Understanding the nature of heavy pentaquarks and searching for them in pion-induced reactions

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Outline

- Early study in 1960s
- Triangle singularity (TS) mechanism
- TS mechanism and the heavy pentaquark “Pc”
- Searching for the heavy pentaquark in $\pi p \to \pi J/\psi p$
- Summary

Triangle Singularity Mechanism

Early study in 1960s

- **Connections between kinematic singularities of the S-matrix elements and resonance-like peaks: e.g. Peierls mechanism**

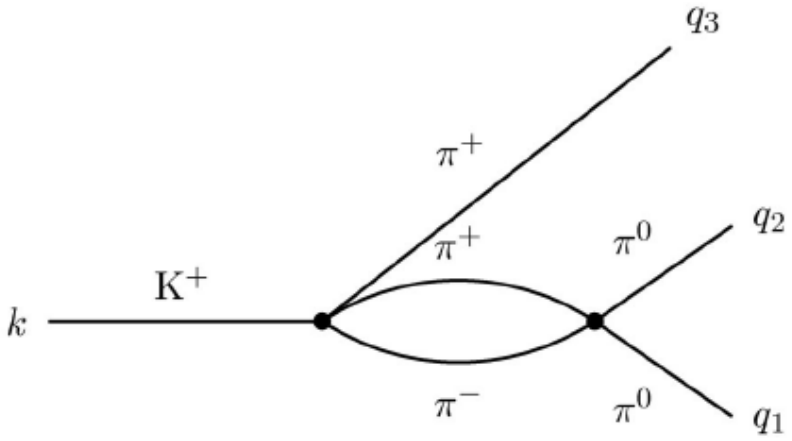
R.F.Peierls, PRL6,641(1961);
R.C.Hwa, PhysRev130,2580(1963);
C.Goebel,PRL13,143(1964);
P.Landshoff&S.Treiman Phys.Rev.127,649(1962);
……

- **Some disadvantages:**
  - ✓ **Few experiments to search for the effects;**
  - ✓ **Low statistics;**
  - ✓ **For the elastic scattering process, the effect of the triangle diagram is nothing more than a multiplication of the singularity from the tree diagram by a phase factor, according to the so-called Schmid theorem**

C.Schmid,Phys.Rev.154,1363(1967);
I. J. R. Aitchison & C. Kacser, Phys.Rev.173,1700(1968); A.V. Anisovich, PLB345,321(1995)

Cusp effect

Resulted from charge-exchange rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$ and the pion mass difference



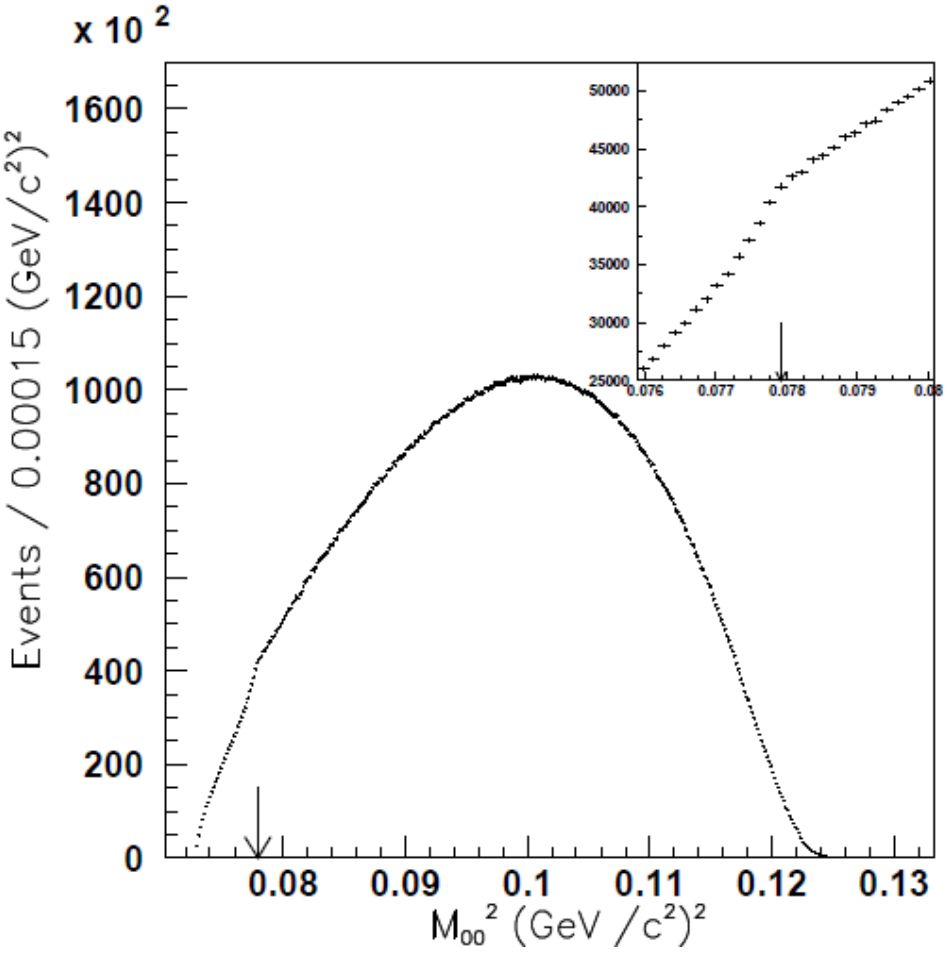
Budini & Fonda, 1961;
Cabibbo, 2004;

Branching ratio

$K^+ \rightarrow \pi^+\pi^-\pi^+$ $((5.59 \pm 0.04)\%)$

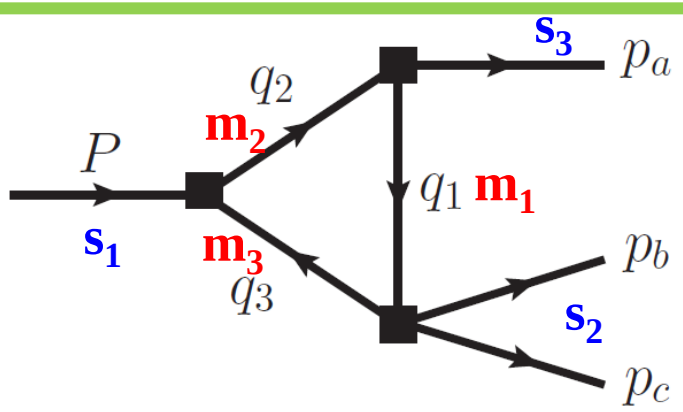
much larger than

$K^+ \rightarrow \pi^0\pi^0\pi^+$ $((1.761 \pm 0.022)\%)$



NA48/2, PLB633,173
6 $\times 10^7$ events

Triangle Singularity Mechanism



$$P^2 = s_1, (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

$$\Gamma_3(s_1, s_2, s_3) = \frac{-1}{16\pi^2} \int_0^1 \int_0^1 \int_0^1 da_1\, da_2\, da_3\, \frac{\delta(1 - a_1 - a_2 - a_3)}{D - i\epsilon}$$

$$D = \sum_{i,j=1}^{3} a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2}\left[m_i^2 + m_j^2 - (q_i - q_j)^2\right]$$

✓ **Singularity in the complex space**

Necessary conditions (Landau Equation)

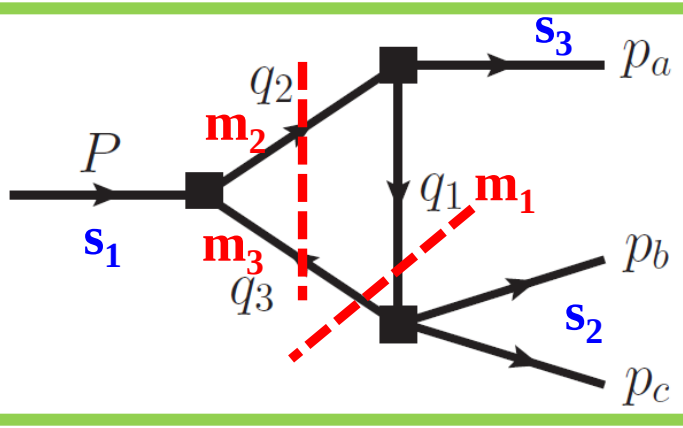
$$D = 0,$$

$$either \;\; a_j = 0 \;\; or \;\; \frac{\partial D}{\partial a_j} = 0.$$

Leading singularity

Landau, Nucl.Phys.13,181(1959)

Triangle Singularity Mechanism



$$P^2 = s_1, (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

✓ **Singularity in the complex space**

The position of the singularity is obtained by solving

$$\det[Y_{ij}] = 0$$

Normal Threshold

$$s_2^{\pm} = (m_1 + m_3)^2 + \frac{1}{2m_2^2}[(m_1^2 + m_2^2 - s_3)(s_1 - m_2^2 - m_3^2) - 4m_2^2 m_1 m_3 \pm \lambda^{1/2}(s_1, m_2^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)], \quad \lambda(x, y, z) \equiv (x - y - z)^2 - 4yz$$

s_1, s_3, $m_{1,2,3}$ fixed

Anomalous Threshold

$$s_1^{\pm} = (m_2 + m_3)^2 + \frac{1}{2m_1^2}[(m_1^2 + m_2^2 - s_3)(s_2 - m_1^2 - m_3^2) - 4m_1^2 m_2 m_3 \pm \lambda^{1/2}(s_2, m_1^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)].$$

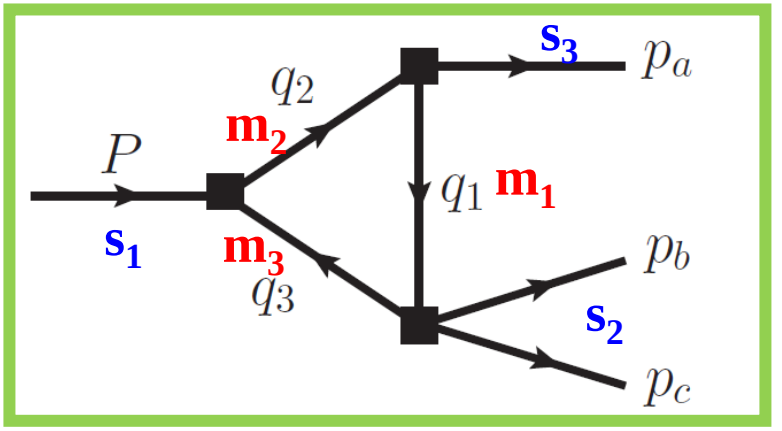
s_2, s_3, $m_{1,2,3}$ fixed

Triangle Singularity Mechanism

Single dispersion relation

$$\Gamma_3(s_1,s_2,s_3)=\frac{1}{\pi}\int_{(m_1+m_3)^2}^{\infty}\frac{ds_2'}{s_2'-s_2-i\epsilon}\,\sigma(s_1,s_2',s_3)$$

$$\sigma(s_1,s_2,s_3)\;=\;\sigma_+-\sigma_-$$



$$\sigma_\pm(s_1,s_2,s_3)\;=\;\frac{-1}{16\pi\lambda^{1/2}(s_1,s_2,s_3)}\log[-s_2(s_1+s_3-s_2+m_1^2+m_3^2-2m_2^2)$$
$$-\;(s_1-s_3)(m_1^2-m_3^2)\pm\lambda^{1/2}(s_1,s_2,s_3)\lambda^{1/2}(s_2,m_1^2,m_3^2)].$$

