Empirical parametrizations of the resonance amplitudes based on the Siegert's theorem

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Plan of the talk

- **Introduction**
  What is the Siegert's theorem, general considerations, ...
- **Empirical parametrizations of the data transition amplitudes:** $\gamma^* N \to R$
  $R = N(1535)\frac{1}{2}^-, N(1520)\frac{3}{2}^-, \Delta(1232)\frac{3}{2}^+$
- **Parametrization for very large** Q^2 (pQCD behavior)
- **Summary and conclusions**

What is the Siegert's theorem ?

- **Siegert's theorem:** $\gamma^* N \to R$ transition
  AJ Buchmann et al, PRC 58, 2478 (1998); Drechsel et al, EJPA 34, 69 (2007)
  The electric amplitude E ($\leftarrow$ transverse amplitudes) and the scalar amplitude S ($\propto S_{1/2}$) are related **at the pseudo-threshold** ($|\mathbf{q}| \to 0$) by

$$E \propto \frac{\omega}{|\mathbf{q}|} S,$$

  ($|\mathbf{q}|$ = photon momentum, ω = photon energy)

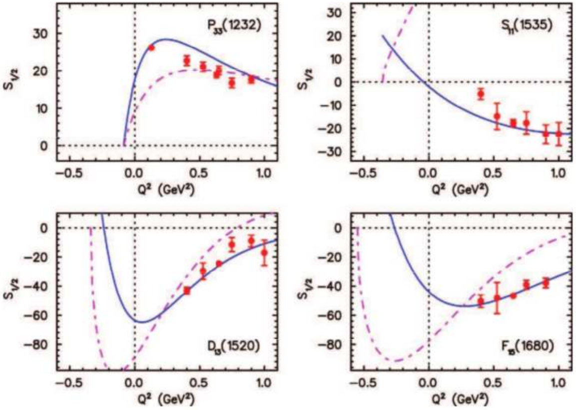
- **Pseudo-threshold:**
  Limit where the nucleon and the resonance are at rest
  R **rest frame:** (M = nucleon mass, M_R = resonances mass)
  $P_N = (E_N, 0, 0, -|\mathbf{q}|)$, $P_R = (M_R, 0, 0, 0)$, thus $q = (\omega, 0, 0, |\mathbf{q}|)$
  In general: $\omega = M_R - E_N$, $E_N = \sqrt{M^2 + |\mathbf{q}|^2}$
  PT limit: $|\mathbf{q}| = 0$, $E_N \to M \Rightarrow$ **nucleon and R at rest**

- At the PT: $Q^2 = -(M_R - M)^2 < 0$, timelike region

transition form factors at low Q^2

empirical fits MAID2007

Siegert Theorem (long-wavelength limit)



problem with S11

violation of LWL ?

Lothar Tiator: Nucleon Resonances: From Photoproduction to High Photon Virtualities, Trento, Italy, October 2015

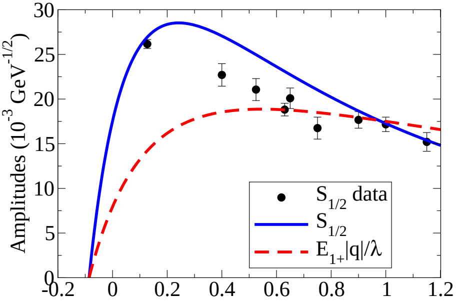
Siegert's theorem **(usual form):**

$$E_{1+} \propto \left(\frac{1}{\sqrt{3}} A_{3/2} - A_{1/2} \right)$$

$$E_{1+} = \sqrt{2}(M_R - M) \frac{S_{1/2}}{|\mathbf{q}|},$$

Test $S_{1/2} \propto E_{1+} |\mathbf{q}|$

$$\lambda = \sqrt{2}(M_R - M)$$



- Why are the electric and the scalar amplitudes correlated at the pseudo-threshold ? $Q^2 = Q^2_{PS} = -(M_R - M)^2$
- ... consequence of the **structure of the transition current:**

Jones and Scadron, Ann. Phy. 81, 1 (1973); Devenish et al, PRD 14, 3063 (1976)

$$G_E(Q^2_{PS}) = \frac{M_R - M}{2M_R} G_C(Q^2_{PS})$$

$\gamma^* N \to N^*$ transition amplitudes – notation

$J^\mu = (\rho, J^i)$ define multipole amplitudes $(S_{l\pm}, M_{l\pm}, E_{l\pm}, L_\pm)$
l is the angular momentum of the meson in the decay $R \to R'm$

Devenish et al, PRD 14, 3063 (1976); Drechsel, Kamalov and Tiator, EPJA 34, 69 (2007)

$N\left(\frac{1}{2}^+\right)$	S_{1-}	–	M_{1-}
$N\left(\frac{1}{2}^-\right)$	S_{0+}	E_{0+}	–
$\Delta\left(\frac{3}{2}^+\right)$	S_{1+}	E_{1+}	M_{1+}
$N\left(\frac{3}{2}^-\right)$	S_{2-}	E_{2-}	M_{2-}

ST is not applicable to the $N\left(\frac{1}{2}^+\right)$ (Roper, $N(1710)$)

Structure of the transition amplitudes

	$N(1535)\frac{1}{2}^-$	$N(1520)\frac{3}{2}^-$	$\Delta(1232)\frac{3}{2}^+$
Siegert	$A_{1/2} = 2b\tilde{F}_1$	$E_{2-} = \mathcal{O}(1)$	$E_{1+} = \mathcal{O}(\lvert\mathbf{q}\rvert)$
	$S_{1/2} = \sqrt{2}b\frac{\lvert\mathbf{q}\rvert}{M_R - M}\tilde{F}_1$	$S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert)$	$S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$
		$M_{2-} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$	$M_{1+} = \mathcal{O}(1)$
	$\tilde{F}_1 = \mathcal{O}(1)$	$\frac{1}{2}E_{2-} = \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert}$	$\frac{E_{1+}}{\lvert\mathbf{q}\rvert} = \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert^2}$
		$A_{1/2} = \frac{1}{\sqrt{3}}A_{3/2}$	
MAID	$A_{1/2} = \mathcal{O}(1)$	$E_{2-} = \mathcal{O}(1)$	$E_{1+} = \mathcal{O}(\lvert\mathbf{q}\rvert)$
	$S_{1/2} = \mathcal{O}(1)$	$S_{1/2} = \mathcal{O}(1)$	$S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$
		$M_{2-} = \mathcal{O}(1)$	$M_{1+} = \mathcal{O}(1)$
		$A_{1/2} \neq \frac{1}{\sqrt{3}}A_{3/2}$	$\frac{E_{1+}}{\lvert\mathbf{q}\rvert} \neq \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert^2}$

$$\gamma^* N \to N(1535)\frac{1}{2}^-$$

$\gamma^* N \to N(1535)\frac{1}{2}^-$ $\quad \lambda = \sqrt{2}(M_R - M)$

- From the analysis of the transition current – singularity-free FF
  Devenish et al, PRD 14, 3063 (1976)

$$J^\mu = F_1 \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{M_R + M}$$

  we obtain the relation ($E \propto A_{1/2}$): $\quad A_{1/2} = \lambda \frac{S_{1/2}}{|\mathbf{q}|}$

