Meson Productions in Neutrino-Nucleon Scattering


Satoshi Nakamura

Osaka University, Japan

Collaborators: H. Kamano (KEK), T. Sato (Osaka Univ.)
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Introduction
Neutrino-nucleus scattering for $\nu$-oscillation experiments

Next-generation exp. $\Rightarrow$ leptonic $CP$, mass hierarchy

$\nu$-nucleus scattering needs to be understood more precisely ($\sim 5\%$)

All $\nu$-oscillation experiments measure $\nu$-flux through $\nu$-nucleus interaction

$$Q^2 = -q^2$$
$$\nu = q^0$$
Neutrino-nucleus scattering for $\nu$-oscillation experiments

Next-generation exp. $\Rightarrow$ leptonic $\mathcal{C}\mathcal{P}$, mass hierarchy
Neutrino-nucleus scattering for $\nu$-oscillation experiments

Next-generation exp. $\Rightarrow$ leptonic $CP$, mass hierarchy
Neutrino-nucleus scattering for $\nu$-oscillation experiments

Next-generation exp. $\Rightarrow$ leptonic $\mathcal{CP}$, mass hierarchy

Wide kinematical region with different characteristic $\Rightarrow$ Combination of different expertise is necessary
Neutrino-nucleus scattering for $\nu$-oscillation experiments

Next-generation exp. $\rightarrow$ leptonic $\mathcal{CP}$, mass hierarchy

http://j-parc-th.kek.jp/html/English/e-index.html

A review article to be published in Reports on Progress in Physics
Neutrino interaction data in resonance region

\[ \nu_\mu p \rightarrow \mu^+ \pi^+ p \]

- Data to fix nucleon axial current \( g_{ANL} \)
- Discrepancy between BNL & ANL data
Neutrino interaction data in resonance region

\[ \nu_{\mu} p \rightarrow \mu^+ \pi^+ p \]

- Data to fix nucleon axial current \((g_{ANL})\)
- Discrepancy between BNL & ANL data
- Recent reanalysis of original data
  \(\rightarrow\) discrepancy resolved (!?)

PRD 90, 112017 (2014)
Neutrino interaction data in resonance region

\[ \nu_\mu p \rightarrow \mu^+ \pi^0 p \]

- Data to fix nucleon axial current \( (g_{AN\lambda}) \)
- Discrepancy between BNL & ANL data
- Recent reanalysis of original data
  \( \rightarrow \) discrepancy resolved (!?)

PRD 90, 112017 (2014)

\[ \nu_\mu CH \rightarrow \mu^0 X \quad \langle E_\nu \rangle = 0.8 \text{ GeV} \]

- Final state interaction (FSI) changes charge, momentum, number of \( \pi \)
- Cross section shape is worse described with FSI
- MINERVA data (PRD92,092008(2015)) favor FSI
  \( \langle E_\nu \rangle = 4.0 \text{ GeV}, \ W < 1.4 \text{ GeV} \)

More data are coming \( \rightarrow \) better understanding of neutrino-nucleus interaction
Relevance to baryon spectroscopy

e.g. $N-\Delta(1232)$ transition form factors

Vector (magnetic) form factor from electron reactions

Axial form factor from neutrino reactions

Sato et al., PRC 63 (2001); PRC 67 (2003)

$G_{M}/G_{D}$

\[ Q^2 \text{ [\(GeV/c\)^2]} \]

\[ G_D = \frac{1}{(1 + Q^2 / M_V^2)^2} \]

\[ M_V = 0.84 \text{ GeV} \]

\[ G_A = \frac{1}{(1 + Q^2 / M_A^2)^2} \]

\[ M_A = 1.02 \text{ GeV} \]
Relevance to baryon spectroscopy

e.g. $N$-$\Delta(1232)$ transition form factors

Sato et al., PRC 63 (2001); PRC 67 (2003)

- Axial structure of baryons can be learned from neutrino reaction data
- Different pion cloud contributions to magnetic and axial form factors (slope)
GOAL: Develop $\nu N$-interaction model in resonance region

