

# Open Charm Effects on the E1 Transition of $\psi(3686)/\psi(3770) \rightarrow \gamma\chi_{cJ}$

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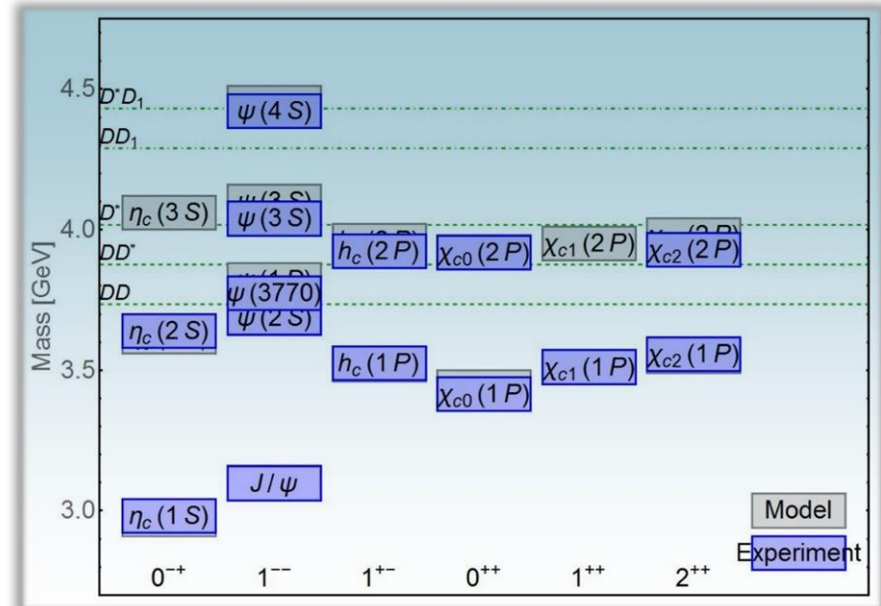
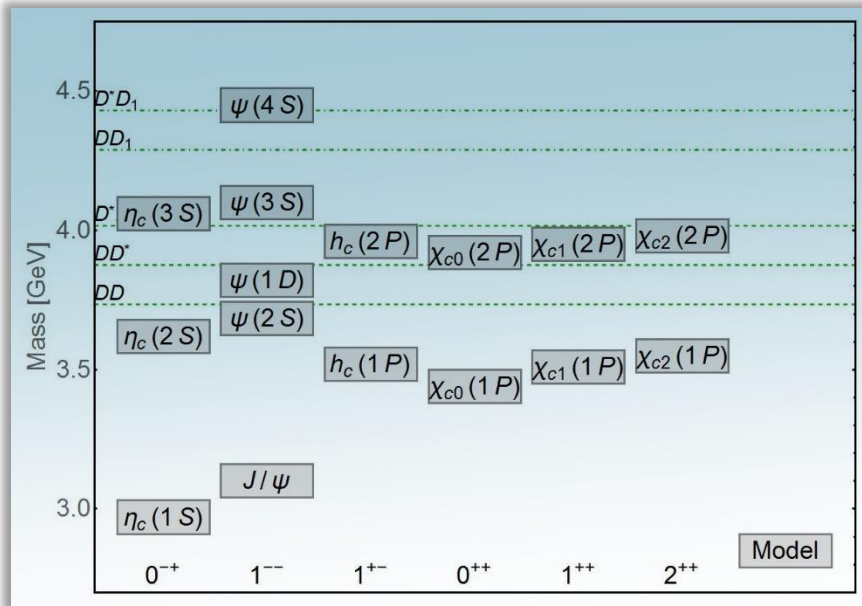
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# CHARMONIUM SPECTRUM



## Charmonium Spectrum: QM Prediction[1] V.S. Exp. Data

- ◆ Quark model works quite good in producing the charmonium spectrum. Even with a “naive” color Coulomb plus linear scalar potential we get a relatively successful description of the spectrum[2].

