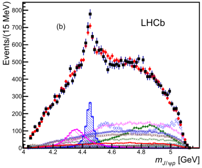


$P_c(4450)$ and triangle singularities

Feng-Kun Guo

Institute of Theoretical Physics, Chinese Academy of Sciences

MENU 2016, 25th – 30th July 2016, Kyoto University

Based on:

FKG, U.-G. Meißner, W. Wang, Z. Yang, Phys. Rev. D **92**, 071502(R) (2015) [arXiv:1507.04950[hep-ph]]

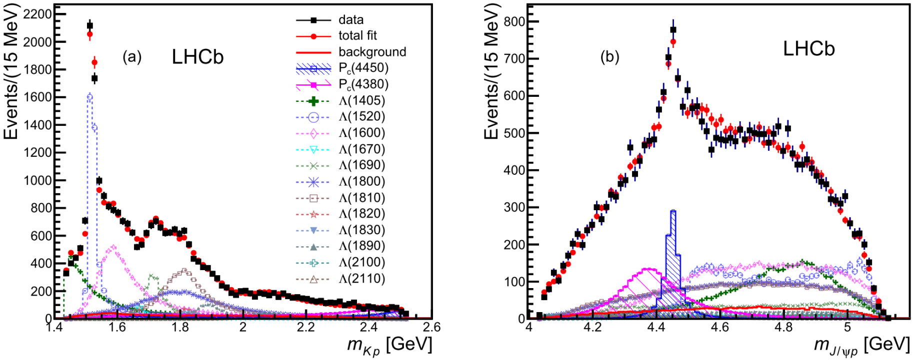
FKG, U.-G. Meißner, J. Nieves, Z. Yang, arXiv:1605.05113[hep-ph]

A. Aceti, M. Bayar, E. Oset, FKG, in preparation

LHCb pentaquark-like structures: Big news one year ago!

PRL115(2015)072001 [arXiv:1507.03414]

appeared on arXiv on 14.07.2015, and accepted by PRL on 24.07.2015!



$$M_1 = (4380 \pm 8 \pm 29)\ \text{MeV}, \qquad \Gamma_1 = (205 \pm 18 \pm 86)\ \text{MeV},$$

$$M_2 = (4449.8 \pm 1.7 \pm 2.5)\ \text{MeV}, \qquad \Gamma_2 = (39 \pm 5 \pm 19)\ \text{MeV}.$$

LHCb pentaquark-like structures (II)

- Quantum numbers not fully determined, for ($P_c(4380), P_c(4450)$):
  $(3/2^-, 5/2^+)$, $(3/2^+, 5/2^-)$, $(5/2^+, 3/2^-)$
- In $J/\psi\, p$ invariant mass distribution, with hidden charm
  $\Rightarrow$ pentaquarks if they are really hadron states
- Narrow pentaquark-like structures with hidden-charm were predicted 5 years ago (07.2010):
  Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV,
  J. J. Wu, R. Molina, E. Oset, B. S. Zou, Phys. Rev. Lett. **105** (2010) 232001.

talks by E. Oset and B.-S. Zou

A flood of short papers

14.07 The LHCb paper appeared on line, arXiv:1507.03414

15.07 R. Chen, X. Liu, X. Q. Li and S. L. Zhu, arXiv:1507.03704 [hep-ph].
H. X. Chen, W. Chen, X. Liu, T. G. Steele and S. L. Zhu, arXiv:1507.03717 [hep-ph].

16.07 L. Roca, J. Nieves and E. Oset, arXiv:1507.04249 [hep-ph].

17.07 A. Mironov and A. Morozov, arXiv:1507.04694 [hep-ph].

18.07 weekend,

19.07 but everybody was working hard (NOT including me). . .

20.07 F.-K. Guo, U.-G. Meißner, W. Wang and Z. Yang, arXiv:1507.04950 [hep-ph].
L. Maiani, A. D. Polosa and V. Riquer, arXiv:1507.04980 [hep-ph].

