Review of GPDs and TMDs

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http://menu2016.riken.jp

Scientific Topics
- Meson-Nucleon Interactions
- Hadron Spectroscopy
- Nucleon Structure
- Few Body Systems
- Fundamental Symmetries
- Electroweak Probes
- Future Facilities and Directions

Why TMDs and GPDs?

Transverse momentum dependent parton distribution functions

(TMDs), and

generalized parton distributions

(GPDs)

encode rich and unique information

on

confined motion, and spatial distribution of quarks and gluons

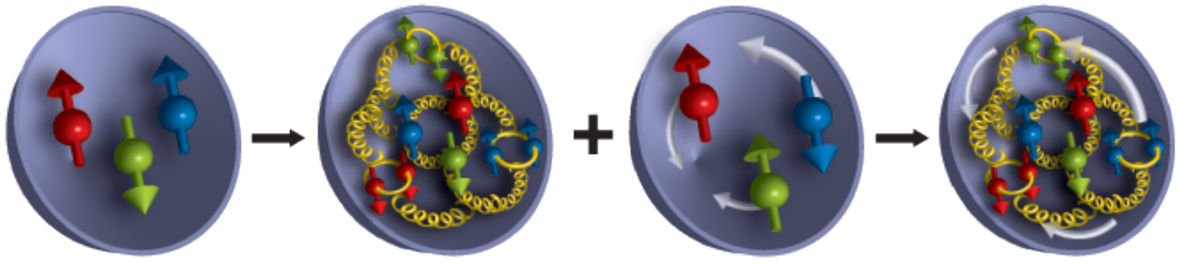
inside a hadron,

respectively

(Gateway to hadron’s 3D partonic structure!)

How to "see" and quantify hadron structure?

- ☐ **Our understanding of hadron evolves**



1970s 1980s/2000s Now

Nucleon is a strongly interacting, relativistic bound state of quarks and gluons

- ☐ **Challenge:**

No modern detector can see quarks and gluons in isolation!

- ☐ **Question:**

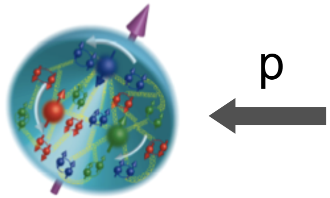
How to quantify the hadron structure if we cannot see quarks and gluons? ***We need the probe!***

- ☐ **Answer:**

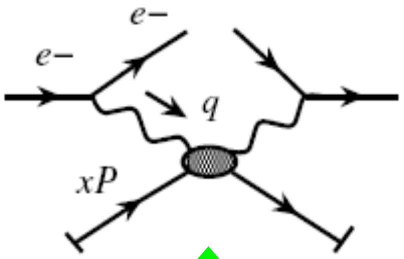
QCD factorization! ***Not exact, but, controllable approximation!***

QCD Factorization: connecting parton to hadron

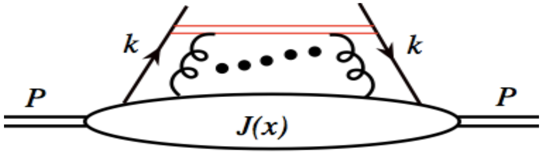
❑ **One hadron:**



$$\sigma_{\rm tot}^{\rm DIS} \sim$$



$$\otimes$$



$$+ O\left(\frac{1}{QR}\right)$$

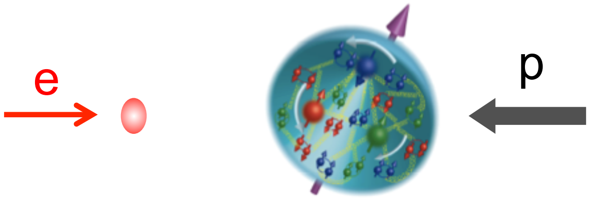
Hard-part
Probe

Parton-distribution
Structure

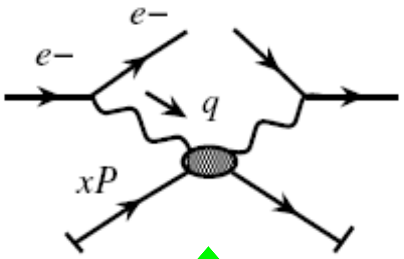
Power corrections
Approximation

QCD Factorization: connecting parton to hadron

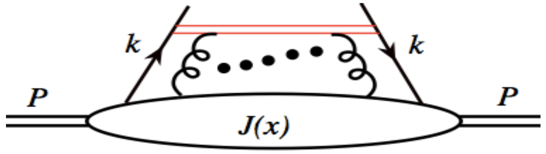
❑ One hadron:



$$\sigma^{\mathrm{DIS}}_{\mathrm{tot}} \sim$$



$$\otimes$$



$$+\,O\left(\frac{1}{QR}\right)$$

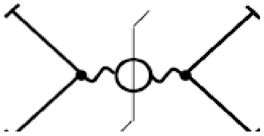
Hard-part
Probe

Parton-distribution
Structure

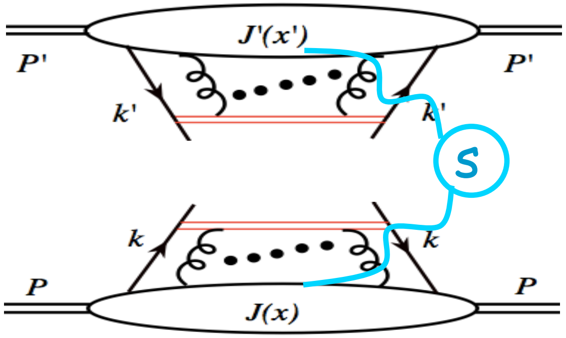
Power corrections
Approximation

❑ Two hadrons:

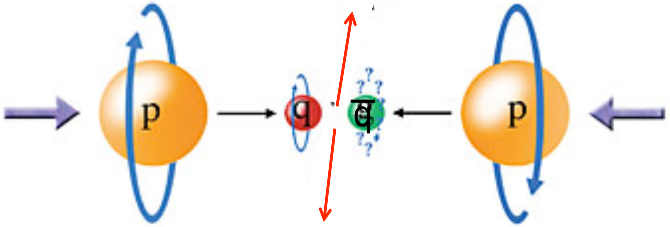
$$\sigma^{\mathrm{DY}}_{\mathrm{tot}} \sim$$



$$\otimes$$



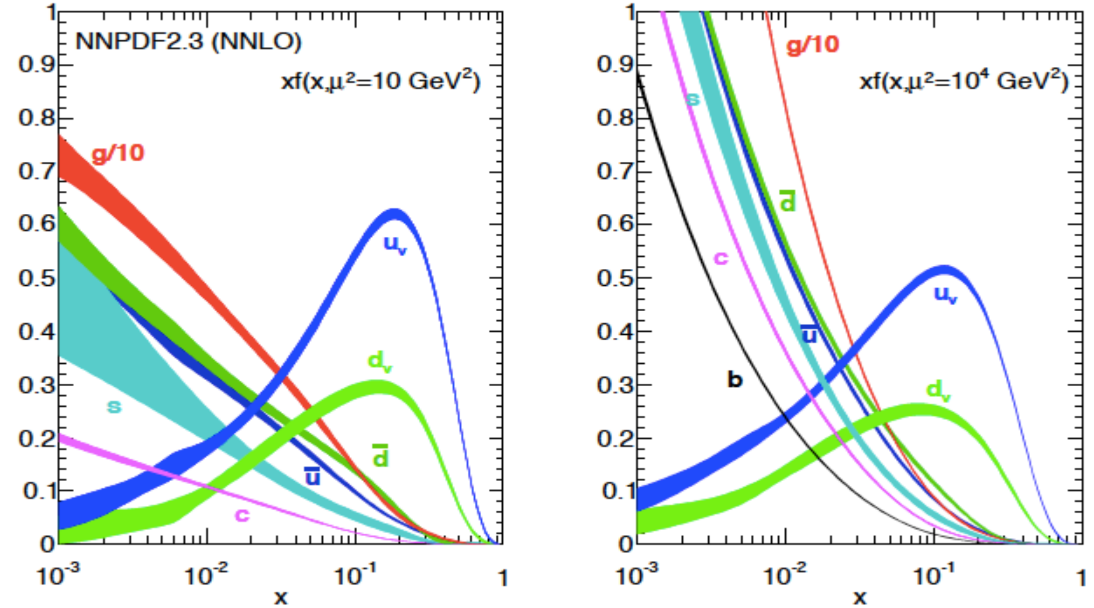
$$+\,O\left(\frac{1}{QR}\right)$$



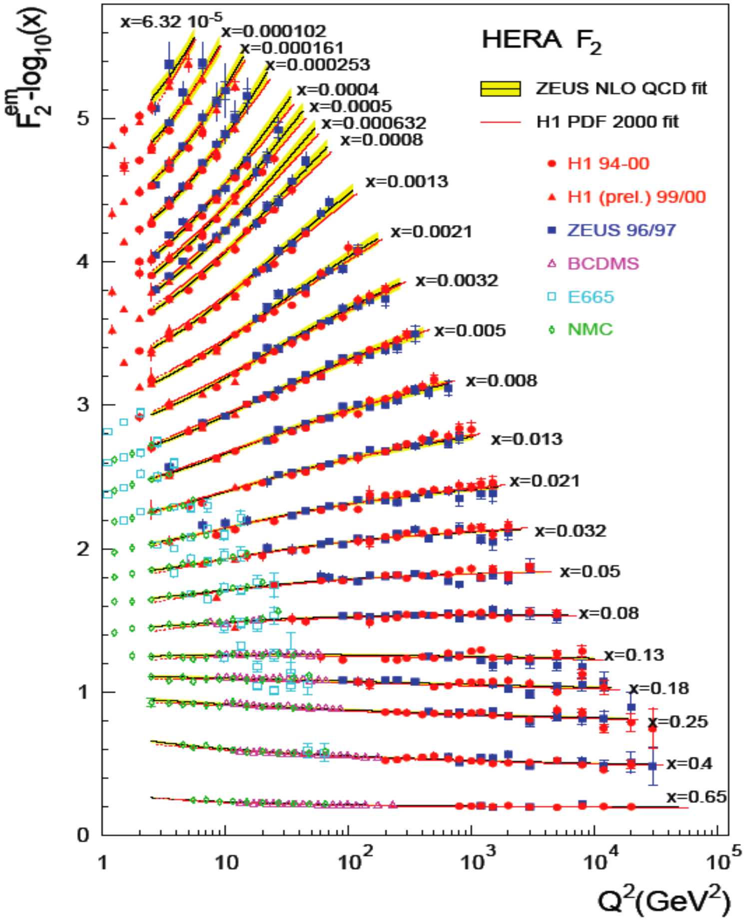
Predictive power:
Universal Parton Distributions

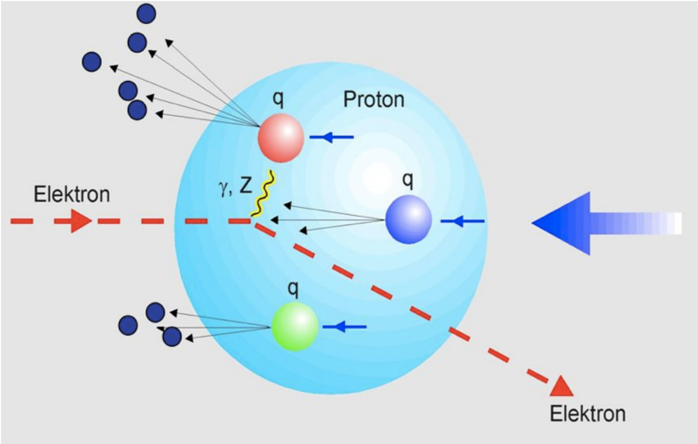
QCD factorization works!

- One set PDFs:



K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)





DIS structure function

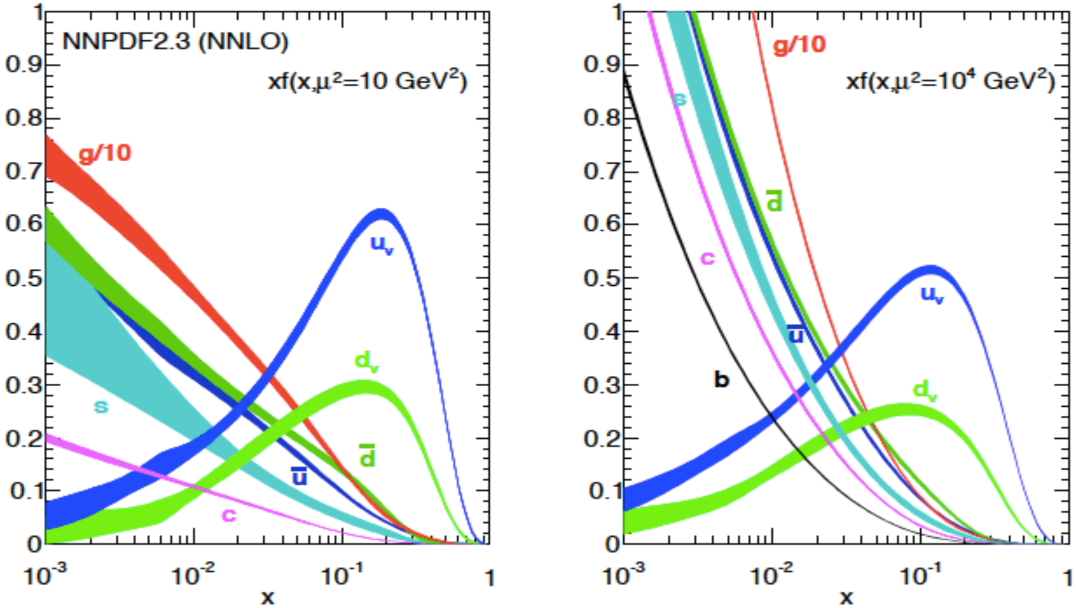
One hadron

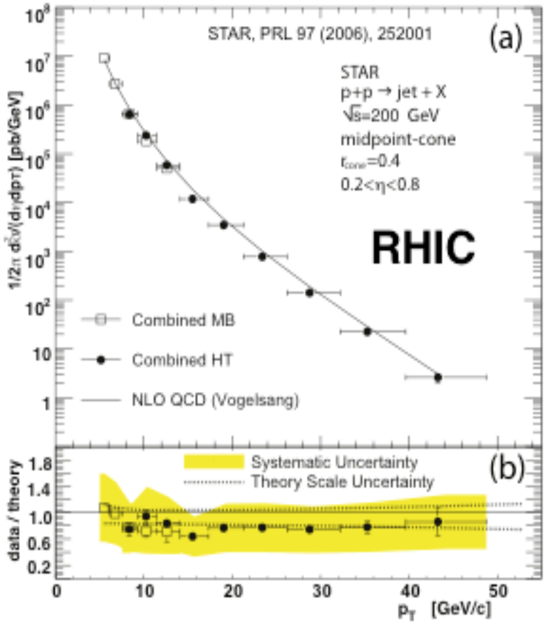
QCD factorization works!