Work in the kinematical region

$$s_1\;\le\;(m_2+m_3)^2,\;s_3\;\le\;(m_2-m_1)^2 \qquad 0<s_2<(m_1+m_3)^2$$

By analytic continuation, it can be extended into the over threshold region

Fronsdal&Norton,J.Math.Phys.5,100(1964)

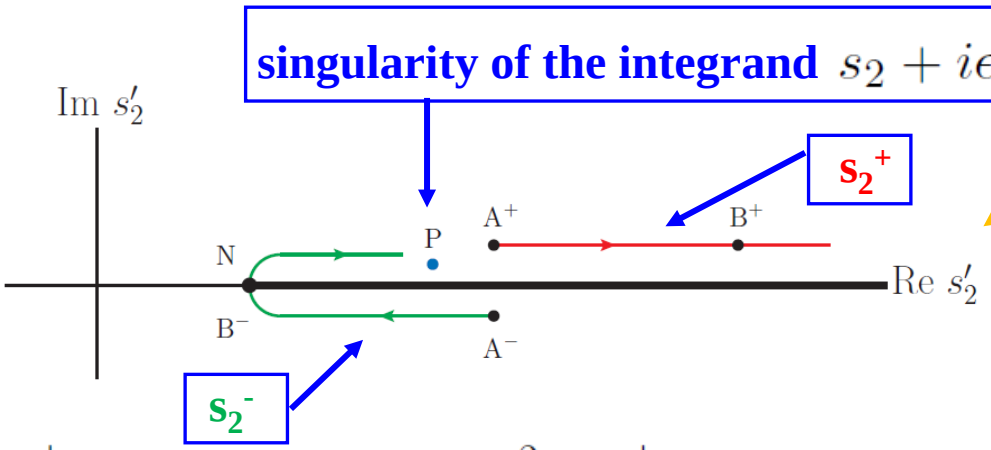
$$s_1\ge(m_2+m_3)^2,\;(m_1+m_3)^2\le s_2\le(\sqrt{s_1}-\sqrt{s_3})^2,\;0\le\sqrt{s_3}\le m_2-m_1$$

Brach points of the log function $s_2^{\pm}$

Triangle Singularity Mechanism

Locations of $s_2^{\pm}$ in the s_2'-plane can be determined by

$$s_2^{\pm}(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^{\pm}}{\partial s_1}$$



Trajectory of the branch points s_2^-, s_2^+ with s_1 increases from normal threshold to infinity in the complex s_2'-plane

$$A^{\pm}: s_1 = (m_2+m_3)^2,\ B^{\pm}:\ s_1 = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2-m_1)^2 - s_3] \quad = s_{1N} \quad = s_{1C}$$

$$A^-:\ s_2^- = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3] - i\epsilon \quad = s_{2C}$$

$$B^-:\ s_2^- = (m_1+m_3)^2 \quad = s_{2N}$$

s_2^- and P will pinch the integral contour

This pinch singularity is the triangle singularity (TS)

Triangle Singularity Mechanism

Locations of $s_2^{\pm}$ in the s_2'-plane can be determined by

$$s_2^{\pm}(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^{\pm}}{\partial s_1}$$

singularity of the integrand $s_2 + i\epsilon$

Trajectory of the branch

"The kinematic conditions for the existence of singularities on the physical boundary are equivalent to the condition that the relevant Feynman diagram be interpretable as a picture of an energy and momentum-conserving process occurring in space-time, with all internal particles real, on the mass shell and moving forward in time."

Coleman&Norton, Nuovo Cimento 38,5018 (1965)

Fronsdal&Norton,J.Math.Phys.5, 100(1964)

……

TS Kinematic Region

Normal threshold and critical point

$$s_{1N} = (m_2+m_3)^2,\ s_{1C} = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2-m_1)^2 - s_3],$$

$$s_{2N} = (m_1+m_3)^2,\ s_{2C} = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3],$$

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Discrepancy between anomalous and normal threshold

When $s_2=s_{2N}$

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$

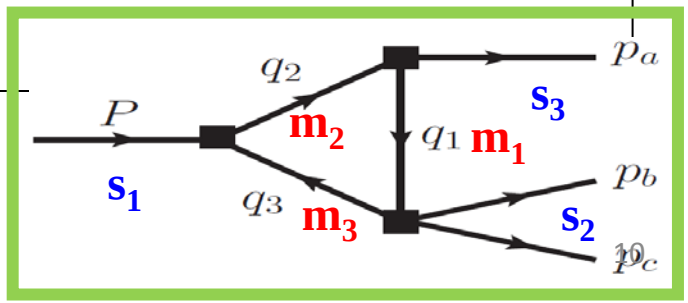
Largest discrepancy

When $s_1=s_{1N}$

$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1+m_3)}[(m_2-m_1)^2 - s_3].$$

Enlarge

How to amplify the discrepancy between normal and anomalous threshold?



Liu, Oka, Zhao, PLB753,297 (2016)
arXiv:1507.01674

TS Kinematic Region

Normal threshold and critical point

$$s_{1N} = (m_2 + m_3)^2,\ s_{1C} = (m_2 + m_3)^2 + \frac{m_3}{m_1}[(m_2 - m_1)^2 - s_3],$$

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$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Discrepancy between anomalous and normal threshold

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2 + m_3)}[(m_2 - m_1)^2 - s_3],$$

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Largest discrepancy

How to amplify the discrepancy between normal and anomalous threshold?

Liu, Oka, Zhao, PLB753,297 (2016)
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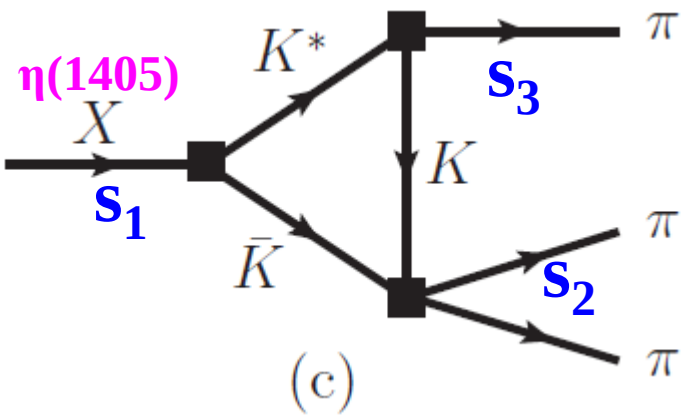
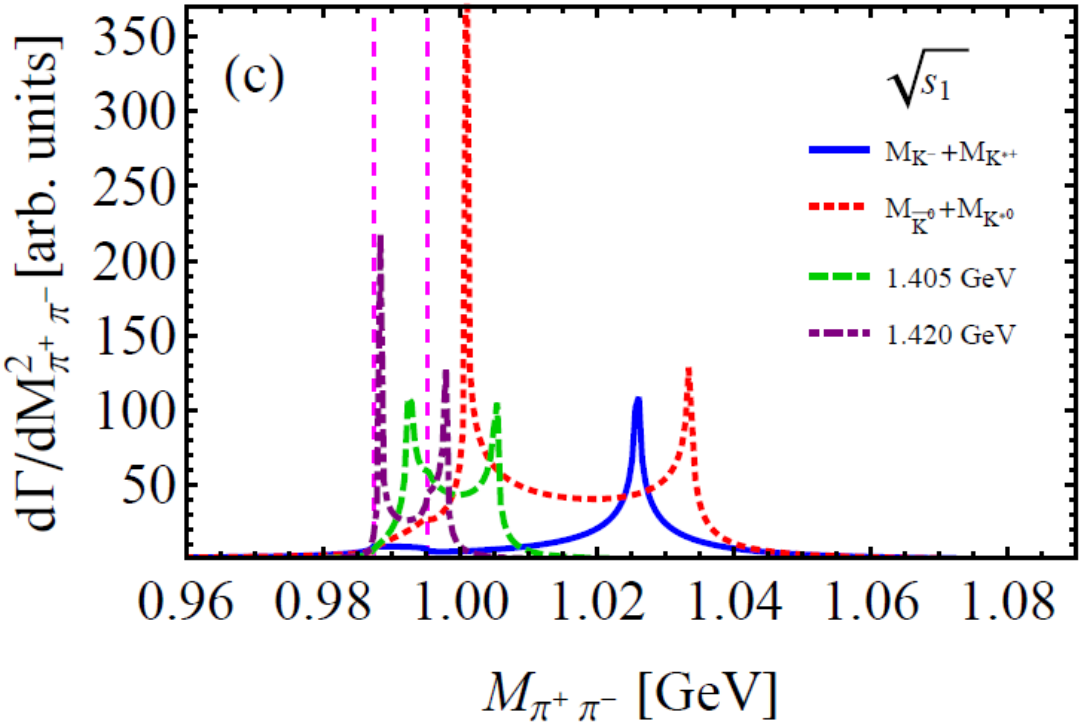
If the discrepancy is larger, maybe it could be used to distinguish the kinematic singularities from genuine particles.

Rescatterings to detect ATS



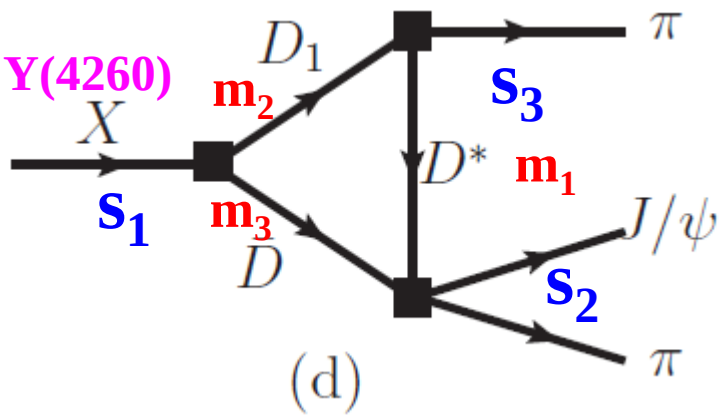


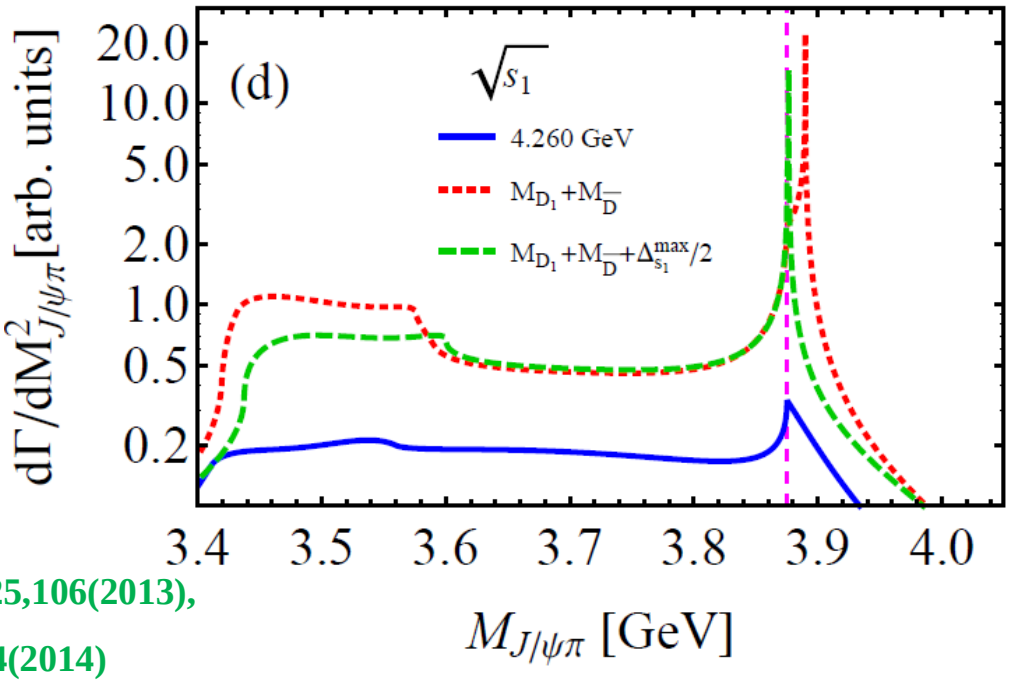
Wu, Liu, Zhao & Zou, PRL108,081803(2012)

