The $B_c \rightarrow J/\psi K D$ weak decay and its relation with the $D_{s0}^*(2317)$ resonance
I. Introduction. A few words about the $D_{s0}^*(2317)$

II. Statement of the problem: weak decay of a heavy hadron

III. Theoretical approach: hadronization and rescattering

IV. Results and observable predictions

V. Summary

Based on:

"$D_{s0}^*(2317)^+ \text{ in the decay of } B_c \text{ into } J/\psi DK$",

I. Introduction. A few words about the $D_{s0}^*(2317)$
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2. Several theoretical interpretations: $c\bar{s}$ state, molecular meson-meson, $K-D$-mixing, two meson and four quark state, four quark state

3. Lattice simulations have found the $D^{*}_{s0}(2317)$ state with a $KD$ component of $\approx 70\%$
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- An experimental test of the molecular nature: $B_s \rightarrow \pi^+ \bar{D}^0 K^-$, $B^0 \rightarrow D^- D^0 K^+$ and $B^+ \rightarrow \bar{D}^0 D^0 K^+$
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$$P(KD) = 70^{+5}_{-6} \pm 4\%$$

Another work to propose: $\bar{B}_s^0 \rightarrow D_s^- (KD)^+$

II. Statement of the problem: weak decay of a heavy hadron
1. The dominant weak mechanism

\[ H_W = \frac{G_F}{\sqrt{2}} V_{Qq_1} V_{q_1q_2} \bar{q}_1 \gamma_\mu (1 - \gamma_5) Q \bar{q}_2 \gamma_\mu (1 - \gamma_5) q_1 \]
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2. We look to a small energy range: smooth energy dependence

3. Amplitude considered as constant: \( V_\rho \)
1. Weak decay of a heavy hadron. 2.
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\[ V_p + V'_p \]

Global constant factor
II. Theoretical approach: hadronization and rescattering.
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u\bar{s} & d\bar{u} & d\bar{d} & d\bar{s} \\
u\bar{c} & d\bar{u} & d\bar{s} & d\bar{c}
\end{pmatrix}
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$$M = \begin{pmatrix}
u\bar{u} & u\bar{d} & u\bar{s} & u\bar{c} \\
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s\bar{u} & s\bar{d} & s\bar{s} & s\bar{c} \\
c\bar{u} & c\bar{d} & c\bar{s} & c\bar{c}
\end{pmatrix} = \begin{pmatrix} u \\
d \\
s \\
c \end{pmatrix} \cdot \begin{pmatrix} \bar{u} \\
\bar{d} \\
\bar{s} \\
\bar{c} \end{pmatrix}$$
II. Theoretical approach: *hadronization*. 2.

\[
M^2 = \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \cdot \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} & \bar{c} \end{pmatrix} \cdot \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \cdot (\bar{u} \quad \bar{d} \quad \bar{s} \quad \bar{c})
\]

\[
= M (u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c}) \rightarrow (M^2)_{4,3} = c\bar{s}(u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s})
\]
II. Theoretical approach: *hadronization*. 2.

\[
\phi = \begin{pmatrix}
\frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} \\
\pi^- \\
K^- \\
D^0 \\
\frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} \\
\overline{K}^0 \\
D_+ \\
\sqrt{\frac{2}{3}} \eta' + \frac{\eta}{\sqrt{3}} \eta_c \\
D_s^+ \\
\eta_c
\end{pmatrix}
\]

\[
(M^2)_{4,3} = c\bar{s} (u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s})
\]

\[
(\phi^2)_{4,3} = D^0 K^+ + D^+ K^0 - \frac{1}{\sqrt{3}} \eta D_s^+ + \sqrt{\frac{2}{3}} D_s^+ \eta' + \eta_c D_s^+ \equiv \sum_j P_j P'_j h_j
\]
II. Theoretical approach: hadronization. 3.

1. \( t_{\text{Decay}} \left( B_c \to J/\Psi P_j P'_j \right) = V_p \times h_j \)
II. Theoretical approach: *hadronization*. 3.