[1]S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985)

[2]E. Eichten et al., Phys. Rev. D 21 (1980) 203

# QUARK MODEL POTENTIAL

## ◆ Nonrelativistic potential model

$$V_{NR}^{c\bar{c}}(r) = \boxed{-\frac{4}{3}\frac{\alpha_s}{r} + br} + \frac{32\pi\alpha_s}{9m_c^2}\tilde{\delta}_\sigma(r)\vec{S}_c \cdot \vec{S}_{\bar{c}}$$

## ◆ Godfrey-Isgur relativized potential model

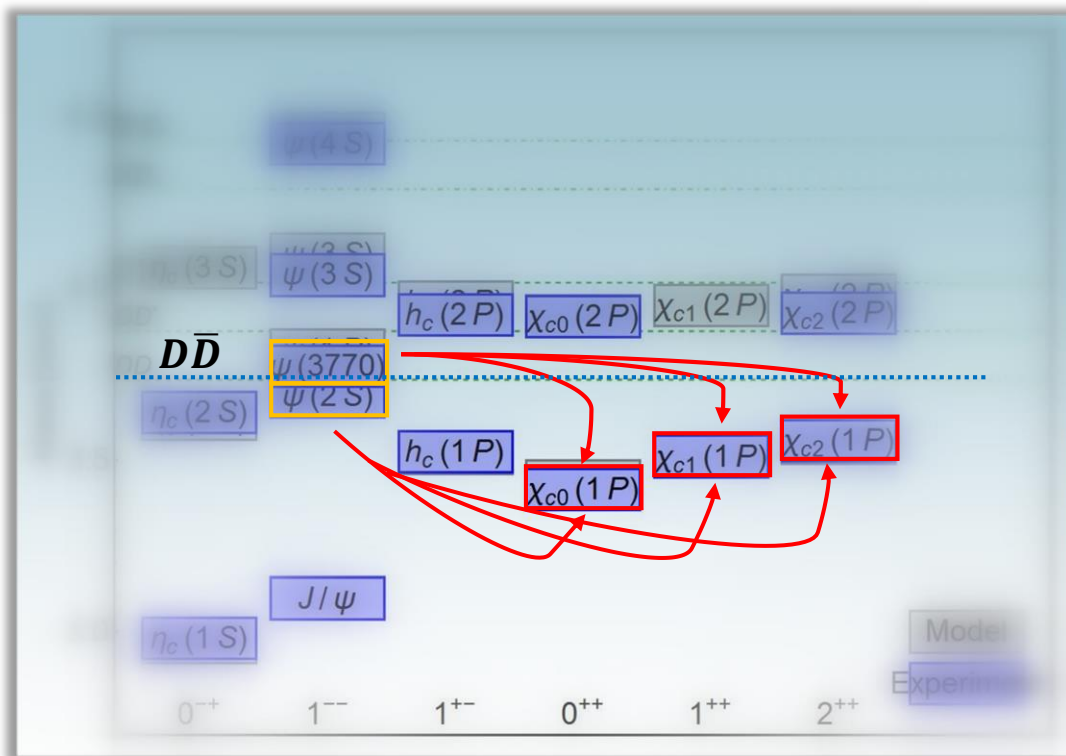
$$V_{GI}^{c\bar{c}}(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + br + H_{c\bar{c}}^{hyp} + H_{c\bar{c}}^{so} + H_A$$

$$H_{c\bar{c}}^{hyp} = \frac{4\alpha_s(r)}{3m_c^2} \left[ \frac{8\pi}{3}\tilde{\delta}_\sigma(r)\vec{S}_c \cdot \vec{S}_{\bar{c}} + \frac{1}{r^3} \left( \frac{3\vec{S}_c \cdot \vec{r}\vec{S}_{\bar{c}} \cdot \vec{r}}{r^2} - \vec{S}_c \cdot \vec{S}_{\bar{c}} \right) \right] \dots$$