21.07 J. He, arXiv:1507.05200 [hep-ph]; X. H. Liu, Q. Wang, Q. Zhao, arXiv:1507.05359 [hep-ph].

22.07 R. F. Lebed, arXiv:1507.05867 [hep-ph].

23.07 Exotic! why no new papers?

24.07 M. Mikhasenko, arXiv:1507.06552 [hep-ph].

28.07 U.-G. Meißner and J. A. Oller, arXiv:1507.07478 [hep-ph].

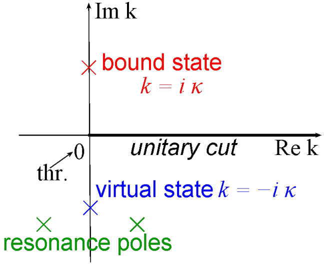
29.07 V. V. Anisovich *et al.*, arXiv:1507.07652 [hep-ph].

30.07 Guan-Nan Li, Min He, Xiao-Gang He, arXiv:1507.08252 [hep-ph].

. . . . [224 citations up to 26.07.2016]

Two kinds of singularities of S matrix

- Poles in the S-matrix: dynamics
  - ☞ bound states (real axis, 1st Riemann sheet (RS) of the complex energy plane)
  - ☞ virtual states (real axis, 2nd RS)
  - ☞ resonances (2nd RS)



- Landau singularities: kinematics
  - ☞ (a): two-body threshold cusp
  - ☞ (b): triangle singularity

…

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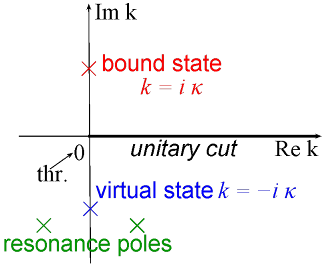
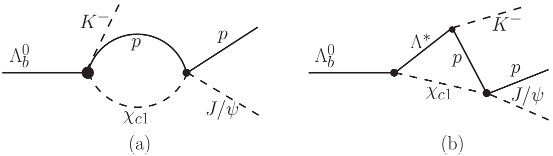


- Landau singularities: kinematics
  - ☞ (a): two-body threshold cusp
  - ☞ (b): triangle singularity

. . .



Triangle singularity – literature

- Some recent work using triangle singularity to explain (part of) peak structures [$\eta(1405/1475)$, $a_1(1420)$, $\ldots$]:

  J. J. Wu, X. H. Liu, Q. Zhao and B. S. Zou, PRL108(2012)081803;

  X. G. Wu, J. J. Wu, Q. Zhao and B. S. Zou, PRD87(2013)014023(2013);

  Q. Wang, C. Hanhart and Q. Zhao, PLB725(2013)106;

  M. Mikhasenko, B. Ketzer and A. Sarantsev, PRD91(2015)094015;

  N. N. Achasov, A. A. Kozhevnikov and G. N. Shestakov, PRD92(2015)036003;

  X. H. Liu, M. Oka and Q. Zhao, PLB753(2016)297;

  A. P. Szczepaniak, PLB747(2015)410; PLB757(2016)61;

  F. Aceti, L. R. Dai and E. Oset, arXiv:1606.06893 [hep-ph];

  ......

  see the talk by Q. Zhao

Triangle singularity – literature

- Very old knowledge from 1960s:

  Classical books:

  R. J. Eden, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne, *The Analytic S-Matrix* Cambridge University Press, 1966.

  T. S. Chang, Introduction to Dispersion Relation (2 volumes, in Chinese, written in 1965), Science Press, Beijing 1980, 1983.

  Recent lecture notes by one of the key players:

  I. J. R. Aitchison, arXiv:1507.02697 [hep-ph]. *Unitarity, Analyticity and Crossing Symmetry in Two- and Three-hadron Final State Interactions*.

Tsung-Sui Chang
(1915–1969)

$P_c(4450)$ is at the $\chi_{c1}p$ threshold

- Mass: $M = (4449.8 \pm 1.7 \pm 2.5)$ MeV

  The LHCb paper says:

  the closest threshold is at (4457.1 ± 0.3) MeV $[\Lambda_c(2595)\bar{D}^0]$

  $\Rightarrow$ difficult to explain with threshold effect

  This is NOT true!

