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# Two-photon exchange effects in $ep \rightarrow eN\pi$ at $\Delta(1232)$ peak

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2016.07.26, Kyoto

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# Outline

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1. Motivation
  2. Introduction
  3. TPE in  $ep \rightarrow eN\pi$  at  $\Delta(1232)$  peak
  4. Numerical results and summary
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# Motivation

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1. It has been known that the two photon exchange effects give large corrections to the ratio of proton's EM form factors  $\mu_p R = \mu_p G_E(Q^2)/G_M(Q^2)$  by Rosenbluth method (extract the ratio from the un-polarized ep scattering data).

A natural question is how about **TPE corrections in the other processes** when we aim at the precise measurement of hadrons' electromagnetic structure?

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# Motivation

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2.  $\Delta(1232)$  is the first excitation of the nucleon, the precise extraction of its deform is an interesting question (is  $\Delta$  spherical? Namely, does it have a D-state?).

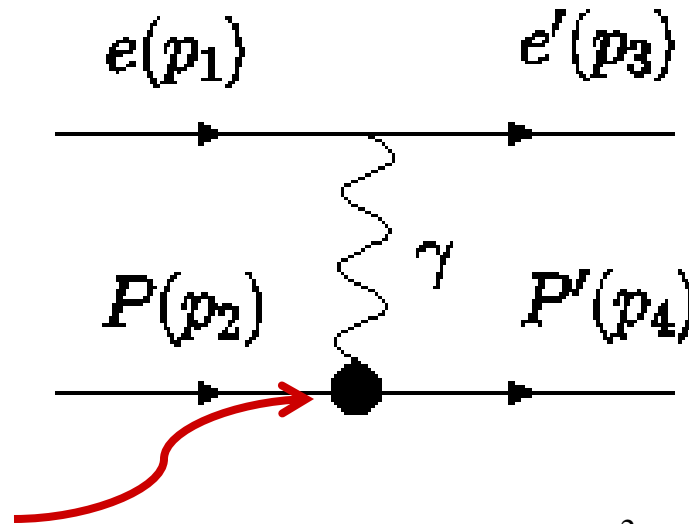
The precise measurement of the cross sections of electromagnetic production of pion on  $\Delta(1232)$  peak, calls for the corresponding precise theoretical estimation on the radiative corrections. The TPE contribution at high  $Q^2$  was first studied in 2006 by using the GPDs methods. This also attracts us to study this question at small  $Q^2$ .

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# Introduction: one photon exchange in ep→ep

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In ep→ep scattering, the one photon exchange approximation (OPE) is used to extract the EM form factors of proton before 2003.



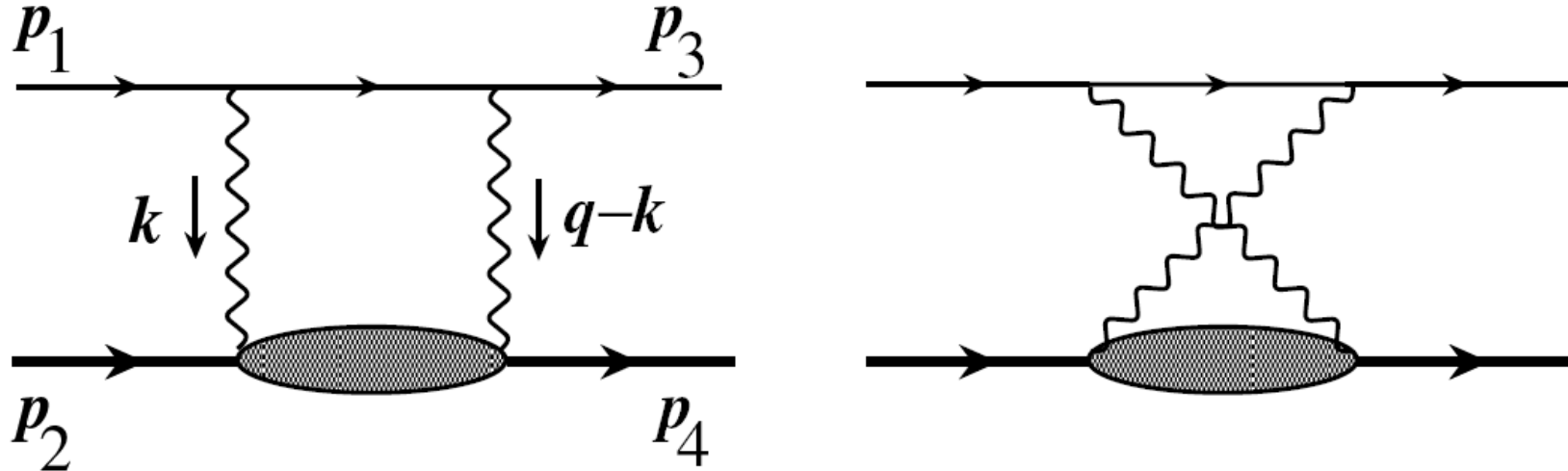
$$\Gamma_{\gamma N \rightarrow N}^{\mu}(q) = ie \left[ F_1(q^2) \gamma_{\mu} + \frac{F_2(q^2)}{m_N} \sigma_{\mu\nu} q^{\nu} \right]$$

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EM radiative corrections are also considered and **soft photon approximation** is used in TPE before 2003.

# Introduction: TPE in unpolarized ep→ep

In 2003, it is found that the TPE contribution (with finite  $k$ ) gives large corrections to the extracted  $\mu_p R$  from the unpolarized ep scattering data [1].