## ◆ E1 transition width

$$\Gamma_{E1} \left( n^{2S+1}L_J \rightarrow n'^{2S'+1}L'_{J'} + \gamma \right) = \frac{4}{3} C_{fi} \delta_{SS'} e_c^2 \alpha |\langle \psi_f | r | \psi_i \rangle|^2 E_\gamma^3 \frac{E_f^{c\bar{c}}}{M_i^{c\bar{c}}}$$

# MOTIVATION



- ◆ We focus on the E1 transitions  $\psi''/\psi' \rightarrow \gamma\chi_{cJ}$  and study the open charm effects as final state interactions for these processes for both  $\psi''$  and  $\psi'$  are close to the  $D\bar{D}$  threshold.

- ◆ Studying these effects help us better understand the long-standing  $\rho\pi$  puzzle and the non- $D\bar{D}$  decay of  $\psi''$ .
- ◆ Similar mechanism has been studied in the M1 transition of  $J/\psi$  and  $\psi'$  [5] comparing to the previous studies in quark model and lattice QCD.

[5]G. Li and Q. Zhao, Phys. Lett. B 670 (2008) 55

# E1 TRANSITION IN QM

Channel	Ref. [2]/keV	Ref. [3]/keV	Ref. [4] /keV	Ref. [4]/keV	Exp. data/keV
$\psi'' \rightarrow \gamma\chi_{c0}$	<i>null</i>	299	403	213	$198.56 \pm 25.54$
$\psi'' \rightarrow \gamma\chi_{c1}$	<i>null</i>	99	125	77	$67.46 \pm 7.85$
$\psi'' \rightarrow \gamma\chi_{c2}$	<i>null</i>	3.88	4.9	3.3	<17.4
$\psi' \rightarrow \gamma\chi_{c0}$	43.2	47	63	26	$29.87 \pm 1.14$
$\psi' \rightarrow \gamma\chi_{c1}$	34	42.8	54	29	$28.55 \pm 1.20$
$\psi' \rightarrow \gamma\chi_{c2}$	23.7	30.1	38	24	$27.24 \pm 1.18$

Table of the E1 decay width of  $\psi''/\psi' \rightarrow \gamma\chi_{cJ}$  predicted by various QM model comparing with exp. data

- ◆ Significant discrepancies between various QM predictions and experimental data.

[2] E. Eichten *et al.*, Phys. Rev. D 21 (1980) 203

[3] N. Brambilla *et al.* [Quarkonium Working Group Collaboration], hep-ph/0412158

[4] T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev. D 72 (2005) 054026

# NONRELATIVISTIC EFFECTIVE FIELD THEORY

**Heavy mesons:**

$$H = P + \vec{\sigma} \cdot \vec{V}$$

**D-mesons:**

$$P_D = (D^0, D^+, D_S); \quad V_D = (D^{*0}, D^{*+}, D_S^*)$$

**S-wave charmonia:**

$$J = \vec{\sigma} \cdot \vec{\psi} + \eta'_c \dots \mathcal{L}_{HH\psi_S} = i \frac{g_2}{2} \langle \bar{H}_a^\dagger \sigma^i \vec{\partial}^i H_a^\dagger J \rangle + H.c.$$

**P-wave charmonia:**

$$\chi^i = \sigma^j \left( -\chi_{c2}^{ij} - \frac{1}{\sqrt{2}} \epsilon^{ijk} \chi_{c1}^k + \frac{1}{\sqrt{3}} \delta^{ij} \chi_{c0} \right) + h_c^i \dots \mathcal{L}_{HH\chi} = i \frac{g_1}{2} \langle \chi^\dagger H_a \sigma^i \bar{H}_a \rangle + H.c.$$