- It is located exactly at the $\chi_{c1}p$ threshold:

$$M_{P_c(4050)} - M_{\chi_{c1}} - M_p = (0.9 \pm 3.1) \text{ MeV}$$

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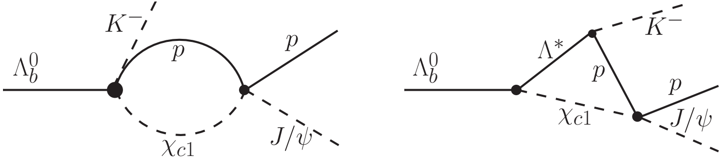
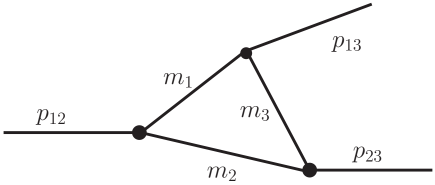


(a) (b)

Landau equation

see the talk by X.-H. Liu



- Triangle singularity: leading Landau singularity for a triangle diagram, anomalous threshold

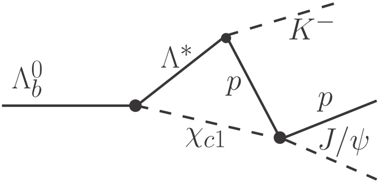
  studied extensively in 1960s

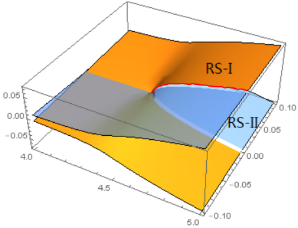
- Solutions of Landau equation: Landau (1959)

$$1 + 2\, y_{12}\, y_{23}\, y_{13} = y_{12}^2 + y_{23}^2 + y_{13}^2, \qquad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_{ij}^2}{2\, m_i\, m_j}$$

  quadratic equation of y_{ij}, always two solutions

- Do they affect the physical amplitude?





- Coleman–Norton theorem: S. Coleman and R. E. Norton, Nuovo Cim. 38 (1965) 438

  The singularity is on the physical boundary if and only if the diagram can be interpreted as a classical process in space-time.

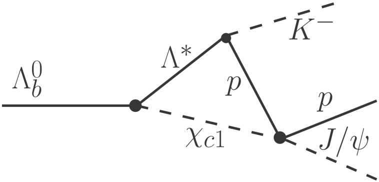
  ☞ physical boundary: upper edge (lower edge) of the unitary cut in the first (second) Riemann sheet

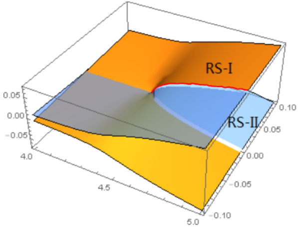
- Translation:

  ☞ all three intermediate states can go on shell

  ☞ $\vec{p} \parallel \vec{p}_{\chi_{c1}}$, the proton can catch up with the χ_{c1} to rescatter

Coleman-Norton theorem





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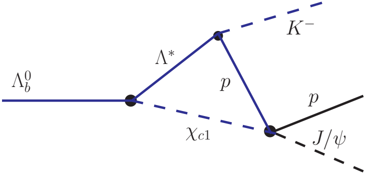
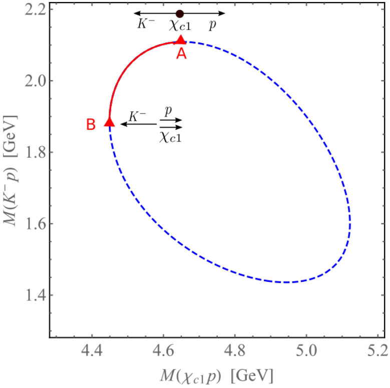


Dalitz plot for $\Lambda_b \to \chi_{c1}\Lambda^* \to \chi_{c1} p \bar{K}$:
Starting from a large Λ^* mass, in Λ_b rest frame

- when $M_{\Lambda^*} > M_{\Lambda_b} - M_{\chi_c 1}$, cannot go on-shell
- at point A, $M_{\Lambda^*} = M_{\Lambda_b} - M_{\chi_c 1}$, χ_{c1} is at rest
- at point B, proton and χ_{c1} has the same velocity
- between A and B, $\vec{p}_p \parallel \vec{p}_{\chi_{c1}}$ and proton moves faster than χ_{c1}



