

Heavy-quark spin symmetry partners of the X(3872) molecule

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Introduction

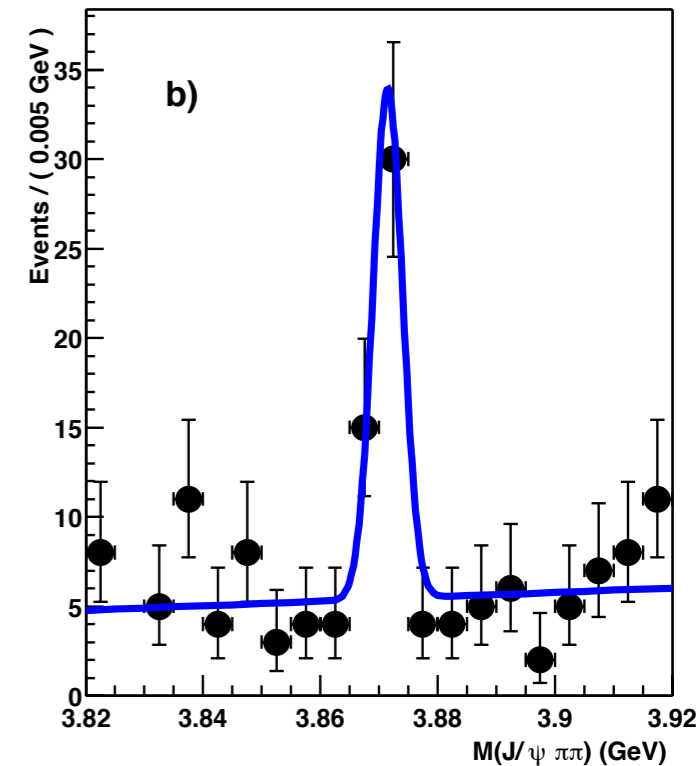
- Plenty of experimentally observed XYZ states do not fit in quark model predictions
 $X(3872)$, $Z_c(3900)$, $Y(4260)$, $Z_b(10610)$, $Z_b(10650)$, ... review article: [Brambilla et al. \(2011\)](#)

Enigmatic example: $X(3872)$ seen by [Belle](#), [CDF](#), [D0](#), [BABAR](#), [LHCb](#), [BESIII](#)

☞ because of its mass, width and quantum numbers does not fit to charmonium interpretation

☞ $X(3872)$ is a $J^{PC} = 1^{++}$ state residing near the $D\bar{D}^*$ threshold:

$$E_X = m_0 + m_0^* - M_X = 0.12 \pm 0.30 \text{ MeV}$$



⇒ Molecular interpretation: bound state of D and \bar{D}^* in an S wave

⇒ Other interpretations: tetraquark, hadrocharmonium, ...

review article: [Brambilla et al. \(2011\)](#)

Heavy quark spin symmetry

The XYZ states contain heavy quark and antiquark \implies employ heavy quark spin symmetry

👉 HQSS implies:

In the limit $\Lambda_{\text{QCD}}/m_Q \rightarrow 0$ strong interactions are independent of HQ spin

👉 Consequences of HQSS — number of states, location and decay properties — are different for different scenarios Cleven et al. (2015)

\implies Search for spin partner states \implies useful insights into the nature of XYZ states

This Talk: **Discuss HQSS predictions for the molecular scenario**

for HQSS predictions in the case of hadrocharmonium see [Cincioglu et al. \(2016\)](#)

HQSS for hadronic molecules

- HQSS molecular partners of isovector $J^{PC}=1^{+-}$ states $Z_b^+(10610)$ and $Z_b^+(10650)$ were predicted
Bondar et al. (2011), Voloshin (2011), Mehen and Powell (2011)
- $J^{PC}=2^{++}$ partner of the $X(3872)$ is predicted as a shallow bound state in the $D^*\bar{D}^*$ system
Nieves and Valderrama (2012), Guo et al. (2013)
- ➡ The width of this state is estimated to be from a few MeV—about a dozen MeV
using an EFT with perturbative pions
Albaladejo et al. (2015)

