Light tetraquarks and the chiral phase transition

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XYZ, P_c states: *strong* evidence for tetraquark & pentaquark states, composed of both light and *heavy* quarks

*Why* do we need heavy quarks to see tetraquark states? Jaffe ’79…Schechter…Close…Tornqvist…Maiani…Giacosa….Peleaz ’15: “the” σ meson is “a” tetraquark (diquark anti-diquark). But situation murky….

Punchline: tetraquarks *must* be included to understand the phase diagram of QCD (versus quark mass, plane of temperature T and baryon chemical potential μ)

Tetraquarks: for three (*not* two) flavors of *very* light quarks, tetraquarks *may* generate a second chiral phase transition

For QCD, in the plane of T & μ plane: *direct* connection between tetraquarks and color superconductivity
Tetraquarks for two light flavors: meh, no big deal
Chiral symmetry for two flavors

Classically, chiral symmetry $G_{\text{cl}} = \text{SU}(2)_L \times \text{SU}(2)_R \times U(1)_A = \text{O}(4) \times \text{O}(2)$

Use $\phi$, complex 4-component vector
Linear $\sigma$ model for exact $\chi$ symmetry:

\[ \phi = (\sigma + i \eta, \vec{a}_0 + i \vec{\pi}) \]

\[ \mathcal{L}^{\text{cl}} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^* \cdot \phi + \lambda (\phi^* \cdot \phi)^2 + \ldots \]

Quantum mechanically, axial anomaly reduces $U(1)_A \to Z(2)_A$:
$\phi \rightarrow (-) \phi$

Simplest term which is only $Z(2)_A$ invariant:

\[ \mathcal{L}^A = + m^2_A (\text{Im} \phi)^2 + \ldots \]

With $G_{\text{cl}}$, $\eta$ degenerate with $\pi$. With axial anomaly, $\eta$ splits from massless $\pi$

Directly induced by instantons + \ldots
Diquarks and tetraquarks for two flavors

Jaffe ’79: most attractive channel for quark-quark scattering is antisymmetric in both flavor and color.

Color: $3 \times 3 = 3$ (antisym) + 6 (sym)

Two flavors: $2 \times 2 = 1$ (antisym) + 3 (sym)

For two flavors diquark is a color triplet, flavor singlet,

$$\chi^A_L = \epsilon^{ABC} \epsilon^{ab} (q^a_L)^T C^{-1} q^b_L$$

(A, B, C = color; a, b = flavor) Also $\chi_R$. $\chi_L$ and $\chi_R$ singlets under $Z(2)_A$.

One complex valued tetraquark field:

$$\zeta = (\chi^A_R)^* \chi^A_L$$
The tetraquark field $\zeta$ is a *singlet* under flavor and $Z(2)_A$.

Split complex $\zeta$ into its real and imaginary parts, $\zeta_r$ and $\zeta_i$.

QCD is even under parity, so only even powers of $\zeta_i$ can appear, forget $\zeta_i$.

But *any* powers of $\zeta_r$ can!

$$V_A^{\zeta_r} = h_r \zeta_r + m_r^2 \zeta_r^2 + \kappa_r \zeta_r^3 + \lambda_r \zeta_r^4$$

Hence $\langle \zeta_r \rangle$ is *always* nonzero!

Couplings to $\phi$ start with:

$$V_{\zeta\phi} = \kappa \zeta \phi^* \cdot \phi + \ldots$$

The tetraquark $\zeta_r$ is just a massive field with a v.e.v.
Chiral transition for two flavors

Assume anomaly is large up to \( T_\chi \).

\[
V_\phi = \frac{m^2}{2} \phi^2 + \lambda \phi^4
\]

\( T = 0: m^2 < 0, \langle \phi \rangle \neq 0 \Rightarrow T \gg f_\pi: m^2 > 0, \langle \phi \rangle = 0 \downarrow \)

Chiral transition second order, \( m^2 = 0 \Rightarrow \) infinite correlation length \( O(4) \) universality class.

Tetraquark field just doesn’t matter: massive field with cubic couplings
Tetraquarks for three light flavors: *must* be included
Chiral symmetry, three flavors

Quark Lagrangian

\[ \mathcal{L}^{q_k} = \bar{q} \not{D} q = \bar{q}_L \not{D} q_L + \bar{q}_R \not{D} q_R , \quad q_{L,R} = \frac{1 \pm \gamma_5}{2} q \]

Classical \( G_{cl} = SU(3)_L \times SU(3)_R \times U(1)_A \):

\[ q_L \rightarrow e^{-i\alpha/2} U_L q_L , \quad q_R \rightarrow e^{+i\alpha/2} U_R q_R \]

Order parameter for \( \chi \) symmetry breaking (a, b... = flavor; A,B... = color)

\[ \Phi^{ab} = \bar{q}^{bA}_L q^{aA}_R \]

Axial anomaly reduces \( U(1)_A \) to \( Z(3)_{A} \).
σ models for χ symmetry

U(1)\(_A\) invariant terms:

\[ \mathcal{V}_\Phi = m^2 \text{tr} (\Phi^\dagger \Phi) - c_A (\det \Phi + \text{c.c.}) + \lambda \text{tr} (\Phi^\dagger \Phi)^2 \]

Drop \((\text{tr } \Phi^\dagger \Phi)^2\), small.