- Consistent with the multipole analysis at PT ($|\mathbf{q}| \to 0$)
  JD Bjorken and JD Walecka, Ann. Phys. 38, 35 (1966); Devenish et al, PRD 14, 3063 (1976); D Drechsel, SS Kamalov and L Tiator, EPJA 34, 69 (2007)

$$A_{1/2} = \mathcal{O}(1), \qquad S_{1/2} = \mathcal{O}(|\mathbf{q}|)$$

- Current conservation: $\quad \tilde{F}_1 = F_1 + \eta F_2$, $\eta = \frac{M_R - M}{M_R + M}$

$$S_{1/2} \propto \tilde{F}_1 |\mathbf{q}|$$

$S_{1/2} \to 0$ at PT: Orthogonality between N and R

$\gamma^* N \to N(1535)\frac{1}{2}^-$ – Amplitudes

R **rest frame** $b = e\sqrt{\frac{Q_+^2}{8M(M_R - M)}}$, $Q_\pm^2 = (M_R \pm M)^2 + Q^2$, $|\mathbf{q}| = \frac{\sqrt{Q_+^2 Q_-^2}}{2M_R}$

$$
\begin{aligned}
A_{1/2}(Q^2) &= 2b\tilde{F}_1(Q^2), \\
S_{1/2}(Q^2) &= -\sqrt{2}b(M_R - M)\frac{|\mathbf{q}|}{Q^2} \times \\
& \left[\tilde{F}_1(Q^2) - \frac{4M_R^2|\mathbf{q}|^2}{(M_R^2 - M^2)Q_+^2}F_2(Q^2)\right],
\end{aligned}
$$

- If $|\mathbf{q}|^2 F_2/\tilde{F}_1 \to 0$:

$A_{1/2} = 2b\tilde{F}_1$, $\qquad S_{1/2} = \sqrt{2}b\frac{|\mathbf{q}|}{M_R - M}\tilde{F}_1$

$\gamma^* N \to N(1535)\frac{1}{2}^-$ – Amplitudes

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- Equivalent to Siegert's theorem if $A_{1/2} \propto \tilde{F}_1 = \mathcal{O}(1)$

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  $A_{1/2} = 2b\tilde{F}_1$, $\qquad S_{1/2} = \sqrt{2}b\frac{|\mathbf{q}|}{M_R-M}\tilde{F}_1$
- Equivalent to Siegert's theorem if $A_{1/2} \propto \tilde{F}_1 = \mathcal{O}(1)$
- **MAID2007**: $|\mathbf{q}|^2 F_2 \propto 1/|\mathbf{q}|$ $\qquad F_2 \propto 1/|\mathbf{q}|^3$ $\qquad [S_{1/2} = \mathcal{O}(1)]$

$\gamma^* N \to N(1535)\frac{1}{2}^-$ – Form factors $\quad \eta = \frac{M_R - M}{M_R + M}$

Write form factors in terms of $\mathcal{R} = A_{1/2} - \lambda \frac{S_{1/2}}{|\mathbf{q}|}$ $\quad$ ($\mathcal{R} \to 0$ at PT)

$$
\begin{aligned}
F_1 &= \frac{1}{2b} \frac{(M_R - M)^2 Q_+^2}{4 M_R^2 |\mathbf{q}|^2} \left[A_{1/2} - \lambda \frac{S_{1/2}}{|\mathbf{q}|} \right] \\
&\quad + \frac{1}{2b} \left[A_{1/2} - \lambda \frac{S_{1/2}}{|\mathbf{q}|} \right], \\
\eta F_2 &= -\frac{1}{2b} \frac{(M_R - M)^2 Q_+^2}{4 M_R^2 |\mathbf{q}|^2} \left[A_{1/2} - \lambda \frac{S_{1/2}}{|\mathbf{q}|} \right] \\
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Note that $\tilde{F}_1 = F_1 + \eta F_2 = A_{1/2}/(2b)$

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- **Siegert's theorem**: $\mathcal{R} = \mathcal{O}(|\mathbf{q}|^2)$, $F_1, -\eta F_2 = \mathcal{O}(1)$

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$\gamma^* N \to N(1535)\frac{1}{2}^-$ – Form factors $\quad \eta = \frac{M_R - M}{M_R + M}$

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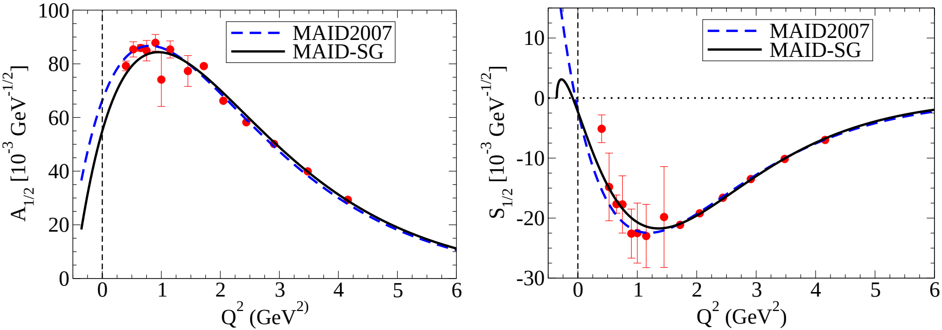
  Because if $A_{1/2}, \frac{S_{1/2}}{|\mathbf{q}|}$ are function of Q^2, there is no term in $|\mathbf{q}|$ on $\mathcal{R}$
  $\frac{dF}{d|\mathbf{q}|} = \frac{4M_R^2 |\mathbf{q}|}{M_R^2 + M^2 + Q^2} \frac{dF}{dQ^2}$ (finite $\frac{dF}{dQ^2}$)

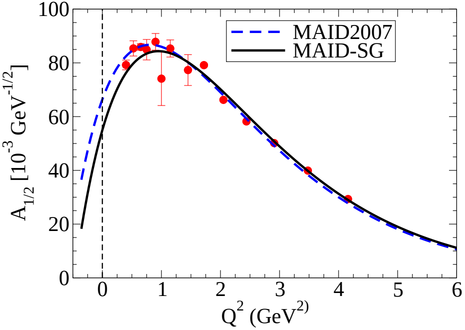
$\gamma^* N \to N(1535)\frac{1}{2}^-$ – MAID-SG

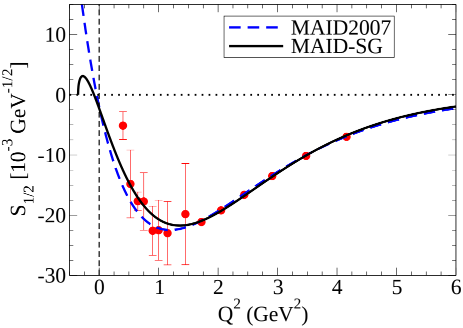


Improved MAID-type parametrizations ($S_{1/2} = \mathcal{O}(|\mathbf{q}|)$); ST $\to$ s_0'

$$A_{1/2} = a_0 \left(1 + a_1 Q^2\right) e^{-a_4 Q^2}, \quad S_{1/2} = \frac{2M_R|\mathbf{q}|}{Q_+^2} s_0' \left(1 + s_1 Q^2 + s_2 Q^4\right) e^{-s_4 Q^2}$$

