Probing Y(4260) as the $D_1\bar{D}+\mathrm{c.c.}$ hadronic molecule state in e^+e^- annihilations

Si-Run Xue

Institute of High Energy Physics, CAS

xuesr@ihep.ac.cn

In collaboration with Prof. Qiang Zhao and Dr.Wen Qin

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Outline

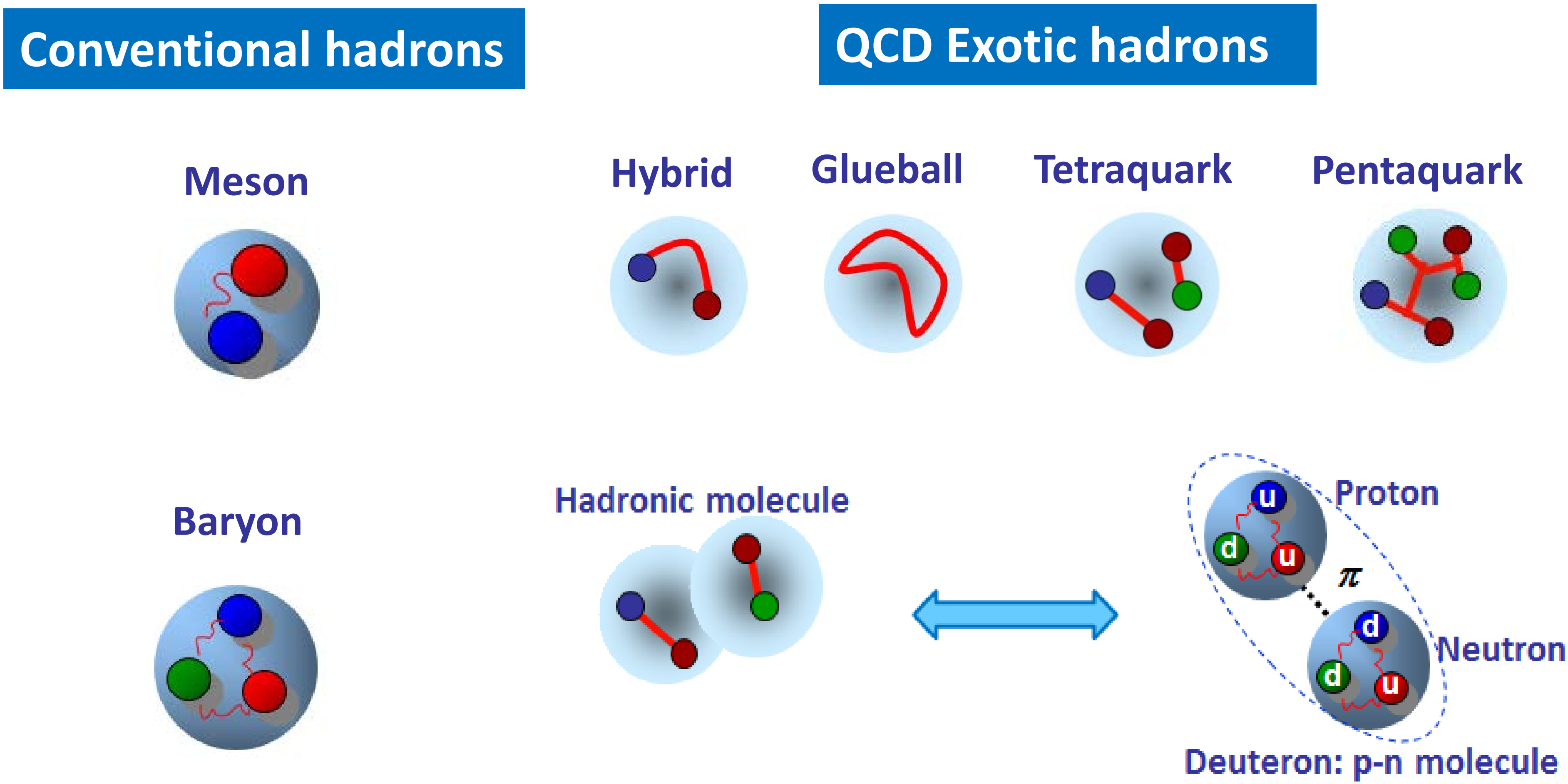
1. Introduction

2. The production mechanism of Y(4260) in e^+e^-annihilations in the framework of hadronic molecules

3. The line shapes of $e^+e^- \to Y(4260) \to D^*\bar{D}\pi + c.c.$ and the decay width of $Y(4260) \to e^+e^-$

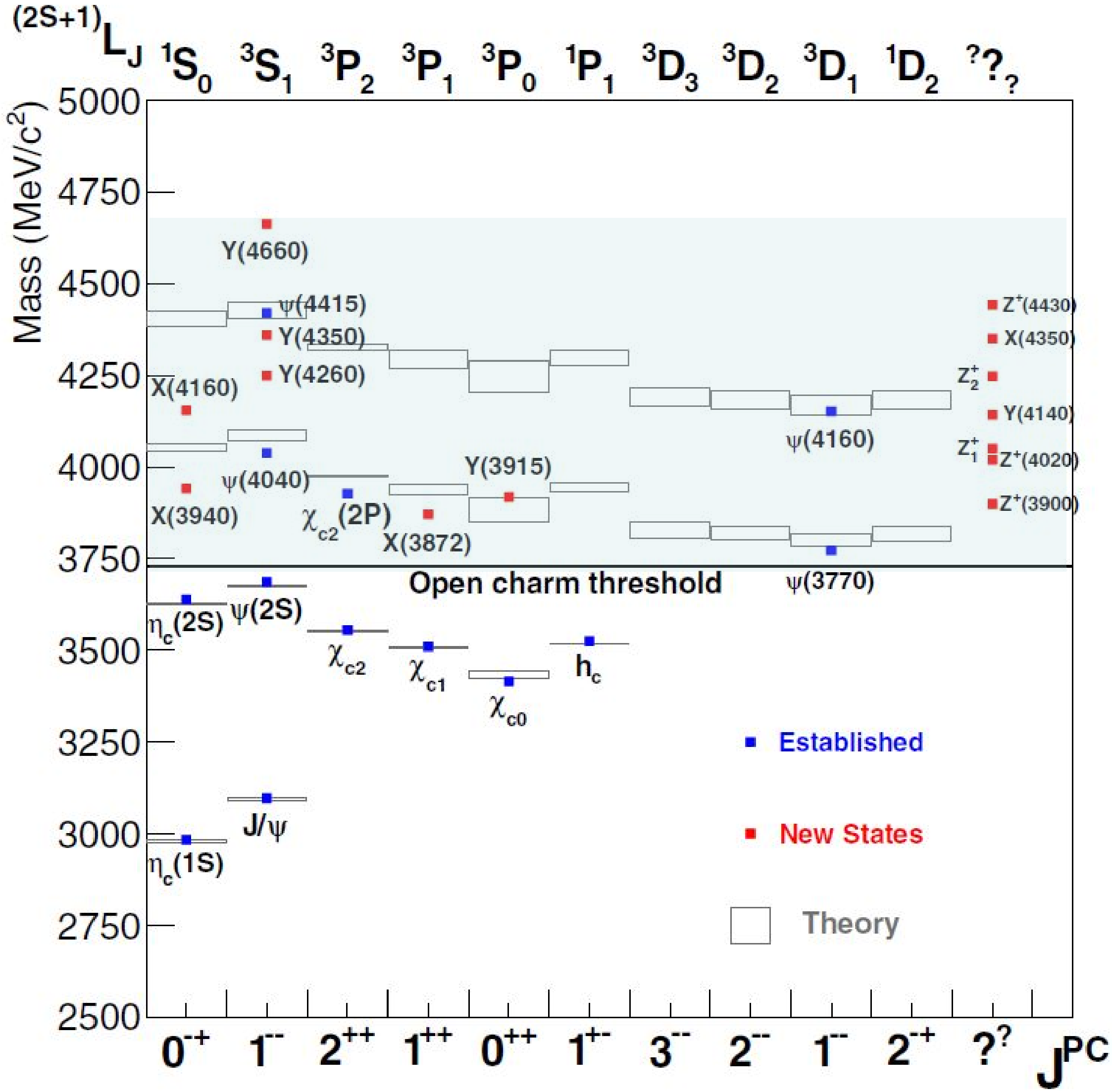
4. Summary

Quantum Chromo-Dynamics (QCD)