See also Q. Zhao's talk

[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s1}^{\max}$	0.089	96	49	16
$\Delta_{s2}^{\max}$	0.087	62	38	15

Rescatterings to detect ATS





Wang,Hanhart,Zhao,PRL111,132003(2013);PLB725,106(2013),

X.H.Liu,G.Li, PRD88,014013(2013);PRD90,074004(2014)

See also Q. Zhao's talk

$$\Delta^{\max}_{s_1} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$

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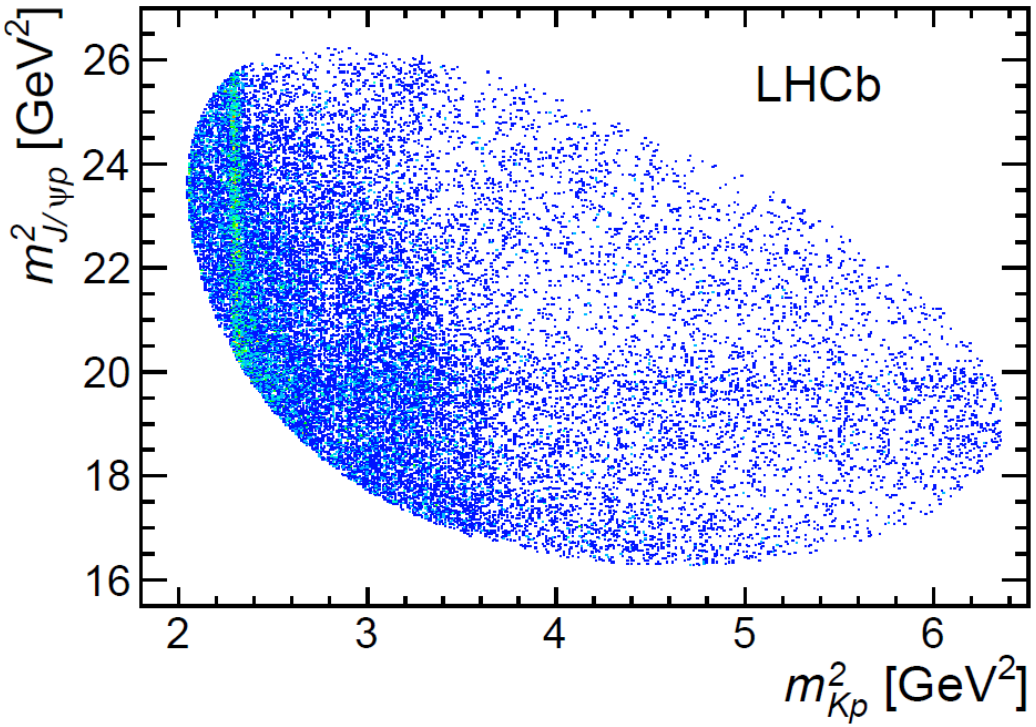
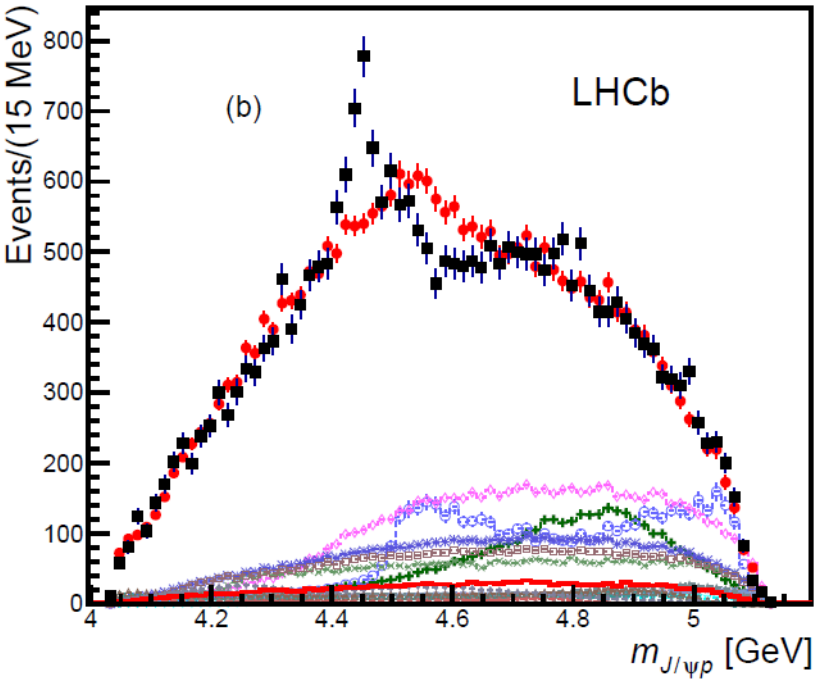
Heavy pentaquark “Pc” observed in LHCb





LHCb, arXiv: 1507.03414

PRL115, 072001 (2015)

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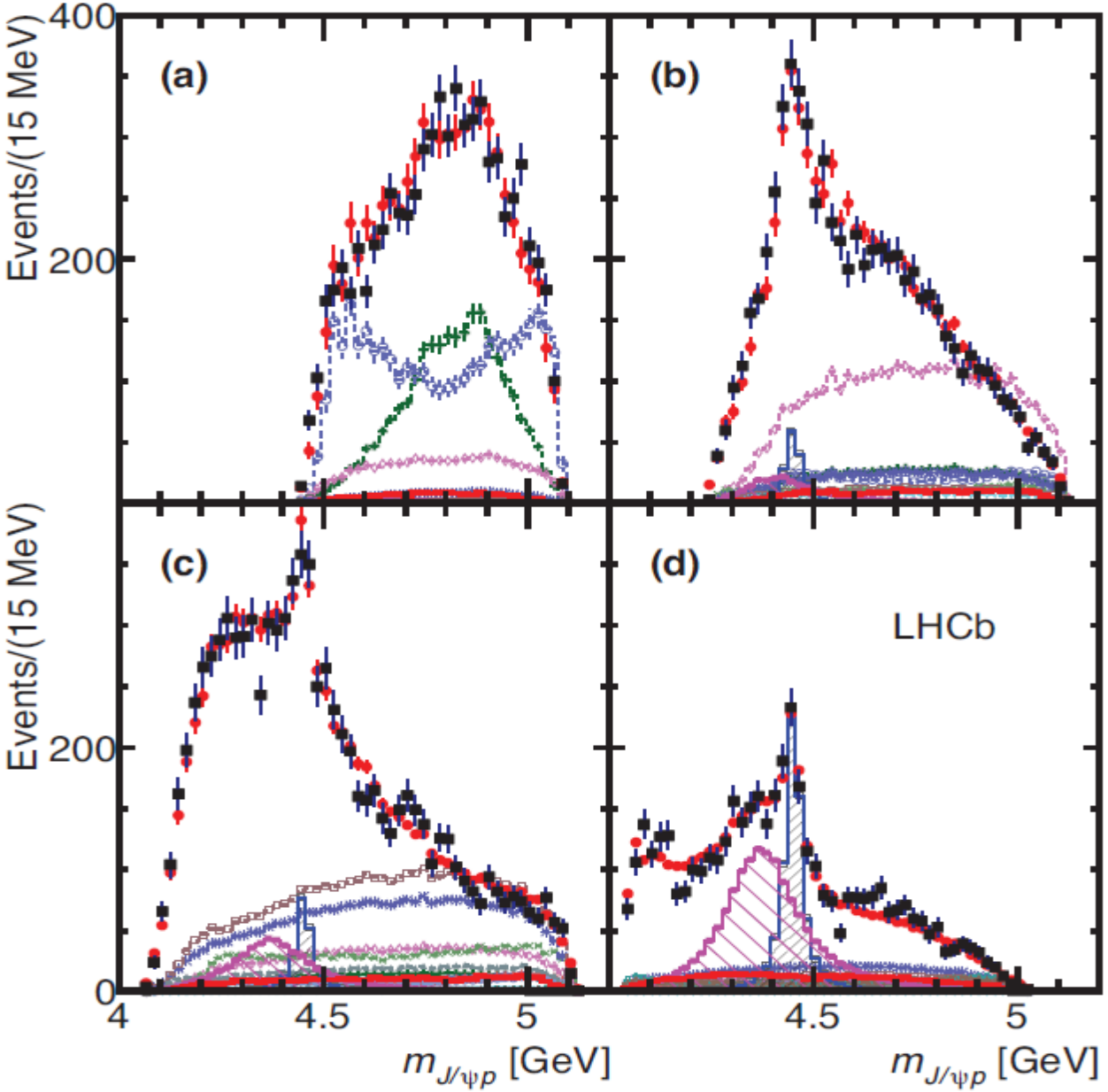
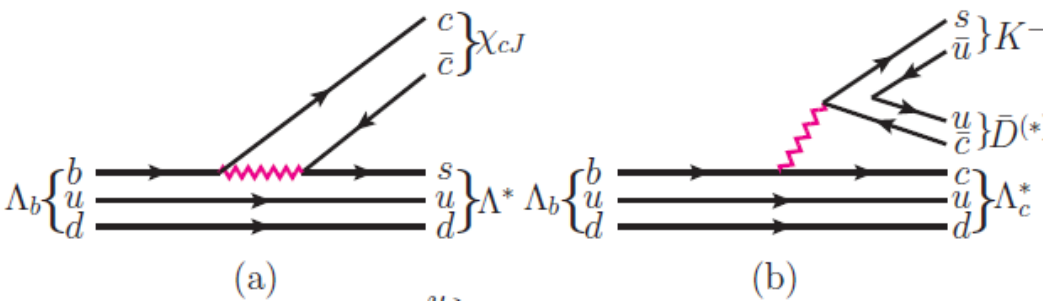
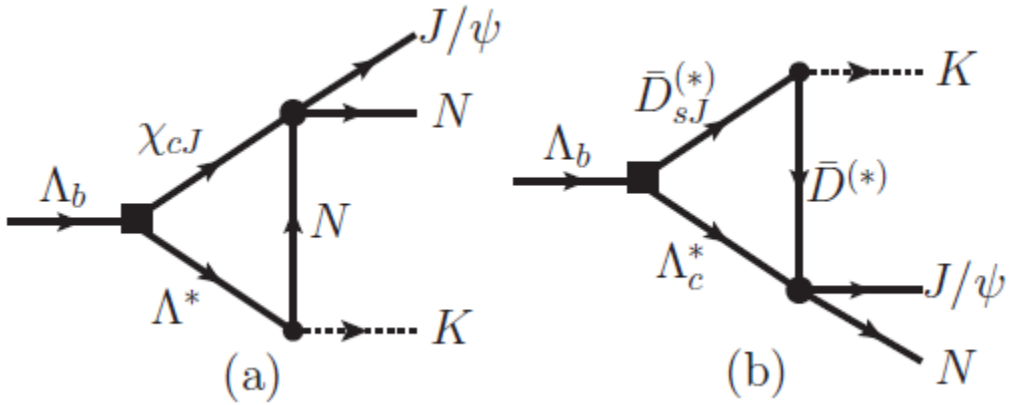
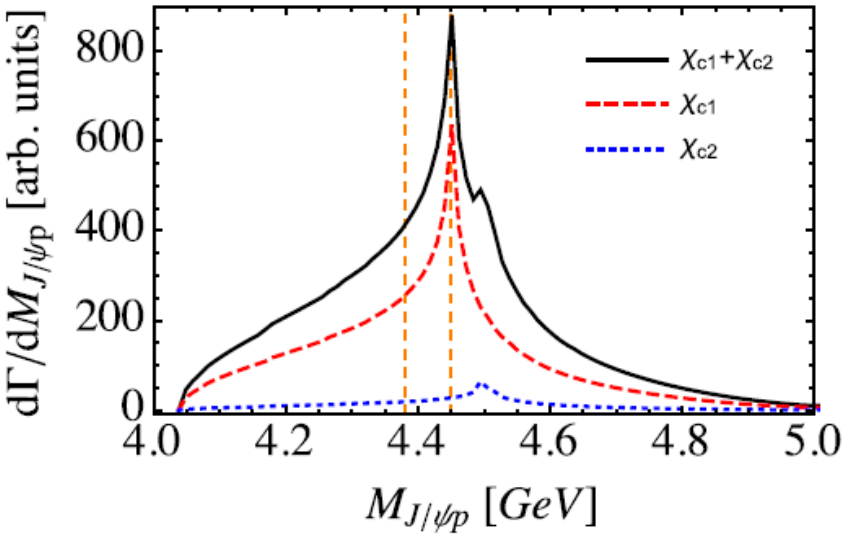


