Exotic-hadron signature by constituent-counting rule in perturbative QCD

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July 25-30, 2016, Kyoto University, Kyoto, Japan
http://menu2016.riken.jp/

July 26, 2016

References

- Constituent-counting rule in photoproduction of hyperon resonances,
  W.-C. Chang, S. Kumano, and T. Sekihara,
  Phys. Rev. D93 (2016) 034006 (arXiv:1512.06647).
- Determination of exotic hadron structure by constituent-counting rule for hard exclusive processes,
  H. Kawamura, S. Kumano, and T. Sekihara,
  Phys. Rev. D88 (2013) 034010 (arXiv:1307.0362).

See also related studies

- Tomography of exotic hadrons in high-energy exclusive processes,
  H. Kawamura and S. Kumano,
  Phys. Rev. D89 (2014) 054007 (arXiv:1312.1596)
- Proposal for exotic-hadron search by fragmentation functions,
  M. Hirai, S. Kumano, M. Oka, and K. Sudoh,
  Phys. Rev. D77 (2008) 017504 (arXiv:0708.1816)
- Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC,
  T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng, S. Sawada, and K. Tanaka,
  Phys. Rev. D93 (2016) 114034 (arXiv:1605.00364)

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- Exotic hadrons
- Constituent-counting rule for exclusive processes

Exotic hadron candidate $\Lambda(1405)$

- Cross section estimate for $\pi^- + p \to K^0 + \Lambda$, $K^0 + \Lambda(1405)$
- Analysis of $\gamma + p \to K^+ + \Lambda$, $K^0 + \Lambda(1405)$

Comments on exotic hadron studies in other high-energy reactions

Exotic hadron in fragmentation functions

- $e^+ + e^- \to h + X$, $h = \pi$, $f^0(980)$, $\cdots$

Tomography for exotic hadrons

- Generalized parton distributions (GPDs)
- Generalized distribution amplitudes (GDAs)

Summary and prospects

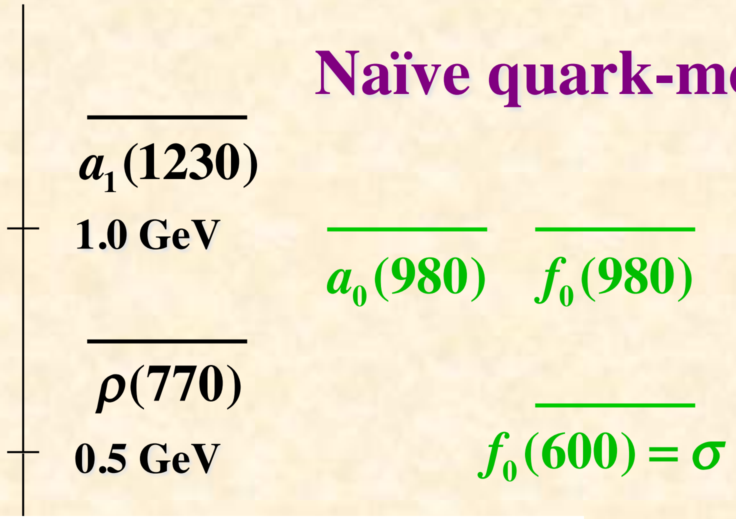
Progress in exotic hadrons

$q\bar{q}$	Meson
q^3	Baryon
$q^2\bar{q}^2$	Tetraquark
$q^4\bar{q}$	Pentaquark
q^6	Dibaryon
…	
$q^{10}\bar{q}$	e.g. Strange tribaryon
…	
gg	Glueball
…	

- $\Theta^+(1540)$???: LEPS
  Pentaquark?
  $uudd\bar{s}$?
- Kaonic nuclei?: KEK-PS, …
  Strange tribaryons, …
  $K^- pnn$, $K^- ppn$?
  $K^- pp$?
- X (3872), Y(3940): Belle
  Tetraquark, $D\bar{D}$ molecule
  $c\bar{c}$
  $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
  $D^+(c\bar{d})D^-(\bar{c}d)$?
- $D_{sJ}(2317)$, $D_{sJ}(2460)$: BaBar, CLEO, Belle
  Tetraquark, DK molecule
  $c\bar{s}$
  $D^0(c\bar{u})K^+(u\bar{s})$
  $D^+(c\bar{d})K^0(d\bar{s})$?
- Z (4430): Belle
  Tetraquark, …
  $c\bar{c}u\bar{d}$, D molecule?
- $P_c(4380)$, $P_c(4450)$: LHCb
- …
  $u\bar{c}udc$, $\bar{D}(u\bar{c})\Sigma_c^*(udc)$, $\bar{D}^*(u\bar{c})\Sigma_c(udc)$ molecule?

Scalar mesons $J^P=0^+$ at $M\sim 1$ GeV

Naïve quark-model



$$\sigma = f_0(600) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$f_0(980) = s\bar{s} \;\rightarrow \text{denote } f_0 \text{ in this talk}$$

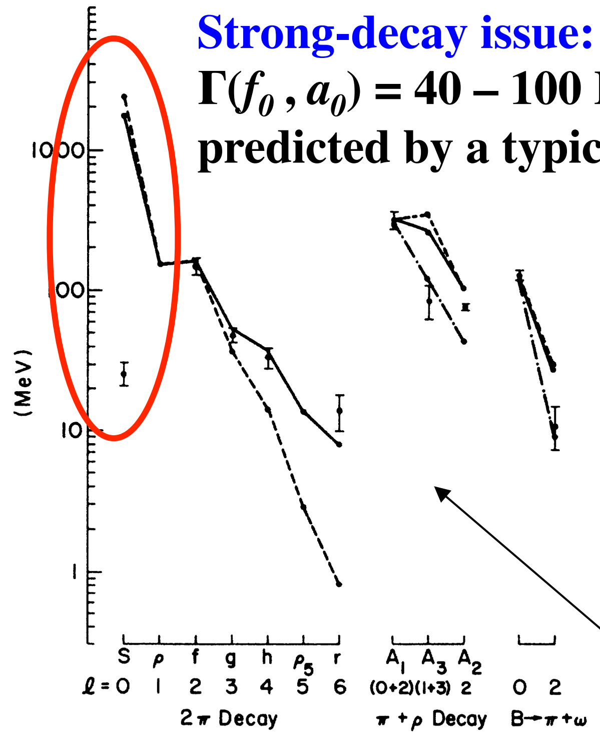
$$a_0(980) = u\bar{d},\quad \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),\quad d\bar{u}$$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

⇕ **contradiction**

Experiment: $m(\sigma) < m(a_0) \sim m(f_0)$

Strong-decay issue: The experimental widths $\Gamma(f_0, a_0)$ = 40 – 100 MeV are too small to be predicted by a typical quark model.



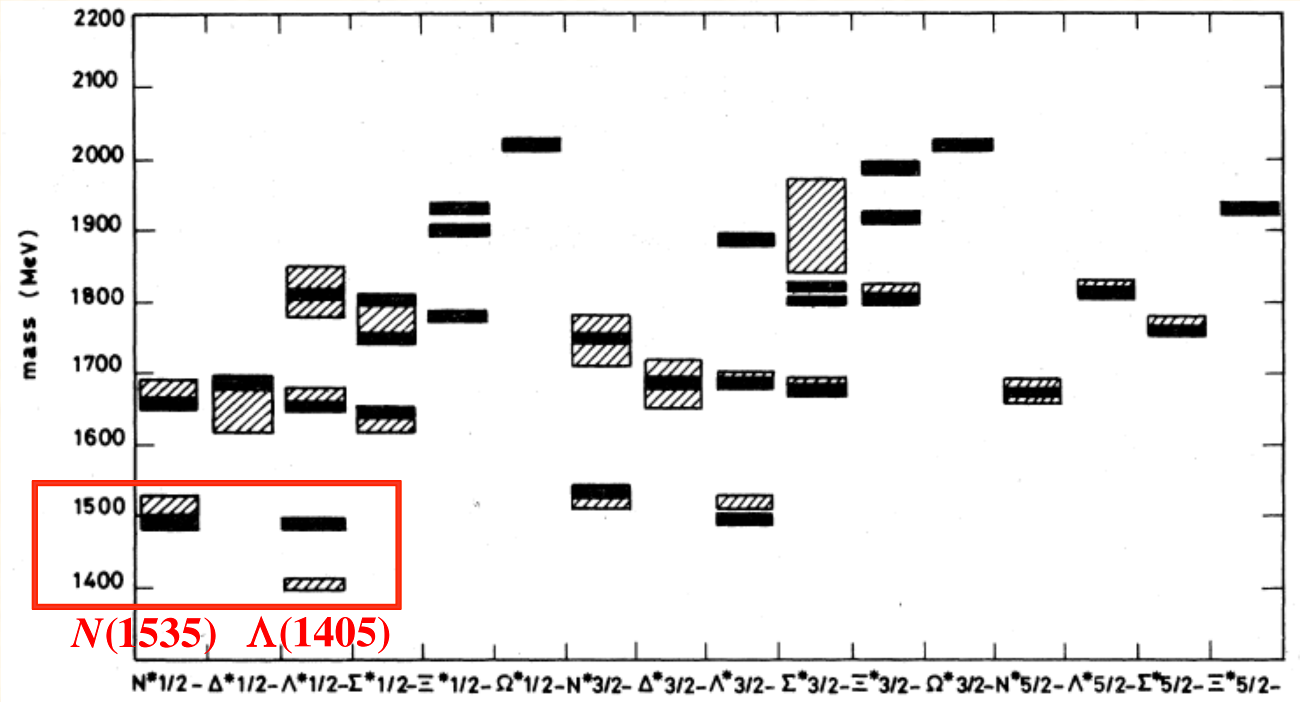
These issues could be resolved if f_0 is a tetraquark ($qq\bar{q}\bar{q}$) or a $K\bar{K}$ molecule, namely an "exotic" hadron.

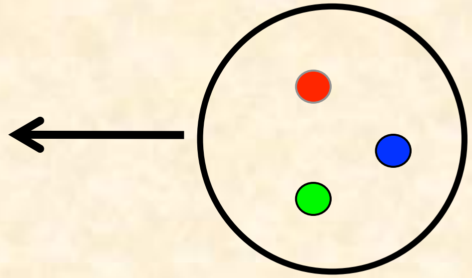
Radiative decay: F. E. Close, N. Isgur, and SK, Nucl. Phys. B389 (1993) 513.

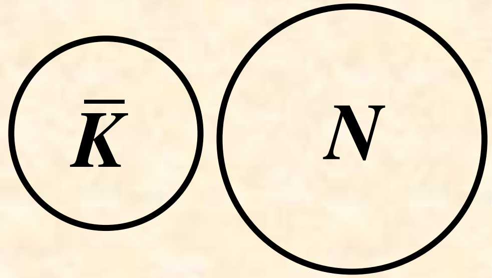
SK and V. R. Pandharipande, Phys. Rev. D38 (1988) 146.

Λ(1405): exotic hadron?

Negative-parity baryons
N. Isgur and G. Karl,
PRD 18 (1978) 4187.

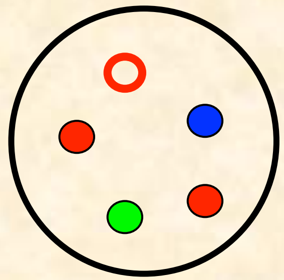






Most spectra agree with the ones by a 3*q*-picture

- Only Λ(1405) deviates from the measurement.
- Difficult to understand the small mass of Λ(1405) in comparison with *N*(1535).
  → $\bar{K}N$ molecure or penta-quark ($qqqq\bar{q}$)?



Constituent-counting rule for exotic hadrons

H. Kawamura, S. Kumano, T. Sekihara,
Phys. Rev. 88 (2013) 034010 (arXiv:1307.0362).

Research purposes

It is not easy to find undoubted evidence for exotic hadrons by global observables (mass, spin, parity, decay width) at low energies.

(1) Determination of internal structure
of exotic hadrons by high energy processes,
where quark-gluon degrees of freedom appear.

Constituent-counting rule could be used
because it counts internal constituents.

(2) Investigation on transition from hadron degrees of freedom
to quark-gluon degrees of freedom for exotic hadrons.

$$\frac{d\sigma_{a+b\to c+d}}{dt} \simeq \frac{1}{16\pi s^2}\overline{\sum_{pol}}\left|M_{a+b\to c+d}\right|^2 \Rightarrow \frac{d\sigma_{a+b\to c+d}}{dt} = \frac{1}{s^{n-2}} f_{a+b\to c+d}(t/s) \quad \text{consituent-counting rule}$$

$$n = n_a + n_b + n_c + n_d$$

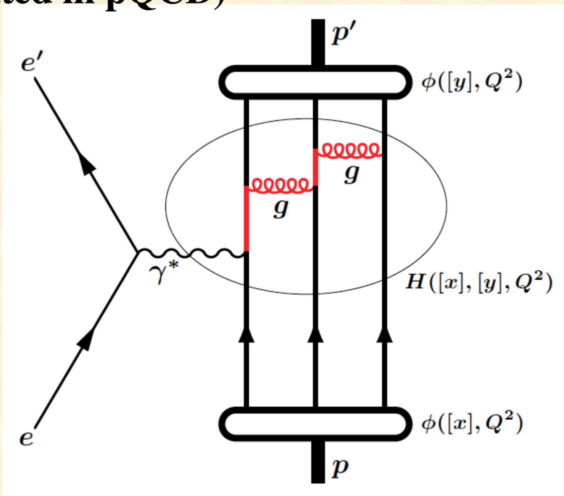
Constituent-counting rule in perturbative QCD: Form factor

Consider the magnetic form factor of the proton

$$\langle p'|J^\mu|p\rangle \simeq \bar{u}(p')\gamma^\mu G_M(Q^2)u(p) \ \text{ at } \ Q^2 = -q^2 \gg m_N^2$$

$$G_M(Q^2) = \int d[x]d[y]\phi_p([y])H_M([x],[y],Q^2)\phi_p([x])$$

ϕ_p = proton distribution amplitude, H_M = hard amplitude (calculated in pQCD)



In the Breit frame with $q = (0, \vec{q}), \quad |\vec{p}| = |\vec{p}'| \equiv P \sim O(Q)$.