Evidence for exotic states provides a great opportunity for understanding the nature of strong QCD. But it is a huge challenge due to the confinement property of QCD in the low-energy region .

Charmonium Spectrum

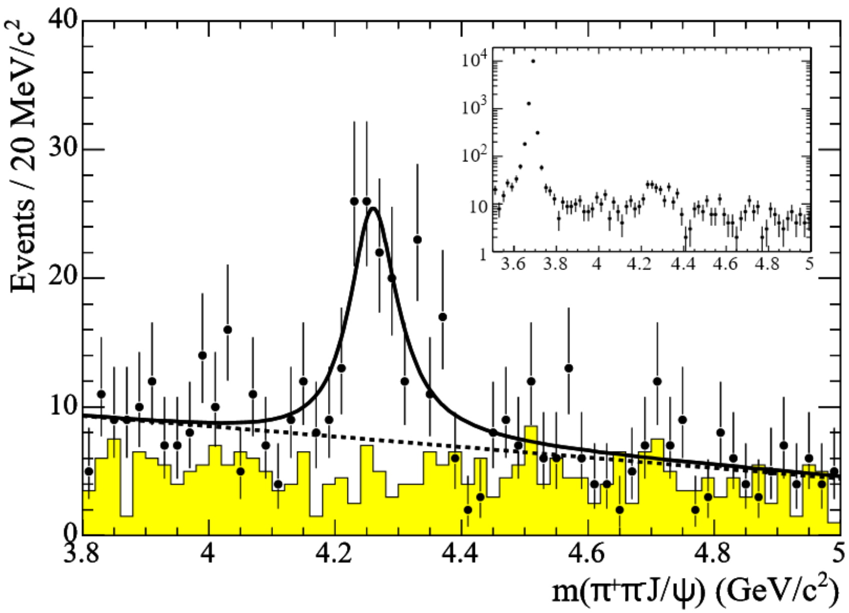


New charmonium-like states observed in experiment are the so-called XYZ states, which are not consistent with the traditional quark model predictions.

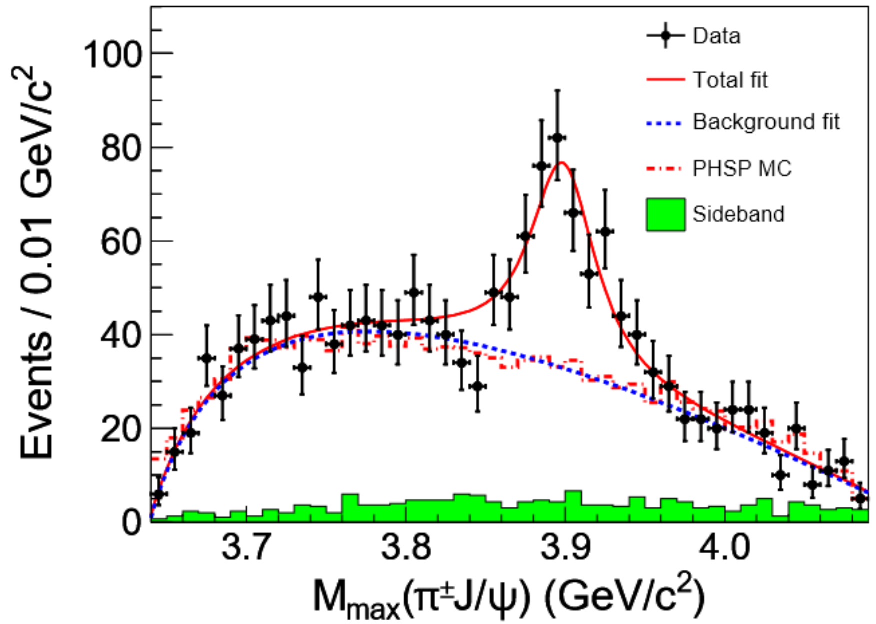
2. The production mechanism of Y(4260) in e^+e^-annihilations in the framework of hadronic molecules