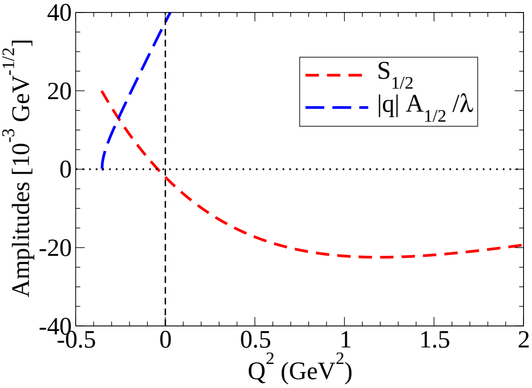
$\gamma^* N \to N(1535)\frac{1}{2}^-$ – MAID-SG



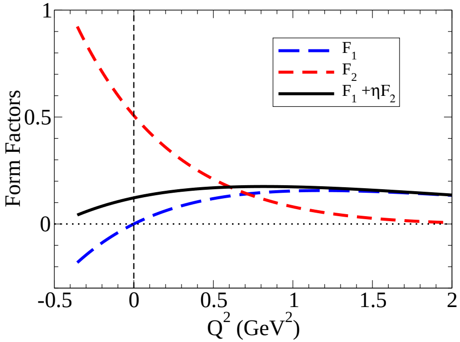


- MAID-SG (Siegert) close to MAID2007 for $Q^2 > 1.5$ GeV^2
- Differences for low Q^2, main difference for $Q^2 < 0$ (see inflection)

$\gamma^* N \to N(1535)\frac{1}{2}^-$ – MAID2007 (optional)



$A_{1/2} = \mathcal{O}(1)$ $\qquad$ $S_{1/2} = \mathcal{O}(1)$

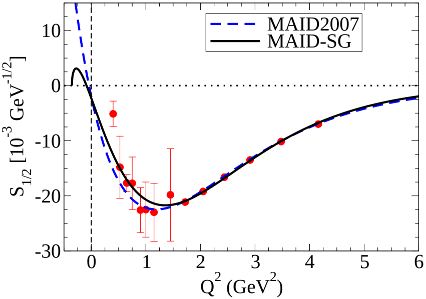


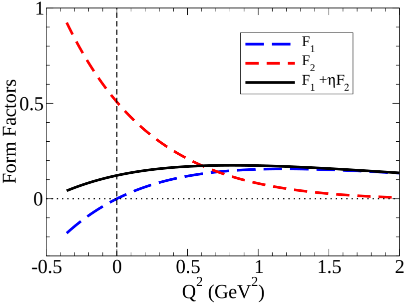
- Check that F_1, $-\eta F_2$ are finite at PT
- F_2 neglegible for large Q^2: $S_{1/2} = -\frac{\sqrt{1+\tau}}{\sqrt{2}} \frac{M_R^2 - M^2}{2 M_R Q} A_{1/2}$

  GR and K Tsushima, PRD 84, 051301 (2011); GR, D Jido and K Tsushima, PRD 85, 093014 (2012)

  balence between valence quark and meson cloud effects

$\gamma^* N \to N(1535)\frac{1}{2}^-$ – MAID-SG – Conclusions





- MAID-SG: parametrization consistent with the **Siegert's theorem** and with the **data** ($S_{1/2} = \mathcal{O}(|\mathbf{q}|)$)
- Form factors free of kinematic singularities at PT

$$\gamma^* N \to N^* \left(\tfrac{3}{2}\right)$$

- $\gamma^* N \to N(1520)$ transition form factors
- $\gamma^* N \to \Delta(1232)$ transition form factors

$\gamma^* N \to N^*(\frac{3}{2}^{\pm})$

Transition current (u_α = Rarita-Schwinger; u = Dirac)

$$J^\mu_{NR} = \langle R| J^\mu |N\rangle = \bar{u}_\alpha(p')\Gamma^{\alpha\mu}(p',p)u(p)$$
$$\Gamma^{\alpha\mu} = \left[G_1 q^\alpha \gamma^\mu + \tfrac{1}{2}G_2 q^\alpha (P_R + P_N)^\mu + G_3 q^\alpha q^\mu\right] \mathbb{1}_P + \ldots$$

$\mathbb{1}_P$ operator parity-deppendent $\mathbb{1}_+ = \gamma_5$, $\mathbb{1}_- = \mathbb{1}$

Scalar amplitude (using current conservation) $\langle J^0 \rangle = \langle S'_z = +\frac{1}{2}| J^0 |S_z = +\frac{1}{2}\rangle$

$$S_{1/2} = \sqrt{\frac{2\pi\alpha}{K}}\langle J^0\rangle \propto G_5^P(\bar{u}_3 \mathbb{1}_P u)|\mathbf{q}|, \qquad K = \frac{M_R^2 - M^2}{2M_R}$$

G_5^P form factor deppendent of the parity $P = \pm$

$$S_{1/2} \propto G_5^P |\mathbf{q}|^n, \qquad n = \begin{cases} 2 & \text{if} \quad P = + \\ 1 & \text{if} \quad P = - \end{cases}$$

Notation: Conversion between Amplitudes and Form Factors ($P = \pm$)

$$F_\pm = \frac{1}{e}\frac{2M}{M_R \pm M}\sqrt{\frac{M M_R K}{Q_\mp^2}}$$

$\gamma^* N \to N(1520)$ $J^P = \frac{3}{2}^-$ $\lambda = \sqrt{2}(M_R - M)$

Devenish et al, PRD 14, 3063 (1976) – G_i $(i = 1, 2, 3)$ free of singularities at PT

$$G_M = -Z_R \frac{4M_R|\mathbf{q}|^2}{Q_+^2} G_1,$$

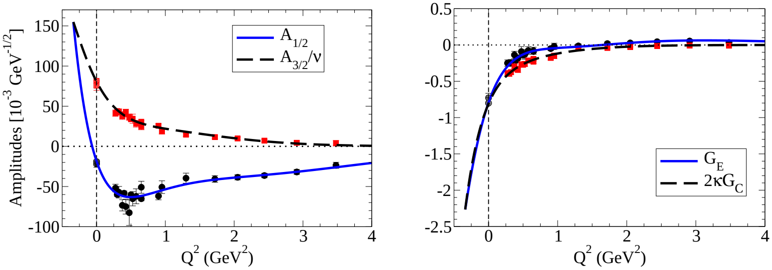
$$G_E = -Z_R \left[4(M_R - M)G_5 - \frac{4M_R^2|\mathbf{q}|^2}{Q_+^2} \left(\frac{G_1}{M_R} + 4G_3 \right) \right],$$

$$G_C = -Z_R \left[4M_R G_5 + \frac{4M_R^2|\mathbf{q}|^2}{Q_+^2} (G_2 - 2G_3) \right],$$

$$G_5 = G_1 + \frac{1}{2}(M_R - M)G_2 + (M_R + M)G_3, \qquad Z_R = \frac{1}{\sqrt{6}} \frac{M}{M_R - M}$$

- Limit PT: $G_M = \mathcal{O}(|\mathbf{q}|^2)$, $G_E = -4(M_R - M)Z_R G_5$, $G_C = -4M_R G_5$
  $G_E = \frac{M_R - M}{M_R} G_C$
- Amplitudes: $G_E = F_- E_{2-}$, $G_C = \frac{2M_R}{|\mathbf{q}|}\sqrt{2} F_- S_{1/2}$, $F_- \propto 1/\sqrt{Q_+^2}$
  $\frac{1}{2} E_{2-} = \lambda \frac{S_{1/2}}{|\mathbf{q}|}$ $\qquad M_{2-} \propto \left(A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2} \right) \propto |\mathbf{q}|^2$