$u^\dagger u = 2E \quad \Rightarrow$ External quark line: $u \sim \sqrt{P} \sim \sqrt{Q}$

$$\langle p'|J^\mu|p\rangle \simeq \bar{u}(p')\gamma^\mu G_M(Q^2)u(p) \sim (\sqrt{Q})^2 G_M(Q^2)$$

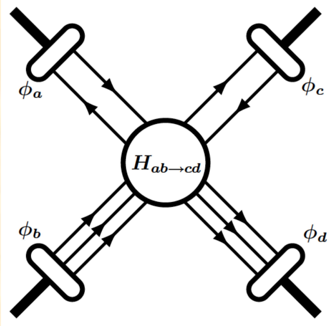
- Two quark propagators: $\dfrac{1}{Q^2}$
- Two gluon propagators: $\dfrac{1}{(Q^2)^2}$
- Six external quark lines: $(\sqrt{Q})^6$

$$\langle p'|J^\mu|p\rangle \sim \frac{1}{Q^2}\frac{\alpha_s(Q^2)}{(Q^2)^2}(\sqrt{Q})^6 = \frac{\alpha_s(Q^2)}{(Q^2)^{3/2}}$$

$$\Rightarrow G_M(Q^2) \sim \frac{1}{(\sqrt{Q})^2}\langle p'|J^\mu|p\rangle \sim \frac{1}{(Q^2)^{1/2}}\frac{\alpha_s(Q^2)}{(Q^2)^{3/2}} = \frac{\alpha_s(Q^2)}{(Q^2)^2} \sim \frac{1}{t^{n_N-1}}, \ \ n_N = 3, \quad -t = Q^2$$

Counting rule for the form factor: $G_M(Q^2) \sim \dfrac{1}{t^{n_N-1}}, \ \ n_N = 3$

Constituent-counting rule in perturbative QCD: Hard exclusive processes $a + b \to c + d$



Consider the hard exclusive hadron reaction $a + b \to c + d$

$$M_{ab\to cd} = \int d[x_a]d[x_b]d[x_c]d[x_d]\phi_c([x_c])\phi_d([x_d])H_M([x_a],[x_b],[x_c],[x_d],Q^2)\phi_a([x_a])\phi_b([x_b])$$

ϕ_p = proton distribution amplitude, H_M = hard amplitude (calculated in pQCD)

Rule for estimating $M_{ab\to cd}$

(1) Feynman diagram: Draw leading and connected Feynman diagram by connecting $n/2$ quark lines by gluons.

(2) Gluon propagators: The factor $1/P^2$ is assigned for each gluon propagator.

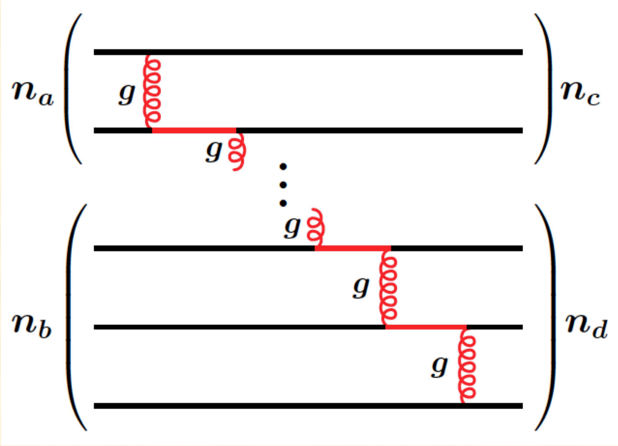
There are $n/2-1$ gluon propagators $\sim 1/(P^2)^{n/2-1}$.

(3) Quark propagators: The factor $1/P$ is assigned for each quark propagator.

There are $n/2-2$ gluon propagators $\sim 1/(P)^{n/2-2}$.

(4) External quarks: The factor $\sqrt{P}$ is assigned for each external quark.

There are n gluon propagators $\sim (\sqrt{P})^n$.



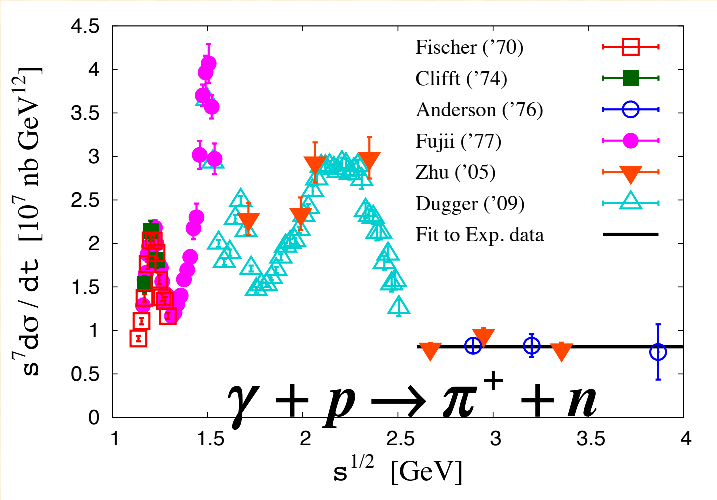
$$M_{ab\to cd} \sim \frac{1}{(P^2)^{n/2-1}}\frac{1}{(P)^{n/2-2}}(\sqrt{P})^n = \frac{(P)^{n/2}}{(P)^{n-2}(P)^{n/2-2}} = \frac{1}{(P)^{n-4}} \sim \frac{1}{s^{n/2-2}}$$

Cross section: $\dfrac{d\sigma_{ab\to cd}}{dt} \simeq \dfrac{1}{16\pi^2}\overline{\sum_{spol}}|M_{ab\to cd}|^2 \sim \dfrac{1}{s^{n-2}}$

Constituent-counting rule, Transition from hadron degrees of freedom to quark-gluon ones

Typical current situation

- Transition from hadron d.o.f to quark d.o.f.
- (Looks like) Constituent-counting scaling



JLab: L.Y. Zhu *et al.*, PRL 91, 022003 (2003);
PRC 71, 044603 (2005);
W. Chen *et al.*, PRL 103, 012301 (2009).

see R. A. Schumacher and M. M. Sargsian,
PRC 83 (2011) 025207 for hyperon production

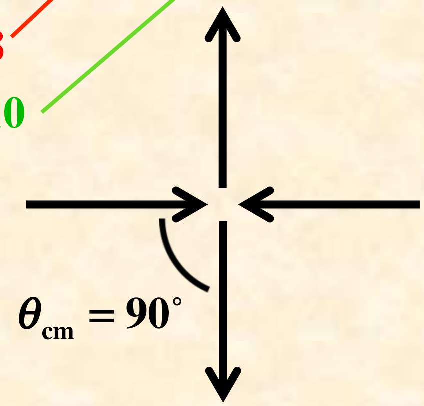
BNL experiment

C. White it et al., PRD 49 (1994) 58.

No.	Interaction	Cross section E838	Cross section E755	n-2 ($\frac{d\sigma}{dt} \sim 1/s^{n-2}$)
1	$\pi^+ p \to p \pi^+$	132 ± 10	4.6 ± 0.3	6.7 ± 0.2
2	$\pi^- p \to p \pi^-$	73 ± 5	1.7 ± 0.2	7.5 ± 0.3
3	$K^+ p \to p K^+$	219 ± 30	3.4 ± 1.4	$8.3^{+0.6}_{-1.0}$
4	$K^- p \to p K^-$	18 ± 6	0.9 ± 0.9	≥ 3.9
5	$\pi^+ p \to p \rho^+$	214 ± 30	3.4 ± 0.7	8.3 ± 0.5
6	$\pi^- p \to p \rho^-$	99 ± 13	1.3 ± 0.6	8.7 ± 1.0
13	$\pi^+ p \to \pi^+ \Delta^+$	45 ± 10	2.0 ± 0.6	6.2 ± 0.8
15	$\pi^- p \to \pi^+ \Delta^-$	24 ± 5	≤ 0.12	≥ 10.1
17	$pp \to pp$	3300 ± 40	48 ± 5	9.1 ± 0.2
18	$\bar{p}p \to p\bar{p}$	75 ± 8	≤ 2.1	≥ 7.5

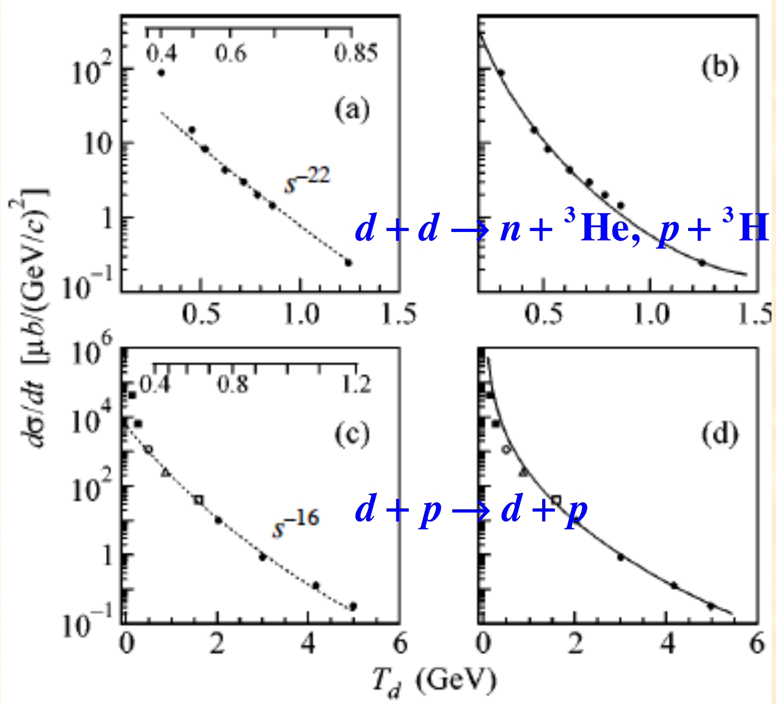
$n-2$: $(2+3+2+3)-2=8$

$(3+3+3+3)-2=10$



Constituent-counting rule for "molecular" systems

Y. N. Uzikov, JETP Lett. 81, 303 (2005).



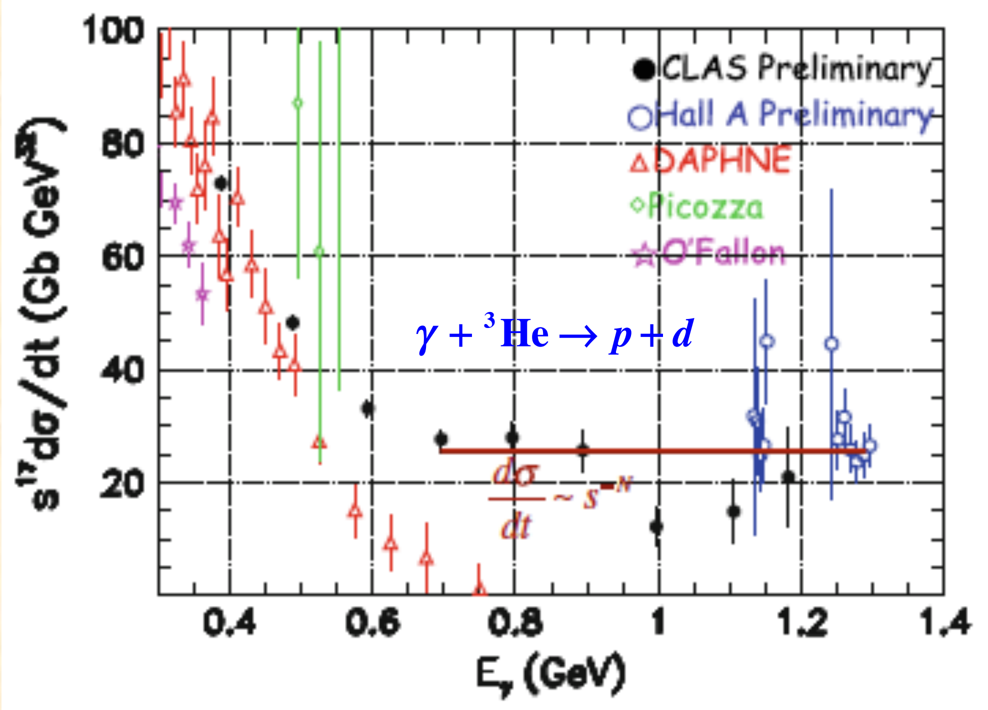
$$d + d \to n + {}^3\text{He},\ \ p + {}^3\text{H}$$