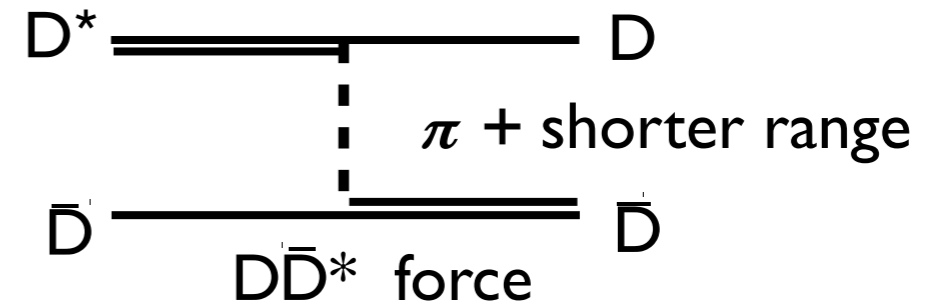
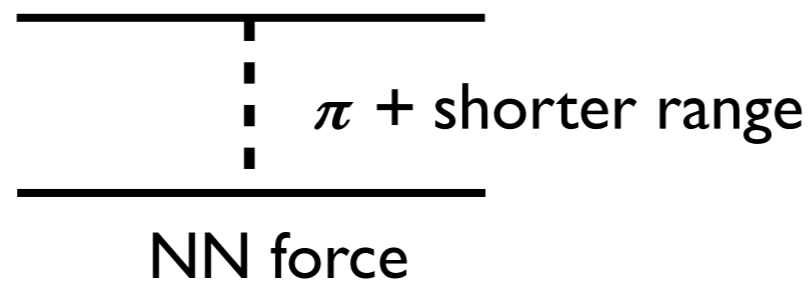
This Talk:

- Revisit *HQSS* predictions for the isoscalar partners of the $X(3872)$ within the molecular scenario
- Explore the role of pions:
 - ➡ how important are they for the location of the spin-partners?
 - ➡ are pionic interactions perturbative or non-perturbative?

X(3872) as a molecular state

- Prediction of the $D\bar{D}^*$ molecular state similar to the deuteron

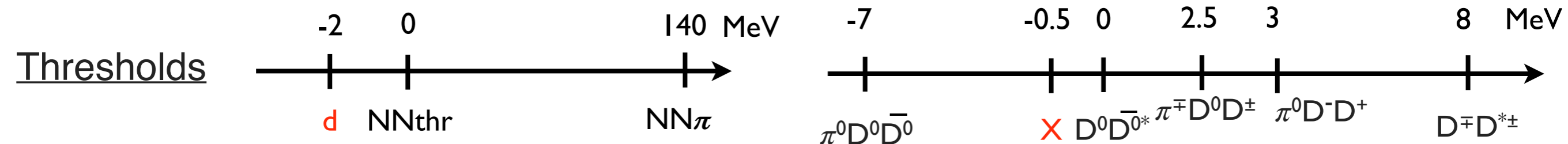
Okun, Voloshin (1976),
Törnqvist (1991)



- Experience from nuclear physics: One pion exchange (OPE) potential governs the lowest energy left-hand singularity of the NN force \implies crucial influence on the NN parameters

Epelbaum, Gegelia (2012), V.B., Filin, Epelbaum, Gegelia (2015)

- OPE between $D\bar{D}^*$ has even richer applications—many three-body thresholds around



👉 OPE is important for the decay rate $X \rightarrow D\bar{D}\pi$ and for chiral extrapolations of the X-pole

Fleming et al. (2007), our works (2010-2015), Jansen et al. (2015), Guo et al. (2014)

\implies Investigate the role of OPE for the molecular partner states

Molecular partners: contact theory

- Basis states J^{PC} made of D and \bar{D}^*

C-parity states: $C = \pm$

$$D\bar{D}^*(\pm) = \frac{1}{\sqrt{2}} (D\bar{D}^* \pm D^*\bar{D})$$

$$\begin{aligned} 0^{++} &: \{D\bar{D}(^1S_0), D^*\bar{D}^*(^1S_0)\}, \\ 1^{+-} &: \{D\bar{D}^*(^3S_1, -), D^*\bar{D}^*(^3S_1)\}, \\ 1^{++} &: \{D\bar{D}^*(^3S_1, +)\}, \\ 2^{++} &: \{D^*\bar{D}^*(^5S_2)\}, \end{aligned}$$