Strategy

- Dynamical coupled-channels (DCC) model for $\gamma N, \pi N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$
- Extension to $\nu N \rightarrow l^- X \ (X = \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma)$

Important improvements over previous models (tree-level, Breit-Wigner)

- Channel-couplings required by unitarity
- Important $2 \pi$ production mechanisms
Dynamical Coupled-Channels model for meson productions
DCC (Dynamical Coupled-Channel) model

Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

\[ \{a, b, c\} = \pi N, \eta N, \pi \pi N, \pi \Delta, \sigma N, \rho N, K \Lambda, K \Sigma \]

By solving the LS equation, coupled-channel unitarity is fully taken into account
**DCC** (Dynamical Coupled-Channel) model

Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

\[ V_{ab} = \begin{array}{c}
\text{bare N*} \\
\hline
\text{Z}
\end{array} \]

- s
- u
- t
- c
**DCC (Dynamical Coupled-Channel) model**

Coupled-channel unitarity is fully taken into account

Kamano et al., PRC 88, 035209 (2013)

In addition, $\gamma$, $W$, $N$, $Z$ channels are included perturbatively.

**Coupled-channel Lippmann-Schwinger equation**

$$ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} $$

$$ V_{ab} = \text{bare } N^* + \text{Z} $$

essential for three-body unitarity
DCC (Dynamical Coupled-Channel) model

Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

\[ G_c = \quad \text{for stable channels} \]

[Diagram showing decay processes for stable and unstable channels]

[Diagram showing coupling between channels]
DCC (Dynamical Coupled-Channel) model

Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

In addition, \( \gamma N, W^{\pm}N, ZN \) channels are included perturbatively
Relation between neutrino and electron (photon) interactions

Charged-current (CC) interaction (e.g. $\nu_\mu + n \rightarrow \mu^- + p$)

$$L^{cc} = \frac{G_F V_{ud}}{\sqrt{2}} [J^{cc}_\lambda \ell^{\lambda}_{cc} + h.c.]$$

$$J^{cc}_\lambda = V_\lambda - A_\lambda$$

$$\ell^{\lambda}_{cc} = \bar{\psi}_\mu \gamma^{\lambda} (1 - \gamma_5) \psi_\nu$$

Electromagnetic interaction (e.g. $\gamma^{(*)} + p \rightarrow p$)

$$L^{em} = e J^{em}_\lambda A^{\lambda}_{em}$$

$$J^{em}_\lambda = V_\lambda + V^{IS}_\lambda$$

$V$ and $V^{IS}$ in $J^{em}$ can be separately determined by analyzing photon ($Q^2=0$) and electron reaction ($Q^2\neq0$) data on both proton and neutron targets, because:

$$<p|V_\lambda|p> = -<n|V_\lambda|n>$$

$$<p|V^{IS}_\lambda|p> = <n|V^{IS}_\lambda|n>$$

Matrix element for the weak vector current is obtained from analyzing electromagnetic processes

$$<p|V_\lambda|n> = 2 <p|V_\lambda|p>$$
DCC model for axial current

Because neutrino reaction data are scarce, axial current cannot be determined phenomenologically.

→ Chiral symmetry and PCAC (partially conserved axial current) are guiding principle.

PCAC relation

\[ <X' | q \cdot A | X > \sim i f_\pi <X' | T | \pi X > \]

\( Q^2 = 0 \)

- **non-resonant mechanisms**
  \[ \partial_\mu \pi \rightarrow f_\pi A_\mu^{external} \]

- **resonant mechanisms**
  \[ A \rightarrow N^* \rightarrow \pi \rightarrow N^* \]

Interference among resonances and background can be uniquely fixed within DCC model.
**DCC model for axial current**

\[ Q^2 \neq 0 \quad F_A(Q^2) : \text{axial form factors} \]

non-resonant mechanisms

\[ F_A(Q^2) = \left( \frac{1}{1 + Q^2 / M_A^2} \right)^2 \]

resonant mechanisms

\[ F_A(Q^2) = \left( \frac{1}{1 + Q^2 / M_A^2} \right)^2 \]

\[ M_A = 1.02 \text{ GeV} \]

