Light tetraquarks and the chiral phase transition

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XYZ, P_c states: *strong* evidence for tetraquark & pentaquark states,
composed of both light and *heavy* quarks

Why do we need heavy quarks to see tetraquark states?
Jaffe '79…Schechter…Close…Tornqvist…Maiani…Giacosa….Peleaz '15:
"the" σ meson is "a" tetraquark (diquark anti-diquark). But situation murky….

Punchline: tetraquarks *must* be included to understand the phase diagram of QCD
(versus quark mass, plane of temperature T and baryon chemical potential μ)

Tetraquarks: for three (*not* two) flavors of *very* light quarks,
tetraquarks *may* generate a second chiral phase transition

For QCD, in the plane of T & μ plane:
direct connection between tetraquarks and color superconductivity

Tetraquarks for two light flavors: meh, no big deal

Chiral symmetry for two flavors

Classically, chiral symmetry $G_{cl} = SU(2)_L \times SU(2)_R \times U(1)_A = O(4) \times O(2)$

Use ϕ, complex 4-component vector
Linear σ model for exact χ symmetry:

$$\phi = (\sigma + i\,\eta\;,\;\vec{a}_0 + i\,\vec{\pi})$$

$$\mathcal{L}^{cl} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\,m^2\,\phi^* \cdot \phi + \lambda\,(\phi^* \cdot \phi)^2 + \ldots$$

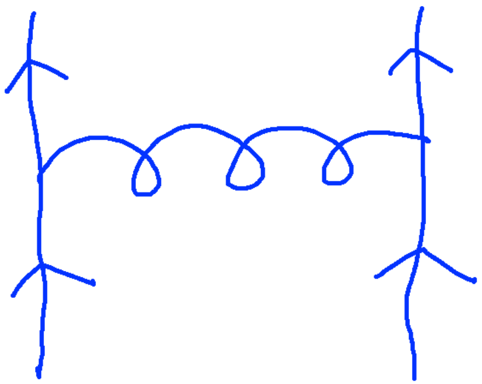
Quantum mechanically, axial anomaly reduces $U(1)_A$ -> $Z(2)_A$: $\phi \rightarrow (-)\,\phi$
Simplest term which is only $Z(2)_A$ invariant:

$$\mathcal{L}^{A} = +\,m_A^2\,(\mathrm{Im}\,\phi)^2 + \ldots$$

With G_{cl} , η degenerate with π . With axial anomaly, η splits from massless π
Directly induced by instantons + …

Diquarks and tetraquarks for two flavors

Jaffe '79: most attractive channel for quark-quark scattering is antisymmetric in *both* flavor and color.



Color: 3 x 3 = <u>3</u> (antisym) + 6 (sym)

Two flavors: 2 x 2 = 1 (antisym) + 3 (sym)

For two flavors diquark is a color triplet, flavor *singlet*,

$$\chi_L^A = \epsilon^{ABC} \, \epsilon^{ab} \, (q_L^{aB})^T \, \mathcal{C}^{-1} \, q_L^{bC}$$

(A, B, C = color; a, b = flavor) Also χ_R. χ_L and χ_R *singlets* under $Z(2)_A$.

One complex valued tetraquark field:

$$\zeta = (\chi_R^A)^* \, \chi_L^A$$

Sigma models and tetraquarks for two flavors

The tetraquark field ζ is a *singlet* under flavor and $Z(2)_A$.

Split complex ζ into its real and imaginary parts, ζ_r and ζ_i.

QCD is even under parity, so only even powers of ζ_i can appears, forget ζ_i.

But *any* powers of ζ_r can!

$$\mathcal{V}^A_{\zeta_r} = h_r\,\zeta_r + m_r^2\,\zeta_r^2 + \kappa_r\,\zeta_r^3 + \lambda_r\,\zeta_r^4$$

Hence $\langle \zeta_r \rangle$ is *always* nonzero!