Observation of Y(4260) in J/ψ ππ spectrum

$e^+ e^- \to J/\psi\ \pi\ \pi$



$e^+ e^- \to Y(4260) \to Zc\ \pi \to J/\psi\ \pi\ \pi$

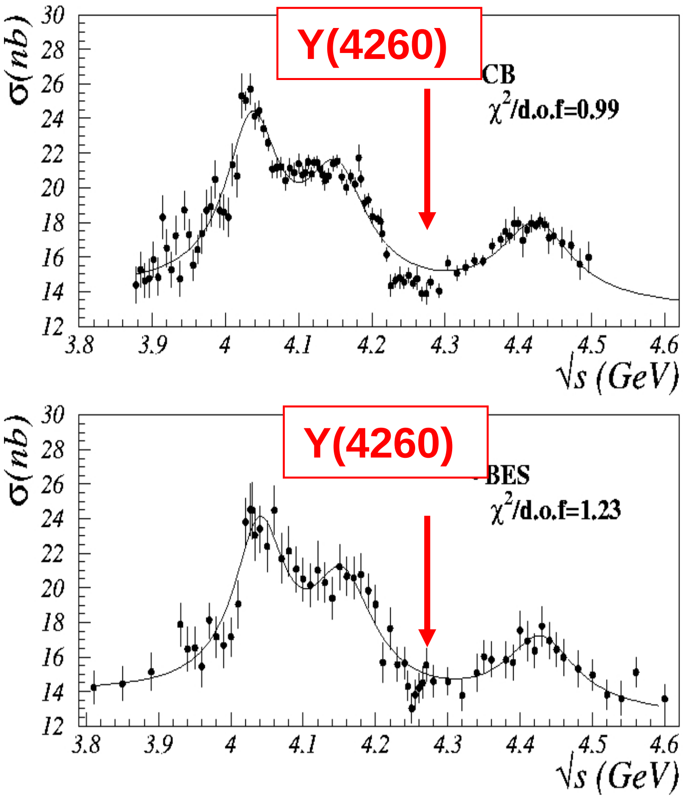


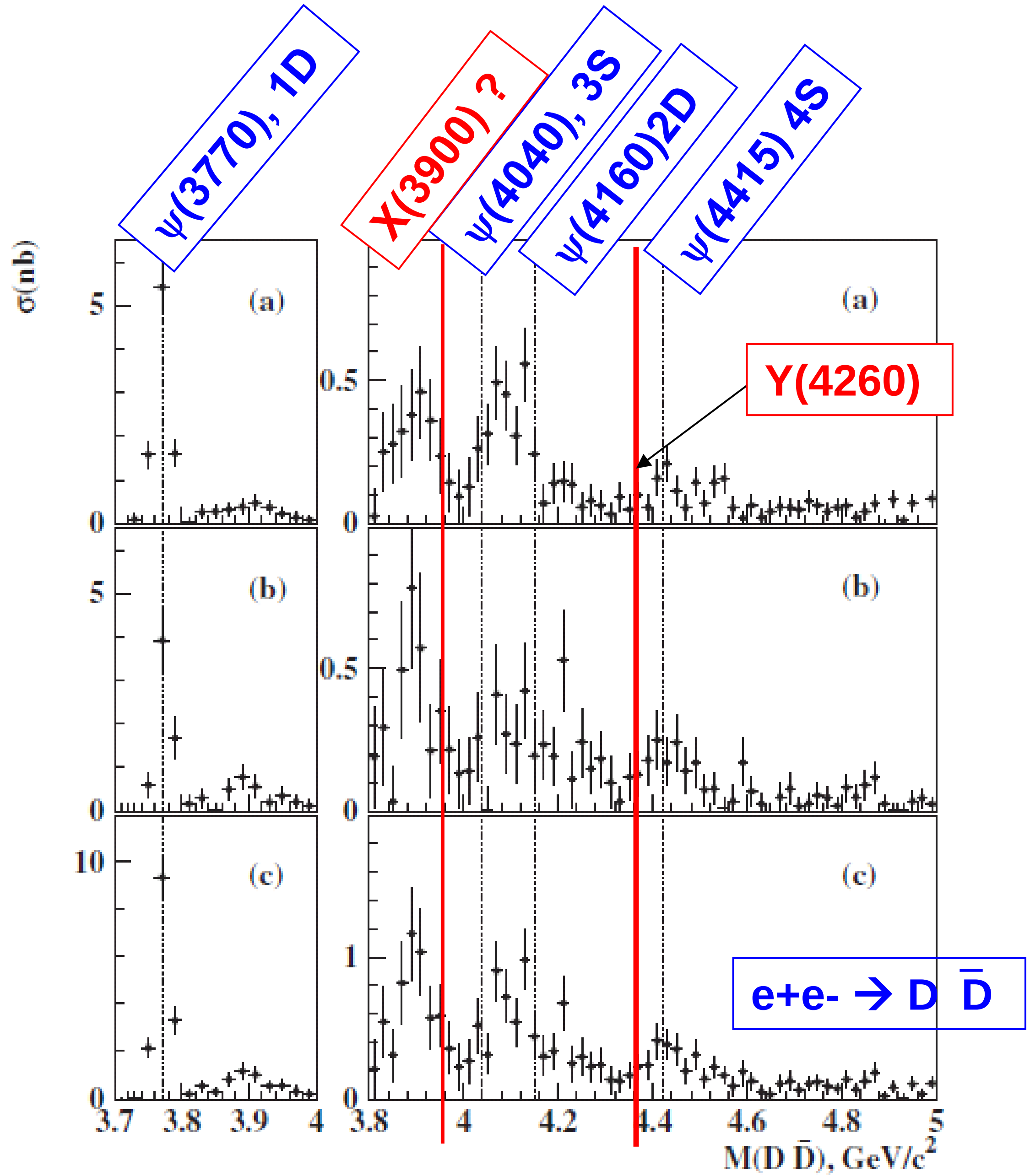
BarBar Collaboration,
Phys. Rev. Lett.,95,.142001(2005)

BESIII Collaboration,
Phys.Rev.Lett.,110,252001(2013)

Belle Collaboration,
Phys.Rev.Lett.,110 ,252002(2013)

$\sigma(e^+e^- \to \text{hadrons})$





Belle PRD77, 011103(2008).

Theoretical models of Y(4260)

Hybrid S. L. Zhu && E.Kou and O.Pene && F. E. Close and P. R.Page

Hadro-charmonium M. B. Voloshin && S. Dubynskiy

Tetraquark L. Maiani et al

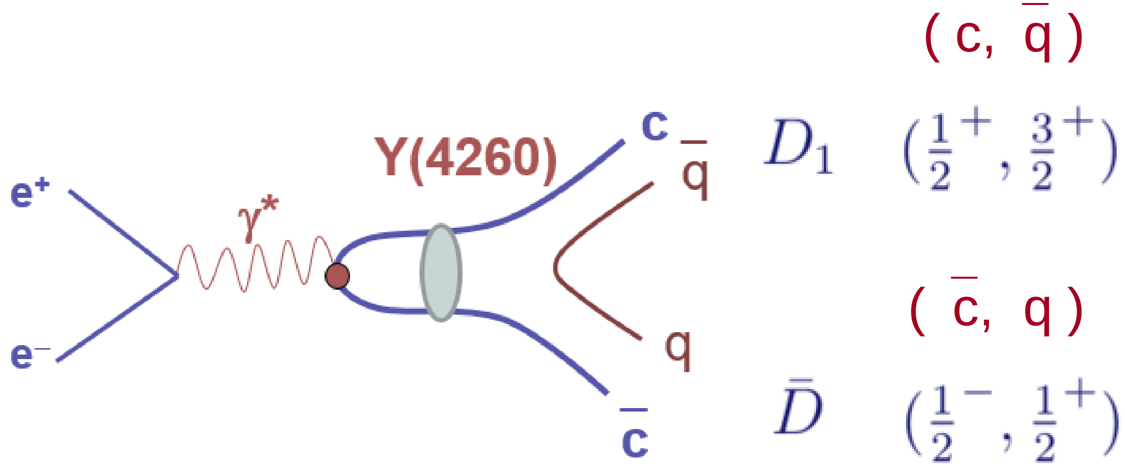
Charmonium F. J. Llanes-Estrada

Hadronic molecule G.J.Ding, C. Hanhart, F. K. Guo, Q. Zhao et al $\bar{D}D_1(2420) + c.c.$

M. Shi, D. L. Yao and H.Q. Zheng $\omega\chi_{c0}$

Molecular picture for Y(4260)

- **Production of Y(4260) in e^+e^-annihilations**



$c\bar{c}$ produced from a virtual photon decouples to an S-wave $D_1\bar{D} + c.c$ in terms of the heavy quark spin symmetry (HQSS),but it is broken due to the finite mass of charm quark [see Q. Wang et al., PRD89, 034001 (2014)].

