Search for beyond standard model and QCD

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(RBC&UKQCD)

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Contents/Collaborators

1. muon g-2 HVP & DWF simulations
   RBC/UKQCD, C. Lehner, M. Spraggs, A. Porttelli, ...

2. muon g-2 HLbL
   T. Blum, N. Christ, M. Hayakawa, L. Jin, C. Jung, C. Lehner,

3. Nucleon EDM
   E. Shintani, T. Blum, A. Soni,
   (dim 5 operators) M. Abramczyk , H. Ohki, S. Syritsyn

Part of calculation are done by resources from
USQCD (DOE), XSEDE, ANL BG/Q, Edinburgh BG/Q,
BNL BG/Q, RIKEN BG/Q and Cluster, HOKUSAI

Support from US DOE, RIKEN, BNL, and JSPS

[RBRC, QCDCQ, 2011-]

The RBC & UKQCD collaborations

BNL and RBRC

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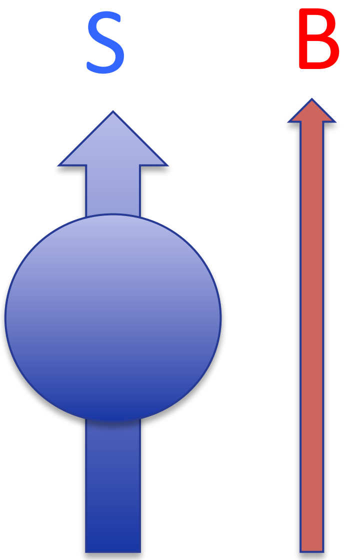
York University (Toronto)

Renwick Hudspith

Anomalous magnetic moment

- Fermion's energy in the external magnetic field:

$$V(x) = -\vec{\mu}_l \cdot \vec{B}$$

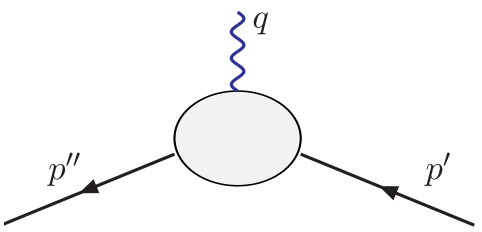


- Magnetic moment and spin g_l : Lande g-factor

  g_l's deviation from tree level value, 2 :

$$\vec{\mu}_l = g_l \frac{e}{2m_l} \vec{S}_l \qquad a_l = \frac{g_l - 2}{2}$$

- Form factor : $\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\, m_l} F_2(q^2)$



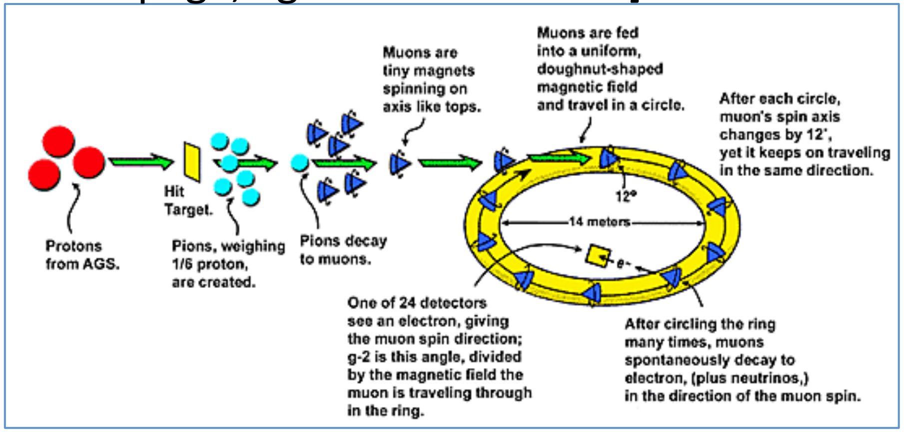
After quantum correction $\Rightarrow a_l = F_2(0)$

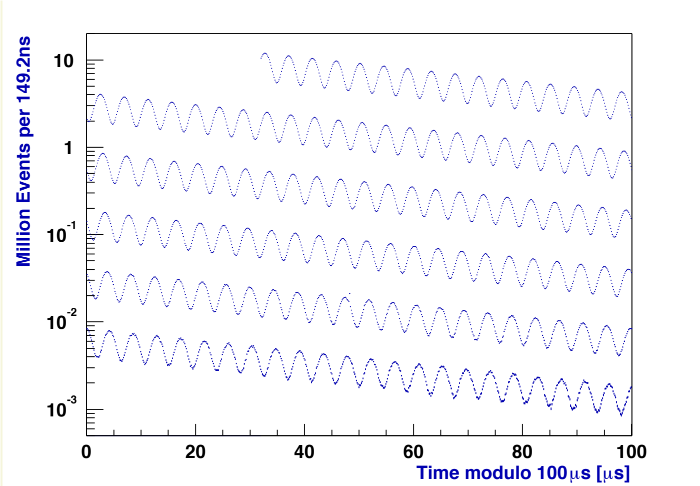
The Muon g-2 experiments
BNL E821 (-2004)

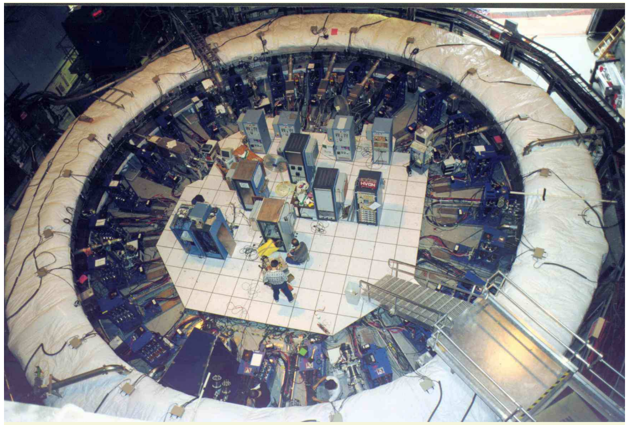
- measure precession of muon spin very accurately

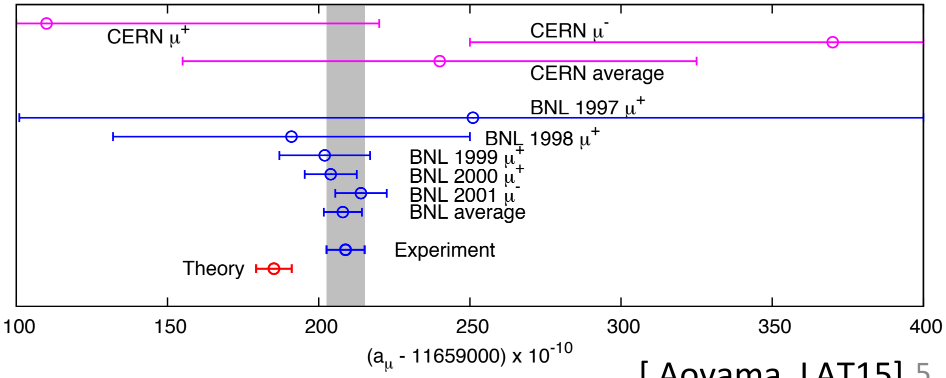
$$N(t) = N_0(E)\ \exp\left(-t/\gamma\tau_\mu\right)\ [1 + A(E)\ \sin(\omega_a t + \phi(E))]$$

[BNL web page,   g-2 collaboration]







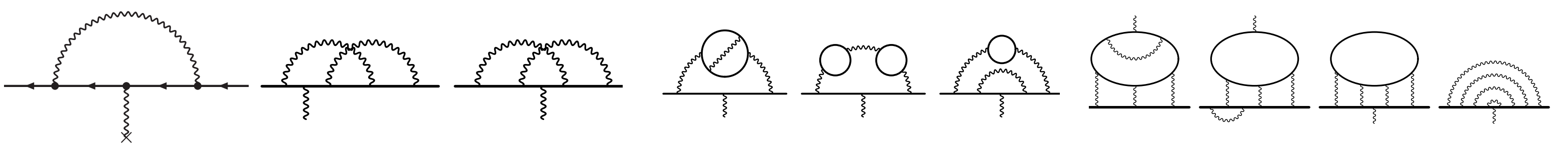


[Aoyama, LAT15]

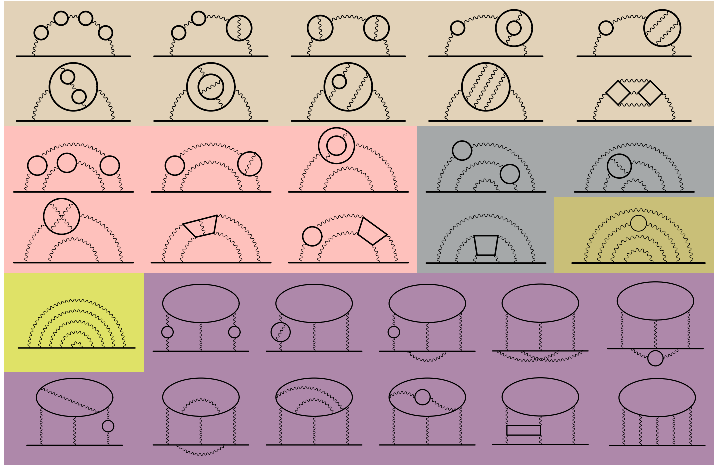
QED calculations

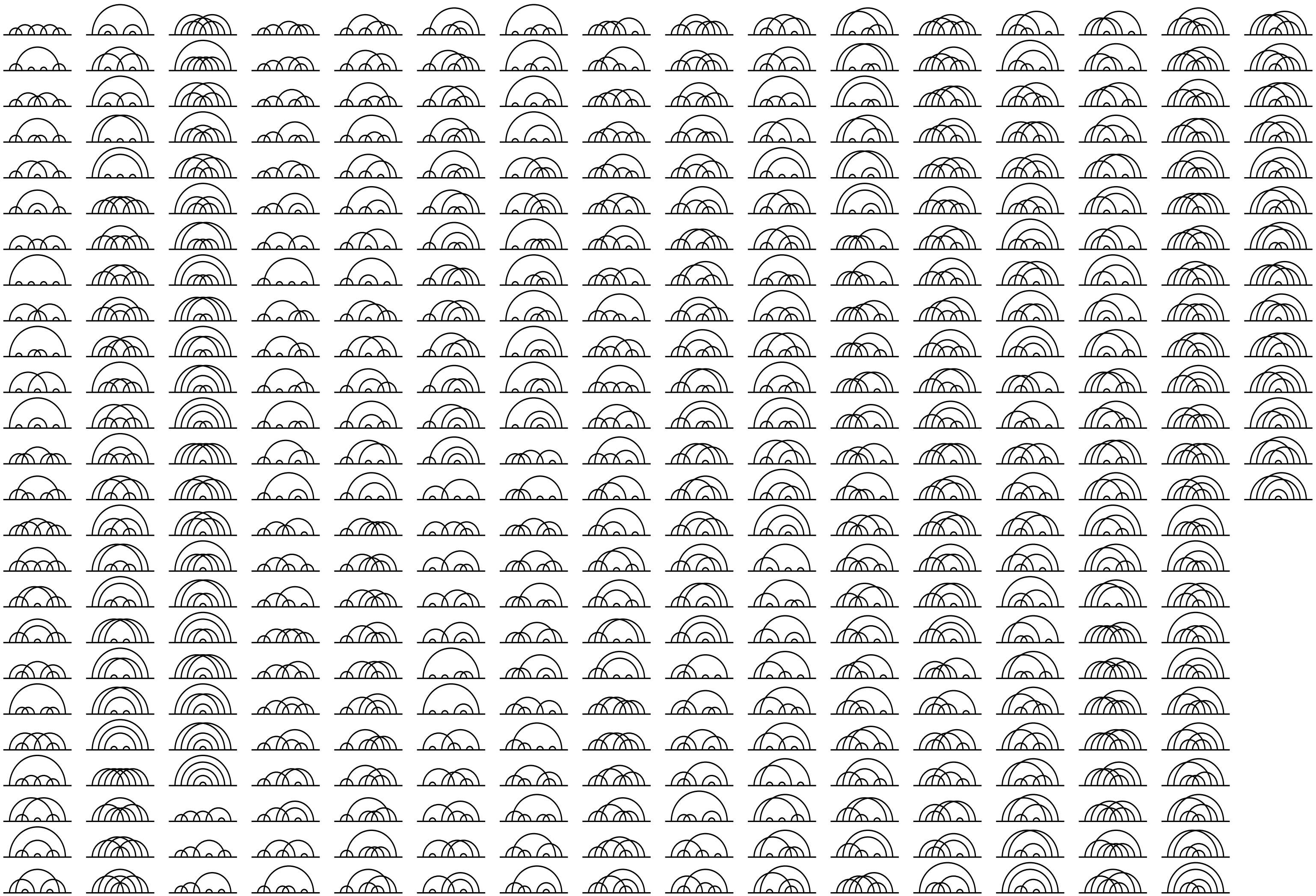
- Fine structure constant α
  Experimental input : anomalous magnetic moment of Electron
  a_e = 0.001 159 652 180 73(28) [0.24 ppb]
  [Hanneke, Fogwell Hoogerheide, Gabrielse, PRA83, 052122 (2011)]

  Theory input: 10th order QED calculation (+ small had+EW)
  [Aoyama, Hayakawa, Kinoshita, Nio Phys. Rev. D 91, 033006 (2015)]
  α^{-1} = 137.035 999 1570 (334) [0.25 ppb]



- 1+7+72+891+12,672
  more than 13,000 diagrams !

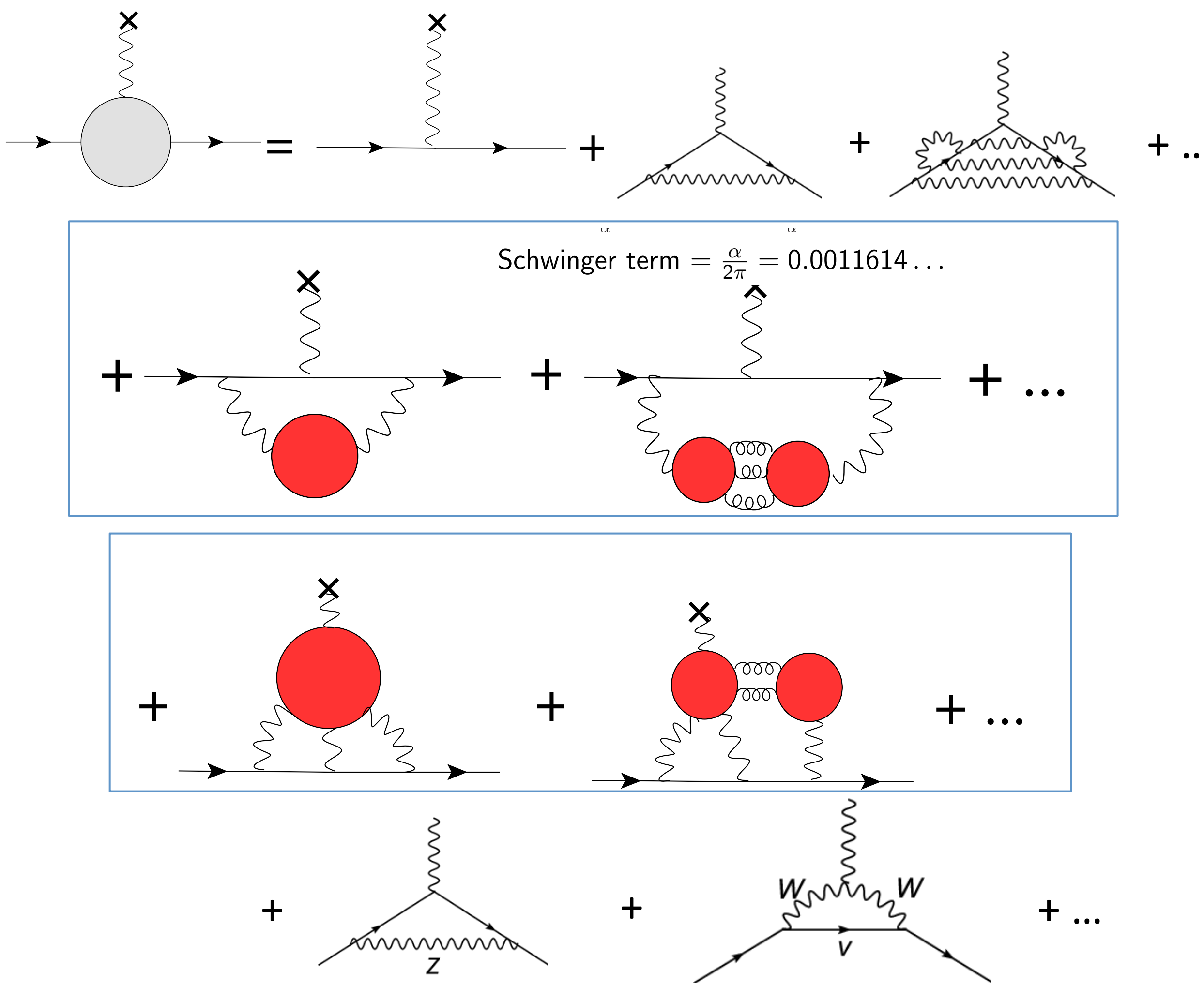




SM Theory

$$\gamma^\mu \rightarrow \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2)\right)$$

- QED, hadronic, EW contributions



QED (5-loop)
Aoyama et al.
PRL109,111808 (2012)

Hadronic vacuum polarization (HVP)

Hadronic light-by-light (Hlbl)

Electroweak (EW)
Knecht et al 02
Czarnecki et al. 02

$(g\text{-}2)_\mu$ SM Theory prediction

- QED, EW, Hadronic contributions

K. Hagiwara et al. , J. Phys. G: Nucl. Part. Phys. 38 (2011) 085003

$$a_\mu^{\rm SM} = (11\ 659\ 182.8 \pm 4.9) \times 10^{-10}$$

$$a_\mu^{\rm QED} = (11\ 658\ 471.808 \pm 0.015) \times 10^{-10}$$
$$a_\mu^{\rm EW} = (15.4 \pm 0.2) \times 10^{-10}$$
$$a_\mu^{\rm had,LOVP} = (694.91 \pm 4.27) \times 10^{-10}$$
$$a_\mu^{\rm had,HOVP} = (-9.84 \pm 0.07) \times 10^{-10}$$
$$a_\mu^{\rm had,lbl} = (10.5 \pm 2.6) \times 10^{-10}$$

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 28.8(6.3)_{\rm exp}(4.9)_{\rm SM} \times 10^{-10} \quad [3.6\sigma]$$

- Discrepancy between EXP and SM is larger than EW!
- Currently the dominant uncertainty comes from HVP, followed by HLbL
- x4 or more accurate experiment FNAL , J-PARC
- Goal : sub 1% accuracy for HVP, and
  → **10% accuracy for HLbL**

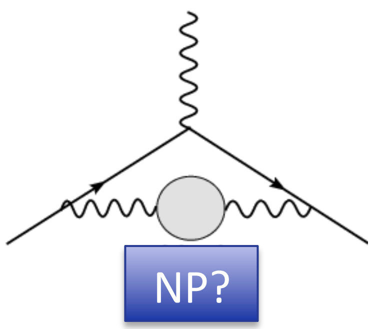
$(g\text{-}2)_\mu$ theory vs experiment

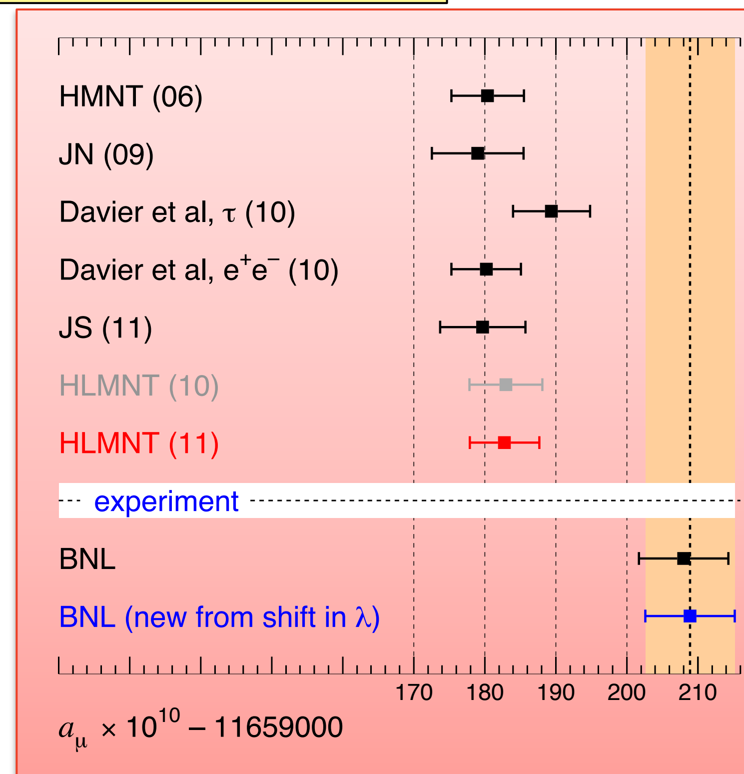
$$a_\mu^{\rm SM} = 116\ 591\ 803(1)_{\rm EW}(42)_{\rm HVP}(26)_{\rm HLbL} \times 10^{-11}$$

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 288(63)_{\rm exp}(49)_{\rm SM} \times 10^{-11} \quad [3.6\sigma]$$

[PDG 2014, Hoecker & Marciano]

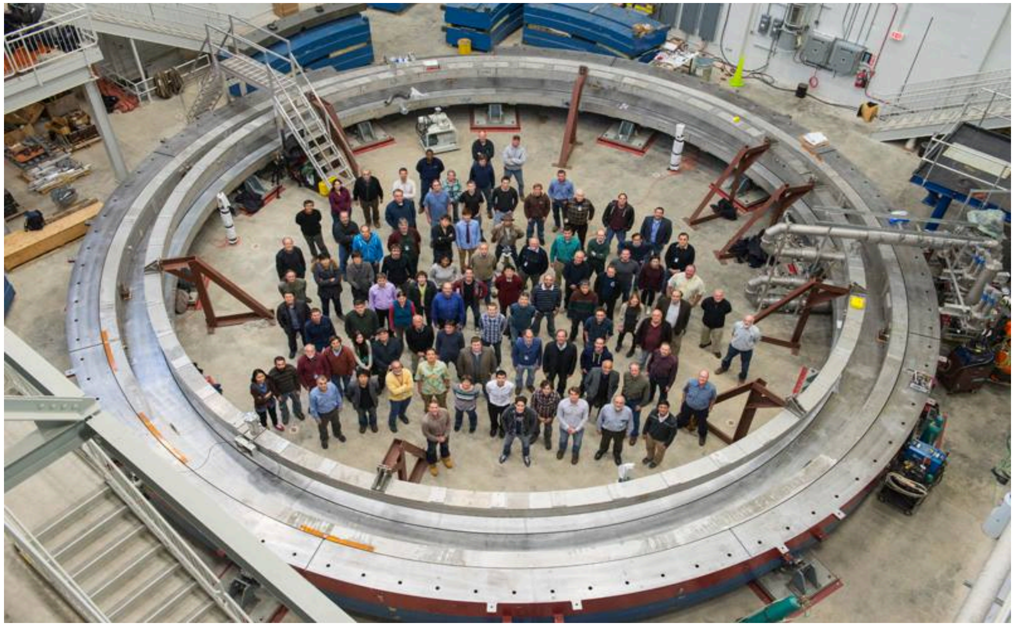
- ~ 3.6 σ discrepancy ?
- SM prediction
- New Physics
- → Hadronic uncertainties ?