Couplings to ϕ start with: $\mathcal{V}_{\zeta\phi} = \kappa\,\zeta\,\phi^* \cdot \phi + \ldots$

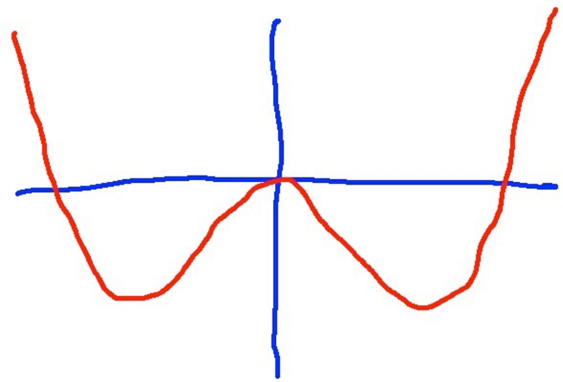
The tetraquark ζ_r is just a massive field with a v.e.v.

Chiral transition for two flavors

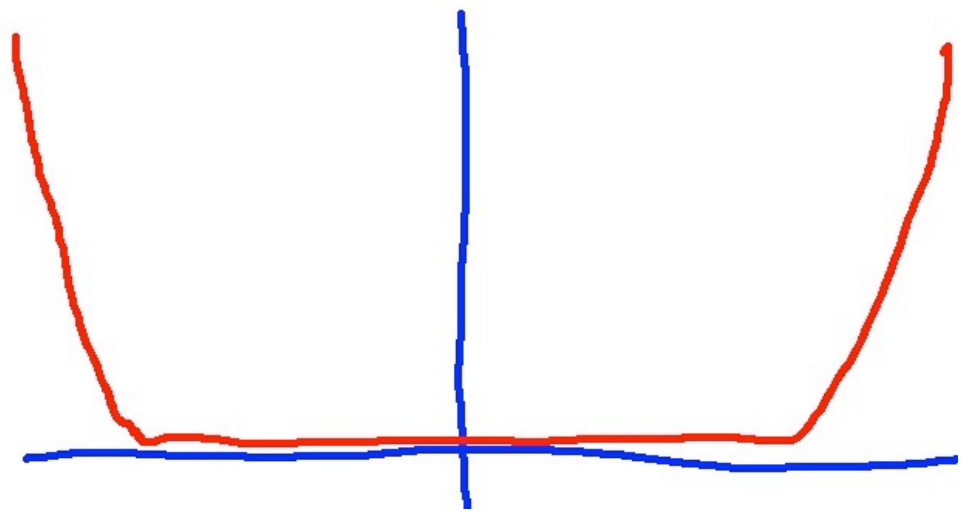
Assume anomaly is large up to T_χ .

$$\mathcal{V}_\phi = \frac{m^2}{2}\,\phi^2 + \lambda\,\phi^4$$

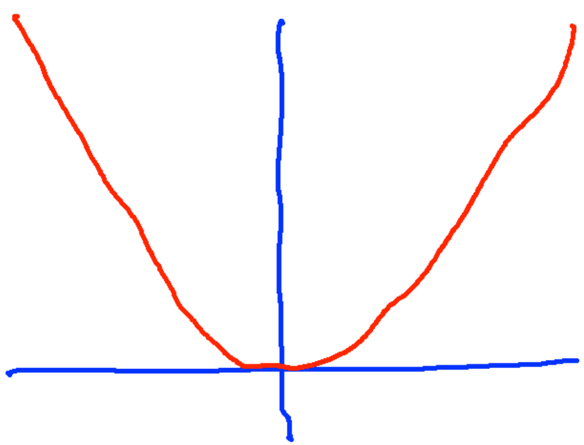
$T = 0$: $m^2 < 0$,
$\langle\phi\rangle \neq 0$ =>



$T = T_\chi$: $m^2 = 0$, $\langle\phi\rangle = 0$ ⇓



$T \gg f_\pi$: $m^2 > 0$,
$\langle\phi\rangle = 0$ =>



Chiral transition second order,
$m^2 = 0$ => infinite correlation length
O(4) universality class.