1. $t_{\text{Decay}} \left( B_c \rightarrow J/\Psi P_j P_j' \right) = V_p \times h_j$

2. The $J/\Psi$ meson interaction will be neglected
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2. The $J/\Psi$ meson interaction will be neglected

3. There are more contributions to the final $J/\Psi P_j P'_j$ state

\[ t_{\text{Decay}}(B_c \rightarrow J/\psi K^0 D^+) = V_p \times \left( h_{K^0 D^+} + \sum_j G_j t_{j, K^0 D^+ h_j} \right) \]
The $t_{i,j}$ is the pseudoscalar interaction vertex relating channels $i$ and $j$. 

$\mathbf{G}$ is a diagonal matrix whose elements are the loop functions of the two mesons $P_i, P'_i$, in each channel $G_{i,i}(s) = \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_i^2 + i\epsilon + (m'_i)^2 + i\epsilon}$.

$V_{i,j}$ is the kernel potential, L.O. S-wave amplitudes ($G$AMERMANN, OSET, STROTTMAN, VICENTE PHYSICAL REVIEW D 76, 074016 (2007)),
The $t_{i,j}$ is the pseudoscalar interaction vertex relating channels $i$ and $j$.

It is obtained solving the scattering in coupled channels $i, j = K^+D^0, K^0D^+, \eta D_s$:

$$t_{i,j} = (1 - V \cdot G)_{i,k}^{-1} \cdot V_{k,j}$$
II. Theoretical approach: rescattering. 5.

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$$G_{i,i}(s = P^2) = \int_{\mathbb{R}^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{(P - q)^2 - m_i^2 + i\epsilon} \frac{1}{q^2 - (m'_i)^2 + i\epsilon}$$
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$\chi$–heavy-meson lagrangian

$$ V_{i,j}(s) = \frac{1}{f_\pi} \left( A_{i,j} (m, m') + B_{i,j} (m, m') s + C_{i,j} (m, m') \frac{1}{s} \right) $$
II. Theoretical approach: rescattering. 5.

- Unitarization of the amplitudes

\[ t_{i,j} = (1 - V \cdot G)^{-1}_{i,k} \cdot V_{k,j} \]
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- **Unitarization of the amplitudes**

\[ t_{i,j} = (1 - V \cdot G)^{-1}_{i,k} \cdot V_{k,j} \]

- **Resummation**

\[ t_{i,j} = (1 + V \cdot G + V \cdot G \cdot V \cdot G + \ldots)_{i,k} \cdot V_{k,j} \]
III. Results and predictions for observables.
Fit the scattering free parameter $\alpha$ in order to have the $D_{s0}^*(2317)$ as a bound state. A pole in the $t$ matrix,

$$t_{i,j} \approx \frac{g_i g_j}{s - (M_{D_{s0}^*(2317)})^2}$$
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The differential decay width $d\Gamma(B_c \to J/\psi K D)/dM_{\text{inv}},$

$$B_c \to J/\psi K^0 D^+$$
$$0^- \to 1^- 0^+$$

$$V_p = \sqrt{3} A p_{J/\psi} \cos \theta$$
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The differential decay width $d\Gamma(B_c \rightarrow J/\Psi K D)/dM_{\text{inv}}$,
$$B_c \rightarrow J/\Psi K^0 D^+$$
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$$\frac{d\Gamma}{dM_{\text{inv}}} = A^2 \frac{p_{J/\Psi}^3 p_{DK}}{(2\pi)^3 4m_{B_c}^2} \left| \frac{t_{\text{Decay}}(B_c \rightarrow J/\Psi D^+ K^0)}{V_p} \right|^2$$
III. Results and predictions for observables. 1.
III. Results and predictions for observables. 2.

- $\Gamma(B_c \rightarrow J/\Psi D^{*}_{s0}(2317))$ Coalescence production of the $D^{*}_{s0}(2317)^+$:
  
  \[ t(B_c \rightarrow J/\Psi R) = V_p \sum_j h_j G_j g_j |_{E=M_R} \]

Diagram:

- $B_c$ to $V_p$ to $D^{*}_{s0}(2317)$ to $J/\Psi$
III. Results and predictions for observables. 2.