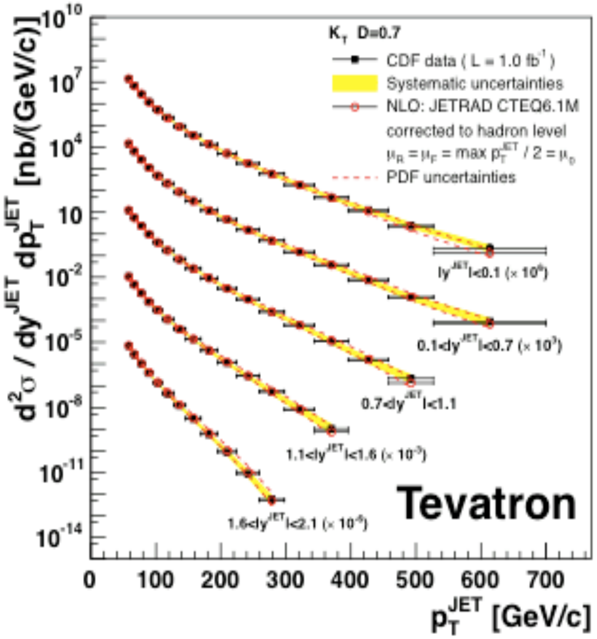
- ❑ **One set PDFs:**

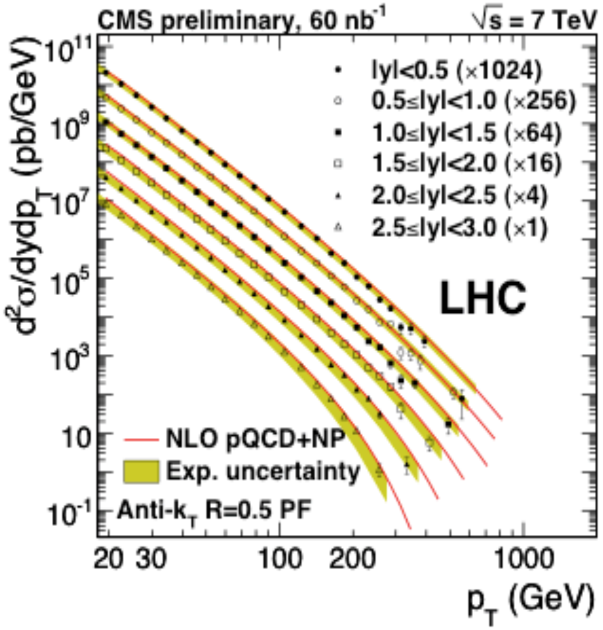
Hadronic jet cross section

Two hadrons



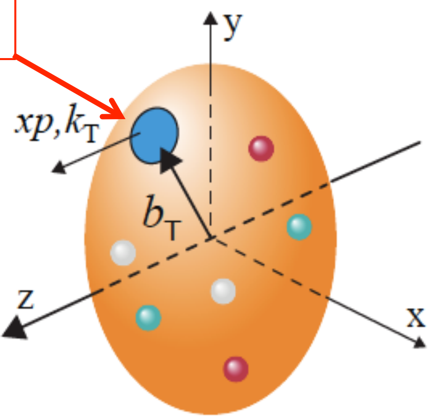






Our knowledge on hadron structure?

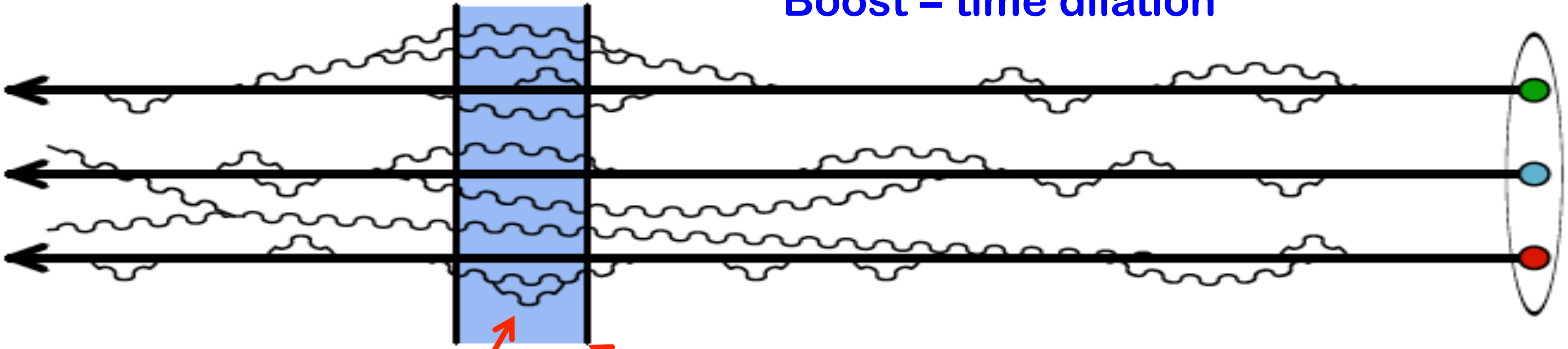
- **QCD has been extremely successful in interpreting and predicting high energy experimental data!**



< 1/10 fm

- **But, we still do NOT know much about hadron structure – work just started!**

- **High energy probes "see" the boosted partonic structure:**



Boost = time dilation

Momentum fraction x

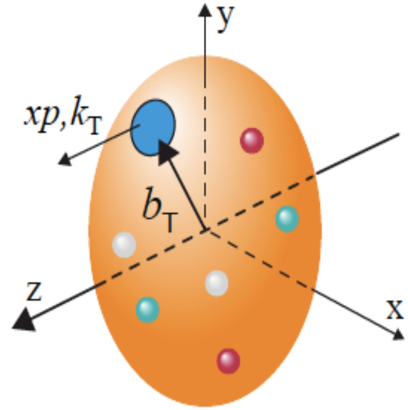
Hard probe: t ~ 1/Q < 1/10 fm

➡ ***Confined motion:*** $1/R \sim \Lambda_{\rm QCD} \ll Q$ ***is too week to be relevant***

Two-momentum-scale observables

☐ Cross sections with two-momentum scales observed:

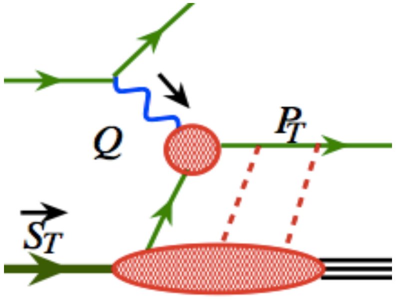
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$



✧ **Hard scale:** Q_1 **localizes the probe to see the quark or gluon d.o.f.**

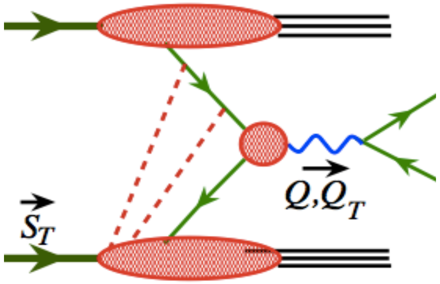
✧ **“Soft” scale:** Q_2 **could be more sensitive to hadron structure, e.g., confined motion**

☐ Two-scale observables with the hadron broken:



SIDIS: $Q \gg P_T$

+



DY: $Q \gg P_T$

+ **Two-jet momentum imbalance in SIDIS, …**

✧ **Natural observables with TWO very different scales**

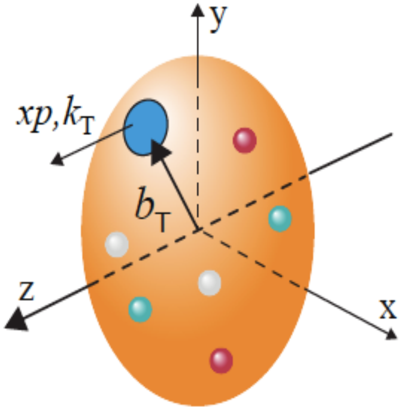
✧ **TMD factorization: partons’ confined motion is encoded into TMDs**

Two-momentum-scale observables

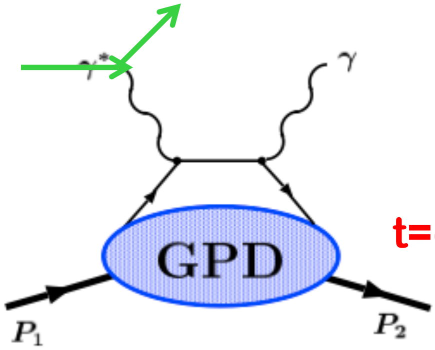
☐ Cross sections with two-momentum scales observed:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

- ✧ **Hard scale:** Q_1 localizes the probe to see the quark or gluon d.o.f.
- ✧ **"Soft" scale:** Q_2 could be more sensitive to hadron structure, e.g., confined motion



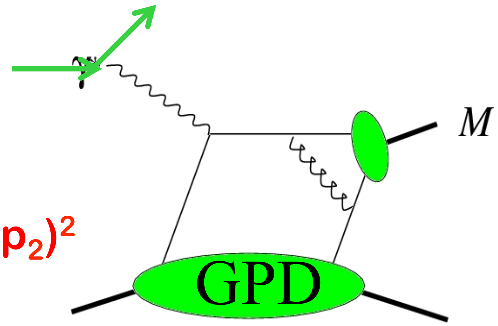
☐ Two-scale observables with the hadron unbroken:



DVCS: $Q^2 >> |t|$

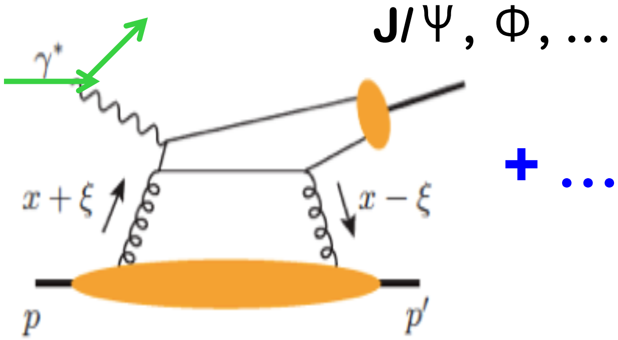
+

$t=(p_1-p_2)^2$



DVEM: $Q^2 >> |t|$

+



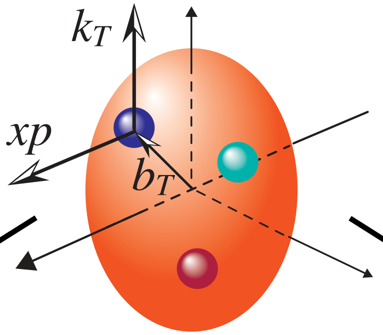
EHMP: $Q^2 >> |t|$

+ ...