**D-wave charmonia:**

$$J^{ij} = \frac{1}{2} \sqrt{\frac{3}{5}} (\sigma^i \psi^j + \sigma^j \psi^i) - \frac{1}{\sqrt{15}} \delta^{ij} \vec{\sigma} \cdot \vec{\psi} \dots \mathcal{L}_{HH\psi_D} = i \frac{g_3}{2} \langle \bar{H}_a^\dagger \sigma^i \vec{\partial}^j H_a^\dagger J^{ij} \rangle + H.c.$$

**Photon  $\gamma$**  .....

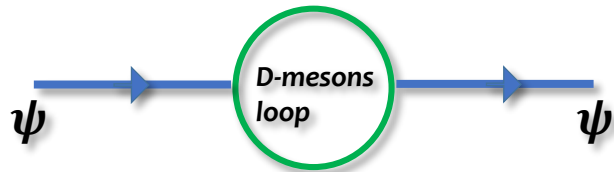
$$\mathcal{L}_{HH\gamma}^m = \frac{e\beta}{2} \langle H_a^\dagger H_b \vec{\sigma} \cdot \vec{B} Q_{ab} \rangle + \frac{eQ'}{2m_Q} \langle H_a^\dagger \vec{\sigma} \cdot \vec{B} H_a \rangle + c.c.$$

$$\mathcal{L}_{HH\gamma}^e = ie \langle A^i H^\dagger \vec{\partial}^i H \rangle + c.c.$$

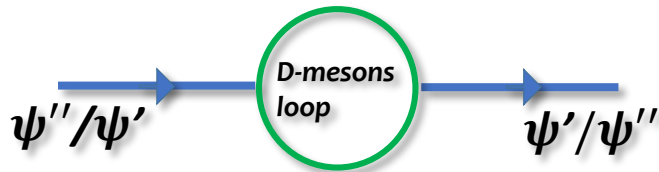
**Not** all the couplings here are clearly known

# OPEN CHARM EFFECTS

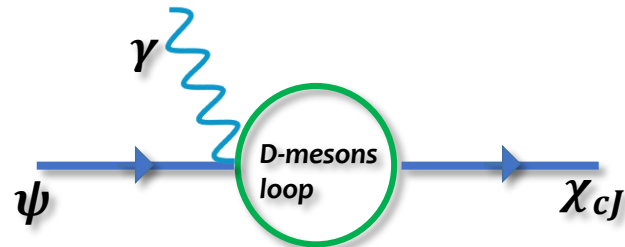
The coupling of  $\psi''/\psi'$  and two  $D$  mesons (pseudoscalar or vector) allows the open charm effects in the radiative E1 transition.