- S-wave derivativeless contact interactions respecting HQSS

$$V_{\text{LO}}^{(0^{++})} = \begin{pmatrix} C_{0a} & -\sqrt{3}C_{0b} \\ -\sqrt{3}C_{0b} & C_{0a} - 2C_{0b} \end{pmatrix},$$

$$V_{\text{LO}}^{(1^{+-})} = \begin{pmatrix} C_{0a} - C_{0b} & 2C_{0b} \\ 2C_{0b} & C_{0a} - C_{0b} \end{pmatrix},$$

$$V_{\text{LO}}^{(1^{++})} = C_{0a} + C_{0b},$$

$$V_{\text{LO}}^{(2^{++})} = C_{0a} + C_{0b},$$

Grinstein et al. (1992),
AlFiky et al. (2006),
Nieves and Valderrama (2012)

two LECs at LO C_{0a} and C_{0b}

$V_{\text{LO}}^{(1^{++})}$ and $V_{\text{LO}}^{(2^{++})}$: the same linear combination

- Lippmann-Schwinger type integral equations:

$$T^{(JPC)}(p, p') = V^{(JPC)}(p, p') - \int dk k^2 V^{(JPC)}(p, k) G(k) T^{(JPC)}(k, p')$$

- position of poles:

$$\det \left[1 + \int dk k^2 V^{(JPC)} G(k) \right] = 0$$

Molecular partners: contact theory

- Strict HQSS: $m = m^* = \bar{m} \implies$ Green functions coincide in all channels

Consequences of HQSS

- Poles of $T^{1^{++}}$ and $T^{2^{++}}$ coincide Nieves, Valderrama (2012)
- By making unitarity transform U for 0^{++} and 1^{+-} potentials $U = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}$

$$\tilde{V}^{(JPC)} = UV^{(JPC)}U^\dagger = \begin{pmatrix} C_0 & 0 \\ 0 & C'_0 \end{pmatrix} \quad \begin{aligned} C_0 &= C_{0a} + C_{0b} \\ C'_0 &= C_{0a} - 3C_{0b} \end{aligned}$$

Coupled-channel LS Eq. decouples into two Eqs. with $V = C_0$ and $V = C'_0$

- ➡ In the strict HQSS limit $T^{(JPC)}$ possess two decoupled solutions:

$$E_{X_{1^{++}}}^{(0)} = E_{X_{2^{++}}}^{(0)} = E_{X_{1^{+-}}}^{(0)} = E_{X_{0^{++}}}^{(0)} \quad \text{and} \quad E_{X'_{0^{++}}}^{(0)} = E_{X'_{1^{+-}}}^{(0)}$$

our finding is in line with Hidalgo-Duque et al. (2013)

- ➡ While the partner states corresponding to $V = C_0$ are predictable using the X(3872) as input, an additional experimental input is needed to determine the location of the states controlled by $V = C'_0$

Contact theory with HQSS breaking: uncoupled case

- Introduce D^* - D mass splitting and average mass:

$$\begin{aligned}\delta &= m_* - m = 141 \text{ MeV} \\ \bar{m} &= \frac{1}{4}(3m_* + m) = 1973 \text{ MeV} \\ \delta/\bar{m} &\simeq 7\%\end{aligned}$$

Uncoupled channels: 1^{++} and 2^{++}

- Leading effect: same binding energies defined relative to $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds

$$M_{X_{2^{++}}} = M_{X_{1^{++}}} + \delta$$

- Relation between the binding momenta:

$$\gamma_{X_{2^{++}}} = \left(1 - \frac{\delta}{2\bar{m}}\right) \gamma_{X_{1^{++}}} + \frac{\delta}{\pi\bar{m}}\Lambda + O\left(\frac{\delta^2\Lambda}{\bar{m}^2}, \frac{\gamma_{X_{1^{++}}}^2}{\Lambda}\right)$$

➡ Correction at $O(\delta)$ is cutoff dependent \Rightarrow extra contact term for renormalization

➡ But variation of Λ from $M_\pi=140$ to 1000 MeV has small impact on the binding energy: $E_{X_{2^{++}}}$ is in a few MeV below $D^*\bar{D}^*$

Contact theory with HQSS breaking: coupled channels

- Green functions of $D\bar{D}$, $D\bar{D}^*$ and $D^*\bar{D}^*$ states are not the same anymore

$$G_{\text{LO}}^{(1+-)}(k) = \begin{pmatrix} (k^2/2\mu_* - E - i0)^{-1} & 0 \\ 0 & (k^2/2\mu_{**} - E - i0)^{-1} \end{pmatrix}, \quad G_{\text{LO}}^{(0++)}(k) = \begin{pmatrix} (k^2/2\mu - E - i0)^{-1} & 0 \\ 0 & (k^2/2\mu_{**} - E - i0)^{-1} \end{pmatrix},$$
$$2\mu = \bar{m} - \frac{3}{4}\delta, \quad 2\mu_* = \bar{m} - \frac{\delta}{4}, \quad 2\mu_{**} = \bar{m} + \frac{1}{4}\delta,$$