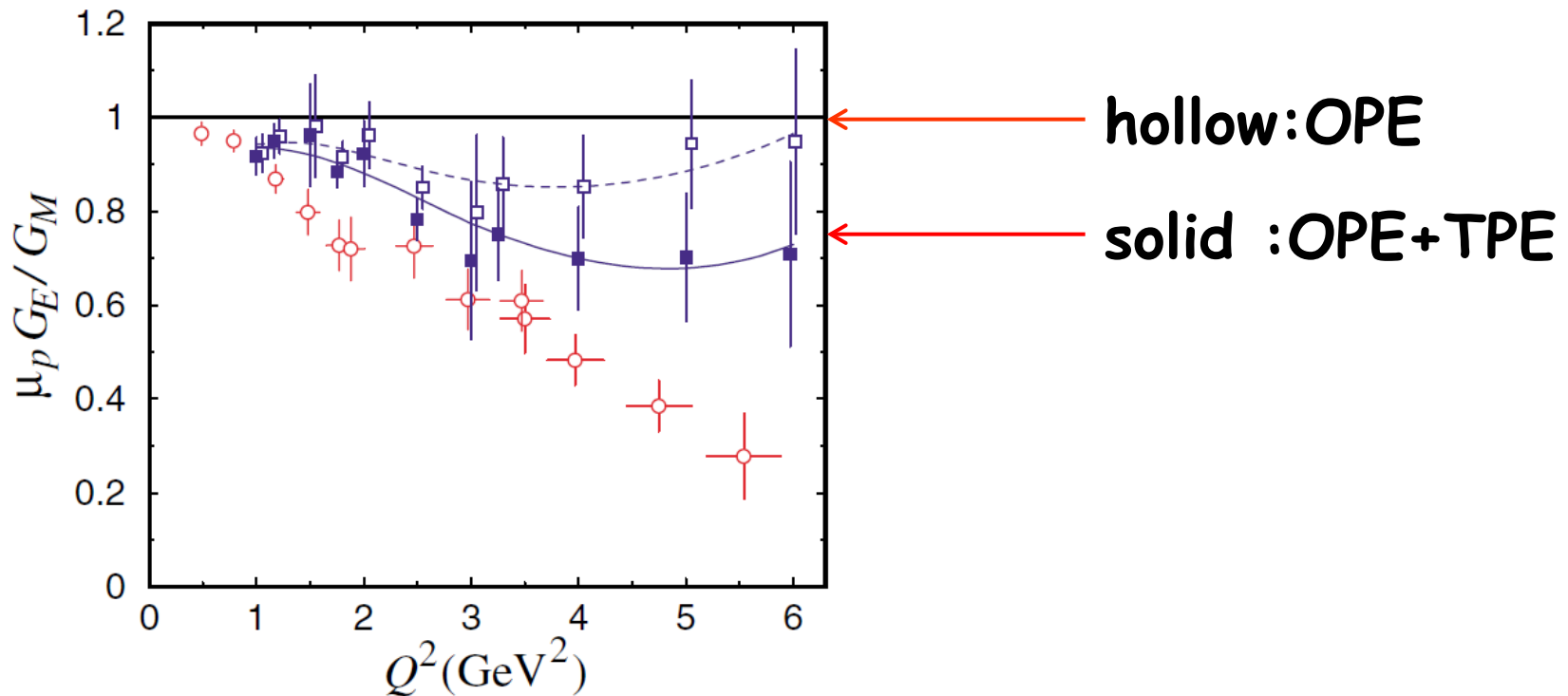


TPE exchange contribution with finite  $k$

[1] P.G. Blunden, W. Melnitchouk and J. A. Tjon, PRL91 (2003) 142304

# Introduction: TPE in unpolarized $ep \rightarrow ep$

numerical results for the TPE corrections to  $\mu_p R$  [1]



[1] P.G. Blunden, W. Melnitchouk and J. A. Tjon, PRL91 (2003) 142304

# Introduction: TPE in ep→ep

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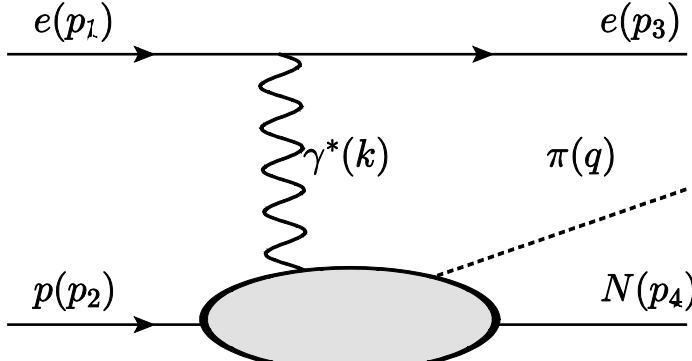
many model dependent methods are used to estimate the TPE effects in the literature.

(1) hadronic model:	J.A.Tjon	etc
(2) GPDs:	M.Vanderhaeghen	etc
(3) dispersion relation:	C. J. Horowitz	etc
(4) pQCD:	M.Vanderhaeghen	etc
(5) SCEF:	M.Vanderhaeghen	etc



# $\gamma^* N \rightarrow \Delta$ FFs from the $ep \rightarrow eN\pi$

In OPE approximation, the amplitude for  $ep \rightarrow eN\pi$  can be written in a general form



$$iM_{ep \rightarrow eN\pi} = \bar{u}(p_3)(-ie\gamma_\mu)u(p_1) \frac{-i}{k^2} (iM_{\gamma^* p \rightarrow N\pi}^\mu)$$

$$iM_{\gamma^* p \rightarrow N\pi}^\mu = \sum_{i=1}^6 \bar{u}(p_4) A_i M_i^\mu u(p_2)$$

where  $M_i$  are 6 special Dirac matrix structures<sup>[1]</sup>, and the invariant amplitudes  $A_i$  can be expressed by the multipoles which are only dependent on the  $W$  and  $Q^2$

$$Q^2 = -(p_1 - p_3)^2, W^2 = (q + p_4)^2$$

[1] B. Pasquini, D. Drechsel, and L. Tiator, Eur. Phys. J. A 34, 387–403 (2007)

## $ep \rightarrow eN\pi$ in OPE

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in detail, the invariant amplitudes  $A_i$  can be expressed as functions of the CGLN amplitudes which can be expanded on the multipoles

$$\mathcal{F}_1 = \sum_{l=0}^{\infty} [(lM_{l+} + E_{l+}) P_{l+1}'(x) + ((l+1)M_{l-} + E_{l-}) P_{l-1}'(x)],$$

where the multipoles  $M_l$ ,  $E_l$  and  $S_l$  are only dependent on  $Q^2$ . And at the  $\Delta(1232)$  peak  $W=M_\Delta$ , the  $E_1^+$ ,  $M_1^+$ ,  $S_1^+$  play the most important roles, which reflect the electromagnetic form factors of  $\gamma^*N \rightarrow \Delta$ .

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# $ep \rightarrow eN\pi$ in OPE

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Models (under OPE approximation) are usually used to extract the  $E_1^+$ ,  $M_1^+$ ,  $S_1^+$  multipoles from the experimental data <sup>[1]</sup>, for example :

MAID2007 and SAID.

In these models, the background are all considered at  $\Delta(1232)$  peak. The including of the background is also important to extract the multipoles.