- **Wave function**

$$|Y(4260)\rangle = \alpha|c\bar{c}\rangle + \beta|\bar{D}D_1 + c.c.\rangle,$$

- **Normalization relation**

$$\alpha^2 + \beta^2 = 1.$$

- **Molecular condition**

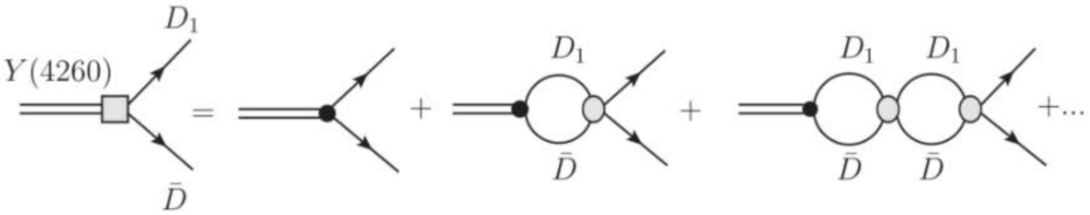
$$\beta^2 \gg \alpha^2$$

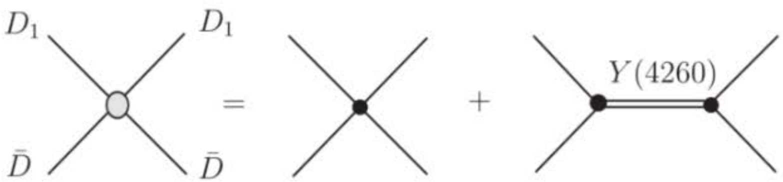
- **Effective Lagrangian for Y(4260)→ $D_1\bar{D}+\mathrm{c.c}$**

$$\mathcal{L}_Y = \frac{y^{\rm bare}}{\sqrt{2}}(\bar{D}_a^{\dagger}Y^iD_{1a}^{i\dagger} - \bar{D}_{1a}^{i\dagger}Y^iD_a^{\dagger}) + g_1\{(D_{1a}^i\bar{D}_a)^{\dagger}(D_{1a}^i\bar{D}_a) + (D_a\bar{D}_{1a}^i)^{\dagger}(D_a\bar{D}_{1a}^i)\} + H.c.,$$

M. Cleven, Q. Wang, F. K. Guo, C. Hanhart, U. G. Meißner and Q. Zhao PhysRevD.90.074039

- **Bubble diagrams for Y(4260)→ $D_1\bar{D}+\mathrm{c.c.}$**





- **Rescattering vertex**

$$\mathcal{T} = ig_1 + \frac{-i(y^{\rm bare})^2/2}{2(E-m_0)}$$

- **Non-relativistic propagator of Y(4260)**

$$\mathcal{G}_Y(E) = \frac{1}{2}\frac{i}{E - m_0 - \Sigma_1(E)},$$

$$\Sigma_1(E) \equiv \Sigma_{D_1\bar{D}}(E)(-i(y^{\rm bare})^2 + 4i(E - m_0)g_1)$$

$$\Sigma_{\bar{D}D_1}(E) = \frac{-1}{4}\int \frac{d^D l}{(2\pi)^D}\frac{1}{(l^0 - m_D - \vec{l}^{\,2}/(2m_D) + i\epsilon)(E - l^0 - m_{D_1} - \vec{l}^{\,2}/(2m_{D_1}) + i\Gamma_{D_1}/2)}.$$

$$\Sigma^{\overline{\rm MS}}_{\bar{D}D_1}(E) = \frac{\mu}{8\pi}\sqrt{2\mu(E - m_D - m_{D_1}) + i\mu\Gamma_{D_1}},$$

- **Renormalization of Y(4260) propagator**

$$\mathcal{G}_Y(E) = \frac{1}{2}\frac{iZ}{E - m_Y - Z\widetilde{\Sigma}_1(E) + i\Gamma^{{\rm non}-\bar{D}D_1}/2},$$

$$\widetilde{\Sigma}_1(E) \equiv \Sigma_1(E) - {\rm Re}(\Sigma_1(m_Y)) - (E - m_Y){\rm Re}(\partial_E\Sigma_1(m_Y))$$

$${\rm Re}(\widetilde{\Sigma}_1(m_Y)) = {\rm Re}(\widetilde{\Sigma}_1'(m_Y)) = 0, \qquad m_Y = m_0 + {\rm Re}\Sigma_1(m_Y).$$

$$Z \equiv 1/[1 - {\rm Re}(\partial_E\Sigma_1(m_Y))]$$

$$m_Y = 4.217 \pm 0.002 \text{ GeV} \qquad \Gamma^{{\rm non}-DD_1} = 0.056 \pm 0.003 \text{ GeV}$$

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- **Weinberg's compositeness theorem in the effective field theory**

$$\boldsymbol{\alpha^2 = |< c\bar{c}|Y(4260)>|^2 = Z}\ ,$$

$$\boldsymbol{\beta^2 = |< D_1\bar{D} + c.c\ |Y(4260)>|^2 = 1 - Z}\ ,$$

$$\boldsymbol{y^{eff} = \sqrt{Z}\ y^{bare}}$$

(1− Z) is the probability of finding Y(4260) in a $\boldsymbol{D_1\bar{D} + c.c}$ molecular state.

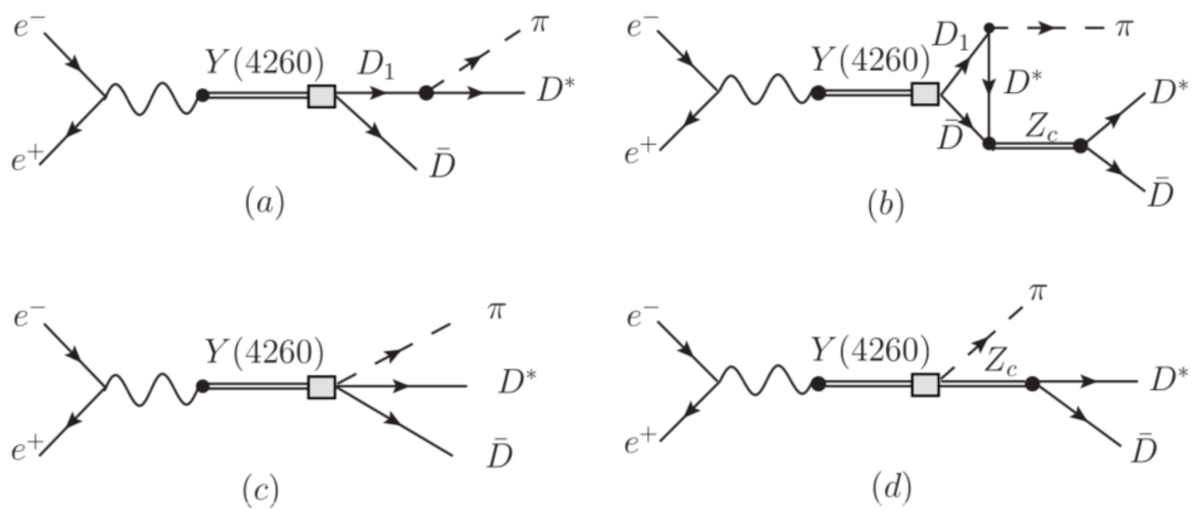
S. Weinberg, Phys. Rev. 130 (1963) 776
V. Baru, C. Hanhart et al Phys.Lett. B586 (2004) 53-61
G.-Y. Chen, Q. Zhao et al Chin.Phys. C39 (2015) no.9, 093101
D. Agadjanov, F.-K. Guo, G. Ríos and A. Rusetsky, JHEP 1501 (2015) 118

The renormalization constant Z is helpful to understand the molecular picture for Y(4260) , which is a function of $g_1,\ y^{bare},\ m_Y$ and can be determined by fitting the experimental data.

3. The line shapes of $e^+e^- \to Y(4260) \to D^*\bar{D}\pi + c.c.$ and the decay width of $Y(4260) \to e^+e^-$

Line shapes of $e^+e^- \to$ Y(4260) $\to D^*\bar{D}\pi + c.c.$

- **Feynman diagrams**



The decay process of Y(4260) $\to D^*\bar{D}\pi + c.c.$ is via the molecular component (a) and (b) in D-wave between D^* and π according to the heavy quark spin symmetry (HQSS) and through compact $c\bar{c}$ component (c) and (d) in S-wave.