$$n - 2 = (6 + 6 + 3 + 9) - 2 = 22$$

$$d + p \to d + p$$

$$n - 2 = (6 + 3 + 6 + 3) - 2 = 16$$

Y. Ilieva, Few Body Syst. 54, 989 (2013).

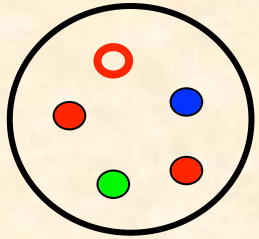


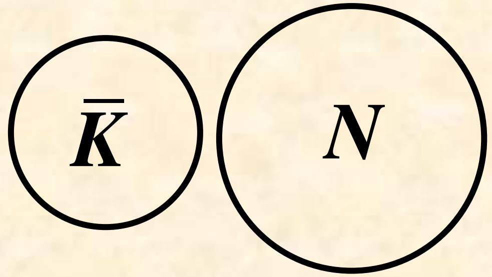
$$\gamma + {}^3\text{He} \to p + d$$

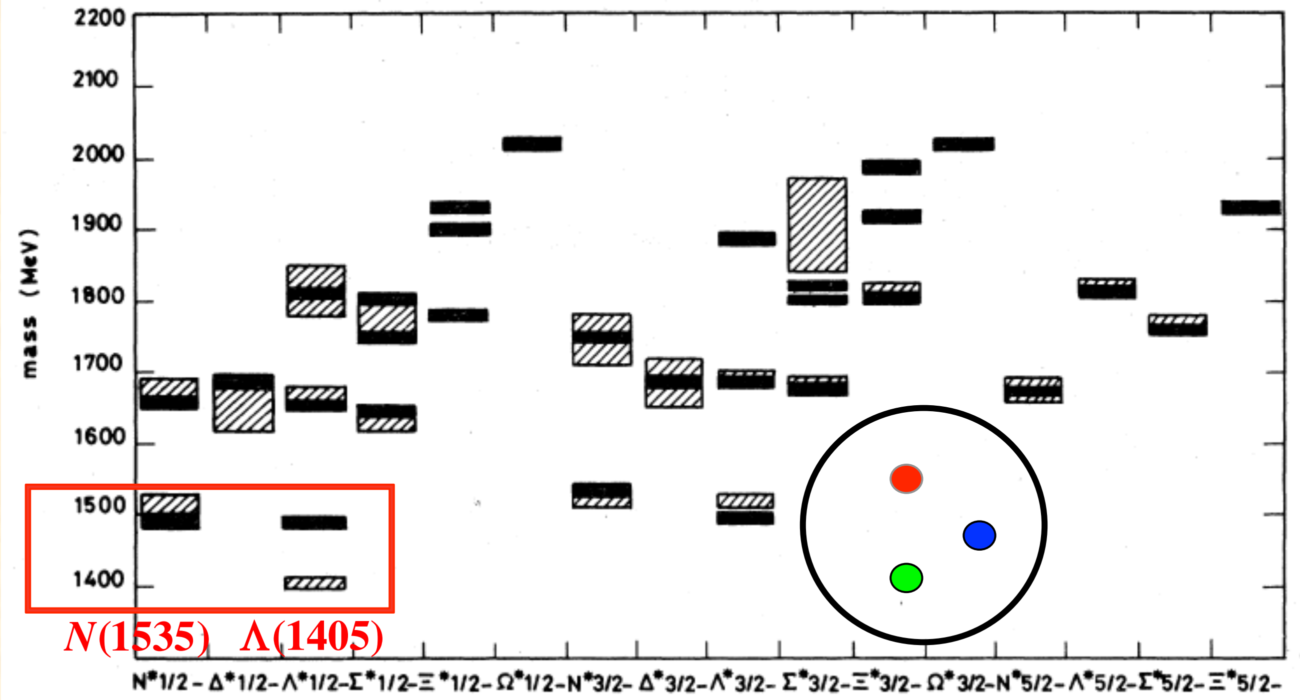
$$n - 2 = (1 + 9 + 3 + 6) - 2 = 17$$

Λ(1405): exotic hadron?

Negative-parity baryons
N. Isgur and G. Karl,
PRD 18 (1978) 4187.







Our proposal:

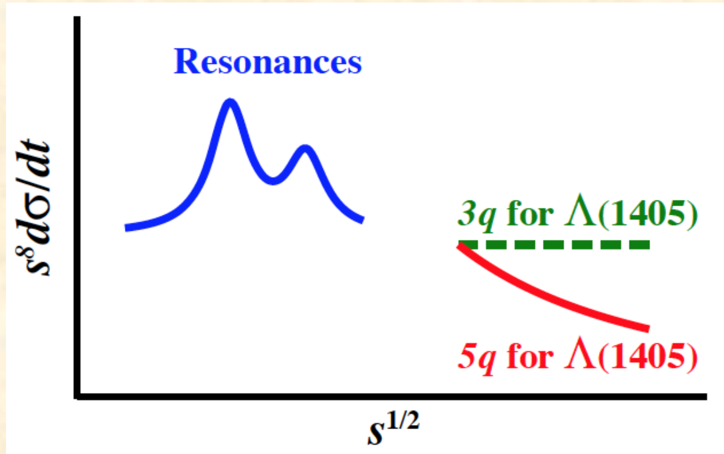
Exotic hadron production

$\pi^- + p \to K^0 + \Lambda(1405)$: J-PARC, COMPASS?

$\gamma + p \to K^+ + \Lambda(1405)$: JLab

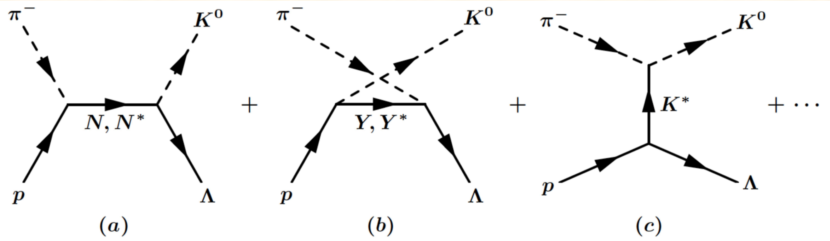
Most spectra agree with the ones by a 3*q*-picture

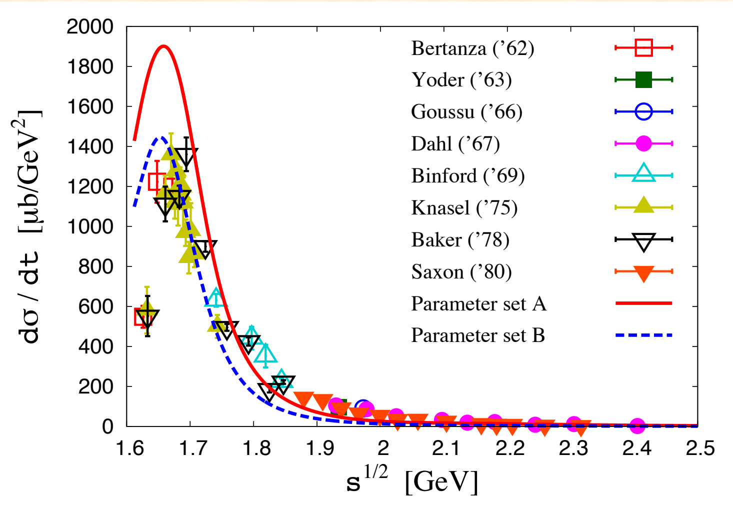
- Only Λ(1405) deviates from the measurement.
- Difficult to understand the small mass of Λ(1405) in comparison with *N*(1535).
  → $\bar{K}N$ molecure or penta-quark ($qqqq\bar{q}$)?



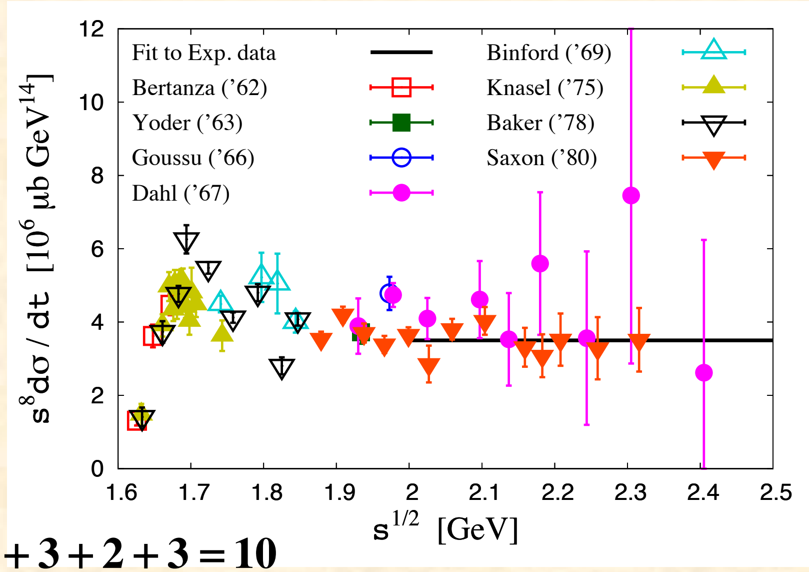
Ordinary-hadron production $\pi^- + p \to K^0 + \Lambda$ as a reference

At low energies





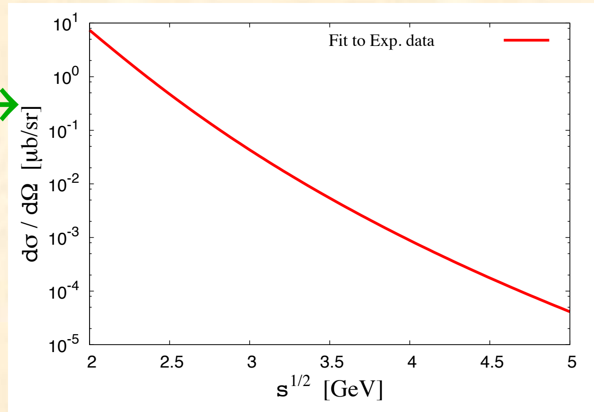
From low to higher energies



$n = 2+3+2+3 = 10$

$$\frac{d\sigma_{ab\to cd}}{dt} = \frac{\text{const}}{s^{n-2}}, \quad n = 10.1 \pm 0.6,$$ **encouraging!**

Our prediction at high energies →



Exotic-hadron production $\pi^- + p \to K^0 + \Lambda(1405)$

Theoretical and experimental situation is no as good as the one for the ground Λ.

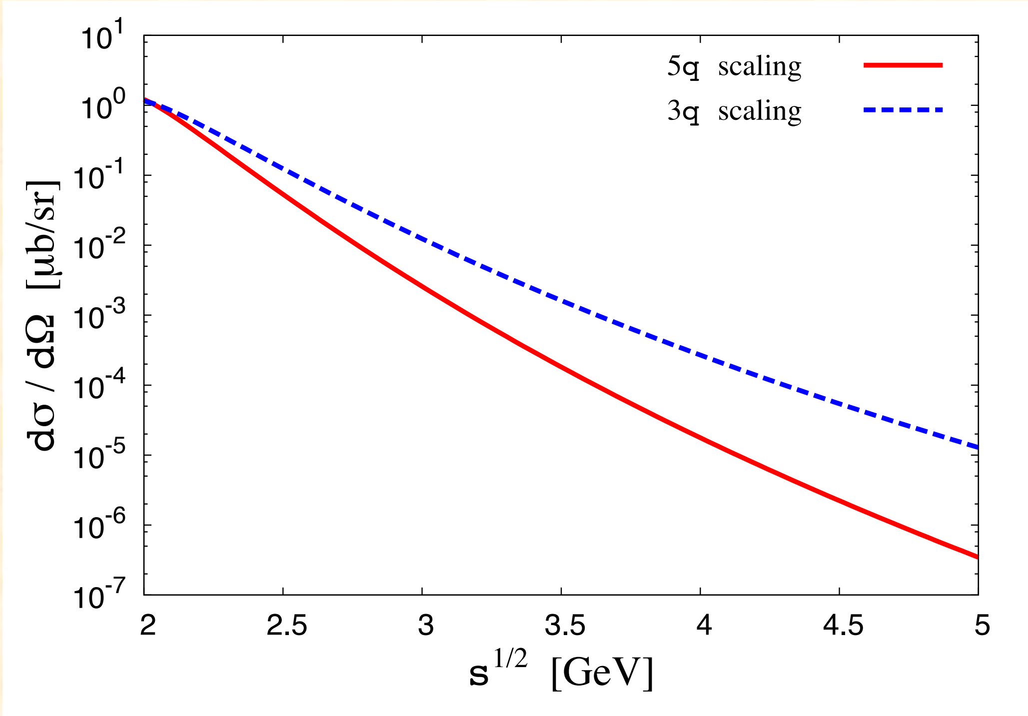
$$n = 2+3+2+3 = 10 \quad \text{if } \Lambda(1405) = \text{three-quark state}$$

$$= 2+3+2+5 = 12 \quad \text{if } \Lambda(1405) = \text{five-quark state}$$

(including $\bar{K}N$ molecule)

$$\frac{d\sigma_{ab\to cd}}{dt} = \frac{\text{const}}{s^{n-2}}, \quad n = 10 \text{ or } 12$$

Our prediction at high energies



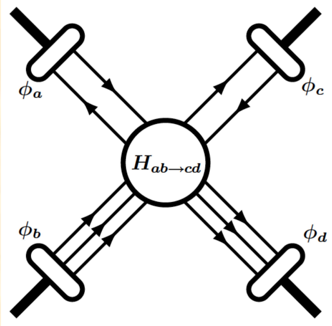
M. Niiyama (Kyoto Univ), J-PARC proposal, under consideration

Interesting project
at 12 GeV JLab, J-PARC, EIC, ...