More neutrino data are necessary to fix axial form factors for \( ANN^* \)

*Neutrino cross sections will be predicted with this axial current*
DCC analysis of $\gamma N, \pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$
and electron scattering data
DCC analysis of meson production data


**Fully combined** analysis of $\gamma N$, $\pi N \rightarrow \pi N$, $\eta N$, $K\Lambda$, $K\Sigma$ data

$d\sigma / d\Omega$ and polarization observables \( (W \leq 2.1 \text{ GeV}) \)

~23,000 data points are fitted

by adjusting parameters \( (N^* \text{ mass, } N^* \rightarrow MB \text{ couplings, cutoffs}) \)

Data for electron scattering on proton and neutron are analyzed by adjusting

$\gamma^* N \rightarrow N^*$ coupling strength at different $Q^2$ values \( (Q^2 \leq 3 \text{ (GeV}/c)^2) \)
DCC analysis results

Caveat

Due to time limitation, small portion of analysis results are presented. More analysis results including extracted resonance parameters, see Kamano’s plenary talk on Friday.
Partial wave amplitudes of $\pi N$ scattering


Previous model
(fitted to $\pi N \rightarrow \pi N$ data only)
[PRC76 065201 (2007)]

Data: SAID $\pi N$ amplitude

Real part

$I = \frac{3}{2}$

Imaginary part
Partial wave amplitudes of $\pi N$ scattering

Constraint on axial current through PCAC


Previous model (fitted to $\pi N \rightarrow \pi N$ data only) [PRC76 065201 (2007)]

Data: SAID $\pi N$ amplitude

Real part

Imaginary part

$I = \frac{3}{2}$
\( \gamma p \rightarrow \pi^0 p \)  

**d\(\sigma/d\Omega\)** for \( W < 2.1 \) GeV

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Vector current (\(Q^2=0\)) for 1\(\pi\)

Production is well-tested by data
Predicted $\pi N \rightarrow \pi\pi N$ total cross sections with our DCC model

Kamano, PRC88(2013)045208
Kamano, Julia-Diaz, Lee, Matsuyama, Sato PRC79(2008)025206
Single $\pi$ production in electron-proton scattering

Purpose: Determine $Q^2$–dependence of vector coupling of $p$-$N^*$: $V_{pN^*}(Q^2)$

$\sigma_T + \varepsilon \sigma_L$ for $Q^2=0.40 \text{ (GeV/c)}^2$ and $W=1.1 – 1.68 \text{ GeV}$

$p(e,e'\pi^0)p$ and $p(e,e'\pi^+\pi^-)n$
Inclusive electron-proton scattering

Data: JLab E00-002 (preliminary)

- Reasonable fit to data for application to neutrino interactions
- Important $2\pi$ contributions for high $W$ region

Similar analysis of electron-neutron scattering data has also been done

DCC vector currents has been tested by data for whole kinematical region relevant to neutrino interactions of $E_\nu \leq 2$ GeV
Neutrino Results
Cross section for $\nu_\mu N \rightarrow \mu^- X$

- $\pi N$ & $\pi\pi N$ are main channels in few-GeV region
- DCC model gives predictions for all final states
- $\eta N$, $KY$ cross sections are $10^{-1} - 10^{-2}$ smaller
Cross section for $\nu_\mu N \rightarrow \mu^- X$

- $\pi N$ & $\pi\pi N$ are main channels in few-GeV region
- DCC model gives predictions for all final states
- $\eta N$, $KY$ cross sections are $10^{-1} - 10^{-2}$ smaller
Comparison with single pion data

\[ \nu_\mu p \rightarrow \mu^+ \pi^- p \quad \nu_\mu n \rightarrow \mu^- \pi^0 p \quad \nu_\mu n \rightarrow \mu^- \pi^+ n \]

DCC model prediction is consistent with data

- DCC model has flexibility to fit data \((ANN^*(Q^2))\)
- Data should be analyzed with nuclear effects
  