Tetraquark field just doesn't matter:
massive field with cubic couplings

Tetraquarks for three light flavors:
must be included

Chiral symmetry, three flavors

Quark Lagrangian

$$\mathcal{L}^{qk} = \overline{q}\, \not{D}\, q = \overline{q}_L \not{D}\, q_L + \overline{q}_R \not{D}\, q_R \quad , \quad q_{L,R} = \frac{1 \pm \gamma_5}{2}\, q$$

Classical $G_{cl} = SU(3)_L \times SU(3)_R \times U(1)_A$:

$$q_L \rightarrow \mathrm{e}^{-i\alpha/2}\, U_L\, q_L \quad , \quad q_R \rightarrow \mathrm{e}^{+i\alpha/2}\, U_R\, q_R$$

Order parameter for χ symmetry breaking (a, b… = flavor; A,B… = color)

$$\Phi^{ab} = \overline{q}_L^{\,bA}\, q_R^{aA} \qquad\qquad \Phi \rightarrow \mathrm{e}^{+i\alpha}\, U_R\, \Phi\, U_L^{\dagger}$$

Axial anomaly reduces $U(1)_A$ to $Z(3)_A$.

σ models for χ symmetry

$U(1)_A$ invariant terms:

$$\mathcal{V}_\Phi = m^2 \,\mathrm{tr}\left(\Phi^\dagger \Phi\right) - c_A \left(\det \Phi + \mathrm{c.c.}\right) + \lambda \,\mathrm{tr}\left(\Phi^\dagger \Phi\right)^2$$

Drop $(\mathrm{tr}\ \Phi^+\Phi)^2$, small.
χ sym. broken by background field:

$$\mathcal{V}_H^0 = -\ \mathrm{tr}\left(H\left(\Phi^\dagger + \Phi\right)\right)$$

To split the η' from the π, K, & η,
add $Z(3)_A$ invariant term

$$\mathcal{V}^A = \kappa \left(\det \Phi + \mathrm{c.c.}\right)$$

Example: SU(3) symmetric case, H = h **1**.
$J^P = 0^-$: octet π=K=η & singlet η'. $J^P = 0^+$: octet $a_0=\varkappa=\sigma_8$ & singlet σ
$h \neq 0$ so π's massive . Satisfy ('t Hooft '86):

$$m_{\eta'}^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2$$

The anomaly moves η' *up* from the π, but also moves singlet σ *down* from the a_0!
Light σ is a problem for σ models

Tetraquarks for three flavors

Three flavors: 3 x 3 = $\underline{3}$ + 6 .
Diquark field flavor *anti-triplet*, $\underline{3}$

$$\chi_L^{aA} = \epsilon^{abc}\, \epsilon^{ABC}\, (q_L^{bB})^T\, \mathcal{C}^{-1}\, q_L^{cC}$$

LR tetraquark field ζ transforms *identically* to Φ under G_χ = $SU(3)_L$ x $SU(3)_R$

$$\zeta^{ab} = (\chi_R^{aA})^*\, \chi_L^{bA}$$

Under $U(1)_A$, Φ has charge +1, ζ charge -2.

Since ζ & Φ in *same* representation of G_χ, *direct* mixing term. $Z(3)_A$ invariant:

Black, Fariborz, Schechter ph/9808415 + ….;
’t Hooft, Isidori, Maiani, Polosa 0801.2288 + ….

$$\mathcal{V}^A_{\zeta\Phi,2} = \widetilde{m}^2\, \mathrm{tr}\left(\zeta^\dagger \Phi + \Phi^\dagger \zeta\right)$$

An extra *dozen* couplings.
E.g., $U(1)_A$ invariant:

$$\mathcal{V}^{\infty}_{\zeta\Phi,3} = \kappa_\infty\, \epsilon^{abc}\, \epsilon^{a'b'c'} \left(\zeta^{aa'}\, \Phi^{bb'}\, \Phi^{cc'} + \text{c.c.}\right)$$

“Mirror” model, T = 0

Spectrum : $\Phi = \pi, K, \eta, \eta'; a_0, \kappa, \sigma_8, \sigma_0$; $\zeta = \widetilde{\pi}, \widetilde{K}, \widetilde{\eta}, \widetilde{\eta}'; \widetilde{a}_0, \widetilde{\kappa}, \widetilde{\sigma}_8, \widetilde{\sigma}_0$.