Hard production of hyperons

W.-C. Chang, S. Kumano, and T. Sekihara
Phys. Rev. D 93 (2016) 034006 (arXiv:1512.06647).

Constituent-counting rule in perturbative QCD: Hard exclusive processes $a + b \to c + d$



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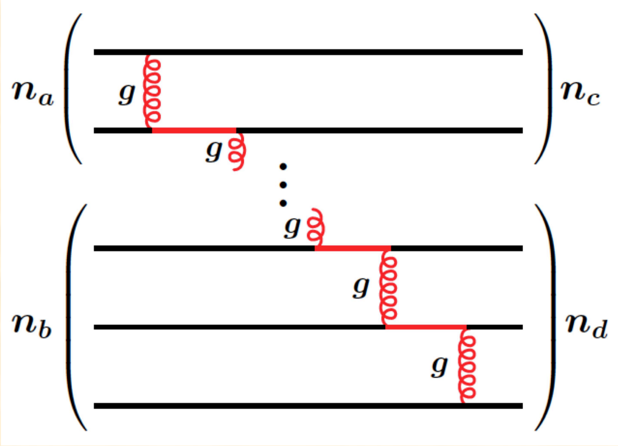
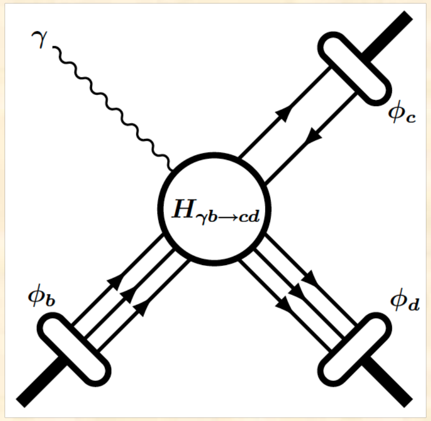
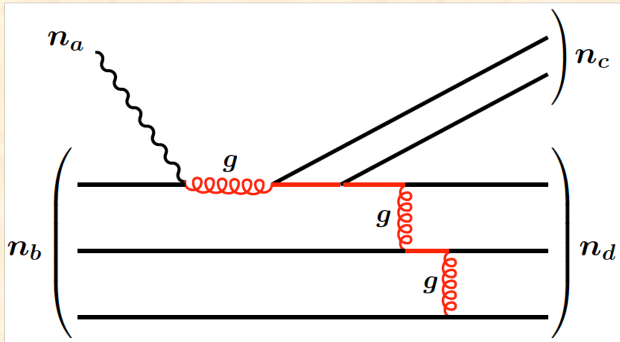


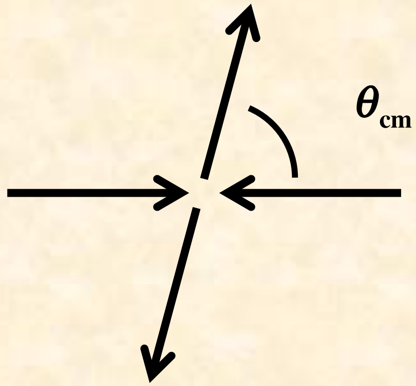
$$M_{ab\to cd} \sim \frac{1}{(P^2)^{n/2-1}}\frac{1}{(P)^{n/2-2}}(\sqrt{P})^n = \frac{(P)^{n/2}}{(P)^{n-2}(P)^{n/2-2}} = \frac{1}{(P)^{n-4}} \sim \frac{1}{s^{n/2-2}}$$

Cross section: $$\frac{d\sigma_{ab\to cd}}{dt} \simeq \frac{1}{16\pi^2}\overline{\sum_{spol}}|M_{ab\to cd}|^2 \sim \frac{1}{s^{n-2}}$$

JLab hyperon productions







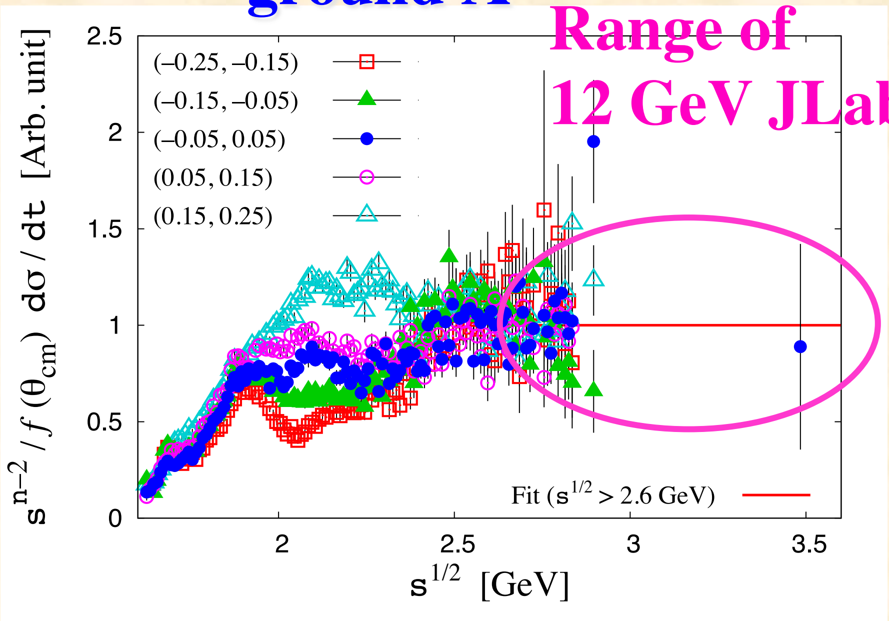
5 bins: $-0.25 < \cos\theta_{cm} < -0.15$, $\cdots$, $0.15 < \cos\theta_{cm} < 0.25$

4 bins: $-0.20 < \cos\theta_{cm} < -0.10$, $\cdots$, $0.10 < \cos\theta_{cm} < 0.20$

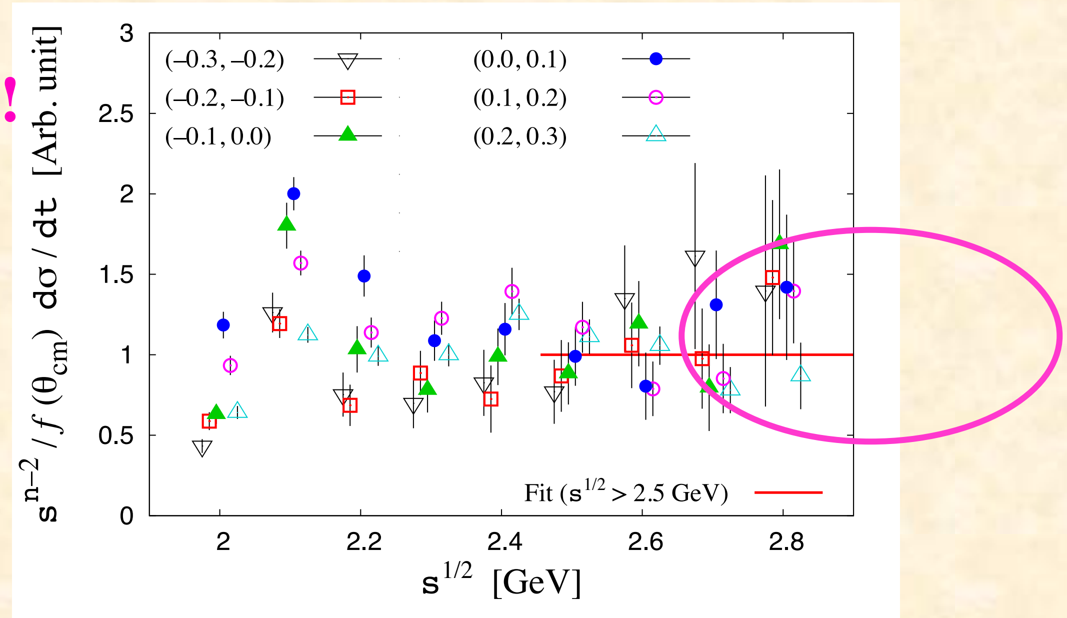
...

1 bin: $-0.05 < \cos\theta_{cm} < +0.05$

ground Λ

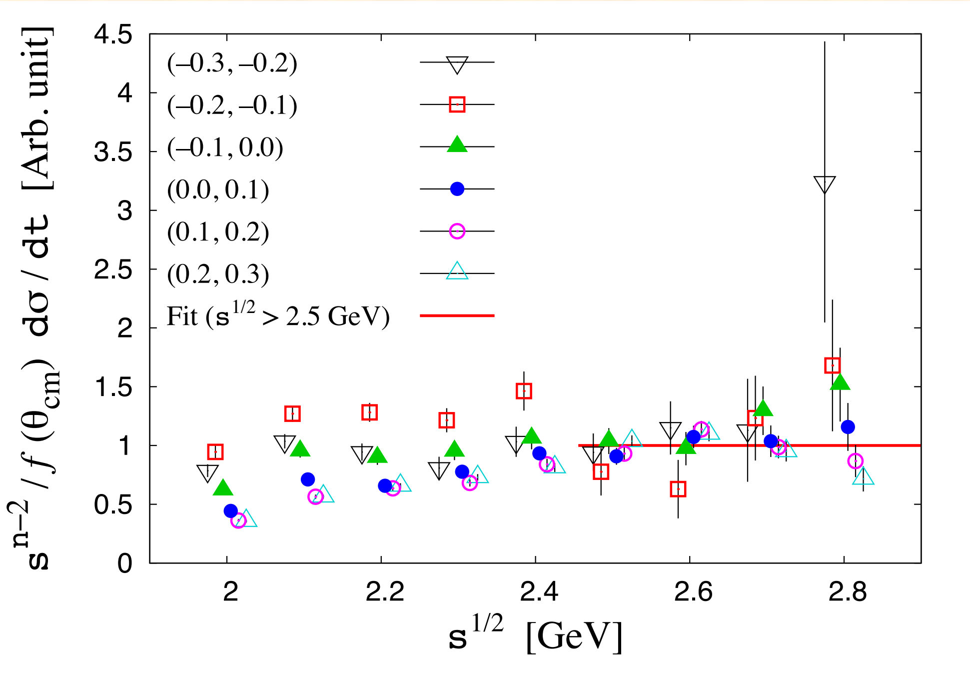


Λ(1405)