(Wu et al., PRC91, 035203 (2015))

ANL Data : PRD 19, 2521 (1979)
BNL Data : PRD 34, 2554 (1986)
Comparison with double pion data

$\nu_\mu p \rightarrow \mu^- \pi^+ \pi^0 p$

$\nu_\mu p \rightarrow \mu^- \pi^+ \pi^+ n$

$\nu_\mu n \rightarrow \mu^- \pi^+ \pi^- p$

Fairly good DCC predications

ANL Data: PRD 28, 2714 (1983)
BNL Data: PRD 34, 2554 (1986)

First dynamical model for 2 $\pi$ production in resonance region
Mechanisms for $\nu_\mu N \rightarrow \mu^- \pi N$

- $\Delta(1232)$ dominates for $\nu_\mu p \rightarrow \mu^- \pi^+ p$ ($I=3/2$) for $E_\nu \leq 2$ GeV
- Non-resonant mechanisms contribute significantly
- Higher $N^*$s becomes important towards $E_\nu \approx 2$ GeV for $\nu_\mu n \rightarrow \mu^- \pi N$
$d\sigma / dW\, dQ^2 \ (\times 10^{-38}\,\text{cm}^2 / \text{GeV}^2)$

$E_{\nu} = 2\,\text{GeV}$

$\nu_{\mu}p \rightarrow \mu^{-}\pi^{+}\,p$

$\nu_{\mu}\,n \rightarrow \mu^{-}\,\pi^{+}\,N$

\[ E_{\nu} = 2\,\text{GeV} \]
$$\frac{d\sigma}{dW \ dQ^2} \ (\times 10^{-38} \text{ cm}^2 / \text{ GeV}^2)$$

$$E_\nu = 2 \text{ GeV}$$

$$\nu_\mu p \rightarrow \mu^- \pi^+ \pi^0 p$$

$$\nu_\mu n \rightarrow \mu^- \pi^+ \pi^- p$$
Conclusion
Development of DCC model for $\nu N$ interaction in resonance region

Start with DCC model for $\gamma N, \pi N \rightarrow \pi N, \pi \pi N, \eta N, KL, K\Sigma$

$\Rightarrow$ extension of vector current to $Q^2 \neq 0$ region, isospin separation
through analysis of $e^- p$ & $e^- \text{'}n$’ data for $W \leq 2 \text{ GeV}, Q^2 \leq 3 (\text{GeV}/c)^2$

$\Rightarrow$ Development of axial current for $\nu N$ interaction; PCAC is maintained

Conclusion

• $\pi N$ & $\pi \pi N$ are main channels in few-GeV region

• DCC model prediction is consistent with BNL data

• $\Delta, N^*$s, non-resonant are all important in few-GeV region (for $\nu \mu n \rightarrow \mu X$)

$\Rightarrow$ essential to understand interference pattern among them

$\Rightarrow$ DCC model can do this; consistency between $\pi$ interaction and axial current
$\nu_\mu p \rightarrow \mu^- \pi^+ p$

$Q^2$ – dependence
Resonance region (single nucleon)

\[ \gamma N \rightarrow X \]

Multi-channel reaction

- \( 2\pi \) production is comparable to \( 1\pi \)
- \( \eta, K \) productions (\( \nu \) case: background of proton decay exp.)
Neutrino interaction data in resonance region

\[ \nu_\mu p \rightarrow \mu^+ \pi^0 p \]

\[ \nu_\mu CH_2 \rightarrow \mu^0 \pi X \quad \langle E_\nu \rangle = 0.8 \text{ GeV} \]

\[ \langle E_\nu \rangle = 4.0 \text{ GeV} \]

- Data to fix nucleon axial current \( g_{A\text{NA}} \)
- Discrepancy between BNL & ANL data
- Recent reanalysis of original data
  \( \rightarrow \) discrepancy resolved (!?)
- Final state interaction (FSI) changes charge, momentum, number of \( \pi \)
- Cross section shape is worse described with FSI
- MINERvA data (arXiv:1406.6415) favor FSI