General model has 20 couplings:

Fariborz, Jora, & Schechter: ph/0506170; 0707.0843; 0801.2552. Pelaez, 1510.00653

Instead study “mirror” model, where Φ and ζ start with *identical* couplings

$$\mathcal{V}_\Phi = m^2 \,\mathrm{tr}\left(\Phi^\dagger \Phi\right) - \kappa\left(\det \Phi + \mathrm{c.c.}\right) + \lambda \,\mathrm{tr}\left(\Phi^+ \Phi\right)^2$$

$$\mathcal{V}_\zeta = m^2 \,\mathrm{tr}\left(\zeta^\dagger \zeta\right) - \kappa\left(\det \zeta + \mathrm{c.c.}\right) + \lambda \,\mathrm{tr}\left(\zeta^+ \zeta\right)^2$$

Assume only ζΦ coupling is mass term: $\mathcal{V}^A_{\zeta\Phi,2} = \widetilde{m}^2 \,\mathrm{tr}\left(\zeta^\dagger \Phi + \Phi^\dagger \zeta\right)$

Simple, because mass *only* mixes: $\pi \leftrightarrow \widetilde{\pi}\,,\; K \leftrightarrow \widetilde{K} \ldots$

Spectrum of the mirror model

In the chiral limit, the mass eigenstates: (need to assume $\widetilde{m}^2 < 0$)

$$\pi, \widetilde{\pi} = 0\,,\; -2\,\widetilde{m}^2 \quad ; \quad \eta', \widetilde{\eta}\,' = 3\,\kappa\,\phi\,,\; 3\,\kappa\,\phi - \; 2\,\widetilde{m}^2$$

$$a_0, \widetilde{a}_0 = m^2 + \kappa\,\phi + 6\,\lambda\,\phi^2 \pm \widetilde{m}^2 \quad ; \quad \sigma_0, \widetilde{\sigma} = m^2 - 2\,\kappa\,\phi + 6\,\lambda\,\phi^2 \pm \widetilde{m}^2$$

All states are mixtures of Φ and ζ. Of course 8 Goldstone bosons.
Satisfy *generalized* 't Hooft relation

$$m^2_{\eta'} + m^2_{\widetilde{\eta}'} - m^2_{\pi} - m^2_{\widetilde{\pi}} = m^2_{a_0} + m^2_{\widetilde{a}_0} - m^2_{\sigma} - m^2_{\widetilde{\sigma}}$$

Since *every* multiplet is doubled, this can easily be satisfied (unlike if just one).

Even with same couplings, all masses are *split* by the mixing term.

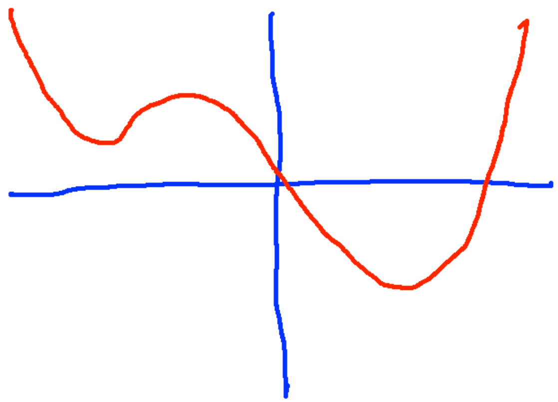
At nonzero T, the thermal masses of the Φ and ζ can*not* be equal!

Chiral transition for three flavors, no tetraquark

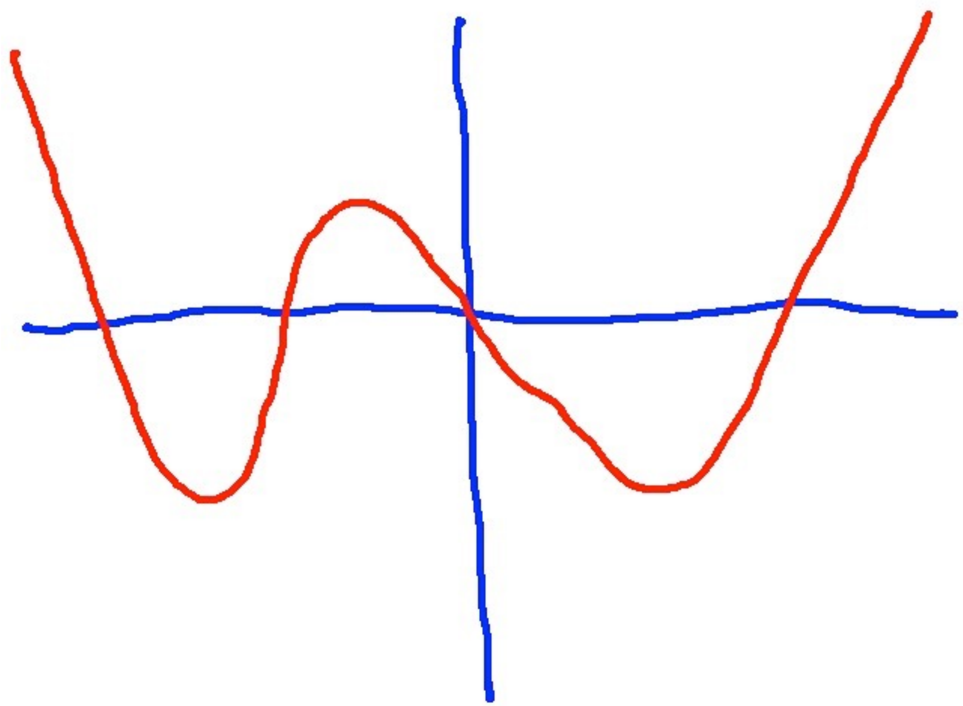
Cubic terms *always* generate *first* order transitions.

