Non-dipolar Wilson links for parton densities

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Outlines

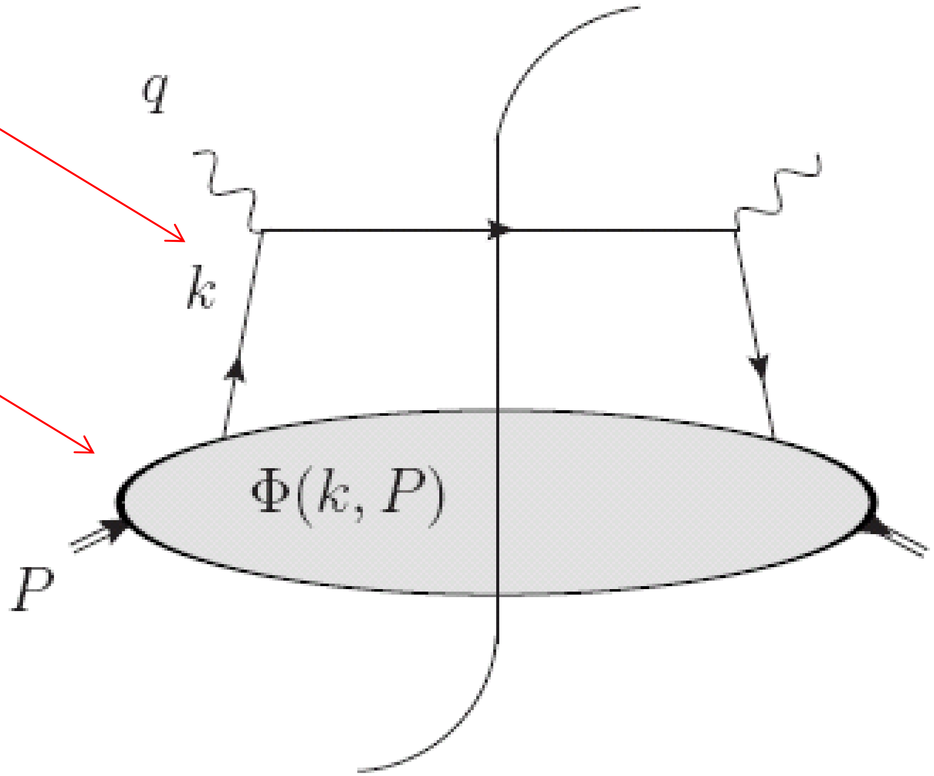
- Introduction
- TMD definitions
- Non-dipolar Wilson links
- Quasi-parton distributions
- Summary

Factorization theorem

- Collinear divergence associated with proton
- Cross section = Hard (H) * Parton distribution function (PDF)
- H = short distance, LO
  PDF = long distance
- Collinear factorization

$$k = (xP^{+}, 0, 0_T)$$

deeply inelastic scattering



k_T factorization

- k_T factorization applies to small x, or high energy region, especially to LHC physics
- Also to final-state spectra at low q_T, like direct photon and jet production
- Keep k_T in hard kernel, $xP^+ \approx k_T\,, q_T \approx k_T$
- Parton $k = (xP^+, 0, k_T)$ enters hard kernel
- Parton k_T is not integrated out in PDF

  $\Rightarrow$ k_T dependent parton density
- Many aspects of k_T factorization not yet investigated in detail

TMD parton density

- Universal transverse-momentum-dependent (TMD) parton density describes probability of parton carrying momentum fraction and transverse momentum
- If neglecting k_T in H, integration over k_T can be worked out, giving

$$\int d^2k_T \Phi_{f/N}(\xi, k_T) \Rightarrow \phi_{f/N}(\xi)$$

TMD definitions

Foundations of perturbative QCD, Collins, 2011

Definition of PDF

- PDF absorbs collinear divergence, i.e., nonperturbative dynamics in high-energy QCD process
- Defined as gauge-invariant hadronic matrix element of nonlocal operator