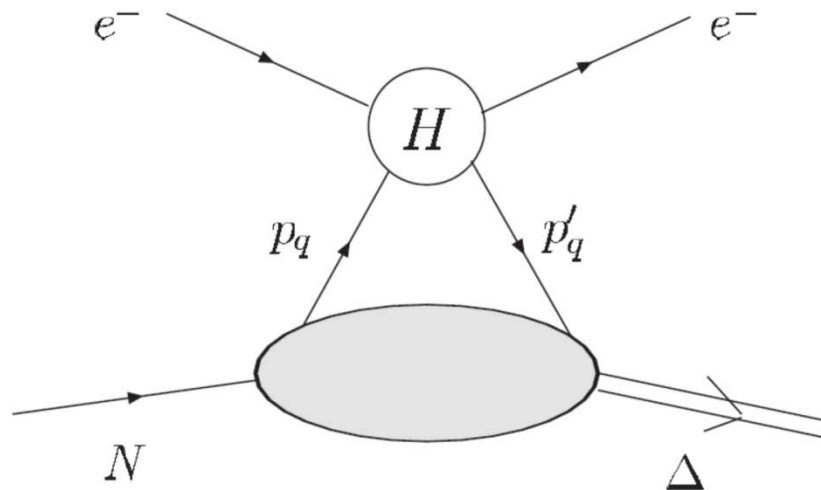
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[1] <http://portal.kph.uni-mainz.de/MAID//>

<http://gwdac.phys.gwu.edu/>

# TPE in the $ep \rightarrow eN\pi$

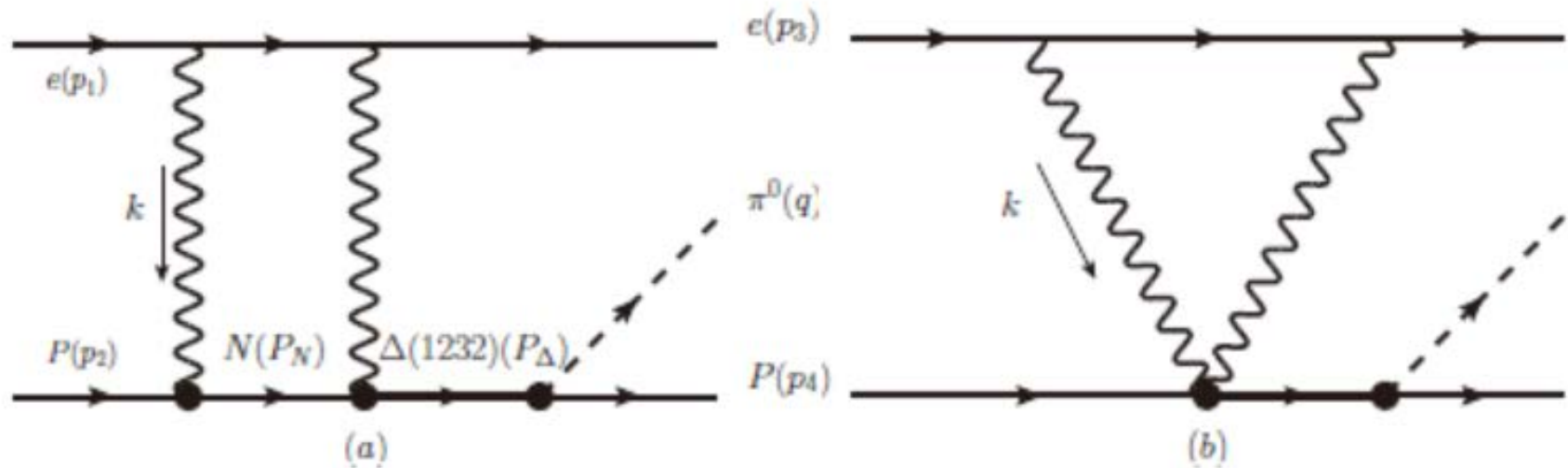
The TPE effect **in the  $ep \rightarrow eN\pi$**  at larger momentum transfer  $Q^2$  was first studied in 2006 by using the GPDs methods <sup>[1]</sup>, where only the  $\Delta(1232)$  peak contribution is considered in both the TPE and OPE.



$ep \rightarrow e\Delta \rightarrow eN\pi$  by GPDs

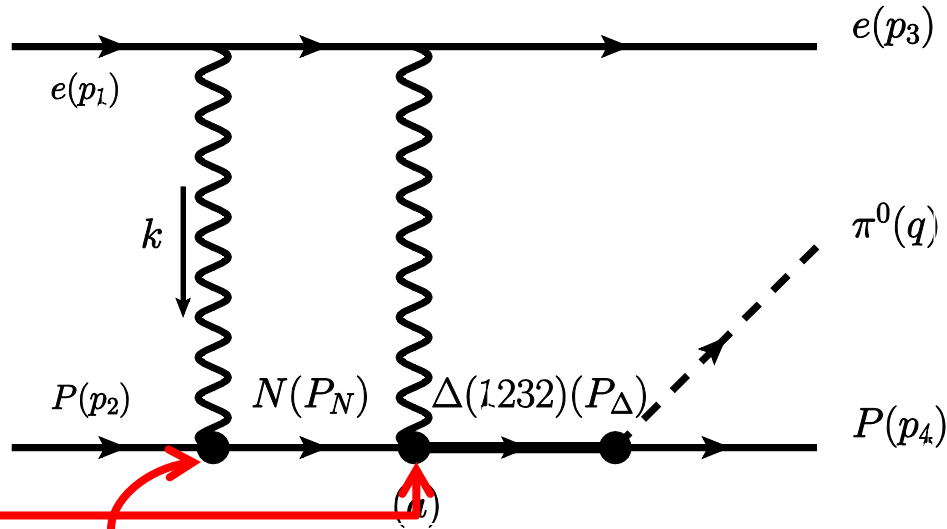
# TPE in $ep \rightarrow eN\pi$

In this talk, we report the TPE effect at low momentum transfer based on a simple hadronic model. We considered the following contributions in the electromagnetic production of pion at  $\Delta(1232)$  peak.



