The $B_c \to J/\Psi K D$ weak decay and its relation with the $D^*_{s0}(2317)$ resonance

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Kyoto, Japan

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Based on:

"$D_{s0}^{*}(2317)^{+}$ IN THE DECAY OF B_c INTO $J/\Psi DK$,"

Z. F. SUN, M. BAYAR, P. FERNANDEZ-SOLER AND E. OSET, PHYS. REV. D **93** (2016) NO.5, 054028

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2. Several theoretical interpretations: $c\bar{s}$ state, molecular meson-meson, $K-D$-mixing, two meson and four quark state, four quark state
3. Lattice simulations have found the $D^*_{s0}(2317)$ state with a KD component of $\approx 70\%$

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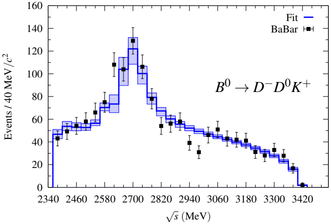

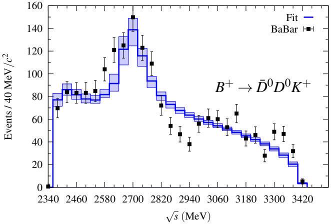

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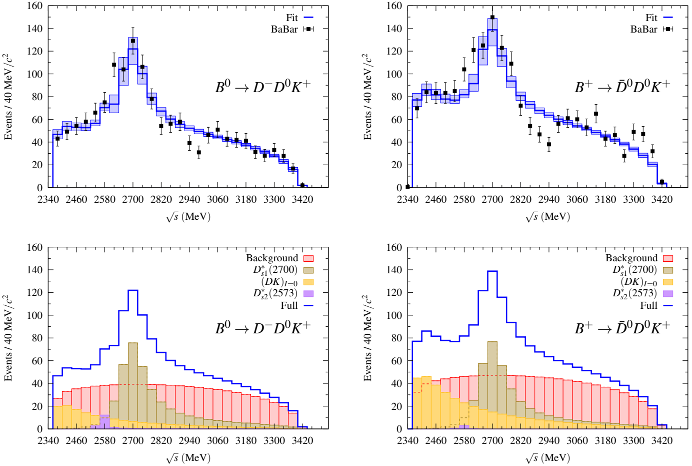

M. Albaladejo, D. Jido, J. Nieves and E. Oset, Eur. Phys. J. C **76** (2016) no.6, 300.

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- Another work to propose: $\bar{B}^0_s \to D^-_s\,(KD)^+$
  M. Albaladejo, M. Nielsen and E. Oset, Phys. Lett. B **746** (2015) 305

II. Statement of the problem: weak decay of a heavy hadron

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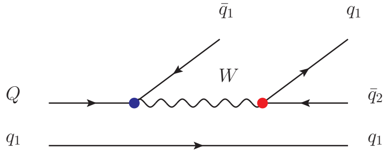

1. The dominant weak mechanism
   $H_W = \frac{G_F}{\sqrt{2}} V_{Qq_1} V_{q_1 q_2} \bar{q}_1 \gamma_\mu (1 - \gamma_5) Q \bar{q}_2 \gamma^\mu (1 - \gamma_5) q_1$

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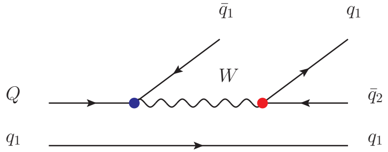

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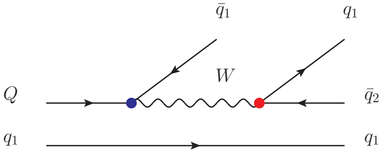

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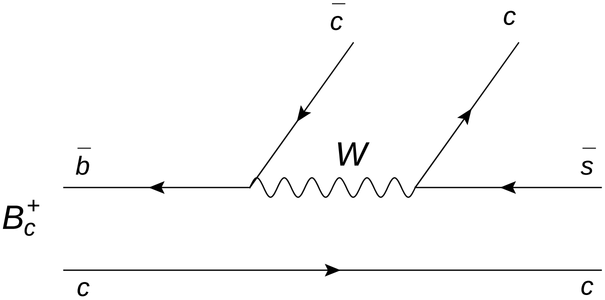

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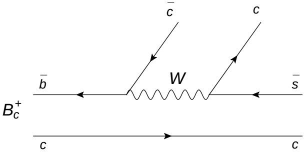