$$\mathcal{V}_\phi = \frac{m^2}{2}\,\phi^2 - \kappa\,\phi^3 + \lambda\,\phi^4$$

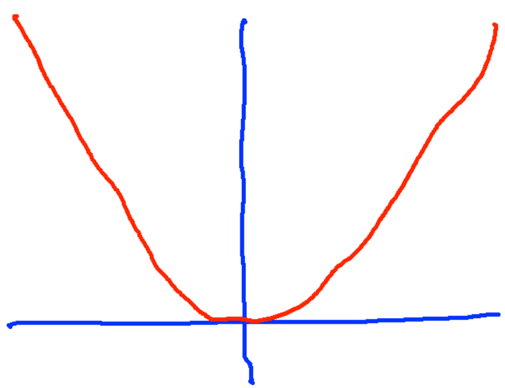
T = 0: $m^2 < 0$, $\langle\phi\rangle \neq 0$ =>



T= T_χ: can*not* flatten the potential =>



$T \gg f_\pi$: $m^2 > 0$, $\langle\phi\rangle = 0$ =>



At T_χ, two degenerate minima, with barrier between them. Transition is first order.

With tetraquarks, *maybe* two chiral transitions

In chiral limit, *may* have have *two* chiral phase transitions. =>

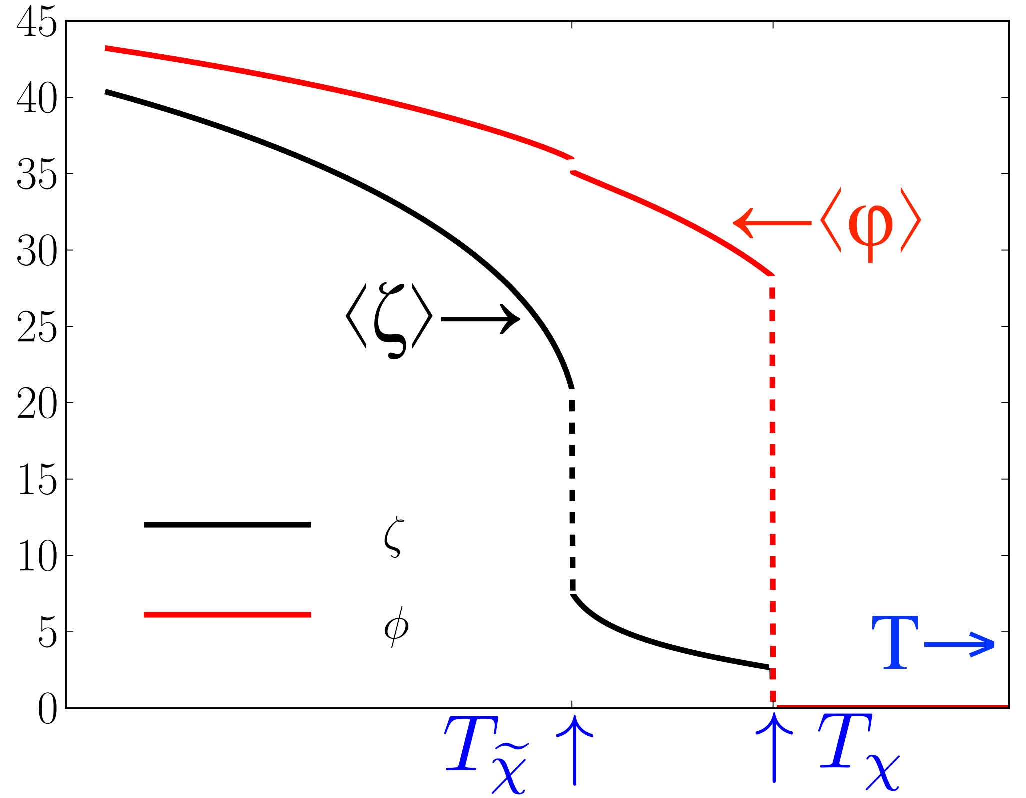
At first, both jump, remain nonzero.