$\Rightarrow VG$ in the Lippmann-Schwinger Eq. can not be diagonalized

☞ poles are determined by both C_0 and C_0' simultaneously — two exp. inputs needed

- C_0' can be fixed assuming the X(3915) to be a $0^{++} D\bar{D}^*$ partner of the X(3872)

Nieves and Valderrama (2012)

But ☞ X(3915) is about 100 MeV below $D\bar{D}^*$

☞ Zhou et al. (PRL 2015) argue that the X(3915) and X(3930) is the same 2^{++} state

Contact theory with HQSS breaking: coupled channels

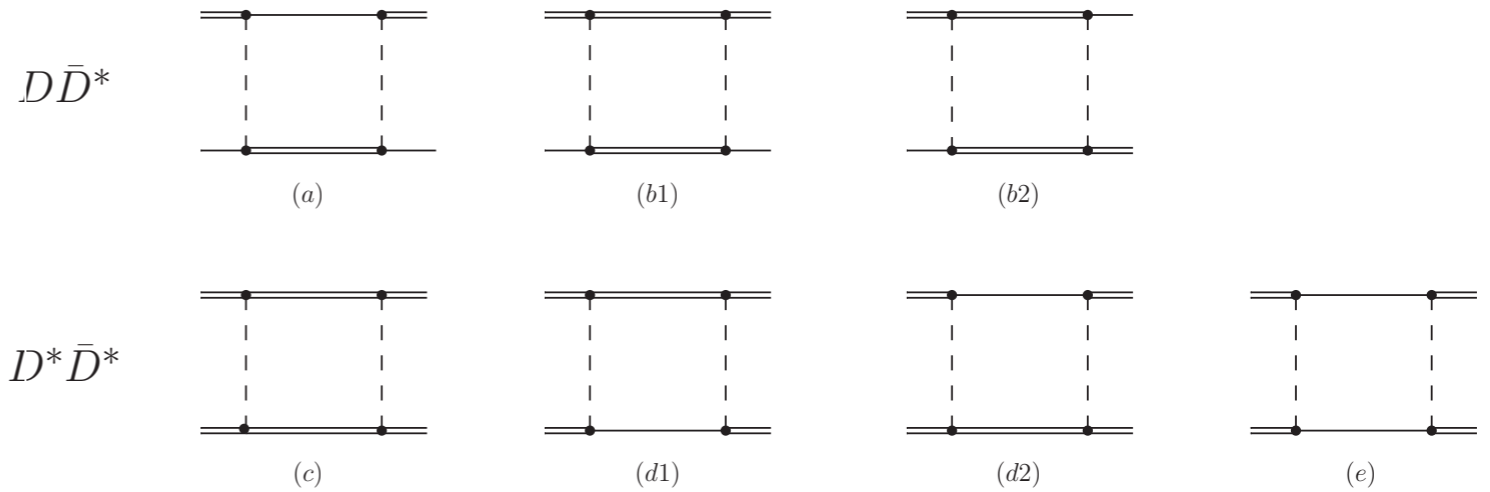
- What if the second input were known?
- Assuming, for example, the existence of a 1^{+-} state X_1 near the $D\bar{D}^*$ threshold
 - ➡ HQSS predicts binding momentum of a heavier $1^{+-} D^*\bar{D}^*$ state X_1'

$$\gamma_{X_1'+-} = \left(1 - \frac{\delta}{2\bar{m}}\right) \gamma_{X_1+-} + \frac{\Lambda\delta}{\pi\bar{m}} - \frac{(\gamma_{X_1+-} - \gamma_{X_1++})^2}{\sqrt{\bar{m}\delta}} + i \frac{(\gamma_{X_1+-} - \gamma_{X_1++})^2}{\sqrt{\bar{m}\delta}} + \dots$$

- ➡ Strong mixing of states that belonged to different multiplets in the strict HQSS limit
- ➡ $\gamma_{X_1'+-}$ has an *Im* part due to coupled-channels $D^*\bar{D}^* \rightarrow D\bar{D}^* \rightarrow D^*\bar{D}^*$
- ➡ Similar predictions for the $D\bar{D}$ and $D^*\bar{D}^*$ 0^{++} states