- ✧ Natural observables with TWO very different scales
- ✧ GPDs: Fourier Transform of t-dependence gives spatial b_T-dependence

Unified view of nucleon structure

- ☐ **Wigner distributions:**



Momentum Space

TMDs

Confined motion

$\int d^2b_T$

$f(x,k_T)$

Two-scales observables

$\int d^2k_T$

$f(x,b_T)$

Coordinate Space

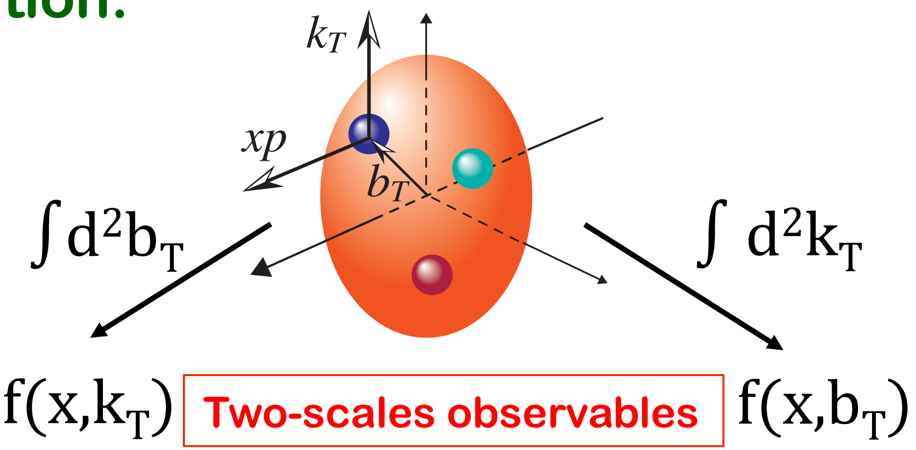
GPDs

Spatial distribution

Unified view of nucleon structure

Also see development of GTMDs

❑ Wigner distribution:



Momentum Space

TMDs

Confined motion

$\int d^2b_T$

$f(x,k_T)$

Two-scales observables

$\int d^2k_T$

$f(x,b_T)$

Coordinate Space

GPDs

Spatial distribution

❑ Note:

- ✧ Partons' confined motion and their spatial distribution are unique – the consequence of QCD
- ✧ But, the TMDs and GPDs that represent them are not unique!
  - – Depending on the definition of the Wigner distribution and QCD factorization to link them to physical observables

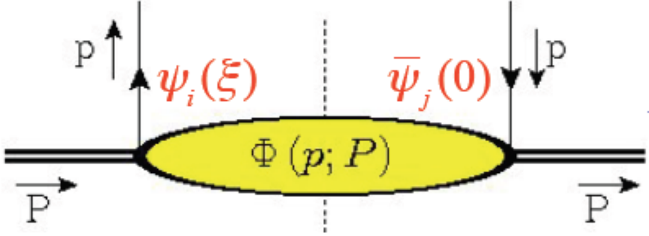
Position *r* x Momentum *p* → Orbital Motion of Partons

Questions/issues for TMDs

❑ Non-perturbative definition:

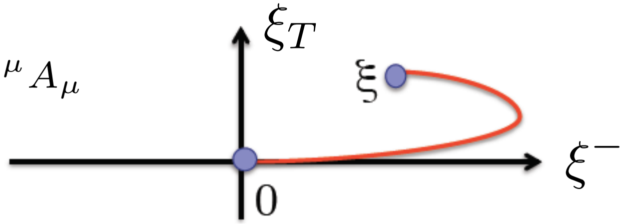
✧ **In terms of matrix elements of parton correlators:**

$$\Phi^{[U]}(x,p_T;n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,p\cdot\xi}\, \langle P,S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P,S\rangle_{\xi^+=0}$$



✧ **Depends on the choice of the gauge link:**

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ **Decomposes into a list of TMDs:**

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5$$
$$+ h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\, \not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

Questions/issues for TMDs

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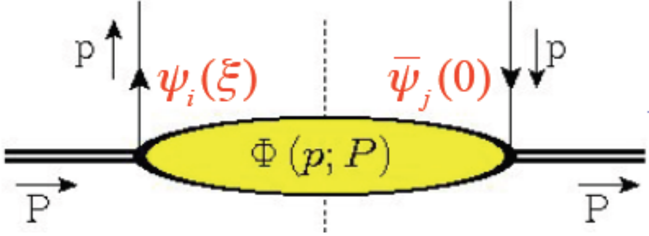

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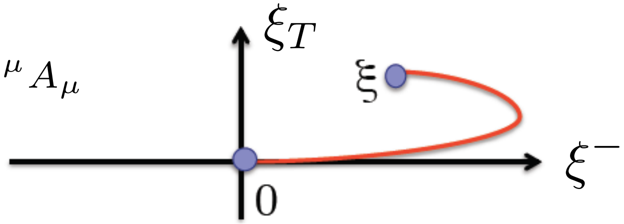

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✧ **IF we knew proton wave function, this definition gives "unique" TMDs!**

But, we do NOT know proton wave function (may calculate it using BSE?)

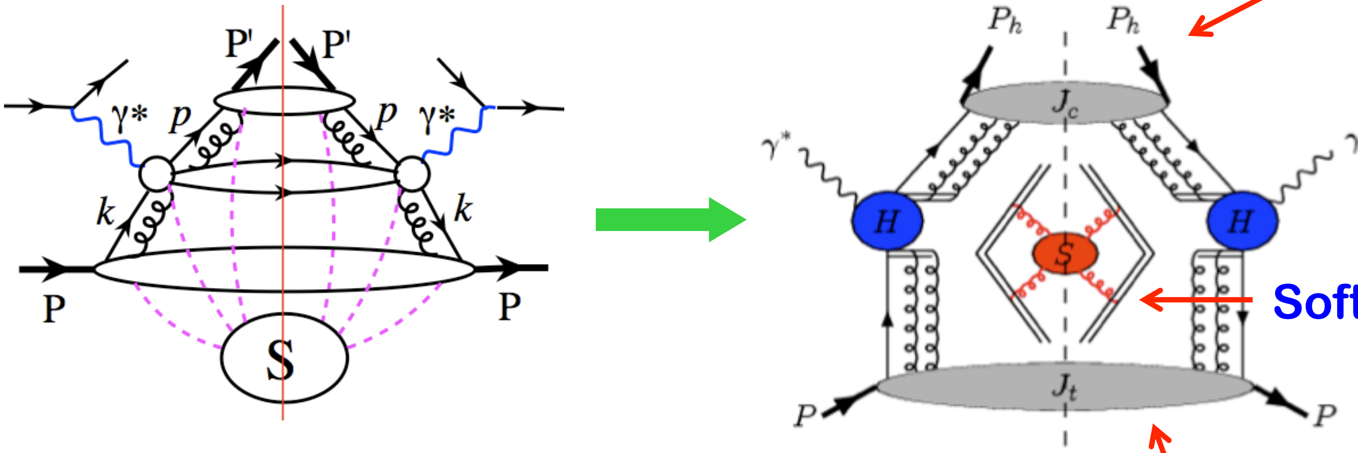
TMDs defined in this way are NOT direct physical observables!

Questions/issues for TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:

TMD fragmentation



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ Extraction of TMDs:

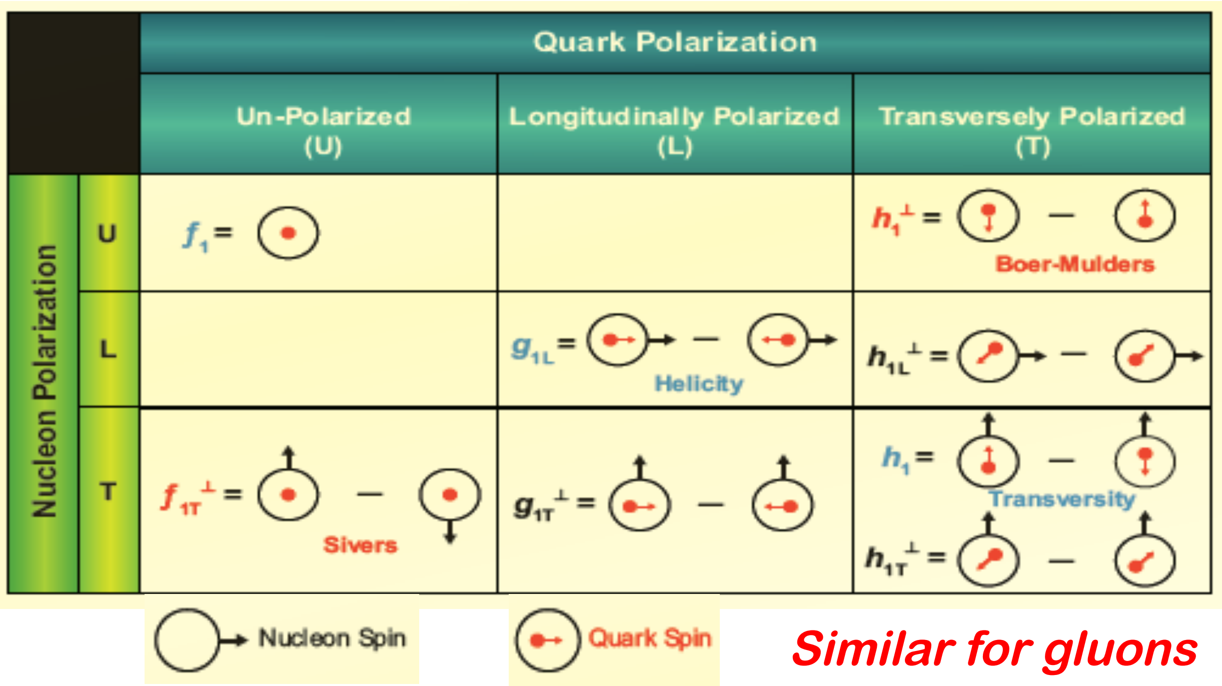
$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

➡ Extracted TMDs are valid only when the $<p^2> << Q^2$

TMDs: confined motion, its spin correlation

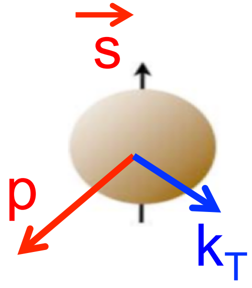
- ❑ **Power of spin – many more correlations:**



		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ − Boer-Mulders
	L		$g_{1L} =$ − Helicity	$h_{1L}^\perp =$ −
	T	$f_{1T}^\perp =$ − Sivers	$g_{1T}^\perp =$ −	$h_1 =$ − Transversity $h_{1T}^\perp =$ −

Nucleon Spin

Quark Spin

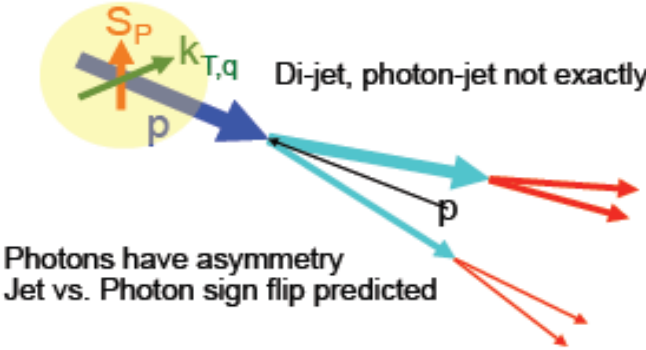


Similar for gluons

Require two Physical scales

More than one TMD contribute to the same observable!

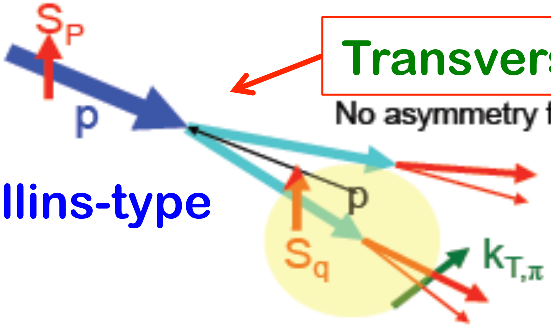
- ❑ **A_N – single hadron production:**



Di-jet, photon-jet not exactly back to back

Photons have asymmetry
Jet vs. Photon sign flip predicted

Sivers-type



Transversity

No asymmetry for the jet axis

Collins-type

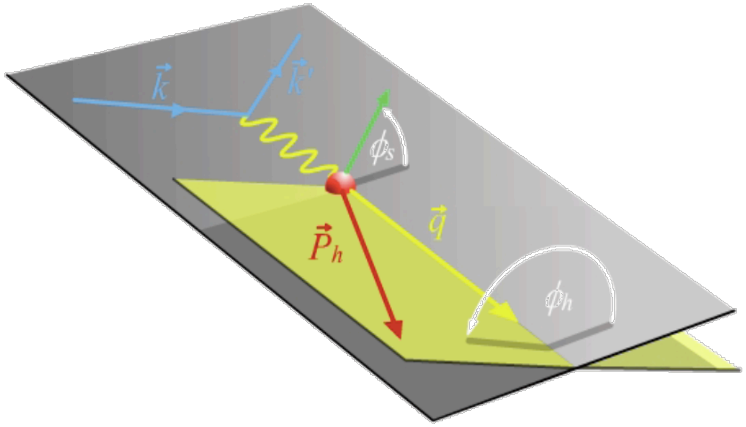
SIDIS is the best for probing TMDs

- **Naturally, two scales & two planes:**

$$A_{UT}(\varphi_h^l,\varphi_S^l)=\frac{1}{P}\frac{N^{\uparrow}-N^{\downarrow}}{N^{\uparrow}+N^{\downarrow}}$$

$$=A_{UT}^{Collins}\sin(\phi_h+\phi_S)+A_{UT}^{Sivers}\sin(\phi_h-\phi_S)$$

$$+A_{UT}^{Pretzelosity}\sin(3\phi_h-\phi_S)$$



- **Separation of TMDs:**

$$A_{UT}^{Collins}\propto\left\langle\sin(\phi_h+\phi_S)\right\rangle_{UT}\propto h_1\otimes H_1^{\perp}$$

$$A_{UT}^{Sivers}\propto\left\langle\sin(\phi_h-\phi_S)\right\rangle_{UT}\propto f_{1T}^{\perp}\otimes D_1$$

$$A_{UT}^{Pretzelosity}\propto\left\langle\sin(3\phi_h-\phi_S)\right\rangle_{UT}\propto h_{1T}^{\perp}\otimes H_1^{\perp}$$

Collins frag. Func. from e$^+$e$^-$ collisions

Hard, if not impossible, to separate TMDs in hadronic collisions

Using a combination of different observables (not the same observable): jet, identified hadron, photon, …

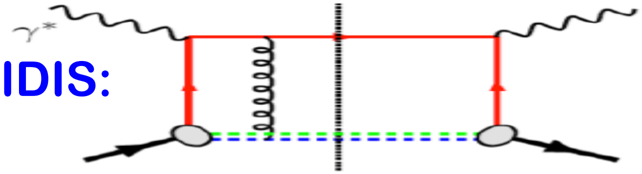
Modified universality for TMDs

❑ Definition:

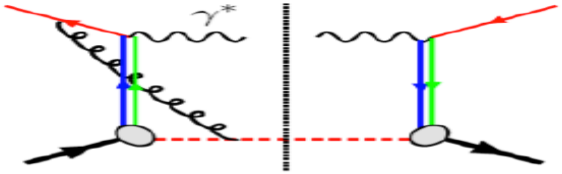
$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\boxed{\text{Gauge link}}\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

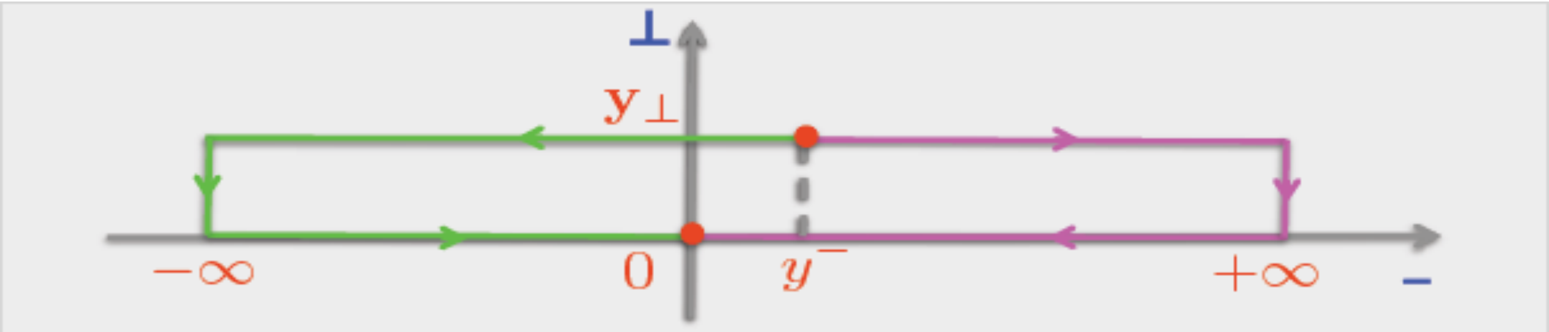
❑ Gauge links:

SIDIS:



DY:





❑ Process dependence:

$$f_{q/h^\uparrow}^{\text{SIDIS}}(x,\mathbf{k}_\perp,\vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x,\mathbf{k}_\perp,\vec{S})$$

Collinear factorized PDFs are process independent

Critical test of TMD factorization

- ☐ **Parity – Time reversal invariance:**

$$f_{q/h^\uparrow}^{\text{SIDIS}}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\text{DY}}(x,\mathbf{k}_\perp,-\vec{S})$$

- ☐ **Definition of Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

- ☐ **Modified universality:**

$$\Delta^N f_{q/h^\uparrow}^{\text{SIDIS}}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\text{DY}}(x,k_\perp)$$

The spin-averaged part of this TMD is process independent,

but, spin-averaged Boer-Mulder's TMD requires the sign change!

Same PT symmetry examination needs for TMD gluon distributions!

Global QCD analysis: extraction of TMDs

❑ QCD TMD factorization:

– Connect cross sections, asymmetries to TMDs

✧ Factorization is known or expected to be valid for SIDIS, Drell-Yan (Υ^*, W/Z, H^0,…), 2-Jet imbalance in DIS, …

✧ *Same level of reliability as collinear factorization for PDFs, up to the sign change*

❑ QCD evolution of TMDs:

– TMDs evolve when probed at different momentum scales

✧ Evolution equations are for TMDs in b_T-space (Fourier Conjugate of k_T)

FACT: QCD evolution does NOT fully fix TMDs in momentum space,
even with TMDs fixed at low Q – large b_T-input!!!

✧ *Very different from DGLAP evolution of PDFs – in momentum space*

FACT: QCD evolution uniquely fix PDFs at large Q, once the PDFs
is determined at lower Q – linear evolution in momentum space

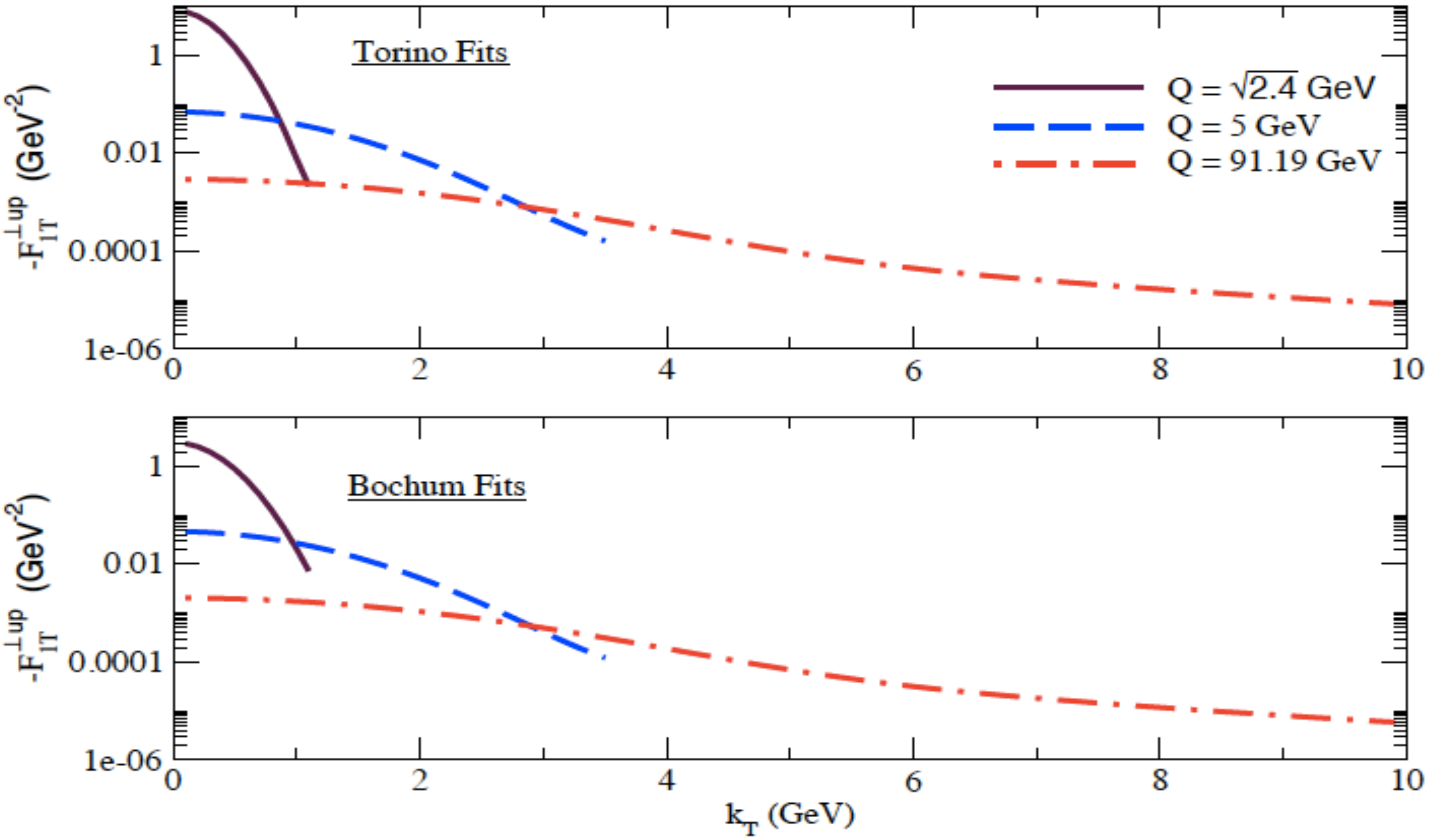
❑ Challenges:

Predictive power, extraction of hadron structure, …

Evolution of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

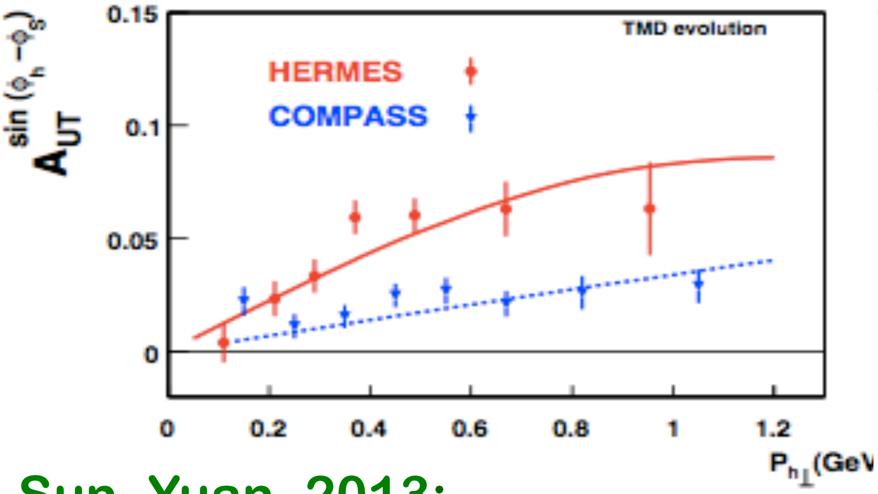
- ❑ **Up quark Sivers function:**

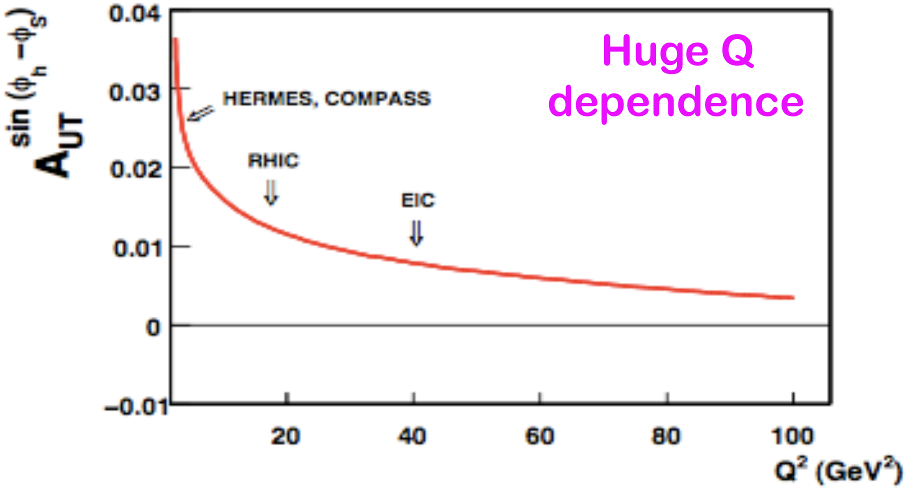


Very significant growth in the width of transverse momentum

Different fits – different Q-dependence

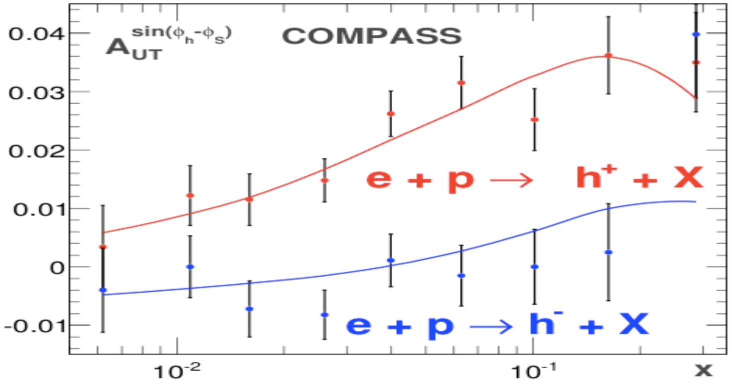
- **Aybat, Prokudin, Rogers, 2012:**

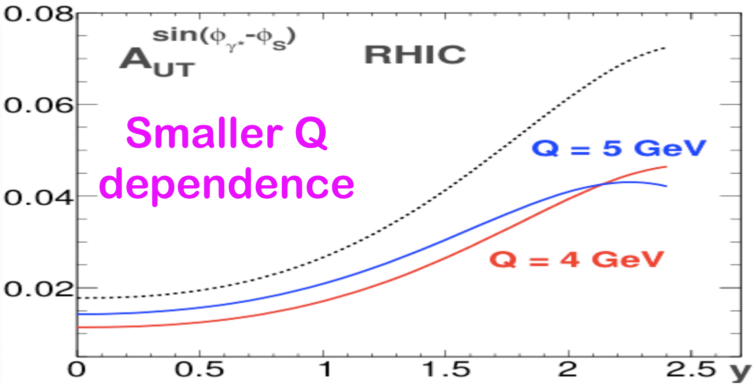




Huge Q dependence

- **Sun, Yuan, 2013:**





Smaller Q dependence

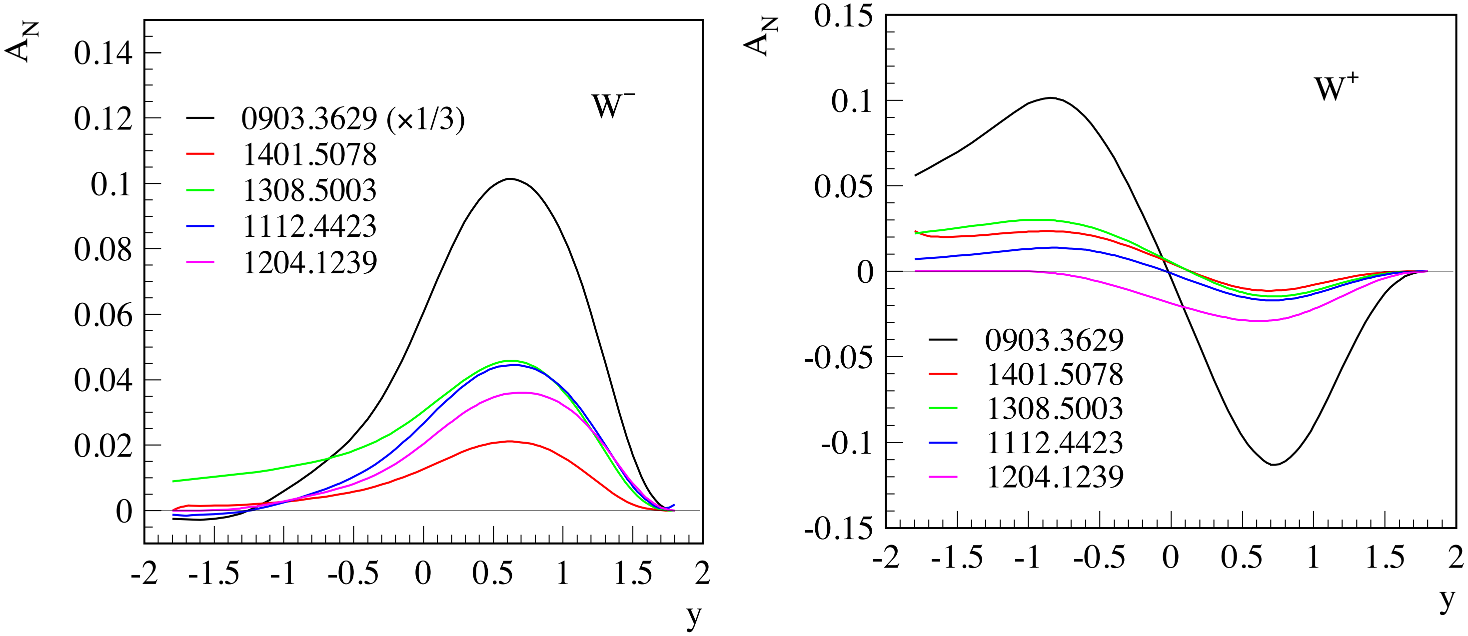
No disagreement on evolution equations!

Issues: Extrapolation to non-perturbative large b-region
Choice of the Q-dependent "form factor"

“Predictions” for A_N of W-production at RHIC?

See also talk by Marcia Quaresma on Wed. for COMPASS

- **Sivers Effect:**
  - Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons
  - QCD Prediction: Sign change of Sivers function from SIDIS and DY
- **Current “prediction” and uncertainty of QCD evolution:**



TMD collaboration proposal: Lattice, theory & Phenomenology

RHIC is the excellent and unique facility to test this (W/Z – DY)!

What happened?

- ☐ **Sivers function:**

Differ from PDFs!

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)=\frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T\, J_1(k_T b_T)\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

Need non-perturbative large b_T information for any value of Q! $Q=\mu$

- ☐ **What is the "correct" Q-dependence of the large b_T tail?**

$$\tilde{F}_{f/P}(x,b_T;Q,Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x},\enclose{circle}{b_*;}\,\mu_b^2,\mu_b,g(\mu_b))f_{j/P}(\hat{x},\mu_b)}^{\text{AA}}$$

$$\times\ \overbrace{\times \exp\left\{\ln\frac{Q}{\mu_b}\tilde{K}\enclose{circle}{(b_*;}\,\mu_b)+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1)-\ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right\}}^{\text{BB}}$$

$$\times\ \boxed{\overbrace{\exp\left\{g_{f/P}(x,b_T)+g_K(b_T)\ln\frac{Q}{Q_0}\right\}}^{\text{CC}}}$$

Nonperturbative "form factor"

$$g_{f/P}(x,b_T)+g_K(b_T)\ln\frac{Q}{Q_0}\equiv-\left[g_1+g_2\ln\frac{Q}{2Q_0}+g_1g_3\ln(10x)\right]b_T^2$$

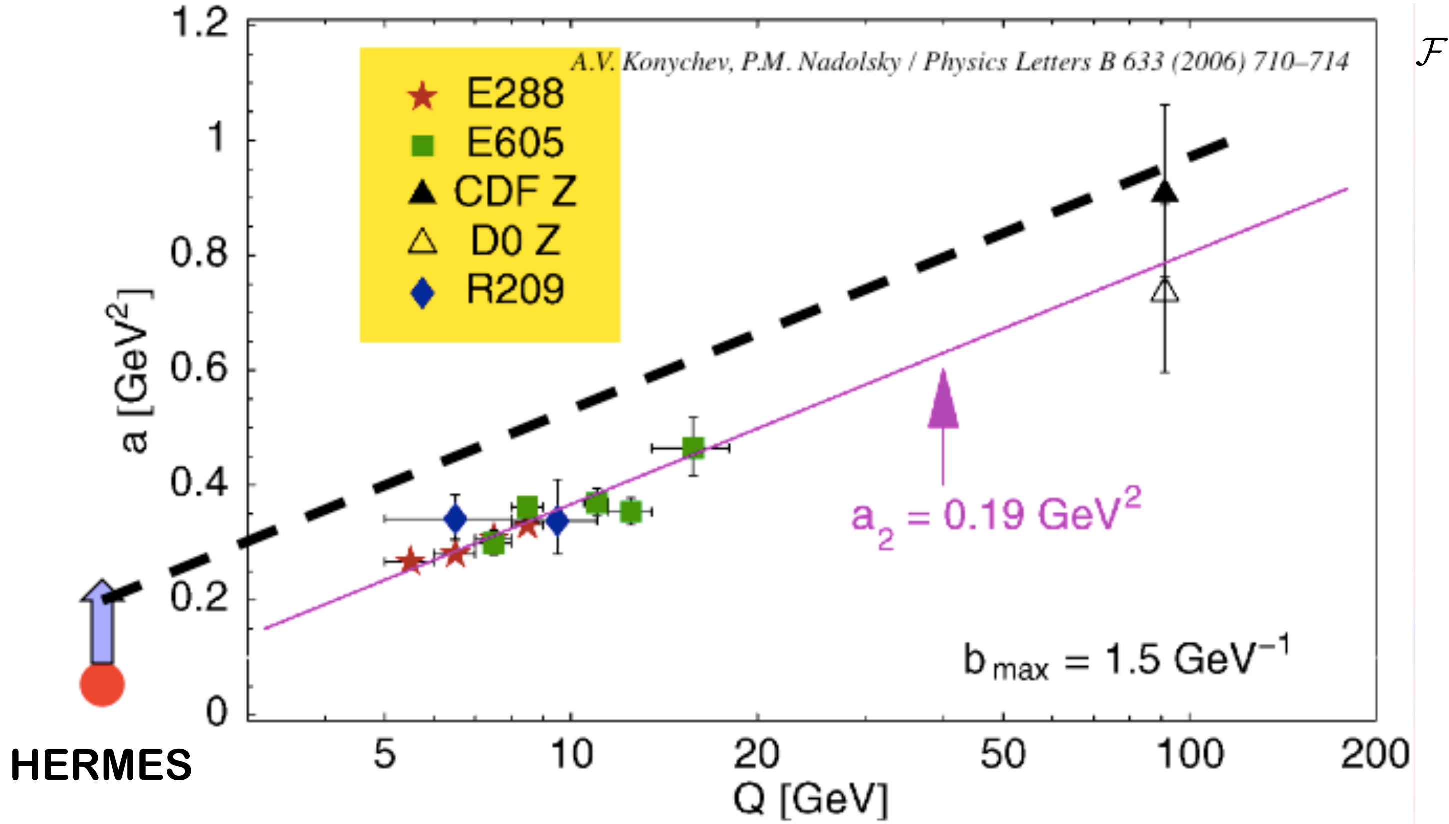
Is the log(Q) dependence sufficient? Choice of g_2 & b_* affects Q-dep.

The "form factor" and b_* change perturbative results at small b_T!

Q-dependence of the “form” factor

- **Q-dependence of the “form factor” :**

Konychev, Nadolsky, 2006



$$\mathcal{F}^{\rm NP}(b,Q) = a(Q^2)\, b^2$$

At Q ~ 1 GeV, ln(Q/Q_0) term may not be the dominant one!

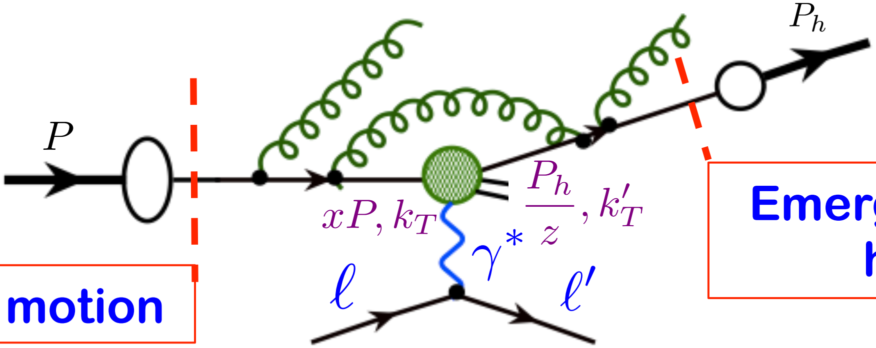
$$\mathcal{F}^{\rm NP} \approx b^2(a_1 + a_2 \ln(Q/Q_0) + a_3 \ln(x_A x_B) + \ldots) + \ldots$$

Power correction? **(Q_0/Q)^n-term?** ***Better fits for HERMES data?***

Parton k_T at the hard collision

❑ Sources of parton k_T at the hard collision:

Gluon shower



Confined motion

Emergence of a hadron
hadronization

❑ Large k_T generated by the shower (caused by the collision):

- ✧ Q^2-dependence – linear evolution equation of TMDs in b-space
- ✧ The evolution kernels are perturbative at small b, but, not large b

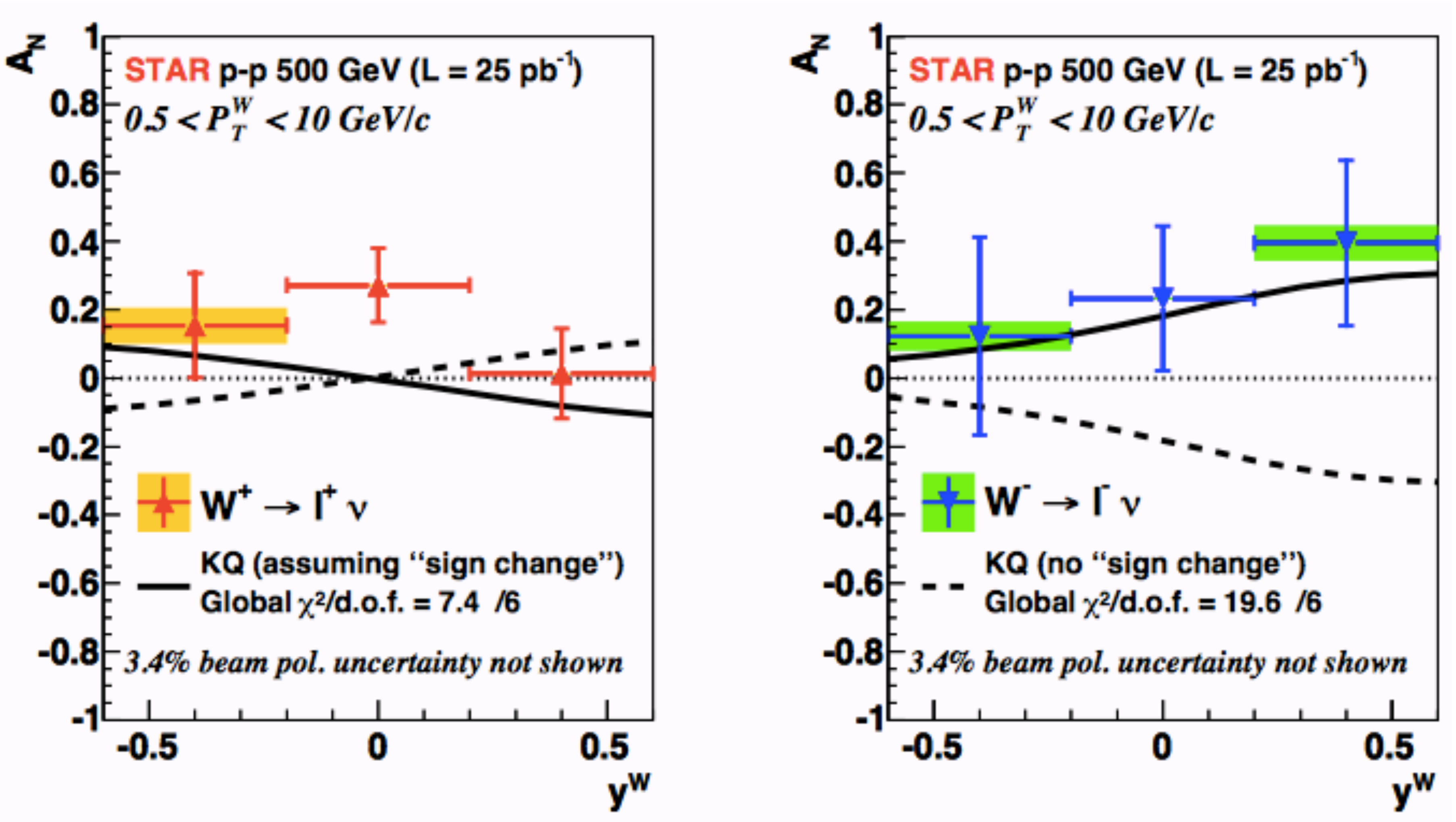
➡ The nonperturbative inputs at large b could impact TMDs at all Q^2

❑ Challenge: to extract the “true” parton’s confined motion:

- ✧ Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs
- ✧ Role of lattice QCD?