Corresponding Lagrangians

- **Y(4260) $D_1\bar{D}$ coupling**

$$\mathcal{L}_Y = \frac{y^{\text{bare}}}{\sqrt{2}}(\bar{D}_a^{\dagger}Y^i D_{1a}^{i\dagger} - \bar{D}_{1a}^{i\dagger}Y^i D_a^{\dagger}) + g_1\{(D_{1a}^i\bar{D}_a)^{\dagger}(D_{1a}^i\bar{D}_a) + (D_a\bar{D}_{1a}^i)^{\dagger}(D_a\bar{D}_{1a}^i)\} + H.c.,$$

- **Zc(3900)$D^*\bar{D}$ coupling**

$$\mathcal{L}_Z = z(\bar{D}_b^{*\dagger i}Z_{ba}^i D_a^{\dagger} - \bar{D}_b^{\dagger}Z_{ba}^i D_a^{*\dagger i}) + \text{H.c.}$$

$$Z_{ba}^i = \begin{pmatrix} \frac{1}{\sqrt{2}}Z^{0i} & Z^{+i} \\ Z^{-i} & -\frac{1}{\sqrt{2}}Z^{0i} \end{pmatrix}_{ba}$$

z=0.77 GeV$^{-1/2}$

M. Cleven, Q. Wang, F. K. Guo, C. Hanhart, U. G. Meißner and Q. Zhao PhysRevD.90.074039

- **D1D*pi coupling**

$$\mathcal{L}_{D_1} = i\frac{h'}{f_\pi}\left[3D_{1a}^i(\partial^i\partial^j\phi_{ab})D_b^{*\dagger j} - D_{1a}^i(\partial^j\partial^j\phi_{ab})D_b^{*\dagger i} - 3\bar{D}_a^{*\dagger i}(\partial^i\partial^j\phi_{ab})\bar{D}_{1b}^j + \bar{D}_a^{*\dagger i}(\partial^j\partial^j\phi_{ab})\bar{D}_{1b}^i\right] + H.c.$$

$|h'| = 0.916 \pm 0.041\ \text{GeV}^{-1}$, determined by the process of $D_1(2420) \to D^*\pi$

- **Y(4260)γ^*coupling**

$$\mathcal{L}_{Y\gamma} = \frac{e m_Y^2}{f_Y} Y_\mu A^\mu ,$$

- **Propagator of Zc(3900)**

$$\mathcal{G}_{Z_c}(E) = \frac{1}{2} \frac{i}{E - m_Z + \Sigma_{\bar{D}D^*}(E) \times [iz^2 - 2i(E - m_Z)g_2] + i\Gamma^{\mathrm{non}-\bar{D}D^*}/2} ,$$

$$\Sigma_{\bar{D}D^*}(E) \equiv \frac{\mu'}{8\pi}(\sqrt{2\mu'\epsilon}\theta(\epsilon) + i\sqrt{-2\mu'\epsilon}\theta(-\epsilon)),$$

- **Parameterization of the coupling of Y(4260)$D^*\bar{D}\pi$ in S-wave through the compact $c\bar{c}$ component.**

$$A_S = a(M_{\bar{D}D^*}^2 + b)\mathcal{G}_{Z_c}(E)$$

a and b are real numbers.

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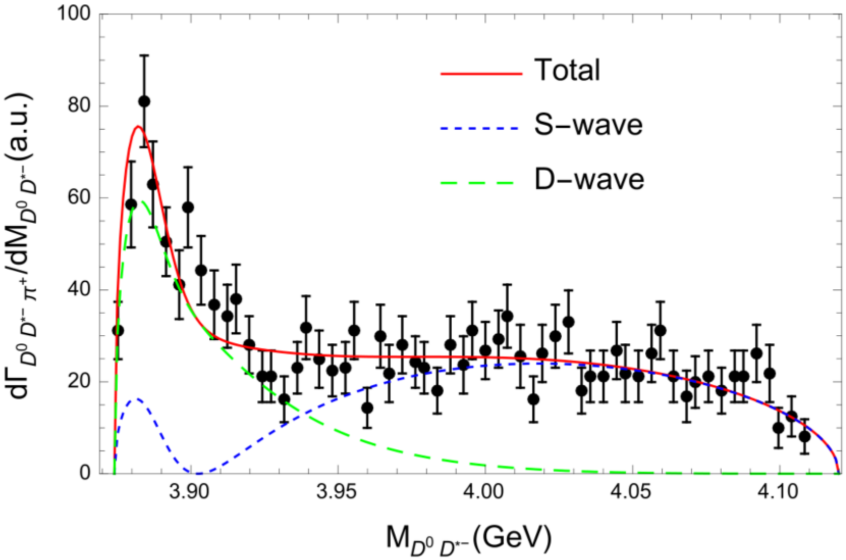
- **Amplitude of $e^+e^- \to$ Y(4260) $\to D^*\bar{D}\pi + c.c.$**

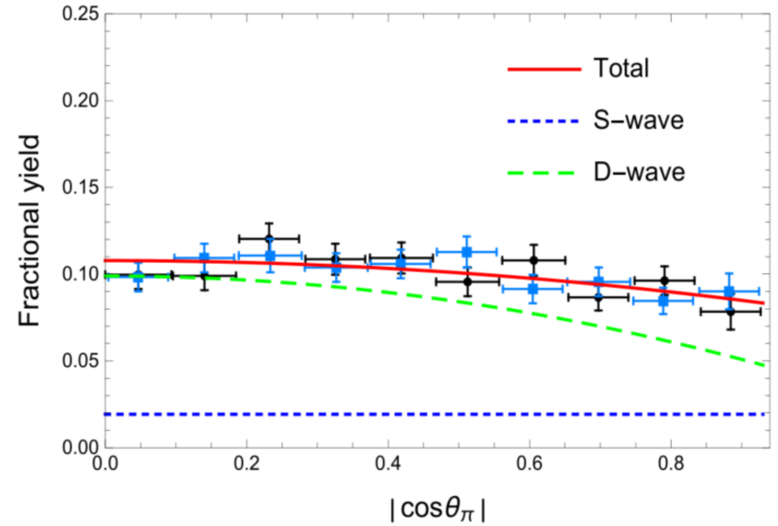
$$
\begin{aligned}
\mathcal{M}_{\bar{D}D^*\pi} &= \bar{v}(p_1)(-ie\gamma^i)u(p_2)\frac{i}{E^2}\frac{iem_Y^2}{f_Y}\frac{1}{2}\frac{iZ}{E-m_Y-Z\widetilde{\Sigma}_1(E)+i\Gamma^{\mathrm{non}-\bar{D}D_1}/2} \\
&\times\Big\{A_S\delta^{ij}+(\frac{iy^{\mathrm{bare}}}{\sqrt{2}})[\frac{i}{s_1-m_{D_1}^2+im_{D_1}\Gamma_{D_1}}+(-z^2)\mathcal{I}(E,\vec{p}_\pi^{\,2})\mathcal{G}_{Z_c}(s_2)]\times\frac{h'}{f_\pi}(3p_\pi^i p_\pi^j-\vec{p}_\pi^{\,2}\delta^{ij})\Big\}\epsilon_{D^*}^j \\
&\equiv \bar{v}(p_1)(\gamma^i)u(p_2)\frac{ie^2m_Y^2}{f_Y E^2}\frac{1}{2}\frac{iZ}{E-m_Y-Z\widetilde{\Sigma}_1(E)+i\Gamma^{\mathrm{non}-\bar{D}D_1}/2} \\
&\times\Big\{A_S\delta^{ij}+(\frac{iy^{\mathrm{bare}}}{\sqrt{2}})A_D(E,M_{\bar{D}D^*},M_{D^*\pi})\times\frac{h'}{f_\pi}(3p_\pi^i p_\pi^j-\vec{p}_\pi^{\,2}\delta^{ij})\Big\}\epsilon_{D^*}^j, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{I}(E,\vec{p}_\pi^{\,2}) &= \frac{1}{8}\int\frac{d^4l}{(2\pi)^4}\frac{i}{(l^0-m_{D_1}-\vec{l}^{\,2}/(2m_{D_1})+i\Gamma_{D_1}/2)}\frac{i}{(E-l^0-m_D-\vec{l}^{\,2}/(2m_D)+i\epsilon)} \\
&\times\frac{i}{(l^0-p_\pi^0-m_{D^*}-(\vec{l}-\vec{p}_\pi)^2/(2m_D)+i\epsilon)}\,.
\end{aligned}
$$