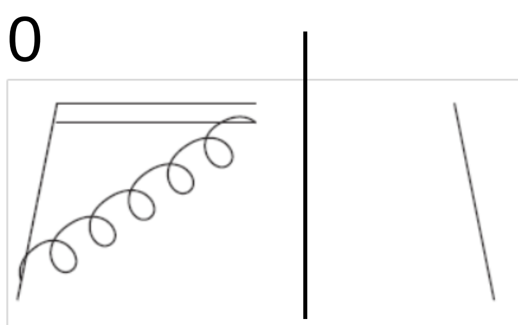
$$
\begin{aligned}
&\phi_{f/N}(\xi,\mu^2) \\
= &\int \frac{dy^-}{2\pi} \exp(-i\xi P^+ y^-) \\
&\times \frac{1}{2}\sum_\sigma \langle N(P,\sigma)|\bar{q}_f(0,y^-,0_T)\frac{1}{2}\gamma^+ W(y^-,0) q_f(0,0,0_T)|N(P,\sigma)\rangle
\end{aligned}
$$

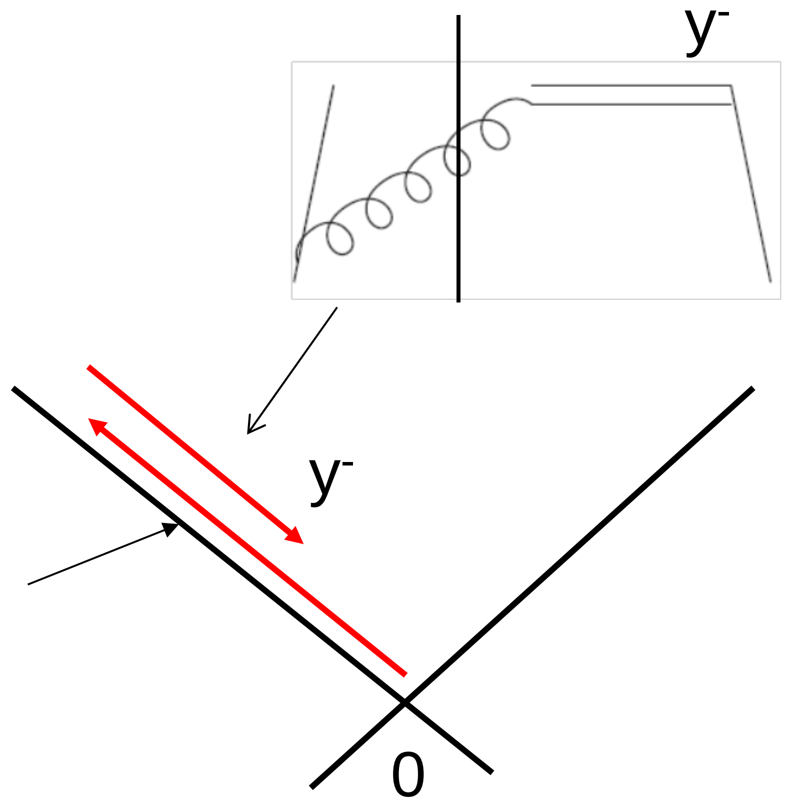
Wilson lines for PDF

$$W(y^-,0) \;=\; W(0)W^\dagger(y^-)$$

$$W(y^-) \;=\; \mathcal{P}\exp\left[-ig\int_0^\infty d\lambda n_- \cdot A(y+\lambda n_-)\right]$$