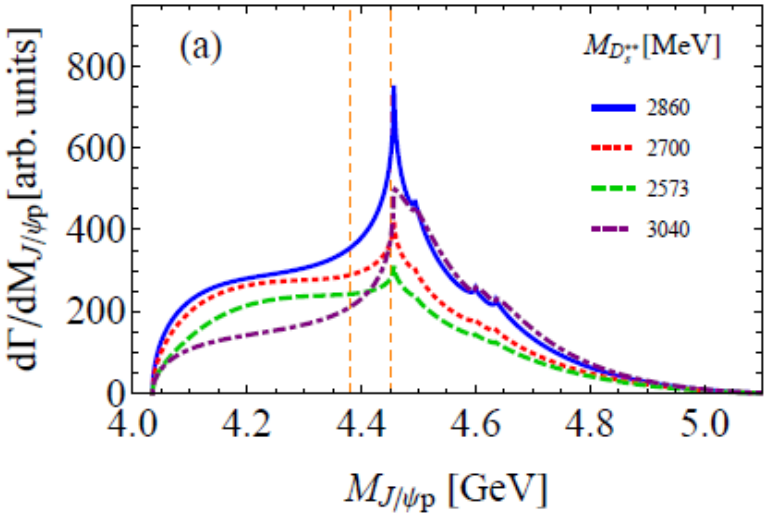
Various intervals of Kp invariant mass for the fit with two Pcs

TS mechanism and the heavy pentaquark "Pc"









See also F.K. Guo's talk

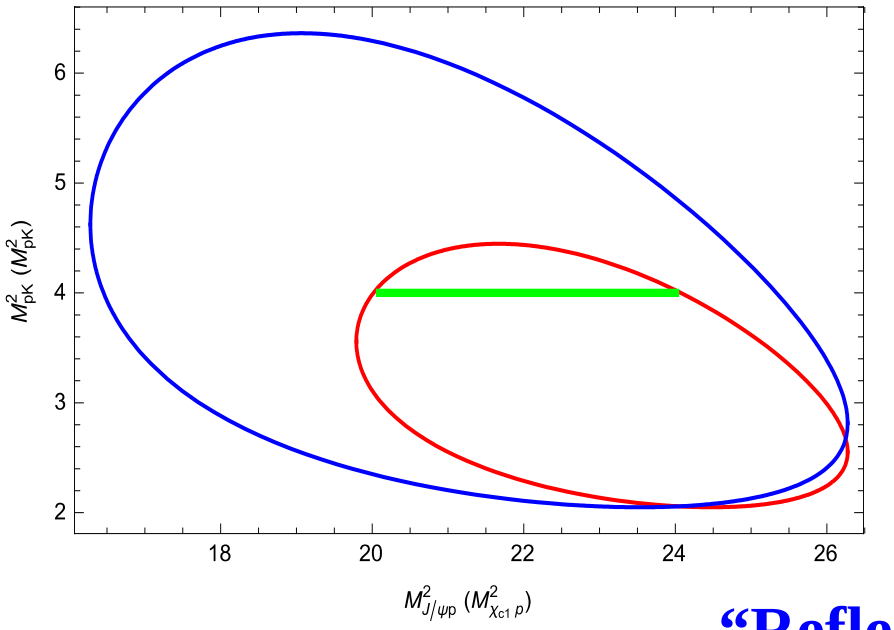
Thresholds [GeV]	$\chi_{c0}(1P)$ 0^+	$\chi_{c1}(1P)$ 1^+	$\chi_{c2}(1P)$ 2^+
p $1/2^+$	4.353	4.449	4.494

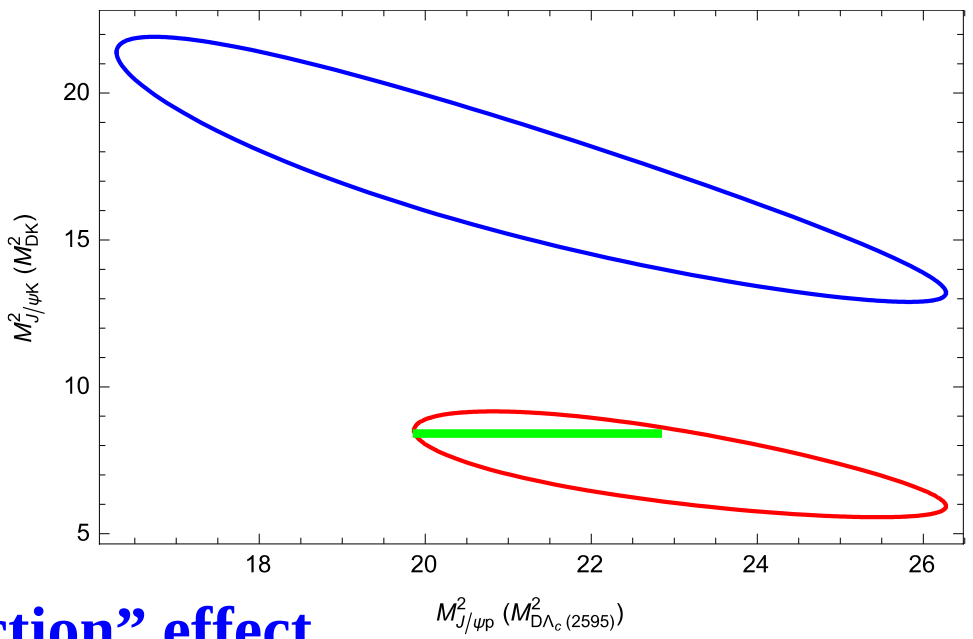
Thresholds [GeV]	$\Lambda_c(2286)$ $1/2^+$	$\Lambda_c(2595)$ $1/2^-$	$\Lambda_c(2625)$ $3/2^-$
$\bar{D}(1865)$ 0^-	4.151	4.457	4.493
$\bar{D}^*(2007)$ 1^-	4.293	4.599	4.635