More data are coming \( \rightarrow \) better understanding of neutrino-nucleus interaction
“Δ” resonances (I=3/2)


\[ J^P(L_{21.21}) \]

-2\text{Im}(M_R) ("width")

Re(M_R)

PDG: 4* & 3* states assigned by PDG2012
AO : ANL-Osaka
J : Juelich (DCC)
[EPJA49(2013)44, Model A]
BG : Bonn-Gatchina (K-matrix)
[EPJA48(2012)5]
Analysis of electron-proton scattering data

Purpose: Determine $Q^2$–dependence of vector coupling of $p-N^*\, VpN^*(Q^2)$

Data:  
- $\pi\pi$ electroproduction

* Empirical inclusive inelastic structure functions $\sigma_T, \sigma_L$  
  Christy et al, PRC 81 (2010)

Database
- $p(e,e'\pi^0)p$
- $p(e,e'\pi^+)n$
- both

region where inclusive $\sigma_T$ & $\sigma_L$ are fitted
Analysis of electron-‘neutron’ scattering data

Purpose : Vector coupling of neutron-$N^*$ and its $Q^2$–dependence : $VnN^* (Q^2)$ ($I=1/2$)

$I=3/2$ part has been fixed by proton target data

Data : * 1π photoproduction ($Q^2$=0)

* Empirical inclusive inelastic structure functions $\sigma_T$, $\sigma_L$ ($Q^2$$\neq$0)

↩ Christy and Bosted, PRC 77 (2010), 81 (2010)

DCC vector currents has been tested by data for whole kinematical region

relevant to neutrino interactions of $E_\nu \leq 2$ GeV
Formalism

Cross section for $\nu N \rightarrow l X \quad (X = \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma)$

$$\theta \rightarrow 0 \quad \frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{G_F^2 E_\ell^2}{2\pi^2} \left( 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \pm W_3 \frac{E_\nu + E_\ell}{m_N} \sin \frac{\theta}{2} \right)$$

$$Q^2 \rightarrow 0 \quad W_2 = \frac{Q^2}{q^2} \sum \left[ \frac{1}{2} \left( (j^x)^2 + (j^y)^2 \right) + \frac{Q^2}{q_c^2} \left| \langle J^0 + \frac{\omega_c}{Q^2} q \cdot J \rangle \right|^2 \right]$$

CVC & PCAC

$$\langle q \cdot J \rangle = \langle q \cdot V \rangle - \langle q \cdot A \rangle = if_\pi m_\pi^2 \langle \hat{\pi} \rangle$$

LSZ & smoothness

$$\langle X | \hat{\pi} | N \rangle = \frac{\sqrt{2}\omega_c}{m_\pi^2} T_{\pi N \rightarrow X}(0) \sim \frac{\sqrt{2}\omega_c}{m_\pi^2} T_{\pi N \rightarrow X}(m_\pi^2)$$

Finally

$$F_2 \equiv \omega W_2 = \frac{2f_\pi^2}{\pi} \sigma_{\pi N \rightarrow X} \quad \sigma_{\pi N \rightarrow X} \text{ is from our DCC model}$$
Results

- Prediction based on model well tested by data (first $\nu N \rightarrow \pi \pi N$)
- $\pi N$ dominates for $W \leq 1.5$ GeV
- $\pi \pi N$ becomes comparable to $\pi N$ for $W \geq 1.5$ GeV
- Smaller contribution from $\eta N$ and $K Y$ $O(10^{-1}) - O(10^{-2})$
- Agreement with SL (no PCAC) in $\Delta$ region
Comparison with Rein-Sehgal model

$\nu_e + p \rightarrow e^- + p + \pi^+$

$\nu_e + n \rightarrow e^- + n + \pi^+$

Comparison in whole kinematical region will be done

after axial current model is developed
$F_2$ from RS model

\[ \nu_e + n \rightarrow e^- + p + \pi^0 \]