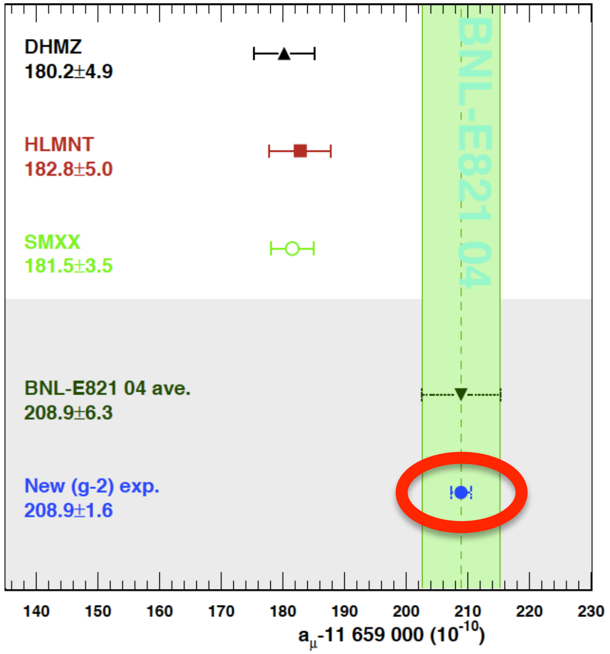


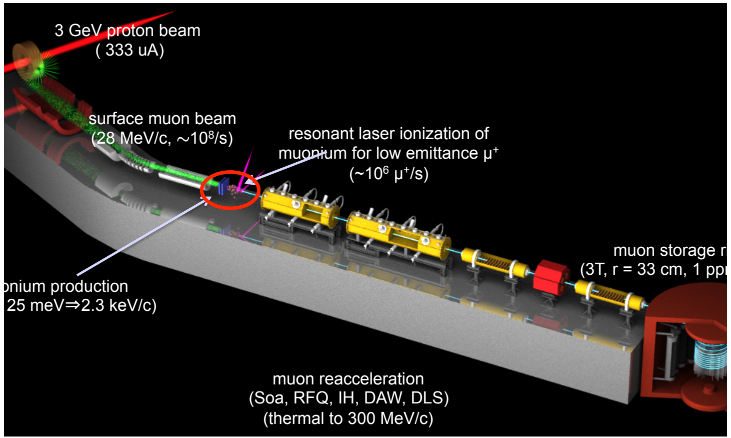


[FNAL, New (g-2) experiment (E989), is scheduled to taking data in 2017, x4 precision]

(near) Future experiments



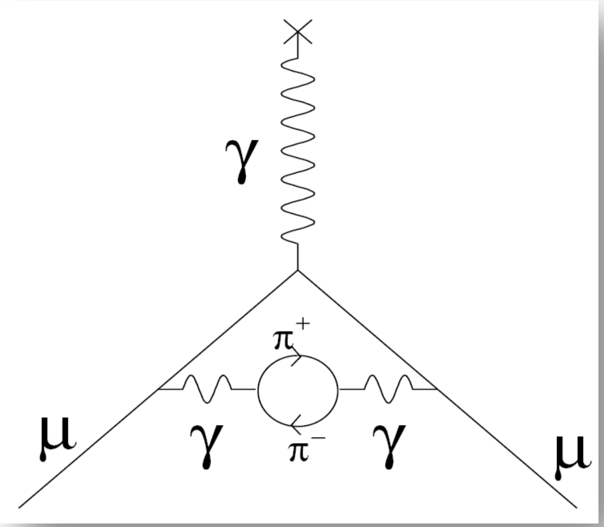




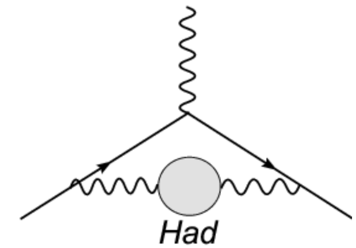
FNAL E989 (2019-)
- move storage ring from BNL
- x4 more precise results, 0.14ppm

J-PARC E34
- ultra-cold muon beam
- table top storage ring

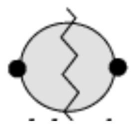
Hadronic Vacuum Polarization (HVP) contribution to g-2



Leading order of hadronic contribution (HVP)



- Hadronic vacuum polarization (HVP)

V_μ  V_ν $= (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi_V(q^2)$

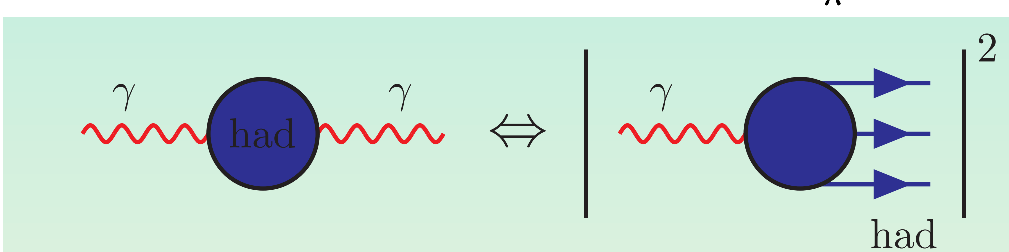
quark's EM current : $V_\mu = \sum_f Q_f \bar{f}\gamma_\mu f$

- Optical Theorem

$$\mathrm{Im}\Pi_V(s) = \frac{s}{4\pi\alpha}\sigma_{\mathrm{tot}}(e^+e^- \to X)$$

- Analycity

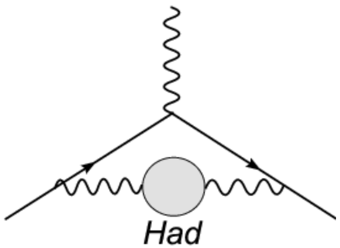
$$\Pi_V(s) - \Pi_V(0) = \frac{k^2}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\Pi_V(s)}{s(s-k^2-i\epsilon)}$$



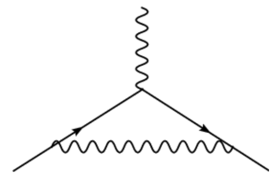
F. Jegerlehner's lecture

Leading order of hadronic contribution (HVP)

- Hadronic vacuum polarization (HVP)



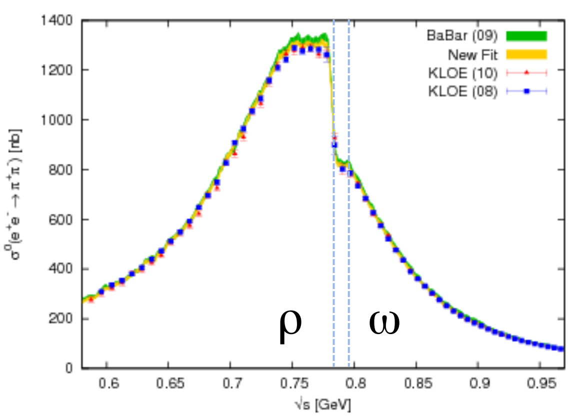
$$a_\mu^{\rm had} = \int ds$$

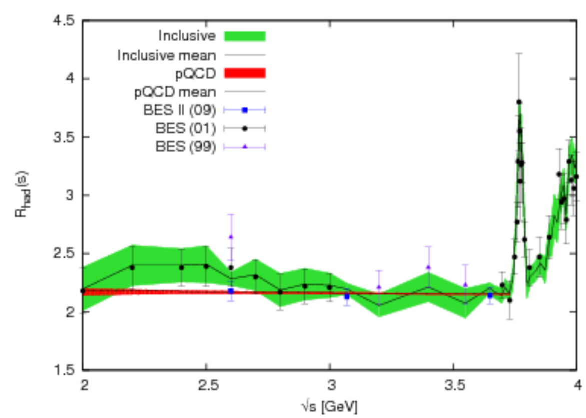


×

$$= \frac{\alpha}{\pi^2}\int_{m_\pi^2}^{\infty}\frac{ds}{s}{\rm Im}\Pi(s)K(s) \qquad K(s) = \int_0^1 dx\frac{x^2(1-x)}{x^2+(s/m_\mu^2)(1-x)}$$

$$= \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2\left[\int_{m_\pi^2}^{s_{\rm cut}} ds\frac{K(s)}{s}R_{\rm had}^{\rm data}(s) + \int_{s_{\rm cut}}^{\infty} ds\frac{K(s)}{s}R_{\rm had}^{\rm pQCD}(s)\right]$$



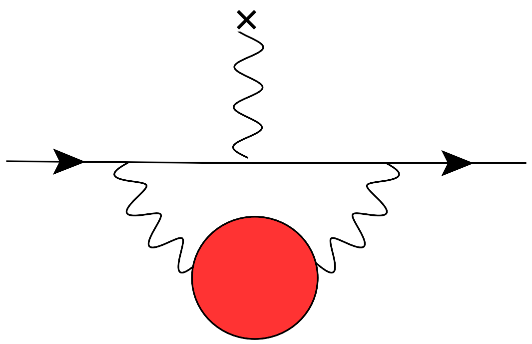


Hagiwara, et al.
J.Phys. G38,085003
(2011)

HVP from experimental data

- From experimental e+ e- total cross section σ_{total}(e+e-) and dispersion relation

$$a_\mu^{\text{HVP}} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds K(s)\sigma_{\text{total}}(s)$$

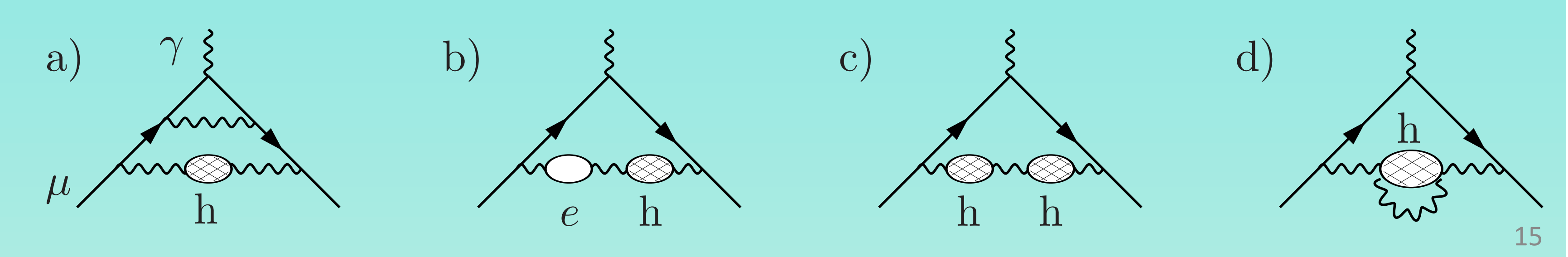


time like $q^2 = s >= 4\, m_\pi^2$

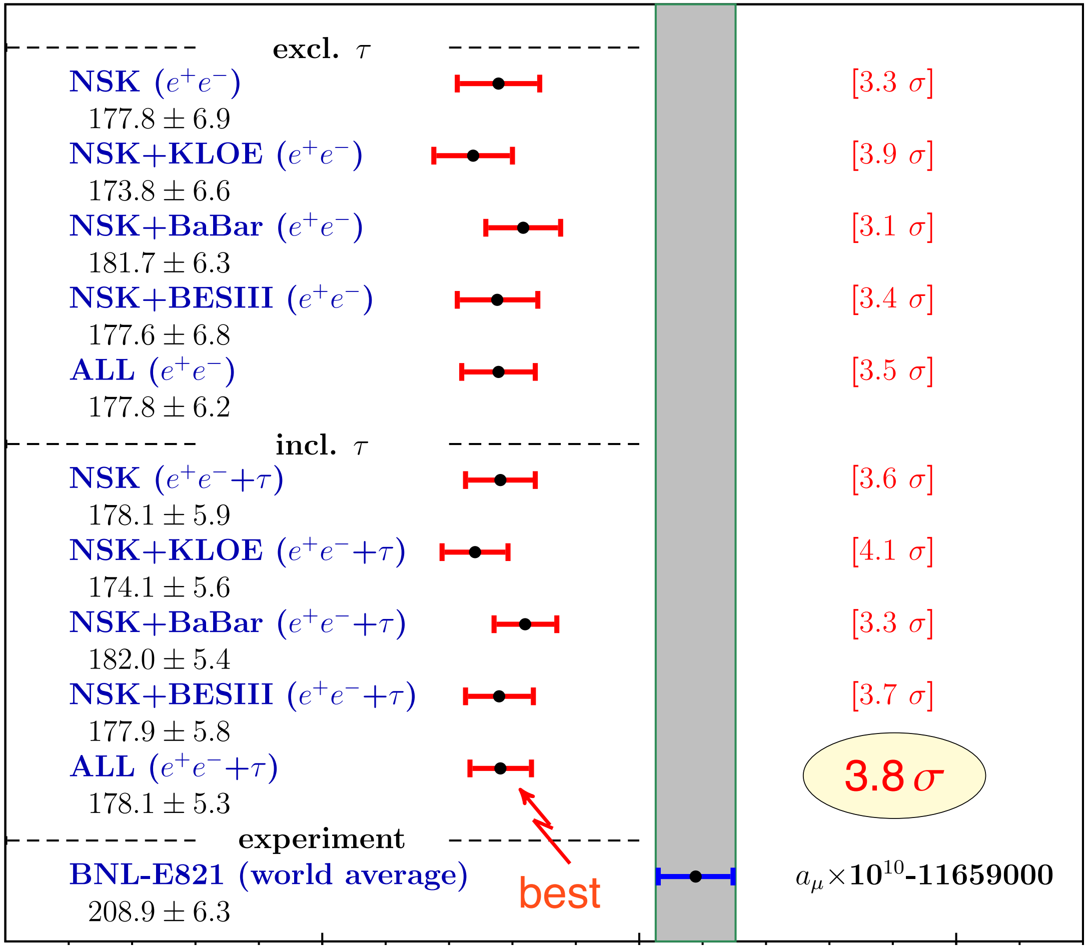
$$a_\mu^{\text{HVP,LO}} = (694.91 \pm 4.27) \times 10^{-10}$$

[~ 0.6 % err]

$$a_\mu^{\text{HVP,HO}} = (-9.84 \pm 0.07) \times 10^{-10}$$



F. Jegerlehner FCCP2015 summary including BES-III



[T. Blum PRL91 (2003) 052001]

HVP from Lattice

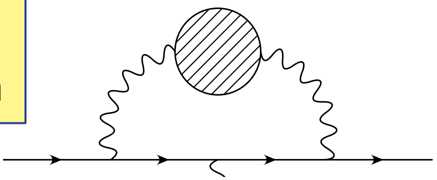
- Analytically continue to Euclidean/space-like momentum $K^2 = -q^2 > 0$
- Vector current 2pt function

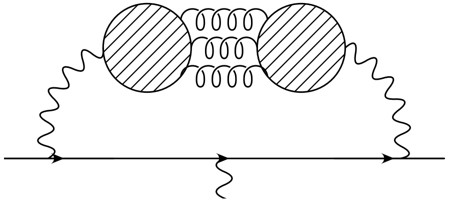
$$a_\mu = \frac{g-2}{2} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2 f(K^2)\hat{\Pi}(K^2)$$

$$\Pi^{\mu\nu}(q) = \int d^4x e^{iqx}\langle J^\mu(x) J^\nu(0)\rangle$$

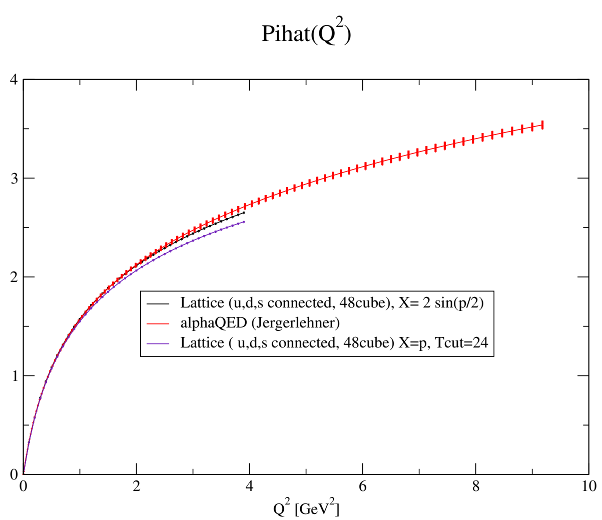
- Low Q2, or long distance, part of Π (Q2) is relevant for g-2

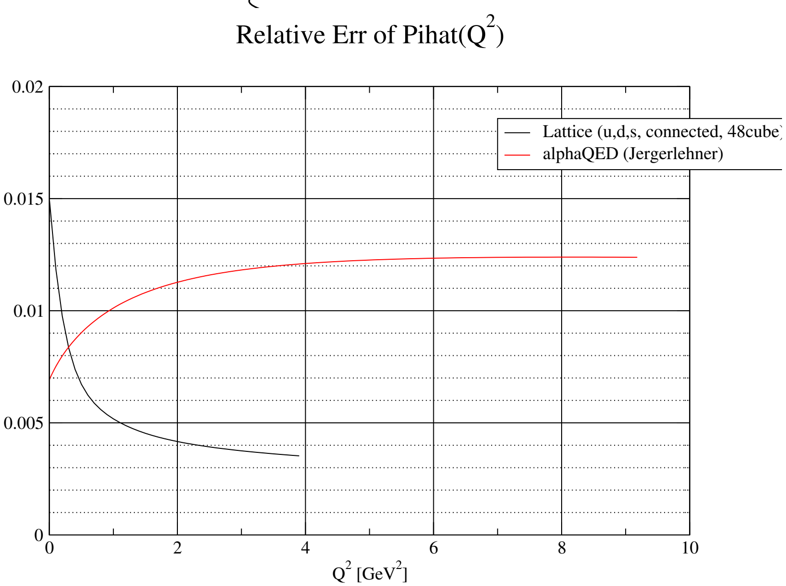
connected contribution





disconnected contribution





Current conservation, subtraction, and coordinate space representation

- Current conservation => transverse tensor

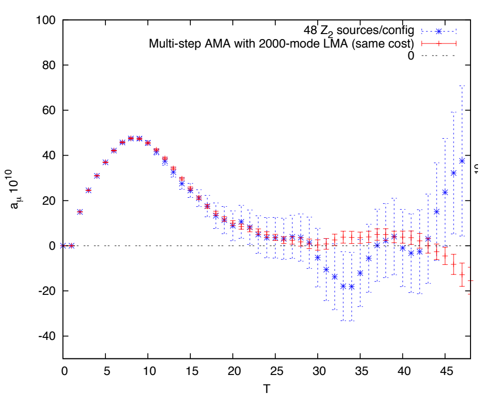
$$\sum_x e^{iQx}\langle J_\mu(x)J_\nu(0)\rangle = (\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu)\Pi(Q^2)$$

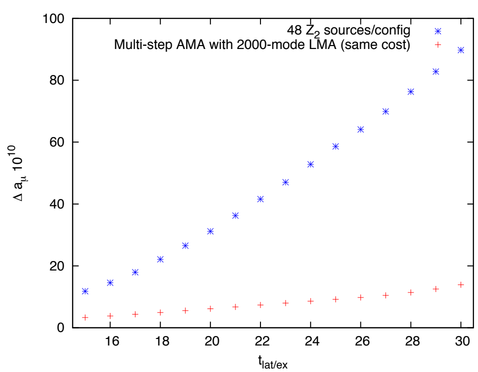
- Coordinate space vector 2 pt Green function C(t) is directly related to subtracted Π (Q2) [Bernecker-Meyer 2011, …]

$$\Pi(Q^2) - \Pi(0) = \sum_t \left(\frac{\cos(qt) - 1}{Q^2} + \frac{t^2}{2} \right) C(t)$$

- g-2 value is also related to C(t) with know kernel w(t) from QED.

$$a_\mu^{\rm HVP} = \sum_t w(t)C(t), \quad w(t) \propto t^4 \cdots$$



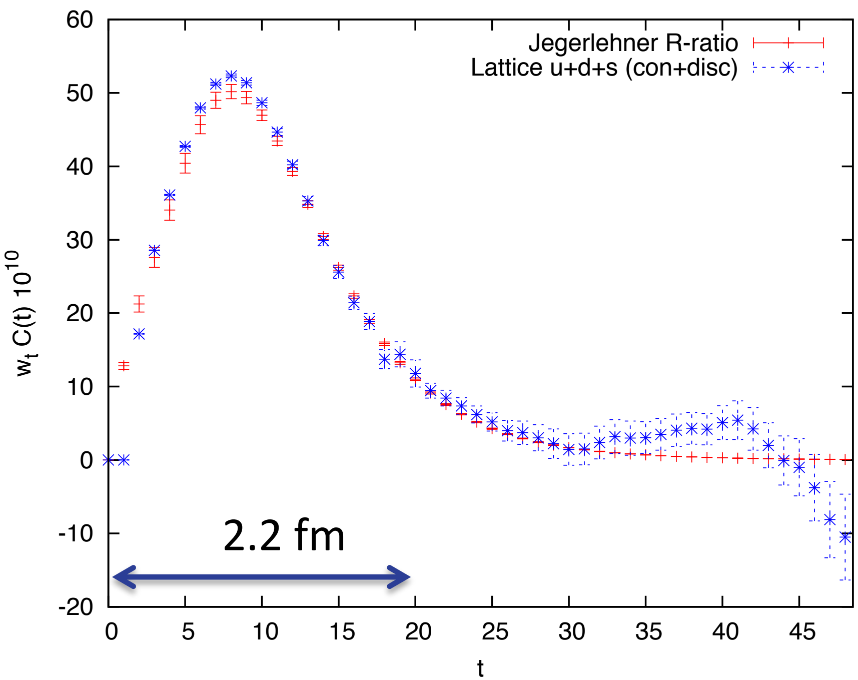


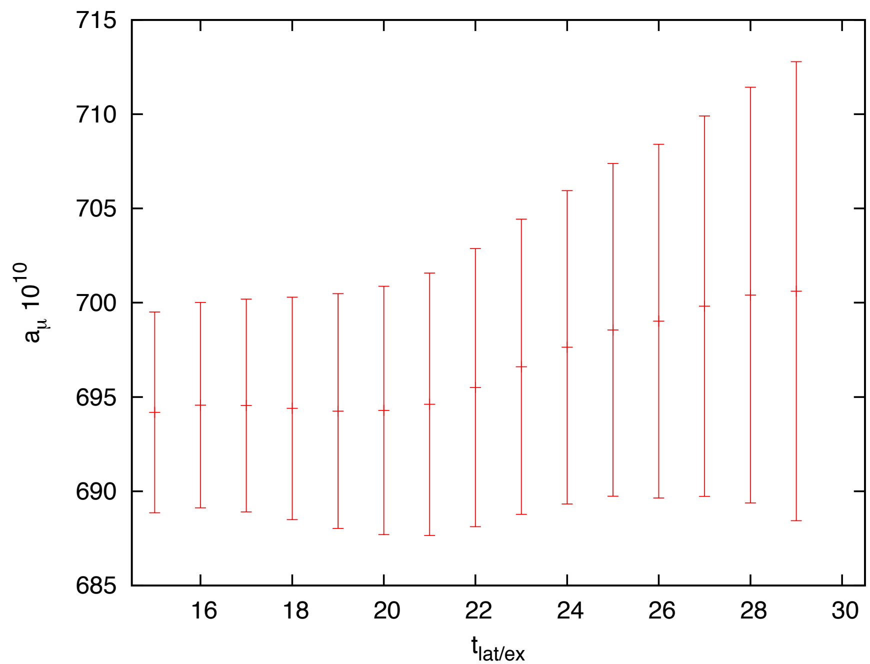
RBC/UKQCD
Chiral Lattice quark DWF
physical point
Quark Propagator Low Mode (A2A)
using All-Mode Averaging (AMA)

(plan B) Interplay between Lattice and Experiment

- Check consistency between Lattice and R-ratio
- Short distance from Lattice, Long distance from R-ratio :
  error <= 1% at $t_{lat/exp}$ = 2fm

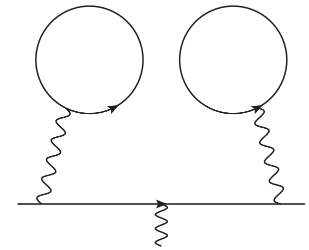
$$a_\mu^{\mathrm{HVP}} = \left[\sum_{t=0}^{t_{\mathrm{lat/exp}}} w(t) C(t)\right]^{\mathrm{LAT}} + \left[\int_{t_{\mathrm{lat/exp}}}^{\infty} dt\, w(t) C(t)\right]^{\mathrm{EXP}}$$





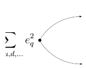
disconnected quark loop contribution

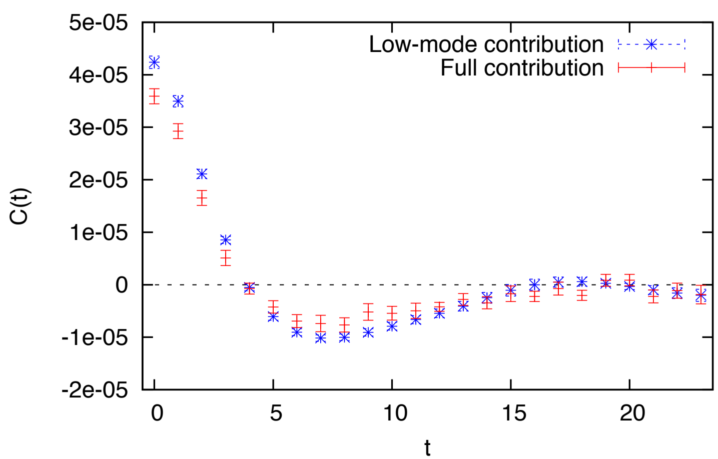
- [C. Lehner et al. (RBC/UKQCD 2015, arXiv:1512.09054, PRL)]
- Very challenging calculation due to statistical noise
- Small contribution, vanishes in SU(3) limit,
  Qu+Qd+Qs = 0

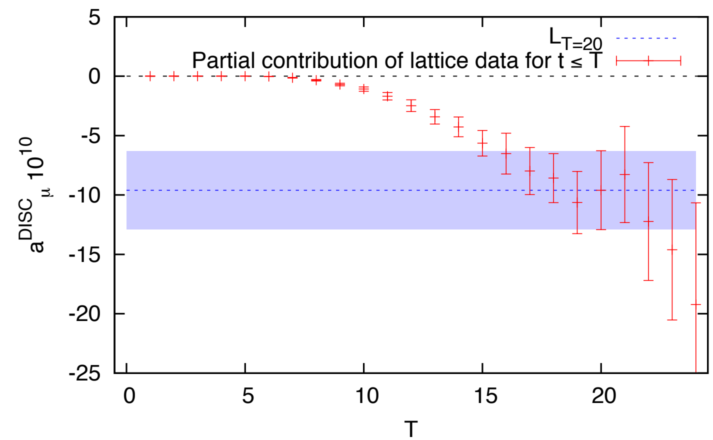


- Use low mode of quark propagator, treat it exactly
  (all-to-all propagator with sparse random source)
- First non-zero signal

$$a_\mu^{\rm HVP\ (LO)\ DISC} = -9.6(3.3)_{\rm stat}(2.3)_{\rm sys} \times 10^{-10}$$