loop momentum flows through the hard kernel

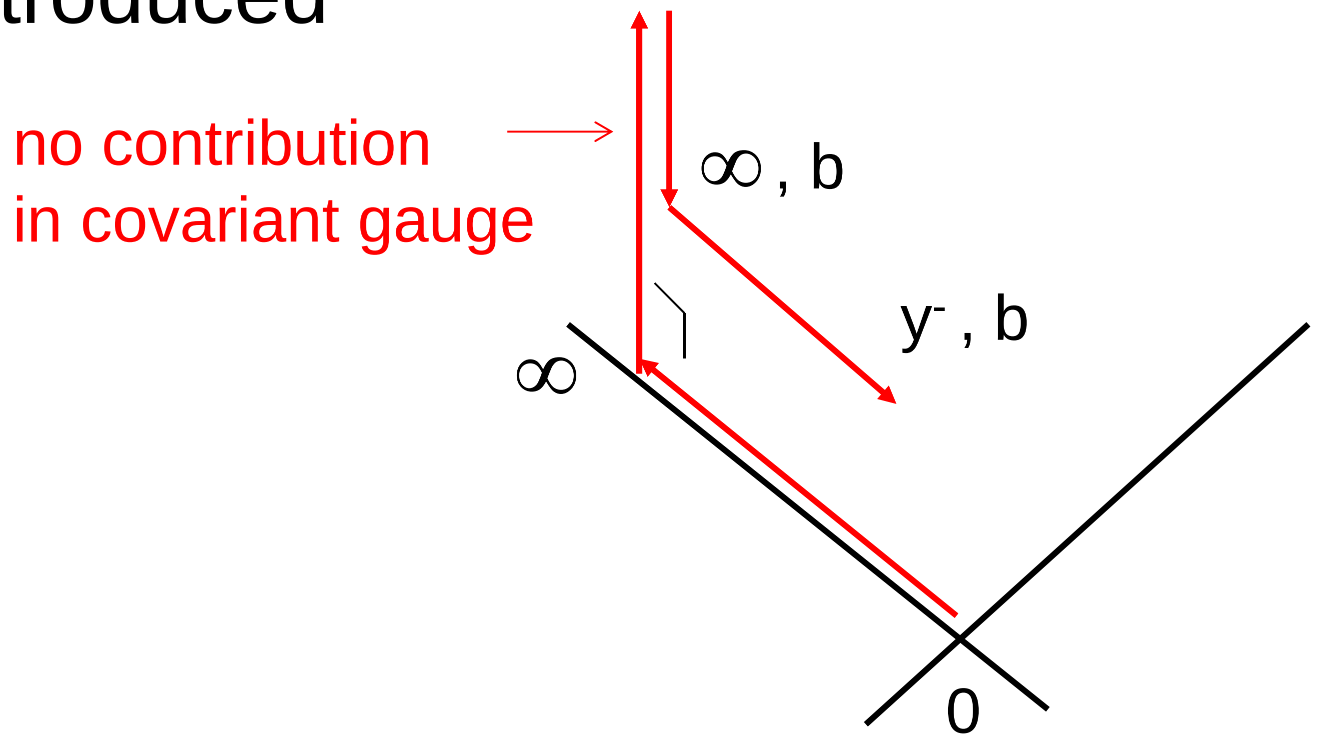




loop momentum does not flow through the hard kernel

Transverse Wilson links

- Suppose TMD factorization established. Quark fields nonlocal in transverse directions. Transverse Wilson links introduced



Light-cone singularity

- Compute $H^{(1)} = G^{(1)} - \phi^{(1)} \otimes H^{(0)}$
- The pole $1/(n_- \cdot l) = 1/l^+$ from Wilson lines in $\phi^{(1)}$ gives light-cone singularity.
- It cancels in collinear factorization