Strict HQSS limit in the presence of pions

- New transitions due to OPE \implies more coupled channels



For example,
at one loop:

- Extended basis states:

$$\begin{aligned}
 0^{++} &: \{D\bar{D}({}^1S_0), D^*\bar{D}^*({}^1S_0), D^*\bar{D}^*({}^5D_0)\}, \\
 1^{+-} &: \{D\bar{D}^*({}^3S_1, -), D\bar{D}^*({}^3D_1, -), D^*\bar{D}^*({}^3S_1), D^*\bar{D}^*({}^3D_1)\}, \\
 1^{++} &: \{D\bar{D}^*({}^3S_1, +), D\bar{D}^*({}^3D_1, +), D^*\bar{D}^*({}^5D_1)\}, \\
 2^{++} &: \{D\bar{D}({}^1D_2), D\bar{D}^*({}^3D_2), D^*\bar{D}^*({}^5S_2), D^*\bar{D}^*({}^1D_2), D^*\bar{D}^*({}^5D_2), D^*\bar{D}^*({}^5G_2)\}
 \end{aligned}$$

- Can one absorb the divergencies of the loop diagrams with different J^{PC} to a single contact term C_0 ? *Yes but only if all coupled-channel transitions are included!*

For each J^{PC} the coefficient in front of the leading divergence is the same once *all coupled-channels* are included

		$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}^*$	<i>Sum</i>
1^{++}	${}^{2S+1}L_J$	—	3S_1 3D_1	5D_1	
	Coeff.	—	$1/9$ $2/9$	$2/3$	1
2^{++}	${}^{2S+1}L_J$	1D_2	3D_2	5S_2 1D_2 5D_2 5G_2	
	Coeff.	$2/15$	$2/5$	$1/9$ $2/45$ $14/45$ 0	1

\implies successful renormalization program

Strict HQSS limit in the presence of pions

- static OPE at LO does not depend on the heavy-quark mass \implies HQSS multiplets (4+2) found in the contact case should hold!

- Unitary transform brings the potential to block-diagonal form:

$$\tilde{V}^{(0^{++})}(3 \times 3) = A(2 \times 2) \oplus B(1 \times 1),$$

$$\tilde{V}^{(1^{+-})}(4 \times 4) = A(2 \times 2) \oplus B(1 \times 1) \oplus C(1 \times 1)$$

$$\tilde{V}^{(1^{++})}(3 \times 3) = A(2 \times 2) \oplus D(1 \times 1),$$

$$\tilde{V}^{(2^{++})}(6 \times 6) = A(2 \times 2) \oplus D(1 \times 1) \oplus E(3 \times 3)$$

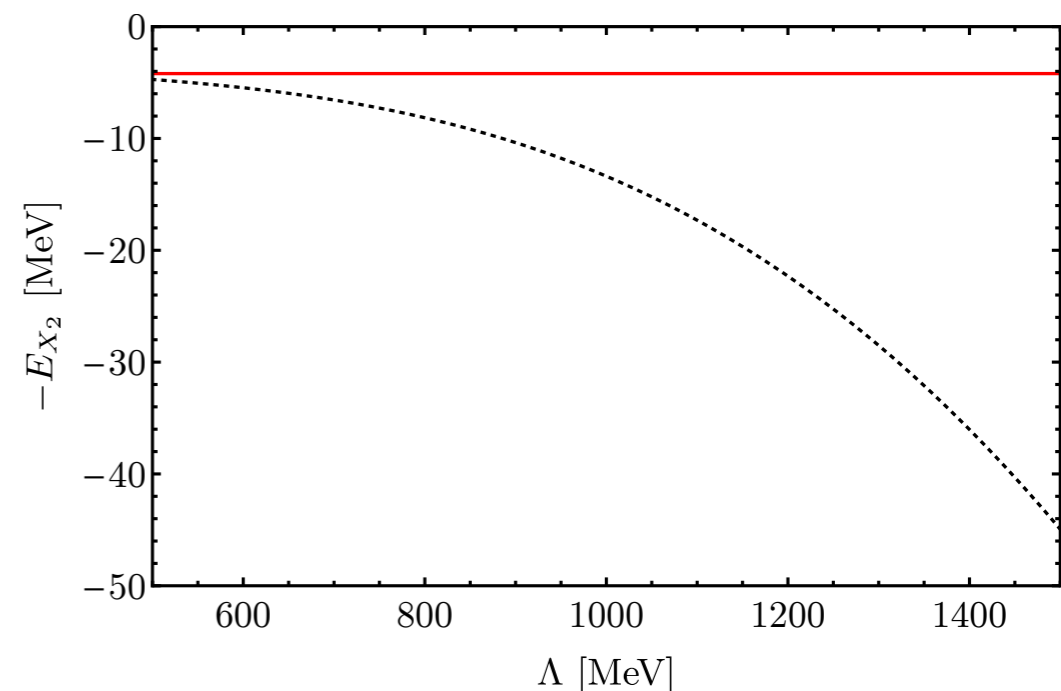
\blacktriangleright $A(2 \times 2)$ is common for all quantum numbers (related to C_0) \implies 4 degenerate states