χ sym. broken by background field:

\[ \mathcal{V}_H^0 = - \text{tr} \left( H (\Phi^\dagger + \Phi) \right) \]

To split the \(\eta'\) from the \(\pi, K, \& \eta\),
add \(Z(3)_A\) invariant term

\[ \mathcal{V}^A = \kappa (\det \Phi + \text{c.c.}) \]

Example: SU(3) symmetric case, \(\mathbf{H} = h \mathbf{1}\).
\(J^p = 0^-\): octet \(\pi=K=\eta\) & singlet \(\eta'\). \(J^p = 0^+\): octet \(a_0=\kappa=\sigma_8\) & singlet \(\sigma\)
\(h \neq 0\) so \(\pi\)'s massive. Satisfy ('t Hooft ’86):

\[ m_{\eta'}^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2 \]

The anomaly moves \(\eta'\) \textit{up} from the \(\pi\), but also moves singlet \(\sigma\) \textit{down} from the \(a_0\)!
Light \(\sigma\) is a problem for \(\sigma\) models
Tetraquarks for three flavors

Three flavors: $3 \times 3 = \mathbf{3} + 6$.

Diquark field flavor *anti-triplet*, $\mathbf{\bar{3}}$

LR tetraquark field $\zeta$ transforms *identically* to $\Phi$ under $G_\chi = SU(3)_L \times SU(3)_R$

Under $U(1)_A$, $\Phi$ has charge $+1$, $\zeta$ charge $-2$.

Since $\zeta$ & $\Phi$ in *same* representation of $G_\chi$, *direct* mixing term. $Z(3)_A$ invariant:

$$V_{\zeta \Phi, 2} = \tilde{m}^2 \text{ tr } (\zeta^\dagger \Phi + \Phi^\dagger \zeta)$$

Black, Fariborz, Schechter ph/9808415 + …;
’t Hooft, Isidori, Maiani, Polosa 0801.2288 + ….

An extra *dozen* couplings.

E.g., $U(1)_A$ invariant:

$$V_{\zeta \Phi, 3} = \kappa_{\infty} \epsilon^{abc} \epsilon^{a'b'c'} (\zeta^{aa'} \Phi^{bb'} \Phi^{cc'} + \text{c.c.})$$
“Mirror” model, $T = 0$

Spectrum: $\Phi = \pi, K, \eta, \eta'; a_0, \kappa, \sigma_8, \sigma_0$; $\zeta = \tilde{\pi}, \tilde{K}, \tilde{\eta}, \tilde{\eta}'; \tilde{a}_0, \tilde{\kappa}, \tilde{\sigma}_8, \tilde{\sigma}_0$.

General model has 20 couplings:

Fariborz, Jora, & Schechter: ph/0506170; 0707.0843; 0801.2552. Pelaez, 1510.00653

Instead study “mirror” model, where $\Phi$ and $\zeta$ start with identical couplings

$$V_{\Phi} = m^2 \text{tr} (\Phi^\dagger \Phi) - \kappa (\det \Phi + \text{c.c.}) + \lambda \text{tr} (\Phi^+ \Phi)^2$$

$$V_{\zeta} = m^2 \text{tr} (\zeta^\dagger \zeta) - \kappa (\det \zeta + \text{c.c.}) + \lambda \text{tr} (\zeta^+ \zeta)^2$$

Assume only $\zeta \Phi$ coupling is mass term: $$V_{\zeta\Phi}^{A,2} = \tilde{m}^2 \text{tr} (\zeta^\dagger \Phi + \Phi^\dagger \zeta)$$

Simple, because mass only mixes: $\pi \leftrightarrow \tilde{\pi}$, $K \leftrightarrow \tilde{K}$ …
Spectrum of the mirror model

