The exclusive Drell-Yan process and deeply virtual pion production

P. Kroll

Fachbereich Physik, Univ. Wuppertal and Univ. Regensburg

MENU2016, Kyoto, July 2016

Outline:

- **Introduction: handbag approach and GPDs**
- **Analysis of pion leptoproduction**

  (the pion pole, transversity, subprocess amplitudes, results)
- **The exclusive Drell-Yan process -** $\pi^- p \to l^+ l^- n$
- **Summary**

The handbag approach

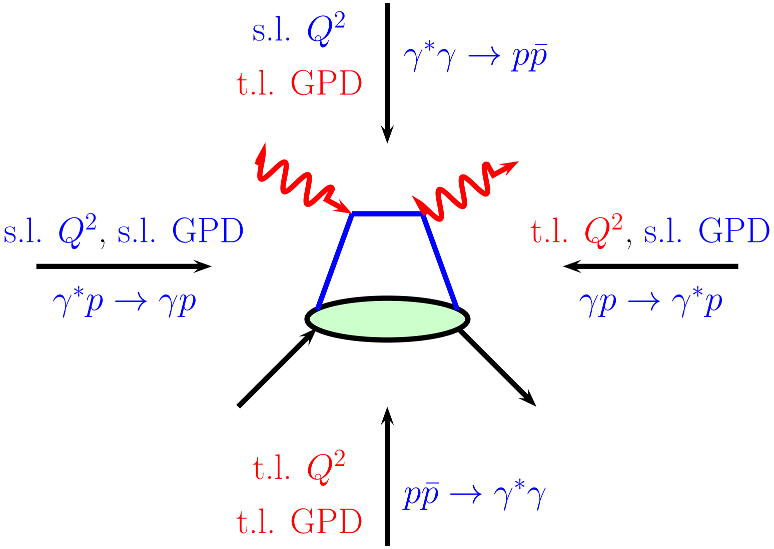
Handbag approach provides a common picture for many hard exclusive processes based on factorization in hard subprocesses and generalized parton distributions (GPDs)

deeply virtual processes

virtually of photon provides the hard scale

in principle all processes are experimentally feasible

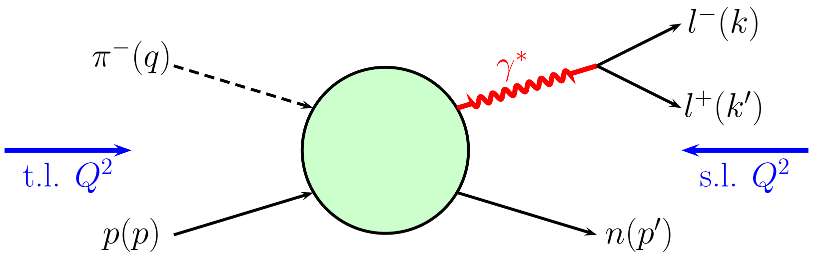
real photon can be replaced by mesons (e.g. $\gamma^* p \to \pi N$, pion-induced DY)



wide-angle processes momentum transfer provides the hard scale
e.g. RCS, photoproduction of mesons, $\gamma\gamma \to h\bar{h}$, $p\bar{p} \to \gamma\gamma, \gamma M$

heavy meson production heavy quark mass provides the hard scale
e.g. $\gamma p \to J/\Psi(\Upsilon)p$, $\pi^- p \to D^- \Lambda_c$,...

$lp \to l\pi^+ n$ and $\pi^- p \to l^- l^+ n$



exclusive Drell-Yan process

directly related to pion production:

- same GPDs
- $\hat{s} - \hat{u}$ $(l - \pi)$ crossed subproces

$$\mathcal{H}^{\pi^- \to \gamma^*}(\hat{u}, \hat{s}) = -\mathcal{H}^{\gamma^* \to \pi^+}(\hat{s}, \hat{u})$$

equivalent to $Q^2 \to -Q'^2$

Idea: Study hard π^+ production first

plenty of data from Jlab and HERMES, more will come (Jlab12 and COMPASS)

learn about relevant GPDs and treatment of subprocesses

apply what has been learned there in a calculation of the Drell-Yan process

GPDs – a reminder

D. Müller et al (94), Ji(97), Radyushkin (97)

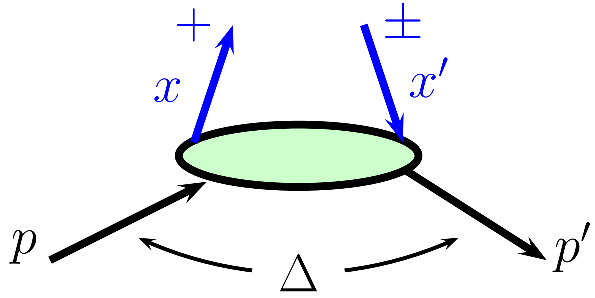
GPDs: $\sim \langle p'|\overline{\Psi}(-z/2)\Gamma\Psi(z/2)|p\rangle \qquad \Gamma = \gamma^+, \gamma^+\gamma_5, \sigma^{+j}$ (encode soft physics)

$$K(\bar{x},\xi,t) = H, E, \widetilde{H}, \widetilde{E}, H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$$

$$x = \frac{\bar{x}+\xi}{1+\xi} \quad x' = \frac{\bar{x}-\xi}{1-\xi} \quad \xi = \frac{(p-p')^+}{(p+p')^+} \simeq \frac{x_B}{2-x_B}$$

for quarks ($\xi < \bar{x} < 1$) and gluons

(antiquarks for $-1 < \bar{x} < -\xi$, $q\bar{q}$ pairs $-\xi < \bar{x} < \xi$)



properties:

reduction formula $H^q(\bar{x},\xi=t=0) = q(\bar{x})$, $\widetilde{H}^q \to \Delta q(\bar{x})$, $H_T^q \to \delta^q(\bar{x})$

sum rules (proton form factors): $F_1^q(t) = \int d\bar{x} H^q(\bar{x},\xi,t)$, $F_1 = \sum e_q F_1^q$

$$E \to F_2,\ \widetilde{H} \to F_A,\ \widetilde{E} \to F_P$$

polynomiality, universality, evolution, positivity constraints

Ji's sum rule $\quad J_q = \frac{1}{2}\int_{-1}^{1} d\bar{x}\, \bar{x}\, [H^q(\bar{x},\xi,t=0) + E^q(\bar{x},\xi,t=0)]$

FT $\mathbf{\Delta} \to \mathbf{b}$ ($\Delta^2 = -t$): information on parton localization in trans. position space