$\gamma^* N \to N(1520)$ $J^P = \frac{3}{2}^-$ Jlab-SG $\kappa = \frac{M_R - M}{2M_R}$ (1)



Jlab-SG parametrization (fit to Jlab data): $D = K/\sqrt{Q_+^2}$ ST $\to$ b_0, c_0

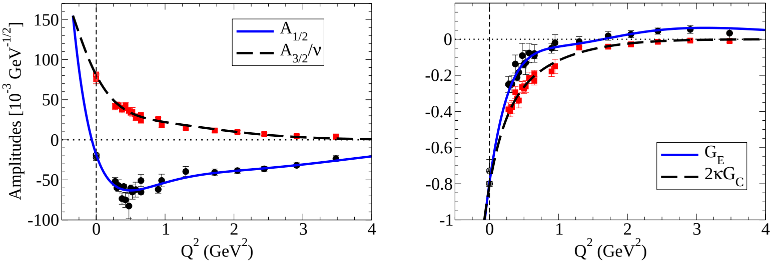
$$A_{1/2} = D a_0 (1 + a_1 Q^2 + a_2 Q^4 + a_3 Q^6) e^{-a_4 Q^2},$$

$$A_{3/2} = D b_0 (1 + b_1 Q^2 + b_2 Q^4 + b_3 Q^6) e^{-b_4 Q^2},$$

$$S_{1/2} = \frac{|\mathbf{q}|}{K} c_0 (1 + c_1 Q^2 + c_2 Q^4 + c_3 Q^6) e^{-c_4 Q^2},$$

V. Mokeev: `https://userweb.jlab.org/~mokeev/resonance_electrocouplings/`

$\gamma^* N \to N(1520)$ $\quad J^P = \frac{3}{2}^-$ $\quad$ Jlab-SG $\kappa = \frac{M_R - M}{2M_R}$ (1')



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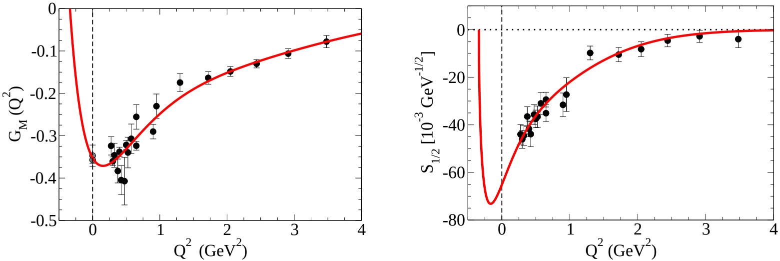

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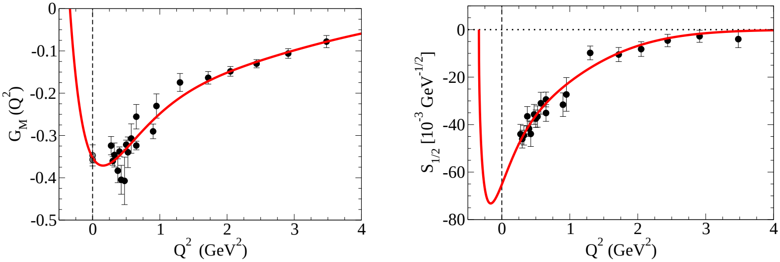
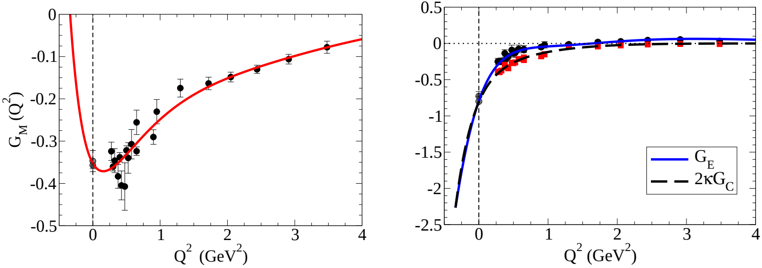
V. Mokeev: https://userweb.jlab.org/~mokeev/resonance_electrocouplings/

$\gamma^* N \to N(1520) \quad J^P = \frac{3}{2}^- \quad$ Jlab-SG (3)



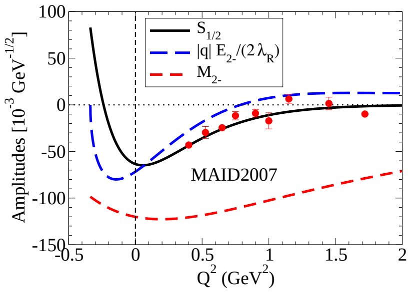
- Jlab-SG: $A_{1/2} = \mathcal{O}(1)$, $A_{3/2} = \mathcal{O}(1)$, $S_{1/2} = \mathcal{O}(|\mathbf{q}|)$
  Jlab-SG: $M_{2-} = \mathcal{O}(|\mathbf{q}|^2)$, $E_{2-} = \mathcal{O}(1)$, $S_{1/2} = \mathcal{O}(|\mathbf{q}|)$
  (MAID2007: $M_{2-} = \mathcal{O}(1)$, $E_{2-} = \mathcal{O}(1)$, $S_{1/2} = \mathcal{O}(1)$)
- At PT: $G_M = \mathcal{O}(|\mathbf{q}|^2) \Rightarrow$ finite slope
  $S_{1/2} = \mathcal{O}(|\mathbf{q}|) \Rightarrow$ infinite slope $\quad \left(\frac{dF}{dQ^2} \propto \frac{1}{|\mathbf{q}|} \frac{dF}{d|\mathbf{q}|} \right)$

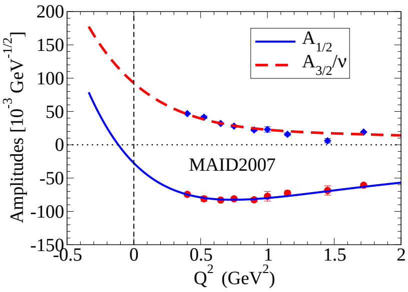
$\gamma^* N \to N(1520) \quad J^P = \frac{3}{2}^-$ – Summary



- **Jlab-SG** gives a good description of the data (Siegert's theorem)
- Not discussed here: differences between **MAID** and **Jlab** analysis
  Different behavior for $Q^2 > 1.5$ GeV^2 PRD 93, 113012 (2016)
- **Jlab-SG** and **MAID-SG**: almost the same behavior for $Q^2 < 0$

$\gamma^* N \to N(1520) \quad J^P = \frac{3}{2}^- \quad$ MAID2007 (optional)





$$S_{1/2} \neq |\mathbf{q}| E_{2-}/(2\lambda) \qquad A_{1/2} \neq \frac{A_{3/2}}{\sqrt{3}}$$

$$M_{2-} \neq 0 \qquad M_{2-} \propto \left(A_{1/2} - \frac{A_{3/2}}{\sqrt{3}} \right) \neq 0$$

$\gamma^* N \to \Delta(1232) \quad J^P = \frac{3}{2}^+ \quad \lambda = \sqrt{2}(M_R - M)$