\blacktriangleright $B(1 \times 1)$ is related to C_0' \implies bring about 2 more degenerate states $0^{++}, 1^{+-}$

- But this conclusion holds only if all particle coupled channels are included:

— Full coupled-channel dynamics: $E_{X_2^{++}} = E_{X_1^{++}}$

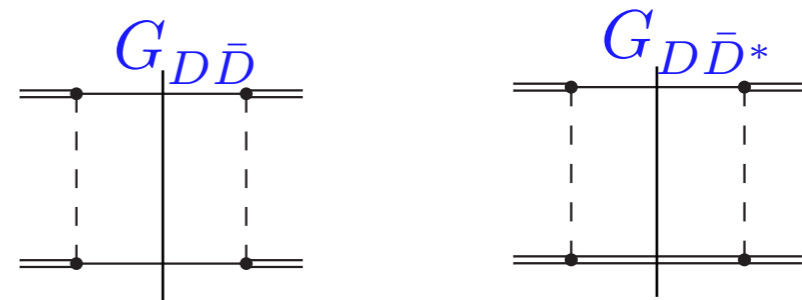
--- Neglecting $D^* \bar{D}^* \rightarrow D \bar{D} \rightarrow D^* \bar{D}^*$ transitions,
 $D^* \bar{D}^* \rightarrow D \bar{D}^* \rightarrow D^* \bar{D}^*$ as done by Nieves, Valderrama (2012), leads to severe violation of HQSS predictions



Contact + OPE interactions: beyond strict HQSS limit

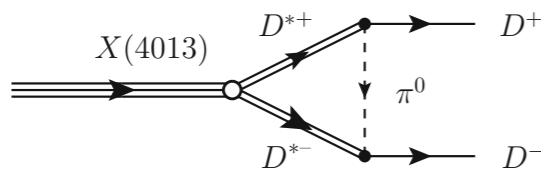
- Switch on D^*-D mass splitting $\implies 2^{++} D^*\bar{D}^*$ state acquires the finite width

Example of transitions which cause the Imaginary part of the amplitude:



- perturbative pions: *the width* $\Gamma_{X_{2^{++}}} \sim 1-20 \text{ MeV}$

For example:



	Without pion-exchange FF		With pion-exchange FF	
	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1 \text{ GeV}$	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1 \text{ GeV}$
$\Gamma(X_2 \rightarrow D^+ D^-)$ [MeV]	$3.3^{+3.4}_{-1.4}$	$7.3^{+7.9}_{-2.1}$	$0.5^{+0.5}_{-0.2}$	$0.8^{+0.7}_{-0.2}$
$\Gamma(X_2 \rightarrow D^0 \bar{D}^0)$ [MeV]	$2.7^{+3.1}_{-1.2}$	$5.7^{+7.8}_{-1.8}$	$0.4^{+0.5}_{-0.2}$	$0.6^{+0.7}_{-0.2}$
$\Gamma(X_2 \rightarrow D^+ D^{*-})$ [MeV]	$2.4^{+2.1}_{-1.0}$	$4.4^{+3.1}_{-1.2}$	$0.7^{+0.6}_{-0.3}$	$1.0^{+0.5}_{-0.2}$
$\Gamma(X_2 \rightarrow D^0 \bar{D}^{*0})$ [MeV]	$2.0^{+2.1}_{-0.9}$	$3.5^{+3.5}_{-1.0}$	$0.5^{+0.6}_{-0.2}$	$0.7^{+0.5}_{-0.2}$

Albaladejo, Guo, Hidalgo-Duque, Nieves, Valderrama (2015)