The effective coupling for Y(4260) to $D_1\bar{D}+$ c.c and a virtual photon can be renormalized as $y^{eff} = \sqrt{Z}\, y^{bare}$ and $\frac{1}{f^{eff}} = \sqrt{Z}\,\frac{1}{f_Y}$, respectively. If Y(4260) is a pure molecular state, which means Z=0, it cannot be produced in e^+e^-annihilations in the HQSS limit.

- **Numerical results**





Distribution of the $D^*\bar{D}$ invariant mass

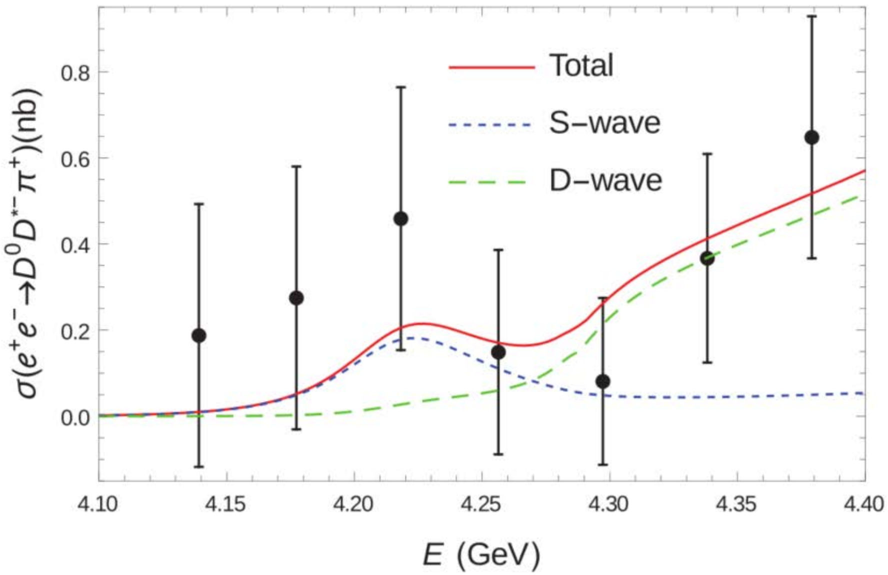
Angular distribution of the pion in the e^+e^- c.m. frame with respect to the beam axis.

M. Ablikim et al. Phys. Rev. Lett., 112:022001

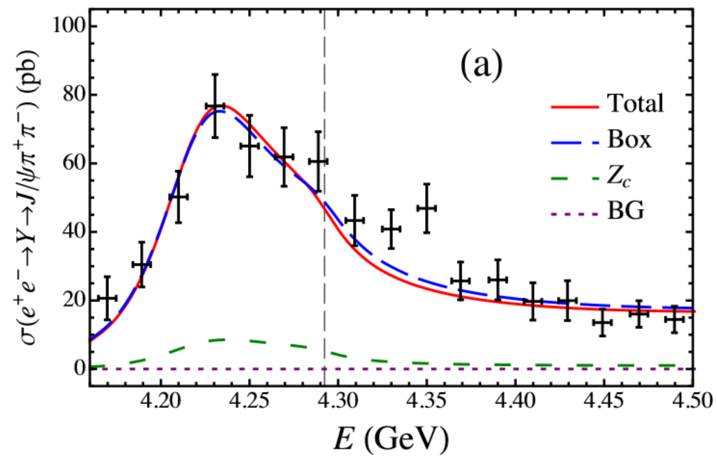
M. Ablikim et al. Phys. Rev., D92(9):092006

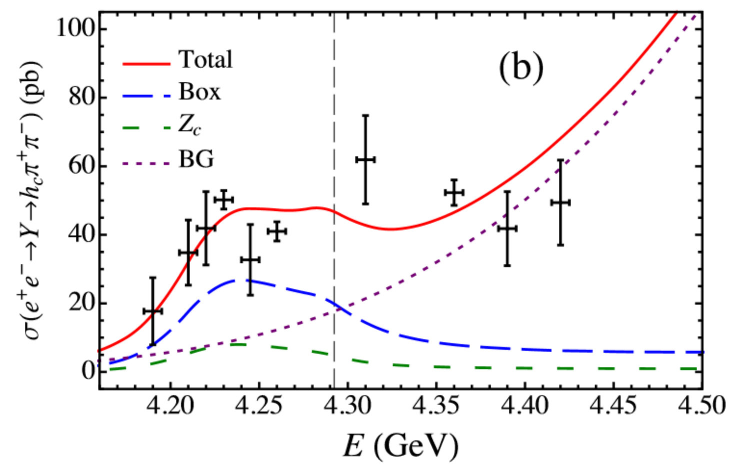
The contribution from the D-wave amplitude is much larger than that from the S-wave for the decay of Y(4260) $\rightarrow D^*\bar{D}\pi + c.c.$ around the threshold of $D^*\bar{D}$, which is consistent with our scenario ,i.e. Y(4260) as the $D_1\bar{D} + \mathrm{c.c.}$ hadronic molecule state dominantly. But note that the D-wave contribute to the cross section smaller than the S-wave around the mass of Y(4260) because of the limited phase space for D-wave in this region. Angular distribution of the pion also supports the molecular picture.

- **Cross section of $e^+e^- \rightarrow$ Y(4260) $\rightarrow D^*\overline{D}\pi + c.c.$**



G. Pakhlova et al Phys. Rev. D, 80:091101





- The line shape is not a simple Briet-Wigner.
- The $D^*\overline{D}\pi + c.c.$ decay channel is far larger than hidden charm channel with both D and S-wave contributions taken into account.
- We need the high accurate experimental data.

Martin Cleven et al. Phys. Rev. D, 90:074039

- **Parameters**

Parameters	Fitted values
$1/f_Y$	0.063 ± 0.011
$\lvert y^{\text{bare}}\rvert$	$(10.88 \pm 0.10)\ \text{GeV}^{-1/2}$
g_1	$(29.50 \pm 0.47)\ \text{GeV}^{-2}$
$\lvert a\rvert$	$(12.67 \pm 0.45)\ \text{GeV}^{-5/2}$
b	$(-15.23 \pm 0.01)\ \text{GeV}^2$
$\chi^2/\text{d.o.f}$	0.92

- **Coupling constants**

$$\frac{1}{f_Y^{\text{eff}}} = \sqrt{Z}\frac{1}{f_Y} = 0.023 \pm 0.004$$

$$y^{\text{eff}} = \sqrt{Z} y^{\text{bare}} = (3.94 \pm 0.04)\ \text{GeV}^{-1/2}$$

$$\Gamma_Y^{\text{total}} = (73.0 \pm 3.5)\ \text{MeV}$$

$$Z = 0.132 \pm 0.003$$

- **Wavefuntion**

$$\alpha^2 = Z = 0.132 \pm 0.003 \quad \beta^2 = 1 - Z = 0.868 \pm 0.003$$

$$|Y(4260)\rangle = 0.363|c\bar{c}\rangle + 0.932|\bar{D}D_1 + c.c.\rangle,$$

$\beta^2 \gg \alpha^2$ supports Y(4260) as the $D_1\bar{D} + \text{c. c.}$ hadronic molecule state.