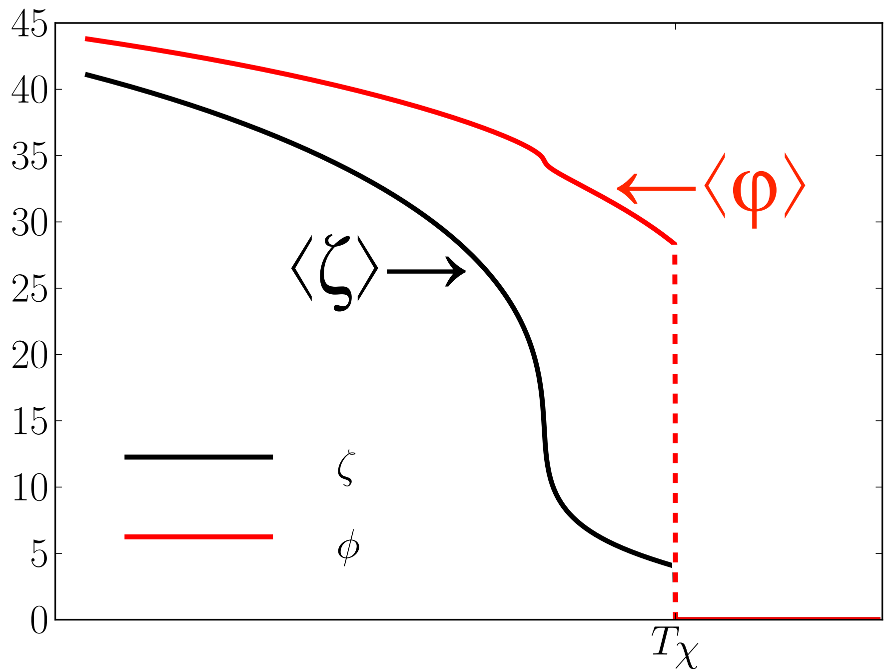
At second, both jump to zero.