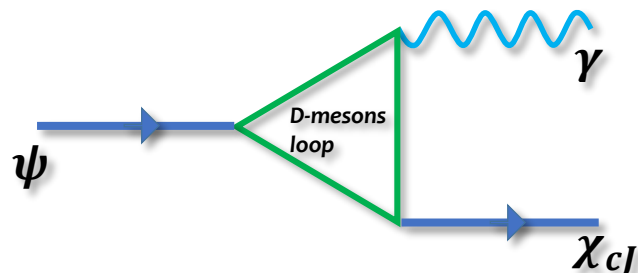
◆ Self-energy corrections



◆ S-D mixing

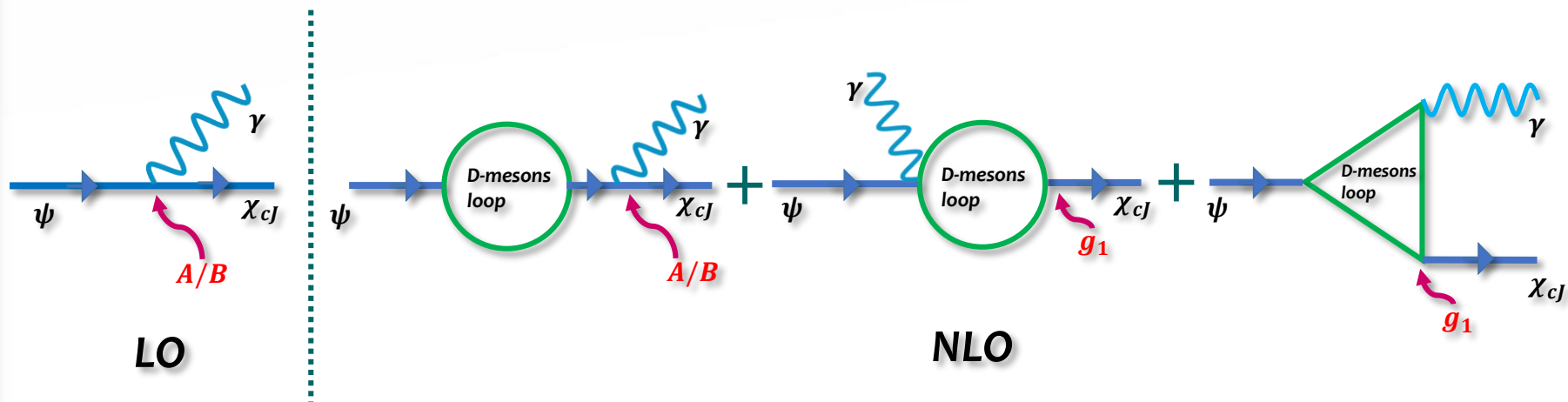


◆ Gauging term from  $\psi H H$  coupling



◆ Charmonium decays, e.g. E1 decay transition of  $\psi''$  and  $\psi'$

# POWER COUNTING AND COUPLINGS



- ◆ For the leading order terms, we use the Lagrangians

$$\mathcal{L}_{SP\gamma} = \mathbf{A} \langle \chi^{\dagger i} J \rangle E^i + h. c. \quad \mathcal{L}_{DP\gamma} = \mathbf{B} \langle \chi^{\dagger i} J^{ij} \rangle E^j + h. c.$$

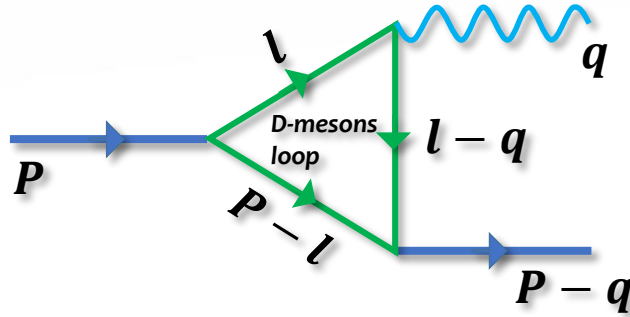
- ◆  $g_S$  is taken from Ref. [6] and  $g_D$  is fitted through the tree process  $\psi'' \rightarrow D\bar{D}$ .
- ◆ the photon magnetic coupling is introduced from Ref. [7].
- ◆  $g_1$  is taken as a free parameter.
- ◆ Loop diagrams have divergence.

[6] G. Y. Chen and Q. Zhao, Phys. Lett. B 718 (2013) 1369

[7] J. Hu and T. Mehen, Phys. Rev. D 73 (2006) 054003



# LOOP INTEGRALS WITH EXP. F.F.



$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

$$\operatorname{erfi}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{t^2} dt$$