The decay width of $Y(4260) \rightarrow e^+e^-$

$$\frac{1}{f_Y^{\mathrm{eff}}} = \sqrt{Z}\frac{1}{f_Y} = 0.023 \pm 0.004$$

$$\Gamma(Y(4260) \rightarrow e^+e^-) \simeq 483 \text{ eV}$$

Resonance	Γ_{ee} (keV)
J/ψ	5.55 ± 0.14
$\psi(3686)$	2.35 ± 0.04
$\psi(3770)$	0.262 ± 0.018
$\psi(4040)$	0.86 ± 0.07
$\psi(4160)$	0.83 ± 0.07
$\psi(4415)$	0.58 ± 0.07

Experimental value: $\Gamma_{Y \rightarrow ee} < 580\text{eV}$

X. H. Mo et al, PLB640:182

$$\Gamma(Y(4260) \rightarrow e^+e^-) \simeq 23 \text{ eV}$$

H. Q. Zheng et al PRD92， 014020

- **The partial decay widths of Y(4260)**

$$\sigma(\sqrt{s}) = \frac{2J+1}{(2S_1+1)(2S_2+1)}\frac{4\pi}{s}[\frac{\Gamma_t^2/4}{(\sqrt{s}-\sqrt{s_0})^2+\Gamma_t^2/4}]Br_{in}Br_{out}$$

Channels	Width(MeV)	Branching ratio
e^+e^-	$(4.83 \pm 1.61) \times 10^{-4}$	$(4.2 \sim 9.3) \times 10^{-6}$
$(\bar{D}D^*\pi)_{S-\mathrm{wave}}$	56.54 ± 4.24	$0.680 \sim 0.875$
$(\bar{D}D^*\pi)_{D-\mathrm{wave}}$	8.37 ± 0.95	$0.097 \sim 0.134$
$\bar{D}D^*\pi$	64.91 ± 4.45	$0.790 \sim 0.998$
$J/\psi\pi\pi$	4.68	$0.061 \sim 0.067$
$h_c\pi\pi$	2.67	$0.035 \sim 0.038$
$\omega\chi_{c0}$	1.60	$0.021 \sim 0.023$
Γ_{Sum}	73.86 ± 4.45	$0.907 \sim 1.1$

- $\Gamma_Y^{\mathrm{total}} = (73.0\pm3.5)$ MeV **is consistent with the** Γ_{Sum} **.**

The HQSS breaking effects in the $^3P_0$ model

$$\begin{pmatrix} |D_1'(2430)\rangle \\ |D_1(2420)\rangle \end{pmatrix} = \begin{pmatrix} |1^+, j_l^p = \frac{1}{2}^+\rangle \\ |1^+, j_l^p = \frac{3}{2}^+\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |^1P_1\rangle \\ |^3P_1\rangle \end{pmatrix}$$

$$\theta = -\arcsin(\sqrt{\tfrac{2}{3}}) = -\arctan(\sqrt{2}) = -54.7^\circ$$ in the HQSS limit.

$$\mathcal{M}[Y(4260) \to (D_1\bar{D} + c.c.)_{S-\text{wave}}] = \frac{-2(\sin\theta + \sqrt{2}\cos\theta)}{9}\gamma\sqrt{E_Y E_{D_1} E_{\bar{D}}}(I_0 + 2I_\pm)$$ in $^3P_0$

γ denotes the strength of the quark-antiquark pair creation from vacuum. I_0 and $I_\pm$ are the corresponding spatial integral functions.

$$|y^{\text{bare}}| = 27.16|(\sin\theta + \sqrt{2}\cos\theta)| \text{ GeV}^{-1/2}$$

$$-54.7^\circ + (-5.7 \pm 4.0)^\circ$$

Belle PRD69,112002

$$\Downarrow$$

$$|y^{\text{bare}}| \simeq 1.36 \sim 7.90 \text{ GeV}^{-1/2}.$$

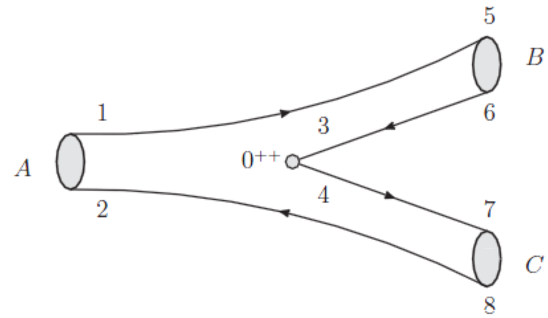
arXive:1605.02407

Summary

- The molecular picture for Y(4260) is consistent with the experimental data available at present.
- The partial decay widths of Y(4260) are obtained. The open charm decay channel is far larger than hidden charm channel with both D and S-wave contributions taken into account.
- We also investigate the HQSS breaking effects in the $^3P_0$ model as an independent check of the HQSS breaking and our scenario is self-consistent.

Thanks for your attention!

$^3P_0$ model



$$T = -3\gamma \sum_m \langle 1\ m; 1\ -m | 0\ 0\rangle \int \mathrm{d}^3\mathbf{k}_3\ \mathrm{d}^3\mathbf{k}_4 \delta^3(\mathbf{k}_3 + \mathbf{k}_4) \mathcal{Y}_{1m}\left(\frac{\mathbf{k}_3 - \mathbf{k}_4}{2}\right)$$
$$\times\ \chi^{34}_{1,-m}\ I^{34}_{00} \varphi^{34}_0\ \omega^{34}_0\ d^\dagger_{3i}(\mathbf{k}_3)\ b^\dagger_{4j}(\mathbf{k}_4)\ , \qquad (2.3)$$