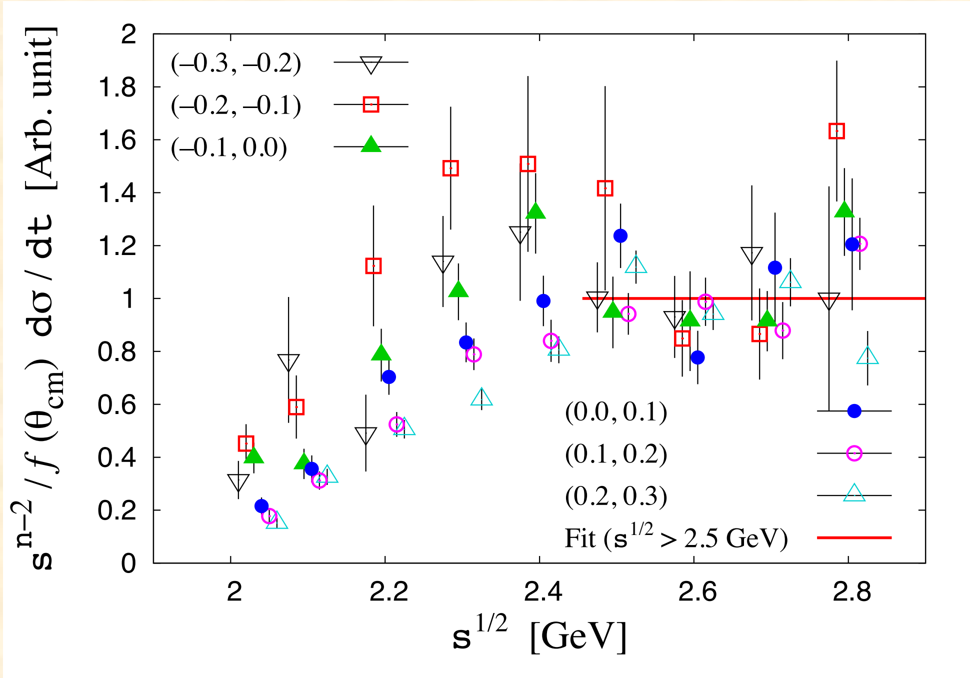


Hyperon productions

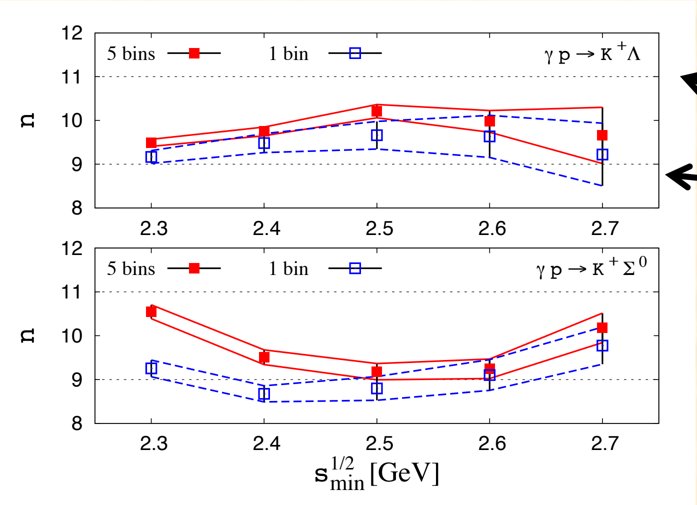
$\Sigma^0(1385)$



$\Lambda(1520)$

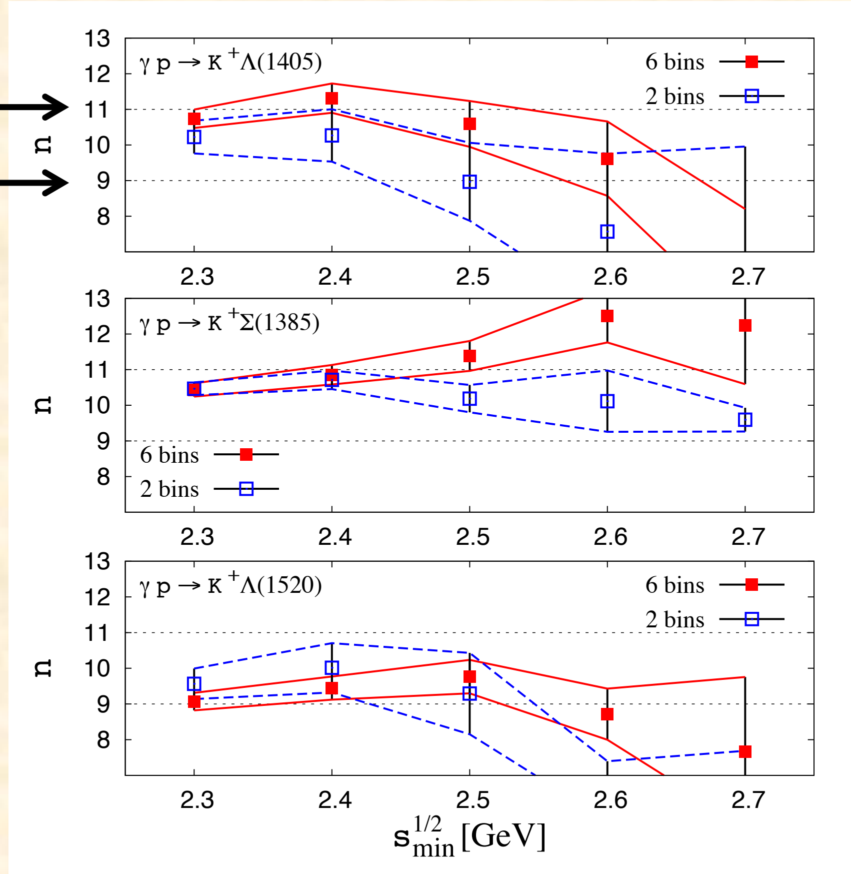


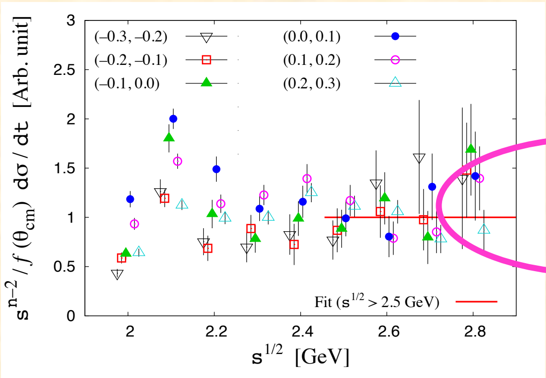
JLab hyperon productions including Λ(1405)



$n_\Lambda = 5$

$n_\Lambda = 3$





Range of
12 GeV JLab!

- Λ. Λ(1520) and Σ seem to be consistent with ordinary baryons with $n = 3$.
- Λ(1405) looks penta-quark at low energies but $n \sim 3$ at high energies???
- Σ(1385) : $n = 5$???

→ In order to clarify the nature of Λ(1405) $\left[qqq,\ \bar{K}N,\ qqqq\bar{q}\right]$,
the JLab 12-GeV experiment plays an important role!

Summary on exotic hadron structure by hard exclusive orocesses

- We propose to use hard exclusive production of exotic hadrons for probing internal quark-gluon structure by the constituent conting rule, $\dfrac{d\sigma}{dt} = \dfrac{\text{const}}{s^{n-2}}$.

- As an example, $\pi^- + p \to K^0 + \Lambda(1405)$ is studied together with $\pi^- + p \to K^0 + \Lambda$ as a reference of an ordinary hadron.

- Exclusive processes of exotic hadrons can be investigated at many facilities in the world.
  For example, J-PARC, LEP, JLab, COMPASS,
  in general any hadron facilities like GSI, Fermilab, RHIC, LHC, ...

Exotic-hadron studies in other high-energy reactions

(1) Fragmentation functions

(2) Tomography by

Generalized Parton Distributions (GPDs)

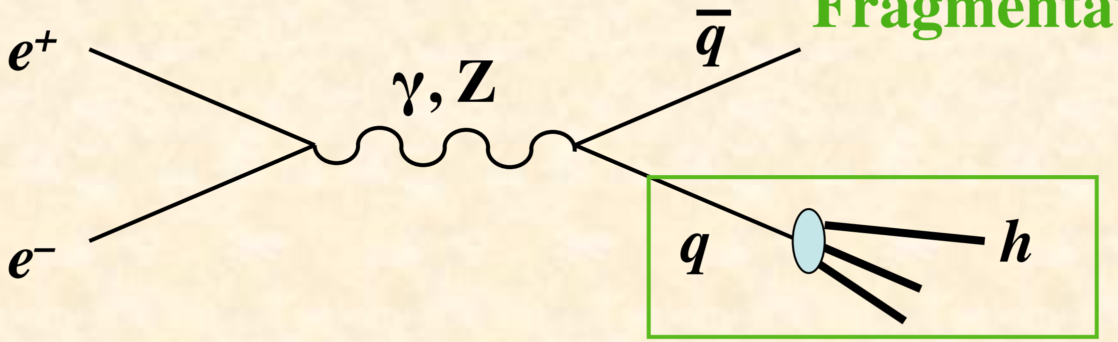
Generalized Distribution Amplitudes (GDAs)

Interesting project at KEK-B, ...

Exotic hadrons in high-energy reactions, *e.g.* fragmentation functions

M. Hirai, S. Kumano, M. Oka, K. Sudoh,
Phys. Rev. D77 (2008) 017504 (arXiv:0708.1816).

Fragmentation Functions



Fragmentation: hadron production from a quark, antiquark, or gluon

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2$$

Total fragmentation function is defined by

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

σ_{tot} = total hadronic cross section

Variable z

- Hadron energy / Beam energy
- Hadron energy / Primary quark energy

A fragmentation process occurs from quarks, antiquarks, and gluons, so that F^h is expressed by their individual contributions:

$$F^h(z,Q^2)_{LO} = \sum_i D_i^h(y,Q^2)$$

$D_i^h(z,Q^2)$ = fragmentation function of hadron h from a parton i

Momentum (energy) sum rule

$D_i^h(z, Q^2) =$ probability to find the hadron h from a parton i with the energy fraction z

Energy conservation: $\sum_h \int_0^1 dz\, z\, D_i^h(z, Q^2) = 1$

$h = \pi^+,\ \pi^0,\ \pi^-,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ p,\ \bar{p},\ n,\ \bar{n},\ \cdots$

Favored and disfavored fragmentation functions

Differences between them could be used for exotic hadron studies.

Simple quark model: $\pi^+(u\bar{d}),\ K^+(u\bar{s}),\ p(uud),\ \cdots$

Favored fragmentation: $D_u^{\pi^+},\ D_{\bar{d}}^{\pi^+}$, ...

(from a quark which exists in a naive quark model)

Disfavored fragmentation: $D_d^{\pi^+},\ D_{\bar{u}}^{\pi^+},\ D_s^{\pi^+}$, ...

(from a quark which does not exist in a naive quark model)

Exotic hadrons by fragmentation functions

"Favored" and "disfavored" (unfavored) fragmentation functions
Possibility of finding exotic hadrons in high-energy processes

Hirai, SK, Oka, Sudoh, PRD 77 (2008) 017504.

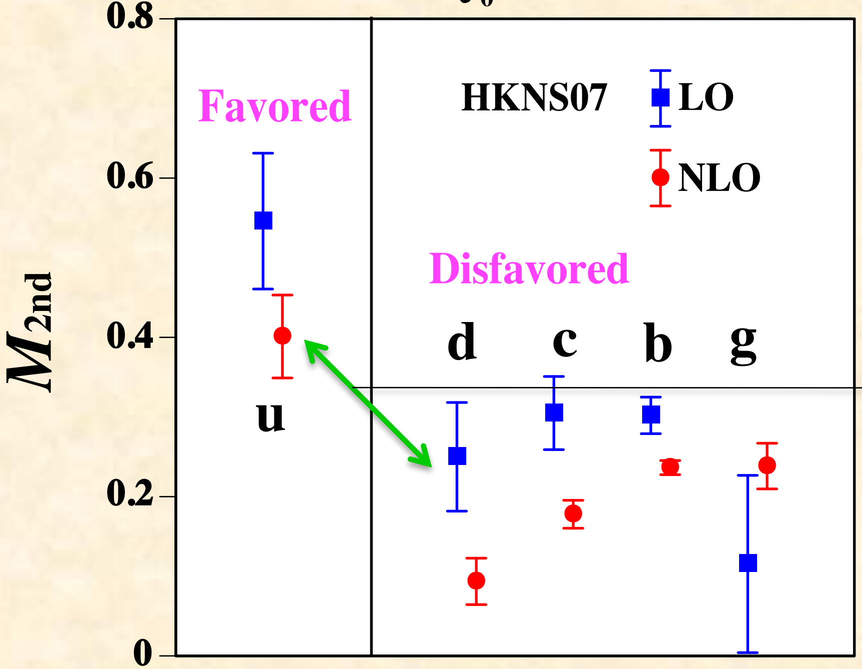
Possibilities for $f_0(980)$: $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$, $s\bar{s}$, $\frac{1}{\sqrt{2}}(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})$, $K\bar{K}$, or gg

$e.g.$ if $f_0(980)=s\bar{s}$: favored $s, \bar{s} \to f_0$; disfavored $u, d, \bar{u}, \bar{d} \to f_0, \cdots$

f_0(980): Belle analysis is possible in principle.

Pion case

$$M_{2nd} = \int_0^1 dz\, z\, D_i^{\pi^+}(z)$$



2nd moments of
M. Hirai, SK, T.-H. Nagai, K. Sudoh,
PRD 75 (2007) 094009.

There are distinct differences between the favored and disfavored 2nd moments.

→ It could be used for exotic-hadron studies.

Criteria for determining f_0 structure by its fragmentation functions

M. Hirai, S. Kumano, M. Oka, K. Sudoh, PRD 77 (2008) 017504.

Possible configurations of $f_0(980)$

(1) ordinary u,d - meson $\quad \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$

Contradicts with experimental widths

$\Gamma_{\text{theo}}(f_0 \to \pi\pi) = 500-1000 \text{ MeV}$
$\quad \gg \Gamma_{\text{exp}} = 40-100 \text{ MeV}$

$\Gamma_{\text{theo}}(f_0 \to \gamma\gamma) = 1.3-1.8 \text{ keV}$
$\quad \gg \Gamma_{\text{exp}} = 0.205 \text{ keV}$

(2) strange meson, $\quad s\bar{s}$

(3) tetraquark ($K\bar{K}$), $\quad \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})$

(4) glueball $\quad gg$

Contradicts with lattice-QCD estimate

$m_{\text{lattice}}(f_0) = 1600 \text{ MeV}$
$\quad \gg m_{\text{exp}} = 980 \text{ MeV}$

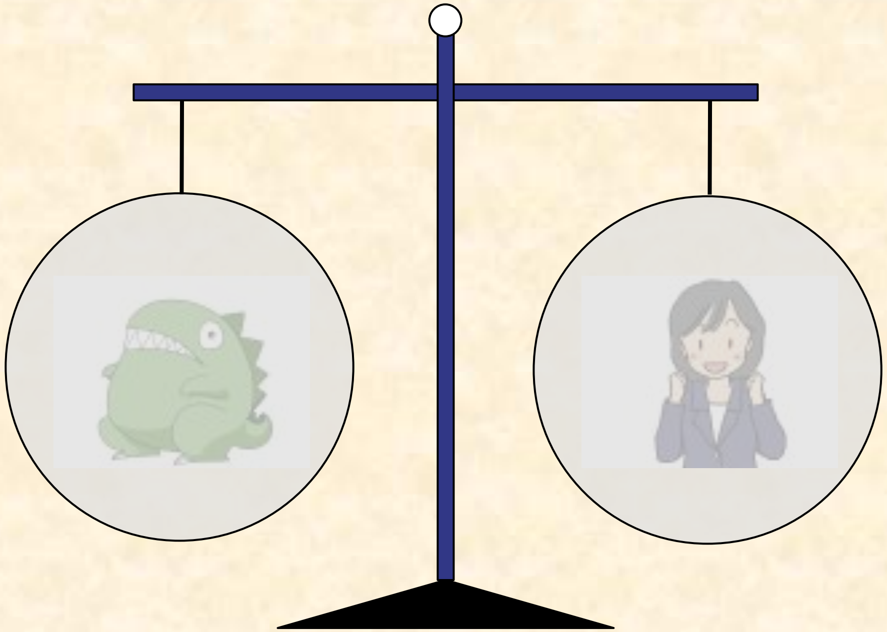
There could difference in fragmentation functions for f_0 depending on its internal structure.