Leptoproduction of pions

Rigorous proofs of collinear factorization in hard subprocesses and GPDs in generalized Bjorken regime of large Q^2, large W but fixed x_B

Collins-Frankfurt-Strikman (96)

leading-twist amplitudes for longitudinally polarized photons

$$\mathcal{M}_{0+0+} = e_0 \sqrt{1-\xi^2} \int_{-1}^{1} dx \mathcal{H}_{0+0+} \big(\widetilde{H} - \frac{\xi^2}{1-\xi^2} \widetilde{E}\big) \qquad \mathcal{M}_{0-0+} = e_0 \frac{\sqrt{-t'}}{2m} \xi \int_{-1}^{1} dx \mathcal{H}_{0+0+} \widetilde{E}$$

$$t' = t - t_0 \qquad t_0 = -4m^2\xi^2/(1-\xi^2)$$

amplitudes for transversally polarized photons are suppressed by $1/Q$

many leading-twist predictions for pion production

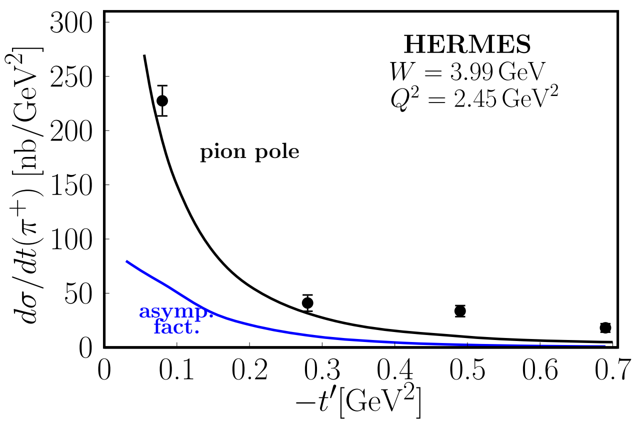
e.g. Mankiewicz et al (98), Frankfurt et al(99), Diehl al et(01), ...

all fail by order of magnitude in comparison with experiment

- strong power corrections to long. amplitudes needed
- contributions from transverse photons are not suppressed see below

The pion pole

For π^+ production - pion pole: (Mankiewicz et al (98), Penttinen et al (99))



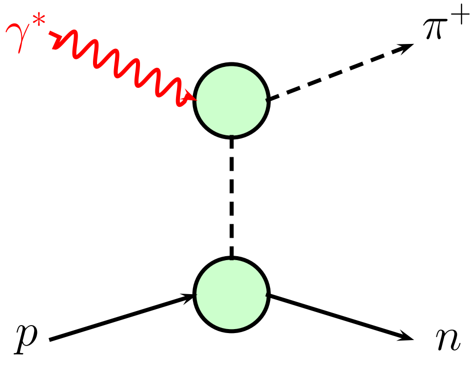
$$\widetilde{E}^u_{\rm pole} = -\widetilde{E}^d_{\rm pole} = \Theta(|x| \leq \xi)\frac{m f_\pi g_{\pi NN}}{\sqrt{2}\xi}\frac{F_{\pi NN}(t)}{m_\pi^2 - t}\Phi_\pi\Big(\frac{x+\xi}{2\xi}\Big)$$

$$\Longrightarrow \frac{d\sigma_L^{\rm pole}}{dt} \sim \frac{-t}{Q^2}\left[\sqrt{2}e_0 g_{\pi NN}\frac{F_{\pi NN}(t)}{m_\pi^2 - t}Q^2 F_\pi^{\rm pert}(Q^2)\right]^2$$

understimates cross ssection (blue line)

$F_\pi^{\rm pert.} \simeq 0.3 - 0.5 F_\pi^{\rm exp.}$

(note: F_π measured in π^+ electroproduction at Jlab)



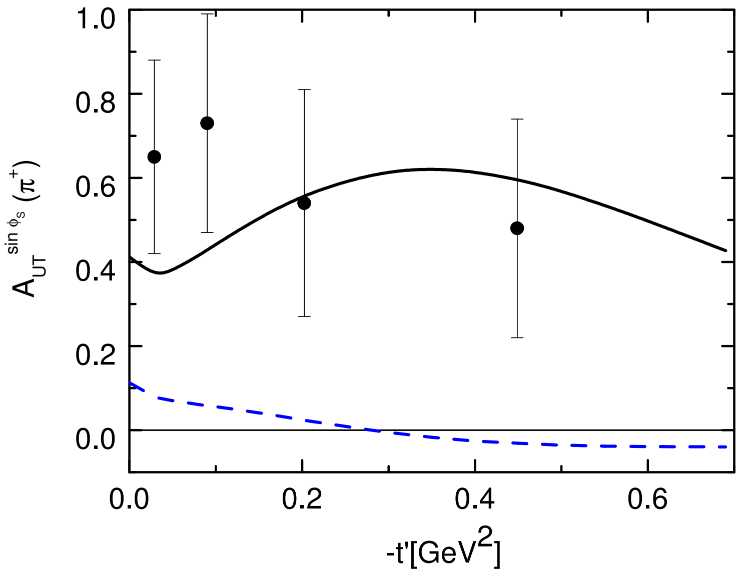
Goloskokov-K(09): $F_\pi^{\rm pert} \to F_\pi^{\rm exp}$

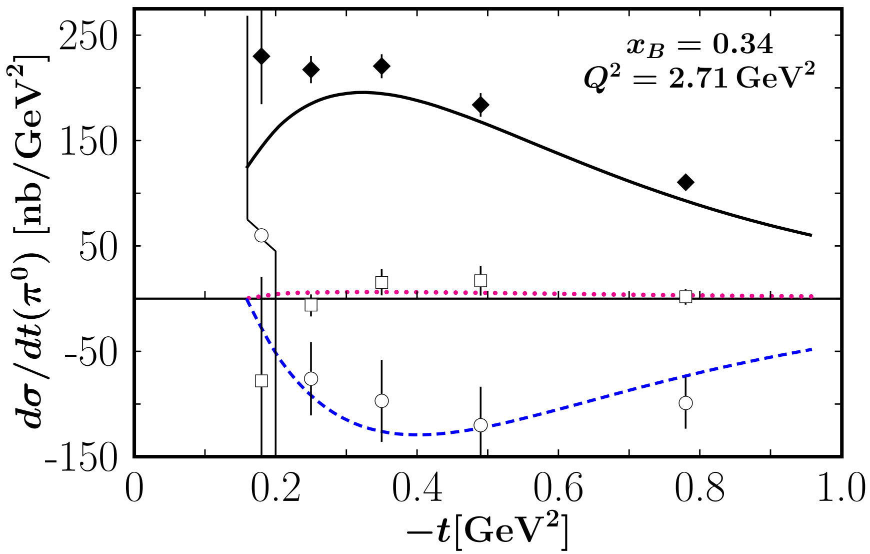
knowledge of the sixties suffices to explain