Devenish et al, PRD 14, 3063 (1976); Jones and Scadron, Ann. Phy. 81, 1 (1973)

G_i $(i = 1, 2, 3)$ free of singularities at PT

$$G_M = Z_R \left[(M_R - M) G_5 + 4 M G_1 + \frac{4 M_R^2 |\mathbf{q}|^2}{Q_+^2} \left(\frac{G_1}{2 M_R} - G_3 \right) \right],$$

$$G_E = Z_R \left[(M_R - M) G_5 - \frac{4 M_R^2 |\mathbf{q}|^2}{Q_+^2} \left(\frac{G_1}{2 M_R} + G_3 \right) \right],$$

$$G_C = Z_R \left[2 M_R G_5 + \frac{4 M_R^2 |\mathbf{q}|^2}{Q_+^2} \left(\frac{1}{2} G_2 - G_3 \right) \right],$$

$$G_5 = G_1 + \frac{1}{2} (M_R + M) G_2 + (M_R - M) G_3, \qquad Z_R = \frac{2M}{3(M_R + M)}$$

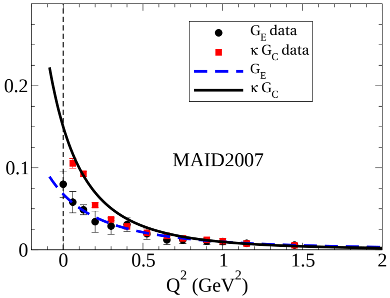
- Limit PT: $G_M = \mathcal{O}(1)$, $G_E = (M_R - M) Z_R G_5$, $G_C = 2 M_R G_5$

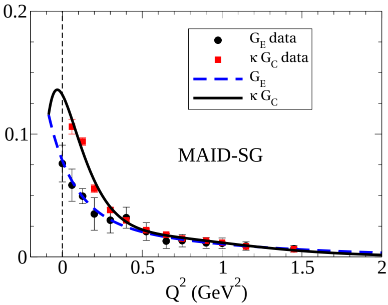
  $G_E = \frac{M_R - M}{2 M_R} G_C$ [Jones and Scadron (1973)]

- Amplitudes: $G_E = F_+ E_{1+}$, $G_C = \frac{2M_R}{|\mathbf{q}|} \sqrt{2} F_+ S_{1/2}$, $F_+ \propto 1/|\mathbf{q}|$

  $\frac{E_{1+}}{|\mathbf{q}|} = \lambda \frac{S_{1/2}}{|\mathbf{q}|^2}$

$\gamma^* N \to \Delta(1232)$ $\quad J^P = \frac{3}{2}^+$ $\quad \lambda = \sqrt{2}(M_R - M)$





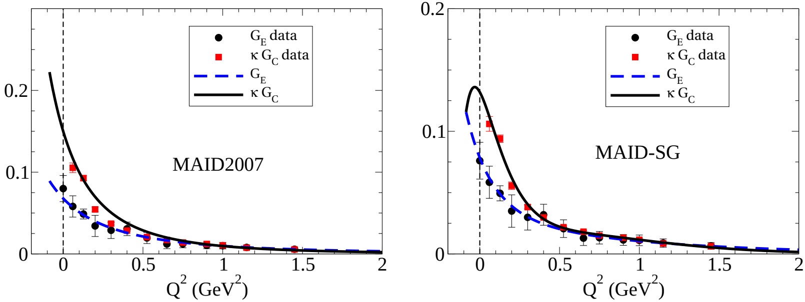
$$G_E \neq \overbrace{\frac{M_R - M}{2M_R}}^{=\kappa} G_C$$

$$E_{1+} = \lambda \frac{S_{1/2}}{|\mathbf{q}|} \to 0$$

$$G_E = \frac{M_R - M}{2M_R} G_C$$

$$\frac{E_{1+}}{|\mathbf{q}|} = \lambda \frac{S_{1/2}}{|\mathbf{q}|^2}$$

$\gamma^* N \to \Delta(1232) \quad J^P = \frac{3}{2}^+$



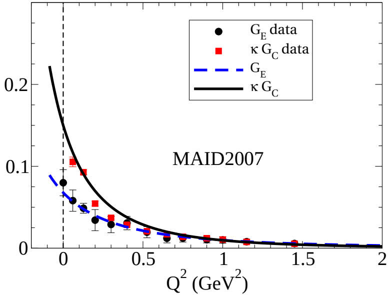
MAID-SG parametrization: $G_D = (1 + Q^2/0.71)^{-2}$, $C_0 = \frac{1}{e}\left(\frac{M^3 K}{M_R}\right)^{1/2}$

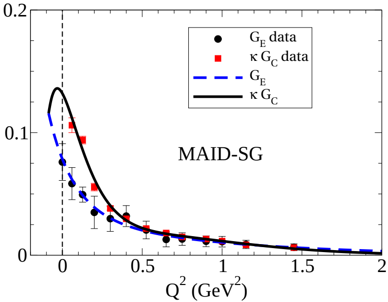
$$G_E = \frac{C_0}{K} b_0 (1 + b_1 Q^2 + b_2 Q^4 + b_3 Q^6) e^{-b_4 Q^2} G_D$$

$$G_C = \frac{C_0}{K}\frac{2M_R}{K} c_0 (1 + c_1 Q^2 + c_2 Q^4 + c_3 Q^6) e^{-c_4 Q^2} G_D,$$

c_0 determined by ST ($\neq$ MAID2007)

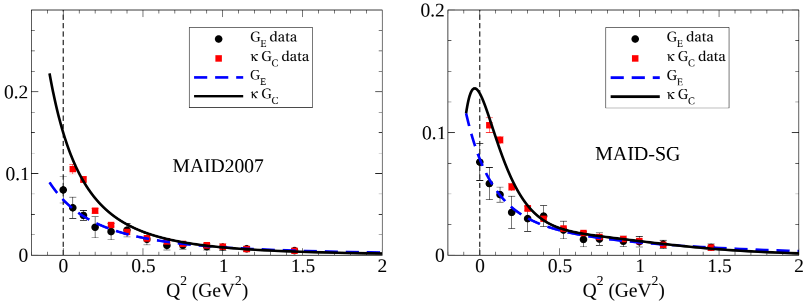
$\gamma^* N \to \Delta(1232) \quad J^P = \frac{3}{2}^+$ – Summary





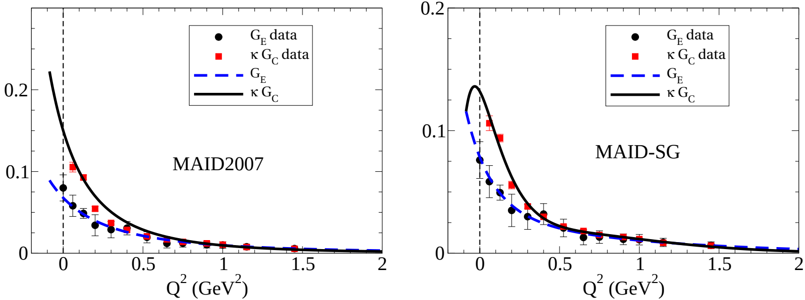
- MAID2007: $E_{1+} = \mathcal{O}(|\mathbf{q}|)$, $S_{1/2} = \mathcal{O}(|\mathbf{q}|^2)$, ...
  but the Sierget's theorem is violated