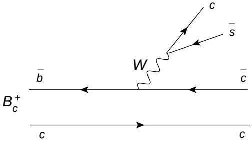

$$V_p + V_p'$$

Global constant factor

II. Theoretical approach: hadronization and rescattering.

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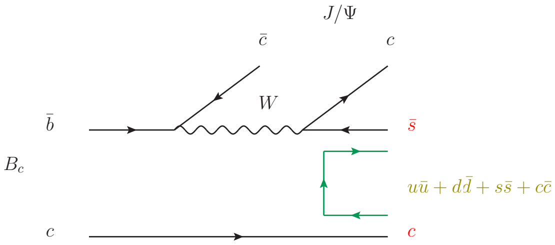

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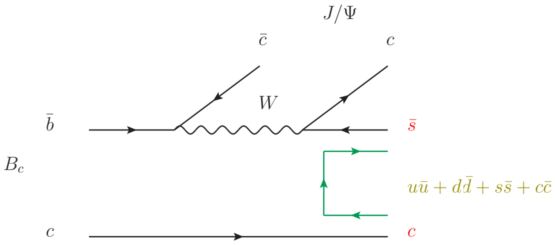

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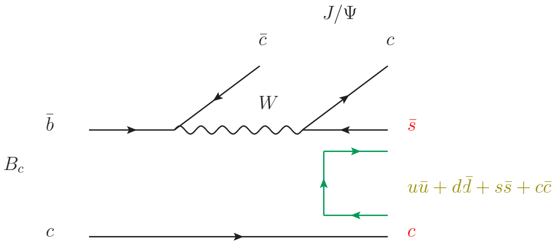

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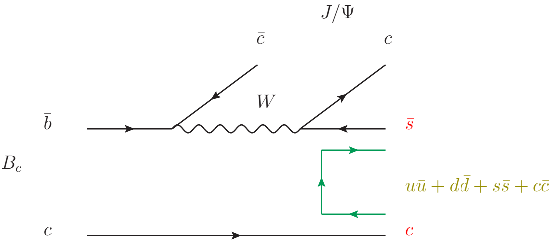

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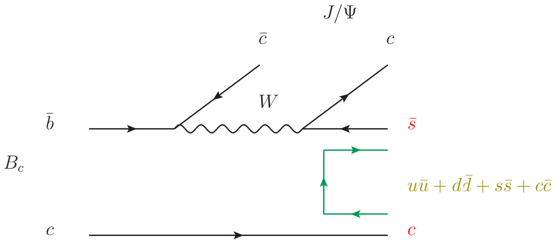

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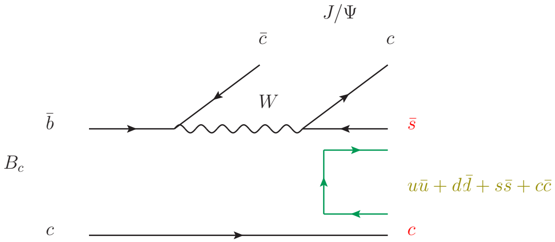

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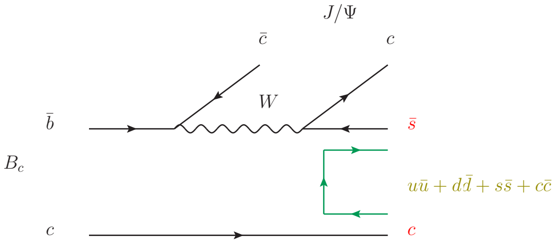

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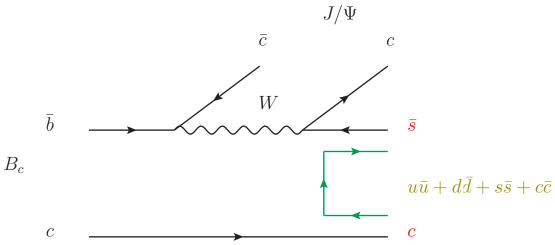

$$M^2 = \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \cdot \left[\begin{pmatrix} \bar{u} & \bar{d} & \bar{s} & \bar{c} \end{pmatrix} \cdot \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \right] \cdot \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} & \bar{c} \end{pmatrix}$$

$$= M \left(u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c} \right) \to (M^2)_{4,3} = c\bar{s}(u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s})$$