$$\phi^{(1)} \otimes H = \int \frac{dl^+}{l^+} [H(x) - H(x + l^+ / P^+)]$$

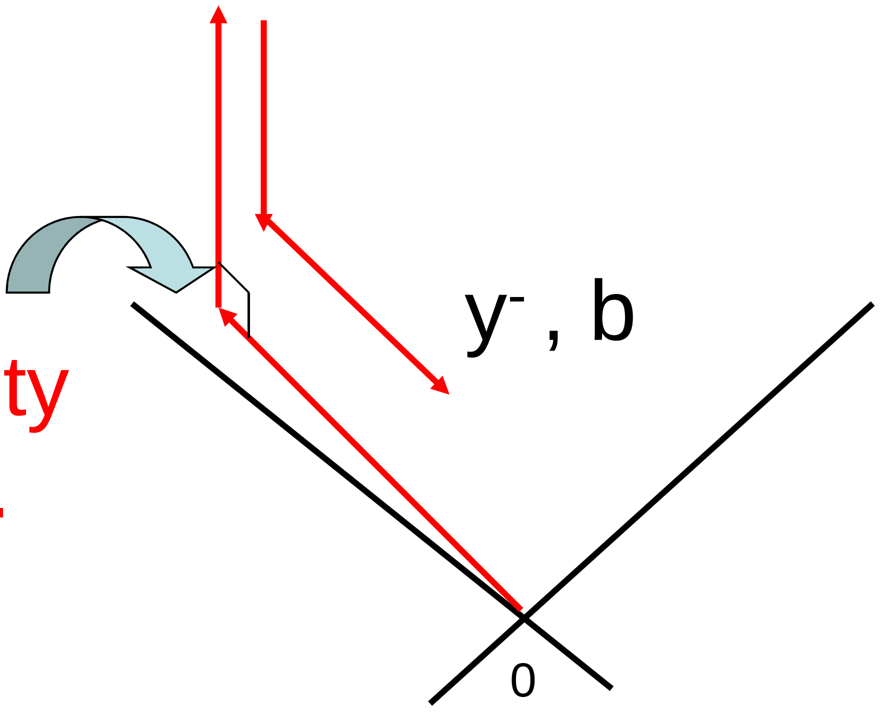
- Difference of $H^{(0)}$ removes singularity.
- It exists in k_T factorization:

$$\int \frac{dl^+}{l^+} [H(x, k_T) - H(x + l^+ / P^+, k_T + l_T)]$$

- because $H(x, k_T) \neq H(x, k_T + l_T)$

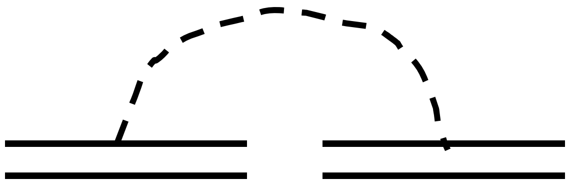
Modified TMD

- Naïve TMD is ill-defined
- Modified definition: $n_{-} \to n, n^2 \neq 0$
- Collins 2003
- Light-cone singularity is regularized by n^2.



New IR singularity

- Self-energy correction to Wilson links appears



- Proportional to n^2, vanishes originally as $n_-^2 = 0$
- Its Feynman integrand $\dfrac{1}{(n\cdot l + i\varepsilon)(n\cdot l - i\varepsilon)}$
- 1st pole $n\cdot l = 0$ leads to pinched singularity from 2nd eikonal propagator
- Off-light-cone Wilson links regularize light-cone singularity, but introduce new one

Non-dipolar Wilson links

Collins' modification

- TMD with light-like Wilson links multiplied by

$$\lim_{\substack{y_1 \to +\infty \\ y_u \to -\infty}} \sqrt{\frac{S(z_T; y_1, y_2)}{S(z_T; y_1, y_u)\, S(z_T; y_2, y_u)}}$$

(+,n)