- Favored and disfavored fragmentation functions
- 2nd moments and functional forms

Hadron tomography for exotic hadrons

H. Kawamura and S. Kumano,
Phys. Rev. D89 (2014) 054007 (arXiv:1312.1596).

Search for exotic hadrons …



It is difficult to determine whether or not a hadron is exotic by low-energy observables, masses, decay widths, …

(Already, history of a half century)

PET (Positron Emission Tomography)

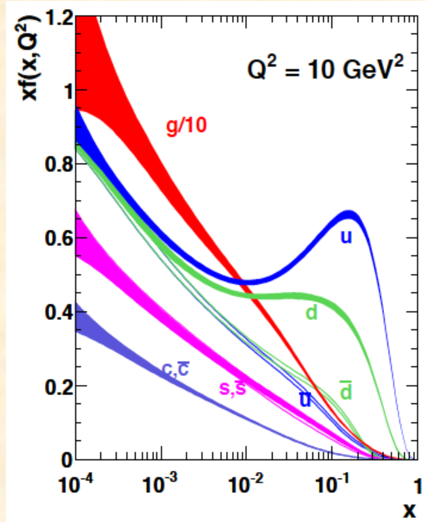
By the tomography, we may determine

 or  .

Wigner distribution and various structure functions

Wigner operator: $\hat{w}(k_+,\vec{k}_\perp,\vec{r}) \equiv \int d\xi_- d^2\xi_\perp e^{i(\xi_- k_+ - \vec{\xi}_\perp \cdot \vec{k}_\perp)} \bar{\psi}\left(\vec{r} - \vec{\xi}/2\right)\psi\left(\vec{r} + \vec{\xi}/2\right)$

Wigner distribution: $W(x,\vec{k}_\perp,\vec{r}) \equiv \int \frac{d^3q}{(2\pi)^3} \langle \vec{q}/2 | \hat{w}(\vec{r},k_+,\vec{k}_\perp) | -\vec{q}/2 \rangle, \quad x = k_+ / p_+$



Form factor ← $\int dx\, d^2k_\perp\, dz$

PDF (Parton Distribution Function) ← $\int d^2k_\perp d^3r$

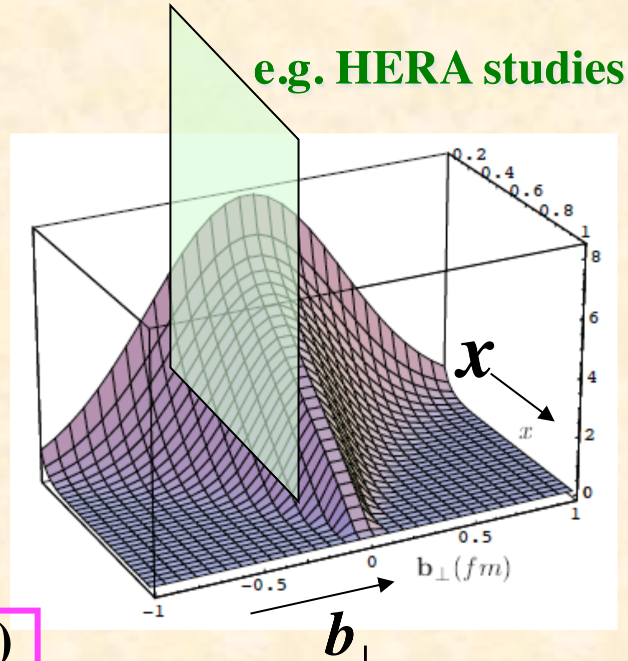
Wigner distribution $W(x,\vec{k}_\perp,\vec{r})$

3D world

$\int d^3r$ → **TMD (Transverse Momentum Dependent) parton distribution**

$\int d^2k_\perp dz$ → **GPD (Generalized Parton Distribution)**

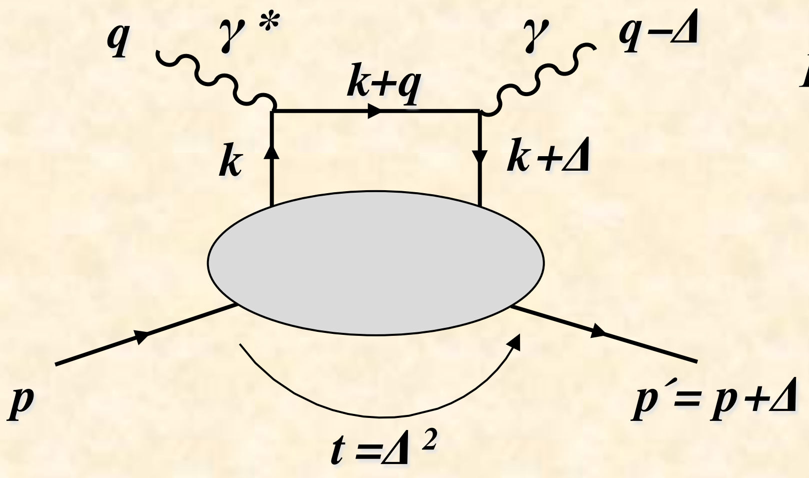
e.g. HERA studies



s-t crossing →

$\gamma\gamma \to h\bar{h}$ **GDA (Generalized Distribution Amplitude)**

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $\quad x = \dfrac{Q^2}{2p \cdot q}$

Momentum transfer squared $\quad t = \Delta^2$

Skewdness parameter $\quad \xi = \dfrac{p^+ - p'^+}{p^+ + p'^+} = -\dfrac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} u(p) \right]$$

Forward limit: PDFs $\quad H(x,\xi,t)\big|_{\xi=t=0} = f(x)$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\quad \int dx\, H(x,\xi,t) = F_1(t), \quad \int dx\, E(x,\xi,t) = F_2(t)$

Second moments: Angular momenta

Sum rule: $J_q = \dfrac{1}{2}\int dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0) \right], \quad J_q = \dfrac{1}{2}\Delta q + L_q$

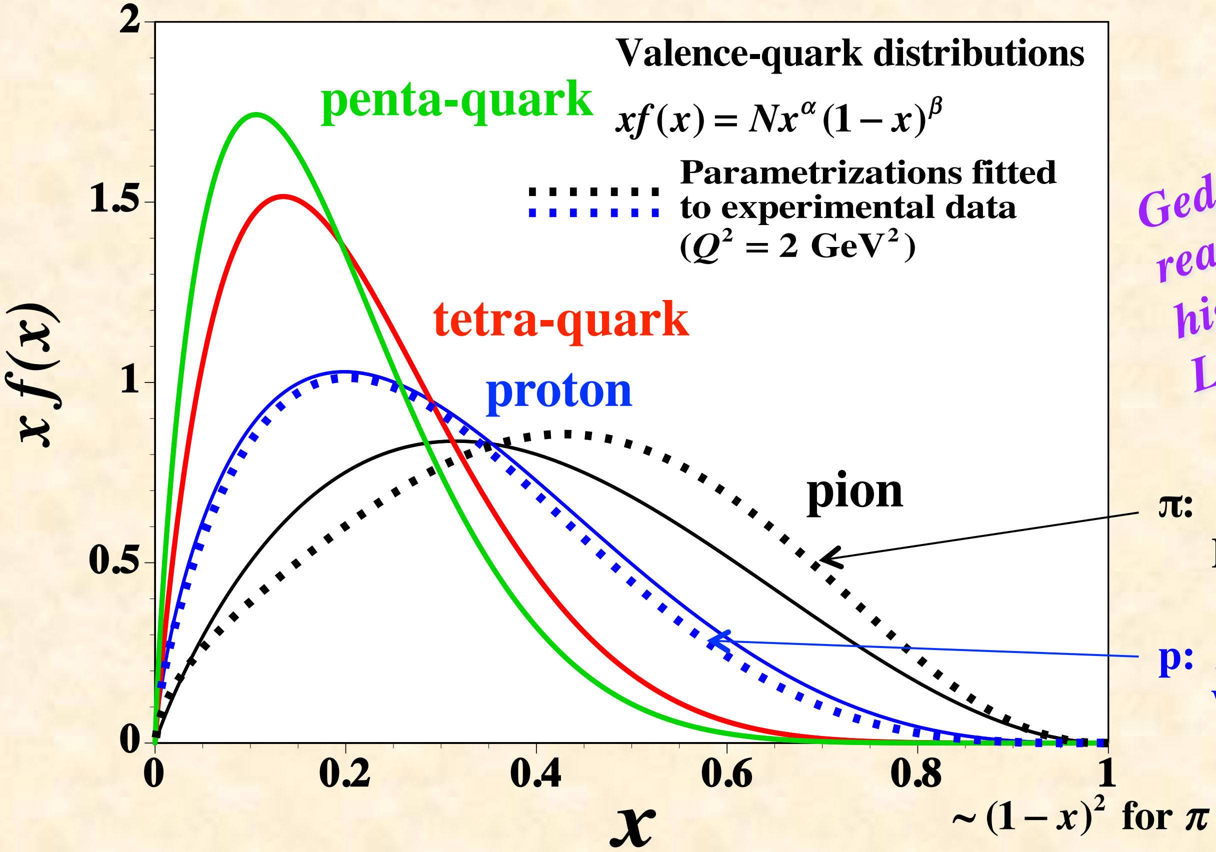
Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaeghen, PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n}(1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx\, f(x) = n$
- Constituent conting rule at $x \to 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx\, x f(x)$



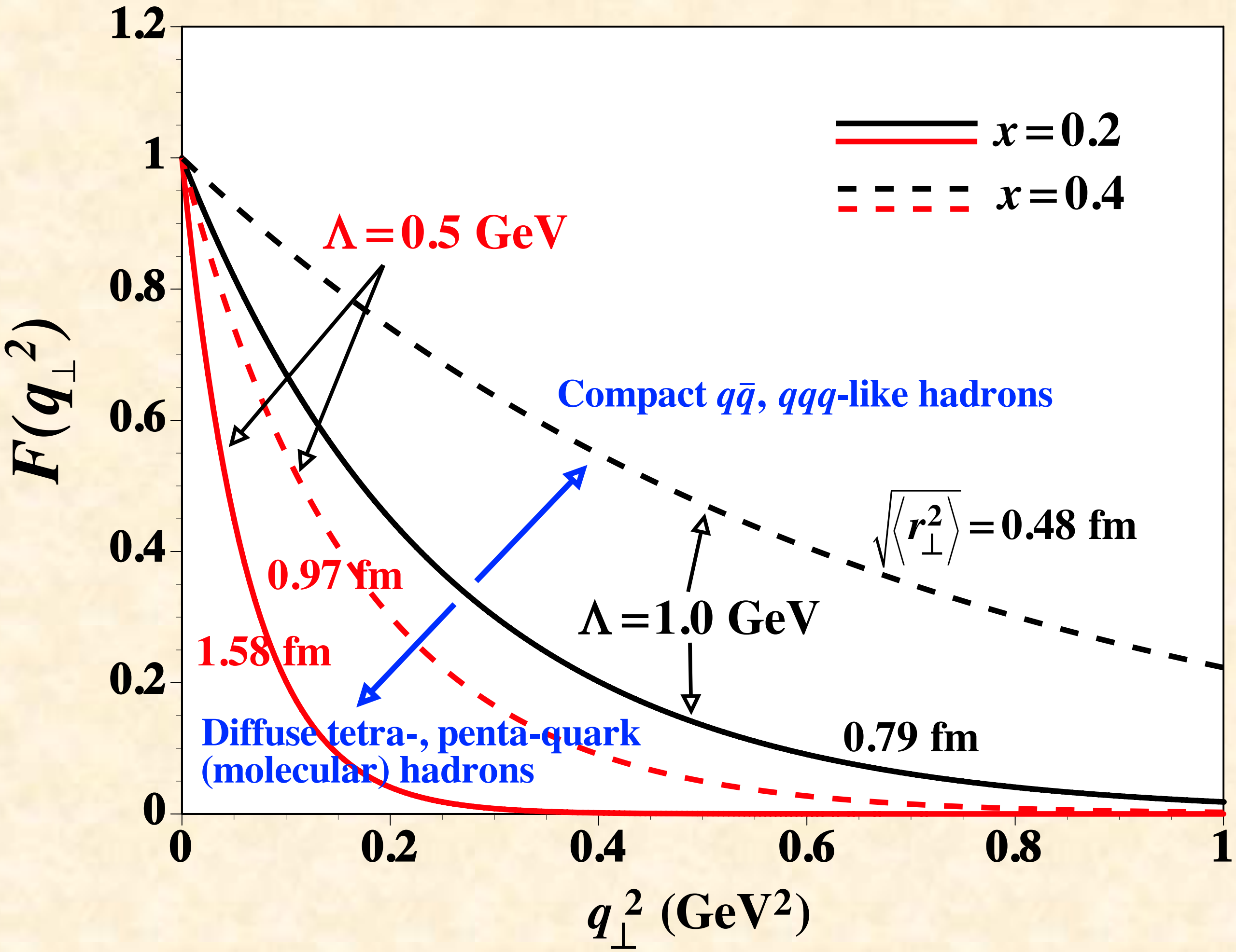
Gedankenexperiment, but … read our paper for studying exotics in high-energy processes at KEK-B, Linear Collider, ….