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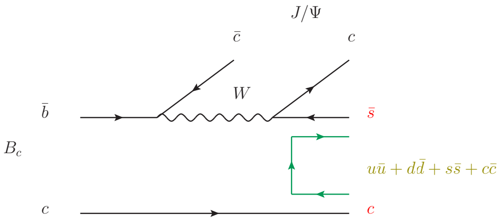
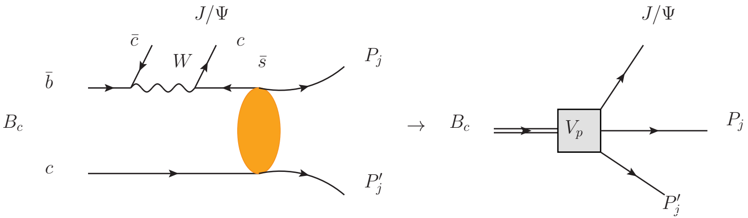


$$\phi = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{2\eta'}{\sqrt{6}} - \frac{\eta}{\sqrt{3}} & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}$$

$$(M^2)_{4,3} = c\bar{s}(u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s})$$

$$(\phi^2)_{4,3} = D^0 K^+ + D^+ K^0 - \frac{1}{\sqrt{3}}\eta D_s^+ + \sqrt{\frac{2}{3}} D_s^+ \eta' + \eta_c D_s^+ \equiv \sum_j P_j P'_j h_j$$

II. Theoretical approach: *hadronization*. 3.



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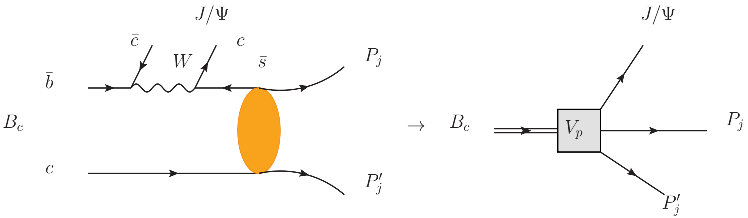

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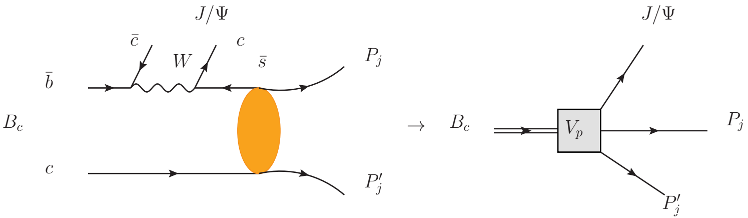
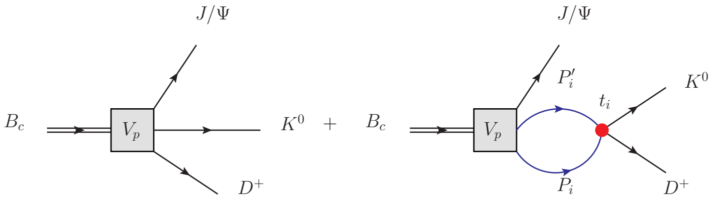


1. $t_{\text{Decay}}\left(B_c \rightarrow J/\Psi P_j P'_j\right) = V_p \times h_j$
2. The J/Ψ meson interaction will be neglected
3. There are more contributions to the final $J/\Psi P_j P'_j$ state

II. Theoretical approach: *hadronization*. 4.



- $t_{\text{Decay}}\left(B_c \to J/\Psi K^0 D^+\right) = V_p \times \left(h_{K^0 D^+} + \sum_j G_j t_{j,K^0 D^+} h_j\right)$

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II. Theoretical approach: *rescattering*. 5.