π^+ data at small $-t$

(detailed comparison Favart et al (16))

Evidence for contributions from transverse photons





HERMES(09)

$Q^2 \simeq 2.5\,\mathrm{GeV}^2$,$W = 3.99\,\mathrm{GeV}$

$\sin\phi_s$ modulation very large

does not vanish for $t' \to 0$

$A_{UT}^{\sin\phi_s} \propto \mathrm{Im}\Big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,0+}\Big]$

n-f. ampl. $\mathcal{M}_{0-,++}$ required

(ϕ_s orientation of target spin vector with respect to lepton plane)

CLAS(12)

unsep. cross sec. $d\sigma_T + \epsilon d\sigma_L$

$d\sigma_{LT}$, $d\sigma_{TT}$

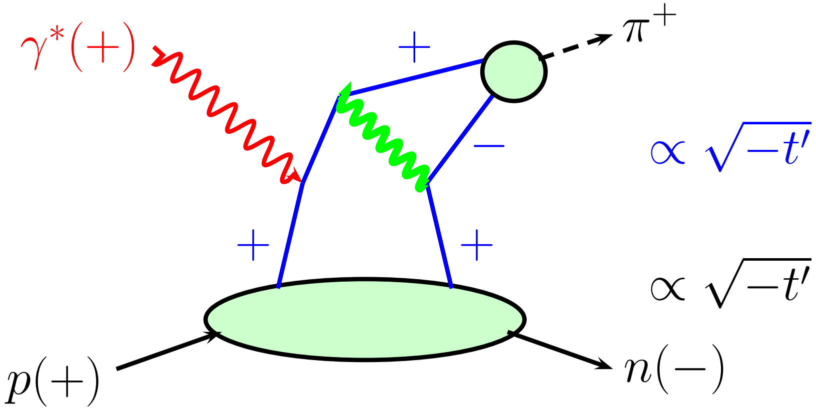
$|d\sigma_{TT}| \simeq 0.5\,d\sigma \Longrightarrow$

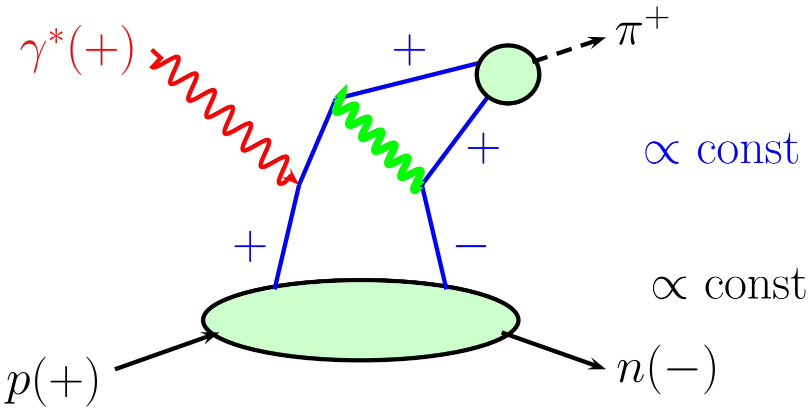
transverse amplitudes are large

How can we model $\mathcal{M}_{0-,++}$ in the handbag?

helicity-non-flip GPDs
$H, E, \widetilde{H}, \widetilde{E}$

helicity-flip (transv.) GPDs
$H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$





lead. twist pion wave fct. $\propto q' \cdot \gamma\gamma_5$
(perhaps including $\mathbf{k}_\perp$)

transversity GPDs required
go along with twist-3 w.f.

$\mathcal{M}_{0-,++} \propto t'$

$\mathcal{M}_{0-,++} \propto$ const

(forced by angular momentum conservation)

The twist-3 pion distr. amplitude

projector $q\bar{q} \to \pi$ (3-part. $q\bar{q}g$ contr. neglected) Beneke-Feldmann (01)

$$\sim q' \cdot \gamma\gamma_5\Phi + \mu_\pi\gamma_5\Big[\Phi_P - \imath\sigma_{\mu\nu}(\ldots\Phi'_\sigma + \ldots\Phi_\sigma\partial/\partial\mathbf{k}_{\perp\nu})\Big]$$

definition: $\langle\pi^+(q') \mid \bar{d}(x)\gamma_5 u(-x) \mid 0\rangle = f_\pi\mu_\pi \int d\tau e^{iq'x\tau}\Phi_P(\tau)$

local limit $x \to 0$ related to divergency of axial vector current

$\Longrightarrow \mu_\pi = m_\pi^2/(m_u + m_d) \simeq 2\,\mathrm{GeV}$ at scale $2\,\mathrm{GeV}$ (conv. $\int d\tau\Phi_P(\tau) = 1$)

Eq. of motion: $\tau\Phi_P = \Phi_\sigma/N_c - \tau\Phi'_\sigma/(2N_c)$

solution: $\Phi_P = 1, \quad \Phi_\sigma = \Phi_{AS} = 6\tau(1-\tau)$ Braun-Filyanov (90)

$H^{\mathrm{twist}-3}_{0-,++}(t=0) \neq 0$, Φ_P dominant, Φ_σ contr. $\propto t/Q^2$

in coll. appr.: $\mathcal{H}^{\mathrm{twist}-3}_{0-,++}$ singular, in $\mathbf{k}_\perp$ factorization (m.p.a.) regular

$$\mathcal{M}_{0-++} = e_0\sqrt{1-\xi^2}\int dx\mathcal{H}^{\mathrm{twist}-3}_{0-++}H_T\,, \qquad \mathcal{M}_{0+\pm+} = -e_0\frac{\sqrt{-t'}}{4m}\int dx\mathcal{H}^{\mathrm{twist}-3}_{0-++}\bar{E}_T$$

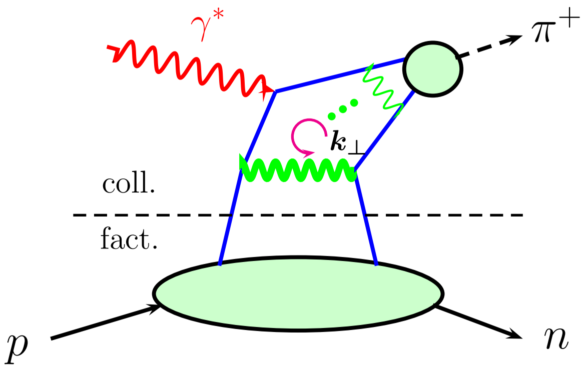
(suppressed by μ_π/Q as compared to $L \to L$ amplitudes) $(\bar{E}_T = 2\widetilde{H}_T + E_T)$

The subprocess amplitude

mod. pert. approach - quark trans. momenta in subprocess

(emission and absorption of partons from proton collinear to proton momenta)

transverse separation of color sources $\Longrightarrow$ gluon radiation



LO pQCD

$+$ quark trans. mom.