Analysis of the kinematics

Dalitz plot for $\Lambda_b \to \chi_{c1}\Lambda^* \to \chi_{c1}p\bar{K}$:
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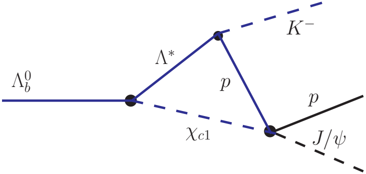

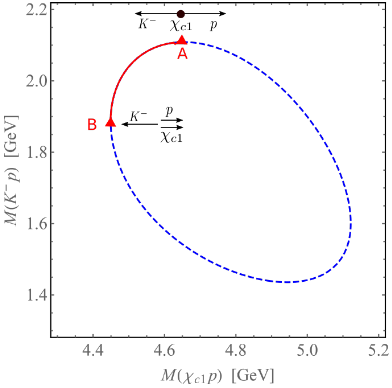

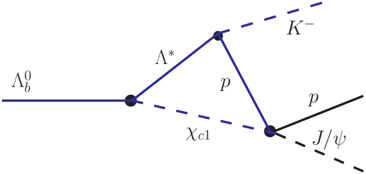

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Starting from a large Λ^* mass, in Λ_b rest frame

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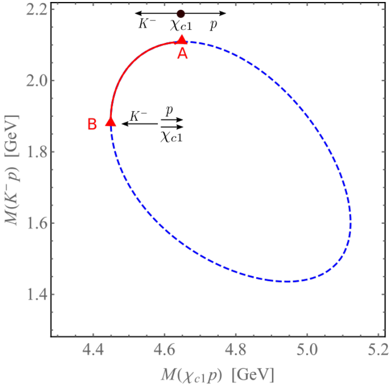

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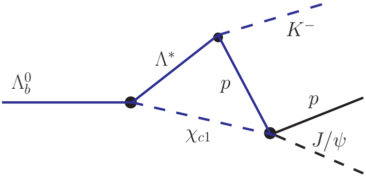

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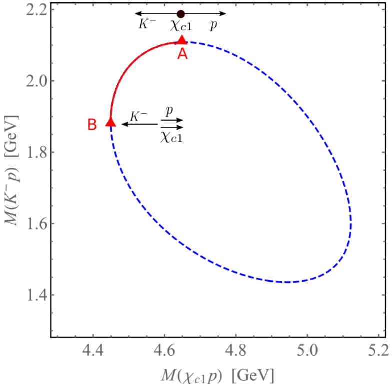
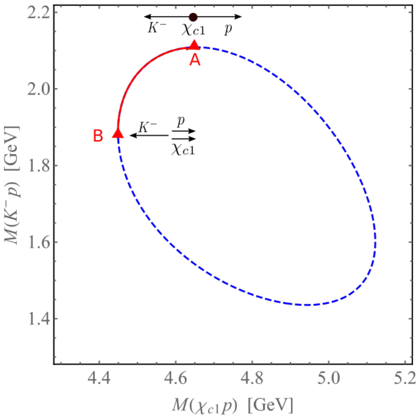
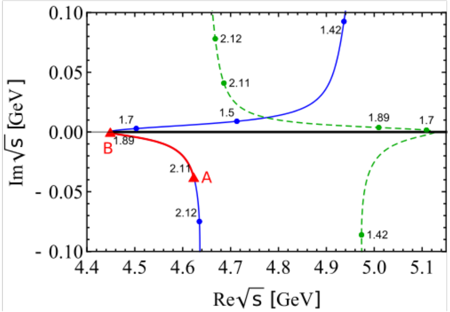


Trajectories of triangle singularities in complex energy plane





numbers: assumed masses for Λ^*

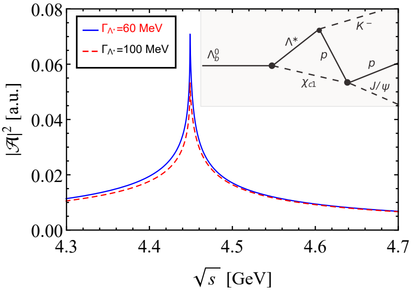
- ☞ blue: proton and χ_{c1} are parallel, in the 2nd Riemann sheet
- ☞ green: proton and χ_{c1} are anti-parallel