$ep \rightarrow e\Delta \rightarrow eN\pi$  TPE in hadronic model

# TPE in $e p \rightarrow e N \pi$ : the detail

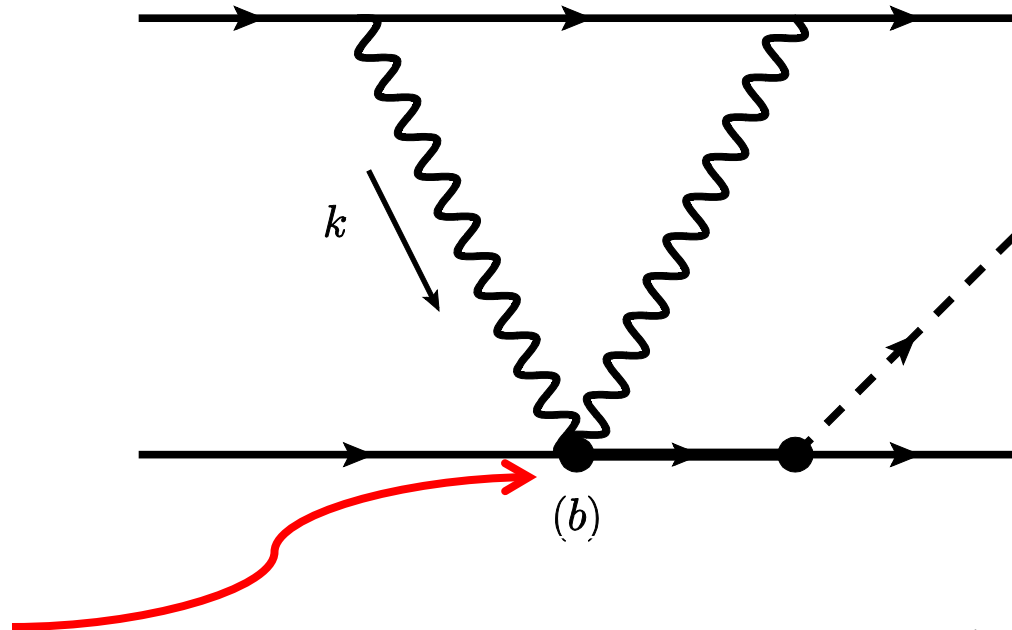


$$\Gamma_{\gamma N \rightarrow N}^\mu(q) = ie[F_1(q^2)\gamma_\mu + \frac{F_2(q^2)}{m_N}\sigma_{\mu\nu}q^\nu]$$

$$\Gamma_{\gamma N \rightarrow \Delta}^{\nu\beta}(p, q) = -i\sqrt{\frac{2}{3}}\frac{e}{2M_\Delta^2}\gamma_5\{g_1(q^2)[g^{\nu\beta}\not{q}\not{p} - p^\nu\not{q}\gamma^\beta - \gamma^\beta\gamma^\nu p\cdot q + \not{p}\gamma^\nu q^\beta]$$

$$+ g_2(q^2)[p^\nu q^\beta - g^{\nu\beta}p\cdot q] - g_3(q^2)/M_\Delta[q^2(p^\nu\gamma^\beta - g^{\nu\beta}\not{p}) + q^\nu(q^\beta\not{p} - \gamma^\beta p\cdot q)]\}$$

# TPE in $e p \rightarrow e N \pi$ : the detail



$$\Gamma_{\gamma N \Delta}^{\nu\nu\beta}(P_{\Delta}, p_2, k, \bar{k}) = e \left\{ (2p_2 + k)^{\nu} \frac{F_1(k)}{(p_2 + k)^2 - M_N^2} \Gamma_{\gamma N \rightarrow \Delta}^{\mu\beta}(P_{\Delta}, \bar{k}) \right. \\ \left. + (2p_2 + \bar{k})^{\nu} \frac{F_1(\bar{k})}{(p_{\Delta} - k)^2 - M_N^2} \Gamma_{\gamma N \rightarrow \Delta}^{\nu\beta}(P_{\Delta}, k) \right\}$$

# unpolarized cross sections for $ep \rightarrow eN\pi$

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The unpolarized cross section in OPE case and OPE+TPE case both can be written as

$$\frac{d\sigma^{1\gamma,2\gamma}}{d\Omega_\pi} = \sigma_0^{1\gamma,2\gamma} + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT}^{1\gamma,2\gamma} \cos\phi + \varepsilon\sigma_{TT}^{1\gamma,2\gamma} \cos 2\phi$$

where  $\varepsilon$  is the transverse polarization of the virtual photon,  $\phi$  is the angle between the electron scattering plane and the reaction plane,  $\theta$  is the angle between the momentums of pion and nucleon.

From the  $\phi$  dependence of the cs, the  $\sigma_{0,LT,TT}$  can be seperated.



# unpolarized cross sections for $ep \rightarrow eN\pi$

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In OPE case: both  $\sigma_{LT,TT}$  are  $\varepsilon$  independent, and  $\theta$  dependent

In OPE+TPE case: both  $\sigma_{LT,TT}$  are  $\varepsilon, \theta$  dependent

which means the TPE corrections to the cross sections are  $\varepsilon$  dependent.

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# from cross sections to multipole

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In this work, the TPE corrections to the cross sections are calculated directly for the TPE diagrams in the hadronic model, and we use MAID2007 and SAID to estimate the OPE amplitude.

To get the corresponding corrections to the multipoles, we fitting the cross sections using the formula under OPE approximation.

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# from cross sections to multipole

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We use the cross sections from the models to simulate the experimental data, and then get the “physical” cross sections after subtracting the TPE correction

$$\sigma_{0,LT,TT}^{Ex} \approx \sigma_{0,LT,TT}^{model} = \sigma_{0,LT,TT} (Z_{l^\pm}^{model})$$

$$\sigma_{0,LT,TT}^{1\gamma} = \sigma_{0,LT,TT}^{Ex} - \sigma_{0,LT,TT}^{2\gamma}$$

Since we only consider the TPE at the  $\Delta(1232)$  peak, which will only give contributions to the  $E_{1+}$ ,  $M_{1+}$ ,  $L_{1+}$ . then we can assume

$$\sigma_{0,LT,TT}^{1\gamma} = \sigma_{0,LT,TT} (Z_{l^\pm}^{model}, E_{1+}^{1\gamma}, M_{1+}^{1\gamma}, S_{1+}^{1\gamma})$$

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# from cross sections to multipole

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In practice, we use the following fitting to get the corresponding multipoles from  $\sigma_{0,LT,TT}^{Ex}$  and  $\sigma_{0,LT,TT}^{1\gamma}$