$+$ Sudakov supp.

$\Rightarrow$ asymp. fact. formula (lead. twist) for $Q^2 \to \infty$

Sudakov factor Sterman et al(93)

$$S(\tau, \mathbf{b}_\perp, Q^2) \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\rm QCD})}{-\ln(b_\perp \Lambda_{\rm QCD})} + \text{NLL}$$

resummed gluon radiation to NLL $\Rightarrow \exp[-S]$

provides sharp cut-off at $b_\perp = 1/\Lambda_{\rm QCD}$

$$\mathcal{H}_{0\pm0+} = \int d\tau d^2 b_\perp \, \hat{\Psi}^\pi_{\pm+}(\tau, -\mathbf{b}_\perp) \, e^{-S} \hat{\mathcal{F}}_{0\pm0+}(\bar{x}, \xi, \tau, Q^2, \mathbf{b}_\perp)$$

$\hat{\Psi}^\pi_{++} \sim \exp[\tau\bar{\tau} b_\perp^2/4a_M^2]$ LC wave fct of pion

$\hat{\mathcal{F}}$ FT of hard scattering kernel

e.g. $\propto 1/[k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi)] \Rightarrow$ Bessel fct

Sudakov factor generates series of power corr. $\sim (\Lambda^2_{\rm QCD}/Q^2)^n$

from intrinsic $k_\perp$ in wave fct: series $\sim (a_M Q)^{-n}$

Parametrizing the GPDs

double distribution ansatz (Mueller *et al* (94), Radyushkin (99))

$$K^i(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho+\xi\eta-x)\, K^i(\rho,\xi=0,t) w_i(\rho,\eta)$$

weight fct $w_i(\rho,\eta) \sim [(1-|\rho|)^2-\eta^2]^{n_i}$ $(n_{\rm sea}=2, n_{\rm val}=1$, generates ξ dep.)

zero-skewness GPD $K^i(\rho,\xi=0,t) = k^i(\rho)\exp\left[(b_{ki}+\alpha'_{ki}\ln(1/\rho))t\right]$

$k=\Delta q, \delta^q$ for $\widetilde{H}, H_T$ and $N_{ki}\rho^{-\alpha_{ki}(0)}(1-\rho)^{\beta_{ki}}$ for $\widetilde{E}, \bar{E}_T$

Regge-like t dep. (for small $-t$ reasonable appr.)

advantages: polynomiality and reduction formulas automatically satisfied
positivity bounds respected (checked numerically)

(no D-term for $\widetilde{H}$, $\widetilde{E}$ and the transversity GPDs)

Details of the parametrization

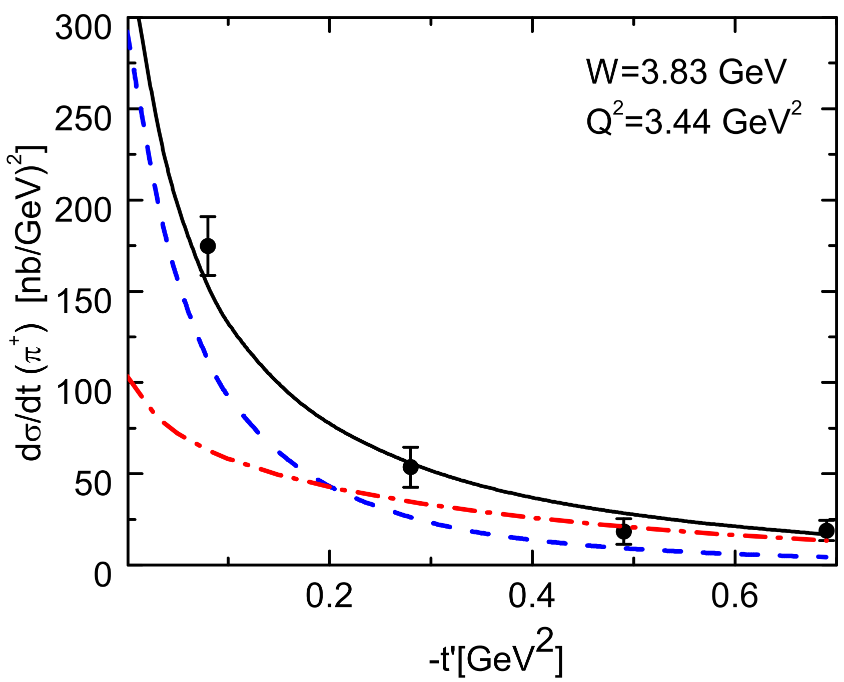
$\widetilde{H}$: taken from analysis of nucleon form factors (sum rules) Diehl-K.(13)

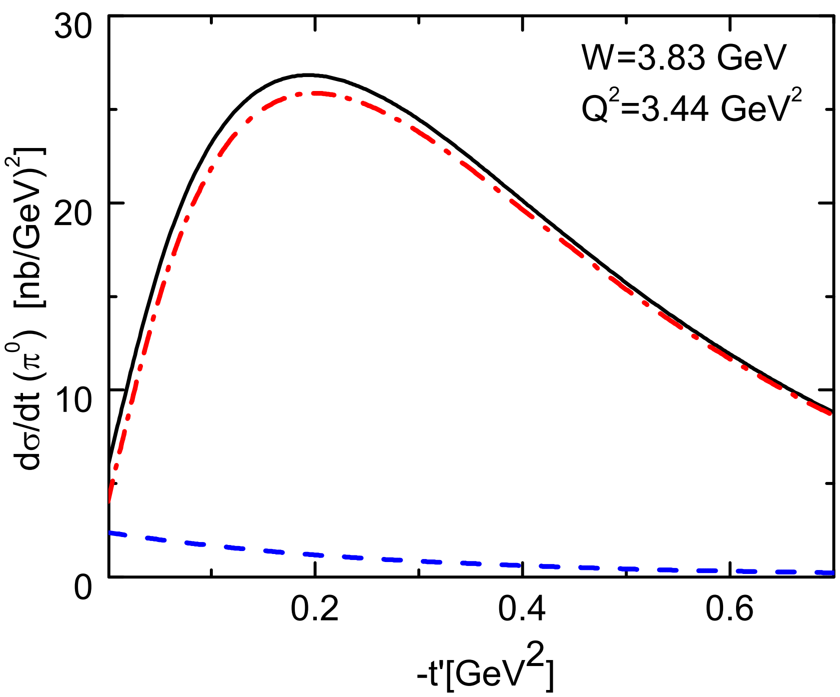
$\widetilde{E}$: fit to π^+ data

H_T: PDFs $\quad \delta^q(x) = N^q_{H_T}\sqrt{x}(1-x)\big[q(x)+\Delta q(x)\big]$ Anselmino et al(09)
opposite sign for u and d quarks, normalized to lattice moments QCDSF-UKQCD(05)