$\gamma^* N \to \Delta(1232) \quad J^P = \frac{3}{2}^+$ – Summary



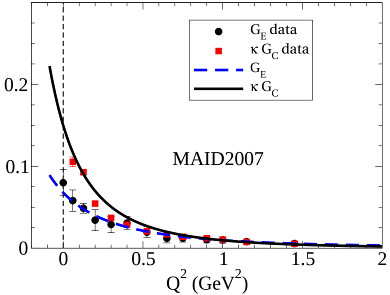
- MAID2007: $E_{1+} = \mathcal{O}(|\mathbf{q}|)$, $S_{1/2} = \mathcal{O}(|\mathbf{q}|^2)$, ...
  but the Sierget's theorem is violated
- MAID-SG: Sierget's theorem OK

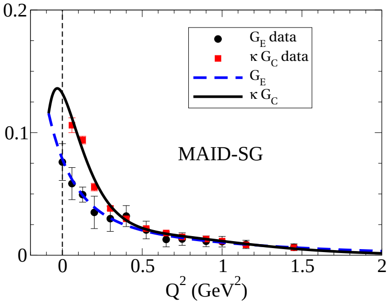
$\gamma^* N \to \Delta(1232) \quad J^P = \frac{3}{2}^+$ – Summary



- MAID2007: $E_{1+} = \mathcal{O}(|\mathbf{q}|)$, $S_{1/2} = \mathcal{O}(|\mathbf{q}|^2)$, ...
  but the Sierget's theorem is violated
- MAID-SG: Sierget's theorem OK
  - smooth functions G_E, G_C at low Q^2

$\gamma^* N \to \Delta(1232) \quad J^P = \frac{3}{2}^+$ – Summary





- MAID2007: $E_{1+} = \mathcal{O}(|\mathbf{q}|)$, $S_{1/2} = \mathcal{O}(|\mathbf{q}|^2)$, ...
  but the Sierget's theorem is violated
- MAID-SG: Sierget's theorem OK
  - smooth functions G_E, G_C at low Q^2
  - dificult to obtain a parametrization consistent with high Q^2
    $\Rightarrow$ Derive alternative parametrizations

$\gamma^* N \to N^*$ – Large Q^2

Parametrizations of form factors compatible with pQCD behavior (large Q^2)

- Start with MAID-type parametrization

$$G = a_0 (1 + a_1 Q^2 + ...) e^{-a_N Q^2} = a_0 \frac{1 + a_1 Q^2 + ...}{e^{a_N Q^2}}$$

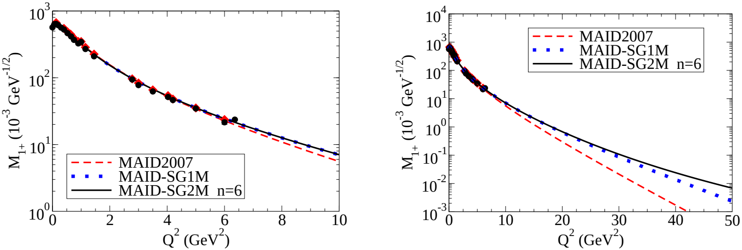
- Replace denominator by the truncated expansion of the exponential series

$$e^{a_N Q^2} \to 1 + a_N Q^2 + \frac{1}{2!} a_N^2 Q^4 ... + \frac{1}{M!} a_N^M Q^{2M}$$

- Then

$$G = a_0 \frac{1 + a_1 Q^2 + a_2 Q^4 ... + a_{M_1} Q^{2M_1}}{1 + a_N Q^2 + \frac{1}{2!} a_N^2 Q^4 ... + \frac{1}{M!} a_N^M Q^{2M}} \propto \frac{1}{(Q^2)^{M - M_1}}$$

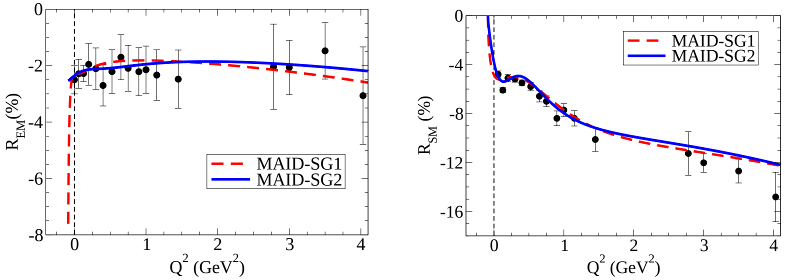
$\gamma^* N \to \Delta(1232)$ – Large Q^2



Graphs for $M_{1+} = G_M / F_+$, $1/F_+ \propto |\mathbf{q}|$,

- **MAID2007:** $G_M = a_0 \frac{(1+a_1 Q^2)}{e^{a_4 Q^2}} \sqrt{1+\tau} G_D$, $\quad G_M \propto e^{-a_4 Q^2}/Q^3$
  $a_1 \simeq 0.01$ GeV^{-2}; $a_4 = 0.23$ GeV^{-2}
- **MAID-SG2M:** $G_M = a_0 \frac{1+a_1 Q^2 + \frac{a_1^2}{2!} Q^4 + \ldots + \frac{a_1^6}{6!} Q^{12}}{1+a_4 Q^2 + \frac{a_4^2}{2!} Q^4 + \ldots + \frac{a_4^6}{6!} Q^{12}} G_D$, $\quad G_M \propto 1/Q^4$
  $a_1 \simeq 0.01$ GeV^{-2}; $a_4 \to a_4 = 0.16$ GeV^{-2}
- $Q^2 < 7$ GeV^2: **MAID2007** $\approx$ **MAID-SG2M**
  Differences for very large Q^2

$\gamma^* N \to \Delta(1232)$: Ratios $R_{EM} = -\frac{G_E}{G_M}$, $R_{SM} = -\frac{|\mathbf{q}|}{2M_R}\frac{G_C}{G_M}$



Parametrization MAID-SG2 (no dipole G_D), $G_E \propto 1/Q^4$, $G_C \propto 1/Q^6$:

$$G_E = \frac{C_0}{K} b_0 \frac{1 + b_1 Q^2 + b_2 Q^4 + b_3 Q^6}{1 + b_4 Q^2 + \frac{1}{2!} b_4^2 Q^4 + \frac{1}{3!} b_4^3 Q^6 + \frac{1}{4!} b_4^4 Q^8 + \frac{1}{5!} b_4^5 Q^{10}},$$

$$G_C = \frac{C_0}{K} \frac{2M_R}{K} c_0 \frac{1 + c_1 Q^2 + c_2 Q^4 + c_3 Q^6}{1 + c_4 Q^2 + \frac{c_4^2}{2!} Q^4 + \frac{c_4^3}{3!} Q^6 + \frac{c_4^4}{4!} Q^8 + \frac{c_4^5}{5!} Q^{10} + \frac{c_4^6}{6!} Q^{12}}$$

Siegert's theorem – Discussion

The Siegert's theorem had been discussed in the literature in the context of constituent quark models and the $\gamma^* N \to \Delta(1232)$ transition

D Drechsel and MM Giannini, PLB 143, 329 (1984); M Weyrauch and HJ Weber, PLB 171, 13 (1986); M Bourdeau and NC Mukhopadhyay, PRL 58, 976 (1987); S Capstick and G Karl, PRD 41, 2767 (1990); AJ Buchmann, PRC 58, 2478 (1998)

- **It was found that:**
  A consistent calculation requires the inclusion of processes beyond the impulse approximation at the quark level (two-body exchange current).
  Inclusion of quark-antiquark states/meson cloud contributions.