$$\begin{aligned} \diamond I^{(0)} &= i \int \frac{d^4 l}{(2\pi)^4} \frac{\exp(-2\vec{l}^2/\Lambda^2)}{(l^2 - m_1^2 + i\epsilon)[(P-l)^2 - m_2^2 + i\epsilon][(l-q)^2 - m_3^2 + i\epsilon]} \\ &= \frac{\mu_{12}\mu_{23}}{2m_1 m_2 m_3} \frac{4\pi}{(2\pi)^3} \int_0^1 dx \int \frac{d^3 \vec{l}}{(2\pi)^3} \frac{\exp(-2\vec{l}^2/\Lambda^2)}{(\vec{l}^2 + \Delta)^2} \\ &= \frac{-\mu_{12}\mu_{23}}{16m_1 m_2 m_3 \Lambda^2 \pi^2} \int_0^1 dx \left\{ 2\Lambda\sqrt{2\pi} + \pi e^{2\Delta/\Lambda^2} \left( 4\sqrt{\Delta} + \frac{\Lambda^2}{\sqrt{\Delta}} \right) \left[ \operatorname{erf}\left(\frac{\sqrt{2\Delta}}{\Lambda}\right) - 1 \right] \right\} \end{aligned}$$

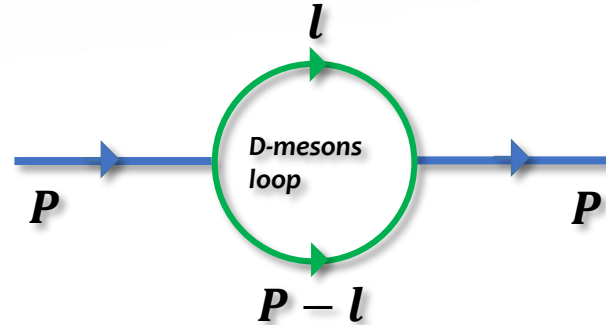
$$\diamond I^{(1)} = \frac{-\mu_{12}\mu_{23}^2}{16m_1 m_2 m_3^2 \Lambda^2 \pi^2} \int_0^1 dx x \left\{ 2\Lambda\sqrt{2\pi} + \pi e^{2\Delta/\Lambda^2} \left( 4\sqrt{\Delta} + \frac{\Lambda^2}{\sqrt{\Delta}} \right) \left[ \operatorname{erf}\left(\frac{\sqrt{2\Delta}}{\Lambda}\right) - 1 \right] \right\}$$

$$\diamond I_1^{(2)} = \frac{\mu_{12}\mu_{23}}{48m_1 m_2 m_3 q_z^2 \Lambda^2 \pi^2} \int_0^1 dx \left\{ \Lambda\sqrt{2\pi}(2\Delta + \Lambda^2) + \pi\sqrt{\Delta} e^{2\Delta/\Lambda^2} (4\Delta + 3\Lambda^2) \left[ \operatorname{erf}\left(\frac{\sqrt{2\Delta}}{\Lambda}\right) - 1 \right] \right\}$$

$$\Delta = x(c' - ax) + (1-x)(c - i\epsilon), \quad c', a \text{ and } c \text{ are defined as Ref[8]}$$

[8] F. K. Guo, C. Hanhart, G. Li, U. G. Meisner and Q. Zhao, Phys. Rev. D 83 (2011) 034013

# LOOP INTEGRALS WITH EXP. F.F.



$$\begin{aligned}
 \diamond I_m &= \int \frac{d^4 l}{(2\pi)^4} \frac{\vec{l}^2 \exp(-2\vec{l}^2/\Lambda^2)}{(l^2 - m_1^2 + i\epsilon)[(P-l)^2 - m_2^2 + i\epsilon]} \\
 &= \frac{i}{4m_1 m_2} \frac{4\pi}{(2\pi)^3} \int_0^\infty dl \frac{\vec{l}^4 \exp(-2\vec{l}^2/\Lambda^2)}{P - m_1 - m_2 - (\vec{l}^2/2\mu_m) + i\epsilon} \\
 &= \frac{-i \exp(-2k^2/\Lambda^2)}{8(m_1 + m_2)\pi^2} \left\{ \sqrt{\frac{\pi}{2}} \Lambda e^{2k^2/\Lambda^2} \frac{\Lambda^2 + 4k^2}{4} + \pi(-k^2 - i\epsilon)^{3/2} \left[ 1 - \operatorname{erf}\left(\frac{\sqrt{-2k^2 - i\epsilon}}{\Lambda}\right) \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 \diamond I_g &= \int \frac{d^4 l}{(2\pi)^4} \frac{\exp(-2\vec{l}^2/\Lambda^2)}{(l^2 - m_1^2 + i\epsilon)[(P-l)^2 - m_2^2 + i\epsilon]} \\
 &= \frac{i}{4m_1 m_2} \left\{ -\frac{\mu\Lambda}{(2\pi)^{\frac{3}{2}}} + \frac{\mu k}{2\pi} e^{-2k^2/\Lambda^2} \left[ \operatorname{erfi}\left(\frac{\sqrt{2}k}{\Lambda}\right) - i \right] \right\}
 \end{aligned}$$