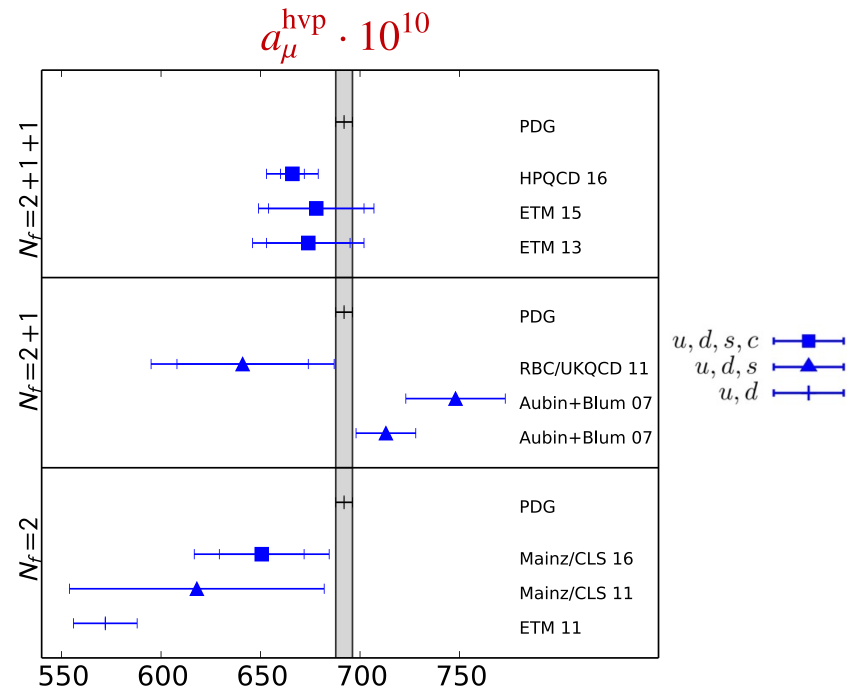
HVP Summary and future prospects

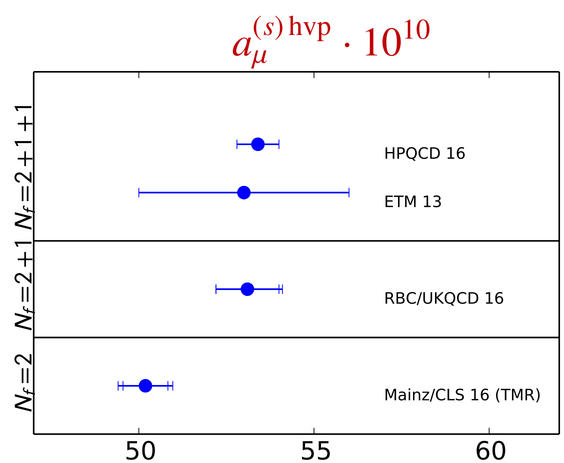
- HVP on Lattice is rapidly progress
- Statistic error is well control
  (low mode, AMA...)
- Disconnected diagram is managed

- Systematic errors

  - *Finite Volume (ππ model ?)*
  - *EM Isospin, ud mass difference*
  - *charm*
  - *discretization error*

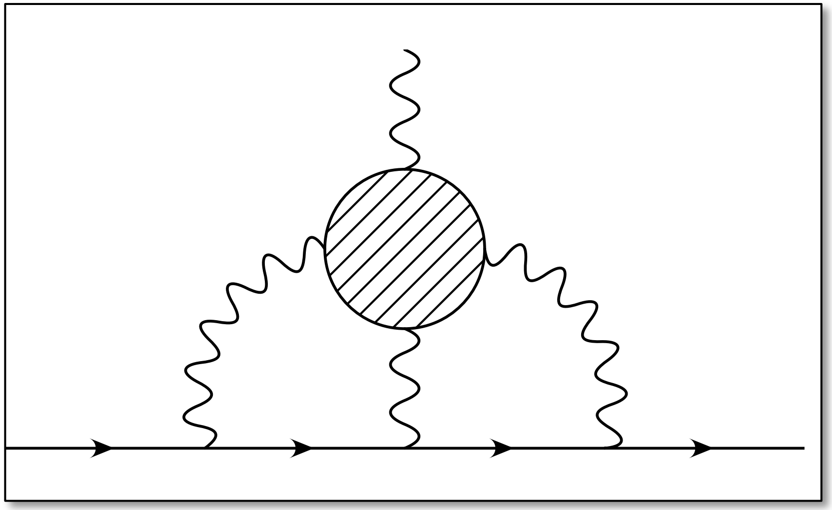
- (Plan-B)
  Interplay between Lattice and R-ratio ?





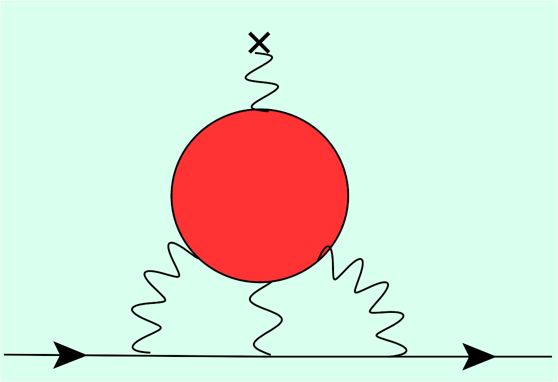
[H. Wittig, LAT16]

Hadronic Light-by-Light (HLbL) contributions



Hadronic Light-by-Light

- 4pt function of EM currents
- No direct experimental data available
- Dispersive approach



$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\,k_2^2\,k_3^2} \times\gamma_\nu S^{(\mu)}(\not p_2+\not k_2)\gamma_\rho S^{(\mu)}(\not p_1+\not k_1)\gamma_\sigma$$

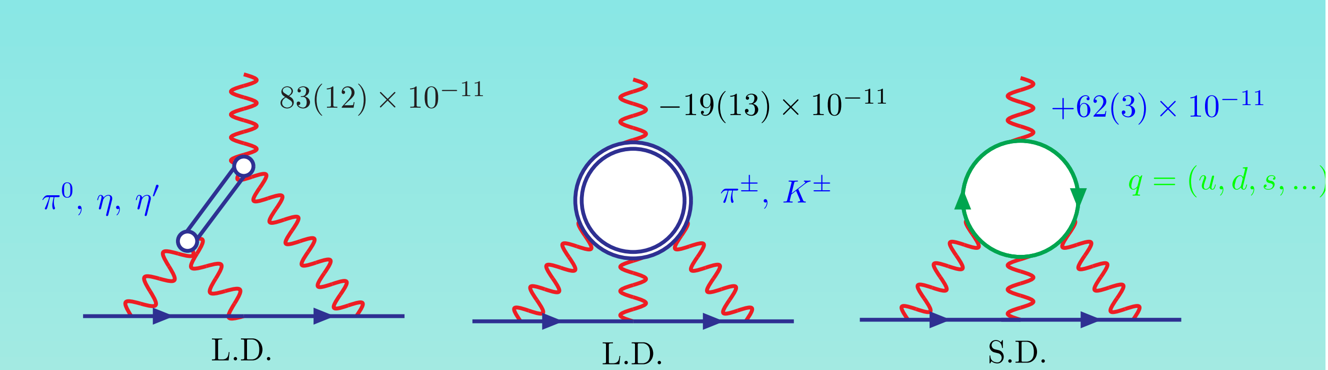
$$\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2) = \int d^4x_1\,d^4x_2\,d^4x_3\,\exp[-i(k_1\cdot x_1+k_2\cdot x_2+k_3\cdot x_3)] \times\langle 0|T[j_\mu(0)j_\nu(x_1)j_\rho(x_2)j_\sigma(x_3)]|0\rangle$$

$$\text{Form factor}: \; \Gamma_\mu(q) = \gamma_\mu\,F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2\,m_l}\,F_2(q^2)$$

HLbL from Models

- Model estimate with non-perturbative constraints at the chiral / low energy limits using anomaly : (9—12) x 10^{-10} with 25-40% uncertainty

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 28.8(6.3)_{\rm exp}(4.9)_{\rm SM} \times 10^{-10} \quad [3.6\sigma]$$



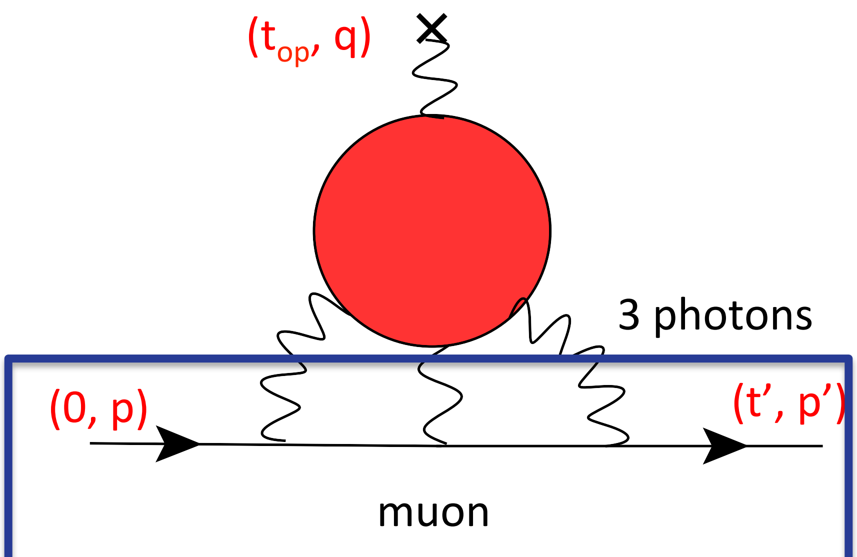
F. Jegerlehner , x 10^{11}

Contribution	BPP	HKS	KN	MV	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	0±10	−19±19	−19±13
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	15±10	22± 5
scalars	−6.8±2.0	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	105±26	116±39

Our Basic strategy : Lattice QCD+QED system

- 4pt function has too much information to parameterize (?)
- Do Monte Carlo integration for QED two-loop with 4 pt function $\pi^{(4)}$ which is sampled in lattice QCD with chiral quark (Domain-Wall fermion)
- Photon & lepton part of diagram is derived either in lattice QED+QCD [Blum et al 2014] (stat noise from QED), or exactly derive for given loop momenta [L. Jin et al 2015] (no noise from QED+lepton).

$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_2,k_3)$$
$$\times\left[S(p_2)\gamma_\nu S(p_2+k_2)\gamma_\rho S(p_1+k_1)\gamma_\sigma S(p_1) + (\mathrm{perm.})\right]$$

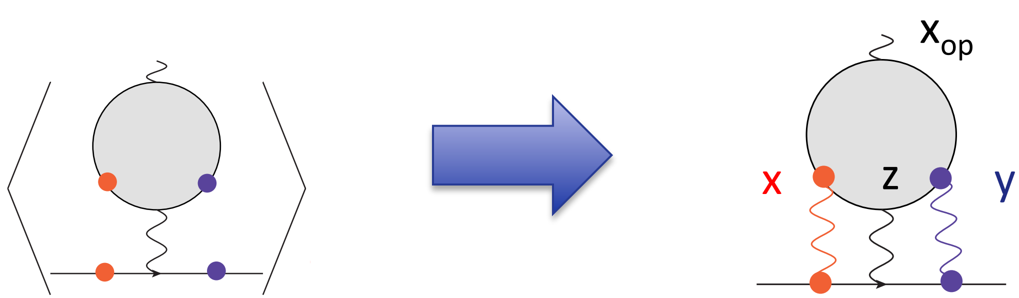


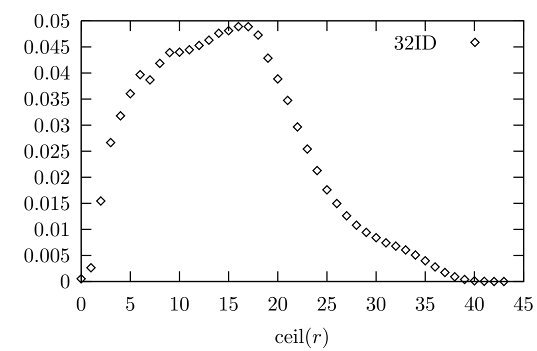
- set spacial momentum for
  - external EM vertex q
  - in- and out- muon p, p'
    q = p-p'
- set time slice of muon source(t=0), sink(t') and operator (t_{op})
- take large time separation for ground state matrix element

Coordinate space Point photon method

[Luchang Jin et al. , PRD93, 014503 (2016)]

- Treat all 3 photon propagators exactly (3 analytical photons) , which makes the quark loop and the lepton line connected :
  disconnected problem in Lattice QED+QCD -> connected problem with analytic photon
- QED 2-loop in coordinate space. Stochastically sample, two of quark-photon vertex location x,y, z and x_{op} is summed over space-time exactly

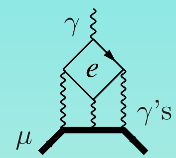




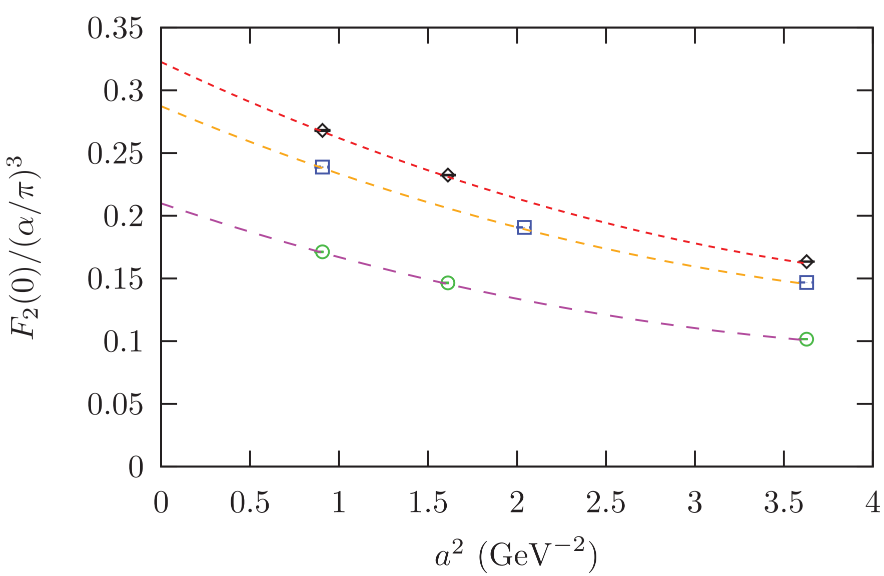
- Short separations, Min[|x-z|,|y-z|,|x-y|] < R ~ O(0.5) fm, which has a large contribution due to confinement, are summed for all pairs
- longer separations, Min[|x-z|,|y-z|,|x-y|] >= R, are done stochastically with a probability shown above (Adaptive Monte Carlo sampling)
- All lepton and photon part produce no noise for given x,y (Ls = ∞ DWF muon)

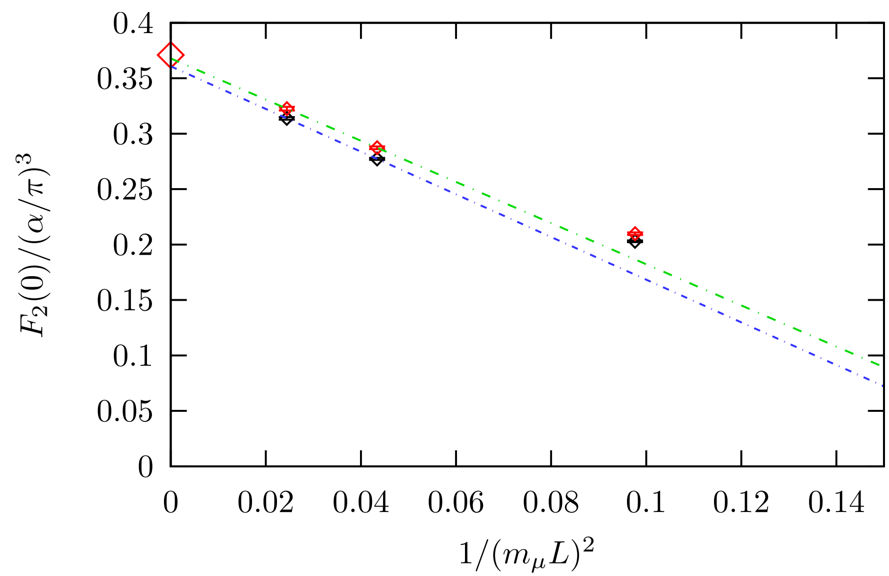
Systematic effects in QED only study

- muon loop, muon line
- a = a m_μ / (106 MeV)
- L= 11.9, 8.9, 5.9 fm
- known result : F2 = 0.371 (diamond) correctly reproduced (good check)



$$a_\mu^{(6)}(\mathrm{lbl}, e) = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \frac{59}{270}\pi^4 - 3\,\zeta(3) - \frac{10}{3}\pi^2 + \frac{2}{3} + O\left(\frac{m_e}{m_\mu}\ln\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^3$$

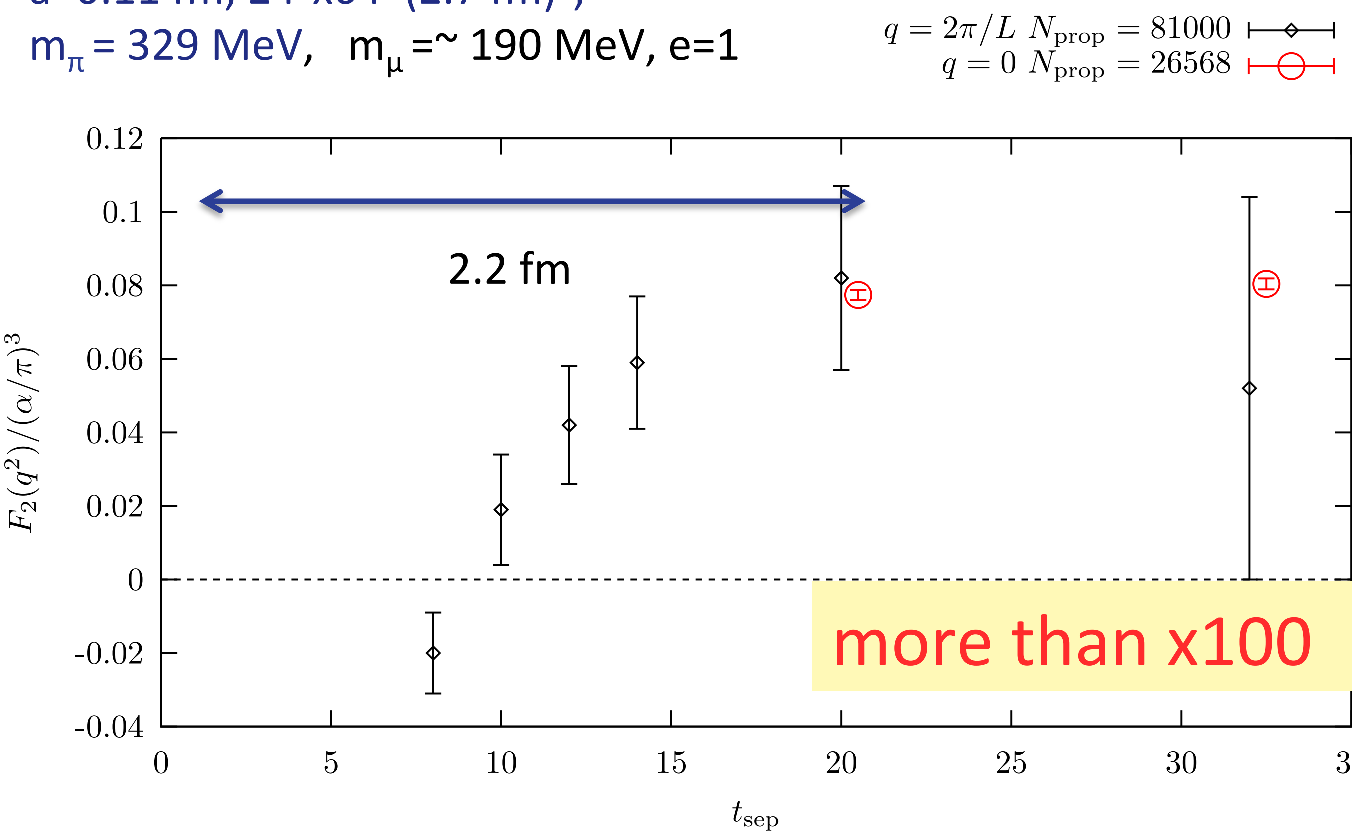


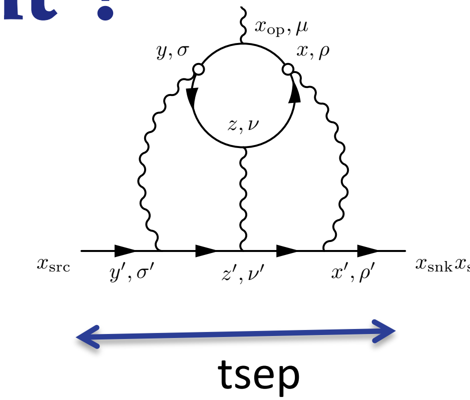


FV and discretization error could be as large as 20-30 %,
similar discretization error seen from QCD+QED study

Dramatic Improvement ! Luchang Jin

a=0.11 fm, 24^3x64 $(2.7\text{ fm})^3$,
m_π = 329 MeV, m_μ =~ 190 MeV, e=1





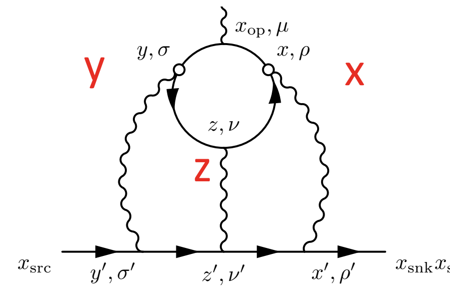
more than x100 reduced cost !

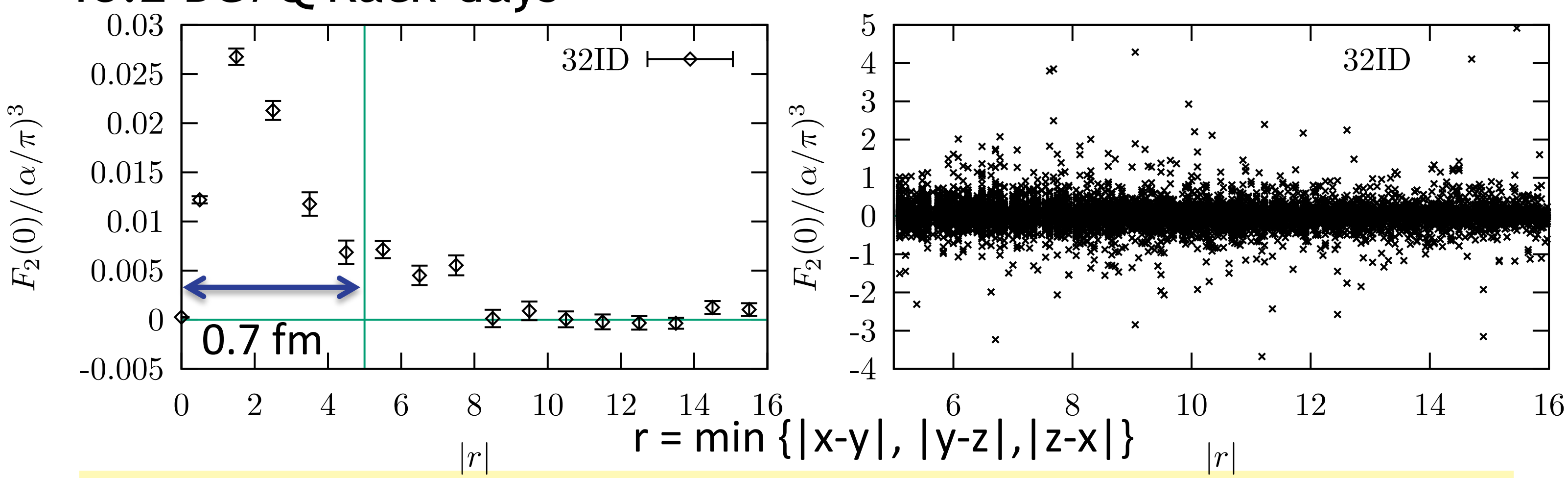
Method	$F_2/(\alpha/\pi)^3$	N_{conf}	N_{prop}	$\sqrt{\text{Var}}$
Conserved	0.0825(32)	12	$(118 + 128) \times 2 \times 7$	0.65
Mom.	0.0804(15)	18	$(118 + 128) \times 2 \times 3$	0.24

M_π=170 MeV cHLbL result

[Luchang Jin et al.,PRD93, 014503 (2016)]

- V=$(4.6\ \text{fm})^3$, a = 0.14 fm, m_μ=130 MeV, 23 conf
- pair-point sampling with AMA (1000 eigV, 100CG) ,
  > 6000 meas/conf
  - |x-y| <= 0.7fm, all pairs, x2-5 samples
    217 pairs (10 AMA-exact)
  - |x-y| > 0.7fm, 512 pairs (48 AMA-exact)
- 13.2 BG/Q Rack-days





$$\frac{(g_\mu - 2)_{\text{cHLbL}}}{2} = (0.1054 \pm 0.0054)(\alpha/\pi)^3 = (132.1 \pm 6.8) \times 10^{-11}.$$

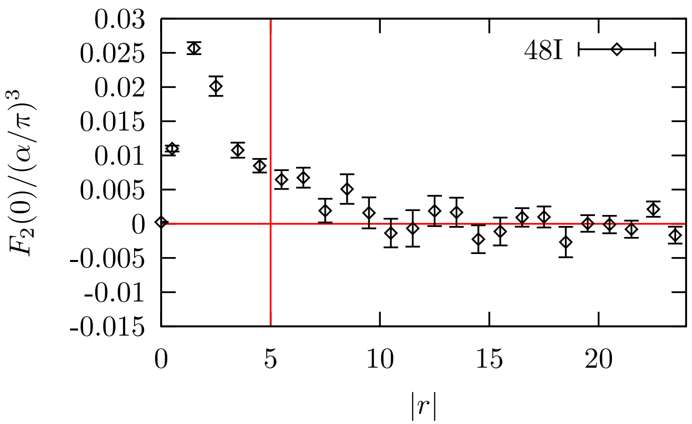
Strange contribution : $(0.0011 \pm 0.005)\ (\alpha/\pi)^3$

physical M_π=140 MeV cHLbL result

[Luchang Jin et al. , preliminary]