π: M. Aicher, A. Schafer, W. Vogelsang, PRL 105 (2010) 252003.

p: A. D. Martin, R. G. Roberts, W. J. Stirling, PLB 636, 259 (2006)

Two-dimensional form factor

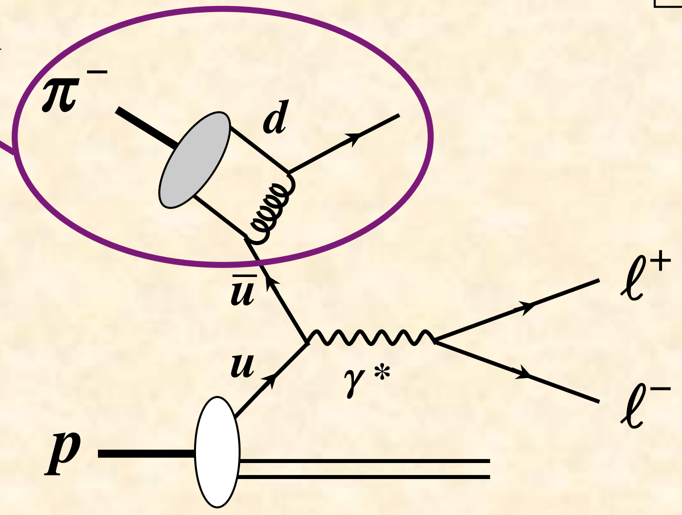
$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$



Toward a new proposal

W.-C. Chang, J.-C. Peng, S. Sawada *et al.*,
possible J-PARC experiment?

pion distribution



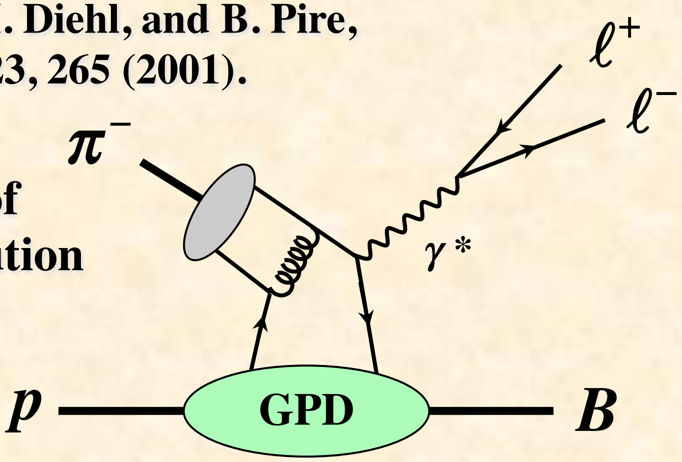
$$\pi^-(\bar{u}s) + p(uud) \rightarrow \ell^+\ell^- + X$$

A. Brandenburg, S. J. Brodsky,
V. V. Khoze, and D. Müller,
Phys. Rev. Lett. 73 (1994) 939.

Investigation of
- Pion distribution amplitude

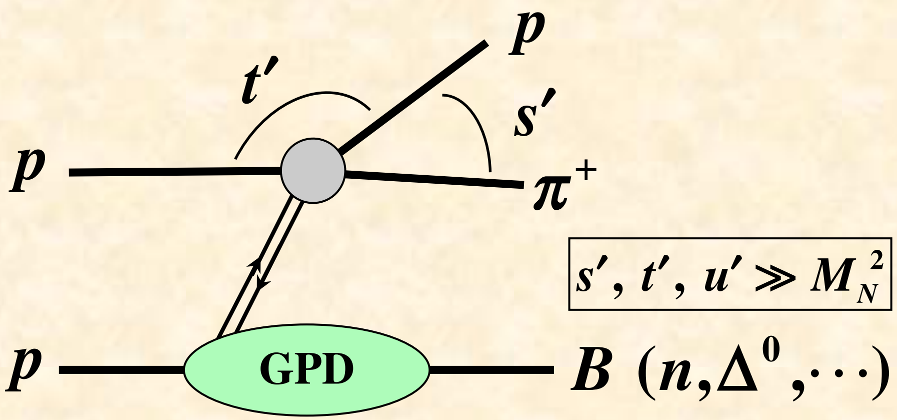
E. R. Berger, M. Diehl, and B. Pire,
Phys. Lett. B 523, 265 (2001).

Investigation of
- Pion distribution amplitude
- GPDs



$$\pi^-(\bar{u}d) + p(uud) \rightarrow B(udd) + \gamma^*(\rightarrow \ell^+\ell^-)$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



Toward a new proposal at J-PARC

K. Tanaka, T. Sawada
@this workshop on July 27

PHYSICAL REVIEW D 93, 114034 (2016)

Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC

Takahiro Sawada[*] and Wen-Chen Chang[†]
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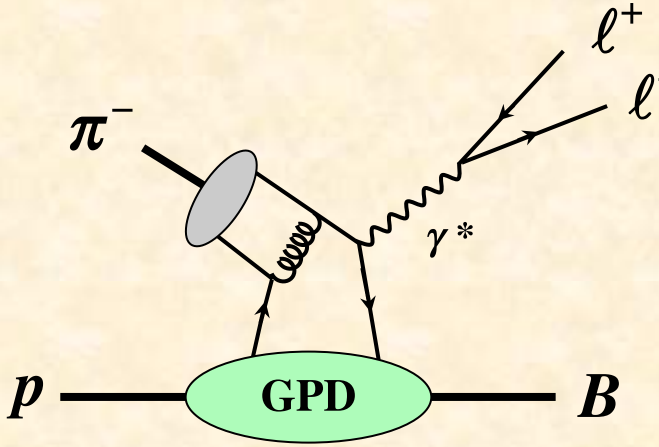
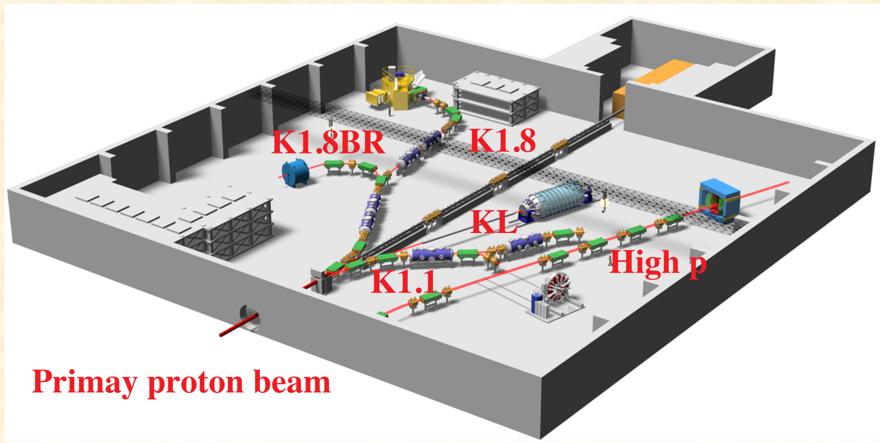
(Received 15 May 2016; published 29 June 2016)

T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng, S. Sawada, and K. Tanaka, PRD93 (2016) 114034.



$$\pi^-(\bar{u}d) + p(uud) \to B(udd) + \gamma^*(\to \ell^+\ell^-)$$

Hadron facility



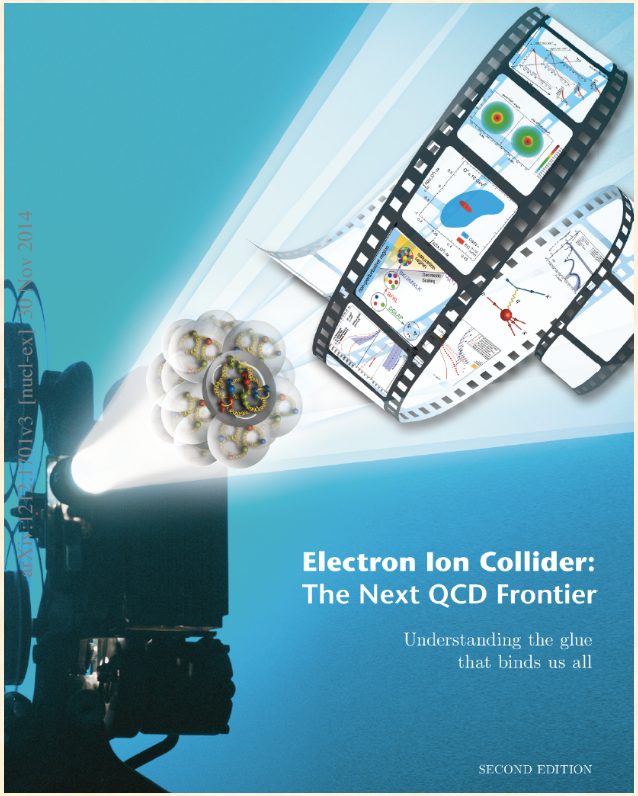
Possibilities of electron-ion collider

The EIC Science case: a report on the joint BNL/INT/JLab program

Gluons and the quark sea at high energies:
distributions, polarization, tomography

arXiv:1108.1713 (551 pages, SK = one of many)

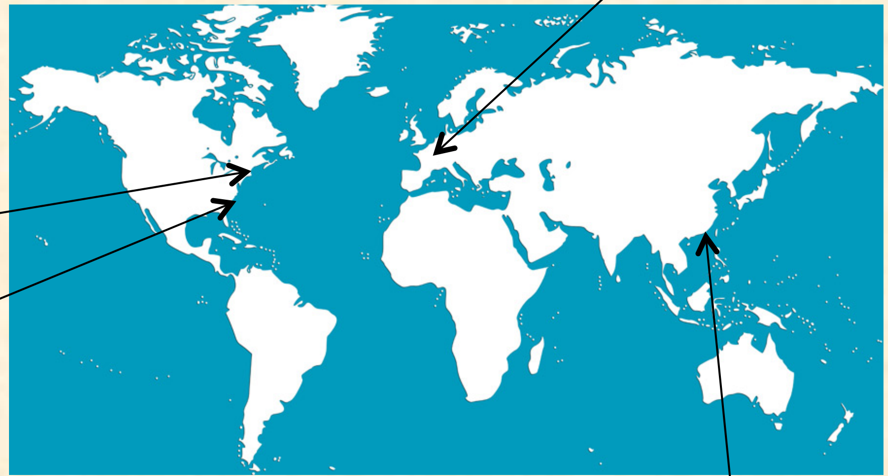
arXiv:1212.1701 (180 pages)



CERN

BNL

JLab



Institute of Modern Physics, Chinese Academy of Sciences

J. Phys. G: Nucl. Part. Phys. 39 (2012) 075001(632 pages)

CERN-OPEN-2012-015
LHeC-Note-2012-002 GEN
Geneva, June 13, 2012

LHeC

A Large Hadron Electron Collider at CERN

Report on the Physics and Design
Concepts for Machine and Detector

LHeC Study Group

GPD $H_q^h(x,\xi,t)$ and GDA $\Phi_q^{h\bar{h}}(z,\zeta,W^2)$

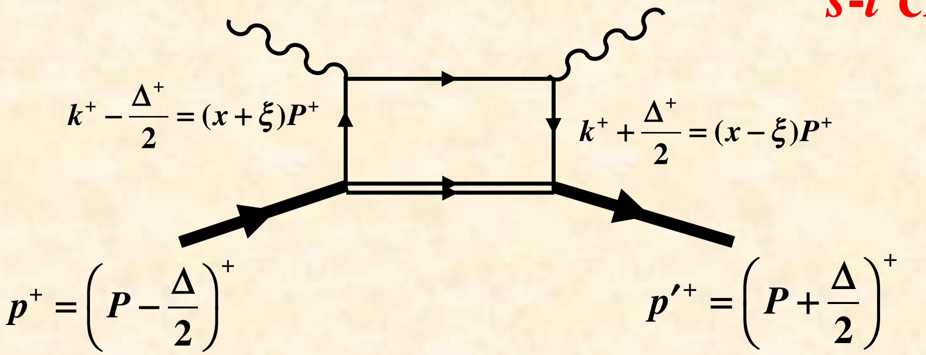
GPD: $H_q(x,\xi,t)=\int\frac{dy^-}{4\pi}\,e^{ixP^+y^-}\langle h(p')|\bar{\psi}(-y/2)\gamma^+\psi(y/2)|h(p)\rangle\Big|_{y^+=0,\vec{y}_\perp=0}$, $\quad P^+=\frac{(p+p')^+}{2}$