$\bar{E}_T$: adjusted to lattice results QCDSF-UKQCD(06)
Large, same sign and almost same size for u and d quarks
Burkardt: related to Boer-Mulders fct $\langle\cos{(2\phi)}\rangle$ in SIDIS $-$ same pattern

The role of H_T and $\bar{E}_T$





unseparated (longitudinal, transverse) cross sections

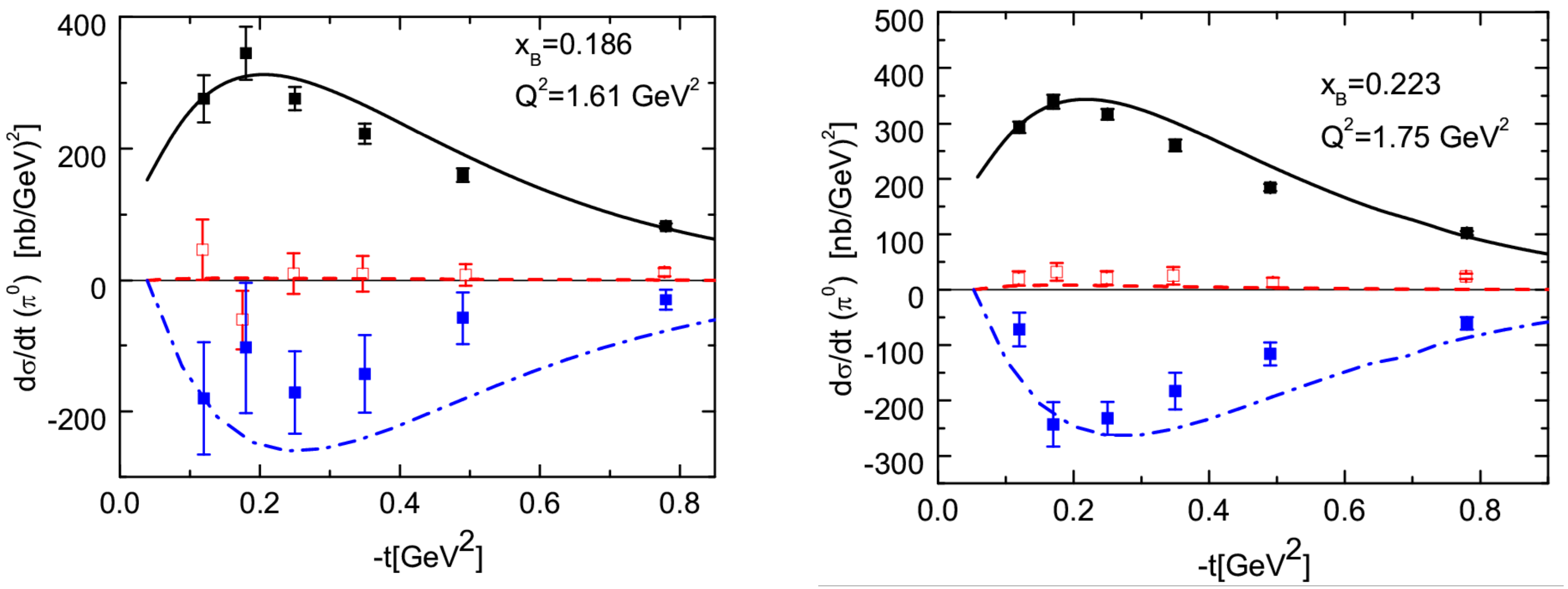
π^+: pion pole and $\propto K^u - K^d$ $\qquad$ π^0: no pion pole and $\propto e_u K^u - e_d K^d$

consider $u - d$ signs: $\widetilde{E}$, $\bar{E}_T$ same, $\widetilde{H}, H_T$ opposite sign