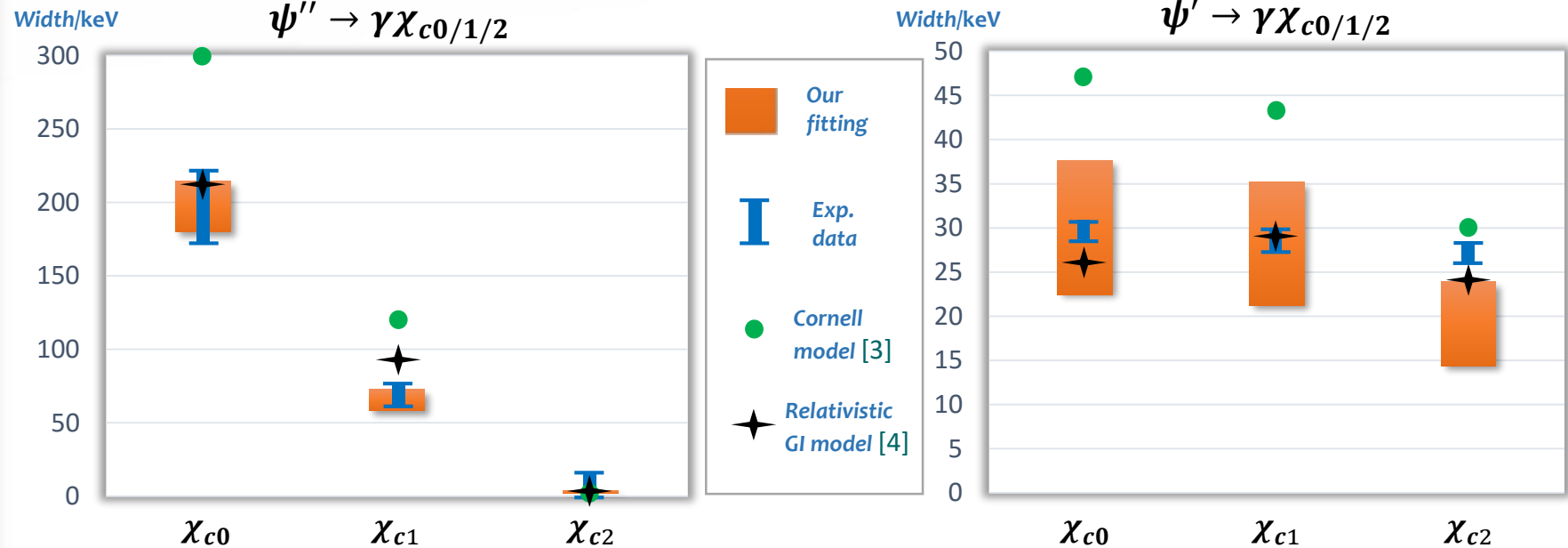
# FITTING PARAMETERS

## ◆ Fitted parameters with $\Lambda$ dependence

	$\Lambda(\text{GeV})$	$\chi^2/\text{d.o.f}$	$A(\text{GeV}^{-1})$	$B(\text{GeV}^{-1})$	$g_1(\text{GeV}^{-1/2})$	$\delta(^{\circ})$
$SU(2)$	0.8	1.95	0.232	0.314	2.12	-119
	0.9	1.56	0.222	0.324	4.44	-114
	1.0	1.20	0.214	0.331	5.38	-109
	1.1	0.94	0.208	0.335	5.33	-106
	1.2	0.78	0.206	0.336	4.48	-105
$SU(3)$	0.8	1.61	0.219	0.326	4.52	-114
	0.9	1.16	0.209	0.332	5.39	-106
	1.0	0.88	0.205	0.335	4.69	-104
	1.1	0.74	0.207	0.335	3.38	-104
	1.2	0.66	0.209	0.334	2.49	-104

# DECAY WIDTHS

$\Lambda = 1.0 \text{ GeV}$ ,  $SU(2)$  scheme



$A = 0.213 \pm 0.055 \text{ GeV}^{-1}$    
  $B = 0.331 \pm 0.025 \text{ GeV}^{-1}$    
 $g_1 = 5.37 \pm 0.51 \text{ GeV}^{-1/2}$    
 $\delta = -109 \pm 14^\circ$

Fitting results in decay width comparing with exp. data and predictions in QM

- ◆ Fitting results for most channels work well.
- ◆ Fitting results for  $\psi' \rightarrow \gamma \chi_{c2}$  can not match the data very well. We finally eliminate this channel when fitting.

[3] N. Brambilla et al. [QWG Collaboration], hep-ph/0412158

[4] T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev. D 72 (2005) 054026

# CONTRIBUTION OF LOOPS

Channel	Fitted width/keV	Without loops/keV
$\psi'' \rightarrow \gamma\chi_{c0}$	197	232
$\psi'' \rightarrow \gamma\chi_{c1}$	65.6	73.2
$\psi'' \rightarrow \gamma\chi_{c2}$	2.9	2.8
$\psi' \rightarrow \gamma\chi_{c0}$	30.1	25.4
$\psi' \rightarrow \gamma\chi_{c1}$	28.2	22.1
$\psi' \rightarrow \gamma\chi_{c2}$	19.1	15.4

- ◆ The open charm effects play an important role in these E1 transitions.