- **Conclusion:**
  Processes beyond impulse approximation are fundamental to describe the form factors data at small Q^2 (near the PT).

  Example: AJ Buchmann, PRL 93, 212301 (2004) – Large N_c
  $G_E(0), G_C(0) \propto r_n^2$ (neutron electric charge radius)^2

$\gamma^* N \to \Delta(1232)$ quadrupole form factors – meson cloud

Large N_c

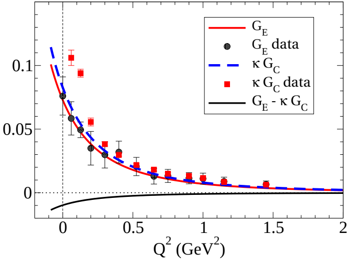
V. Pascalutsa and M. Vanderhaeghen, PRD 76, 111501 (2007);

P. Grabmayr and A. J. Buchmann, PRL 86, 2237 (2001)

$$\tilde{G}_{En} \equiv G_{En}/Q^2$$

$$G_E(Q^2) = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \tilde{G}_{En}(Q^2)$$

$$G_C(Q^2) = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En}(Q^2),$$



Imcompatible with ST

$$\mathcal{R}_{pt} = G_E(Q^2_{pt}) - \kappa G_C(Q^2_{pt}) \neq 0$$

$$Q^2_{pt} = -(M_\Delta - M)^2 = \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$$\mathcal{R}_{pt} \simeq -\left(\frac{M}{M_\Delta}\right)^{3/2} \frac{r_n^2}{12\sqrt{2}} Q^2_{pt} \simeq \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

Large N_c

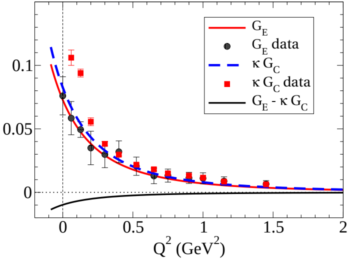
V. Pascalutsa and M. Vanderhaeghen, PRD 76, 111501 (2007);

P. Grabmayr and A. J. Buchmann, PRL 86, 2237 (2001)

$$\tilde{G}_{En} \equiv G_{En}/Q^2 \qquad G_{En}(Q^2) \simeq -\tfrac{1}{6} r_n^2 Q^2$$

$$G_E(Q^2) = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \tilde{G}_{En}(Q^2)$$

$$G_C(Q^2) = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En}(Q^2),$$



Imcompatible with ST

$$\mathcal{R}_{pt} = G_E(Q_{pt}^2) - \kappa G_C(Q_{pt}^2) \neq 0$$

$$Q_{pt}^2 = -(M_\Delta - M)^2 = \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$$\mathcal{R}_{pt} \simeq -\left(\frac{M}{M_\Delta}\right)^{3/2} \frac{r_n^2}{12\sqrt{2}} Q_{pt}^2 \simeq \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$\gamma^* N \to \Delta(1232)$ quadrupole form factors – meson cloud

Large-N_c

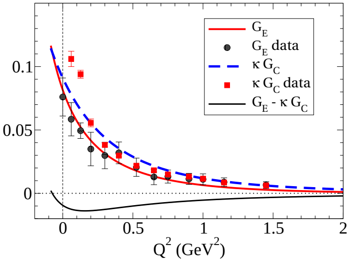
Approximated ST parametrization:

$$G_E \to \frac{G_E}{1+\frac{Q^2}{M_\Delta^2 - M^2}}$$

[same result for $G_E(0)$]

$$G_E = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \frac{\tilde{G}_{En}}{1+\frac{Q^2}{M_\Delta^2 - M^2}}$$

$$G_C = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En},$$



Almost compatible with ST

$$\mathcal{R}_{pt} = G_E(Q^2_{pt}) - \kappa G_C(Q^2_{pt})$$
$$= \mathcal{O}\left(\frac{1}{N_c^4}\right)$$

$$\mathcal{R}_{pt} \simeq \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta - M}{2M} \frac{r_n^2}{12\sqrt{2}} Q^2_{pt}$$

Large-N_c

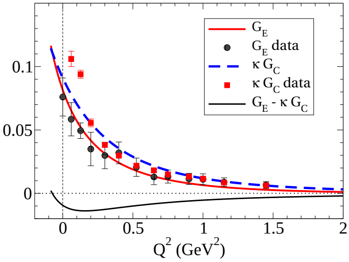
Approximated ST parametrization:

$$G_E \to \frac{G_E}{1+\frac{Q^2}{M_\Delta^2 - M^2}}$$

[same result for $G_E(0)$]

$$G_E = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \frac{\tilde{G}_{En}}{1+\frac{Q^2}{M_\Delta^2 - M^2}}$$

$$G_C = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En},$$



Almost compatible with ST

$$\mathcal{R}_{pt} = G_E(Q^2_{pt}) - \kappa G_C(Q^2_{pt}) = \mathcal{O}\left(\frac{1}{N_c^4}\right)$$

$$\mathcal{R}_{pt} \simeq \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta - M}{2M} \frac{r_n^2}{12\sqrt{2}} Q^2_{pt}$$

Include valence quark contribution

$\gamma^* N \to \Delta(1232)$ quadrupole form factors – valence quarks

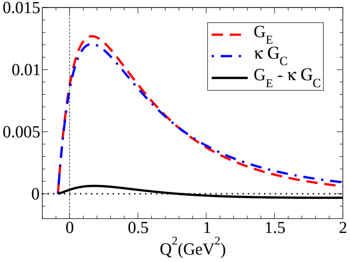
GR and MT Peña, PRD 80, 013008 (2009)

$$G_E^B(Q^2_{pt}) \propto \int_{\Omega_k} Y_{20}(\hat{\mathbf{k}})$$

$$G_C^B(Q^2_{pt}) \propto \int_{\Omega_k} Y_{20}(\hat{\mathbf{k}})$$

Compatible with ST

$$G_E^B(Q^2_{pt}) = G_C^B(Q^2_{pt}) = 0$$



$$\mathcal{R}^B_{pt} = 0$$

Valence quarks

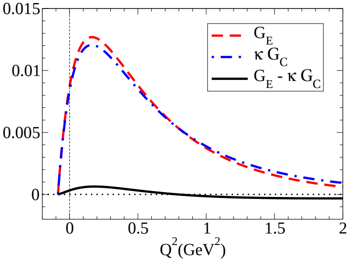
GR and MT Peña, PRD 80, 013008 (2009)

$$G_E^B(Q^2_{pt}) \propto \int_{\Omega_k} Y_{20}(\hat{\mathbf{k}})$$

$$G_C^B(Q^2_{pt}) \propto \int_{\Omega_k} Y_{20}(\hat{\mathbf{k}})$$

Compatible with ST

$$G_E^B(Q^2_{pt}) = G_C^B(Q^2_{pt}) = 0$$



$$\mathcal{R}^B_{pt} = 0$$

Valence quarks

Valence quark component **compatible** with the ST

$\gamma^* N \to \Delta(1232)$ quadrupole form factors – valence + mc

GR and MT Peña, PRD 80, 013008 (2009)