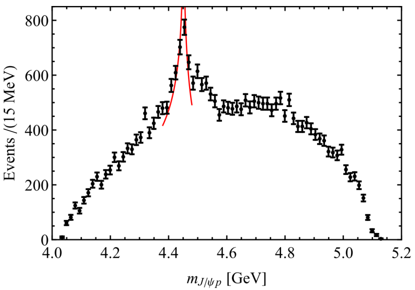
$M_{\Lambda_b} = 5.62$ GeV, $M_{\chi_{c1}} = 3.51$ GeV, $\quad \sqrt{s} \equiv M(\chi_{c1} p)$

$$M_{K^-p,A} = M_{\Lambda_b} - M_{\chi_{c1}}, \qquad M_{K^-p,B} = \sqrt{\frac{M_{\Lambda_b}^2 M_p + M_K^2 M_{\chi_{c1}}}{M_{\chi_{c1}} + M_p} - M_{\chi_{c1}} M_p}$$

Triangle singularity for $P_c(4450)$

- When $M_{\Lambda^*} = 1.89$ GeV, the effective triangle singularity is located exactly at the $\chi_{c1}p$ threshold, 4.449 GeV!
- Coincidentally, four-star baryon $\Lambda(1890)$: $J^P = 3/2^+$, $\Gamma:\ 60 - 200$ MeV
- triangle loop with S-wave $\chi_{c1}p$:

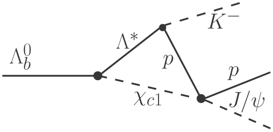


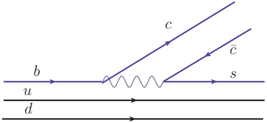


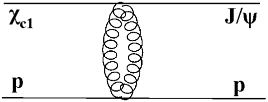
More comments

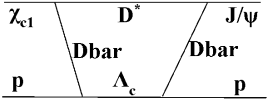
Strength of the triangle singularity is determined by

- couplings:
  - ☞ $\Lambda_b \to \Lambda^* \chi_{c1}$ is from $b \to c\bar{c}s$, not measured, but should be easy:
    Br$(B^+ \to J/\psi K^+) \simeq 1 \times 10^{-3}$,
    Br$(B^+ \to \chi_{c1} K^+) \simeq 0.5 \times 10^{-3}$
  - ☞ $\Lambda^*(1890) \to N\bar{K}$: largest branching fraction, Br$= 20 - 35\%$
  - ☞ $\chi_{c1} p \to J/\psi p$: OZI suppressed,
    $\mathcal{O}\left(1/N_c\right)$ [recall: OZI suppressed meson-meson scattering: $\mathcal{O}\left(1/N_c^2\right)$]









lattice QCD predicts possible $c\bar{c}$-nucleus bound states at $M_\pi = 805$ MeV

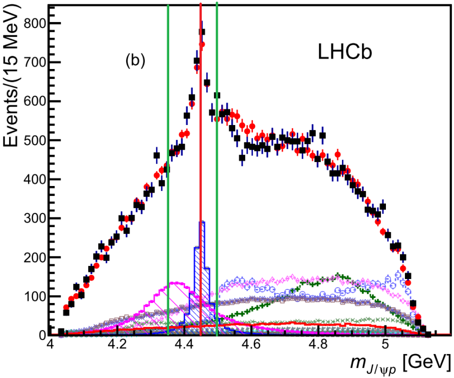
NPLQCD, PRD91(2015)114503

More triangle singularities?