- ◆ Loops diagrams give destructive contributions in  $\psi' \rightarrow \gamma\chi_{cJ}$  channels and constructive contributions in  $\psi'' \rightarrow \gamma\chi_{cJ}$  channels.

- ◆ Mixing angle  $\theta_\psi$  between  $\psi'$  and  $\psi''$  can be related via  $|\xi_\psi(s)| = |\sin\theta_\psi|$  [9] where

$$|\xi_i(s)| = \left| \frac{D_{\psi'\psi''}}{D_i} \right| = \left| \frac{A_{\psi''(\psi')}^{mixing}}{A_{\psi'(\psi')}^{tree}} \right| .$$

In this way we find that  $\theta_{\psi'} = 8.1^\circ$  which is consistent with those extracted in Refs. [9][10].

[9] J. L. Rosner, Annals Phys. 319 (2005) 1

[10] Y. J. Zhang and Q. Zhao, Phys. Rev. D 81 (2010) 034011

# SUMMARY

- ◆ *We present a detailed study of the E1 transitions for  $\psi' / \psi'' \rightarrow \gamma \chi_{cJ}$  in the NREFT where the subleading corrections arising from the charmed meson loops are consistently taken into account in the same framework.*
- ◆ *We find that the intermediate meson loops play an important role by introducing important interferences to the tree-level transitions.*
- ◆ *The open charm contribution from the triangle processes appears to be a general phenomenon that has brought a lot of interesting insights into the understanding of recent “XYZ” states. Meanwhile, as we have shown in this work, it can also produce sizable effects on processes where the dominant contribution is from the potential quark model.*
- ◆ *Further precise measurement of  $\psi' \rightarrow \gamma \chi_{c2}$  will help clarify the underlying dynamics.*

**THANK YOU!**    **ありがとうございます!**