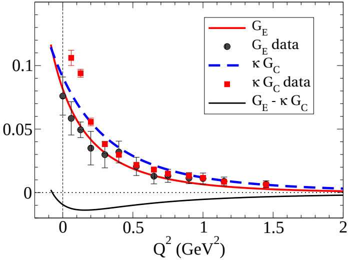
$$G_E^B(Q^2_{pt}) \propto \int_{\Omega_k} Y_{20}(\hat{\mathbf{k}})$$

$$G_C^B(Q^2_{pt}) \propto \int_{\Omega_k} Y_{20}(\hat{\mathbf{k}})$$

Almost compatible with ST

$$G_E^B(Q^2_{pt}) = G_C^B(Q^2_{pt}) = 0$$

$$\mathcal{R}_{pt} = \mathcal{O}\left(\frac{1}{N_c^4}\right)$$



$$\mathcal{R}_{pt} \simeq \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta - M}{2M} \frac{r_n^2}{12\sqrt{2}} Q^2_{pt}$$

Valence + meson cloud

$\gamma^* N \to \Delta(1232)$ quadrupole form factors – valence + mc

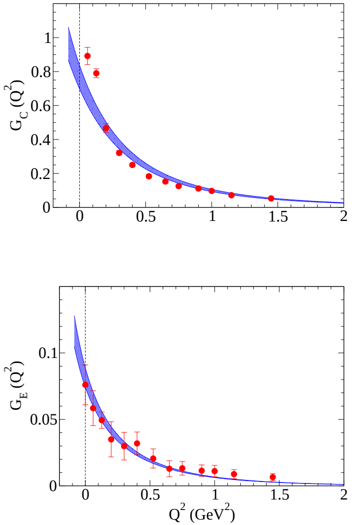
GR, arXiv:1606.03042

Combining **Quark Model** with **Pion Cloud**

parametrization *compatible* with ST:

$$G_E = G_E^B + G_E^\pi$$
$$G_C = G_C^B + G_C^\pi$$

- Sum *almost* compatible with ST $\mathcal{R}_{pt} = \mathcal{O}\left(\frac{1}{N_c^4}\right)$
- Good agreement with the data



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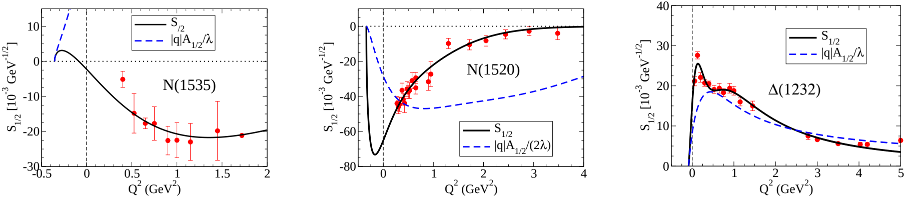
- $\Delta(1232)$: proposed **large-N_c** parametrization for the quadrupole (G_E and G_C) pion cloud – **compatible with ST**

Summary and Conclusions ... ($S_{1/2}$ vs $E_{l\pm}|\mathbf{q}|$)



Thank you

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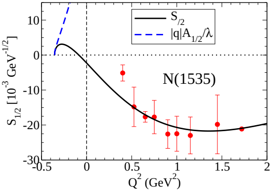

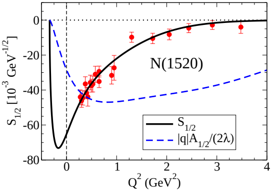

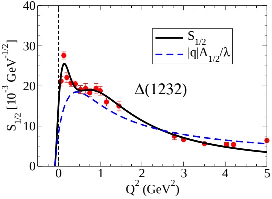

Arigatou/Obrigado

More ...

Additional information

Structure of the transition amplitudes

	$N(1535)\frac{1}{2}^-$	$N(1520)\frac{3}{2}^-$	$\Delta(1232)\frac{3}{2}^+$
Siegert	$A_{1/2} = 2b\tilde{F}_1$ $S_{1/2} = \sqrt{2}b\frac{\lvert\mathbf{q}\rvert}{M_R - M}\tilde{F}_1$	$E_{2-} = \mathcal{O}(1)$ $S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert)$ $M_{2-} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$	$E_{1+} = \mathcal{O}(\lvert\mathbf{q}\rvert)$ $S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$ $M_{1+} = \mathcal{O}(1)$
	$\tilde{F}_1 = \mathcal{O}(1)$	$\frac{1}{2}E_{2-} = \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert}$ $A_{1/2} = \frac{1}{\sqrt{3}}A_{3/2}$	$\frac{E_{1+}}{\lvert\mathbf{q}\rvert} = \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert^2}$
MAID	$A_{1/2} = \mathcal{O}(1)$ $S_{1/2} = \mathcal{O}(1)$	$E_{2-} = \mathcal{O}(1)$ $S_{1/2} = \mathcal{O}(1)$ $M_{2-} = \mathcal{O}(1)$	$E_{1+} = \mathcal{O}(\lvert\mathbf{q}\rvert)$ $S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$ $M_{1+} = \mathcal{O}(1)$
		$A_{1/2} \neq \frac{1}{\sqrt{3}}A_{3/2}$	$\frac{E_{1+}}{\lvert\mathbf{q}\rvert} \neq \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert^2}$

Example: Nucleon elastic form factors (optional)

- Nucleon form factors:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- Consider the threshold of $e^+ e^- \to p\bar{p}$
- At the threshold: $Q^2 = -4M^2$:

$$G_M(Q^2) = G_E(Q^2)$$

In this case the form factors can be measured.

$\gamma^* N \to N(1535)\frac{1}{2}^-$ $\lambda = \sqrt{2}(M_R - M)$ (optional)

- From the analysis of the transition current – singularity-free FF
  Devenish et al, PRD 14, 3063 (1976)

$$J^\mu = F_1 \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{M_R + M}$$

  we obtain the relation ($E \propto A_{1/2}$): $A_{1/2} = \lambda \frac{S_{1/2}}{|\mathbf{q}|}$

- Consistent with the multipole analysis at PT ($|\mathbf{q}| \to 0$)
  JD Bjorken and JD Walecka, Ann. Phys. 38, 35 (1966); Devenish et al, PRD 14, 3063 (1976); D Drechsel, SS Kamalov and L Tiator, EPJA 34, 69 (2007)

$$A_{1/2} = \mathcal{O}(1), \qquad S_{1/2} = \mathcal{O}(|\mathbf{q}|)$$

- Charge density: $\langle J^0 \rangle$ = projection in states $S_z = S'_z = +\frac{1}{2}$
- Current conservation: $\tilde{F}_1 = F_1 + \eta F_2$, $\eta = \frac{M_R - M}{M_R + M}$

$$S_{1/2} \propto \langle J^0 \rangle = \tilde{F}_1 (\bar{u}_R \gamma_5 u) \propto \tilde{F}_1 |\mathbf{q}|$$

- $S_{1/2} = \mathcal{O}(|\mathbf{q}|) \Rightarrow$ Orthogonality between N and R ($\langle J^0 \rangle \to 0$)
- ... but, only $\tilde{F}_1 = \mathcal{O}(1)$ is compatible with $S_{1/2} = \mathcal{O}(|\mathbf{q}|)$