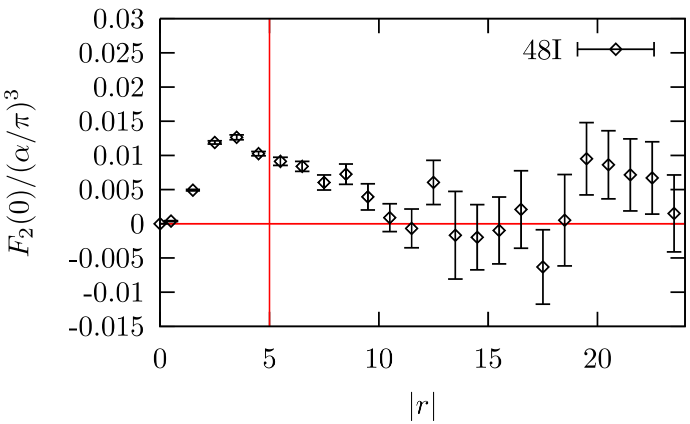
- V=(5.5 fm)^3, a = 0.11 fm, m_μ=106 MeV, 69 conf [RBC/UKQCD]
- Two stage AMA (2,000 eigV, 200CG and 400 CG) using zMobius, ~4500 meas/conf
- 160 BG/Q Rack-days

0.6 fm

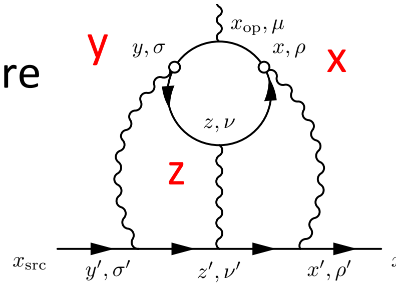


r = min {|x-y|, |y-z|,|z-x|}

integrand safely suppressed before reaching r ~ L/2



r = max{|x-y|, |y-z|,|z-x|}

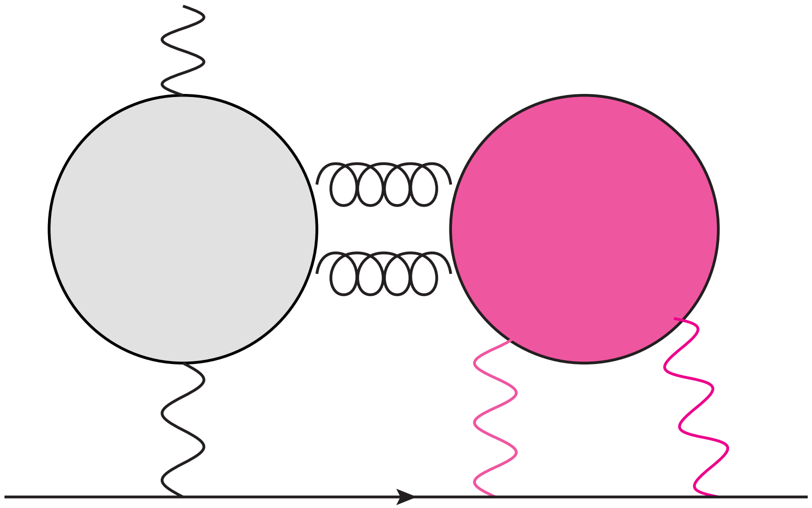


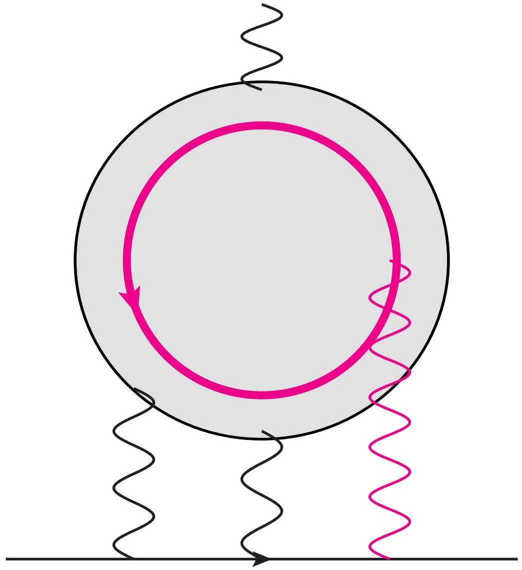
$$\frac{(g_\mu - 2)_{\rm cHLbL}}{2} = (0.933 \pm 0.0073)(\alpha/\pi)^3 = (116.9 \pm 9.1) \times 10^{-11}$$

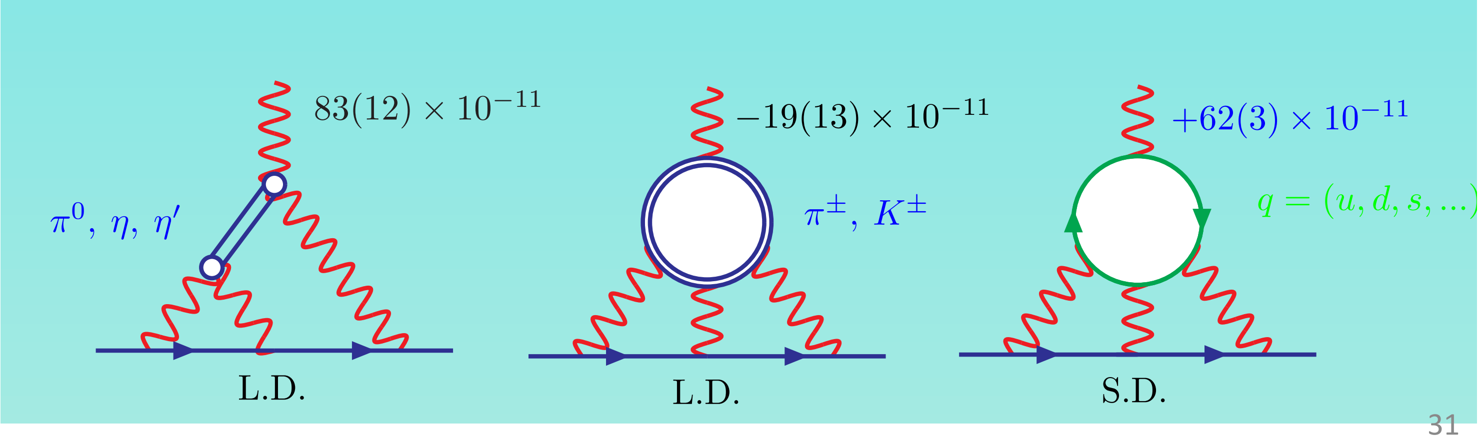
(preliminary, connected, stat err only)

Disconnected diagrams in HLbL

- Disconnected diagrams

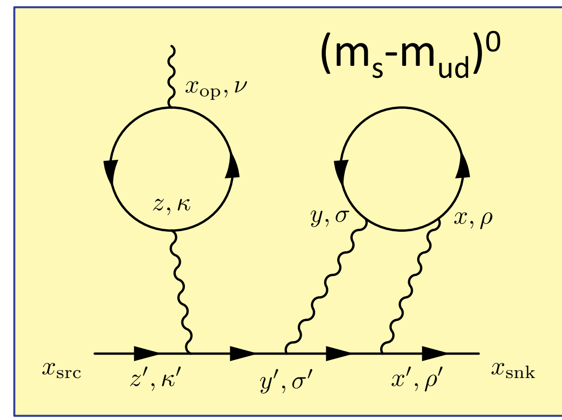




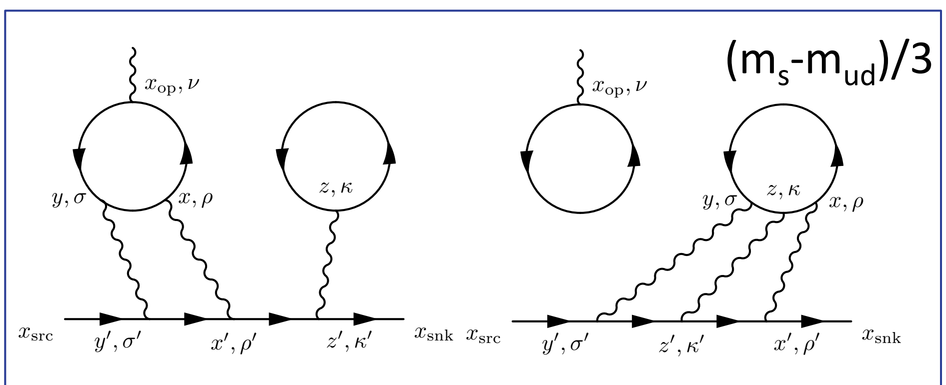


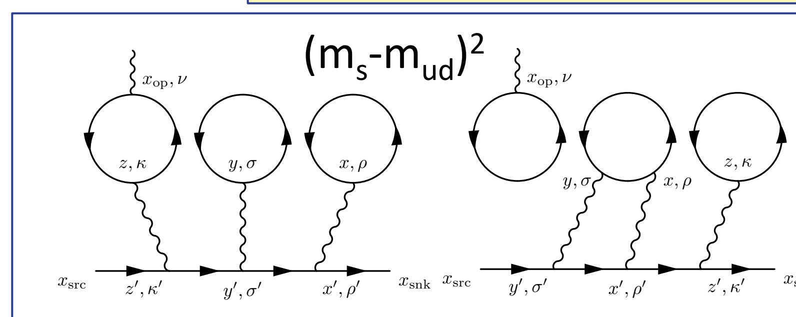
SU(3) hierarchies for d-HLbL

- At $m_s = m_{ud}$ limit, following type of disconnected HLbL diagrams survive $Q_u + Q_d + Q_s = 0$
- Physical point run using similar techniques to c-HLbL.



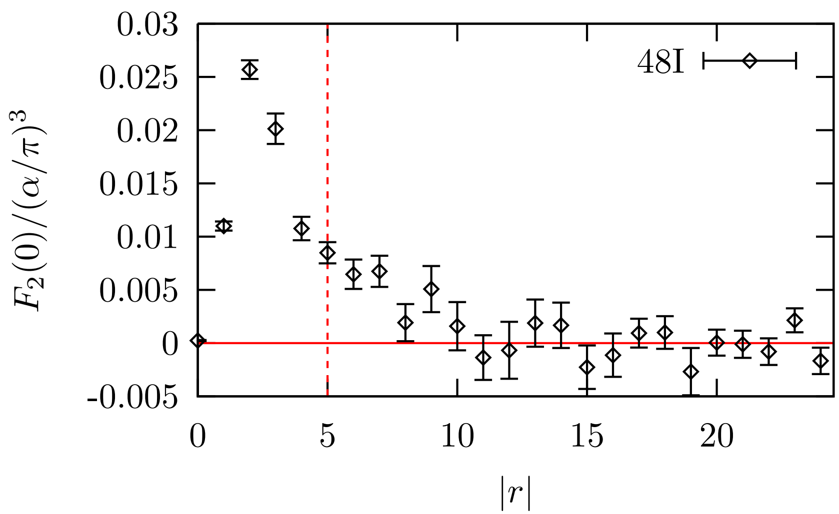
- other diagrams suppressed by
  $O(m_s - m_{ud})/3$ and $O((m_s - m_{ud})^2)$

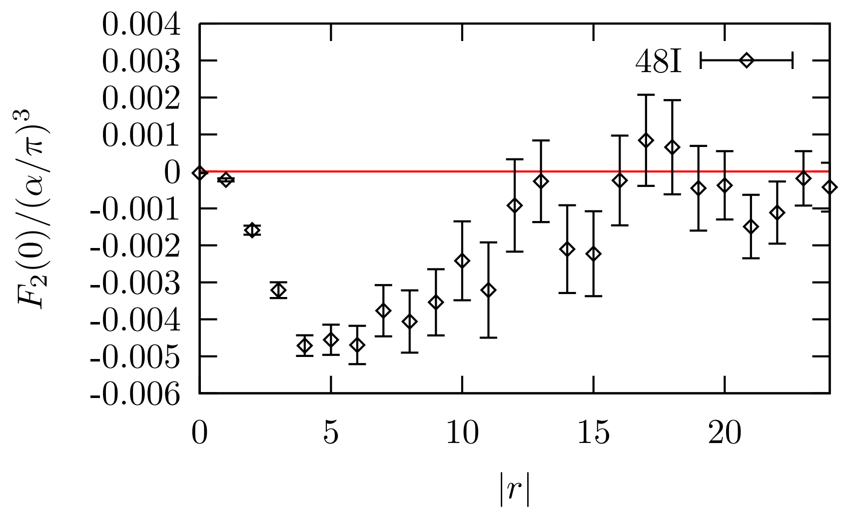




139 MeV Pion, connected and disconnected LbL results (preliminary)

- left: connected, right : leading disconnected





- Using AMA with 2,000 zMobius low modes, AMA

(Preliminary, statistical error only)

$$\left.\frac{g_\mu - 2}{2}\right|_{\mathrm{cHLbL}} = (0.0926 \pm 0.0077) \times \left(\frac{\alpha}{\pi}\right)^3 = (11.60 \pm 0.96) \times 10^{-10}$$

$$\left.\frac{g_\mu - 2}{2}\right|_{\mathrm{dHLbL}} = (-0.0498 \pm 0.0064) \times \left(\frac{\alpha}{\pi}\right)^3 = (-6.25 \pm 0.80) \times 10^{-10}$$

$$\left.\frac{g_\mu - 2}{2}\right|_{\mathrm{HLbL}} = (0.0427 \pm 0.0108) \times \left(\frac{\alpha}{\pi}\right)^3 = (5.35 \pm 1.35) \times 10^{-10}$$

g-2 Summary

- Lattice calculation for g-2 calculation is improved very rapidly
- HLbL including leading disconnected diagrams :
  Many orders of magnitudes improvements
  -> 8 % stat error in connected, 13 % stat error in leading disconnected
  - coordinate-space integral using analytic photon propagator with adaptive probability (point photon method)
  - config-by-config conserved external current
  - take moment of relative coordinate to directly take q→0
  - AMA, zMobius, 2000 low modes

(preliminary, con+L-discon, stat err only)

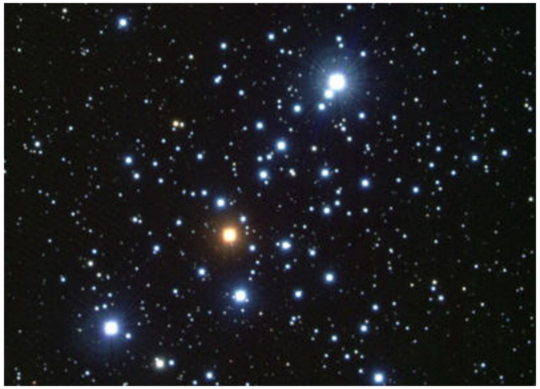
$$a_\mu^{\text{LbL, con}} = (11.60 \pm 0.96) \times 10^{-10}, \quad a_\mu^{\text{LbL, L-dcon}} = (-6.25 \pm 0.80) \times 10^{-10}$$

$$a_\mu^{\text{LbL, c+Ld}} = (0.0427 \pm 0.0108) \times \left(\frac{\alpha}{\pi}\right)^3 = (5.35 \pm 1.35) \times 10^{-10}$$

- Still large systematic errors (missing disconnected, FV, discr. error, ...)
- Also direct 4pt method [Mainz group] and Dispersive analysis [Colangelo et al. 2014, 2015, Pauk&Vanderhaeghen 2014]
- Goal : HVP sub 1%, HLbL 10% error

Nucleon Eclectic dipole Moments

[R. Picker's talk]



Violation of C (charge conjugation symmetry) and **CP** (parity and C)

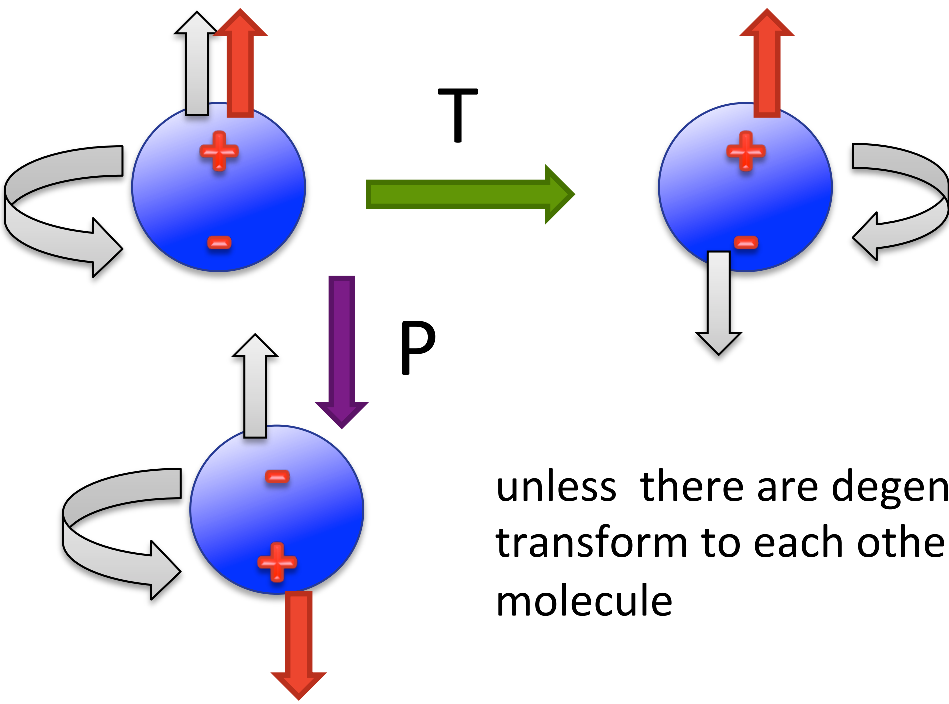
P & CP violation and Electric Dipole Moments (EDM)

- Electric Dipole Moment d

  energy shifts in an electric field E

$$\Delta H = \vec{d} \bullet \vec{E}$$

- A nonzero EDM is a signature of P and T (CP through CPT) violation



exp: $\Delta H \sim 10^{-6}$ Hz $\sim 10^{-21}$ eV
→ $|d| < \Delta H/E \sim 10^{-25}$ e cm

if theo: $d \sim 10^{-2} \times 1$ MeV / Λ^2_{CP}
→ Λ_{CP} >~ O(1) TeV

unless there are degenerate ground states transform to each other by Parity c.f. Water molecule

Sources of CP violation

- θ term in the QCD Lagrangian:

$$\mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M$$

renormalizable and CP-violation comes due to topological charge density.

- also higher dimension CP violating operators

$$i d_q \bar{q}\sigma_{\mu\nu}\gamma_5 F_{\mu\nu} q \qquad i\tilde{d}_q \bar{q}\sigma_{\mu\nu}\gamma_5 G_{\mu\nu} q$$

- EDM experiment provides very strong constraint on
  ⇒ θ and $\arg\det$ M need to be unnaturally canceled !
  strong CP problem

$$|d_N^{\exp}| < 2.9 \times 10^{-26}\,\mathrm{e\cdot cm} \qquad \bar{\theta} < 10^{-9}$$

- up quark mass less ? Axion ? (θ term case only) ?

CP violation on lattice : Reweighting

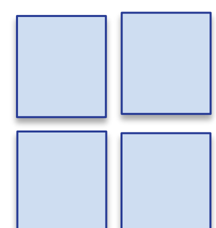
- Source of CP violation (Θ in our case)

$$S_\theta = i\frac{\theta}{32\pi^2}\int d^4x\ \mathrm{tr}[\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}G^{\mu\nu}]$$

$$= i\theta\ Q_{\mathrm{top}}$$

- Topological charge is measured either by gluonic observable GG^* or by counting zero mode of chiral fermions

$Q \rightarrow \Sigma\ G_{12}G_{34},\quad G_{\mu\nu} = \mathrm{Im}$



- $\Theta=0$ lattice QCD ensemble is generated, then each sample of QCD vacuum are reweighted using topological charge

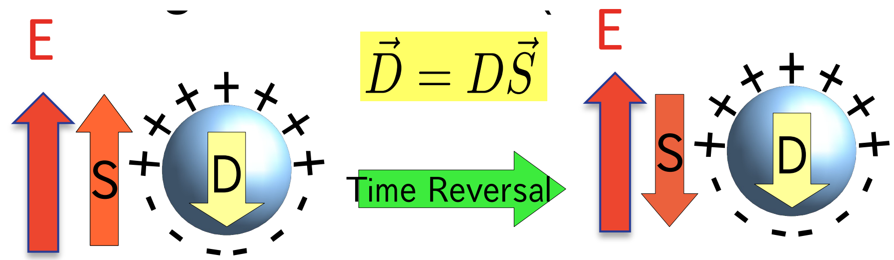
$$\langle\mathcal{O}\rangle_\theta = \langle\mathcal{O}e^{i\theta Q}\rangle_{\theta=0}$$

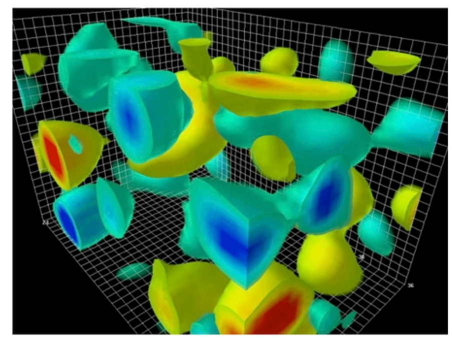
EDM Computations on Lattice

- Measure energies with external Electric field

$$\frac{\langle N_\uparrow(t)\bar{N}_\uparrow(t_0)\rangle}{\langle N_\downarrow(t)\bar{N}_\downarrow(t_0)\rangle} \to Ce^{\Delta M t}$$

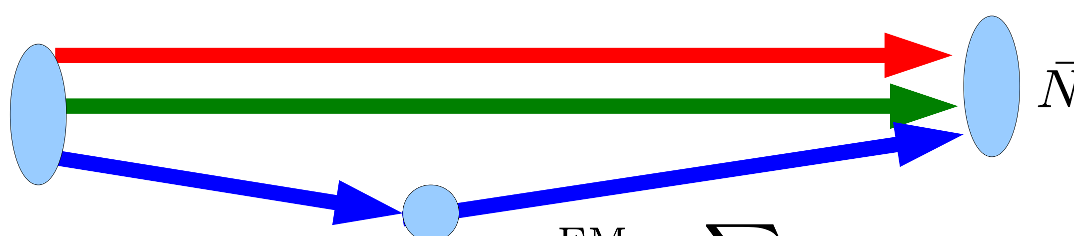
$$\begin{aligned}\Delta M &= M_N(E,\uparrow) - M_N(E,\downarrow)\\ &= -2D_N(\theta) S\cdot E\end{aligned}$$





- Form factors

$$\langle N(p')|V_\mu^{\mathrm{EM}}(q)|N(p)\rangle = F_1(q^2)\gamma_\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} + F_3(q^2)\frac{\sigma^{\mu\nu}q^\nu\gamma^5}{2m_N}$$

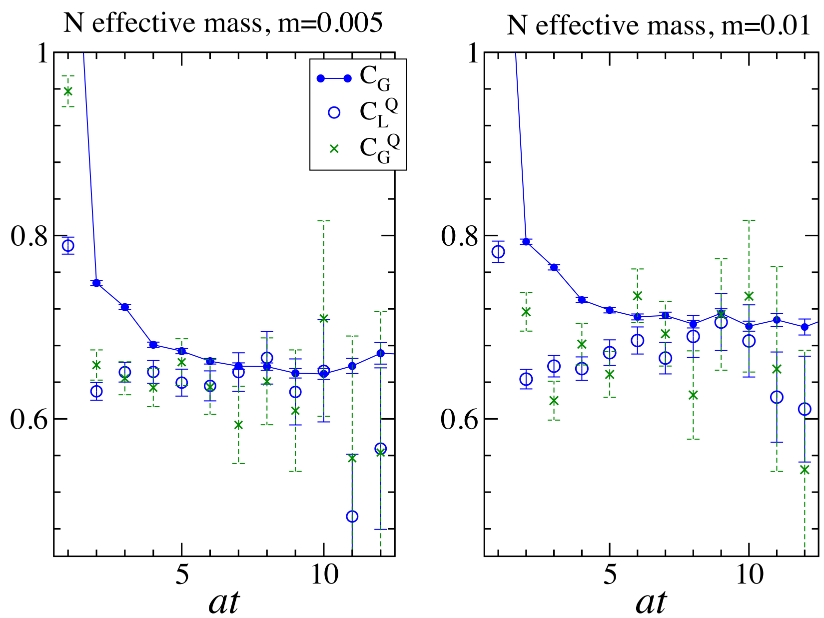


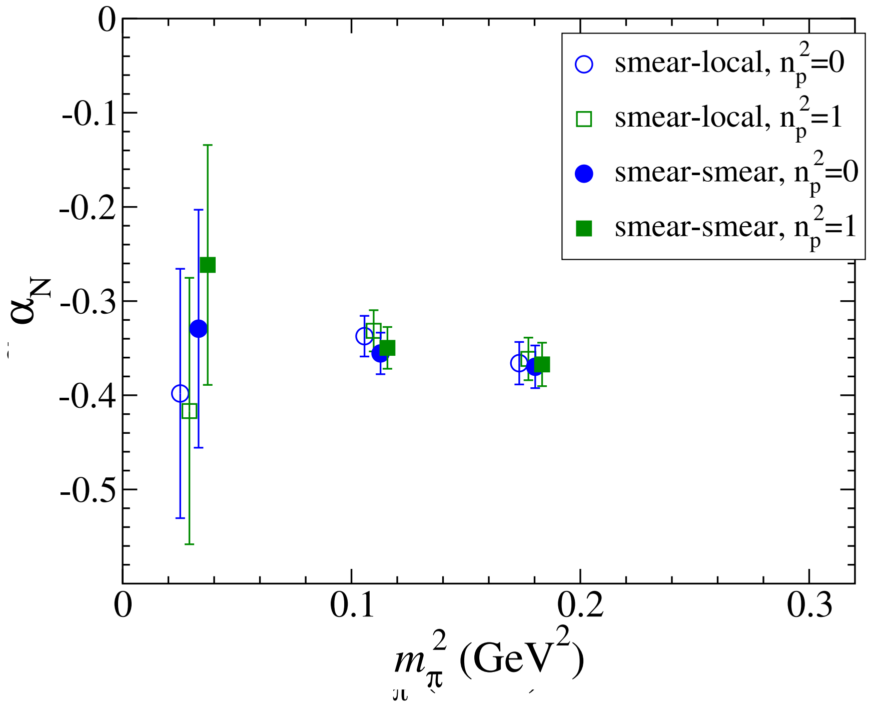
$$V_\mu^{\mathrm{EM}} = \sum_q e_q\bar{q}\gamma_\mu q$$

$$d_N = \lim_{Q^2\to 0} F_3(Q^2)/2m_N$$

CP even and odd 2point functions

effective nucleon mass





- need to take into account CP even/odd mixing for Nucleon spinor, 2pt functions
- CP odd/even has a common mass