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- It is obtained solving the scattering in coupled channels $i, j = K^+D^0, K^0D^+, \eta D_s$:

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- G is a diagonal matrix whose elements are the loop functions of the two mesons P_i, P_i', in each channel

$$G_{i,i}(s = P^2) = \int_{\mathbb{R}^4} \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_i^2 + \mathrm{i}\epsilon} \frac{1}{q^2 - (m_i')^2 + \mathrm{i}\epsilon}$$

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- $V_{i,j}$ is the kernel potential, L.O. S-wave amplitudes (Gamermann, Oset, Strottman, Vicente Vacas. PHYSICAL REVIEW D 76, 074016 (2007)), $\chi-$heavy-meson lagrangian

$$V_{i,j}(s) = \frac{1}{f_\pi}\left(A_{i,j}(m, m') + B_{i,j}(m, m')\, s + C_{i,j}(m, m')\, \frac{1}{s}\right)$$

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- Resummation

$$t_{i,j} = (1 + V \cdot G + V \cdot G \cdot V \cdot G + \ldots)_{i,k} \cdot V_{k,j}$$

III. Results and predictions for observables.

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- Fit the scattering free parameter α in order to have the $D^*_{s0}(2317)$ as a bound state. A pole in the t matrix, $t_{i,j} \approx \frac{g_i g_j}{s-(M_{D^*_{s0}(2317)})^2}$

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- The differential decay width $d\Gamma(B_c \to J/\Psi K D)/dM_{inv}$,

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$$\frac{\mathrm{d}\Gamma}{\mathrm{d}M_{\mathrm{inv}}} = A^2 \frac{p^3_{J/\Psi} p_{DK}}{(2\pi)^3 4 m^2_{B_c}} \left| \frac{t_{\,\mathrm{Decay}} \left(B_c \to J/\Psi D^+ K^0\right)}{V_p} \right|^2$$

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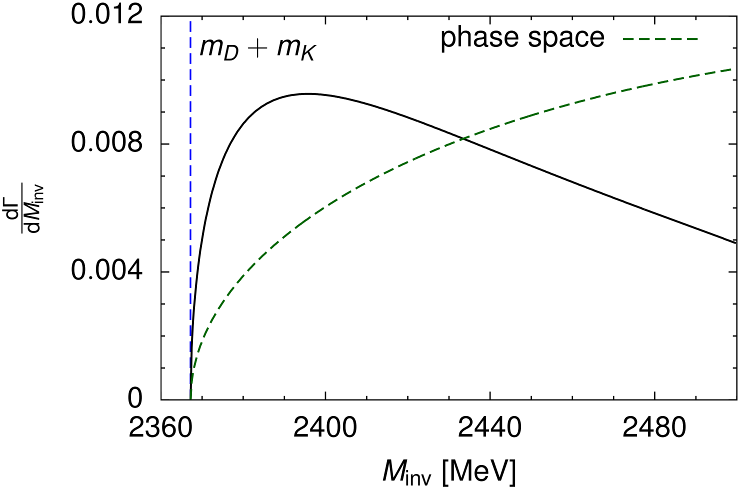
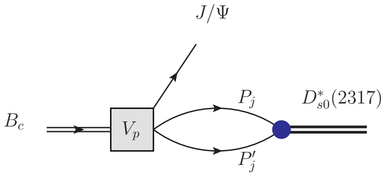


- $\Gamma(B_c \to J/\Psi D^*_{s0}(2317))$ Coalescence production of the $D^*_{s0}(2317)^+$: $t(B_c \to J/\Psi R) = V_p \sum_j h_j G_j g_j \Big|_{E=M_R}$



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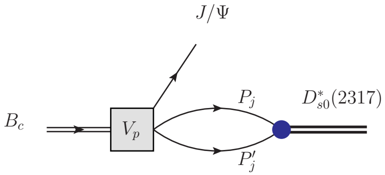

- $\frac{d\tilde{\Gamma}(B_c \to J/\Psi KD)}{dM_{inv}} = \frac{d\Gamma(B_c \to J/\Psi KD)}{dM_{inv}} \frac{1}{p^3_{J/\Psi} p_{DK}} \cdots$

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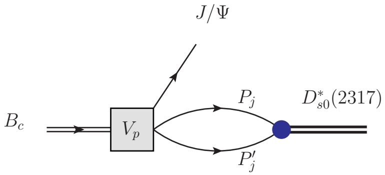

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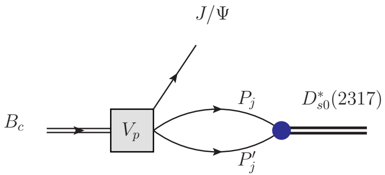

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$$\frac{d\Gamma(B_c \to J/\Psi KD)}{dM_{inv}} \frac{1}{p^3_{J/\Psi} p_{DK}} \times \left[\frac{1}{\Gamma(B_c \to J/\Psi R)}\right] \times p^3_{J/\Psi}\Big|_{E=M_R} M_R^2$$
dimensionless quantity