Task of the DOE supported TMD collaboration

Hint of the sign change: A_N of W production



Data from STAR collaboration on A_N for W-production are consistent with a sign change between SIDIS and DY

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)

Boosted 3D nucleon structure

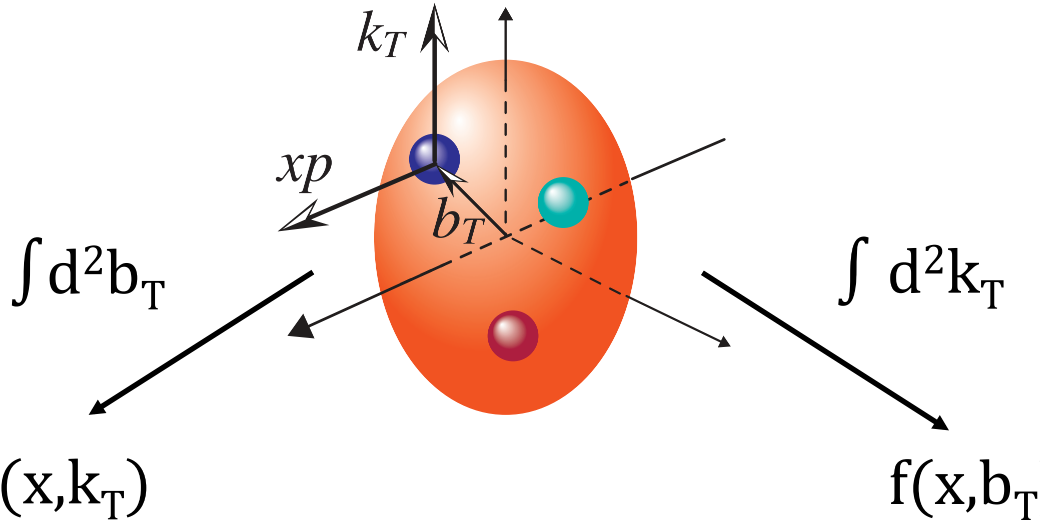
- ☐ **High energy probes "see" the boosted partonic structure:**

Momentum Space

TMDs

Coordinate Space

GPDs



$\int d^2 b_T$

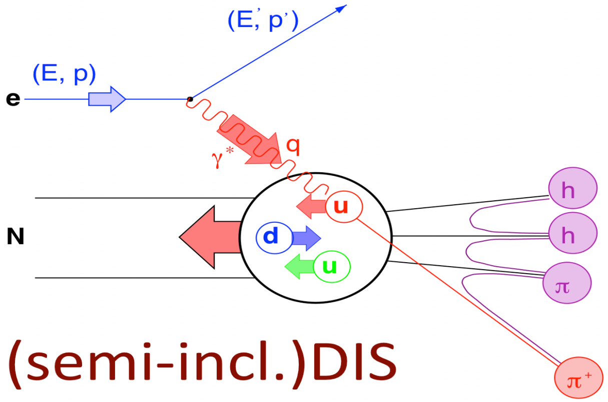
$\int d^2 k_T$

$f(x,k_T)$

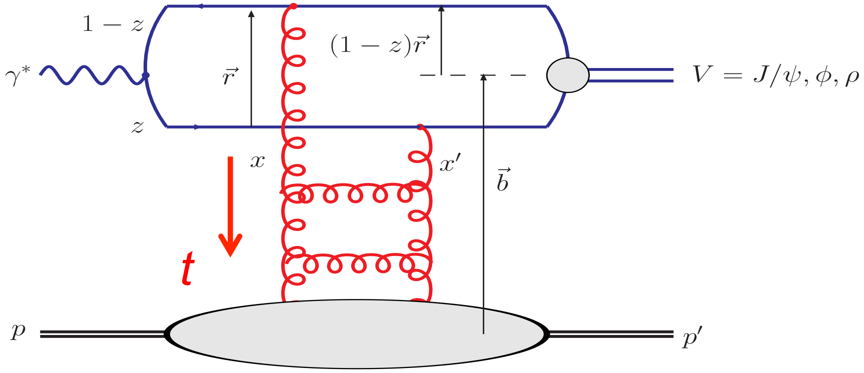
$f(x,b_T)$

3D momentum space images

2+1D coordinate space images



(semi-incl.)DIS



JLab12 for large-x, EIC for medium to small-x

GPDs – its role in solving the spin puzzle

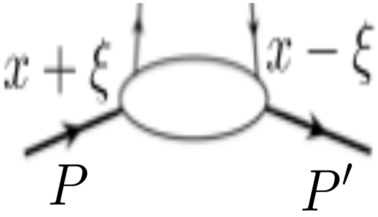
See also Hatta’s talk on Tuesday

❑ Quark “form factor”:

$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} \mathrm{e}^{-ix\lambda} \langle P'|\bar{\psi}_q(\lambda/2)\frac{\gamma\cdot n}{2P\cdot n}\psi_q(-\lambda/2)|P\rangle$$

$$\equiv \; H_q(x,\xi,t,\mu^2)\,\left[\bar{\mathcal{U}}(P')\gamma^\mu\mathcal{U}(P)\right]\frac{n_\mu}{2P\cdot n}$$

$$+ \; E_q(x,\xi,t,\mu^2)\left[\bar{\mathcal{U}}(P')\frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M}\mathcal{U}(P)\right]\frac{n_\mu}{2P\cdot n}$$



with $\xi = (P'-P)\cdot n/2$ and $t=(P'-P)^2 \Rightarrow -\Delta_\perp^2$ if $\xi\to 0$

$\tilde{H}_q(x,\xi,t,Q)$, $\tilde{E}_q(x,\xi,t,Q)$ **Different quark spin projection**

❑ Total quark’s orbital contribution to proton’s spin:

Ji, PRL78, 1997

$$J_q = \frac{1}{2}\lim_{t\to 0}\int dx\, x\,\left[H_q(x,\xi,t) + E_q(x,\xi,t)\right]$$

$$= \frac{1}{2}\Delta q + L_q$$

❑ Connection to normal quark distribution:

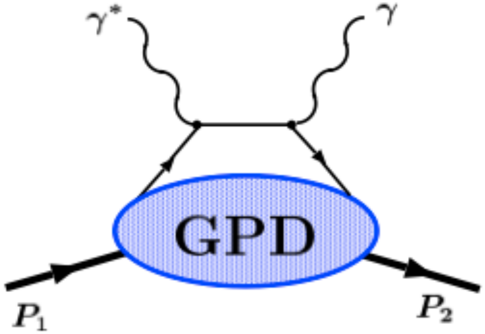
$H_q(x,0,0,\mu^2) = q(x,\mu^2)$ **The limit when** $\xi\to 0$

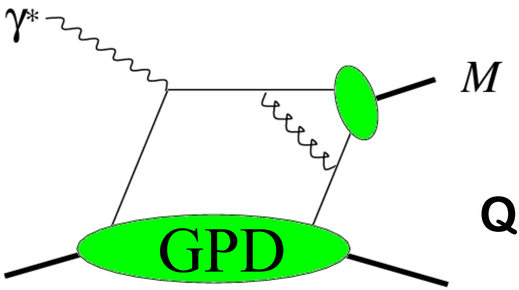
Exclusive DIS: Hunting for GPDs

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

- **Experimental access to GPDs:**
  - **Diffractive exclusive processes – high luminosity:**

    DVCS: Deeply virtual Compton Scattering
    DVEM: Deeply virtual exclusive meson production





Require

$Q^2 >> (-t), \Lambda^2_{QCD}, M^2$

  - **No factorization for hadronic diffractive processes – EIC is ideal**

- **Much more complicated – (x, ξ, t) variables:**

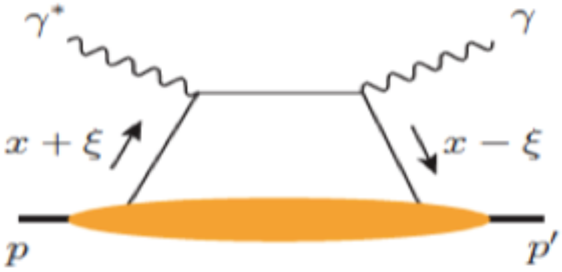
  Challenge to derive GPDs from data

- **Great experimental effort:**

  HERA, HERMES, COMPASS, JLab → JLab12, COMPASS-II, EIC

Deep virtual Compton scattering

❑ The LO diagram:



$$\xi = Q^2/(2\bar{P}\cdot q)$$
$$P' = P + \Delta$$

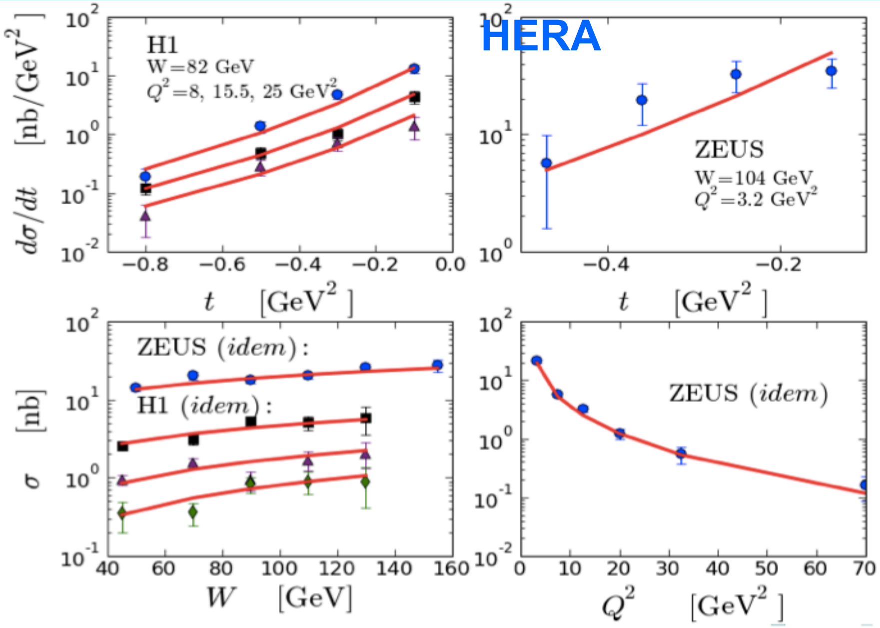
❑ Scattering amplitude:

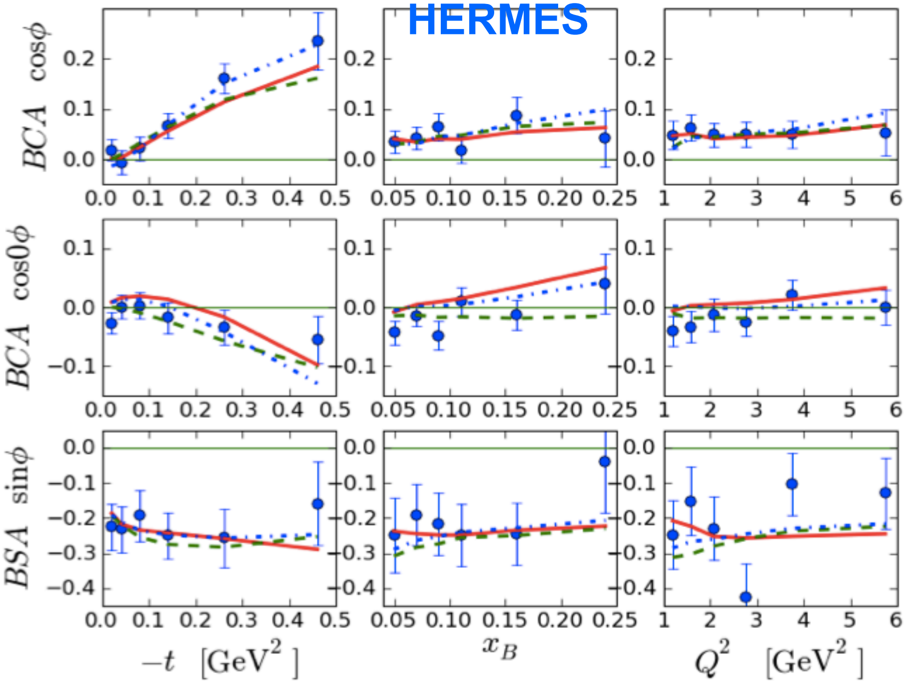
$$T^{\mu\nu}(P,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu})\int dx\left(\frac{1}{x-\xi/2+i\epsilon}+\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}U(P)+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$
$$-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta\int dx\left(\frac{1}{x-\xi/2+i\epsilon}-\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[\tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}\gamma_5 U(P)+\tilde{E}(x,\Delta^2,\Delta\cdot n)\frac{\Delta\cdot n}{2M}\bar{U}(P')\gamma_5 U(P)\right]$$