$\Longrightarrow$ $\widetilde{H}$ and H_T large for π^+, small for π^0

$\widetilde{E}$ and $\bar{E}_T$ small for π^+, large for π^0

Results for π^0

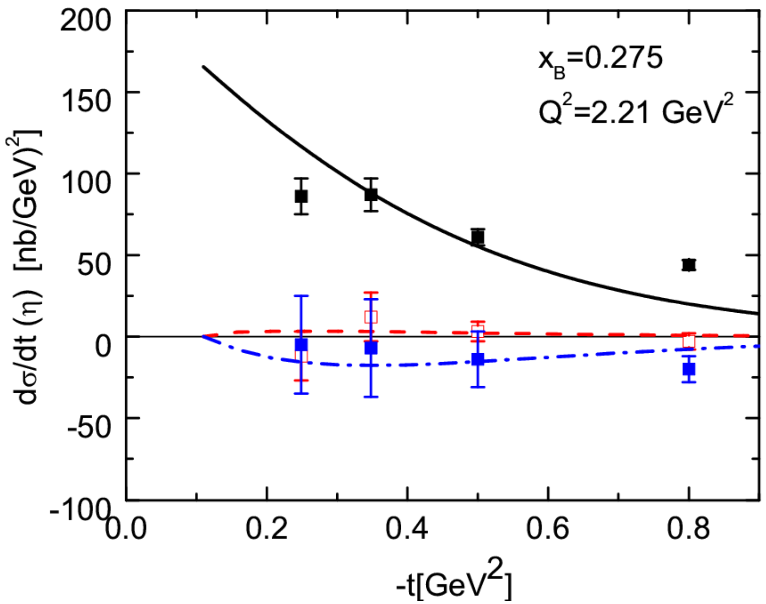


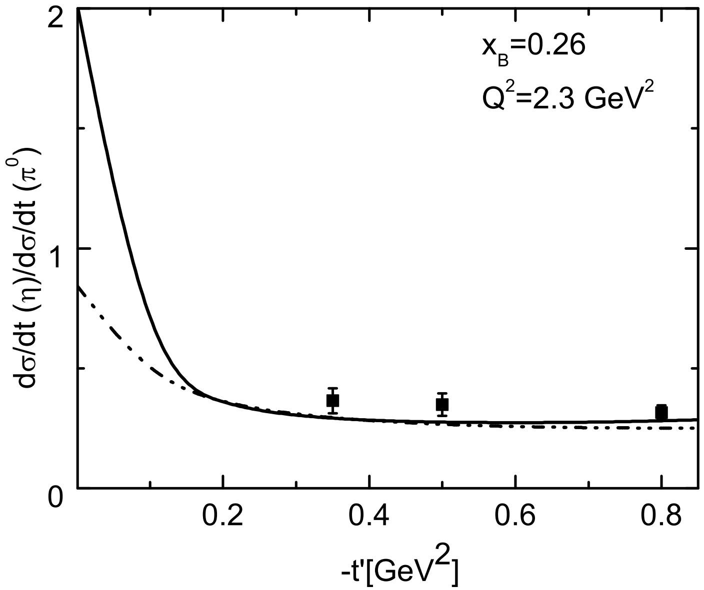
data: Bedlinsky et al (12)

curves: Goloskokov-K(11)

transversity GPDs in pion production also studied by Goldstein et al (12)

η production





data CLAS (prel.) unseparated (longitudinal, transverse) cross sections

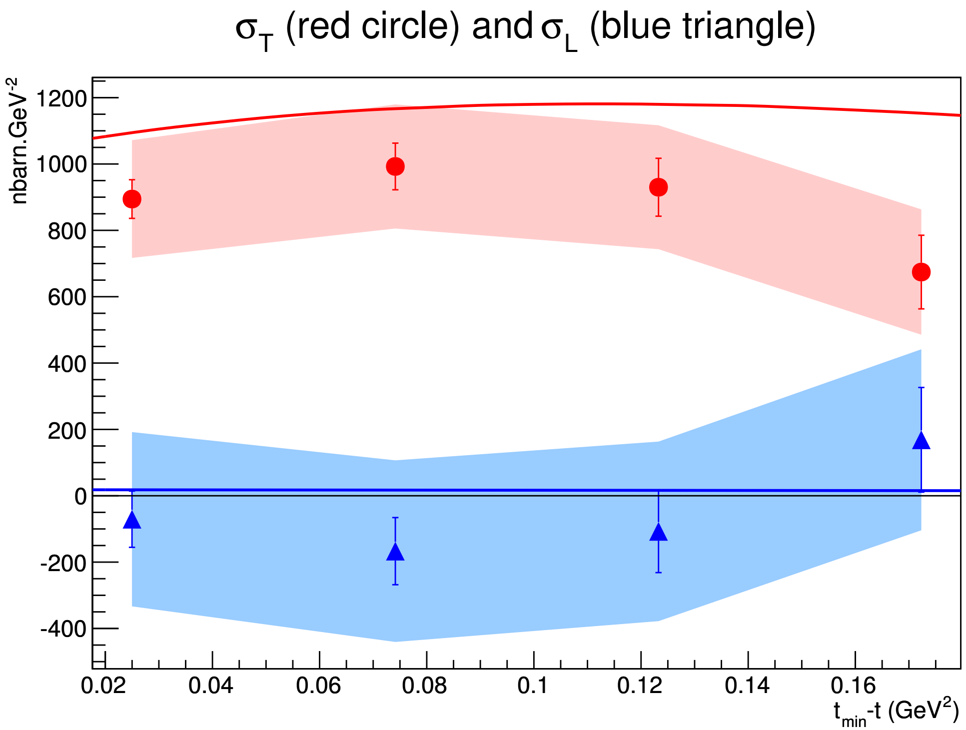
$$\frac{d\sigma(\eta)}{d\sigma(\pi^0)} \simeq \left(\frac{f_\eta}{f_\pi}\right)^2 \frac{1}{3} \left|\frac{e_u\langle K^u\rangle + e_d\langle K^d\rangle}{e_u\langle K^u\rangle - e_d\langle K^d\rangle}\right|^2 \qquad (f_\eta \simeq 1.26 f_\pi)$$

if K^u and K^d have opposite sign: $\eta/\pi^0 \gtrsim 1$ same sign: $\eta/\pi^0 < 1$

$t' \simeq 0$ $\widetilde{H}, H_T$ dominant (see also Eides et al(98) assuming dominance of $\widetilde{H}$ for all t')

$t' \neq 0$ $\bar{E}_T$ dominant

The separated π^0 cross section



data: Hall A preliminary

$Q^2 = 1.76\,\mathrm{GeV}^2$, $x_B = 0.36$

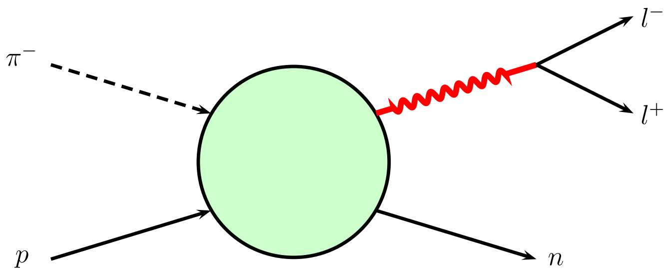
solid lines:

Goloskokov-K(11) predictions

$$d\sigma_T \gg d\sigma_L$$

like expectations for $Q^2 \to 0$ and not for $Q^2 \to \infty$

The exclusive Drell-Yan process



Berger-Diehl-Pire (01): leading-twist, LO analysis of long. cross section
(i.e. exploiting asymp. factor. formula)
(detailed reanalysis Sawada et al, 1605.00364)

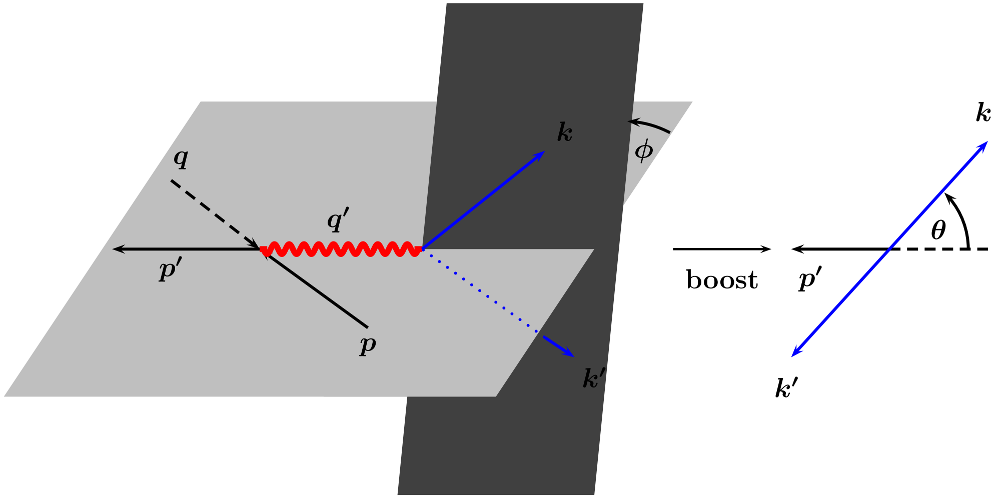
we know that leading-twist analysis of π^+ production fails with JLAB, HERMES data by order of magnitude