$$m_\phi^2(T) = 3\,T^2 + m^2$$

$$m_\zeta^2(T) = 5\,T^2 + m^2$$

$$\widetilde{m}^2 = -(100)^2$$





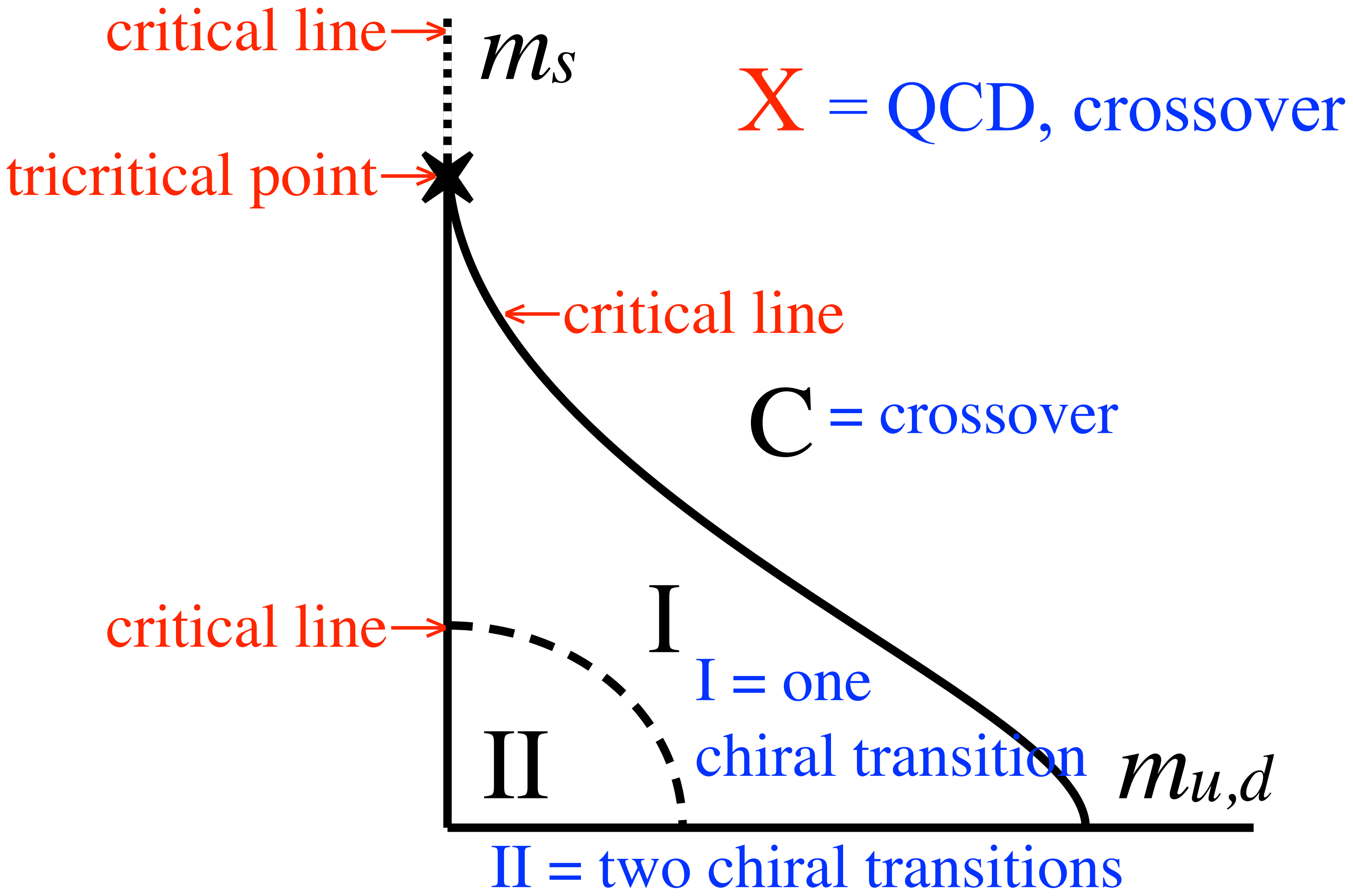
<= Also possible to have single chiral phase transition, tetraquark crossover