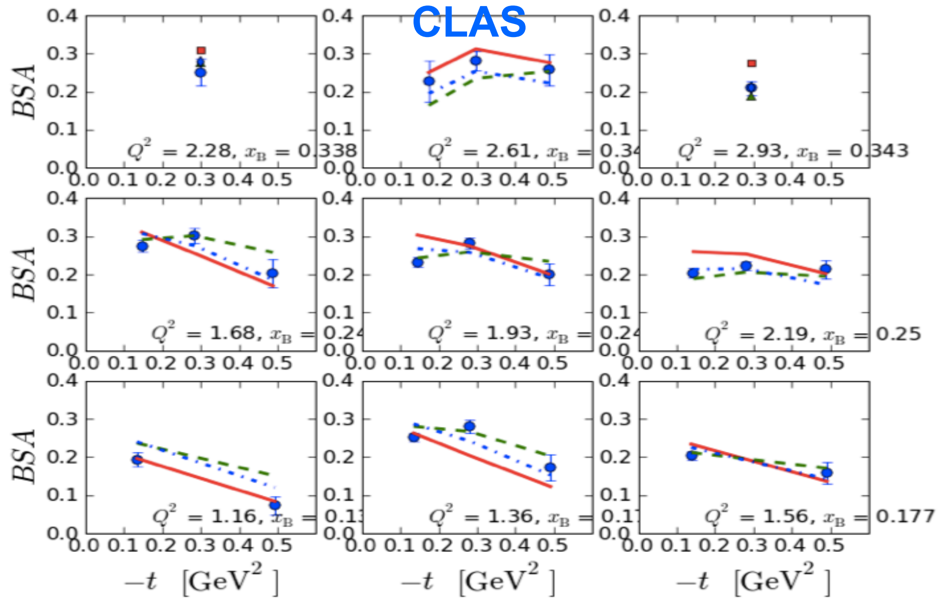
❑ GPDs:

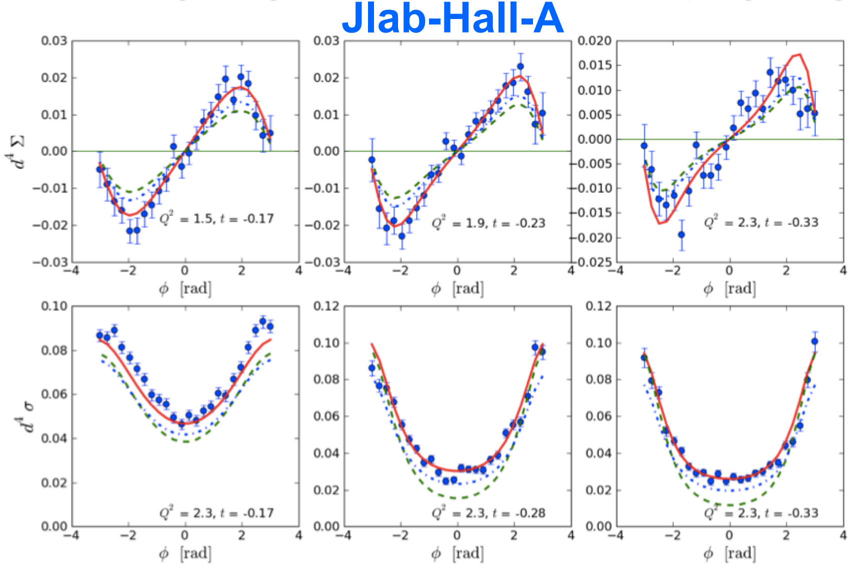
$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P)$$
$$+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P)+\ldots$$
$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu\gamma_5 U(P)$$
$$+\tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P)+\ldots$$

GPDs: just the beginning


HERA

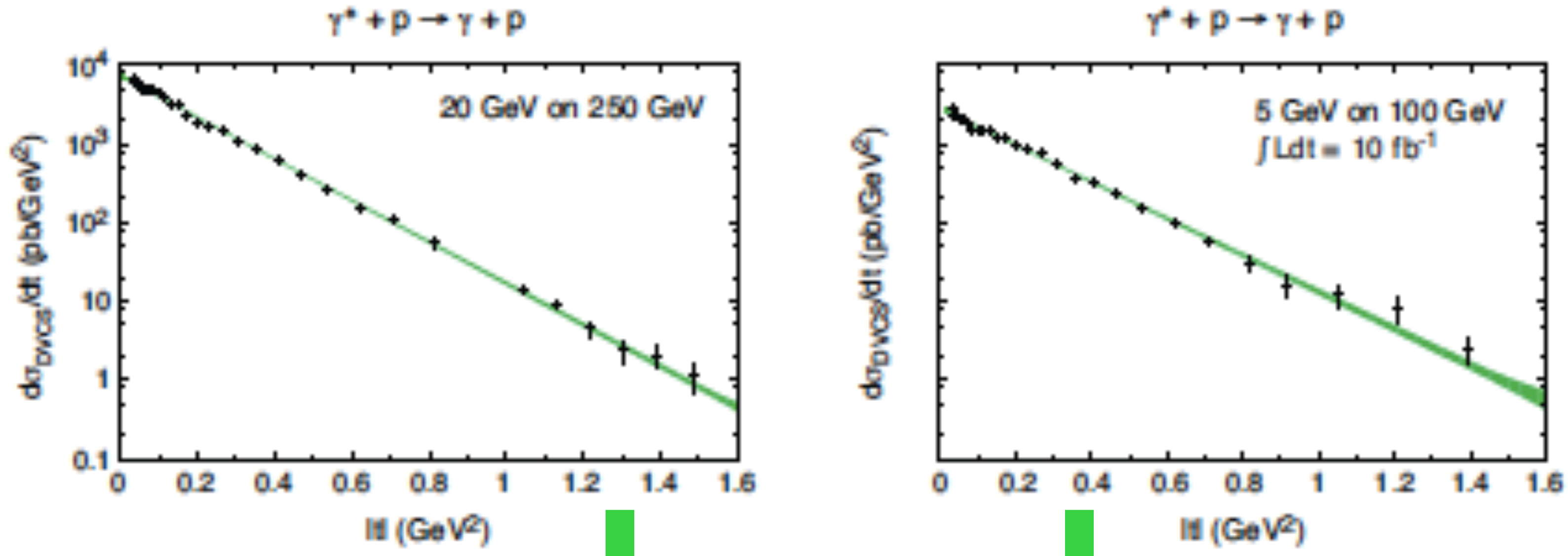

HERMES


CLAS

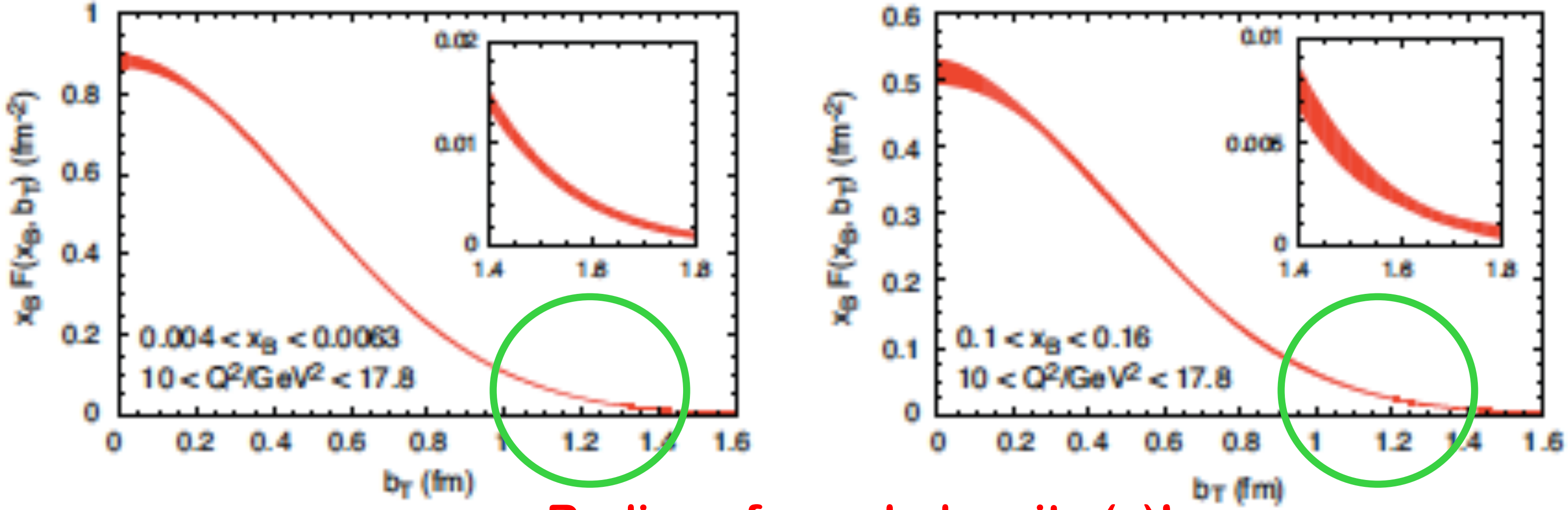

Jlab-Hall-A

DVCS @ EIC

❑ **Cross Sections:**



❑ **Spatial distributions:**



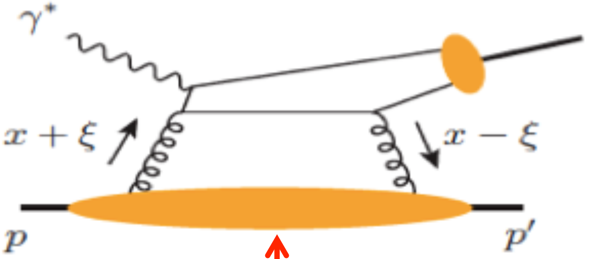
Radius of quark density (x)!

Spatial distribution of gluons

❑ Exclusive vector meson production:

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

EIC-WhitePaper



J/Ψ, Φ, …

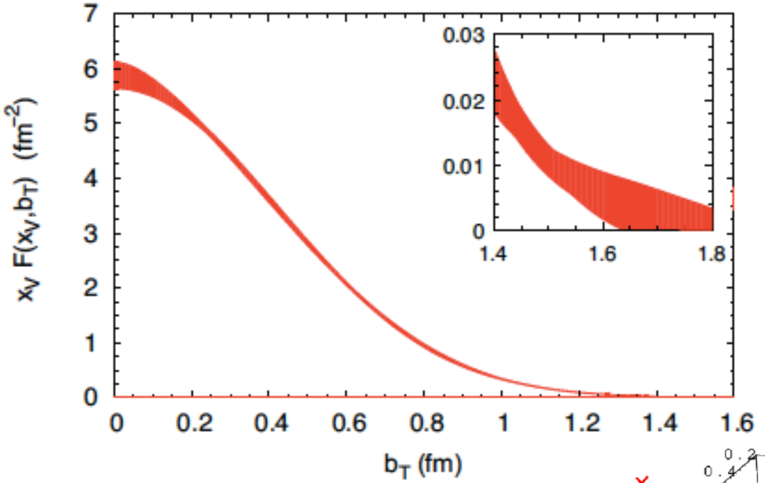
↑ t-dep

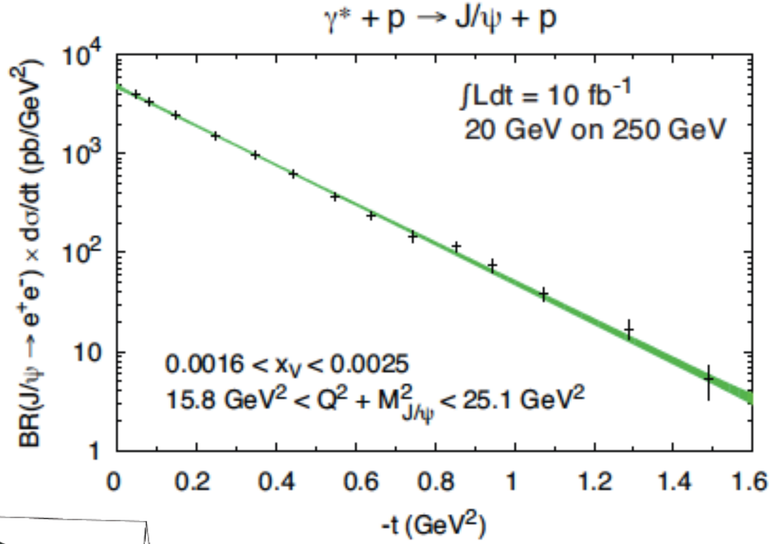
✧ Fourier transform of the t-dep

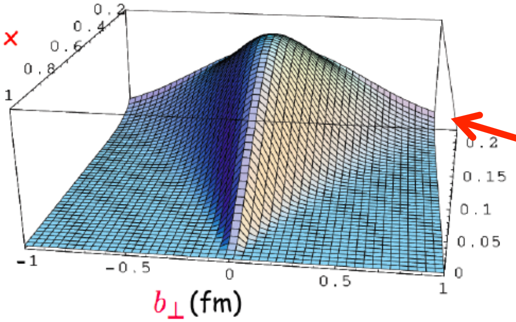
➡ *Spatial imaging of glue density*

✧ Resolution ~ 1/Q or $1/M_Q$

❑ Gluon imaging from simulation:







Only possible at the EIC

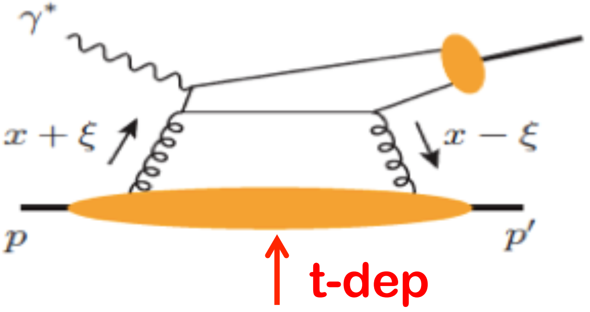
"Proton" radius?

Radius of gluon density (x)!