(Pospelov, Ritz 1998, S. Aoki *et al.* 2005)

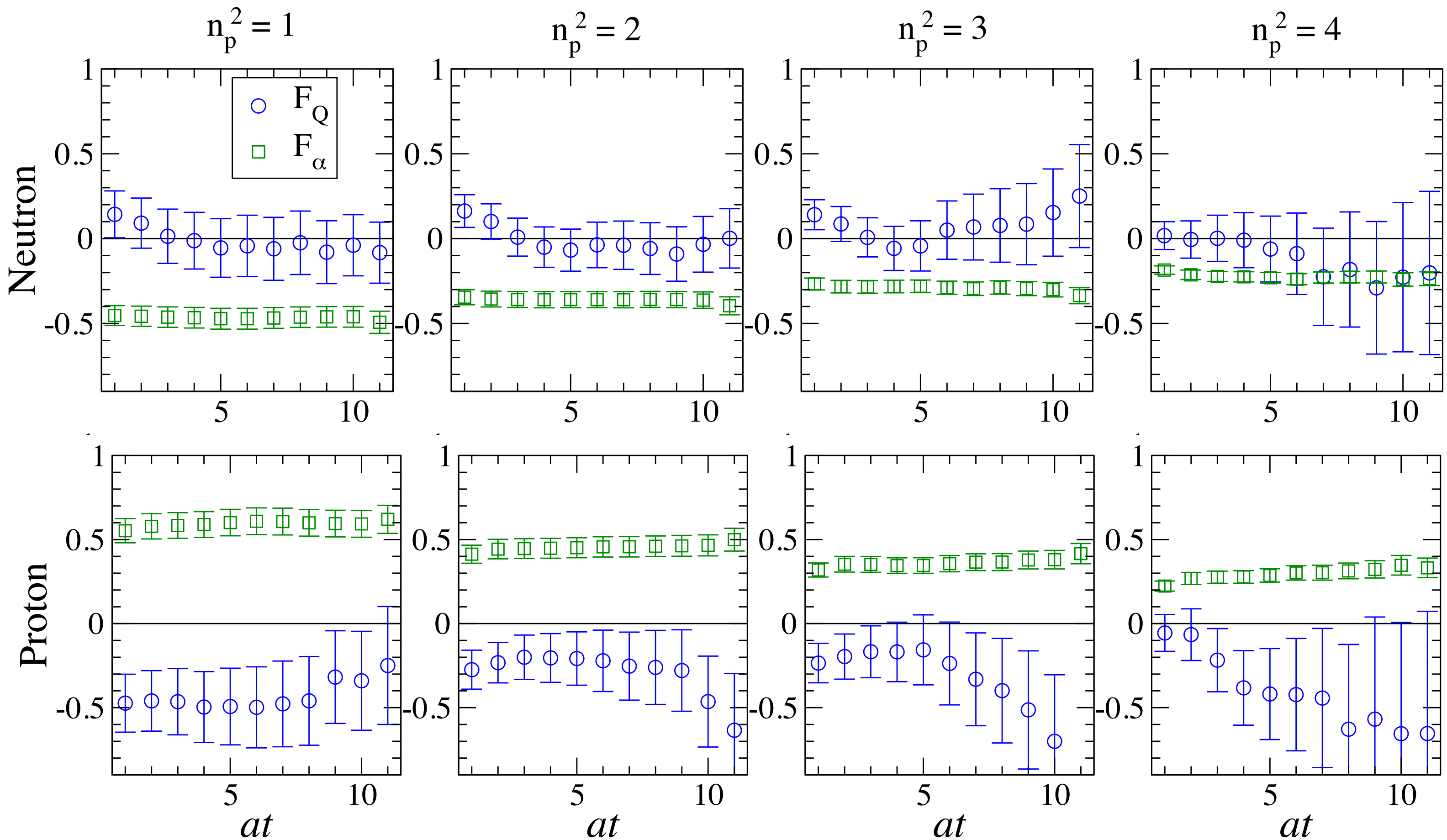
$$\sum_{s,s'} u_{s',\theta}(\vec{p})\bar{u}_{s,\theta}(\vec{p}) = E(\vec{p})\gamma_t - i\vec{\gamma}\cdot\vec{p} + me^{2i\alpha\gamma_5},$$

$$\approx E(\vec{p})\gamma_t - i\vec{\gamma}\cdot\vec{p} + m(1+2i\alpha\gamma_5)$$

where $u_\theta = \exp i\alpha\gamma_5 u$.

F3 unsubtracted @ Mpi=300 MeV

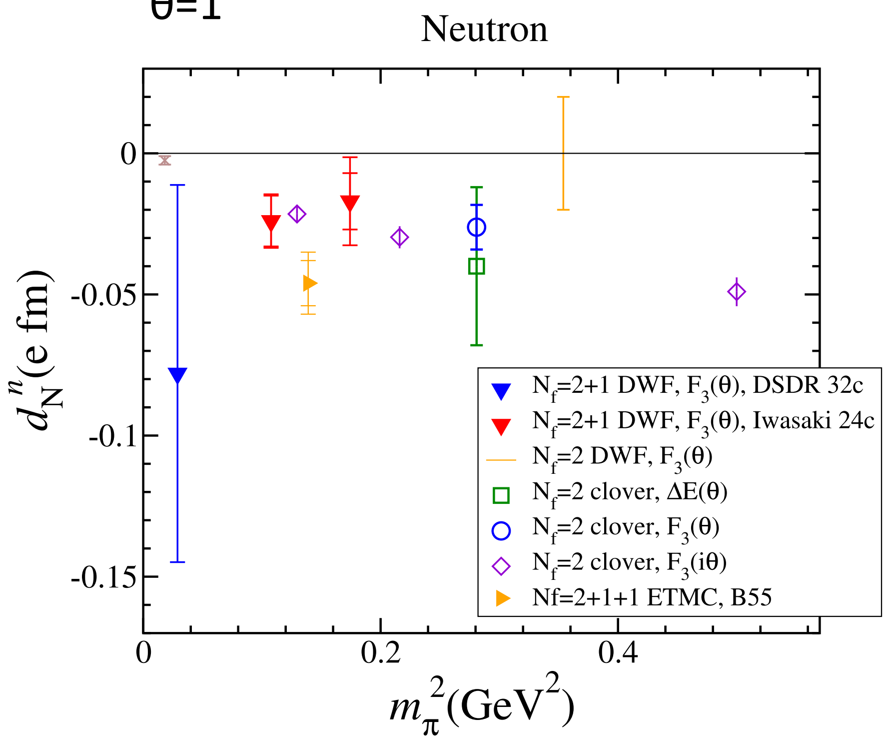
F3 = FQ + Fα
FQ : CP-odd 3pt function contribution
Fα : α x CP-even 3pt function (F1 and F2)

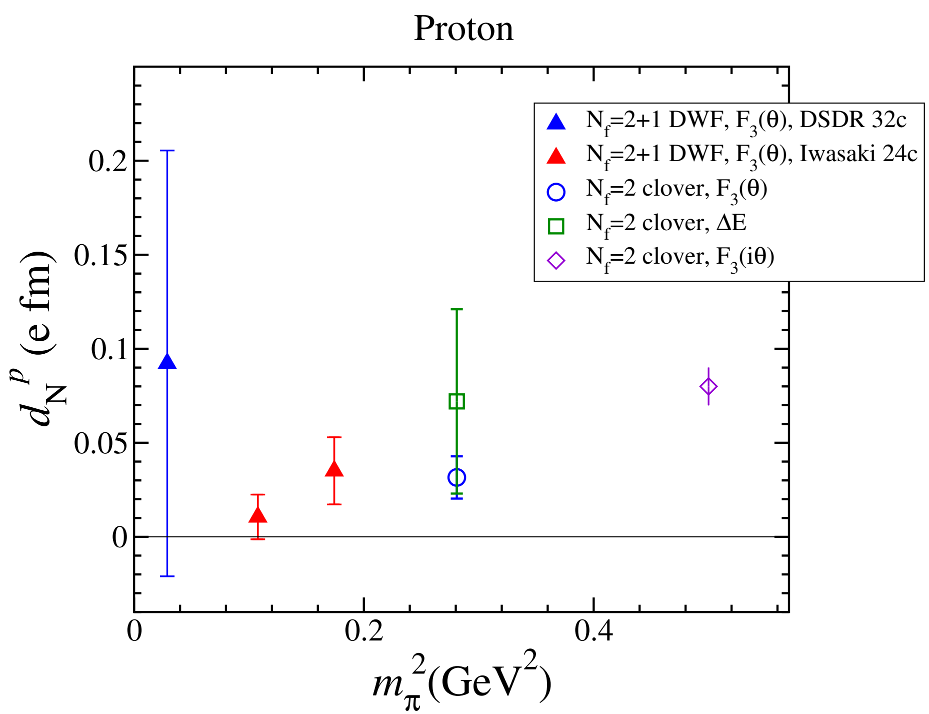


Nucleon EDMs from Lattice
θ case, summary

In θ case, dN ~ mq,
quark mass dependence is important

θ=1





New : DWF (3 ensemble), imaginary Θ (2 ensemble), ETMC (1 ensemble) + quenched

Summary

- g-2 HVP : goal sub 1%
- g-2 HLbL : goal 10%

- Nucleon EDM for proton & Neutron
  - Results for Θ term with large noise or heavy quark mass
  - Higher dim CP violating operators are also being computed
  - Very challenging

- Rapid progress, some of calculations still needs new ideas and techniques are still needed to reduce error further to reach goal.

  Important moments for Lattice QCD

g-2 Future plans

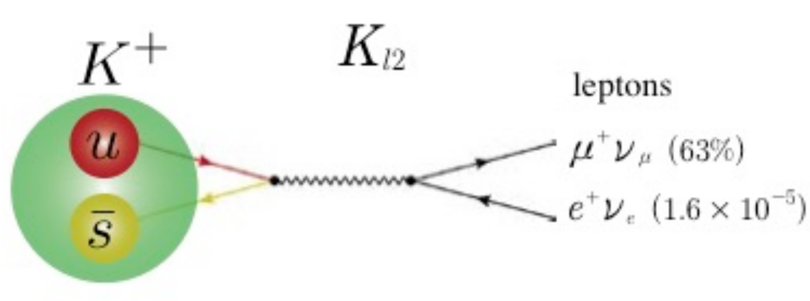
- (discretization error) Nf=2+1 DWF/ Mobius ensemble at physical point, L=5.5 fm, a=0.083 fm, $(64)^3$ at Mira, ALCC @Argonne started to run
- (FV study) QCD box in QED box at physical point
- Disconnected diagrams

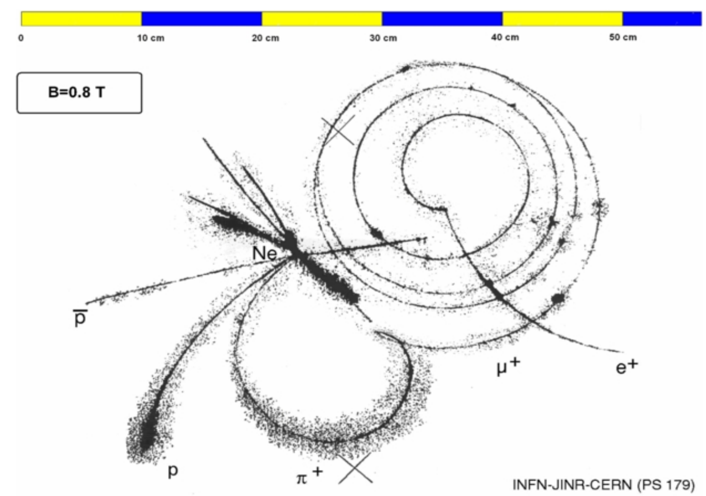
Backup slides / for discussion

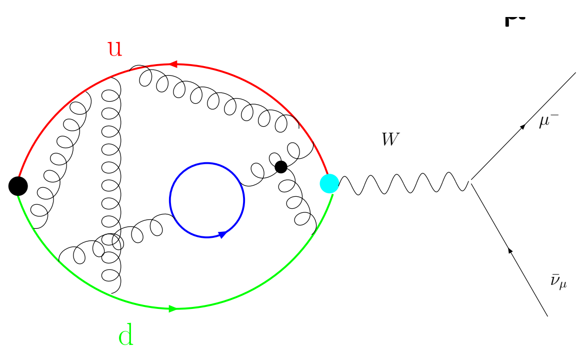
Sub-percent accuracy on Physical point

- now on-physical point (M_π=135 MeV),
  a few lattice spacing a^{-1} = 1.7 and 2.4 GeV, V~(5.5 fm)^3

$$f_\pi = 0.1298(9)(0)(2)\ \mathrm{GeV}[0.7\%]$$
$$f_K = 0.1556(8)(0)(2)\ \mathrm{GeV}[0.5\%]$$







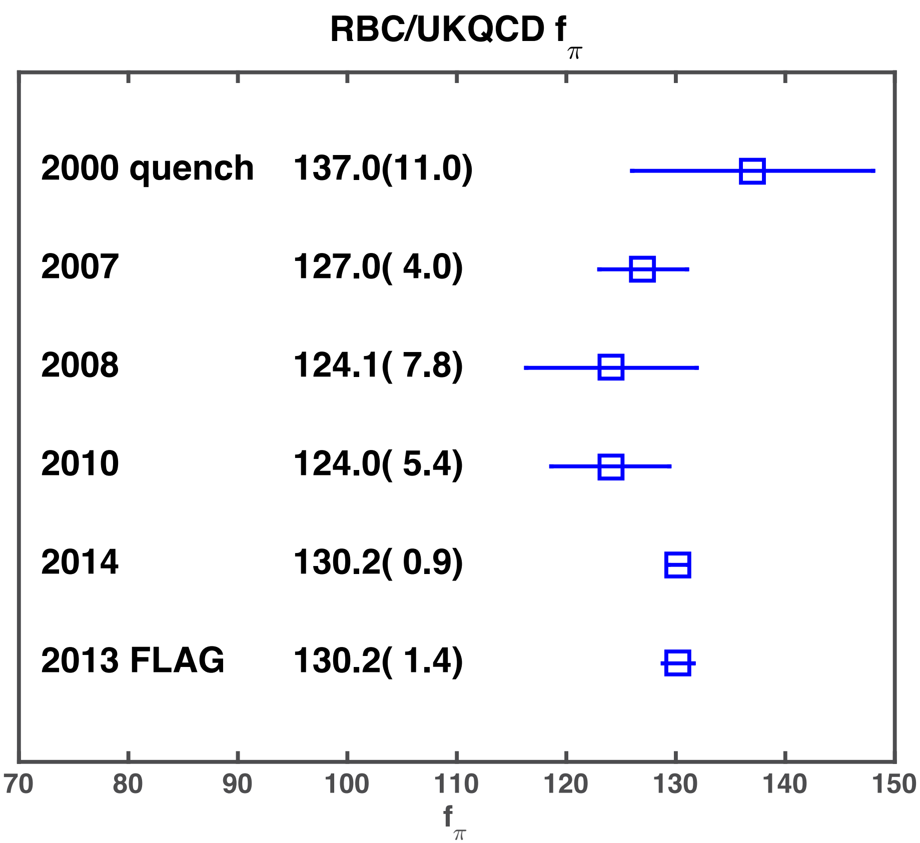
$$\langle 0|\bar{d}\gamma_5 u(0)|\pi\rangle \frac{e^{ipx}}{\sqrt{2E}} \langle \pi|\bar{u}\gamma_m u\gamma_5 d|0\rangle \times G_F V_{ud} m_\mu \bar{\nu}(1-\gamma_5)\mu$$

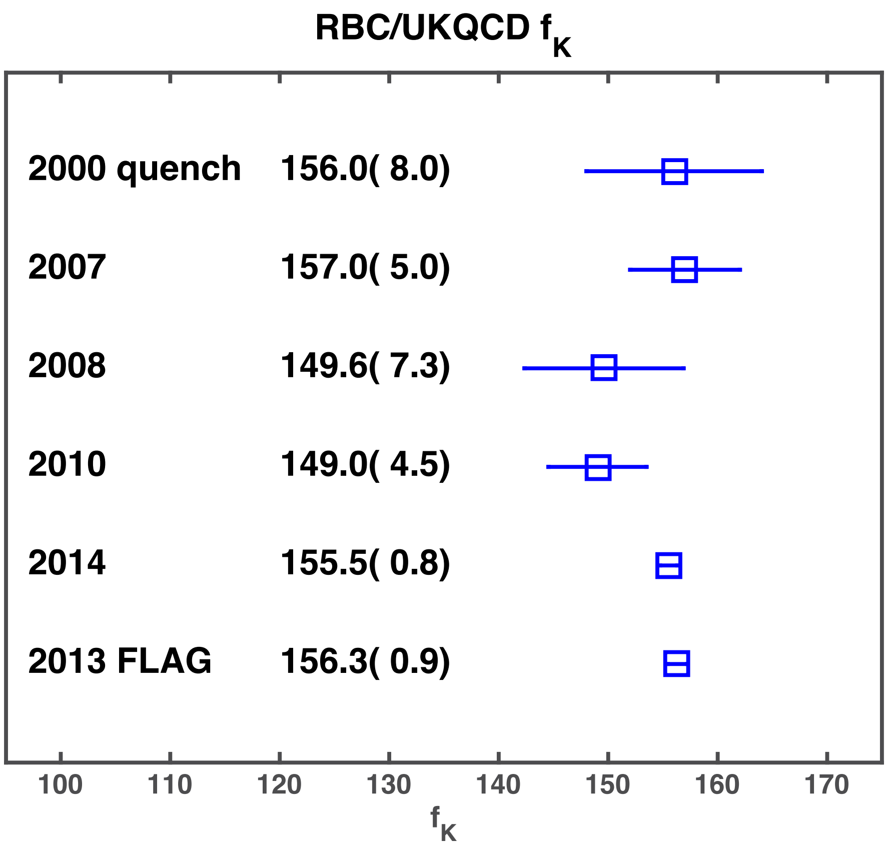
$$\mathcal{M}(\pi\to\mu\bar{\nu}) \sim i f_\pi q_\mu \times G_F V_{ud} m_\mu (\bar{\nu}\mu)_L$$

$$= \langle \pi(q)|\bar{u}\gamma_\mu\gamma_5 d(0)|0\rangle \times G_F V_{ud} m_\mu (\bar{\nu}\mu)_L$$

Sub-percent accuracy on Physical point

- now adding on-physical point (M_π=135 MeV),
  2 lattice spacing a^{-1} = 1.7 and 2.4 GeV, V~(5.5 fm)^3 !





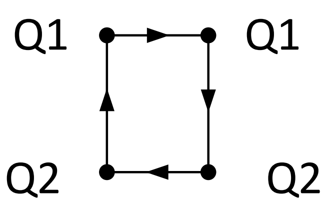
[R. Mawhinney]

Direct 4pt calculation for selected kinematical range

[J. Green et al. Mainz group, Phys. Rev. Lek 115, 222003(2015)]

- Compute connected contribution of 4 pt function in momentum space
- Forward amplitudes related to γ *(Q1) γ *(Q2) -> hadron cross section via dispersion relation

$$\begin{aligned} & \mathcal{M}_{\text{had}}\left(\gamma^*(Q_1)\gamma^*(Q_2) \to \gamma^*(Q_1)\gamma^*(Q_2)\right) \\ \leftrightarrow \quad & \sigma_{0,2}\left(\gamma^*(Q_1)\gamma^*(Q_2) \to \text{had.}\right) \end{aligned}$$



$$\nu = -Q_1 \cdot Q_2$$

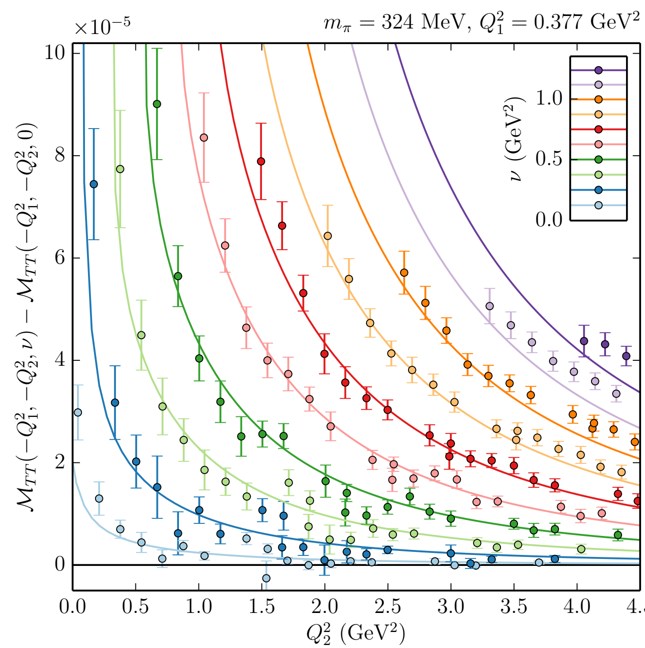


FIG. 3. The forward scattering amplitude $\mathcal{M}_{\rm TT}$ at a fixed virtuality $Q_1^2 = 0.377\text{GeV}^2$, as a function of the other photon virtuality Q_2^2, for different values of ν. The curves represent the predictions based on Eq. (10), see the text for details.