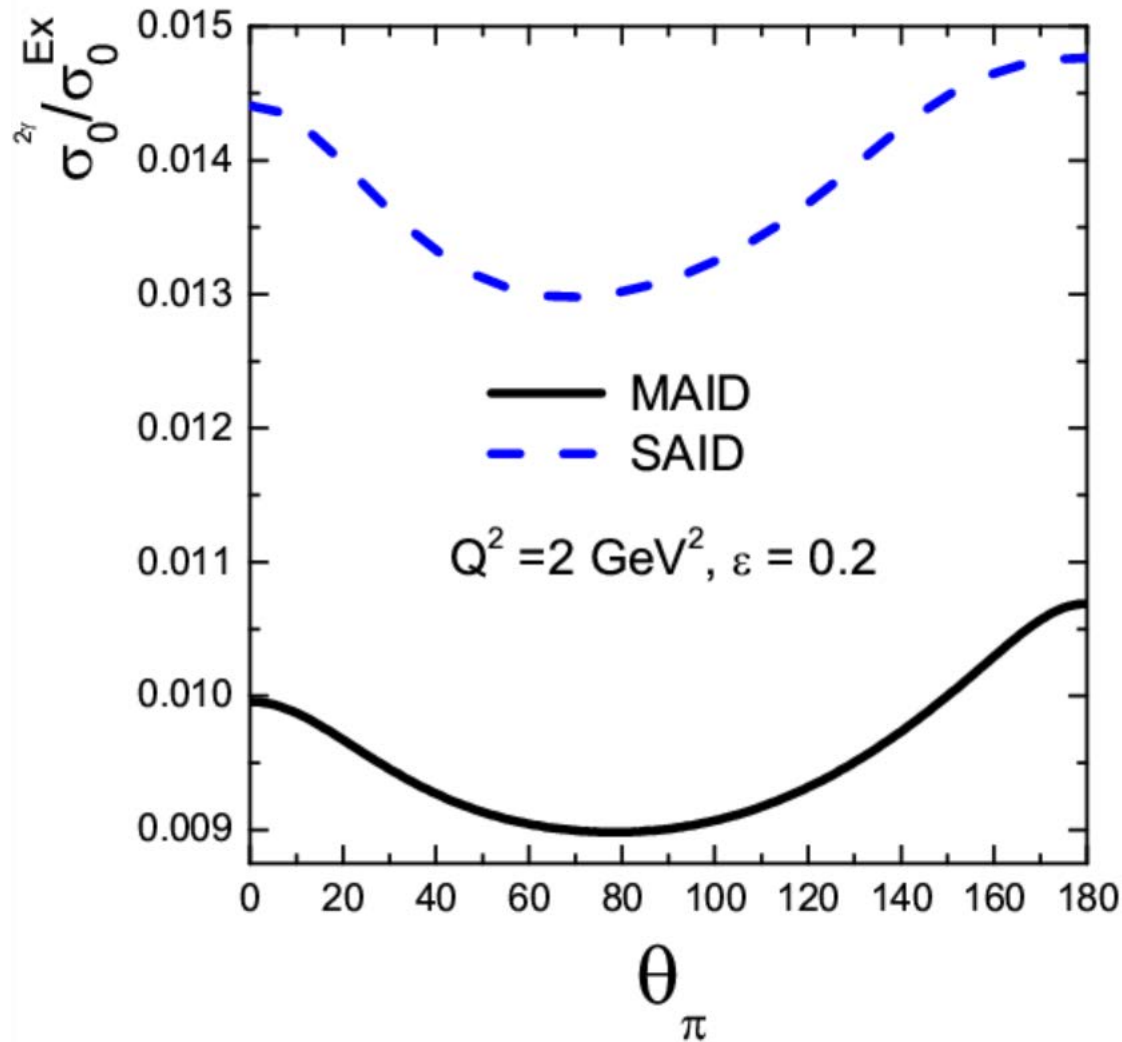
$$\chi^2 = \sum_{\theta=1}^{179} (\sigma_i^{Ex} - \sigma_i (|E_{1^+}^{Ex}|, |M_{1^+}^{Ex}|, |S_{1^+}^{Ex}|))$$

$$\chi^2 = \sum_{\theta=1}^{179} (\sigma_i^{1\gamma} - \sigma_i (|E_{1^+}^{1\gamma}|, |M_{1^+}^{1\gamma}|, |S_{1^+}^{1\gamma}|))$$

Here the contributions from the background have been included in the OPE, and the  $M_{1^+}$  dominance approximation is not used to extract the multipoles.

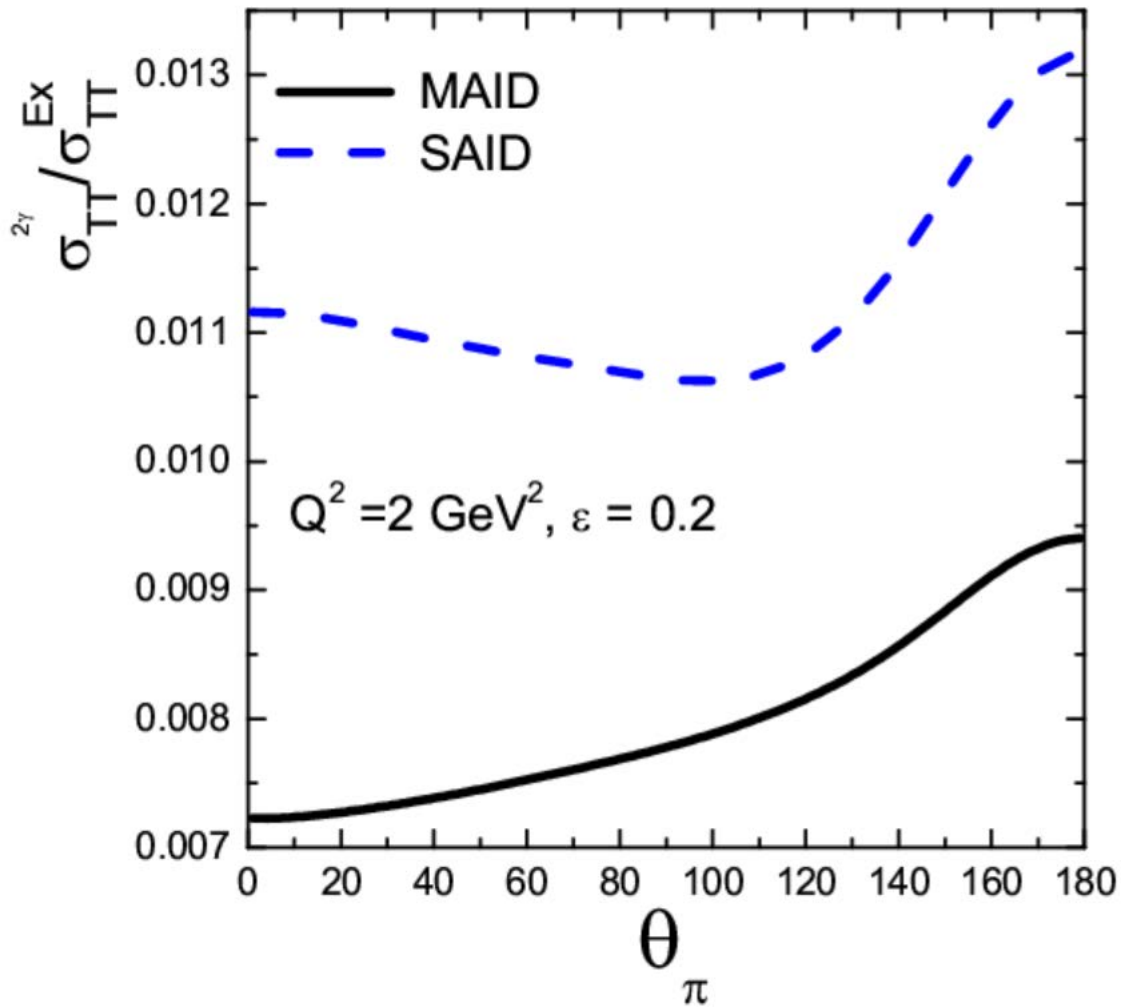
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# TPE correction to the cross sections