How spread
at small-x?
Color confinement

Spatial distribution of gluons

❑ Exclusive vector meson production:



J/Ψ, Φ, …

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

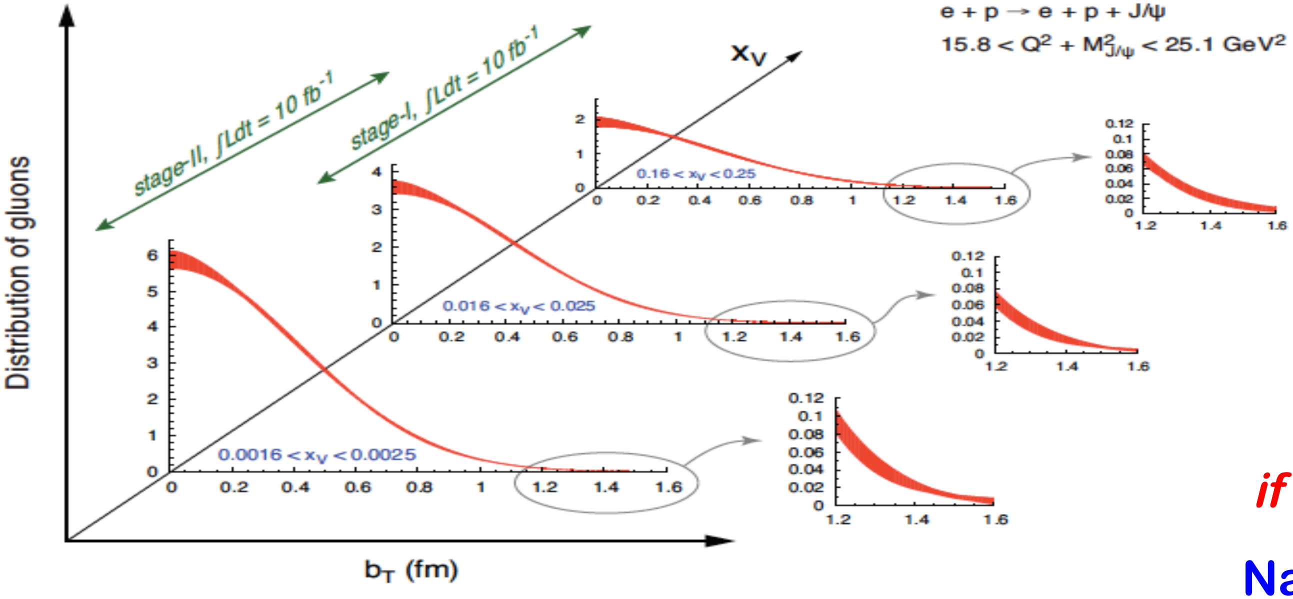
EIC-WhitePaper

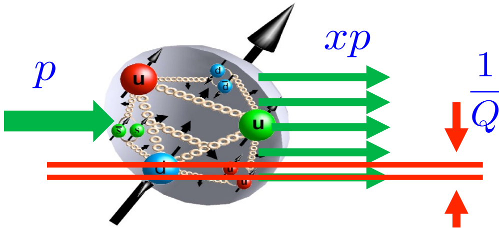
- ✧ **Fourier transform of the t-dep**

➡ ***Spatial imaging of glue density***

- ✧ **Resolution ~ 1/Q or 1/M_Q**

❑ Gluon imaging from simulation:





**Images of gluons
from exclusive
J/ψ production**

***Proton's radius
if only seeing gluon?***

Nature of pion cloud?

Model dependence – parameterization?

EIC simulation

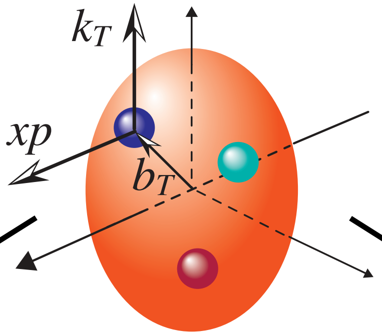
Unified view of nucleon structure

❑ **Wigner distributions:**

Momentum Space

TMDs

Confined motion



$\int d^2b_T$ $\int d^2k_T$

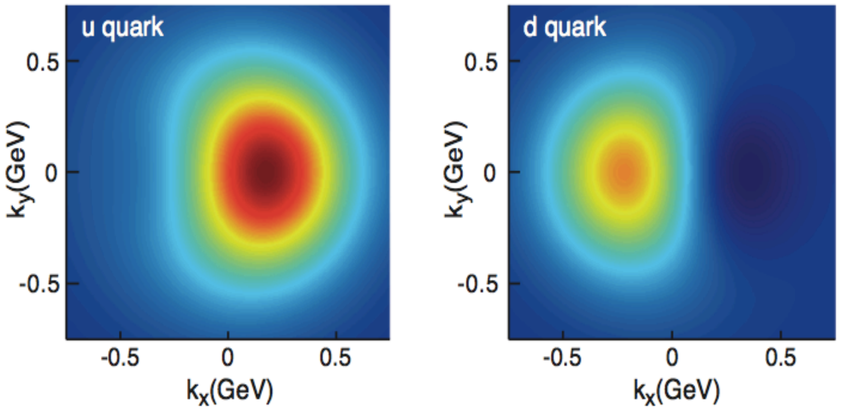
$f(x,k_T)$ **Two-scales observables** $f(x,b_T)$

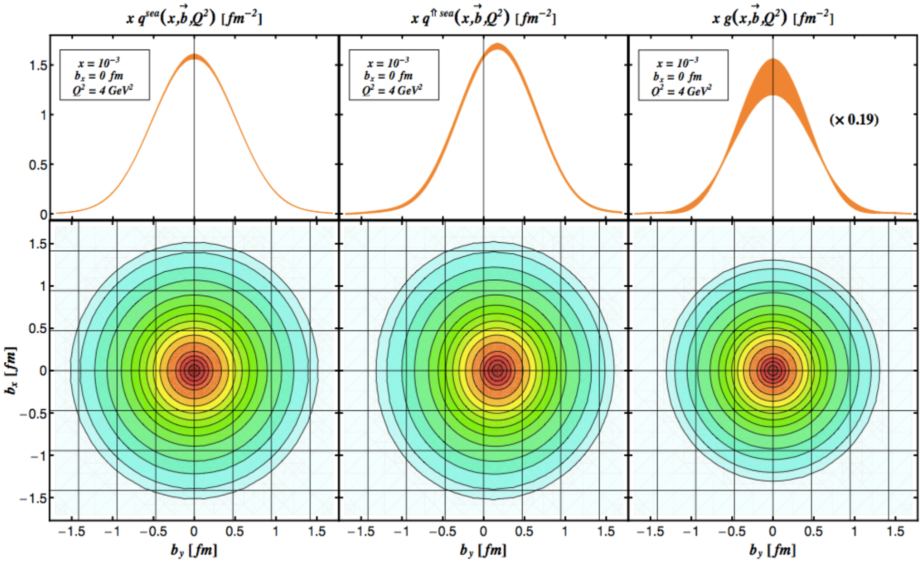
Coordinate Space

GPDs

Spatial distribution

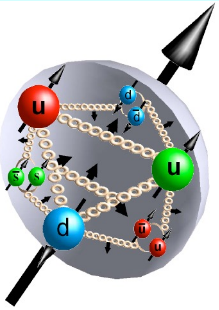
Sivers Functions





Position *r* x Momentum *p* → Orbital Motion of Partons

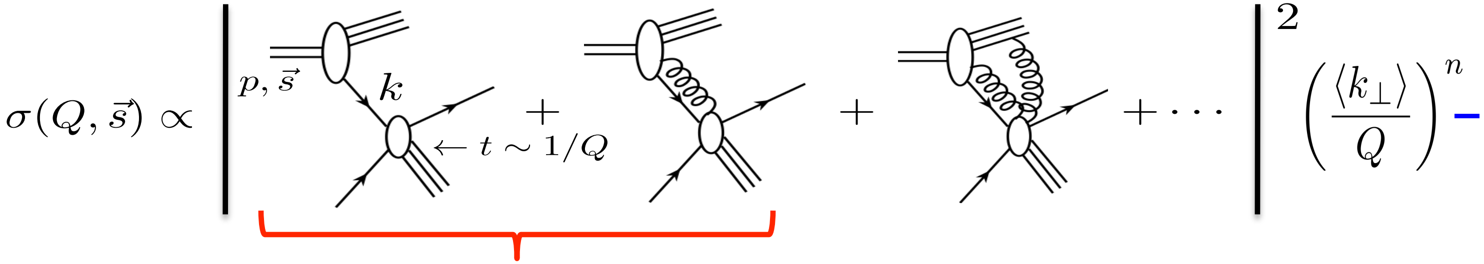
PDFs, TMDs, GPDs, and hadron structure



- **What do we need to know for full hadron structure?**
  - ✧ In theory: $\langle P, S|\mathcal{O}(\overline{\psi}, \psi, A^\mu)|P, S\rangle$ – Hadronic matrix elements

    with ALL possible operators: $\mathcal{O}(\overline{\psi}, \psi, A^\mu)$
  - ✧ In fact: *None of these matrix elements is a direct physical observable in QCD – color confinement! – need probes!!!*
  - ✧ In practice: Accessible hadron structure
    = hadron matrix elements of quarks and gluons, which

    1) can be related to physical cross sections of hadrons and leptons with controllable approximation – factorization;
    2) can be calculated in lattice QCD
- **Multi-parton correlations – beyond single parton distributions:**



– Expansion

Quantum interference ➡ 3-parton matrix element – not a probability!

Summary

- ☐ TMDs and GPDs are NOT direct physical observables – could be defined differently

- ☐ Knowledge of nonperturbative inputs at large b_T is crucial in determining the TMDs from fitting the data

- ☐ QCD factorization is necessary for any controllable "probe" for hadron's quark-gluon structure!

- ☐ Jlab12, COMPASS, … will provide rich information on hadron structure via TMDs and/or GPDs in years to come!

- ☐ EIC is a ultimate QCD machine, and will provide answers to many of our questions on hadron structure, in particular, the confined transverse motions (TMDs), spatial distributions (GPDs), and multi-parton correlations, …

Thank you!

Backup slides

Evolution equations for TMDs

J.C. Collins, in his book on QCD

❑ **TMDs in the b-space:**

$$\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F) = \tilde{F}^{\mathrm{unsub}}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; y_P - (-\infty))\sqrt{\frac{\tilde{S}_{(0)}(\mathrm{b_T}; +\infty, y_s)}{\tilde{S}_{(0)}(\mathrm{b_T}; +\infty, -\infty)\tilde{S}_{(0)}(\mathrm{b_T}; y_s, -\infty)}} Z_F Z_2$$

❑ **Collins-Soper equation:**

Renormalization of the soft-factor

$$\frac{\partial \tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)$$

$$\zeta_F = M_P^2 x^2 e^{2(y_P - y_s)}$$

$$\tilde{K}(b_T; \mu) = \frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)}\right)$$

Introduced to regulate the rapidity divergence of TMDs

❑ **RG equations:**

Wave function Renormalization

$$\frac{d\tilde{K}(b_T; \mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$

Evolution equations are only valid when $b_T << 1/\Lambda_{QCD}$!

$$\frac{d\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{d\ln\mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F).$$

❑ **Momentum space TMDs:**

Need information at large b_T for all scale μ !

$$F_{f/P^\uparrow}(x, \mathrm{k_T}, S; \mu, \zeta_F) = \frac{1}{(2\pi)^2}\int d^2\mathrm{b}_T\, e^{i\mathrm{k}_T\cdot\mathrm{b}_T}\, \tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu, \zeta_F)$$

Evolution equations for Sivers function

Aybat, Collins, Qiu, Rogers, 2011

❑ Sivers function:

$$F_{f/P^\uparrow}(x,k_T,S;\mu,\zeta_F) = F_{f/P}(x,k_T;\mu,\zeta_F) - F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)\frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

❑ Collins-Soper equation:

Its derivative obeys the CS equation

$$\frac{\partial \ln \tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)$$

$$\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp f}(x,b_T;\mu,\zeta_F)}{\partial b_T}$$

❑ RG equations:

$$\frac{d\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \Longrightarrow \quad \frac{\partial\gamma_F(g(\mu);\zeta_F/\mu^2)}{\partial\ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

❑ Sivers function in momentum space:

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T J_1(k_T b_T)\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

JI, Ma, Yuan, 2004
Idilbi, et al, 2004,
Boer, 2001, 2009,
Kang, Xiao, Yuan, 2011
Aybat, Prokudin, Rogers, 201
Idilbi, et al, 2012,
Sun, Yuan 2013, …

Extrapolation to large b_T

☐ CSS b*-prescription:

Aybat and Rogers, arXiv:1101.5057
Collins and Rogers, arXiv:1412.3820

$$\begin{aligned}
\tilde{F}_{f/P}(x, \mathrm{b}_T; Q, Q^2) &= \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\mathrm{AA}} \\
&\times \overbrace{\times \exp\left\{ \ln\frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln\frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\mathrm{BB}} \\
&\times \boxed{\overbrace{\exp\left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln\frac{Q}{Q_0} \right\}}^{\mathrm{CC}}}
\end{aligned}$$

Nonperturbative “form factor”

$$\boldsymbol{b}_* = \frac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2/b_\mathrm{max}^2}} \qquad \text{with } b_\mathrm{max} \sim 1/2\ \mathrm{GeV}^{-1}$$

☐ Nonperturbative fitting functions

Various fits correspond to different choices for $g_{f/P}(x, b_T)$ **and** $g_K(b_T)$
e.g.

$$g_{f/P}(x, b_T) + g_K(b_T) \ln\frac{Q}{Q_0} \equiv -\left[g_1 + g_2 \ln\frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

Different choice of g_2 & b_* could lead to different over all Q-dependence!