- solid curve: model prediction
- π0 exchange is seen to be not dominant, possibly due to heavy quark mass in the simulation (Mπ = 324 MeV)
- disconnected quark diagram loop

in progress in 2016

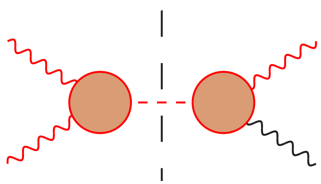
Dispersive approach for HLbL

[Colangelo et al. 2014, 2015, Pauk&Vanderhaeghen 2014]

- Using crossing symmetry, gauge invariance, 138 form factors are reduced 12 relevant for HLbL

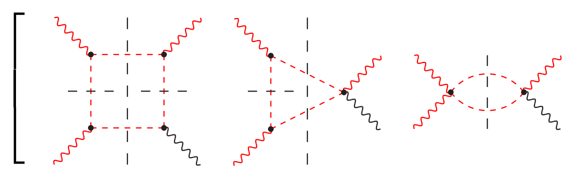
$$a_\mu^{\rm HLbL} = -e^6 \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{1}{q_1^2q_2^2(q_1+q_2)^2}\frac{1}{(p+q_1)^2-m_\mu^2}\frac{1}{(p-q_2)^2-m_\mu^2} \times \sum_{j=1}^{12}\xi_j \hat{T}_{i_j}(q_1,q_2;p)\hat{\Pi}_{i_j}(q_1,q_2,-q_1-q_2),$$

- π0, η , η ’ exchange, pion-loop (exactly scalar QED with pion Form factor)



$$= \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)F_{\pi^0\gamma^*\gamma^*}(q_3^2,q_4^2)}{s-M_\pi^2}$$

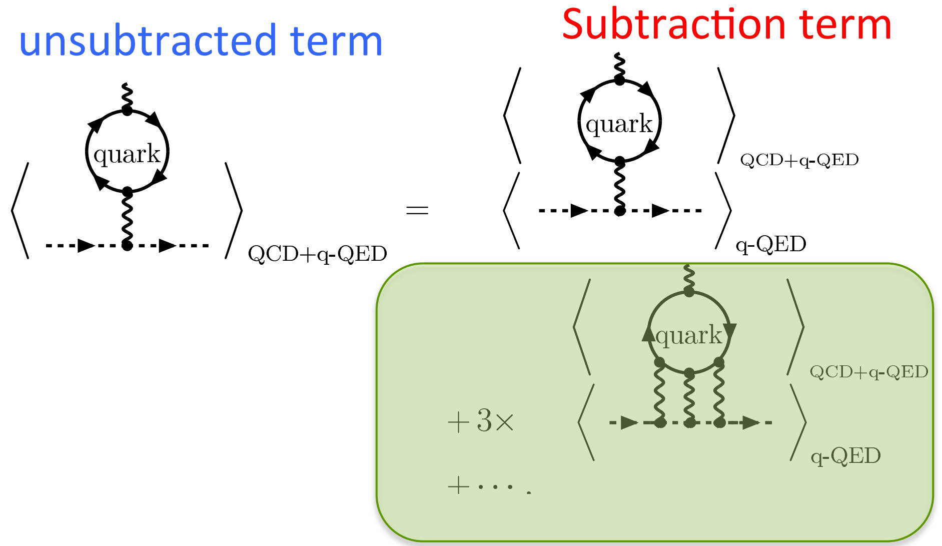
$$\Pi^{\rm FsQED}_{\mu\nu\lambda\sigma} = F_\pi^V(q_1^2)F_\pi^V(q_2^2)F_\pi^V(q_3^2)\times$$



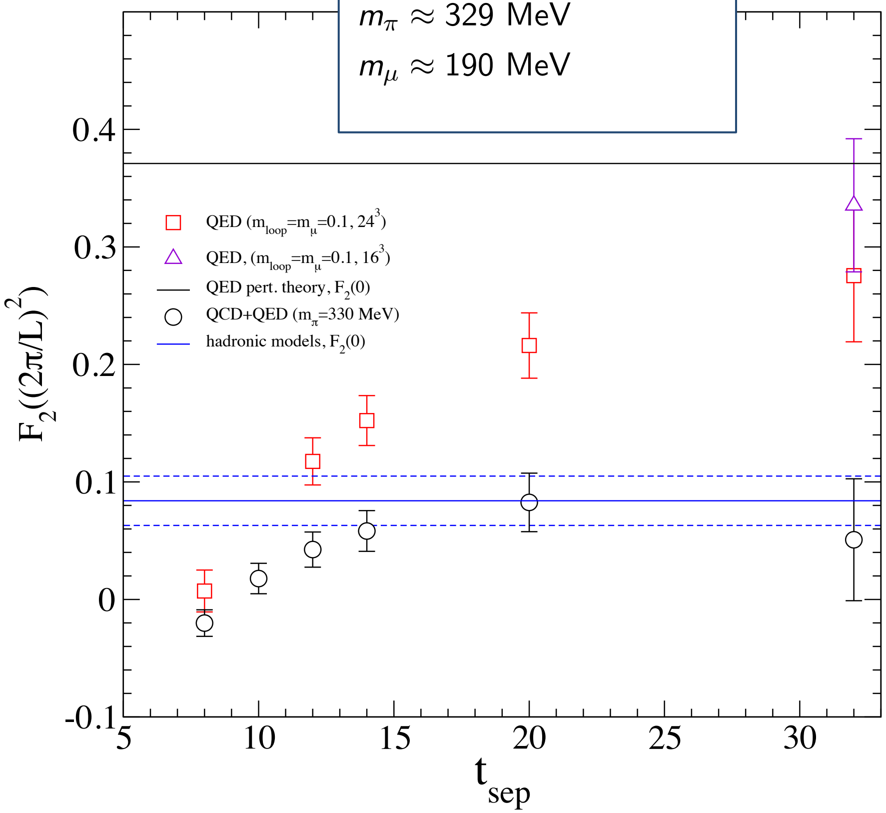
- other contribution is neglected

QCD+QED method [Blum et al 2015]

- One photon is treated analytically
- other two sampled stochastically
- needs subtraction
- use AMA for error reduction
- use Furry's theoretm to reduce α^2 noise



24^3 lattice size
$Q^2 = 0.11$ and 0.18 GeV^2
$m_\pi \approx 329$ MeV
$m_\mu \approx 190$ MeV



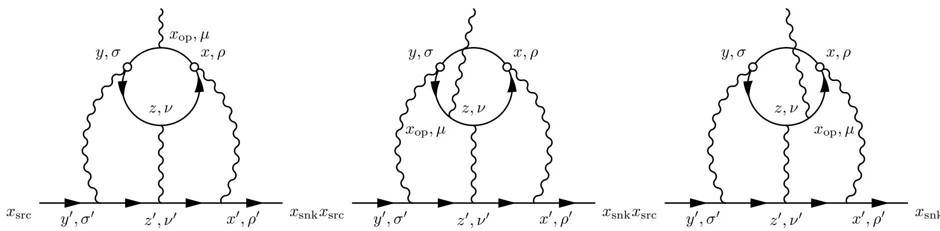
- Connected part only

- QED only calculation consistent with QED loop calculation for larger volume

- QED+QCD
  - ball park of model values
  - significant exited state effects ?

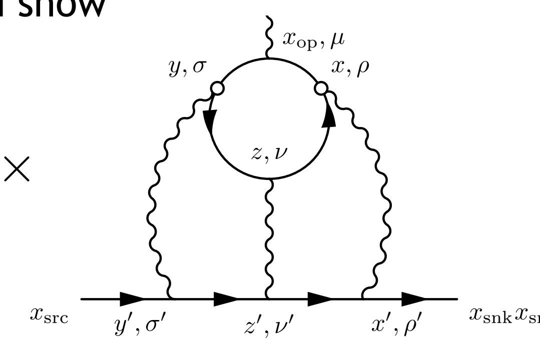
Conserved current & moment method

- [conserved current method at finite q2] To tame UV divergence, one of quark-photon vertex (external current) is set to be conserved current (other three are local currents). All possible insertion are made to realize conservation of external currents config-by-config.



- [moment method , q2→0] By exploiting the translational covariance for fixed external momentum of lepton and external EM field, q->0 limit value is directly computed via the first moment of the relative coordinate, xop – (x+y)/2, one could show

$$\frac{\partial}{\partial q_i}\mathcal{M}_\nu(\vec{q})|_{\vec{q}=0} = i \sum_{x,y,z,x_{\rm op}} (x_{\rm op} - (x+y)/2)_i \times$$

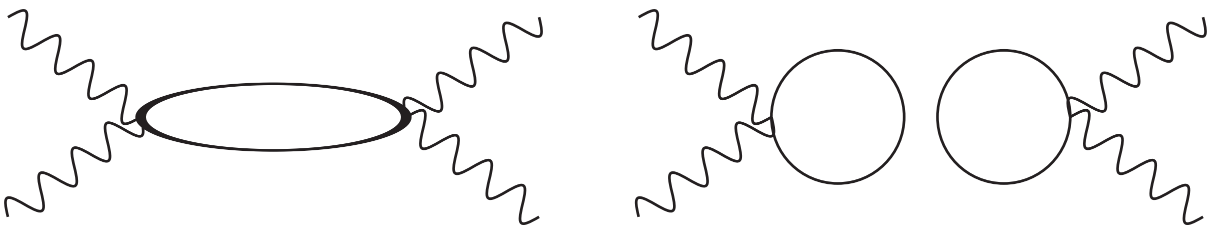


to directly get $F_2(0)$ without extrapolation.

$$\text{Form factor}: \; \Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\, m_l} F_2(q^2)$$

Disconnected HLbL would be non-negligible

- The major contribution, single pi0 (and η , η ')exchange diagrams through 2 $\gamma \rightarrow \pi 0$, would have both connected and disconnected contributions.



- A quark model consideration for LbL pi0 exchange turns out to be Con : DisCon roughly same size with opposite sign (L. Jin)
- Good news : it's not η ' (only), so S/N would not grow [P. Lepage] exponentially with the propagation length.
- Bad news : it's disconnected quark loops, and many of them.

Systematic errors

- Missing disconnected diagrams
  → compute them

- Finite volume

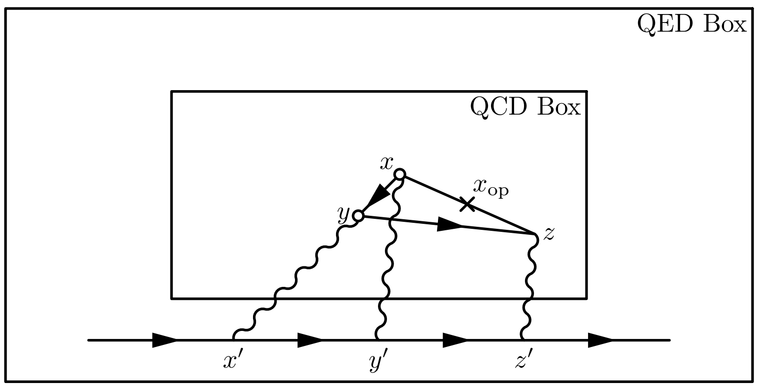
- Discretization error

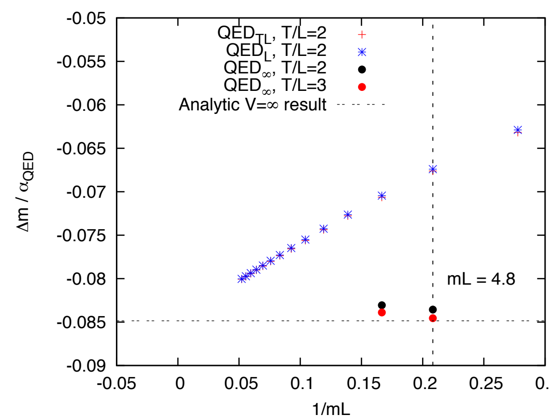
→ a scaling study for 1/a = 2.7 GeV, 64 cube lattice at physical quark mass is proposed to ALCC at Argonne

- …

QCD box in QED box

- FV from quark is exponentially suppressed ~ exp(- M_π L_{QCD})
- Dominant FV effects would be from photon
- Let photon and muon propagate in larger (or infinite) box than that of quark



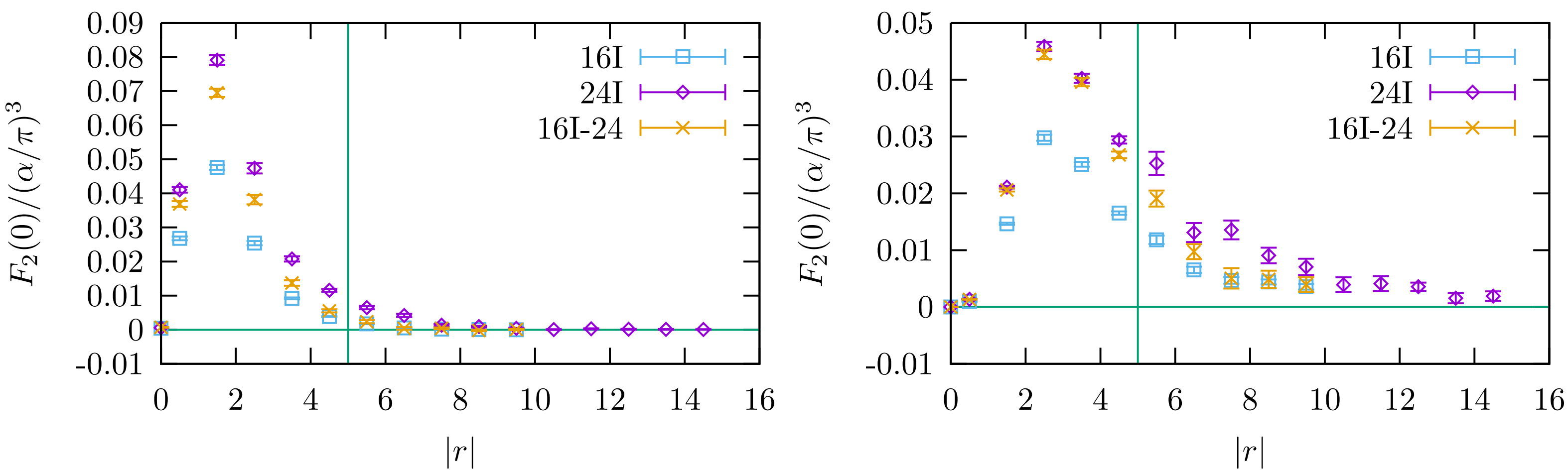


- We could examine different lepton/photon in the off-line manner
e.g. QED_L (Hayakwa-Uno 2008) with larger box,
Twisting Averaging [Lehner TI LATTICE14] or
Infinite Vol. Photon propagators [C. Lehner, L.Jin, TI LATTICE15]

QED box in QCD box (contd.)

- Mπ=420 MeV, mμ=330 MeV, 1/a=1.7 GeV
- $(16)^3 = (1.8\ \mathrm{fm})^3$ QCD box in $(24)^3 = (2.7\ \mathrm{fm})^3$ QED box

Ensemble	$m_\pi L$	QCD Size	QED Size	$\frac{F_2(q^2=0)}{(\alpha/\pi)^3}$
16I	3.87	$16^3 \times 32$	$16^3 \times 32$	0.1158(8)
24I	5.81	$24^3 \times 64$	$24^3 \times 64$	0.2144(27)
16I-24		$16^3 \times 32$	$24^3 \times 64$	0.1674(22)



r = min {|x-y|, |y-z|,|z-x|}   r = max{|x-y|, |y-z|,|z-x|}

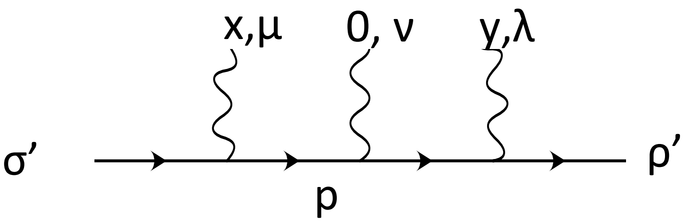
Continuum Infinite Volume (a.k.a HVP way)

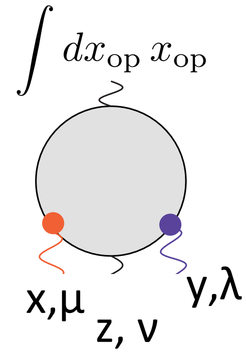
$$a_\mu^{\rm HVP} = \sum_t w(t) C(t), \quad w(t) \propto t^4 \cdots$$

- One could also use infinite volume/continuum lepton&photon diagram in coordinate space

  [J. Green et al. Mainz group, LAT16 proceedings]

$$\mathcal{L}_{\mu\nu\lambda\sigma\rho}(x, y; p)$$





- Techniques in continuum model calculation [Knect Nyffeler 2002; Jegerlehner Nyffeler 2009] : angle average over muon momentum, and carry out angle of two virtual photons

$$L(x_1, x_2) = \sum_{m,l} \sum_{\substack{k=|l-m| \\ \text{step}=2}}^{l+m} (-1)^k C_k(\hat{x}_1 \hat{x}_2)$$

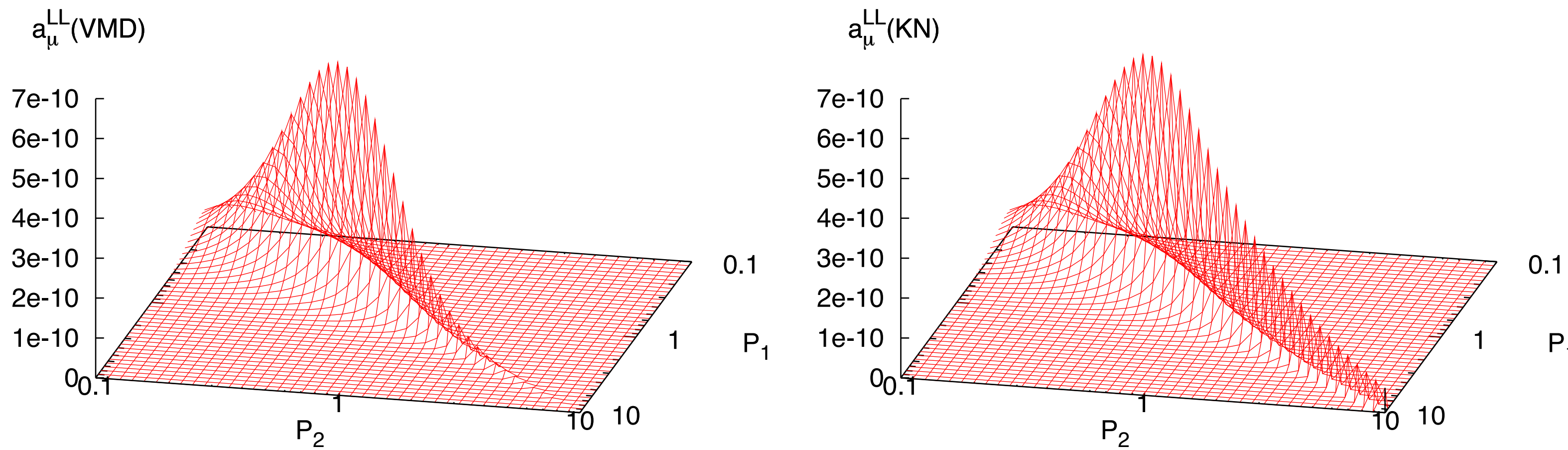
$$\times \int dQ_1 dQ_2 \frac{4 Z_1 Z_2}{m^2 Q_1 Q_2 X_1 X_2} \frac{(-Z_1 Z_2)^l}{l+1} J_{k+1}(Q_1 X_1) J_{k+1}(Q_2 X_2)$$

$$\times \left[\frac{\theta(1 - Q_2/Q_1)}{Q_1^2} \left(\frac{Q_2}{Q_1}\right)^m + \frac{\theta(1 - Q_1/Q_2)}{Q_2^2} \left(\frac{Q_1}{Q_2}\right)^m \right]$$

Can Lattice produce a counter part ?

[J. Bijnens]

- Which momentum regimes important studied: JB and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170]
- $a_\mu = \int dl_1 dl_2 a_\mu^{LL}$ with $l_i = \log(P_i/GeV)$



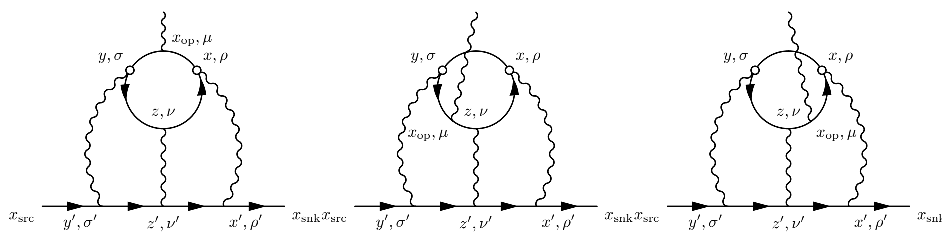
Which momentum regions do what:
volume under the plot $\propto a_\mu$

M_π=170 MeV cHLbL result (contd.)

"Exact" … q = 2pi / L,

"Conserved (current)" … q=2pi/L, 3 diagrams

"Mom" … moment method q->0, with AMA



Method	$F_2/(\alpha/\pi)^3$	$N_{\rm conf}$	$N_{\rm prop}$	$\sqrt{\rm Var}$	$r_{\rm max}$	SD	LD	ind-pair
Exact	0.0693(218)	47	$58 + 8 \times 16$	2.04	3	−0.0152(17)	0.0845(218)	0.0186
Conserved	0.1022(137)	13	$(58 + 8 \times 16) \times 7$	1.78	3	0.0637(34)	0.0385(114)	0.0093
Mom. (approx)	0.0994(29)	23	$(217 + 512) \times 2 \times 4$	1.08	5	0.0791(18)	0.0203(26)	0.0028
Mom. (corr)	0.0060(43)	23	$(10 + 48) \times 2 \times 4$	0.44	2	0.0024(6)	0.0036(44)	0.0045
Mom. (tot)	0.1054(54)	23						

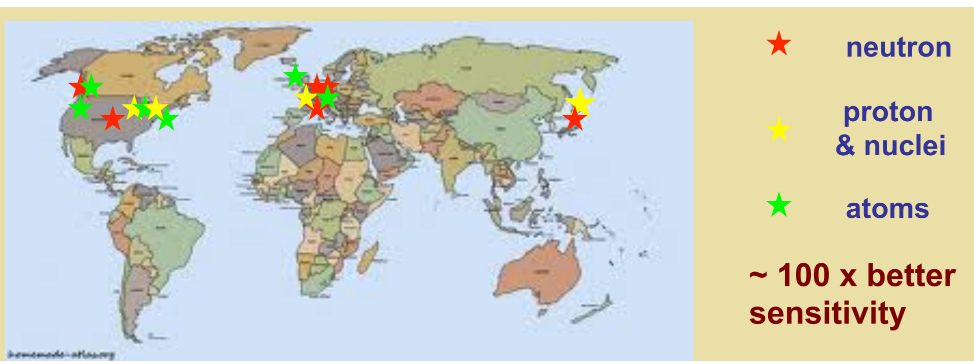
EDM Experiments

- The present and future experiments are aiming to check/exclude of MSSM

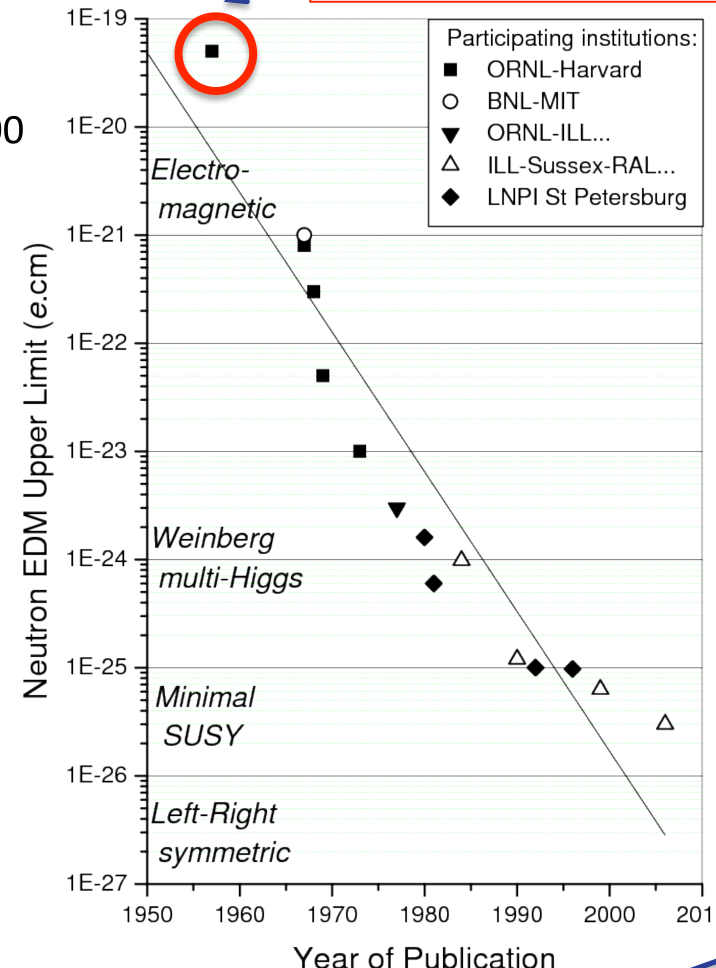
  pEDM @ BNL
  nEDM @ ORNL, PSI, ILL, J-PARC, TRIUMF ,FNAL, FRM2, …
  charged hadrons @ COSY

  Harris, 0709.3100

  ⇒ a sensitivity of 10^{-29} e·cm !



1957 ORNL-Harvard



- Current theoretical estimations are based on quark model, sum rules, …

  non-perturbative computations of EDM $d_n(\theta, d_q, d_q^c, \ldots)$ are necessary

nEDM ORNL SNS

QCD ensemble for NEDM

- 1/a = 1.73 GeV
- V=(2.7 fm)^3
- Mpi = 330, 400 MeV
- 750 configurations

- 1/a = 1.37 GeV
- V=(4.6 fm)^3
- Mpi = 170 MeV
- 39 configurations

Chiral symmetry & EDM

$$q(x) \to e^{i\gamma_5\theta} q(x)$$
$$\bar{q}(x) \to \bar{q}(x) e^{-i\gamma_5\theta}$$

- Chiral symmetry is broken by lattice systematic error

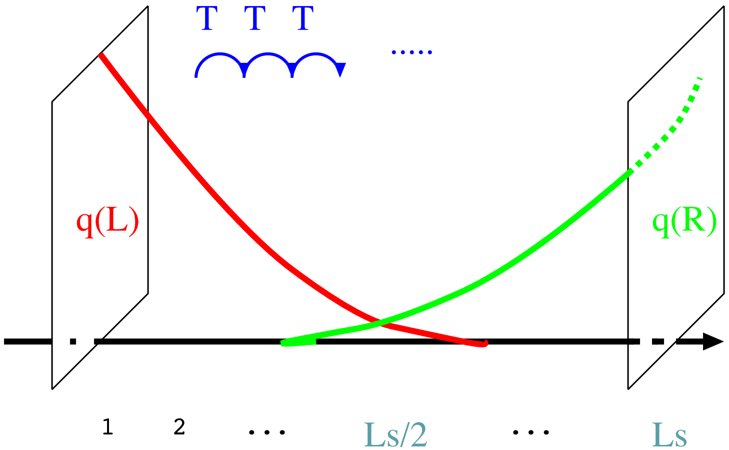
  for Wilson-type quarks, which has "wrong" Pauli term by O(a)

$$\mathcal{L}_{\text{Wilson}} = \mathcal{L}_{\text{QCD}} + ca\bar{q}\sigma_{\mu\nu} \cdot F_{\mu\nu} q$$

- CP violation from θ or other BSM operators introduce extra artificial CP violation in simulation.