Liu,Wang,Zhao, arXiv:1507.05359

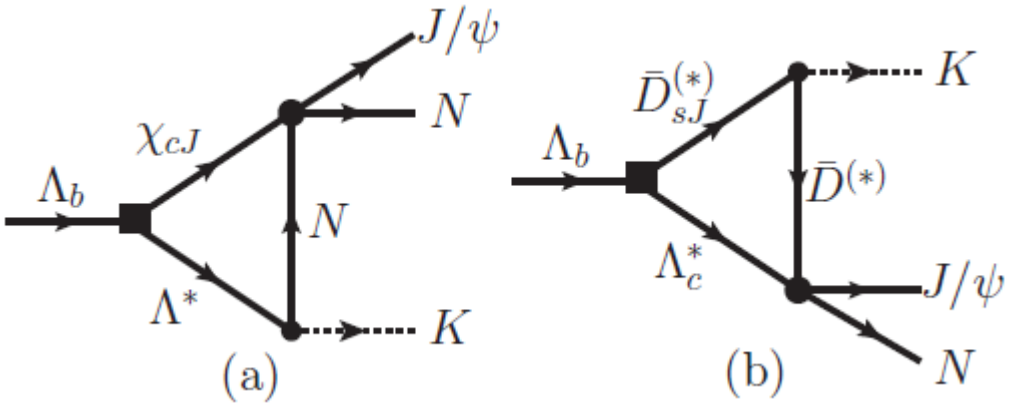
Guo et al, arXiv:1507.04950

TS mechanism: Dalitz plot

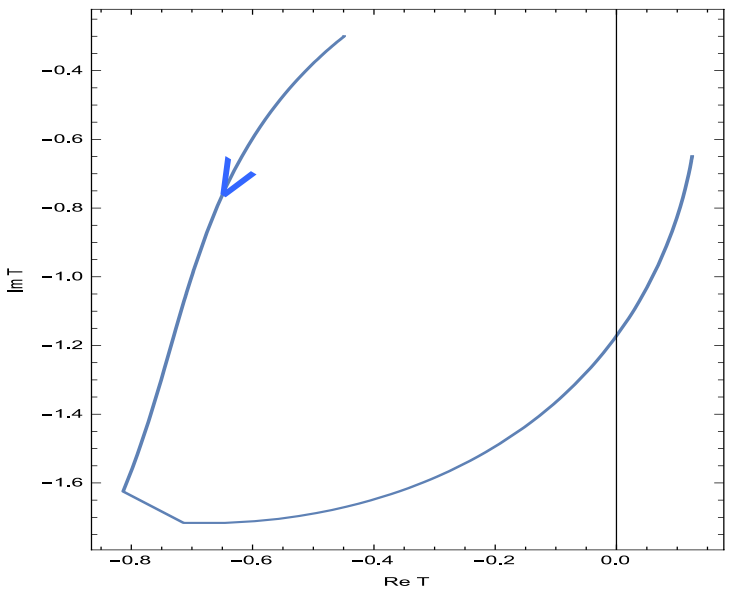


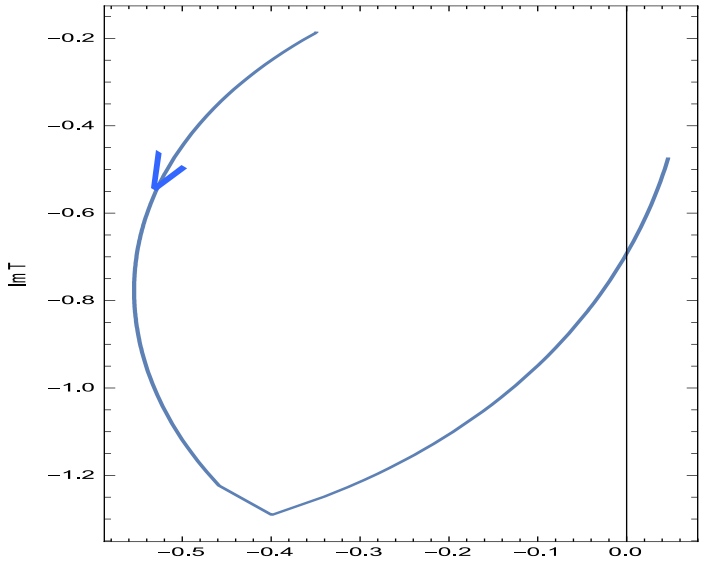


"Reflection" effect



TS mechanism: Argand diagram





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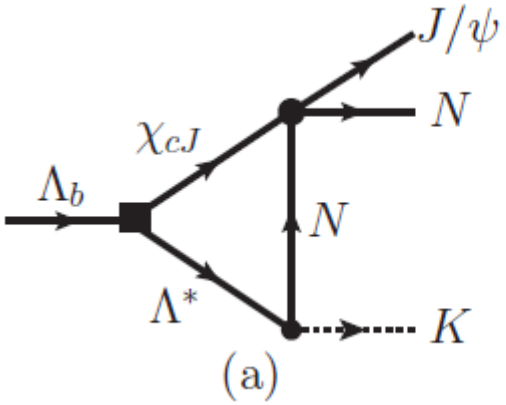
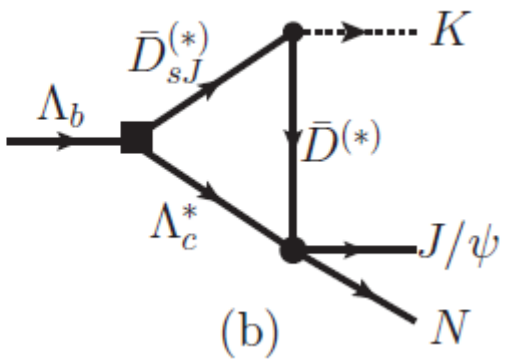
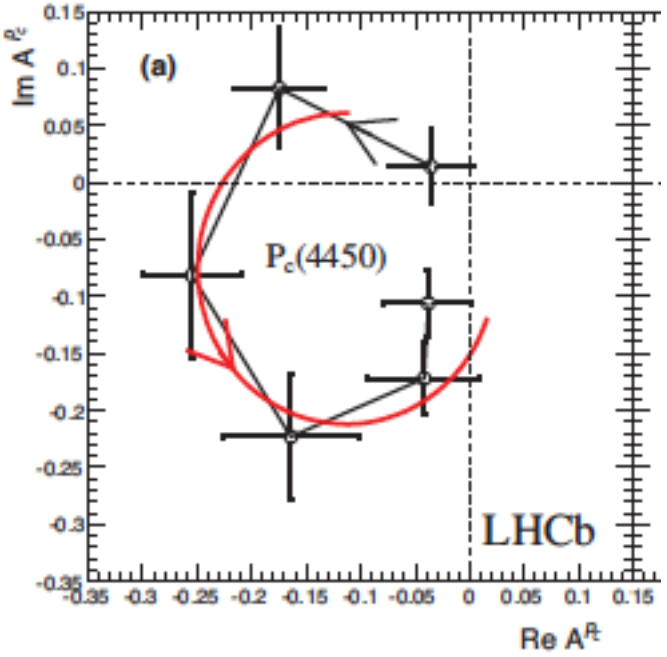
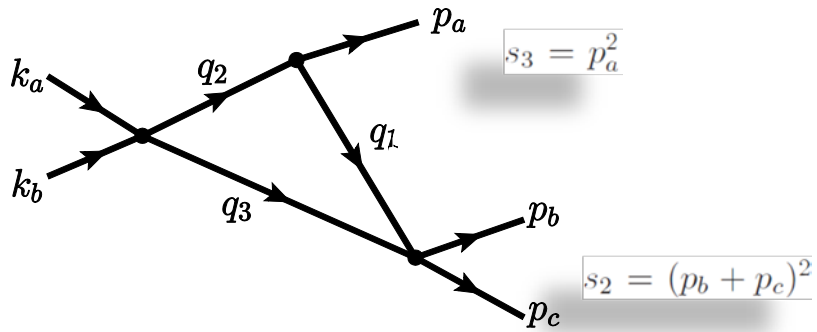






Searching for the heavy pentaquark in πp→πJ/ψp

TS mechanism is a highly process-dependent mechanism. Different kinds of processes are required to check it.



$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$

$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1+m_3)}[(m_2-m_1)^2 - s_3].$$

Requirements:

- **The quantity $[(m_2-m_1)^2 - s_3]$ should be larger (phase space should be larger);**
- **Particle q_2 should not be too broad;**
- **Balance between the phase space and width.**

Width Effect of the Intermediate State

Adopt the Breit-Wigner type propagator in the triangle loop integral

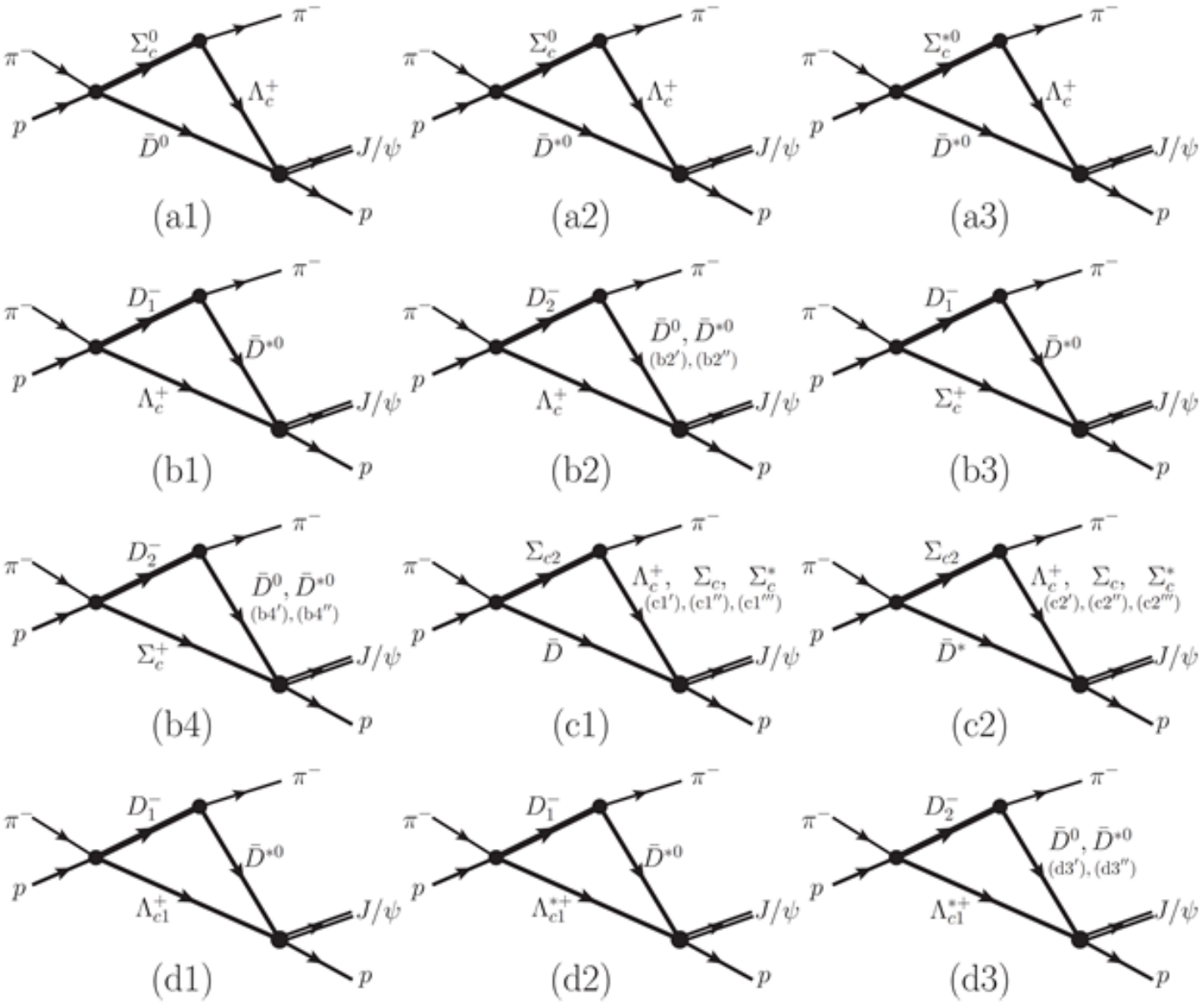
$$G_{\mathcal{D}}^{(0)} = \frac{i}{q_{\mathcal{D}}^2 - m_{\mathcal{D}}^2 + i\, m_{\mathcal{D}}\Gamma_{\mathcal{D}}},$$

$$G_{Y_c}^{(\frac{1}{2})} = \frac{i\, m_{Y_c}(1+\not{v})}{q_{Y_c}^2 - m_{Y_c}^2 + i\, m_{Y_c}\Gamma_{Y_c}},$$

Rarita-Schwinger wave functions for higher spin particles

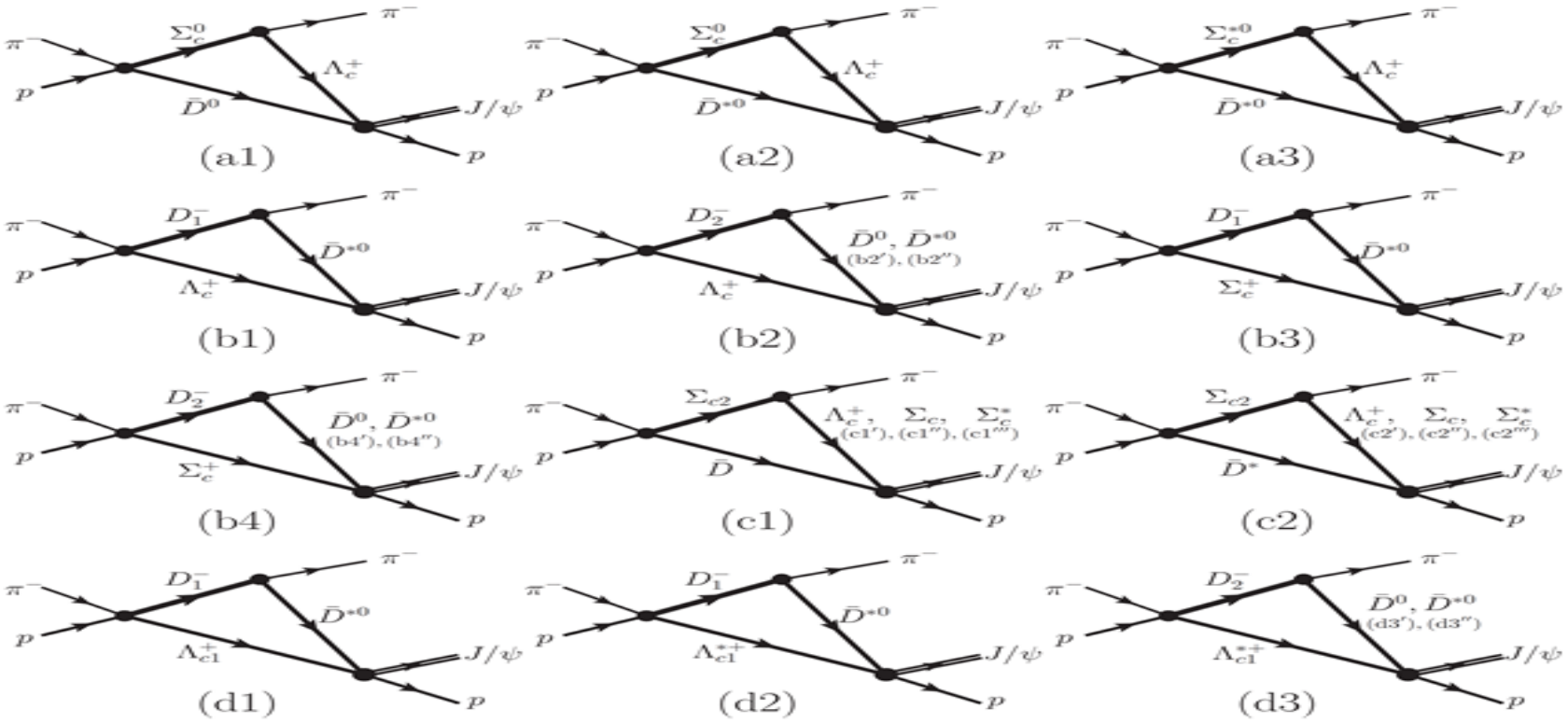
BW type propagator will remove the TS from the physical boundary by a small distance, if the corresponding decay width is smaller. I.J.R. Aitchison & C. Kacser, PR133, B1239 (1964)