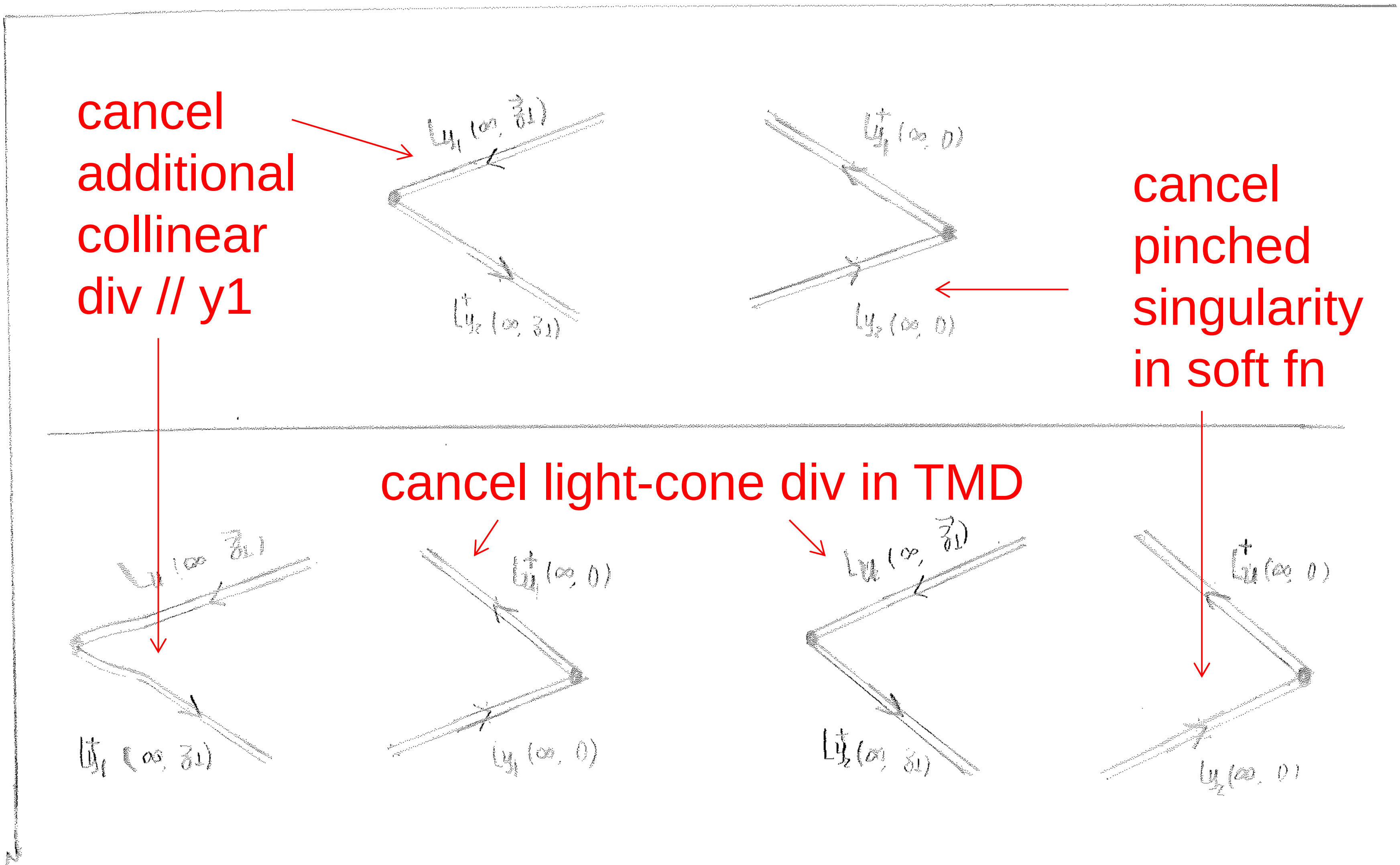
(+,-) (n,-)

Wilson-line rapidity

$$n_2 = (e^{y_2}, e^{-y_2}, \mathbf{0_T})$$

- u and n1 on light cone, n2 off light cone
- Off-light-cone Wilson links move into soft function
- Square root renders calculation difficult