In the chiral limit, the mass eigenstates: (need to assume $\tilde{m}^2 < 0$)

\[ \pi, \tilde{\pi} = 0, -2 \tilde{m}^2 \quad ; \quad \eta', \tilde{\eta}' = 3 \kappa \phi, 3 \kappa \phi - 2 \tilde{m}^2 \]

\[ a_0, \tilde{a}_0 = m^2 + \kappa \phi + 6 \lambda \phi^2 \pm \tilde{m}^2 \quad ; \quad \sigma_0, \tilde{\sigma} = m^2 - 2 \kappa \phi + 6 \lambda \phi^2 \pm \tilde{m}^2 \]

All states are mixtures of $\Phi$ and $\zeta$. Of course 8 Goldstone bosons. Satisfy generalized 't Hooft relation

\[ m_{\eta'}^2 + m_{\tilde{\eta}'}^2 - m_\pi^2 - m_{\tilde{\pi}}^2 = m_{a_0}^2 + m_{\tilde{a}_0}^2 - m_\sigma^2 - m_{\tilde{\sigma}}^2 \]

Since every multiplet is doubled, this can easily be satisfied (unlike if just one).

Even with same couplings, all masses are split by the mixing term.

At nonzero T, the thermal masses of the $\Phi$ and $\zeta$ cannot be equal!
Chiral transition for three flavors, no tetraquark

Cubic terms *always* generate *first* order transitions.

\[ V_\phi = \frac{m^2}{2} \phi^2 - \kappa \phi^3 + \lambda \phi^4 \]

\( T = 0 \): \( m^2 < 0 \), \( \langle \phi \rangle \neq 0 \) \( \Rightarrow \)

\( T = T_\chi \): *cannot* flatten the potential \( \Rightarrow \)

\( T \gg f_\pi \): \( m^2 > 0 \), \( \langle \phi \rangle = 0 \) \( \Rightarrow \)

At \( T_\chi \), two degenerate minima, with barrier between them.
Transition is first order.
With tetraquarks, *maybe* two chiral transitions

In chiral limit, *may* have *two* chiral phase transitions. => At first, both jump, remain nonzero. At second, both jump to zero.

\[ m_\phi^2(T) = 3T^2 + m^2 \]
\[ m_\zeta^2(T) = 5T^2 + m^2 \]
\[ \tilde{m}^2 = -(100)^2 \]

<= Also possible to have single chiral phase transition, tetraquark crossover

\[ \tilde{m}^2 = -(120)^2 \]
"Columbia" phase diagram for light quarks

Lattice: chiral transition crossover in QCD
If two chiral phase transitions for three massless flavors, persist for nonzero range. Implies new phase diagram in the plane of $m_u = m_d$ versus $m_s$:

I = one chiral transition

II = two chiral transitions

Critical line

Tricritical point

X = QCD, crossover

C = crossover
Tetraquarks in the plane of $T$ and $\mu$

Diquark fields are *identical* to the order parameters for color superconductivity. Tetraquark condensate = gauge invariant square of CS condensate. Suggests:

\[ \chi \rightarrow \text{tetraquark crossover} \]

\[ \widetilde{\chi} \rightarrow \text{chiral crossover line} \]

\[ \rightarrow \text{color superconducting line} \]

Line for chiral crossover *might* end, meet line for *first* order chiral transition at Critical EndPoint (CEP). Massless $\sigma$ at CEP. Rajagopal, Shuryak & Stephanov, ’99

In *effective* models, to find the CEP, *must* include tetraquarks: need the *right* $\sigma$!
**Four flavors and hexaquarks**

Four flavors, three colors: diquark is 2-index antisymmetric tensor:

\[ \chi_L^{(ab)A} = \epsilon^{abcd} \epsilon^{ABC} \left( q_L^c B \right)^T C^{-1} q_L^d \]

So LR tetraquark is same:

\[ \zeta^{(ab), (cd)} = \left( \chi_R^{(ab)A} \right)^\dagger \chi_L^{(cd)A} \]

Tetraquark couples to usual \( \Phi \) through cubic, quadratic terms, so what.

Instead, consider triquark field:

\[ \chi_L^a = \epsilon^{abcd} \epsilon^{ABC} q_L^b A \left( q_L^c B \right)^T C^{-1} q_L^d \]

Triquark is a color singlet, fundamental rep. in flavor. Hence a LR hexaquark field is just like the usual \( \Phi \), and mixes *directly* with it.

\[ \xi^{ab} = \left( \chi_R^a \right)^\dagger \chi_L^b \]

Like color superconductivity, the analysis for general numbers of flavors and colors is *not* trivial.
Why is the lattice afraid to measure the $0^+$ multiplet?

Admittedly, difficult: need to include disconnected diagrams.
Signal/noise much smaller. Still, possible:

LatKMI collaboration: Yasumichi Aoki et al, 1403.5000
Three colors, eight flavors. Measure the masses of the $\sigma, \pi, \eta : m_\sigma \approx m_\pi \ll m_\eta$. 