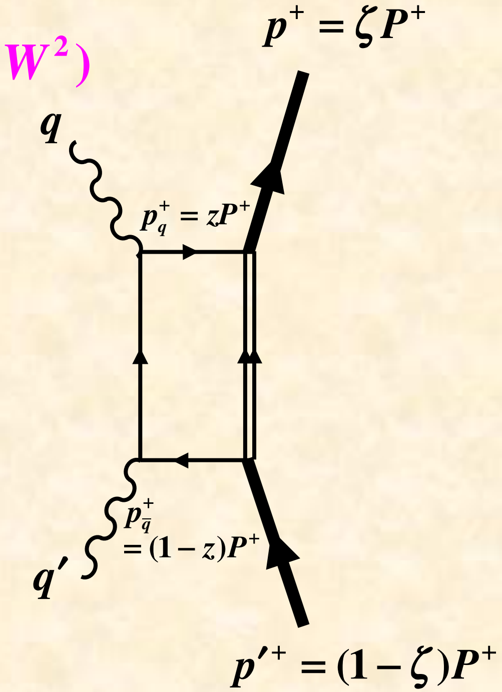
GDA: $\Phi_q(z,\zeta,s)=\int\frac{dy^-}{2\pi}\,e^{izP^+y^-}\langle h(p)\bar{h}(p')|\bar{\psi}(-y/2)\gamma^+\psi(y/2)|0\rangle\Big|_{y^+=0,\vec{y}_\perp=0}$

DA: $\Phi_q^h(z,\zeta,s)=\int\frac{dy^-}{2\pi}\,e^{izP^+y^-}\langle h(p)|\bar{\psi}(-y/2)\gamma^+\gamma_5\psi(y/2)|0\rangle\Big|_{y^+=0,\vec{y}_\perp=0}$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

***s*-*t* crossing**





$P=\frac{p+p'}{2},\ \Delta=p'-p$

Bjorken variable: $x=\frac{Q^2}{2p\cdot q}$

Momentum transfer squared: $t=\Delta^2$

Skewness parameter: $\xi=\frac{p^+-p'^+}{p^++p'^+}=-\frac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z=\frac{Q^2}{2q\cdot q'}$

Light-cone momentum ratio for h in $h\bar{h}$: $\zeta=\frac{p^+}{P^+}=\frac{1+\beta\cos\theta}{2}$

Invariant mass of $h\bar{h}$: $W^2=(p+p')^2$

Cross section: form factor dependence

$$\Phi_q^{h\bar{h}(I=0)}(z,\zeta,W^2) \propto F_h(W^2)$$

$$F_h(W^2) = \frac{1}{\left[1+(W^2-4m_h^2)/\Lambda^2\right]^{n-1}}$$

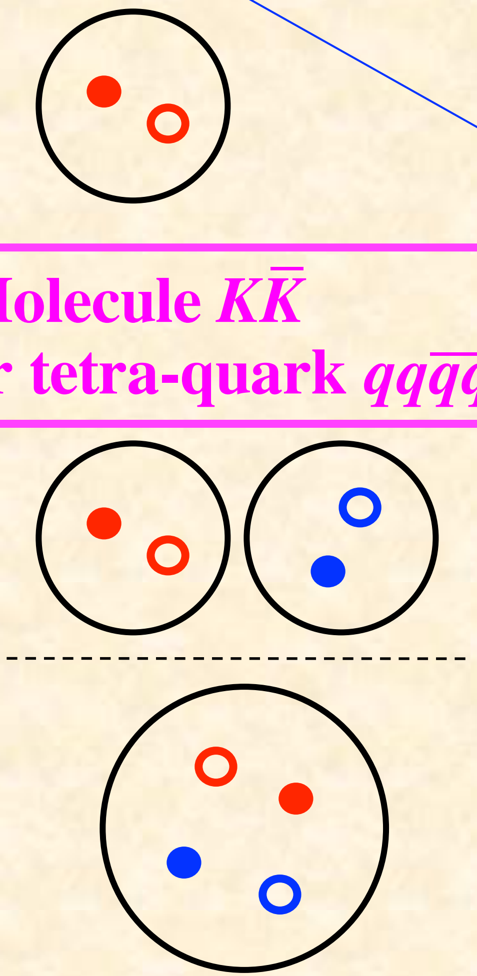
Constituent-counting rule

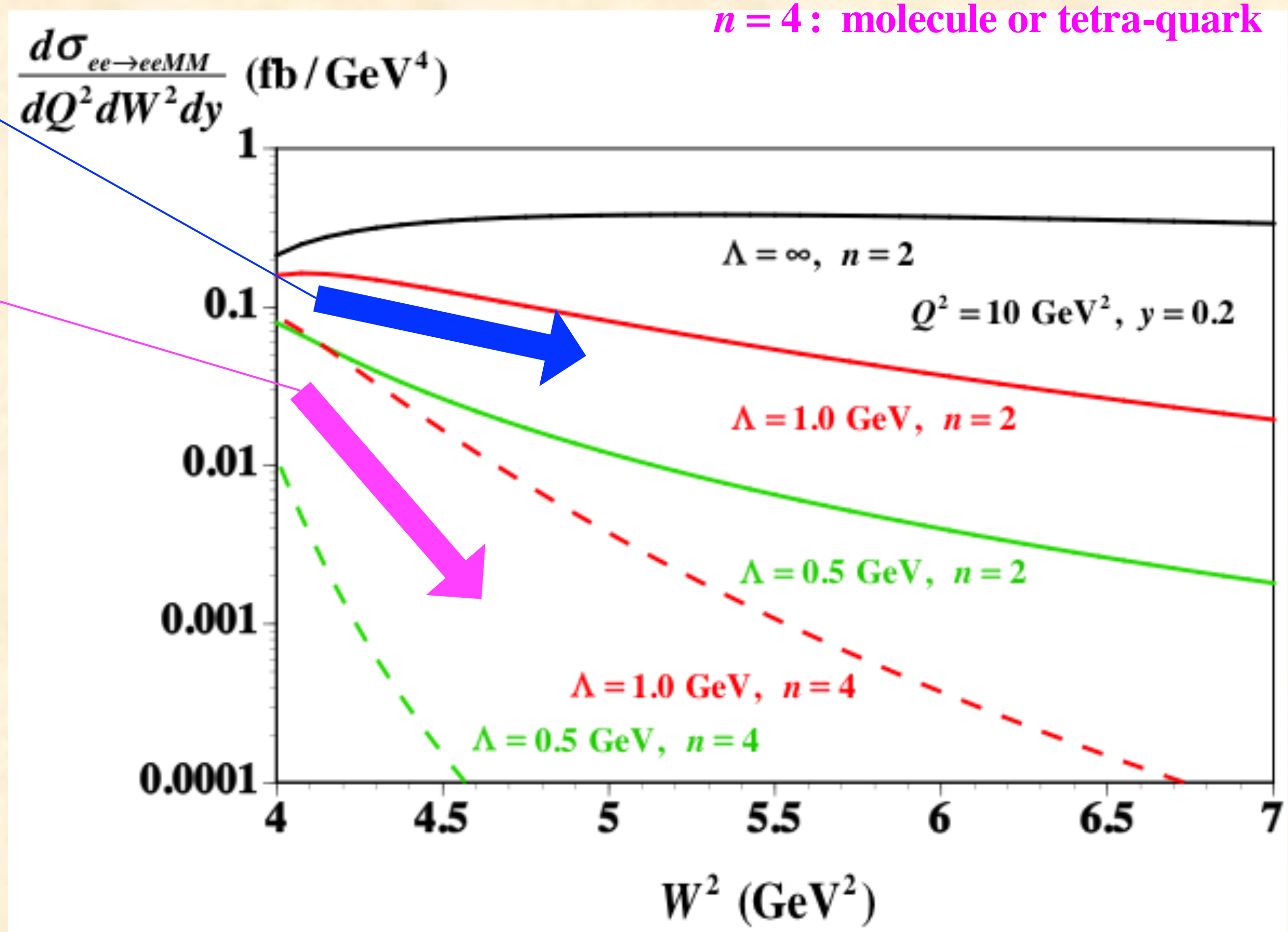
$n=2:$ ordinary meson

$n=4:$ molecule or tetra-quark

Ordinal $q\bar{q}$

Molecule $K\bar{K}$
or tetra-quark $qq\bar{q}\bar{q}$

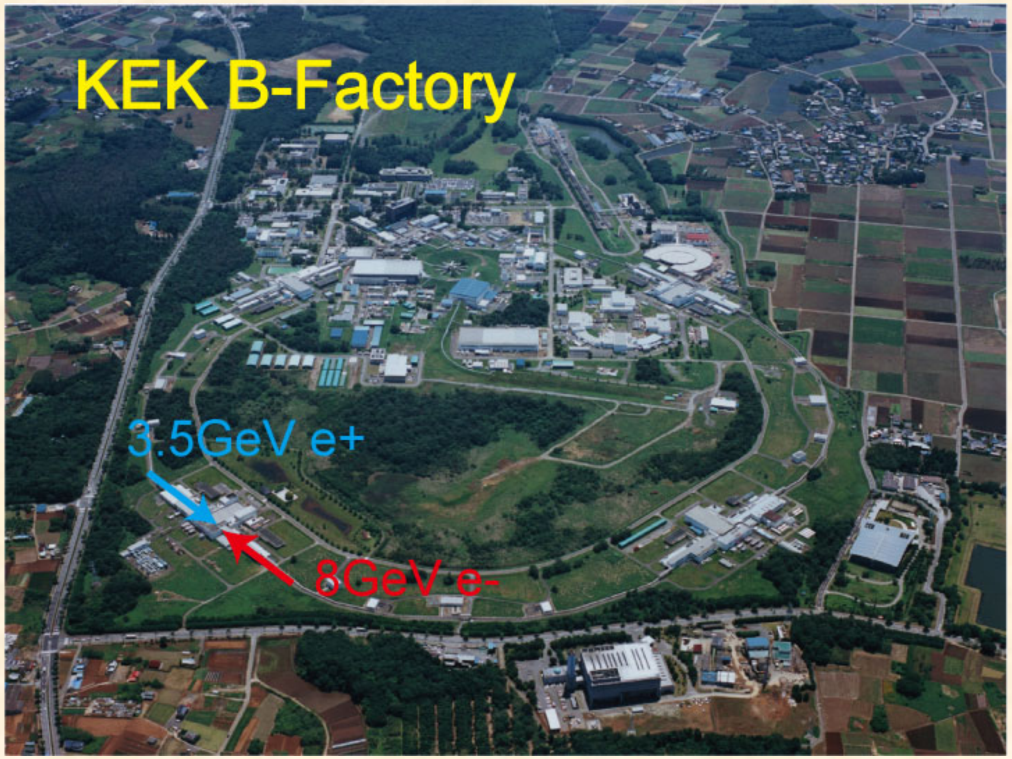




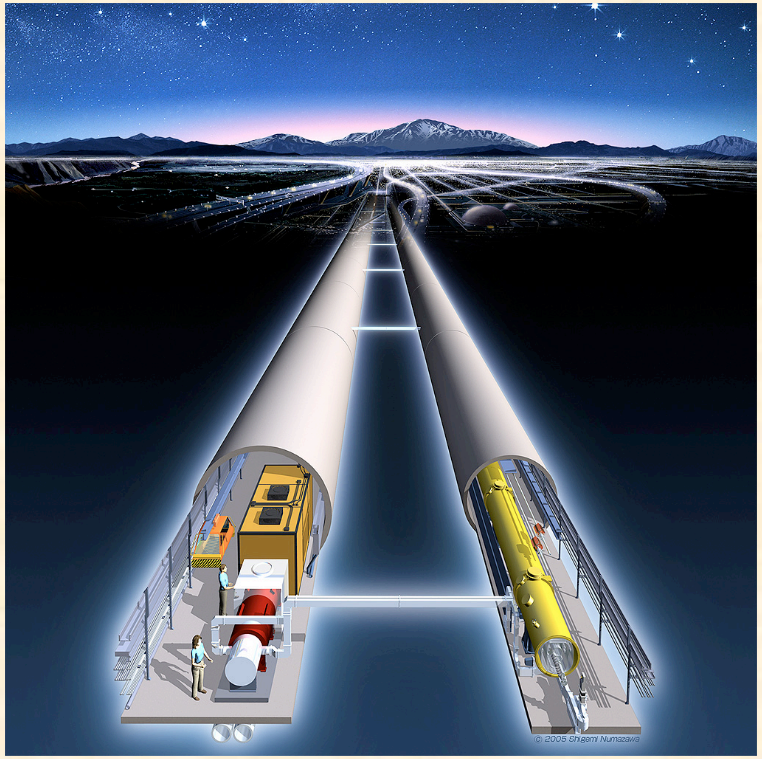
Experimental studies of GDAs in future

$\gamma\gamma \to h\bar{h}$ for internal structure of exotic hadron candidate h

KEK B-factory



Linear Collider ?



Summary

Constituent-counting rule for exotic hadrons

High energies = Quark and gluon degrees of freedom
It could be appropriate to use high-energy processes
for determination of internal configurations
for exotic-hadron candidates.

Other high-energy reactions for exotic hadrons

Fragmentation functions, GPDs, GDAs, ...

Future experimental projects

JLab: trying to get in touch with experimentalists.
J-PARC: M. Niiyama, under consideration.
COMPASS/GSI/EIC: ...

The End

The End