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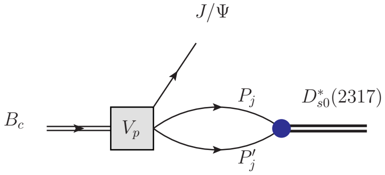
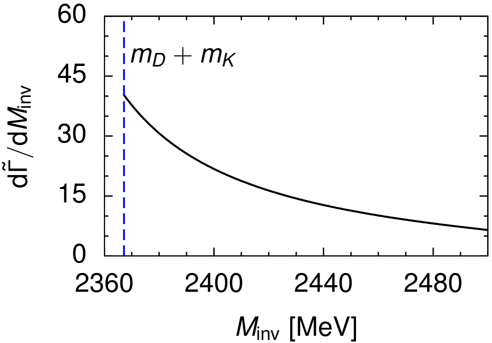


- $\frac{d\tilde{\Gamma}(B_c \to J/\Psi KD)}{dM_{inv}} =$
$\frac{d\Gamma(B_c \to J/\Psi KD)}{dM_{inv}} \frac{1}{p^3_{J/\Psi} p_{DK}} \times \left[\frac{1}{\Gamma(B_c \to J/\Psi R)}\right] \times p^3_{J/\Psi}\Big|_{E=M_R} M_R^2$
$= \frac{M_R^2}{4\pi^2} \frac{\left|h_{D^+K^0} + \sum_i h_i G_i t_{i,D^+K^0}\right|^2}{\left|\sum_i h_i G_i g_i\right|^2\Big|_{\text{pole}}},$
dimensionless quantity

III. Results and predictions for observables. 2.

- $\Gamma(B_c \to J/\Psi D^*_{s0}(2317))$ Coalescence production of the $D^*_{s0}(2317)^+$: $t(B_c \to J/\Psi R) = V_p \sum_j h_j G_j g_j \Big|_{E=M_R}$
- $$\frac{d\tilde{\Gamma}(B_c \to J/\Psi KD)}{dM_{inv}} = \left[M_R^2 \frac{p^3_{J/\Psi}\big|_{E=M_R}}{\Gamma(B_c \to J/\Psi R)} \right] \times \frac{1}{p^3_{J/\Psi} p_{DK}} \frac{d\Gamma(B_c \to J/\Psi KD)}{dM_{inv}}$$



III. Results and predictions for observables. 3.

Possible $q\bar{q}$ component to the $D^*_{s0}(2317)$ generation (A. Martínez Torres, E. Oset, S. Prelovsek and A. Ramos, JHEP **1505** (2015) 153):

- The amount of KD from the lattice data

$$P(KD) = (72 \pm 14)\% \leftrightarrow P(KD) = - \sum_{i=K^+D^0, K^0D^+} g_i^2 \frac{\partial G_i}{\partial s}\bigg|_{\rm pole}$$

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- We add a *Castillejo-Dalitz-Dyson* pole to the potential

$$V_{i,j} \to V_{i,j} + \delta V, \quad \delta V = \frac{\gamma}{M_{inv} - M_{q\bar{q}}}$$

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Possible $q\bar{q}$ component to the $D_{s0}^*(2317)$ generation (A. Martínez Torres, E. Oset, S. Prelovsek and A. Ramos, JHEP **1505** (2015) 153):

- The amount of *KD* from the lattice data

$$P(KD) = (72 \pm 14)\% \leftrightarrow P(KD) = - \sum_{i=K^+D^0, K^0D^+} g_i^2 \left.\frac{\partial G_i}{\partial s}\right|_{\text{pole}}$$

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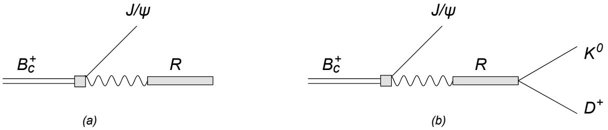
$$V_{i,j} \to V_{i,j} + \delta V, \quad \delta V = \frac{\gamma}{M_{inv} - M_{q\bar{q}}}$$

- We consider direct coupling to this component in the B_c decay



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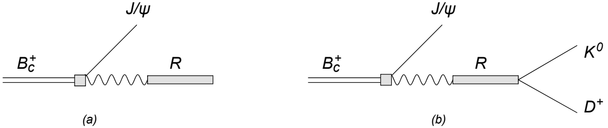