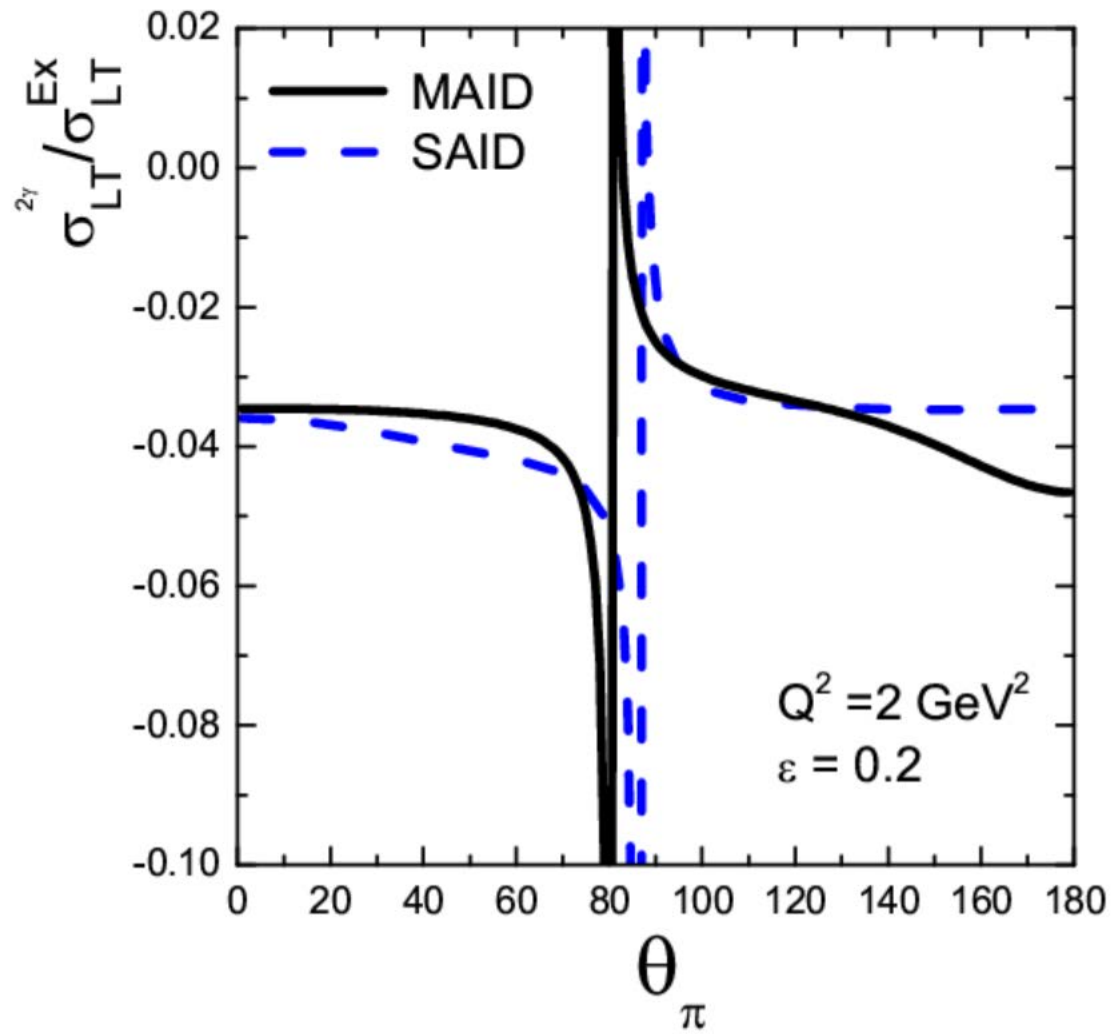
# TPE correction to the cross sections

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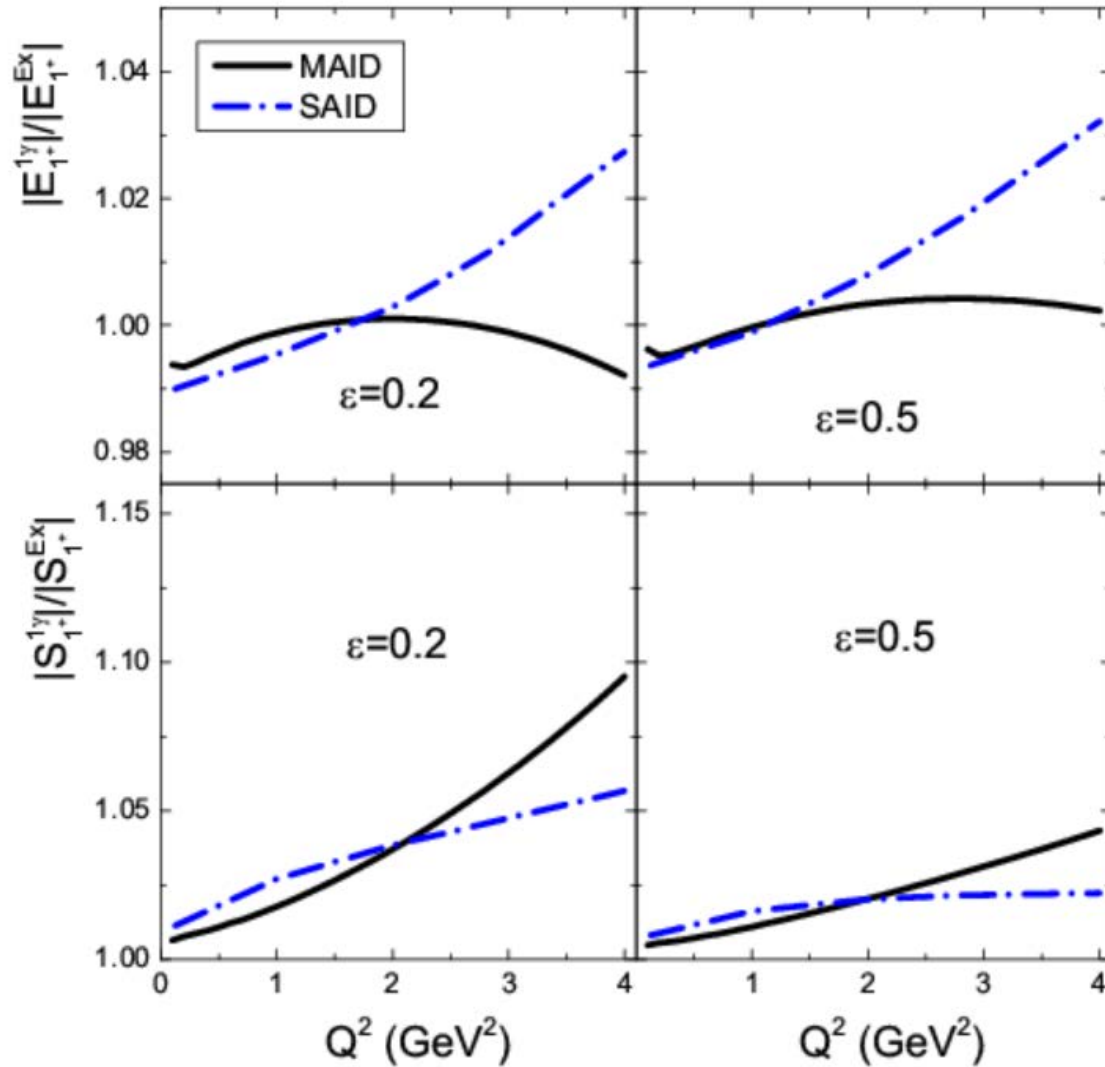


