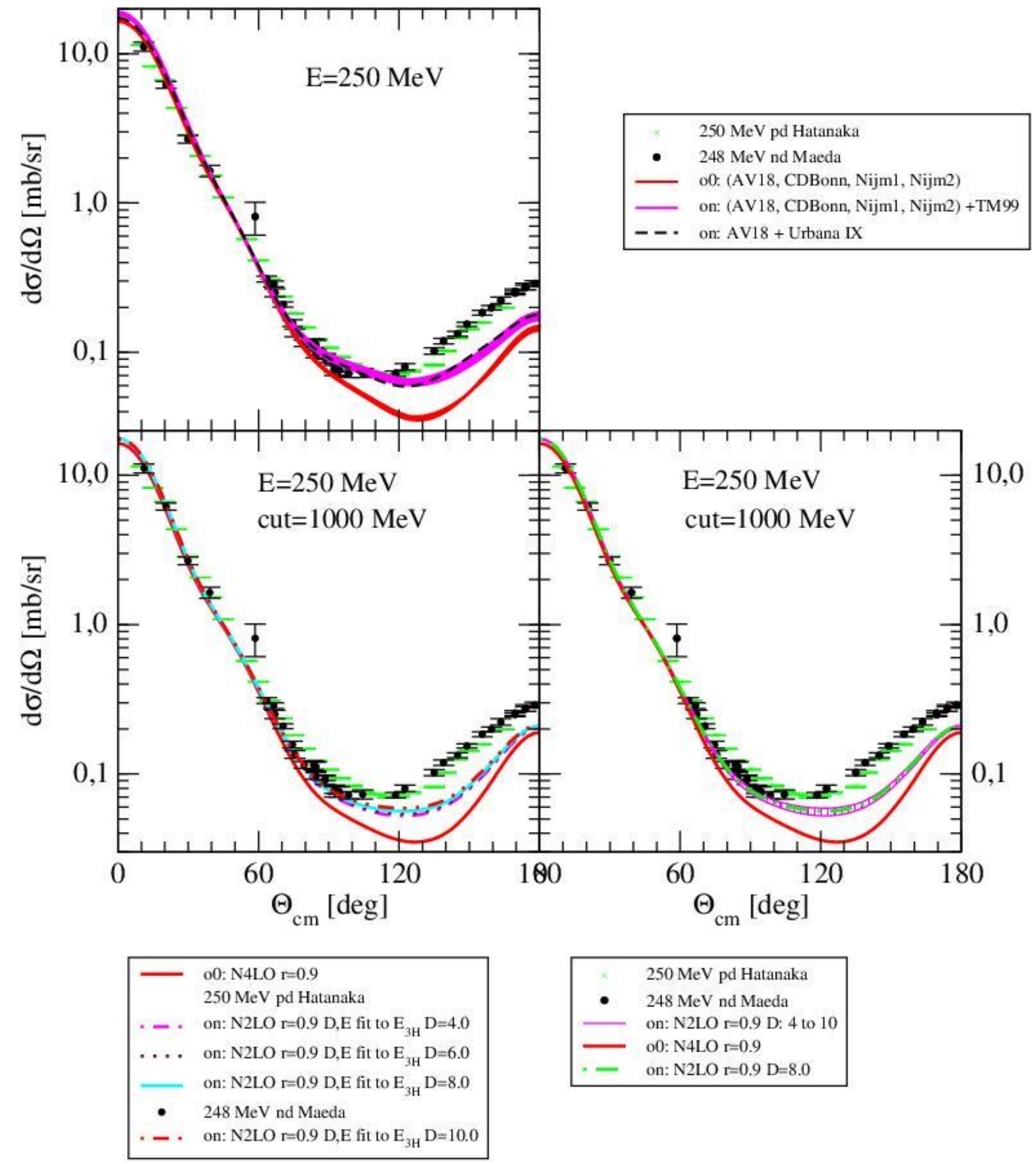


FIG. 1: (color online) The dependence of c_E on c_D for versions 201 (a) and 204 (b) of the N^3LO chiral NN Hamiltonian under the condition that the experimental binding energy of ^3H is reproduced. In c) and d) the corresponding values for doublet nd scattering length are shown. The experimental value of the doublet nd scattering length is $^2a_{nd} = 0.645(7)$ fm [39].



NN N^4LO + 3NF N^2LO
D=+6.6679+-0.40112
(27-50: Θ_{cm}=80.05° - 148.77°)
χ^2
D
E=65 MeV
cut=1000 MeV
dσ/dΩ [mb/sr]
Θ_{cm} [deg]
o0: N4LO r=0.9
on: N2LO r=0.9 D,E fit to E_{3H} D=0.0
on: N2LO r=0.9 D,E fit to E_{3H} D=2.0
on: N2LO r=0.9 D,E fit to E_{3H} D=4.0
on: N2LO r=0.9 D,E fit to E_{3H} D=6.0
on: N2LO r=0.9 D,E fit to E_{3H} D=8.0
65 MeV pd H. Shimizu NP A382(82)242
on: N2LO r=0.9 D,E fit to E_{3H} D=10.0
65 MeV pd H. Shimizu NP A382(82)242
on: N2LO r=0.9 D: 0 to +10
o0: N4LO r=0.9
on: N2LO r=0.9 D=6.0



E=250 MeV
250 MeV pd Hatanaka
248 MeV nd Maeda
o0: (AV18, CDBonn, Nijm1, Nijm2)
on: (AV18, CDBonn, Nijm1, Nijm2) +TM99
on: AV18 + Urbana IX
dσ/dΩ [mb/sr]
E=250 MeV
cut=1000 MeV
Θ_{cm} [deg]
o0: N4LO r=0.9
250 MeV pd Hatanaka
on: N2LO r=0.9 D,E fit to E_{3H} D=4.0
on: N2LO r=0.9 D,E fit to E_{3H} D=6.0
on: N2LO r=0.9 D,E fit to E_{3H} D=8.0
248 MeV nd Maeda
on: N2LO r=0.9 D,E fit to E_{3H} D=10.0
250 MeV pd Hatanaka
248 MeV nd Maeda
on: N2LO r=0.9 D: 4 to 10
o0: N4LO r=0.9
on: N2LO r=0.9 D=8.0

Summary:

- Nd elastic scattering and deuteron breakup reaction reveal large sensitivity to underlying nuclear forces --> good tools to test nuclear Hamiltonian
- it is clear, that based on the present day NN forces additional term in nuclear Hamiltonian is needed --> 3NF
- 3NF models, derived independently from NN potentials, can account in some cases for discrepancies between theory and data
- call for consistency between 2N and 3N forces: support and guidance --> chiral perturbation theory
- Big challenge: application of chiral N^3LO forces (2- and 3-body) to 3N continuum at higher energies