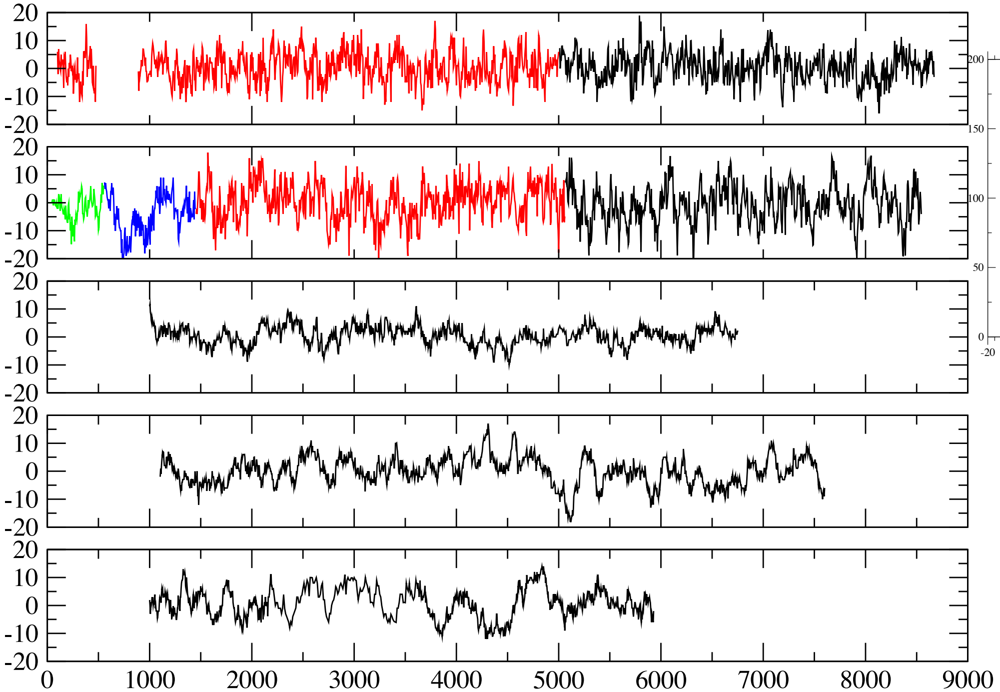
- In fact, chiral rotation of valence quark is not observable in continuum theory, and the EDM signal measured in Wilson quark due to valence quark's θ is unphysical, which should be carefully removed by taking continuum limit a → 0 [S. Aoki-Gockschu, Manohar, Sharpe et al. Phys.Rev.Lett. 65 (1990) 1092-1095 (1990)]

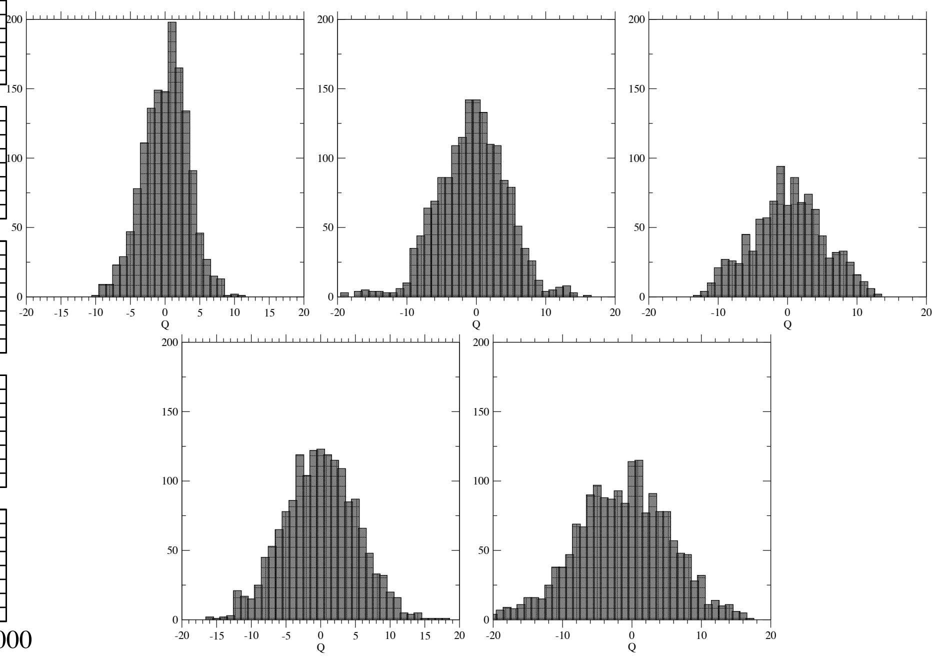
→ Our choice : chiral lattice quark called domain-wall fermions (DWF)
[97 Blum Soni, 99 CP-PACS, 00- RBC, 05 RBC/UKQCD...]



Qtop on lattice (Θ=0)

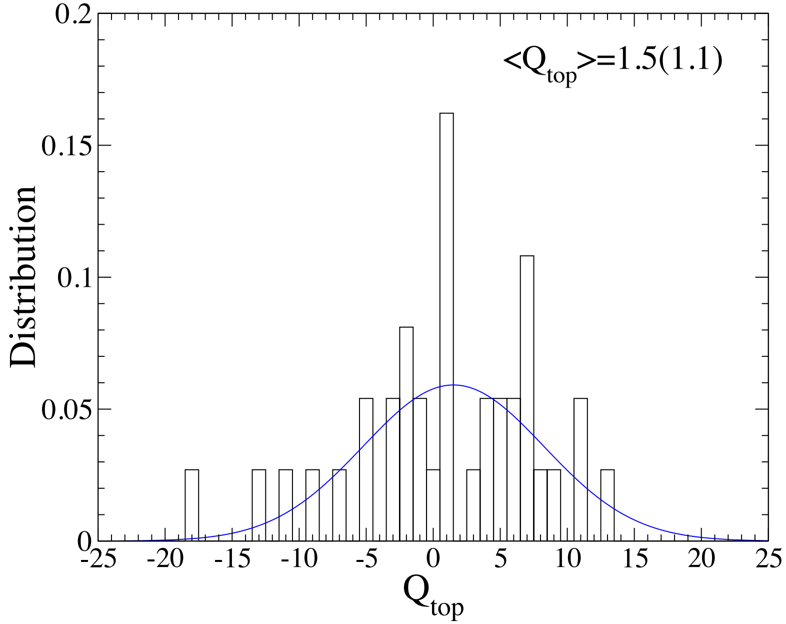
- Qtop history in simulation Nf=2+1 DWF, [RBC/UKQCD]
- 1/a= 1.73, 2.28 GeV
- mps =290 – 420 MeV



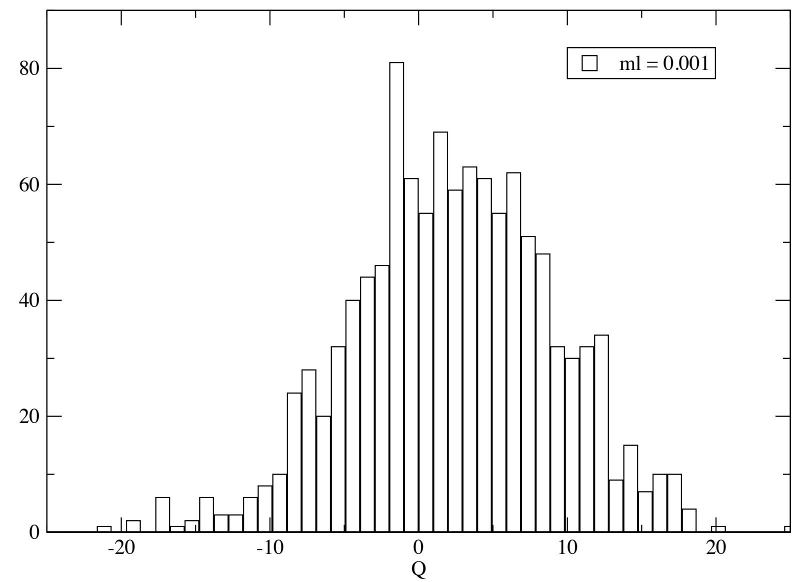


Mpi=330, 420 MeV

Qtop at Mpi=170 MeV ensemble



Used



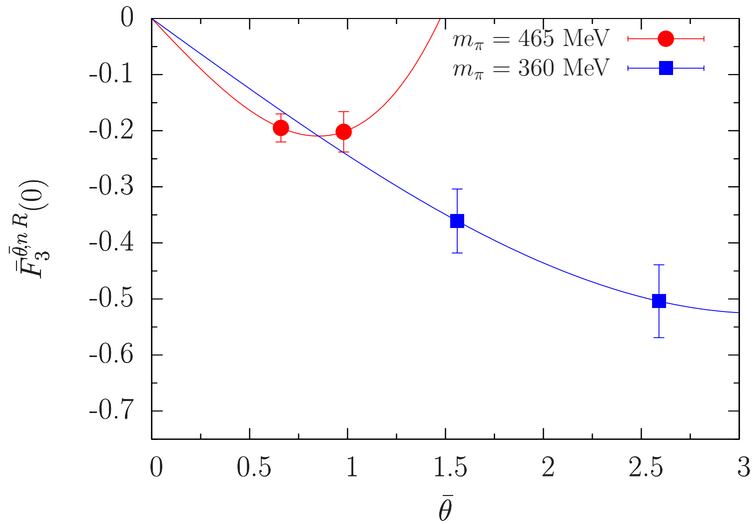
Full ensemble

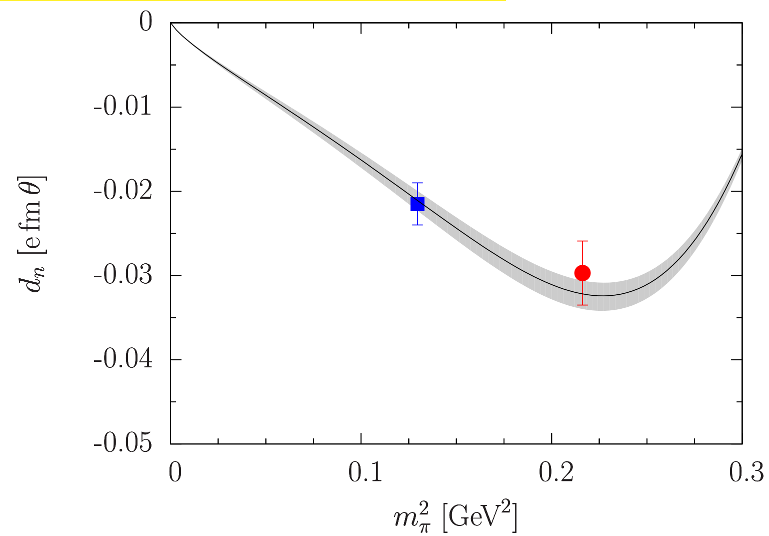
Imaginary Θ simulation QCDSF
arXiv:1502.02295

- Perform dynamical simulation with imaginary

$$\theta = i\bar{\theta}\,.$$

$$S_\theta = \bar{\theta}\,\frac{m_\ell\, m_s}{2m_s + m_\ell}\,a^4 \sum_x \left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s\right)$$





$$d_n = -0.0039(2)(9)\,[e\,\mathrm{fm}\,\theta]\,.$$

m_π [MeV]	m_K [MeV]	d_n [e fm θ]
465(13)	465(13)	−0.0297(38)
360(10)	505(14)	−0.0215(25)

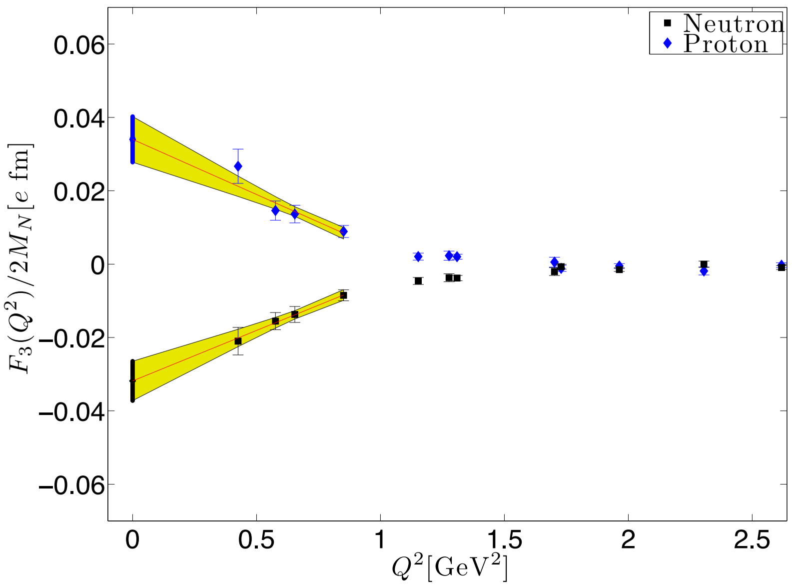
Pregress !
but systematic error under control ?
(chiral breaking, csw, large Θ, ...)

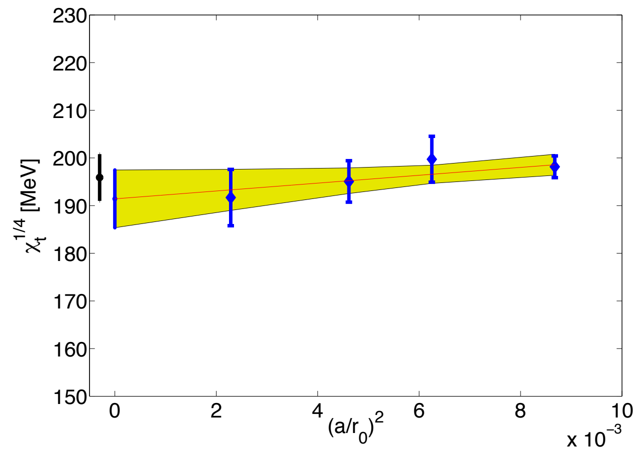
Quenched calculation using gradient flow topological charge

Andrea Shindler [Thu, 9:30]

- 4 beta, a=0.1-0.05 fm, L ~ 1.5 fm
- gradient flow for Q_top
- non-perturbatively improved Wilson
- Mps ~ 800 MeV

$$[\chi_t]^{1/4} = 195.9(4.9)\ \mathrm{MeV}$$





$$d_{\mathrm{P}} = 0.0340(62)\ \theta\ e \cdot \mathrm{fm}$$

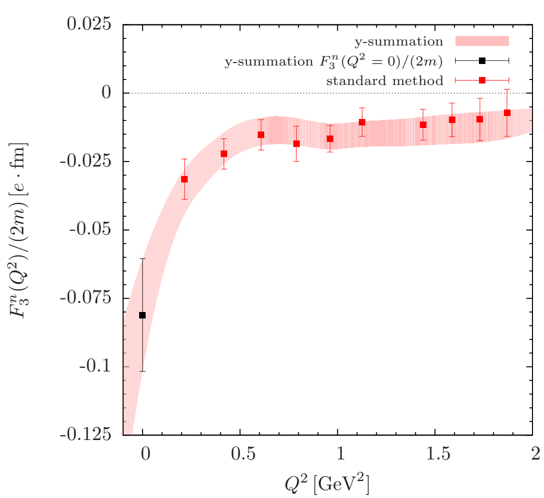
$$d_{\mathrm{N}} = -0.0318(54)\ \theta\ e \cdot \mathrm{fm}$$

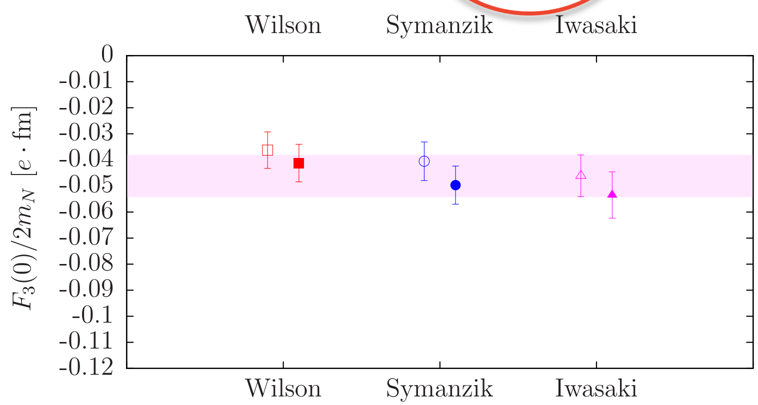
@ Mps ~ 800 MeV

ETMC NEDM
Andreas Athenodorou [Thu, 11:00]

- ETMC Nf=2+1+1, B55 ensemble
- a=0.082fm, Mpi = 373 MeV, L = 2.6 fm
- Qtop from various cooling and gradient flow
- F3(q2) extrapolation by dipole fit, coordinate space methods (~ moment method) W. Wilcox's concern hep-lat/0204024 ?

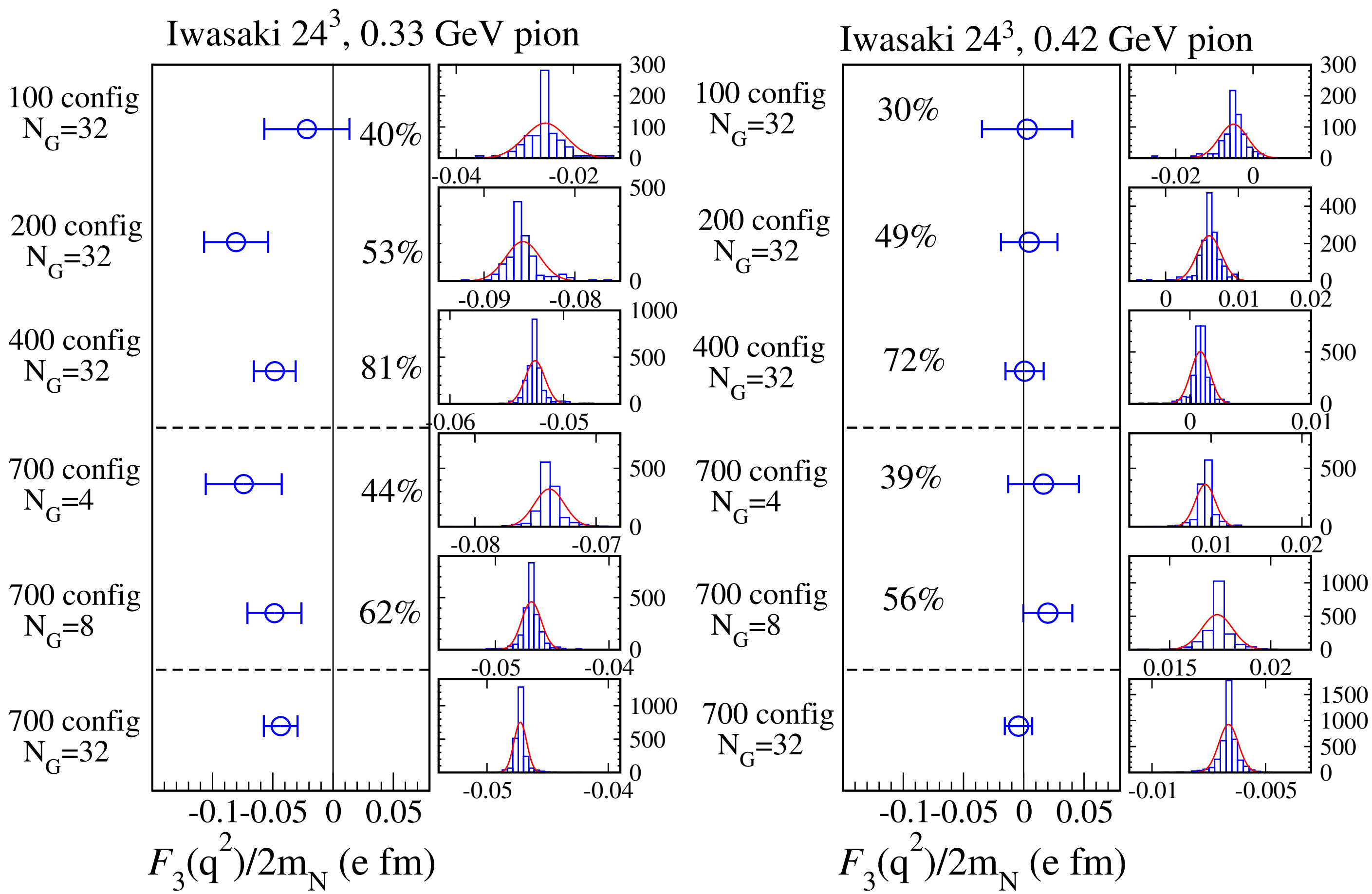
$$\left.\frac{G_{pJ_jp}(t_2,t_1;(\vec{x}_1)_i,\Gamma_k)}{G_{pp}(t_2;\vec{p},\Gamma_4)}\right|_S \xrightarrow{EETL} \epsilon_{ijk}\frac{e^{(m_N-E_1)\,t_1}}{E_1}G_m(Q_1^2),$$



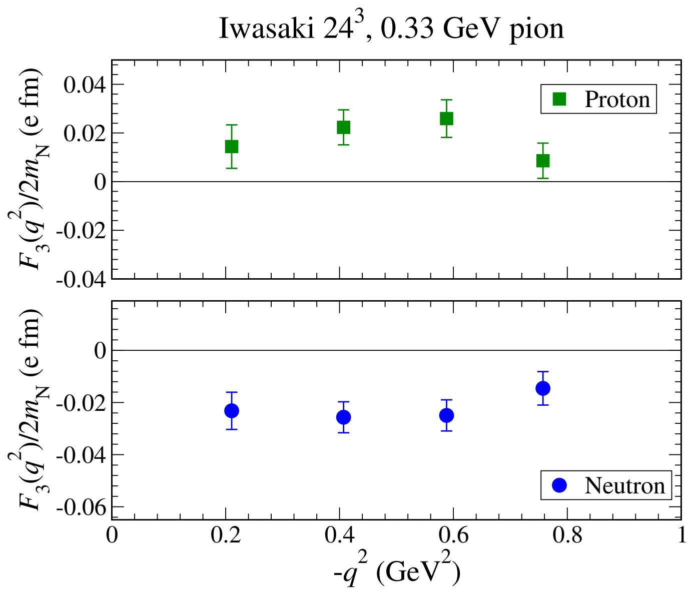


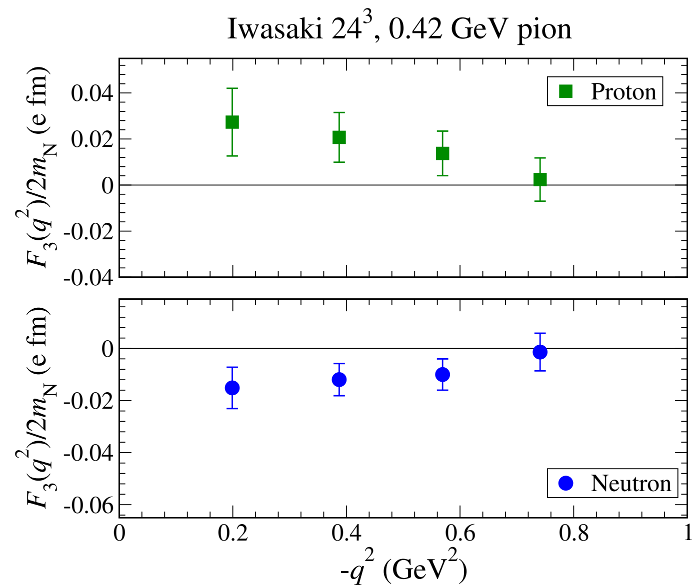
Final Result $\frac{F_3(0)}{2m_N} = -0.046(8)(7)$ using a total of $\mathcal{O}(4700)$ measurements.

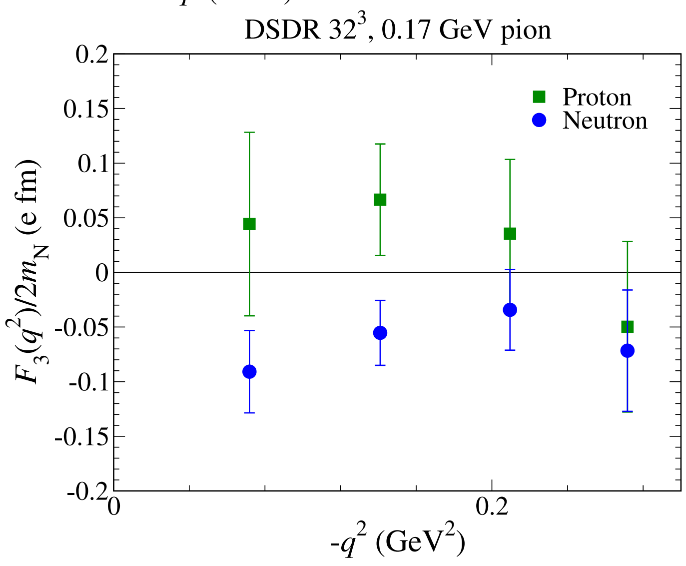
statistical checks



F3 form factor, q2 dependence







$$F_3(q^2)/2m_N = d_N + S'q^2 + \mathcal{O}(q^4),$$

tensor charge & quark EDM

- BSM operators, such as quark EDM, or quark chromo EDM, are also interesting/important besides Θ term

$$i d_q \bar{q} \sigma_{\mu\nu} \gamma_5 F_{\mu\nu} q \qquad i\tilde{d}_q \bar{q} \sigma_{\mu\nu} \gamma_5 G_{\mu\nu} q$$

$$d_W f^{abc} G^a G^b \tilde{G}^c$$

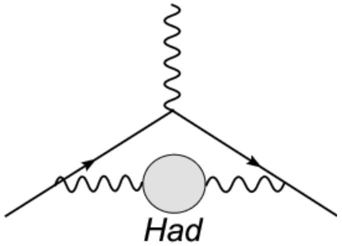
- Quark EDM is related to the tensor charge of QCD (arXiv:1502.07325, 1506.04196, 1506.06411)

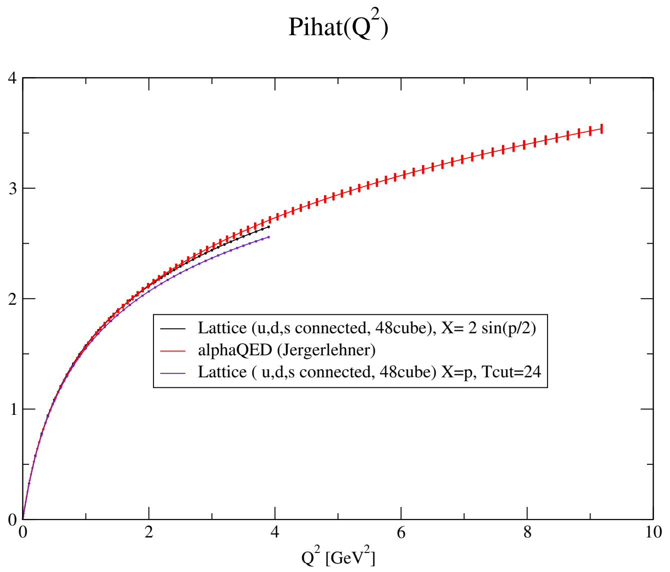
- Operator renormalization/mixing will be also discussed.