Therefore . . . (Goloskokov-K. 1506.04619)

a reanalyis of the exclusive Drell-Yan process seems appropriate
making use of what we have learned from analysis of pion production

- treating pion-pole contribution as an OPE term
- take into account transverse photons and transversity GPDs
- retaining quark transverse momenta in the subprocess (the MPA)

Cross section



k momentum of l^-

$\tau = Q'^2/(s - m^2)$

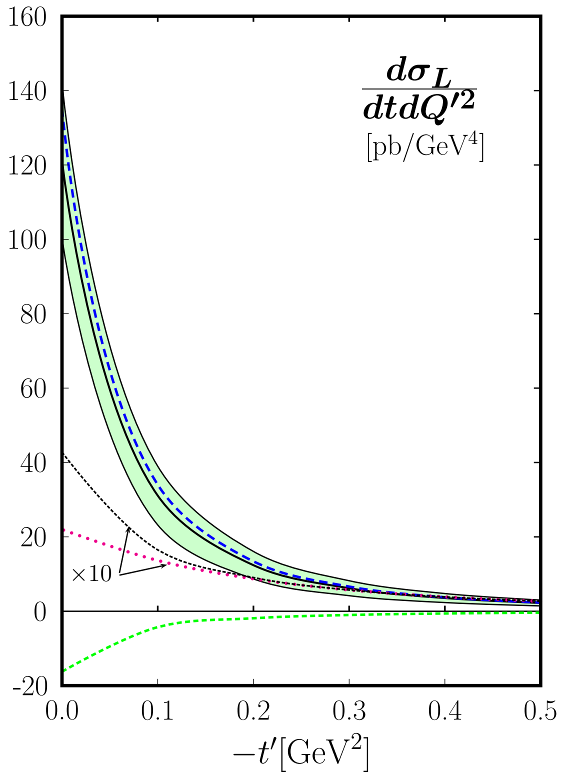
the time-like analogue of x_B

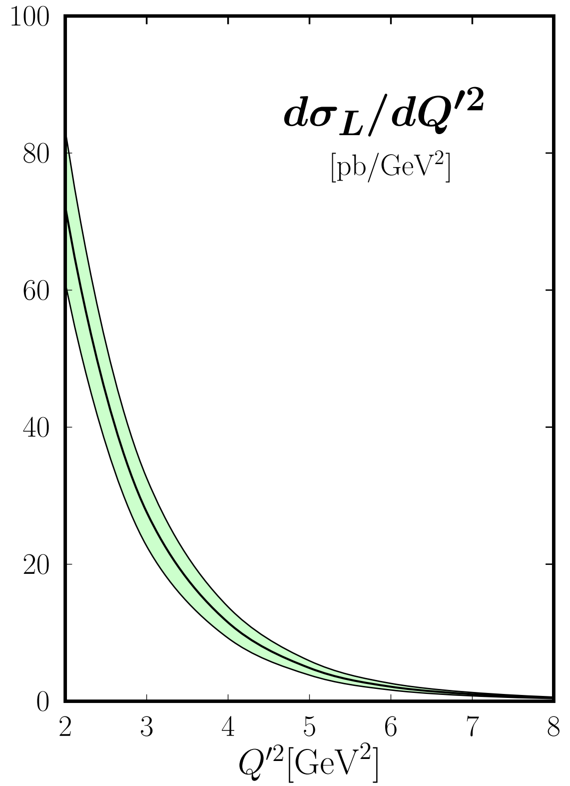
$$\frac{d\sigma}{dt dQ'^2 d\cos\theta d\phi} = \frac{3}{8\pi}\left\{\sin^2\theta\,\frac{d\sigma_L}{dtdQ'^2} + \frac{1+\cos^2\theta}{2}\,\frac{d\sigma_T}{dtdQ'^2}\right.$$
$$\left. + \frac{1}{\sqrt{2}}\sin(2\theta)\cos\phi\,\frac{d\sigma_{LT}}{dtdQ'^2} + \sin^2\theta\cos(2\phi)\,\frac{d\sigma_{TT}}{dtdQ'^2}\right\}$$

$$\frac{d\sigma_L}{dtdQ'^2} = \frac{\alpha_{\rm elm}}{48\pi^2}\frac{\tau^2}{Q'^6}\sum_{\nu'}|\mathcal{M}_{0\nu',0+}|^2 \qquad \frac{d\sigma_T}{dtdQ'^2} = \frac{\alpha_{\rm elm}}{48\pi^2}\frac{\tau^2}{Q'^6}\sum_{\mu=\pm1,\nu'}|\mathcal{M}_{\mu\nu',0+}|^2$$

partial cross sections analogous to pion production

Results on the longitudinal cross section





$Q'^2 = 4\,\mathrm{GeV}^2$ and $s = 20\,\mathrm{GeV}^2$ solid lines with error bands: full result

pion pole, $|\langle \widetilde{H}^{(3)} \rangle|^2$, interference, short dashed: leading-twist contribution

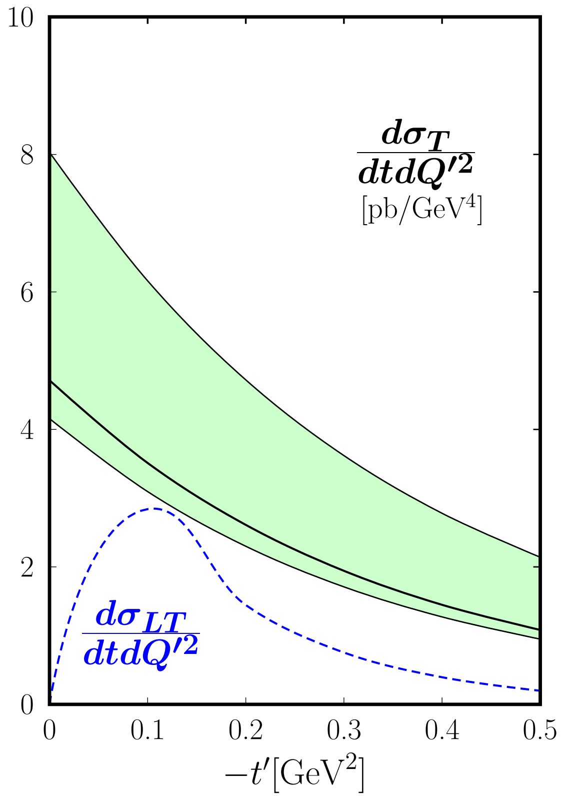
time-like pion FF: $Q'^2|F_\pi(Q'^2)| = 0.88 \pm 0.04\,\mathrm{GeV}^2$ (CLEO, BaBar, $J/\Psi \to \pi^+\pi^-$)