# TPE correction to the cross sections

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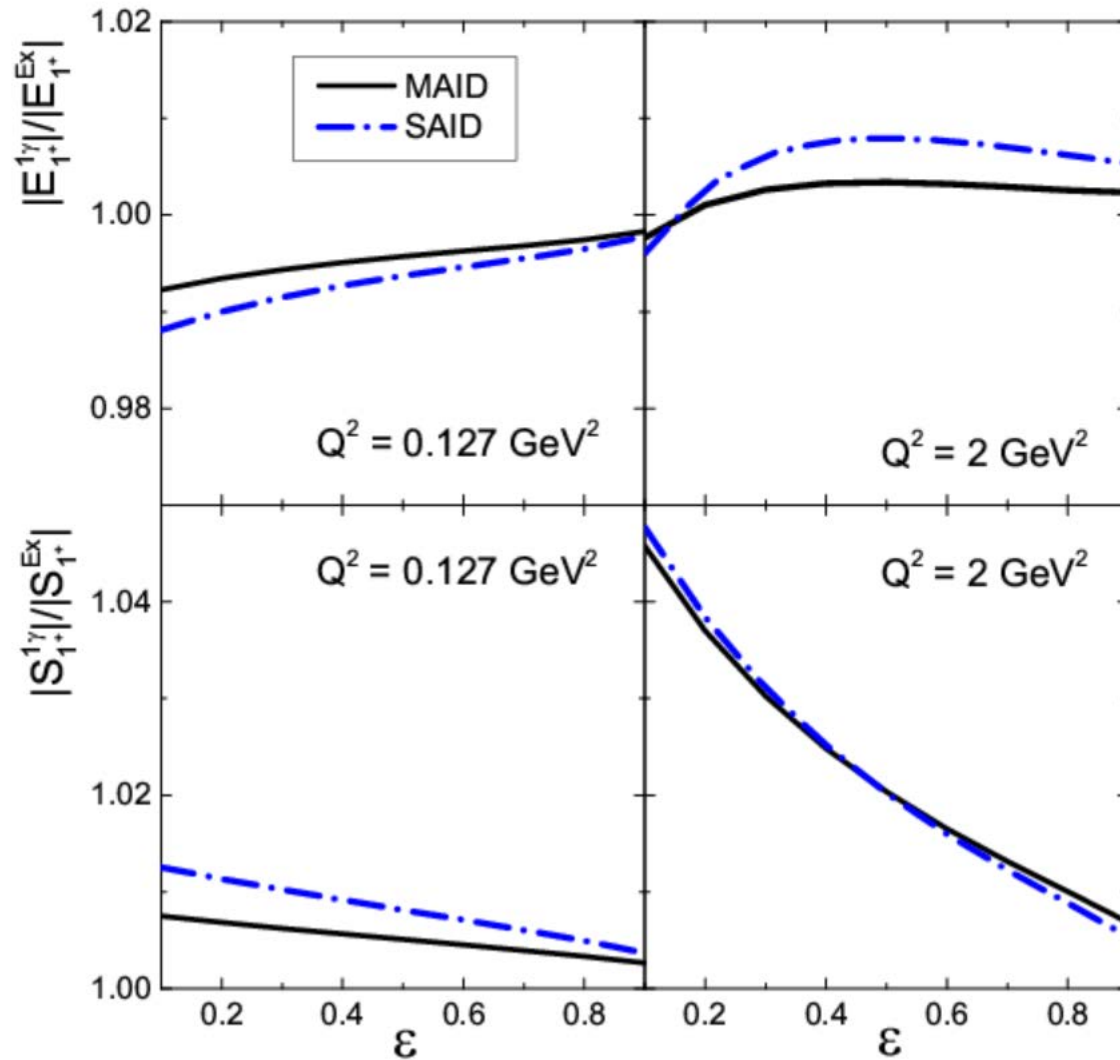
# TPE correction to the multipoles