πp→πJ/ψp via the open-charm loops



Liu, Oka, arXiv: 1602.07069

$\pi p \to \pi J/\psi p$ via the open-charm loops



I) (a1)-(a3): Both $\bar{\mathcal{D}}$ and Y_c are S-wave states,

II) (b1)-(b4): $\bar{\mathcal{D}}$ is a P-wave state and Y_c is an S-wave state,

III) (c1)-(c2): $\bar{\mathcal{D}}$ is an S-wave state and Y_c is a P-wave state,

IV) (d1)-(d3): Both $\bar{\mathcal{D}}$ and Y_c are P-wave states.

$\pi p \to \pi J/\psi p$ via the open-charm loops

Particle in PDG	Notation	J^P	Mass [MeV]	Width [MeV]
Λ_c^+	Λ_c^+	$\frac{1}{2}^+$	2286.46±0.14	
$\Lambda_c(2595)^+$	Λ_{c1}^+	$\frac{1}{2}^-$	2592.25± 0.28	2.6±0.6
$\Lambda_c(2625)^+$	Λ_{c1}^{*+}	$\frac{3}{2}^-$	2628.11±0.19	<0.97
$\Sigma_c(2455)^{++}$	Σ_c^{++}	$\frac{1}{2}^+$	2453.98±0.16	2.26±0.25
$\Sigma_c(2455)^+$	Σ_c^+	$\frac{1}{2}^+$	2452.9±0.4	<4.6
$\Sigma_c(2455)^0$	Σ_c^0	$\frac{1}{2}^+$	2453.74±0.16	2.16±0.26
$\Sigma_c(2520)^{++}$	Σ_c^{*++}	$\frac{3}{2}^+$	2517.9±0.6	14.9±1.5
$\Sigma_c(2520)^+$	Σ_c^{*+}	$\frac{3}{2}^+$	2517.5±2.3	<17
$\Sigma_c(2520)^0$	Σ_c^{*0}	$\frac{3}{2}^+$	2518.8±0.6	14.5±1.5
$\Sigma_c(2800)^{++}$	Σ_{c2}^{++}	$\frac{3}{2}^-$?	2801^{+4}_{-6}	75^{+22}_{-17}
$\Sigma_c(2800)^+$	Σ_{c2}^+	$\frac{3}{2}^-$?	2792^{+14}_{-5}	62^{+60}_{-40}
$\Sigma_c(2800)^0$	Σ_{c2}^0	$\frac{3}{2}^-$?	2806^{+5}_{-7}	72^{+22}_{-15}
D^0	D^0	0^-	1864.84±0.07	
$D^\pm$	$D^\pm$	0^-	1869.61±0.10	
$D^*(2007)^0$	D^{*0}	1^-	2006.96±0.10	<2.1
$D^*(2010)^\pm$	$D^{*\pm}$	1^-	2010.26±0.07	0.0834± 0.0018
$D_1(2420)^0$	D_1^0	1^+	2421.4±0.6	27.4±2.5
$D_2(2460)^0$	D_2^0	2^+	2462.6±0.6	49.0±1.3
$D_2(2460)^\pm$	$D_2^\pm$	2^+	2464.3±1.6	37±6

Notations of the charmed hadrons in relevant with our discussion

$\pi p \to \pi J/\psi p$ via the open-charm loops

Diagram #	$\sqrt{s_{1N}}$	$\sqrt{s_{1C}}$	$\Delta_{s_1}^{\max}$	$\sqrt{s_{2N}}$	$\sqrt{s_{2C}}$	$\Delta_{s_2}^{\max}$
(a1)	4317.7	4318.5	0.773	4151.4	4152.1	0.75
(a2)	4459.9	4460.7	0.806	4293.5	4294.3	0.78
(a3)	4524.5	4527.7	3.28	4293.5	4296.7	3.14
(b1)	4708.0	4726.3	18.4	4293.5	4310.2	16.7
(b2′)	4750.9	4794.5	43.7	4151.4	4189.2	37.8
(b2″)	4750.9	4773.5	22.7	4293.5	4314.0	20.4
(b3)	4874.3	4893.4	19.1	4459.9	4477.1	17.3
(b4′)	4917.2	4962.4	45.2	4317.7	4356.7	39.0
(b4″)	4917.2	4940.7	23.5	4459.9	4481.0	21.1
(c1′)	4665.8	4687.2	21.4	4151.4	4171.0	19.6
(c1″)	4665.8	4674.1	8.28	4317.7	4325.6	7.83
(c1‴)	4665.8	4670.7	4.83	4382.3	4387.0	4.62
(c2′)	4808.0	4830.3	22.3	4293.5	4313.9	20.4
(c2″)	4808.0	4816.6	8.65	4459.9	4468.0	8.16
(c2‴)	4808.0	4813.0	5.05	4524.5	4529.3	4.82
(d1)	5013.7	5033.2	19.6	4599.2	4616.9	17.7
(d2)	5049.5	5069.2	19.7	4635.1	4652.9	17.8
(d3′)	5092.4	5139.2	46.8	4493.0	4533.1	40.2
(d3″)	5092.4	5116.7	24.3	4635.1	4656.8	21.8

TS Kinematic Region

Formalism

The model is built in the framework of heavy hadron chiral perturbation theory (HHChPT)

Charmed Meson Superfiled:

$$H_{1a} = \frac{1+\not{v}}{2}[D^*_{a\mu}\gamma^\mu - D_a\gamma_5],$$

$$H_{2a} = [\bar{D}^*_{a\mu}\gamma^\mu + \bar{D}_a\gamma_5]\frac{1-\not{v}}{2},$$

$$T^\mu_{1a} = \frac{1+\not{v}}{2}\left\{ D^{\mu\nu}_{2a}\gamma_\nu - \sqrt{\frac{3}{2}}D_{1a\nu} \times\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\right\},$$

$$T^\mu_{2a} = \left\{ \bar{D}^{\mu\nu}_{2a}\gamma_\nu + \sqrt{\frac{3}{2}}\bar{D}_{1a\nu} \times\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\right\}\frac{1-\not{v}}{2},$$

$$\bar{H}_{1a,2a} = \gamma^0 H^\dagger_{1a,2a}\gamma^0,\ \bar{T}_{1a,2a} = \gamma^0 T^\dagger_{1a,2a}\gamma^0,$$

Formalism

Charmed Baryon Superfiled:

$$\mathcal{T}_i = \frac{1}{2}\epsilon_{ijk}\frac{1+\not{v}}{2}(B_{\bar{3}})_{jk},$$

$$\mathcal{R}_\mu = \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5\frac{1+\not{v}}{2}\Lambda_{c1}^{+} + \frac{1+\not{v}}{2}\Lambda_{c1\mu}^{*+},$$

$$\mathcal{S}_\mu^{ij} = \frac{1+\not{v}}{2}B_{6\mu}^{*ij} + \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5\frac{1+\not{v}}{2}B_6^{ij},$$

$$\mathcal{X}_{\mu\nu}^{ij} = \frac{1}{\sqrt{10}}\left[(\gamma_\mu + v_\mu)\gamma_5 g_\nu^\alpha + (\gamma_\nu + v_\nu)\gamma_5 g_\mu^\alpha\right]X_\alpha^{ij},$$

$$(B_{\bar{3}})_{ij} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}_{ij} \qquad (B_6)_{ij} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c^{\prime+} \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c^{\prime0} \\ \frac{1}{\sqrt{2}}\Xi_c^{\prime+} & \frac{1}{\sqrt{2}}\Xi_c^{\prime0} & \Omega_c^0 \end{pmatrix}_{ij}$$

$$(X)_{ij} = \begin{pmatrix} \Sigma_{c2}^{++} & \frac{1}{\sqrt{2}}\Sigma_{c2}^+ & \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime+} \\ \frac{1}{\sqrt{2}}\Sigma_{c2}^+ & \Sigma_{c2}^0 & \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime0} \\ \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime+} & \frac{1}{\sqrt{2}}\Xi_{c2}^{\prime0} & \Omega_{c2}^0 \end{pmatrix}_{ij}$$

Formalism

Effective Lagrangian, combing the chiral symmetry and the heavy quark spin symmetry

$$\begin{aligned}
\mathcal{L}_{\rm meson} &= i\frac{h'}{\Lambda_\chi}{\rm Tr}\left[\bar{H}_{2a}T^{\mu}_{2b}\gamma^{\nu}\gamma_5(D_\mu\mathcal{A}_\nu+D_\nu\mathcal{A}_\mu)_{ba}\right] + \text{ h.c.},\\
\mathcal{L}_{\rm baryon} &= -\sqrt{3}g_2{\rm Tr}\left[\bar{B}_{\bar{3}}\mathcal{A}^{\mu}\mathcal{S}_\mu+\bar{\mathcal{S}}_\mu\mathcal{A}^{\mu}B_{\bar{3}}\right]\\
&+ ih_{10}\epsilon_{ijk}\bar{\mathcal{T}}_i(D_\mu\mathcal{A}_\nu+D_\nu\mathcal{A}_\mu)_{jl}\mathcal{X}^{\mu\nu}_{kl}\\
&+ h_{11}\epsilon_{\mu\nu\sigma\lambda}v^{\lambda}{\rm Tr}\left[\bar{\mathcal{S}}^{\mu}(D^{\nu}\mathcal{A}_\alpha+D_\alpha\mathcal{A}^{\nu})\mathcal{X}^{\alpha\sigma}\right],
\end{aligned}$$

Cho, PRD50,3295 (1994); PLB285,145(1992); Prijol&Yan, PRD56,5483(1997); Cheng&Chua, PRD75,014006

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger\right)$$

$$\xi = e^{i\mathcal{M}/f_\pi}\ ,\ \mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0\\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

$$D_\mu=\partial_\mu+\mathcal{V}_\mu \qquad \mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger\right)$$

Formalism

Coupling constants can be determined by the corresponding decay widths

$$h' = 0.43,$$
$$g_2 = 0.565,$$
$$|h_{10}| = 0.85.$$

By means of the quark model

$$|h_{11}| = \sqrt{2}|h_{10}|$$

Prijol&Yan, PRD56,5483(1997)

D-wave decay modes (advantage for the TS mechanism)

$$\bar{D}_1 \to \bar{D}^*\pi,\ \bar{D}_2 \to \bar{D}^{(*)}\pi \quad \Sigma_{c2} \to \Lambda_c\pi/\Sigma_c^{(*)}\pi$$

Formalism

Effective Lagrangian for $\bar{\mathcal{D}} Y_c \to J/\psi p$

$$\mathcal{L}_{\rm ct} = g_{\Lambda_c} \bar{N} H_2 \bar{J} \mathcal{T}_3 + g_{\Sigma_c} \bar{N} \gamma_\mu \gamma_5 H_2 \bar{J} \mathcal{S}^\mu + i g_{\Lambda_{c1}} \partial_\mu \bar{N} H_2 \bar{J} \mathcal{R}^\mu,$$

$$J = \frac{1+\not{v}}{2} [\psi(nS)^\mu \gamma_\mu - \eta_c(nS)\gamma_5] \frac{1-\not{v}}{2}$$

$$\bar{J} = \gamma^0 J \gamma^0.$$

For these short range interactions, from the dimensional analysis, it is expected that the couplings g_{Λ_c} (g_{Σ_c}) and $g_{\Lambda_{c1}}$ are of the order of magnitude of m_D^{-2} and m_D^{-3} respectively, where m_D is the mass of the D meson. Notice that in HHChPT, the heavy filed H_2 (J) in Eq. (26) will contain a factor $\sqrt{M_{H_2}}$ ($\sqrt{M_J}$) for normalization. In addition, we will estimate the scattering amplitudes in the static limit , which means the four velocity is set to $v = (1, 0, 0, 0)$.