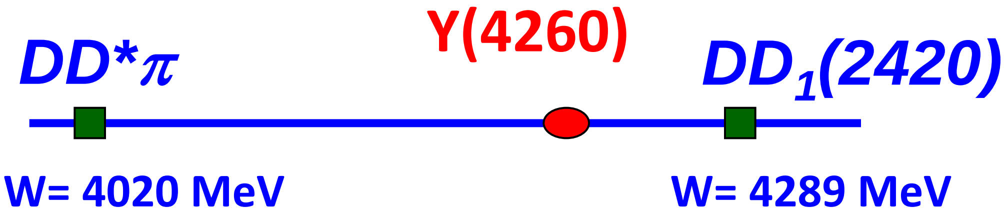
$$|A(n_A^{2S_A+1} L_{A J_A M_{J_A}})(\mathbf{K}_A)\rangle = \sqrt{2E_A} \sum_{M_{L_A}, M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A}\rangle$$
$$\times \int \mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2 \delta^3\left(\mathbf{K}_\mathrm{A} - \mathbf{k}_1 - \mathbf{k}_2\right)\ {}_{\mathrm{n_A L_A M_{L_A}}} \left(\frac{\mathrm{m}_2 \mathbf{k}_1 - \mathrm{m}_1 \mathbf{k}_2}{\mathrm{m}_1 + \mathrm{m}_2}\right)$$
$$\times\ \chi^{12}_{S_A M_{S_A}} I^{12}_A \varphi^{12}_A \omega^{12}_A |\ q_1\left(\mathbf{k}_1\right) \bar{q}_2\left(\mathbf{k}_2\right)\rangle,$$

$$\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{K}) = \gamma\sqrt{8E_A E_B E_C} \sum_{\substack{M_{L_A}, M_{S_A} \\ M_{L_B}, M_{S_B} \\ M_{L_C}, M_{S_C}, m}} \langle 1\ m; 1\ -m|\ 0\ 0\rangle$$
$$\times \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A}\rangle \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B}\rangle$$
$$\times \langle L_C M_{L_C} S_C M_{S_C} | J_C M_{J_C}\rangle \langle \varphi^{13}_B \varphi^{24}_C | \varphi^{12}_A \varphi^{34}_0\rangle \langle I^{13}_B I^{24}_C | I^{12}_A I^{34}_0\rangle$$
$$\times \langle \chi^{13}_{S_B M_{S_B}} \chi^{24}_{S_C M_{S_C}} | \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1 -m}\rangle I^{M_{L_A}, m}_{M_{L_B}, M_{L_C}}(\mathbf{K})\ . \qquad (2.8)$$

$$I^{M_{L_A}, m}_{M_{L_B}, M_{L_C}}(\mathbf{K}) = \int d^3\mathbf{k}_3 \Psi^*_{n_B L_B M_{L_B}}\left(\frac{m_3 \mathbf{K}}{m_1 + m_3} - \mathbf{k}_3\right) \Psi^*_{n_C L_C M_{L_C}}\left(\frac{-m_3 \mathbf{K}}{m_2 + m_3} + \mathbf{k}_3\right)$$
$$\times\ \Psi_{n_A L_A M_{L_A}}(\mathbf{K} - \mathbf{k}_3)\, \mathcal{Y}_{1m}(\mathbf{k}_3),$$

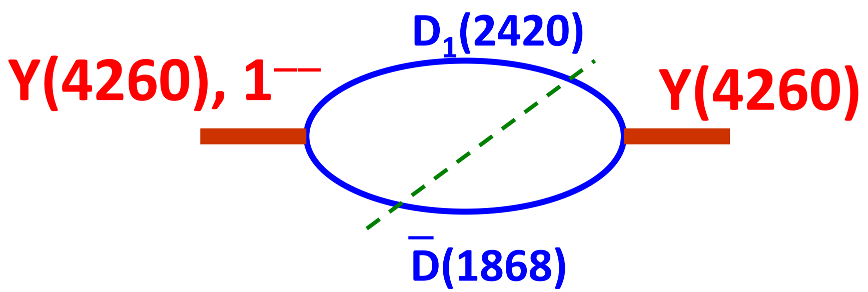
$$\Psi_{nLM}(\mathbf{k}) = \mathcal{N}_{nL} \exp\left(-\frac{R^2 \mathbf{k}^2}{2}\right) \mathcal{Y}_{LM}(\mathbf{k})\, \mathcal{P}(\mathbf{k}^2),$$

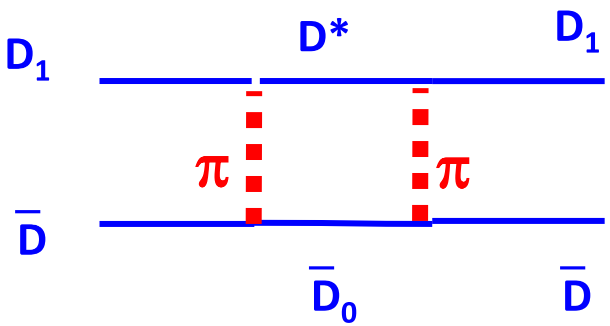
Y(4260) could be a hadronic molecule made of $DD_1(2420)$



- $D\ (c\bar{q})$, $J^P=0^-$;
- $D^*(c\bar{q})$, $J^P=1^-$;
- $D_1\ (c\bar{q})$, $J^P=1^+$.

"threshold state"





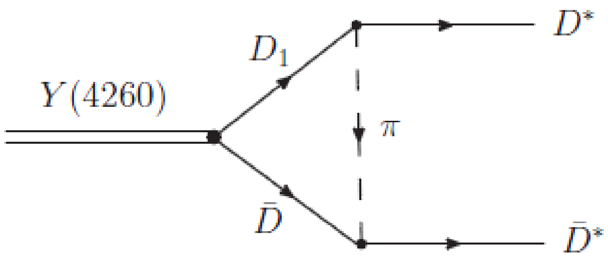
Q. Wang, C. Hanhart, QZ, PRL111, 132003 (2013); PLB(2013)

$$D_1 \qquad (\tfrac{1}{2}^+, \tfrac{3}{2}^+)$$

$$\bar{D} \qquad (\tfrac{1}{2}^-, \tfrac{1}{2}^+)$$

$$\psi_{10} = 1_H^{--} \otimes 0_L^{++}, \psi_{11} = 1_H^{--} \otimes 1_L^{++}, \psi_{12} = 1_H^{--} \otimes 2_L^{++}, \psi_{01} = 0_H^{-+} \otimes 1_L^{+-}$$

$$|D_1 D(^3S_1) + c.c.\rangle^{1^{--}} = \frac{1}{2}\psi_{01} + \frac{1}{2\sqrt{2}}\psi_{12} + \frac{\sqrt{5}}{2\sqrt{2}}\psi_{11}$$



The Feynman diagram for the process of $Y(4260) \to D^* \bar{D}^*$

$$\mathcal{L}_{YD_1\bar{D}} = \frac{y}{\sqrt{2}}(\bar{D}_a^\dagger Y^i D_{1a}^{i\dagger} - \bar{D}_{1a}^{i\dagger} Y^i D_a^\dagger) + H.c.,$$

$$\mathcal{L}_{D_1 D^*\pi} = i\frac{h'}{f_\pi}\Big[3D_{1a}^i(\partial^i\partial^j\phi_{ab})D_b^{*\dagger j} - D_{1a}^i(\partial^j\partial^j\phi_{ab})D_b^{*\dagger i} - 3\bar{D}_a^{*\dagger i}(\partial^i\partial^j\phi_{ab})\bar{D}_{1b}^j + \bar{D}_a^{*\dagger i}(\partial^j\partial^j\phi_{ab})\bar{D}_{1b}^i\Big] + H.c.,$$

$$\mathcal{L}_{\bar{D}^*\bar{D}\pi} = g_\pi \bar{D}^{*i\dagger}\bar{D}\partial^i\pi + H.c.,$$

$$i\mathcal{M}_{D^*\bar{D}^*} = \frac{3yh'g_\pi}{\sqrt{2}f_\pi}\epsilon_Y^i\epsilon_{D^*}^{j*}\epsilon_{\bar{D}^*}^{k*}\int\frac{d^4l}{(2\pi)^4}\frac{-3l^il^jl^k + \delta^{ij}l^k\vec{l}^{\,2}}{[(p_1+l)^2 - m_{D_1}^2 + i0^+][(p_2-l)^2 - m_D^2 + i0^+][l^2 - m_\pi^2 + i0^+]}$$