- $\chi_{c0,c1,c2}\, p \to J/\psi\, p$ are related through heavy quark spin symmetry
- Weak decay $b \to c\bar{c}s$, $V - A \overset{\text{Fierz}}{\Rightarrow} \bar{c}\gamma^\mu(1-\gamma_5)c$
  - ☞ $\Lambda_b \to \Lambda^*\, J/\psi$ and $\Lambda_b \to \Lambda^*\, \chi_{c1}$ are easy
  - ☞ for χ_{c2}: strongly suppressed, χ_{c0}: also suppressed; for $B^+ \to \chi_{cJ} K^+$

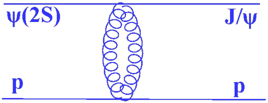
$$\text{Br}_1 \simeq 5\times10^{-4} > \text{Br}_0 \simeq 1.5\times10^{-4} \gg \text{Br}_2 \simeq 1.1\times10^{-5}$$

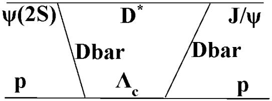
$\Rightarrow$ no obvious peak around the $\chi_{c0}\, p$ or $\chi_{c2}\, p$ threshold



Considering other possible $c\bar{c}$–Λ^* combinations:

- $h_c, \eta_c(1S,2S)$: spin-singlet
  $\Rightarrow h_c[\eta_c(1S,2S)]p \to J/\psi p$ breaks heavy quark spin symmetry, suppressed relative to $\chi_{c1}p \to J/\psi p$
- J/ψ:
  $J/\psi p \to J/\psi p$: elastic, no peak will show up (due to Schmid theorem)
- $\psi(2S)$: radial excitation different from J/ψ





in comparison with $\chi_{c1}p \to J/\psi p$

left: strongly suppressed; right: might be slightly suppressed, not very clear

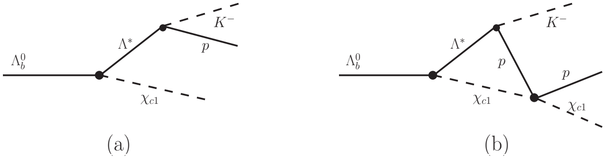
For possible triangle singularities for $\Lambda_b \to J/\psi pK$, the χ_{c1}–$\Lambda^*(1890)$ seems the most prominent one among all $c\bar{c}$–Λ^* combinations

How to distinguish triangle-singularity from genuine resonance?

- Schmid theorem: C. Schmid, Phys. Rev. 154 (1967) 1363

  see also, A. V. Anisovich, V. V. Anisovich, Phys. Lett. B 345 (1995) 321

  Triangle singularity cannot produce an additional peak in the invariant mass distribution of the elastic channel when neglecting inelasticity



  Nearby the effective singularity: $\mathcal{A}_{(a)+(b)}(s) \sim e^{2i\,\delta_{\chi_{c1}p}(s)}\mathcal{A}_{(a)}(s)$

  here $\delta_{\chi_{c1}p}$ is the elastic $\chi_{c1}p$ scattering phase shift

- corrections from coupled channels A. Szczepaniak, PLB757(2016)61

How to distinguish triangle-singularity from genuine resonance?

- Method-1: measuring the process $\Lambda_b^0 \to \chi_{c1}\, p\, K^-$
  - ☞ if a narrow near-threshold peak in $\chi_{c1}\, p \Rightarrow$ a real exotic resonance
  - ☞ otherwise, cannot conclude $P_c(4450)$ to be an exotic hadron
- Method-2: processes (such as photoproduction) with a different kinematics

  Q. Wang, X.-H. Liu, Q. Zhao, PRD92(2015)034022;

  V. Kubarovsky, M. Voloshin, PRD92(2015)031502;

  M. Karliner, J. L. Rosner, PLB752(2015)329; . . .

talk by X.-H. Liu

Summary

- Two coincidences for the LHCb $P_c(4450)$ structure:
  - ☞ located exactly at the $\chi_{c1}\,p$ threshold
  - ☞ four-star $\Lambda(1890)$ makes a triangle singularity exactly at the same position
- To control the strength, we need:
  - ☞ Br$(\Lambda_b \to \Lambda^*(1890)\chi_{c1}) \Leftarrow$ LHCb
  - ☞ $\chi_{c1}p \to J/\psi p$, might get information from lattice QCD
- More measurements are necessary to reveal the nature of the $P_c(4450)$
  - ☞ J^P unambiguously
  - ☞ $\Lambda_b \to \chi_{c1}pK$
  - ☞ searching for $P_c(4450)$ in processes with a different kinematics

THANK YOU FOR YOUR ATTENTION!