- But the *relevant momentum scales* stem from coupled-channels induced by OPE

$$q_1 = \sqrt{2\delta\bar{m}} \approx 700 \text{ MeV} \quad \text{from} \quad G_{D\bar{D}} = \frac{1}{k^2/2\mu - 2\delta - E - i0}$$

$$q_2 = \sqrt{\delta\bar{m}} \approx 500 \text{ MeV} \quad \text{from} \quad G_{D\bar{D}^*} = \frac{1}{k^2/2\mu_* - \delta - E - i0}$$

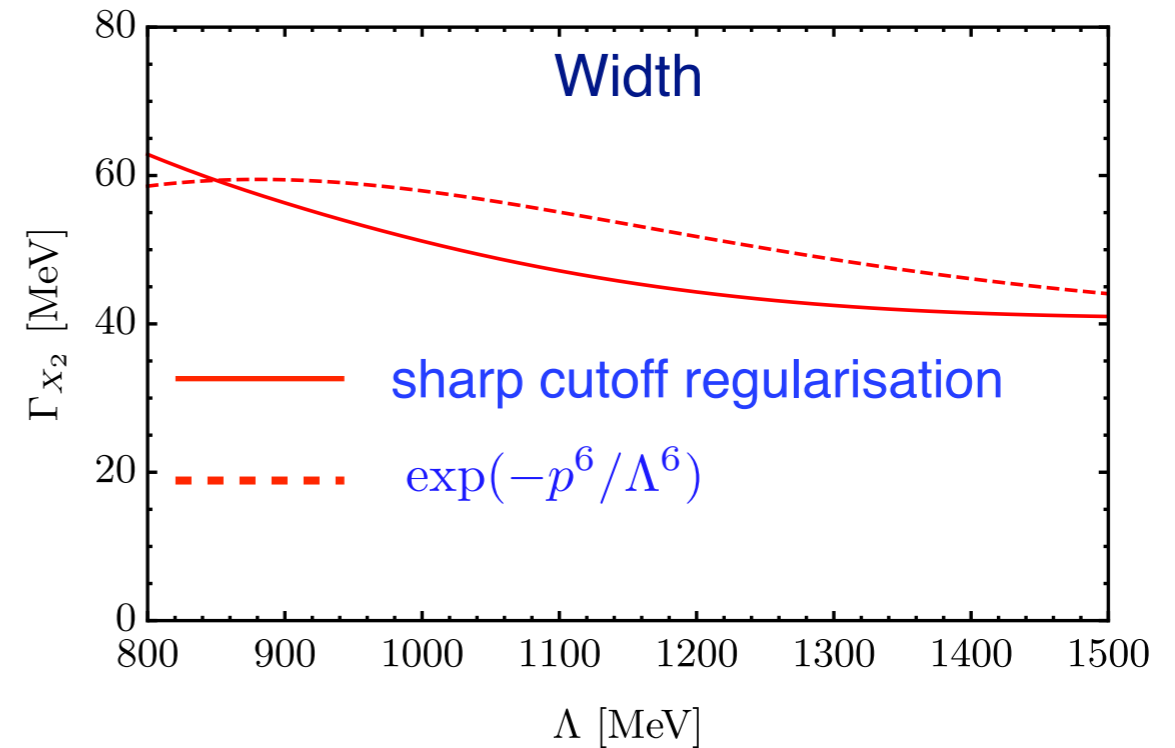
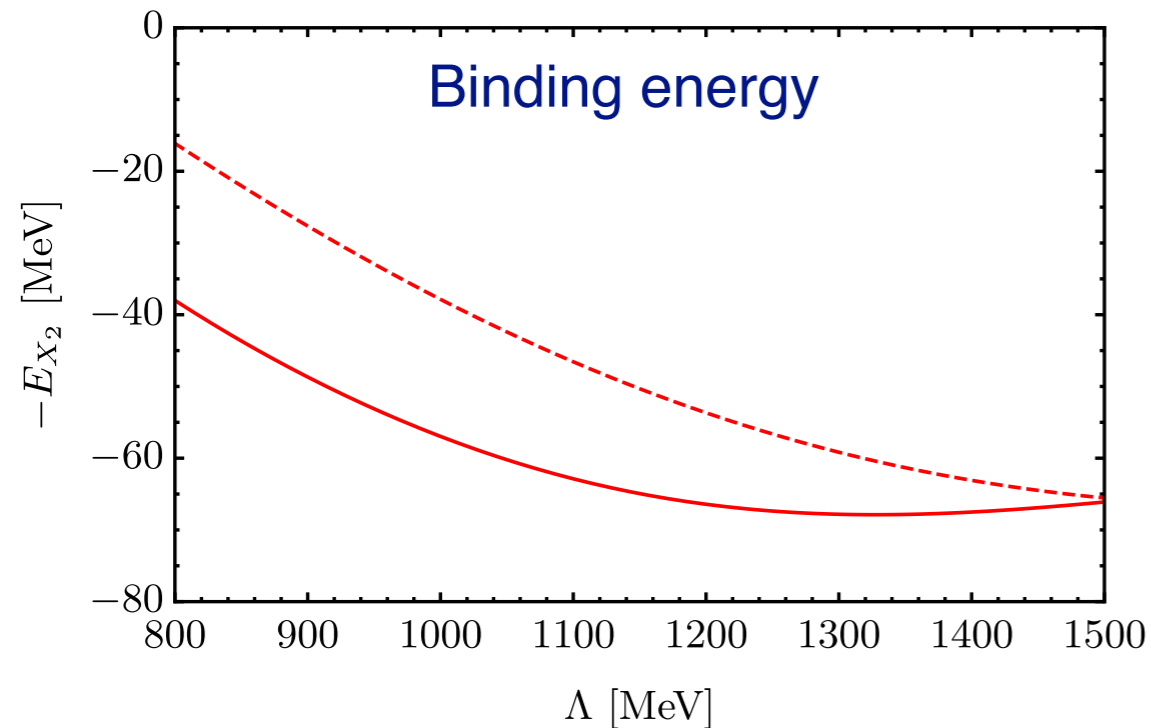
\implies D-wave coupled-channel transitions are not suppressed relative to S-wave ones