$$\widetilde{m}^2 = -(120)^2$$

"Columbia" phase diagram for light quarks

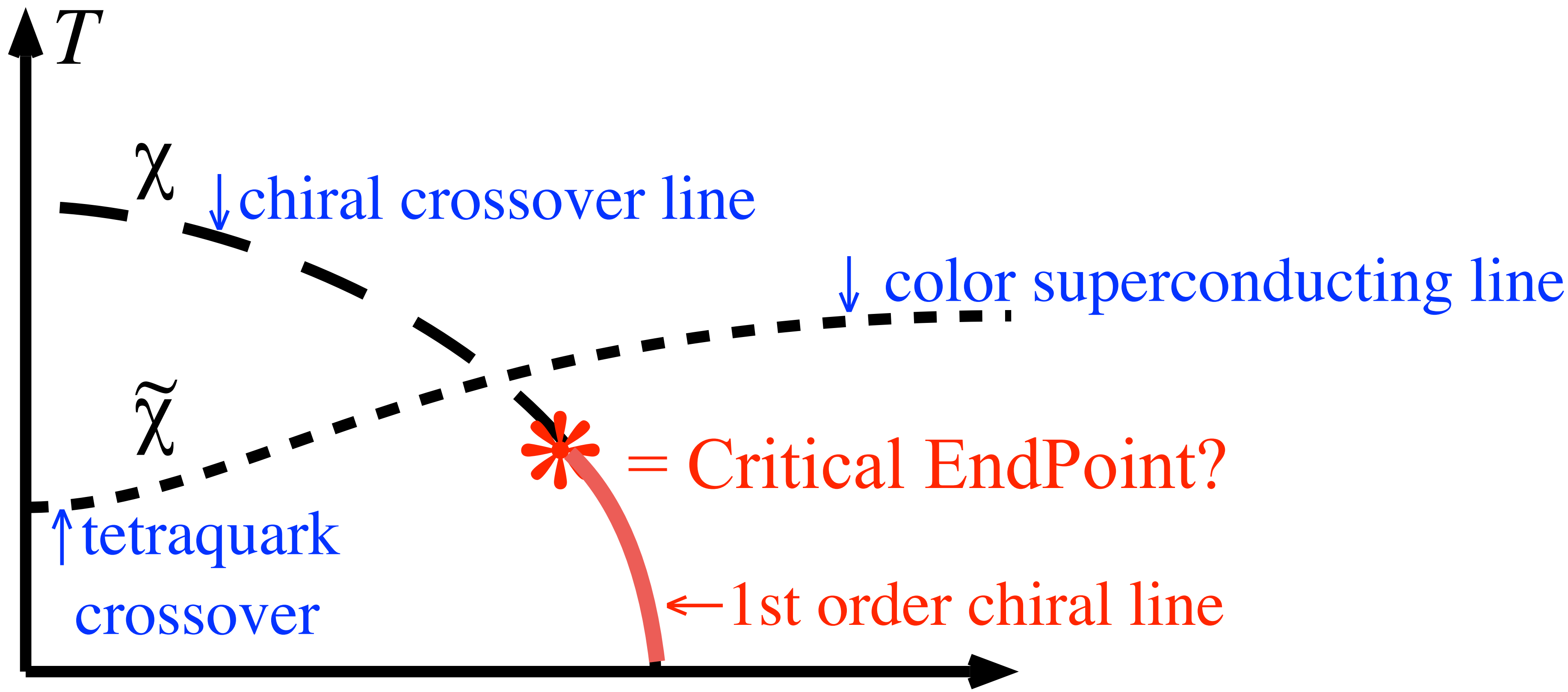
Lattice: chiral transition crossover in QCD

If two chiral phase transitions for three massless flavors, persist for nonzero range. Implies new phase diagram in the plane of $m_u = m_d$ versus m_s:



Tetraquarks in the plane of T and μ

Diquark fields are *identical* to the order parameters for color superconductivity.
Tetraquark condensate = gauge invariant square of CS condensate. Suggests:



Line for chiral crossover *might* end, meet line for *first* order chiral transition at *Critical EndPoint* (CEP). Massless σ at CEP. Rajagopal, Shuryak & Stephanov, '99

In *effective* models, to find the CEP, *must* include tetraquarks: need the *right* σ!

Four flavors and *hexa*quarks

Four flavors, three colors: diquark is 2-index antisymmetric tensor:

$$\chi_L^{(ab)A} = \epsilon^{abcd}\,\epsilon^{ABC}\,(q_L^{cB})^T\,\mathcal{C}^{-1}\,q_L^{dC}$$

So LR tetraquark is same:

$$\zeta^{(ab),(cd)} = \left(\chi_R^{(ab)A}\right)^\dagger\,\chi_L^{(cd)A}$$

Tetraquark couples to usual Φ through cubic, quadratic terms, so what.

Instead, consider triquark field:

$$\chi_L^a = \epsilon^{abcd}\,\epsilon^{ABC}\,q_L^{bA}\,(q_L^{cB})^T\,\mathcal{C}^{-1}\,q_L^{dC}$$

Triquark is a color singlet, fundamental rep. in flavor.
Hence a LR *hexa*quark field is just like the usual Φ,
and mixes *directly* with it.

$$\xi^{ab} = (\chi_R^a)^\dagger\,\chi_L^b$$

Like color superconductivity, the analysis for general numbers of flavors and colors is *not* trivial.

Why is the lattice afraid to measure the 0^+ multiplet?

Admittedly, difficult: need to include disconnected diagrams.
Signal/noise much smaller. Still, possible:

LatKMI collaboration: Yasumichi Aoki et al, 1403.5000

Three colors, *eight* flavors. Measure the masses of the σ, π, η : $m_\sigma \simeq m_\pi \ll m_{\eta'}$.