![Graph showing $F_2$ vs. W (GeV) with two curves: one labeled "w/o non-res" in red and another labeled "w non-res" in blue. The graph has labeled axes: $F_2$ on the y-axis and $W (\text{GeV})$ on the x-axis.](image)
SL model applied to $\nu$–nucleus scattering

$1\pi$ production

\[ \nu_e + ^{12}\text{C} \rightarrow e^- + X \ (E_{\nu} = 1 \text{ GeV}) \quad \text{and} \quad e^- + ^{12}\text{C} \rightarrow e^- + X \ (E_e = 1.1 \text{ GeV}) \]

\[ \text{Szczerbinska et al. (2007)} \]
SL model applied to $\nu$–nucleus scattering

coherent $\pi$ production

$$\gamma + ^{12}\text{C} \rightarrow \pi^0 + ^{12}\text{C}$$

$$\nu_\mu + ^{12}\text{C} \rightarrow \mu^- + \pi^0 + ^{12}\text{C}$$

Nakamura et al. (2010)
Previous models for $\nu$-induced $1\pi$ production in resonance region

resonant only

\[ \sum_i \left\{ \begin{array}{c}
N^*_i \\
\end{array} \right\} \]

Rein et al. (1981), (1987); Lalalulich et al. (2005), (2006)

+ non-resonant (tree-level)

Hernandez et al. (2007), (2010); Lalakulich et al. (2010)

\[ \sum_i \left\{ \begin{array}{c}
N^*_i \\
\end{array} \right\} + \left\{ \begin{array}{c}
\end{array} \right\} + \left\{ \begin{array}{c}
\end{array} \right\} + \ldots \]

+ rescattering ($\pi N$ unitarity)

Sato, Lee (2003), (2005)

\[ \left\{ \begin{array}{c}
\Delta \\
\end{array} \right\} + \left\{ \begin{array}{c}
\end{array} \right\} + \ldots + \left\{ \begin{array}{c}
\end{array} \right\} + \ldots \]
Eta production reactions

$\pi^- p \rightarrow \eta n$
KY production reactions

\[ d\sigma/d\Omega \ (\mu b/sr) \]

\[ \pi^+ p \rightarrow K^+ \Sigma^+ \]

1732 MeV

1845 MeV

1985 MeV

2031 MeV

\[ \pi^- p \rightarrow K^0 \Lambda^0 \]

1757 MeV

1966 MeV

2059 MeV

\[ \pi^- p \rightarrow K^0 \Sigma^0 \]

1792 MeV

1966 MeV

2059 MeV

Kamano, Nakamura, Lee, Sato, 2012
\[ \pi N \rightarrow \pi \pi N \]

(parameters had been fitted to \( \pi N \rightarrow \pi N \))

Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC79 025206 (2009)

\[ \sigma (\text{mb}) \]

- \( \pi^+ p \rightarrow \pi^+ \pi^+ n \)
- \( \pi^- p \rightarrow \pi^+ \pi^- n \)
- \( \pi^- p \rightarrow \pi^- \pi^0 p \)

\[ W (\text{GeV}) \]

\[ \sigma (\text{mb}) \]

- \( \pi^+ p \rightarrow \pi^+ \pi^0 p \)
- \( \pi^- p \rightarrow \pi^0 \pi^0 n \)

Full

C.C. effect off
Vector current ($Q^2=0$) for $\eta$ Production is well-tested by data
Vector current ($Q^2=0$) for $K$ Production is well-tested by data

\[ \gamma N \rightarrow \pi \pi N \]

(parameters had been fitted to \( \pi N, \gamma N \rightarrow \pi N \))

Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC80 055203 (2009)

\[ \gamma p \rightarrow \pi^+ \pi^- p \]
\[ \gamma p \rightarrow \pi^0 \pi^0 p \]
\[ \gamma p \rightarrow \pi^+ \pi^0 n \]

\[ \sigma (\mu b) \]

\[ W (\text{MeV}) \]

\[ \frac{d\sigma}{dM} (\mu b/\text{GeV}) \]

\[ M (\pi p) \text{ (GeV)} \]
\[ M(\pi^+ p) \text{ (GeV)} \]
\[ M(\pi^+ \pi^-) \text{ (GeV)} \]