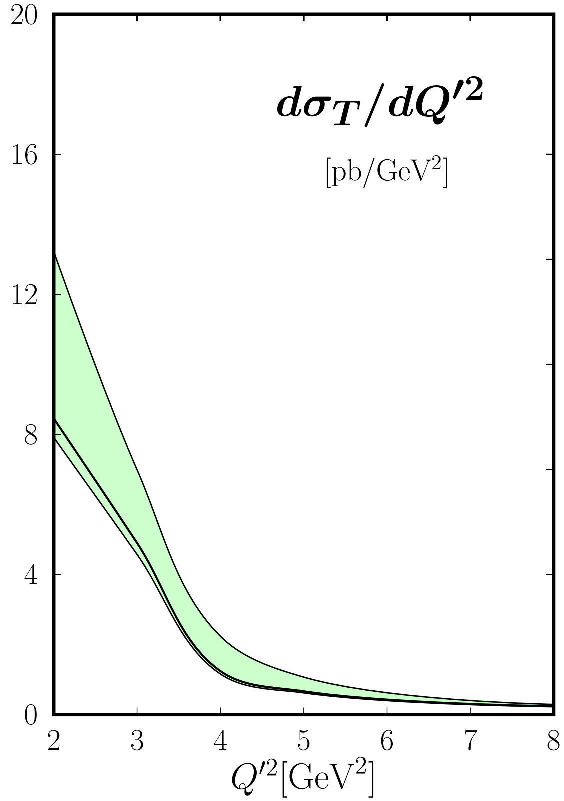
phase $(\exp[i\delta(Q'^2)])$ from disp. rel. Belicka et al(11) for $Q'^2 < 8.9\,\mathrm{GeV}^2$

$\delta = 1.014\pi + 0.195(Q'^2/\mathrm{GeV}^2 - 2) - 0.029(Q'^2/\mathrm{GeV}^2 - 2)^2$

for $Q'^2 \geq 8.9\,\mathrm{GeV}^2$: $\delta = \pi$, the LO pQCD result

Results on the transversal cross section





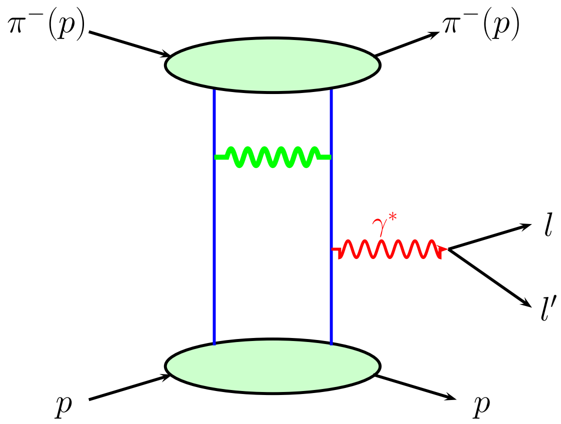
$Q'^2 = 4\,\mathrm{GeV}^2$ and $s = 20\,\mathrm{GeV}^2$
dominated by H_T

$\bar{E}_T^u - \bar{E}_T^d$ small

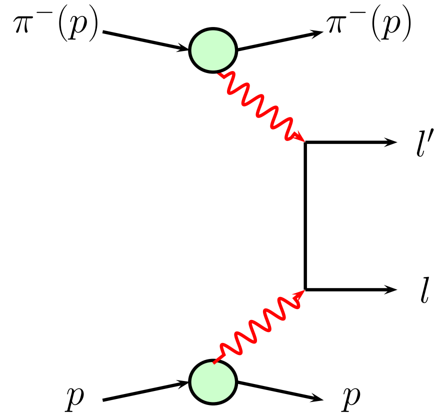
(COMPASS: $s = 360\,\mathrm{GeV}^2$ pb $\rightarrow$ fb)

Lepton-pair production in exclusive hadron-hadron collisions

Pivovarov-Teryaev (14): double handbag



access to pion GPD



elm. contribution

$\sim F^{\pi(p)}_{\mathrm{elm}} F^{p}_{\mathrm{elm}}$

Remarks on processes with time-like virtual photons

- time-like excl. processes difficult to understand theoretically
  e.g. no satisfactory explanation of time-like elm form factors
  within pert. QCD

- Drell-Yan process $\pi^- p \to l^+ l^- X$
  large K-factor needed (larger than NLO corr. Sutton et al (92))
  now understood as 'threshold logs' $(Q'^2/(x_1 x_2 s) \to 1)$
  (gluon radiation resummed to NLL Sterman(87), Catani-Trentadue(89))
  leading finally to reasonable fits of data and extraction of PDFs for the
  pion with plausible behavior for $x \to 1$ Aicher-Schäfer-Vogelsang (11)

- hard exclusive scattering processes with time-like virtual photons
  no data as yet but predictions
  time-like DVCS (Pire et al (13)) and $\pi^- p \to l^+ l^- n$
  does factorization hold in time-like region? (Qui(15))
  experimental verification of predictions important

Summary

- asymptotia is far away
  interpretation of data on pion leptoproduction requires strong power corrections from the pion pole and from transverse photons
- within handbag approach $\gamma_T^* \to \pi$ transitions are related to transversity (helicity-flip) GPDs accompanied by a twist-3 pion wave fct.
- making use of what we have learned from pion leptoproduction we evaluated the long. and transverse cross sections for the exclusive Drell-Yan process
- long. cross section dominated by the pion pole
  (not subject to evolution and pert. corrections)
  transverse cross section fed by H_T ($\bar{E}_T$ small)
  Φ-dependence: various interference terms
- t.l. π FF: $l^+l^- \to \pi^+\pi^-$ (CLEO, BaBar) versus $\pi^-\pi^{+*} \to l^+l^-$