Formalism

Effective Lagrangian for $\pi^- p \to \bar{D} Y_c$, with the fewest derivatives

$$\mathcal{L}_{\rm a1} = \frac{g_{a1}}{2m_D}\,\mathbb{D}\,\tau\cdot\pi\,\tau\cdot\bar{\mathbf{\Sigma}}_{\mathbf{c}}\,N,$$

$$\mathcal{L}_{\rm a2} = \frac{g_{a2}}{2m_D}\,\mathbb{D}^*_\mu\,\tau\cdot\pi\,\tau\cdot\bar{\mathbf{\Sigma}}_{\mathbf{c}}\gamma_5\gamma^\mu N,$$

$$\mathcal{L}_{\rm a3} = \frac{g_{a3}}{2m_D}\,\mathbb{D}^*_\mu\,\tau\cdot\pi\,\tau\cdot\bar{\mathbf{\Sigma}}^{*\mu}_{\mathbf{c}}\,N,$$

$$\mathcal{L}_{\rm b1} = \frac{ig_{b1}}{\sqrt{2}m_D^2}\,\mathbb{D}^\mu_1\,\tau\cdot\partial_\mu\pi\,\bar{\Lambda}_c\,N,$$

$$\mathcal{L}_{\rm b2} = \frac{ig_{b2}}{\sqrt{2}m_D^2}\,\mathbb{D}^{\mu\nu}_2\,\tau\cdot\partial_\mu\pi\,\bar{\Lambda}_c\gamma_5\gamma_\nu N,$$

$$\mathcal{L}_{\rm b3} = \frac{ig_{b3}}{2m_D^2}\,\mathbb{D}^\mu_1\,\tau\cdot\partial_\mu\pi\,\tau\cdot\bar{\mathbf{\Sigma}}_{\mathbf{c}}\,N,$$

$$\mathcal{L}_{\rm b4} = \frac{ig_{b4}}{2m_D^2}\,\mathbb{D}^{\mu\nu}_2\,\tau\cdot\partial_\mu\pi\,\tau\cdot\bar{\mathbf{\Sigma}}_{\mathbf{c}}\gamma_5\gamma_\nu N,$$

$$\mathcal{L}_{\rm c1} = \frac{ig_{c1}}{2m_D^2}\,\mathbb{D}\,\tau\cdot\partial_\mu\pi\,\tau\cdot\bar{\mathbf{\Sigma}}^\mu_{\mathbf{c2}}\,N,$$

$$\mathcal{L}_{\rm c2} = \frac{ig_{c2}}{2m_D^2}\,\mathbb{D}^*_\mu\,\tau\cdot\partial_\nu\pi\,\tau\cdot\bar{\mathbf{\Sigma}}^\mu_{\mathbf{c2}}\gamma_5\gamma^\nu N,$$

$$\mathcal{L}_{\rm d1} = \frac{g_{d1}}{\sqrt{2}m_D}\,\mathbb{D}^\mu_1\,\tau\cdot\pi\,\bar{\Lambda}_{c1}\gamma_5\gamma_\mu N,$$

$$\mathcal{L}_{\rm d2} = \frac{g_{d2}}{\sqrt{2}m_D}\,\mathbb{D}^\mu_1\,\tau\cdot\pi\,\bar{\Lambda}^*_{c1\mu}\,N,$$

$$\mathcal{L}_{\rm d3} = \frac{g_{d3}}{\sqrt{2}m_D}\,\mathbb{D}^{\mu\nu}_2\,\tau\cdot\pi\,\bar{\Lambda}^*_{c1\mu}\gamma_5\gamma_\nu N,$$

$$\mathbb{D} = (D^0,\ D^+)$$

Open-charm Production

Experimental and theoretical constraints on the open-charm productions in πN collisions:

$\sigma(\pi^- p \rightarrow D^{*-}\Lambda_c^+)$ and $\sigma(\pi^- p \rightarrow D^{*-}\Sigma_c^+)$
upper limit at 13 GeV is about 7 nb BNL, PRL55, 154(1985)

Effective Lagrangian method and Regge approach: at the order of 1 nb S.H. Kim et al, PRD92, 094001 (2015); arXiv: 1405.3445

Generalized parton picture: at the order of nb
S. Kofler et al, PRD91, 054027 (2015)

Our assumptions:

$\sigma(\pi^- p \rightarrow \bar{D}Y_c)$ is 1 nb at 20 GeV pion-beam energy

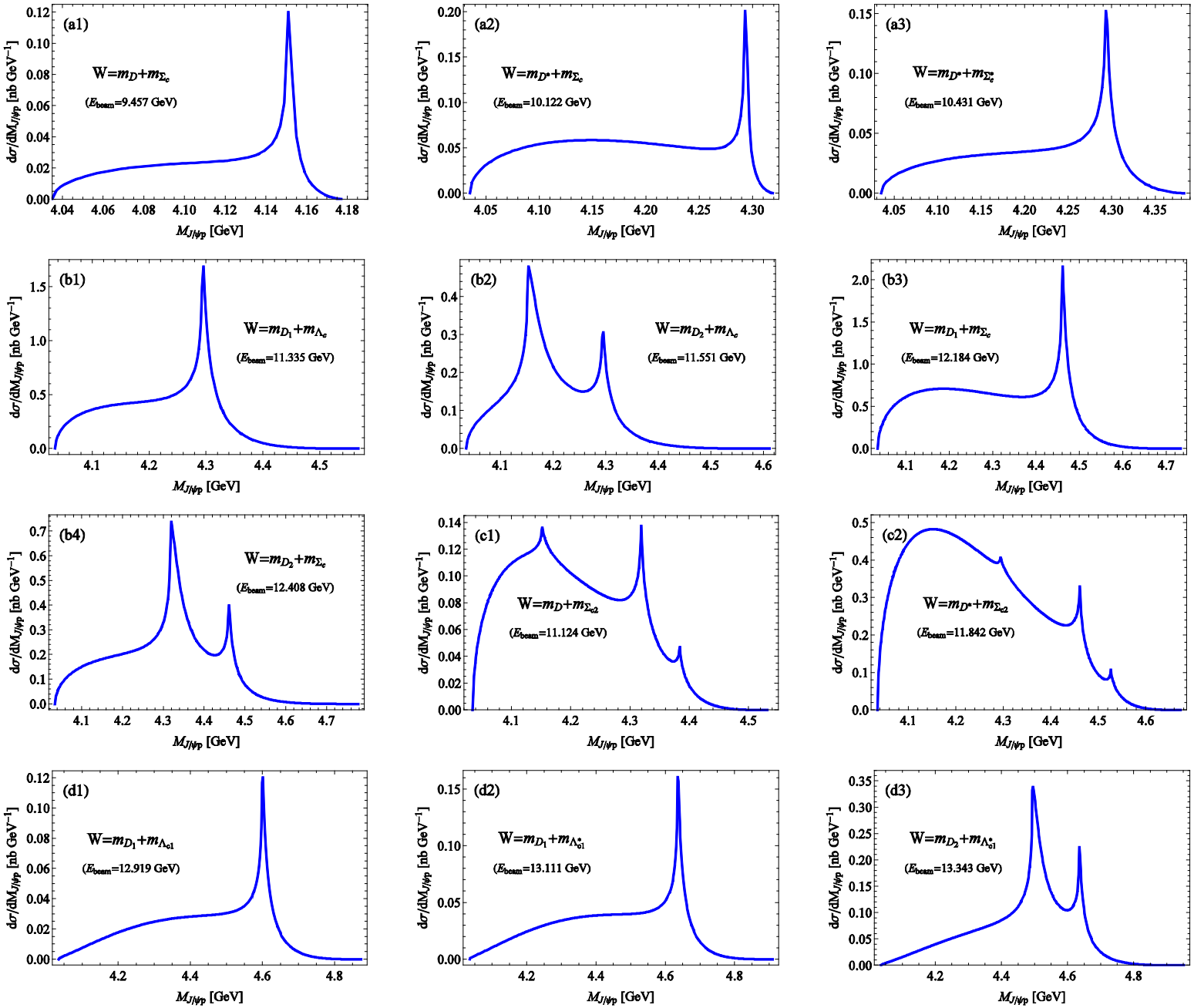
Open-charm Production

Coupling constants according to the assumpions

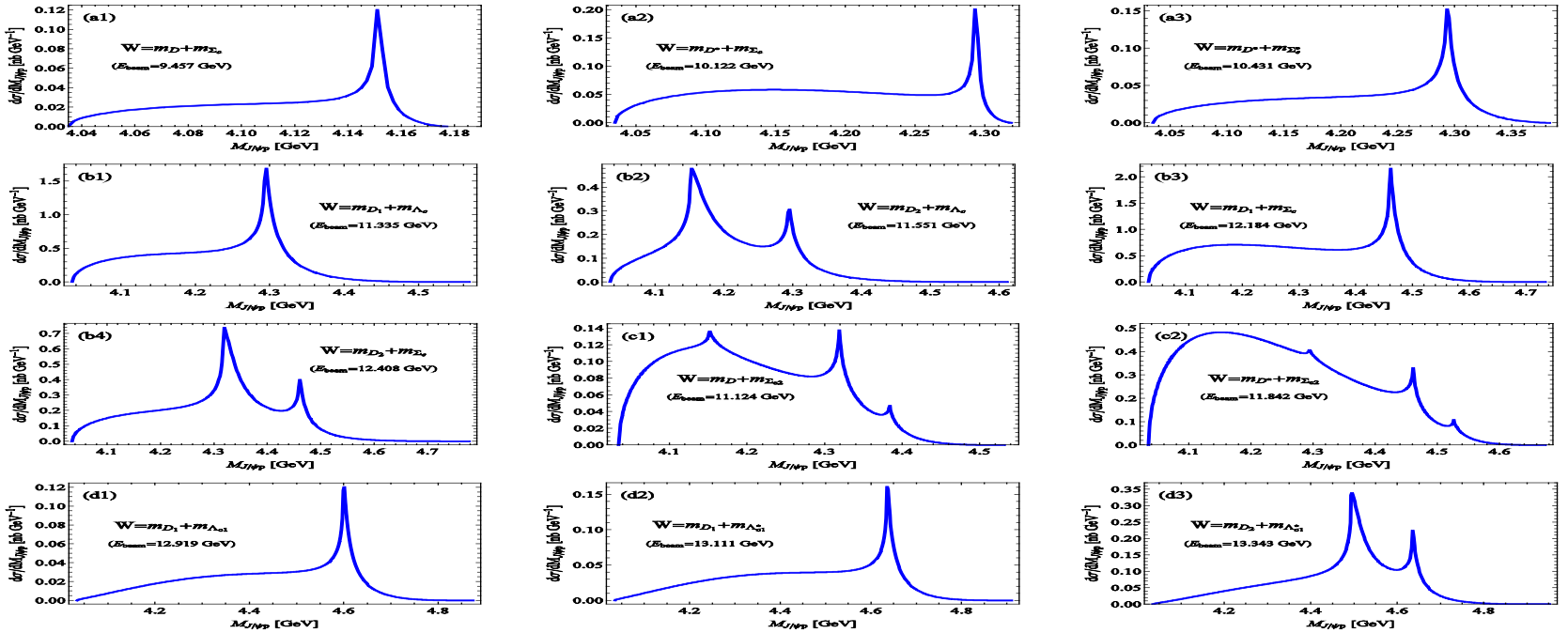
$\sigma(\pi^- p \to \bar{D} Y_c)$ **is 1 nb at 20 GeV pion-beam energy**