* Good description near threshold
* Good shape of invariant mass distribution
* Total cross sections overestimate data for \( W \geq 1.5 \) GeV
Analysis result

\[ Q^2 = 0.16 \, (\text{GeV}/c)^2 \]

\[ \sigma_T \ & \ \sigma_L \quad \text{(inclusive inelastic)} \]

\[ \sigma_T, \ \sigma_L \quad \text{(inclusive inelastic)} \]

- \[ \sigma_T \] (red line)
- \[ \sigma_L \] (blue line)

**DCC**
- Christy et al PRC 81
- **Green box**: region where inclusive \[ \sigma_T \ & \ \sigma_L \] are fitted
Analysis result

$Q^2 = 0.40 \text{ (GeV/c)}^2$

$\sigma_T$ & $\sigma_L$ (inclusive inelastic)
$Q^2 = 2.95 \, (\text{GeV}/c)^2$

Analysis result

$\sigma_T \, \& \, \sigma_L$ (inclusive inelastic)
DCC model for neutrino interaction

$\nu N \rightarrow l X \quad (X = \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma)$

at forward limit $Q^2 = 0$


$$\frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{G_F^2}{2\pi^2} E_\ell^2 W_2$$

via PCAC

$$F_2 \equiv \omega W_2 = \frac{2f_\pi^2}{\pi} \sigma_{\pi N \rightarrow X}$$

$\sigma_{\pi N \rightarrow X}$ is from our DCC model
$F_2(Q^2=0)$ from DCC model and PCAC

DCC model keeps good consistency with PCAC
Comparison with LPP model


- Large difference beyond $\Delta(1232)$ region
- Importance of consistency between axial-current and $\pi N$ interaction
Analysis result (single $\pi$)

$Q^2=0$

d$\sigma$ / d$\Omega$ ($\gamma n \rightarrow \pi p$) for $W=1.1 - 2.0$ GeV
Analysis result (inclusive $e^-d$)

- Our calculation: $\left[ \sigma(e^-p) + \sigma(e^-n) \right] / 2$
- Too sharp resonant peaks → fermi motion smearing, other nuclear effects needed
- Reasonable starting point for application to neutrino interactions

For application to neutrino interactions

Analysis of electron scattering data

- \( V_{pN^*}(Q^2) \) & \( V_{nN^*}(Q^2) \) fixed for several \( Q^2 \) values
- Parameterize \( V_{pN^*}(Q^2) \) & \( V_{nN^*}(Q^2) \) with simple analytic function of \( Q^2 \)

\[ I=3/2 \quad : \quad V_{pN^*}(Q^2) = V_{nN^*}(Q^2) \quad \Rightarrow \quad \text{CC, NC} \]
\[ I=1/2 \quad \text{isovector part} \quad : \quad \left( V_{pN^*}(Q^2) - V_{nN^*}(Q^2) \right) / 2 \quad \Rightarrow \quad \text{CC, NC} \]
\[ I=1/2 \quad \text{isoscalar part} \quad : \quad \left( V_{pN^*}(Q^2) + V_{nN^*}(Q^2) \right) / 2 \quad \Rightarrow \quad \text{NC} \]

\( DCC \) vector currents has been tested by data for whole kinematical region relevant to neutrino interactions of \( E_\nu \leq 2 \) GeV
Analysis result (single $\pi$)

$Q^2 = 1.76$ (GeV/c)$^2$

$\sigma_T + \varepsilon \sigma_L$ for $W = 1.10 - 2.01$ GeV

$p(e,e'\pi^0)p$
Analysis result (single $\pi$)

$Q^2=2.91-3.00 \text{ (GeV}/c)^2$

$\sigma_T + \varepsilon \sigma_L$ for $W=1.10 - 1.67$ GeV

$p(e,e'\pi^0)p$ vs $p(e,e'\pi^+)n$