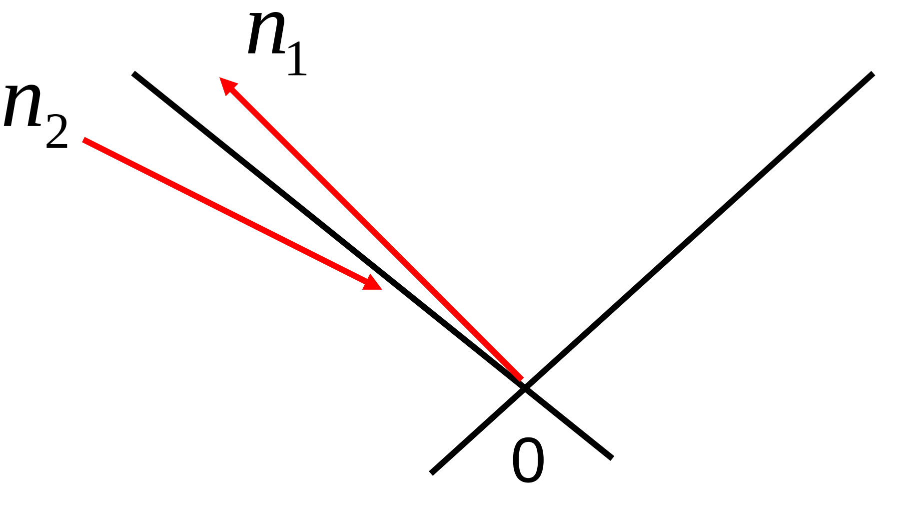
IR cancellations



Our modification (Wang, Li 2014)

- Choose orthogonal gauge vectors for off-light-cone Wilson links
- Same light-cone limit as Collins' definition
- Pinched singularity disappears
- soft subtraction is not needed

$$\frac{n_1 \cdot n_2 = 0}{(n_1 \cdot l + i\varepsilon)(n_2 \cdot l - i\varepsilon)}$$



Check IR behavior

- Take pion transition form factor as an example, whose hard kernel is simple
- Our definition gives the same collinear logarithm, the same as in QCD diagrams
- It realizes k_T factorization at small x

$$\int_{-\infty}^{+\infty} dk'_+ \int_{-\infty}^{+\infty} d^{2-2\epsilon} k'_T \; \phi^{C(1)}(k'_+, k'_T, y_2)\, H^{(0)}(k'_+, k'_T)$$
$$= -\frac{\alpha_s C_F}{4\pi} \left[\ln\left(\frac{k_+}{p_+}\right) + 2 \right] \ln k_T^2 \, H^{(0)}(k_+, k_T) + \cdots ,$$

x

Quasi-parton distributions

Difficulty on lattice (see Hatta's talk)

- Ordinary light-cone PDF

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

- Involve time dependence, not suitable for lattice calculation in Euclidean space
- Reason why only (few) moments (local objects) can be calculated by lattice

Quasi PDF definition

- To facilitate direct lattice calculation of PDF, i.e., all-moment calculation, modify definition in large Pz limit (Ji, 2013)

$$\tilde{q}_n(x,\mu,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\psi(w) W_n^\dagger(w)\gamma^z W_n(0)\psi(0)|P\rangle$$

$$W_n(w) = P\exp\left[-ig\int_0^\infty d\lambda T^a\, n\cdot A^a(\lambda n + w)\right]$$