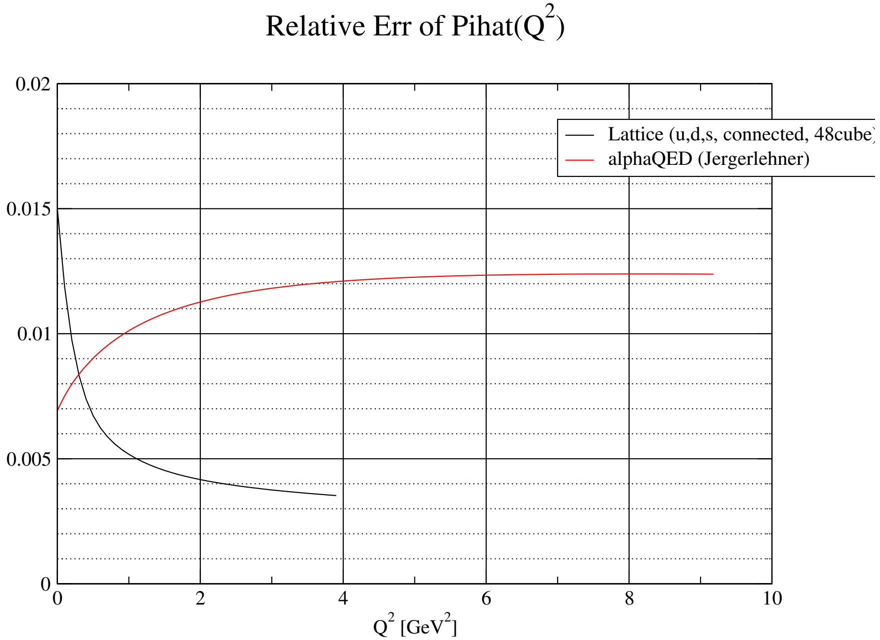
(plan B) Interplays between lattice and dispersive approach g-2

- R-Ratio error ~ 0.6%, HPQCD error ~ 2%
- Goal would be ~ 0.2 %
- Dispersive approach from R-ratio R(s)

$$\hat{\Pi}(Q^2) = \frac{Q^2}{3} \int_{s_0} ds \frac{R(s)}{s(s+Q^2)}$$







also [ETMC, Mainz, ...]

- Can we combine dispersive & lattice and get more precise (g-2)HVP than both ? [2011 Bernecker Meyer]
- Inverse Fourier trans to Euclidean vector correlator
- Relevant for g-2 $Q^2 = (m_\mu/2)^2 = 0.0025\ \mathrm{GeV}^2$
- It may be interesting to think

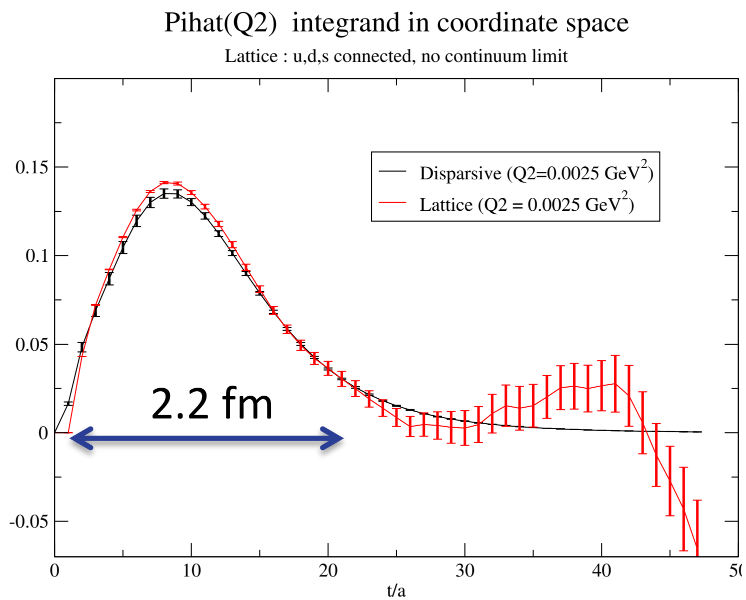
$$\frac{\hat{\Pi}(Q^2)}{Q^2} = \left[\frac{\hat{\Pi}(Q^2)}{Q^2} - \frac{\hat{\Pi}(P^2)}{P^2}\right]^{\mathrm{Exp}} + \left[\frac{\hat{\Pi}(P^2)}{P^2}\right]^{\mathrm{Lat}}$$

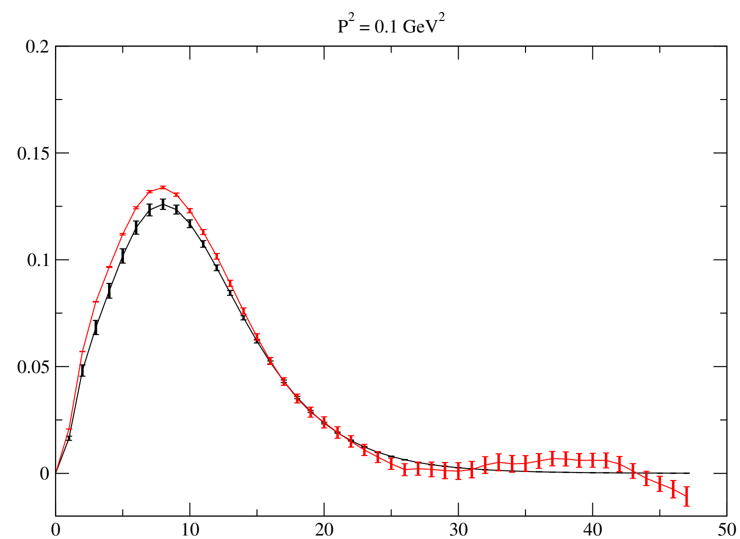
$$a_\mu^{\mathrm{HVP}} = \sum_t w(t)C(t), \quad w(t) \propto t^4 \cdots$$

$$\hat{\Pi}(Q^2) = \frac{Q^2}{3}\int_{s_0} ds \frac{R(s)}{s(s+Q^2)}$$

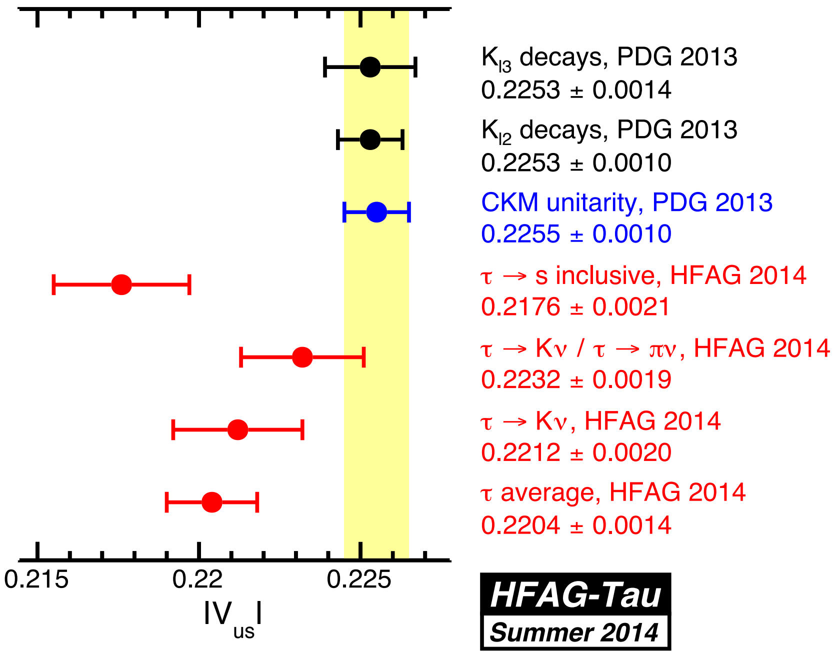
Black : R-ratio , alpha QED (Jegerlehner)

Red : Lattice (DWF)





V_{us} extraction strangeness tau inclusive decay



Tau decay

- $\tau \to \nu + had$ through V-A vertex
- Apply the optical theorem to related to VV and AA hadronic vacuum polarization (HVP)
- For hadrons with strangeness -1, CKM matrix elements V_{us} is multiplied
- ν takes energy away, makes differential cross section is related to the HVPs (c.f. in e^+e^- case, the total cross section is directly related to HVP)

$$R_{ij} = \frac{\Gamma(\tau^- \to \text{hadrons}_{ij}\ \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{12\pi |V_{ij}^2| S_{EW}}{m_\tau^2} \int_0^{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \underbrace{\left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im}\Pi^{(1)}(s) + \text{Im}\Pi^{(0)}(s)\right]}_{\equiv\ \text{Im}\,\Pi(s)}$$

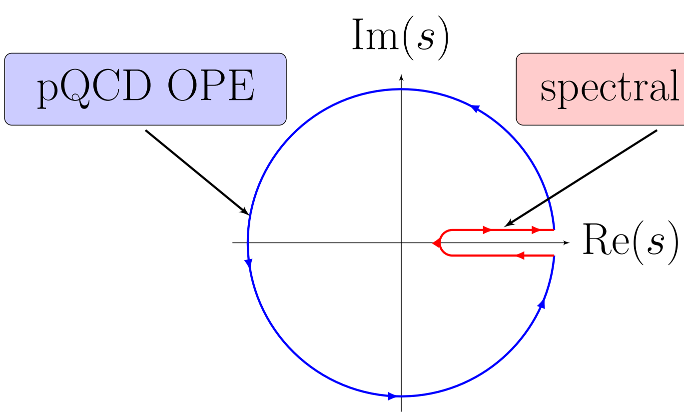
- The Spin=0 and 1, vacuum polarization, Vector(V) or Axial (A) current-current two point

$$\Pi^{\mu\nu}_{ij;V/A}(q^2) = i \int d^4x e^{iqx} \left\langle 0 | T J^\mu_{ij;V/A}(x) J^{\dagger\mu}_{ij;V/A}(0) | 0 \right\rangle = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(1)}_{ij;V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij;V/A}$$

Finite Energy Sum Rule (FESR)

- Do the finite radius contour integral
- Real axis integral from experimental R_τ
- Use pQCD and OPE for the large circle integral
- Any analytic weight function $w(s)$

$$\int_{s_{th}}^{s_0} \text{Im}\Pi(s)w(s) = \frac{i}{2}\oint_{|s|=s_0} ds\Pi(s)w(s)$$



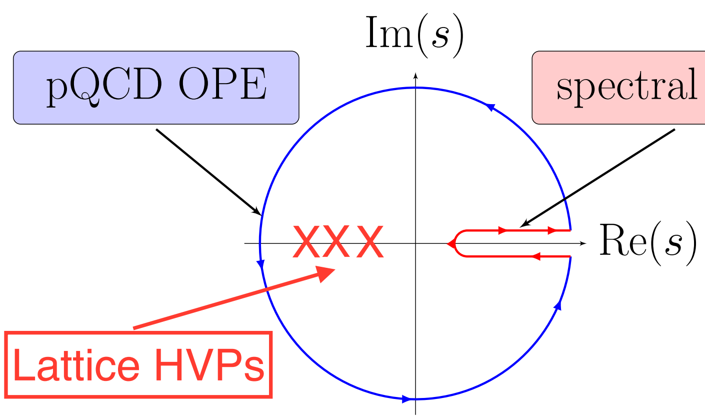
Combining FESR and Lattice

- If we have a reliable estimate for $\Pi(s)$ in Euclidean (space-like) points, $s = -Q_k^2 < 0$, we could extend the FESR with weight function $w(s)$ to have poles there,

$$\int_{s_{th}}^{\infty} w(s)\mathrm{Im}\Pi(s) = \pi \sum_{k}^{N_p} \mathrm{Res}_k[w(s)\Pi(s)]_{s=-Q_k^2}$$

$$\Pi(s) = \left(1 + 2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s) \propto s \quad (|s| \to \infty)$$

- For $N_p \geq 3$, the $|s| \to \infty$ circle integral vanishes.



weight function $w(s)$

- Example of weight function

$$w(s) = \prod_k^{N_p} \frac{1}{(s+Q_k^2)} = \sum_k a_k \frac{1}{s+Q_k^2}, \quad a_k = \sum_{j \neq k} \frac{1}{Q_k^2 - Q_j^2}$$

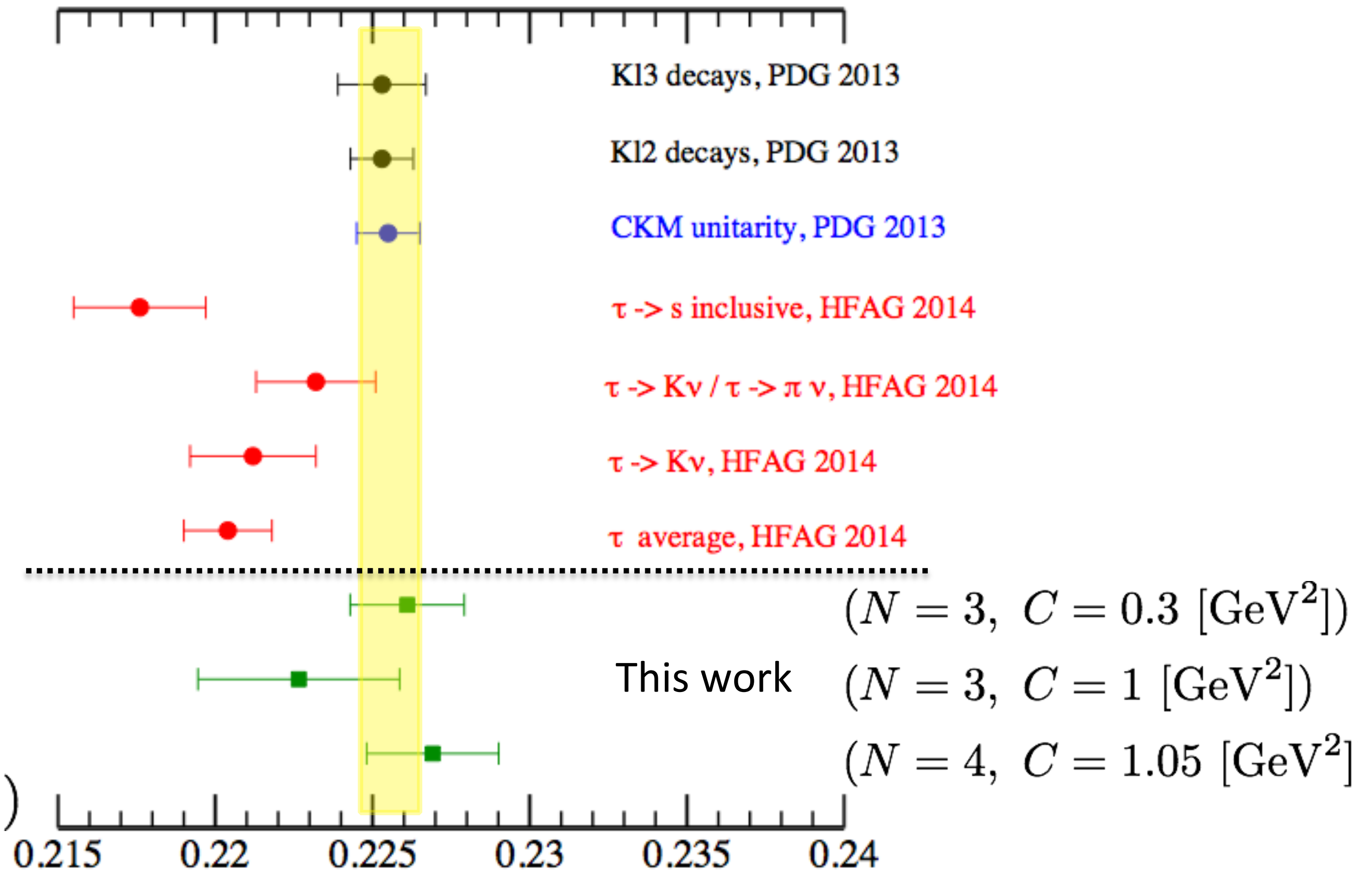
$$\Longrightarrow \sum_k (Q_k)^M a_k = 0 \quad (M = 0, 1, \cdots, N_p - 2)$$

- The residue constraints automatically subtracts $\Pi^{(0,1)}(0)$ and $s\Pi^{(1)}(0)$ terms.
- For experimental data, $w(s) \sim 1/s^n, n \geq 3$ suppresses
  - ▷ *larger error from higher multiplicity final states at larger* $s < m_\tau^2$
  - ▷ *uncertanties due to pQCD+OPE at* $m_\tau^2 < s$
- For lattice, Q_k^2 should be not too small to avoid large stat. error, $Q^2 \to 0$ extrapolation, Finite Volume error(?). Also not too larger than m_τ^2 to make the suppression in time-like $0 < s < m_\tau^2$ working.
- Other $w(s)$ could be useful to enhance some region $s > 0$ which may be usable for $(g-2)_\mu$ HVP (?)
- c.f. HPQCD's HVP moments works

Preliminary results

[H. Ohki, A. Juttner, C. Lehner, K. Maltman et al.]

very preliminary



Our result

for all channels

$(V_1 + V_0 + A_1 + A_0)$

All our results (C<1, N=3,4) are consistent with each other.

Note : Other systematic errors of sea quark mass chiral extrapolation, lattice O(a^4) discretization,

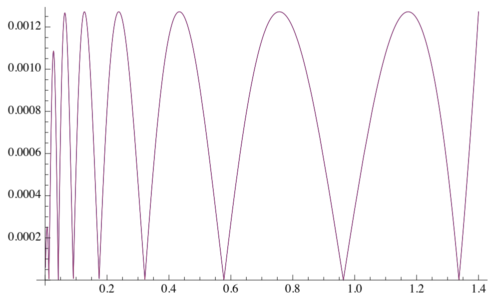
and higher order OPE have not been included. These must be assessed in a future study.

AMA+MADWF(fastPV)+zMobius accelerations

- We utilize complexified 5d hopping term of Mobius action [Brower, Neff, Orginos], zMobius, for a better approximation of the sign function.

$$\epsilon_L(h_M) = \frac{\prod_s^L (1+\omega_s^{-1} h_M) - \prod_s^L (1-\omega_s^{-1} h_M)}{\prod_s^L (1+\omega_s^{-1} h_M) + \prod_s^L (1-\omega_s^{-1} h_M)}, \qquad \omega_s^{-1} = b + c \in \mathbb{C}$$

- 1/a~2 GeV, Ls=48 Shamir ~ Ls=24 Mobius (b=1.5, c=0.5) ~ Ls=10 zMobius (b_s, c_s complex varying) ~5 times saving for cost AND memory



Ls	|eps(48cube) – eps(zMobius)|
6	0.0124
8	0.00127
10	0.000110
12	8.05e-6

- The even/odd preconditioning is optimized (sym2 precondition) to suppress the growth of condition number due to order of magnitudes hierarchy of b_s, c_s [also Neff found this]

$$\texttt{sym2}: \quad 1 - \kappa_b M_4 M_5^{-1} \kappa_b M_4 M_5^{-1}$$

- Fast Pauli Villars (mf=1) solve, needed for the exact solve of AMA via MADWF (Yin, Mawhinney) is speed up by a factor of 4 or more by Fourier acceleration in 5D [Edward, Heller]
- All in all, sloppy solve compared to the traditional CG is 160 times faster on the physical point 48 cube case. And ~100 and 200 times for the 32 cube, Mpi=170 MeV, 140, in this proposal (1,200 eigenV for 32cube) .

$$\underbrace{\frac{20,000}{600}}_{\text{MADWF+zMobius+deflation}} \times \underbrace{\frac{600 * 32/10}{300}}_{\text{AMA+zMobius}} = 33.3 \times 6.4 = \underline{210 \text{ times faster}}$$

[Blum, TI, Shintani PRD 88 (2013) 094503]

Covariant Approximation Averaging (CAA) a new class of Error reduction techniques

Original

$$\mathcal{O} = \mathcal{O}^{(\mathrm{appx})} + \mathcal{O}^{(\mathrm{rest})}$$

Lattice Symmetry

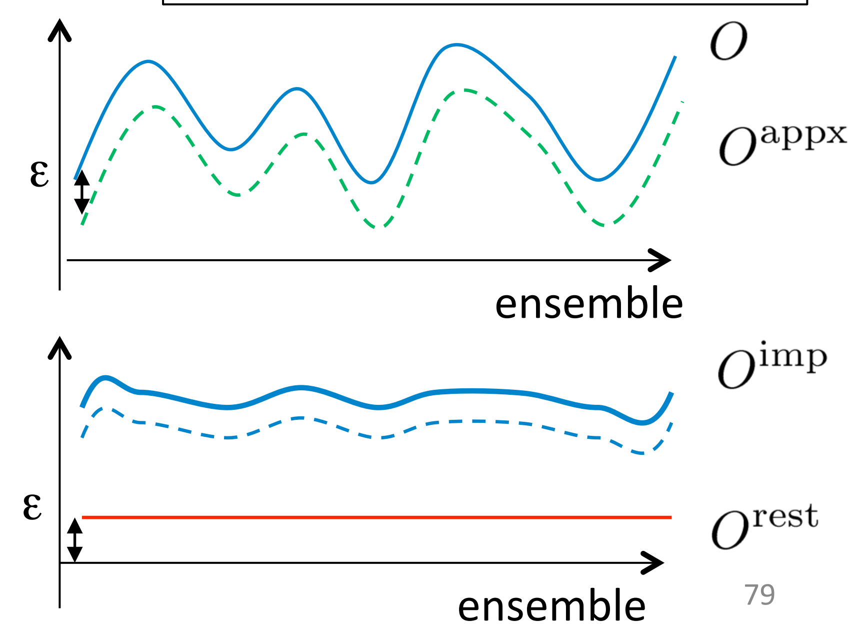
unbiased imporved

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

Expensive : infrequently measured

Cheap : frequently measured

- $O^{(\mathrm{imp})}$ has smaller error
  $O^{(\mathrm{appx})}$ need to be cheap & not to be too accurate
  N_G suppresses the bulk part of noise cheaply



New bias-free estimator even without covariant approximation by a stochastic choice of source location for the exact/rest computation is now available : Appendix D of arXiv:1402.0244

Examples of Covariant Approximations (contd.)

- All Mode Averaging **AMA**

  Sloppy CG or Polynomial approximations

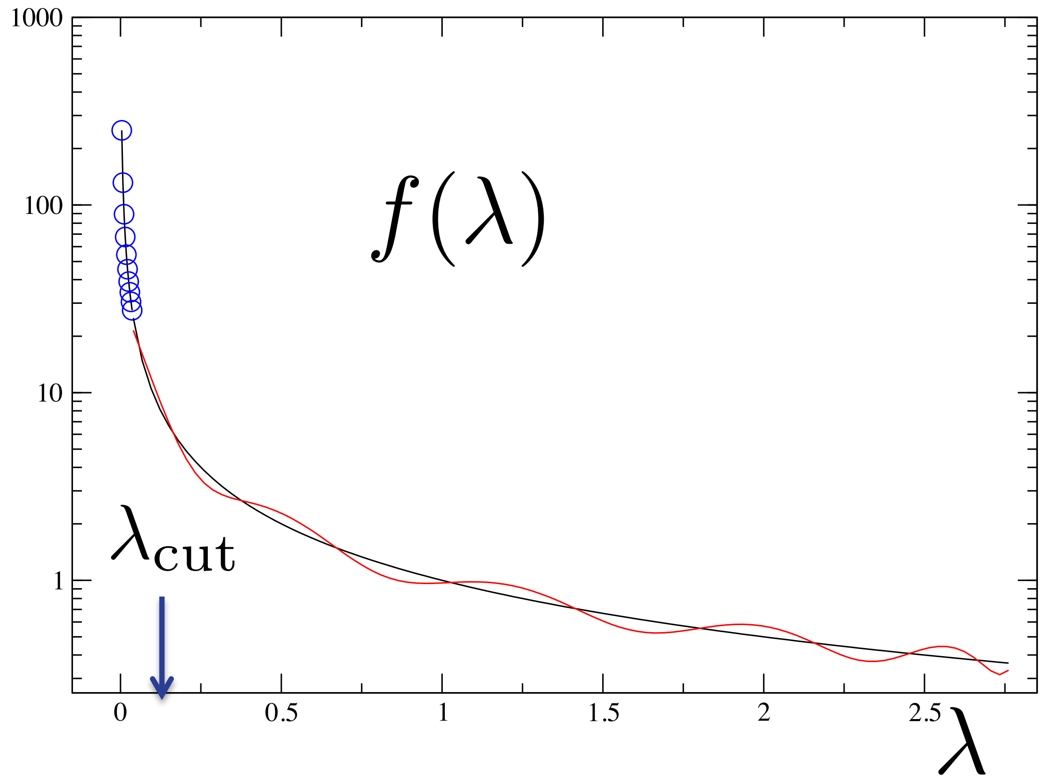
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$

If quark mass is heavy, e.g. ~ strange, low mode isolation may be unneccesary



accuracy control :

- low mode part : # of eig-mode
- mid-high mode : degree of poly.