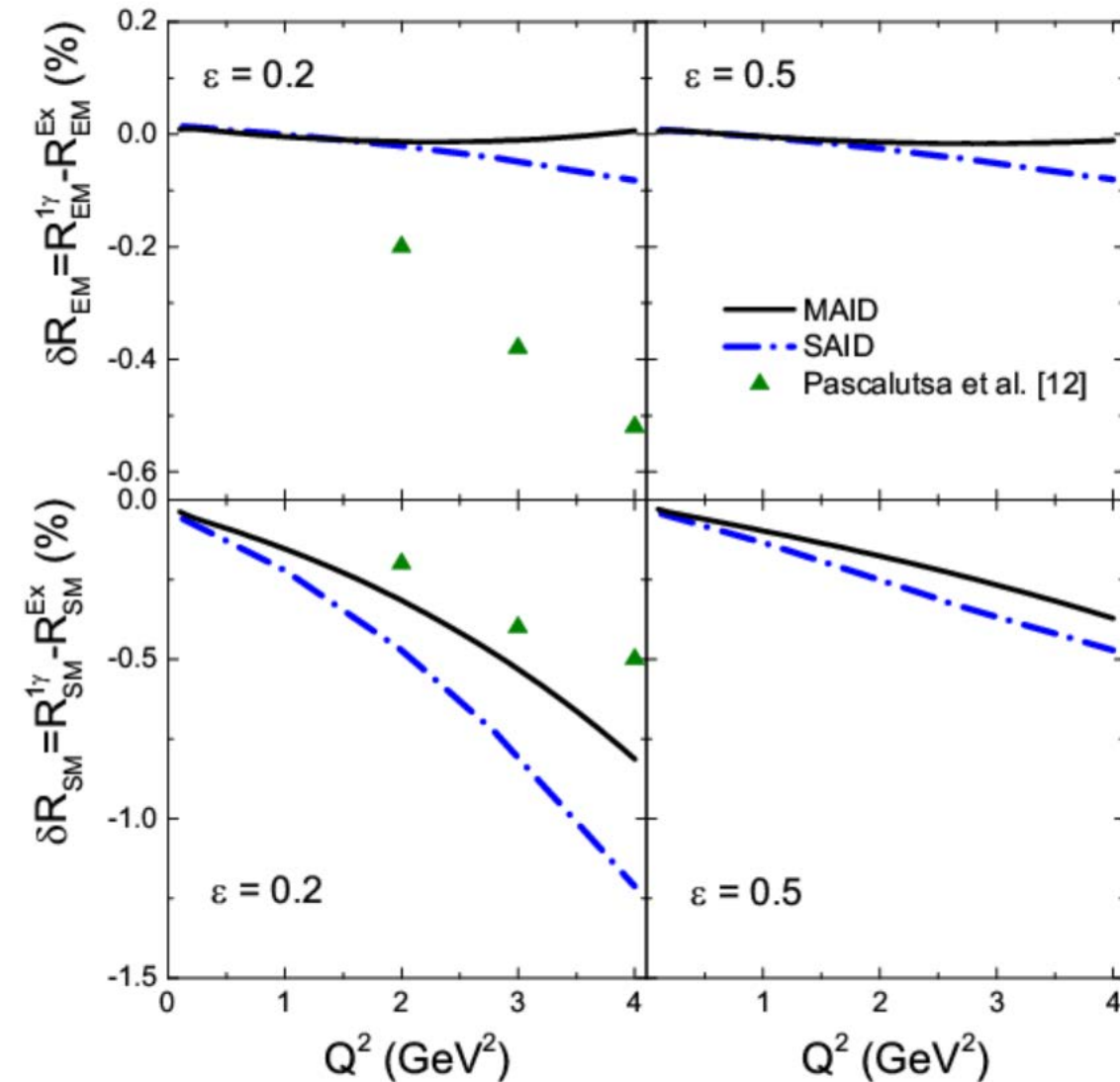
The TPE corrections to  $M_{1+}$  are as small as 0.1% and are not showed.



# TEP correction to the multipoles



# TPE corrections to the $R_{EM,SM}$



$$R_{EM} \equiv \frac{\text{Im}[E_{1+}^{(3/2)}]}{\text{Im}[M_{1+}^{(3/2)}]}$$

$$R_{SM} \equiv \frac{\text{Im}[S_{1+}^{(3/2)}]}{\text{Im}[M_{1+}^{(3/2)}]}$$

# Summary and conclusion

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We calculate the TPE contributions in  $ep \rightarrow eN\pi$  at  $\Delta(1232)$  peak with  $Q^2 < 4\text{GeV}^2$ , and find

1. the TPE corrections to the multipoles are from -1% to 6% in, and decrease when  $\varepsilon$  increase.
  2. the model dependence of the TPE corrections is moderate especially when  $Q^2 > 2\text{GeV}^2$ .
  3. our results are very different with those given by GPDs at  $Q^2 = 2-4\text{GeV}^2$ ,  $\delta R_{EM}$  are much smaller and  $\delta R_{SM}$  are much larger
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Thanks!

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# Appendix: experimental data

Table 2. Extracted values for  $\sigma_0$ ,  $\sigma_{LT}$ , and  $\sigma_{TT}$  at  $Q^2 = 0.127 \text{ (GeV}/c)^2$ . The uncertainties correspond to the statistical and the systematic uncertainties, respectively.

$W$ (MeV)	$\theta_{pq}^*$ °	$\sigma$	$\sigma$ ( $\mu\text{b}/\text{sr}$ )
1232	28	$\sigma_0$	$15.51 \pm 0.20 \pm 0.65$
1232	28	$\sigma_{LT}$	$1.62 \pm 0.13 \pm 0.16$
1232	28	$\sigma_{TT}$	$-3.98 \pm 0.16 \pm 0.32$

Table 1. Extracted values for  $\sigma_0$ ,  $\sigma_{LT}$ ,  $\sigma_{TT}$ , and  $\sigma_{LT'}$  at  $Q^2 = 0.20$  (GeV/c)<sup>2</sup>. The uncertainties correspond to the statistical and the systematic uncertainties, respectively.

$W$ (MeV)	$\theta_{pq}^*$ °	$\sigma$	$\sigma$ ( $\mu\text{b}/\text{sr}$ )
1205	34	$\sigma_0$	$18.75 \pm 0.21 \pm 0.69$
1205	34	$\sigma_{LT}$	$1.69 \pm 0.05 \pm 0.13$
1205	34	$\sigma_{TT}$	$-4.62 \pm 0.15 \pm 0.30$
1205	34	$\sigma_{LT'}$	$1.94 \pm 0.21 \pm 0.34$
1221	34	$\sigma_0$	$18.86 \pm 0.21 \pm 0.68$
1221	34	$\sigma_{LT}$	$1.94 \pm 0.06 \pm 0.12$
1221	34	$\sigma_{TT}$	$-5.41 \pm 0.16 \pm 0.32$
1221	34	$\sigma_{LT'}$	$2.10 \pm 0.22 \pm 0.35$
1232	34	$\sigma_0$	$17.10 \pm 0.20 \pm 0.68$
1232	34	$\sigma_{LT}$	$1.85 \pm 0.06 \pm 0.13$
1232	34	$\sigma_{TT}$	$-5.37 \pm 0.16 \pm 0.33$
1232	34	$\sigma_{LT'}$	$1.98 \pm 0.21 \pm 0.34$