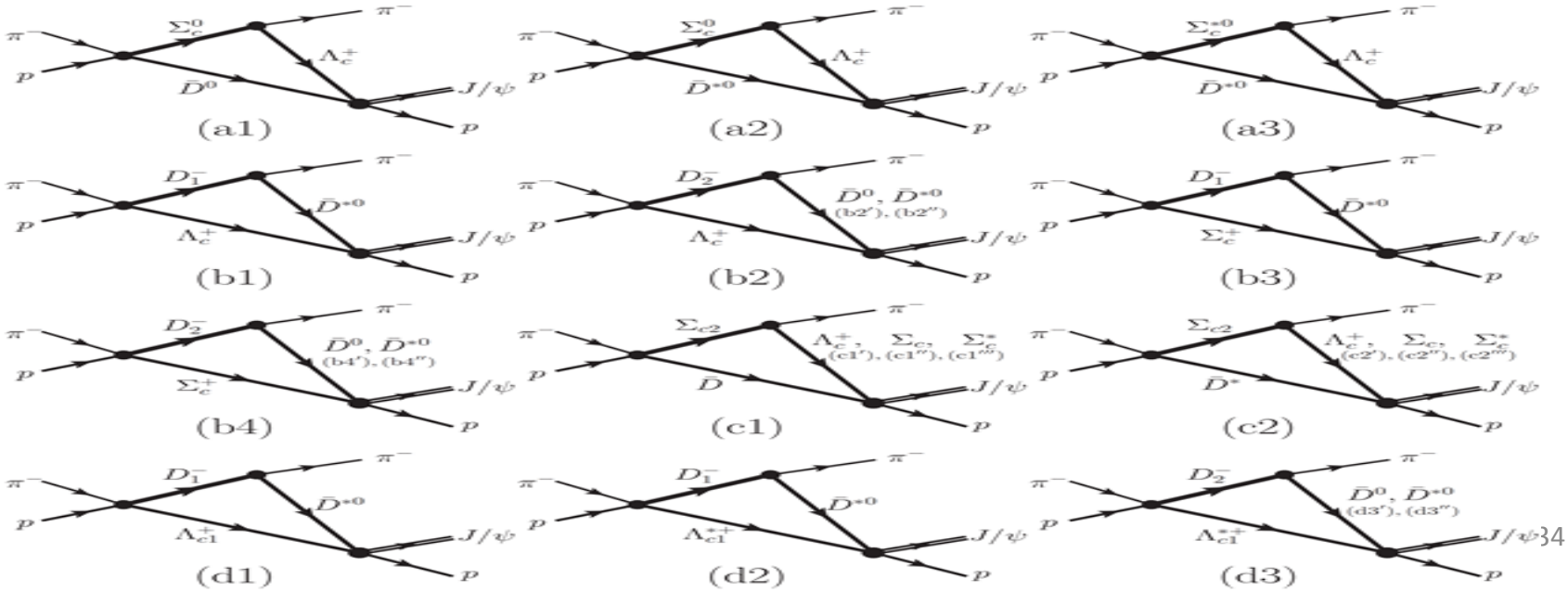
g_{a1}	g_{a2}	g_{a3}	g_{b1}	g_{b2}	g_{b3}
0.035	0.020	0.025	0.024	0.018	0.023
g_{b4}	g_{c1}	g_{c2}	g_{d1}	g_{d2}	g_{d3}
0.017	0.027	0.009	0.024	0.026	0.029

Numerical Results: J/ψp Invariant Mass Distribution



Numerical Results: J/ψp Invariant Mass Distribution





Summary

- **Kinematic singularities (TS) of the rescattering amplitude will behave themselves as peaks in the invariant mass distribution, which may imply that non-resonance interpretations for some resonance-like structures is possible.**
- **Being different from the genuine resonances, the TS mechanism is a highly process-dependent mechanism, and very sensitive to the kinematic configurations.**
- **The forthcoming J-PARC pion-induced experiment may offer us a good opportunity to check different kinematic or dynamic mechanisms and clarify the ambiguities, with its high luminosity.**

Thanks!

Backup

Amplitude in HQET

The weak amplitude $\left\langle \Lambda_{c1}^{(*)} \bar{D}_{sJ}^{(*)} | H_{\rm eff} | \Lambda_b \right\rangle$ **can be factorized into**

$$\left\langle \Lambda_{c1}^{(*)} | (V-A)_\mu | \Lambda_b \right\rangle \left\langle \bar{D}_{sJ}^{(*)} | (V-A)^\mu | 0 \right\rangle$$

At the leading order

$$\left\langle \Lambda_{c1}^{(*)} | (V-A)_\mu | \Lambda_b \right\rangle = \xi(\omega) v_\alpha \bar{\mathcal{R}}_{v'}^{\alpha} (V-A)_\mu \Lambda_{bv}$$

$$\mathcal{R}_v^\alpha = \frac{1}{\sqrt{3}} (\gamma^\alpha + v^\alpha) \gamma_5 \frac{1+\not{v}}{2} \Lambda_{c1} + \frac{1+\not{v}}{2} \Lambda_{c1}^{*\alpha}$$

For the contact interaction

$$\mathcal{L}_{\rm ct} = i g_{\Lambda_{c1}} \partial_\mu \bar{N} H_2 \bar{J} \mathcal{R}^\mu$$

$\Lambda_{c1}^{(*)} \bar{D}_{sJ}^{(*)}$ loops

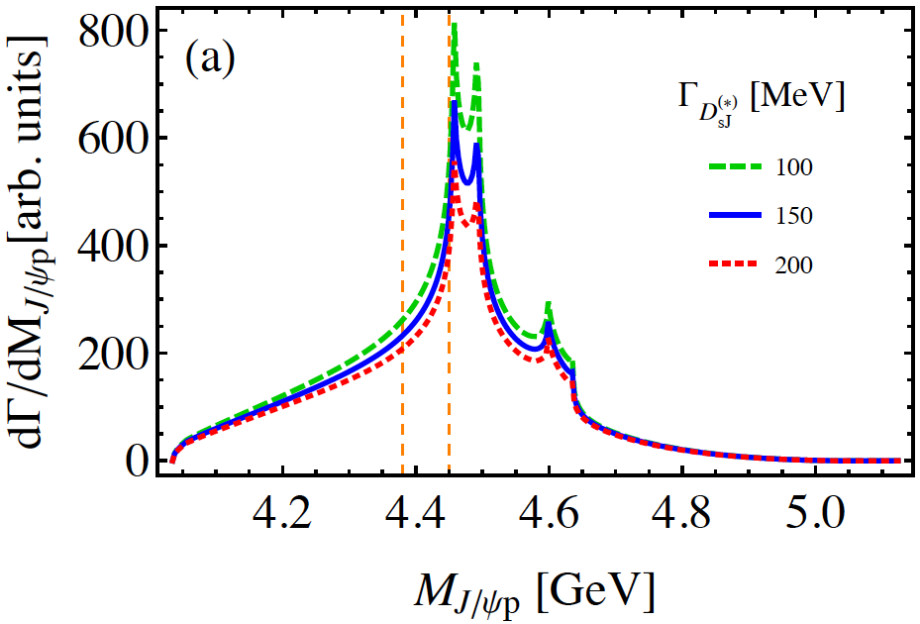
suppressed by heavy quark symmetry

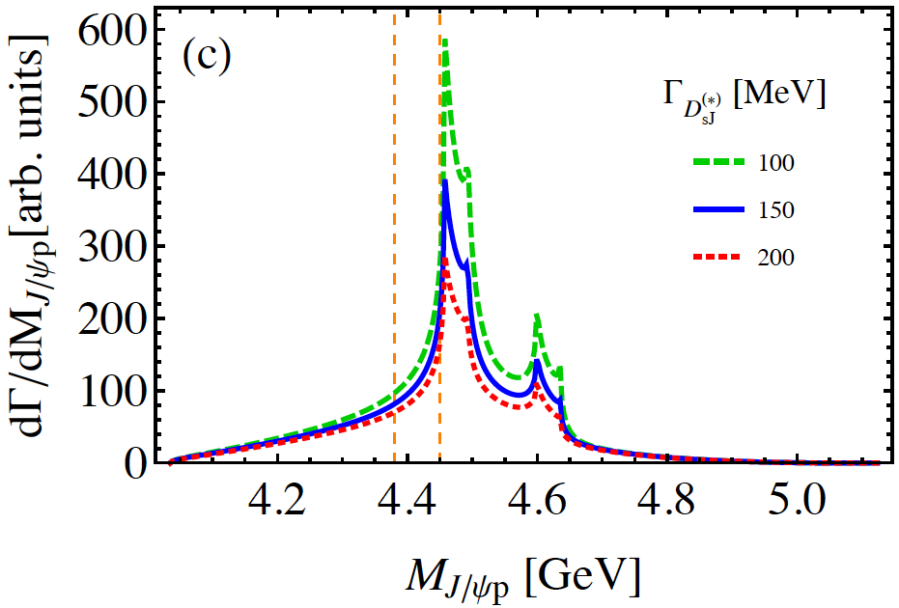
competitive

$\chi_{cJ} \Lambda^*$ loops

OZI suppressed

Amplitude in HQET





Mass of $D_{sJ}^{(*)}$ **2.86 GeV** **2.95 GeV**

The broad $D_{s1}(2860)$ is accepted as a D-wave charmed-strange meson

$\Gamma(D_{s1} \to D\,K)/\Gamma(D_{s1} \to D^*\,K)$ is approximately 2.

Numerical Results: Total Cross Section

Diagram #	Cross Section [nb]
(a1)	0.003
(a2)	0.015
(a3)	0.011
(b1)	0.153
(b2)	0.056
(b3)	0.330
(b4)	0.113
(c1)	0.032
(c2)	0.166
(d1)	0.016
(d2)	0.024
(d3)	0.056