- $\Gamma(B_c \rightarrow J/\Psi D^{*}_{s0}(2317))$ Coalescence production of the $D^{*}_{s0}(2317)^{+}$: $t(B_c \rightarrow J/\Psi R) = V_P \sum_j h_j G_j g_j \Big|_{E=M_R}$

- $\frac{d\tilde{\Gamma}(B_c \rightarrow J/\Psi KD)}{dM_{inv}} = \frac{d\Gamma(B_c \rightarrow J/\Psi KD)}{dM_{inv}} \frac{1}{p_{J/\Psi}^3 p_{DK}} \ldots$
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\[ J/\Psi \]

\[ D_{s0}^*(2317) \]

\[ B_c \]

\[ V_p \]

\[ P_j \]

\[ P_{j}' \]

\[ d\tilde{\Gamma}(B_c \to J/\Psi KD) \]

\[ = \frac{d\Gamma(B_c \to J/\Psi KD)}{dM_{inv}} \frac{1}{p_{J/\Psi}^3 p_{DK}} \times \left[ \frac{1}{\Gamma(B_c \to J/\Psi R)} \right] \ldots \]
III. Results and predictions for observables. 2.

\[ \Gamma(B_c \to J/\Psi D_{s0}^*(2317)) \]

Coalescence production of the \( D_{s0}^*(2317)^+ \):

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\[ \Gamma(B_c \to J/\Psi KD) \]

\[ \frac{d\Gamma(B_c \to J/\Psi KD)}{dM_{inv}} \bigg|_{\rho_{j/\psi} \rho_{DK}} \times \left[ \frac{1}{\Gamma(B_c \to J/\Psi R)} \right] \times \rho_{j/\psi}^3 \bigg|_{E=M_R} M_R^2 \]

dimensionless quantity
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\[ \Gamma(B_c \to J/\psi D_{s0}^*(2317)) \] Coalescence production of the \( D_{s0}^*(2317)^+ \):

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\[ = \frac{M_R^2}{4\pi^2} \left| \frac{h_{D+K^0} + \sum_i h_i G_i t_i, D+K^0}{\sum_i h_i G_i g_i} \right|^2 \bigg|_{\text{pole}}, \]

dimensionless quantity
\[\Gamma(B_c \to J/\psi D_{s0}^*(2317))\] Coalescence production of the \(D_{s0}^*(2317)^+\): \(t(B_c \to J/\psi R) = V_p \sum_j h_j G_j g_j \bigg|_{E=M_R}\)

\[\frac{d\tilde{\Gamma}(B_c \to J/\psi KD)}{dM_{inv}} = \left[M^2 \frac{p^3_{J/\psi}}{R \Gamma(B_c \to J/\psi R)}\bigg|_{E=M_R}\right] \times \frac{1}{p^3_{J/\psi} p_{DK}} \frac{d\Gamma(B_c \to J/\psi KD)}{dM_{inv}}\]
III. Results and predictions for observables. 3.

Possible $q\bar{q}$ component to the $D_{s0}^*(2317)$ generation (A. Martínez Torres, E. Oset, S. Prelovsek and A. Ramos, JHEP 1505 (2015) 153):

- The amount of $KD$ from the lattice data

\[
P(KD) = (72 \pm 14)\% \leftrightarrow P(KD) = -\sum_{i=K^+D^0, K^0D^+} g_i^2 \frac{\partial G_i}{\partial s} \bigg|_{\text{pole}}
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- We add a Castillejo-Dalitz-Dyson pole to the potential

\[
V_{i,j} \rightarrow V_{i,j} + \delta V, \quad \delta V = \frac{\gamma}{M_{\text{inv}} - M_{q\bar{q}}}
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$$V_{i,j} \rightarrow V_{i,j} + \delta V, \quad \delta V = \frac{\gamma}{M_{\text{inv}} - M_{q\bar{q}}}$$

- We consider direct coupling to this component in the $B_c$ decay
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We consider direct coupling to this component in the $B_c$ decay

![Diagram](image)

For different cases $P(KD) = 58, 72, 86\%$, we consider a $q\bar{q}$ coupling to the resonance such that $\Rightarrow 0, 21, 44\%$ of increase in $\Gamma(B_c \rightarrow J/\psi D_{s0}^*(2317))$
III. Results and predictions for observables. 3.

(a) $P(KD) = 0.58$

(b) $P(KD) = 0.72$

(c) $P(KD) = 0.86$
V. Summary
The enhancement of events seen close to the $KD$ threshold in $B$ decays can be an experimental test of the molecular nature of the $D_{s0}^*(2317)$.
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We have proposed an unmeasured weak decay process where, with the molecular $KD$ hypothesis, this feature is also observed and due to the generation of the $D_{s0}^*(2317)$. 

V. Summary.
Thank you for your attention


[hep-ph/0005253].