\implies Non-perturbative pion dynamics is expected to be important

Contact + OPE interactions at LO: Non-perturbative Results

- 2^{++} partner from the nonperturb. solution of the LS equation with $V = C_0 + V_{\text{OPE}}$

$$V_{\text{OPE}}(\vec{p}, \vec{p}') \sim -\frac{g_c^2}{(4\pi f_\pi)^2} \frac{(\vec{A}_1^* \cdot \vec{q})(\vec{A}_2 \cdot \vec{q})}{\vec{q}^2 + M_\pi^2}$$



- Significant shift of $E_{X_2^{++}}$ and large width $\Gamma_{X_2^{++}} \simeq 50 \pm 10$ MeV
 - Relatively Λ independent due to unitarity
 - much larger than in the perturbative study
- Albaladejo et al. (2015)

- Cutoff variation \implies estimate of a higher-order contact term at $O(\delta)$

Open Questions and Theory To-Do List

- Relatively small separation of scales may call the convergence of the EFT into question
 - ➡ include explicitly the members of SU(3) pseudoscalar octet as well as vector mesons
- Investigate the role of three-body effects in the OPE potential
 - ➡ Since the main contribution to the width of the 2^{++} D^*D^* state stems from coupled channels, three-body effects are not expected to change the picture qualitatively
 - ➡ Bring additional Imaginary parts from the right-hand cut
 - ➡ Bring additional HQSS corrections due to D , D^* energies
- Estimate HQSS violating contact terms more reliably

Summary

- We confirm that in the *strict HQSS* limit there are two degenerate multiplets of isoscalar molecular partner states with

$$E_{X_{1^{++}}}^{(0)} = E_{X_{2^{++}}}^{(0)} = E_{X_{1^{+-}}}^{(0)} = E_{X_{0^{++}}}^{(0)} \quad \text{and} \quad E_{X'_{0^{++}}}^{(0)} = E_{X'_{1^{+-}}}^{(0)}$$

- This conclusion holds in the presence of OPE interactions if and only if all particle coupled-channel transitions are included
- Leading HQSS breaking correction stems from D^* - D mass splitting
 - ➡ Molecular states reside in the vicinity of their thresholds
 - ➡ Predictions for the 2^{++} partner of the the $X(3872)$ are possible
 - ➡ One additional experimental input is needed to predict 0^{++} and 1^{+-} partners
- Non-perturbative pion dynamics leads to a large shift of the 2^{++} partner state and its significant broadening
 - ➡ The predicted width $\Gamma_{X_{2^{++}}} \simeq 50 \pm 10 \text{ MeV}$ stems from unitarity and CC dynamics and therefore has only minor cutoff dependence