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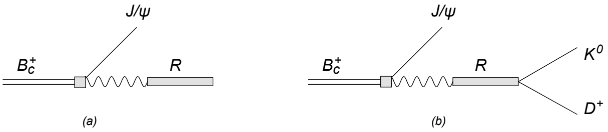
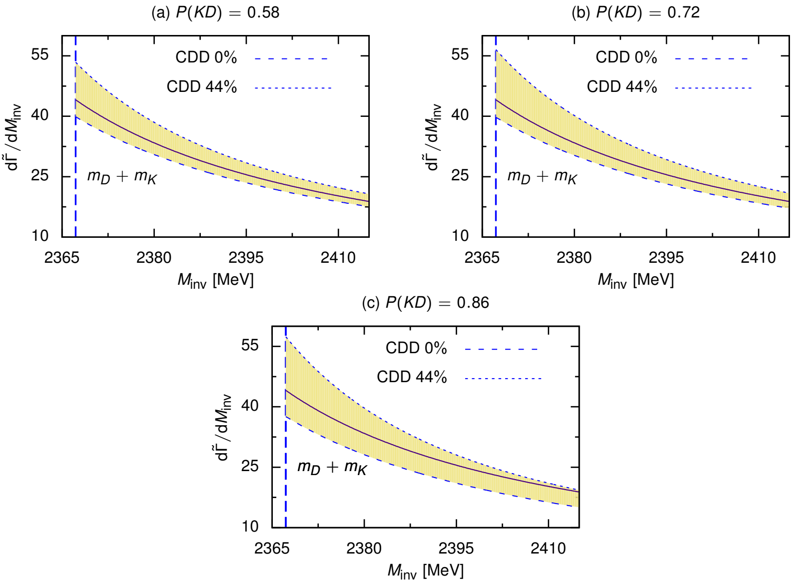


- For different cases $P(KD) = 58, 72, 86\%$, we consider a $q\bar{q}$ coupling to the resonance such that $\Rightarrow 0,\ 21,\ 44\%$ of increase in $\Gamma(B_c \to J/\Psi D_{s0}^*(2317))$

III. Results and predictions for observables. 3.



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- The enhancement of events seen close to the *KD* threshold in *B* decays can be an experimental test of the molecular nature of the $D_{s0}^*(2317)$
- We have proposed an unmeasured weak decay process where, with the molecular *KD* hypothesis, this feature is also observed and due to the generation of the $D_{s0}^*(2317)$

Thank you for your attention

Z. F. Sun, M. Bayar, P. Fernandez-Soler and E. Oset, Phys. Rev. D **93** (2016) no.5, 054028 doi:10.1103/PhysRevD.93.054028 [arXiv:1510.06316 [hep-ph]].

B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **90** (2003) 242001 doi:10.1103/PhysRevLett.90.242001 [hep-ex/0304021].

L. Liu, K. Orginos, F. K. Guo, C. Hanhart and U. G. Meissner, Phys. Rev. D **87** (2013) no.1, 014508 doi:10.1103/PhysRevD.87.014508 [arXiv:1208.4535 [hep-lat]].

M. Albaladejo, M. Nielsen and E. Oset, Phys. Lett. B **746** (2015) 305 doi:10.1016/j.physletb.2015.05.019 [arXiv:1501.03455 [hep-ph]].

M. Albaladejo, D. Jido, J. Nieves and E. Oset, Eur. Phys. J. C **76** (2016) no.6, 300 doi:10.1140/epjc/s10052-016-4144-3 [arXiv:1604.01193 [hep-ph]].

G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125 doi:10.1103/RevModPhys.68.1125 [hep-ph/9512380].

E. Oset *et al.*, Int. J. Mod. Phys. E **25** (2016) 1630001 doi:10.1142/S0218301316300010 [arXiv:1601.03972 [hep-ph]].

U. G. Meissner and J. A. Oller, Nucl. Phys. A **679** (2001) 671 doi:10.1016/S0375-9474(00)00367-5 [hep-ph/0005253].

D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, Phys. Rev. D **76** (2007) 074016 doi:10.1103/PhysRevD.76.074016 [hep-ph/0612179].

A. Martínez Torres, E. Oset, S. Prelovsek and A. Ramos, JHEP **1505** (2015) 153 doi:10.1007/JHEP05(2015)153 [arXiv:1412.1706 [hep-lat]].