- Then match Quasi PDF to light-cone PDF

$$\tilde{q}_n(x,\mu,P^z) = \int \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu)$$

lattice data

calculable in perturbation

extracted

Linear divergence

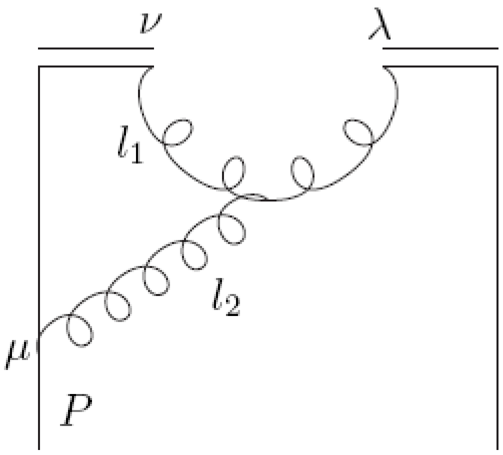
- Quasi PDF and light-cone PDF have the same collinear divergence as expected
- Rotation of Wilson links does not change collinear structure: $1/n\cdot l \approx 1/n^- l^+$
- But linear power divergence exists in quasi PDF from large zero component

$$
\begin{aligned}
\tilde{q}_n^{1c} &= -\frac{1}{2} g^2 C_F \int \frac{d^2 l_T}{(2\pi)^3} \frac{P^z}{l^0 (l^z)^2} \\
&= -\frac{\alpha_s}{2\pi} C_F \frac{\mu}{(1-x)^2 P^z}.
\end{aligned}
$$

Breakdown of factorization

for details, see my talk on July 31

- Linear divergence induces additional collinear divergence at two loops, from the region $l^0 \sim l_T >> l^z$
- This collinear divergence comes from loop momentum perpendicular to z axis
- This region is usually power suppressed, but enhanced by linear divergence



Modified quasi PDF

- Non-dipolar Wilson links also resolve linear divergence

$$\tilde{q}(x,\mu,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\psi(w) W_{n_2}^{\dagger}(w)\gamma^z W_{n_1}(0)\psi(0)|P\rangle$$

$$n_1 = (0,1,0,1) \text{ and } n_2 = (0,-1,0,1)$$

- $\tilde{q}^{1c} = 0$
- Subleading-power divergence is really power suppressed and neglected
- Factorization can be generalized to all orders

Summary

- k_T factorization, more complicated than collinear factorization, has many difficulties
- TMD definition is one of them
- Rotation of Wilson links can be used to define TMD and quasi-PDF,
- Non-dipolar Wilson links remove linear divergences in TMD and quasi-PDF, facilitate proof of factorization theorem
- Should put modified quasi-PDF on lattice

Back-up slides

Eikonal approximation

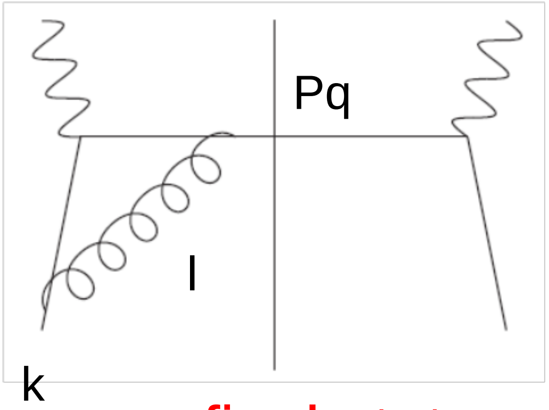
$$\not{P}_q \gamma_\nu \frac{\not{P}_q + \not{l}}{(P_q + l)^2} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^\nu, \quad k \propto k^+, \quad P_q \propto P_q^-$$

$$\approx \not{P}_q \gamma^- \frac{\not{P}_q + \not{l}}{(P_q + l)^2} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^+, \quad l \propto l^+$$

$$\approx \not{P}_q \gamma^- \frac{\not{P}_q}{2P_q \cdot l} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^+$$

$$\approx \not{P}_q \frac{2P_q^- - \not{P}_q \gamma^-}{2P_q \cdot l} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^\nu, \quad \not{P}_q \not{P}_q = P_q^2 = 0$$

$$\approx \not{P}_q \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^\nu \frac{n_{-\nu}}{n_- \cdot l}$$



final-state cut

Feynman rules represented by Wilson lines

NLO diagrams